IV – 2 The Higgs Sector

(i) $W^\pm, Z^0$ are massive, e.g. $m_Z = 91.1882(22)\,\mathrm{GeV}$

$m_W^2 = m_Z^2 \cos^2\theta_W$

(ii) Matter-fields are massive

(i) $\sim m_W^2 \,\mathrm{tr}\, W^2$

(ii) $\sim -m_\psi \bar\psi \psi = m_\psi \bar\psi_R \psi_L + m_\psi \bar\psi_L \psi_R$

(i),(ii) are not gauge invariant: $\psi \to e^{i(1-\gamma_5)/2\, \overbrace{\omega}^{\omega^a I^a}} \psi$

$$m_W^2 \,\mathrm{tr}\, W^2 \to -\frac{i}{g} m_W^2 \,\mathrm{tr}\, D W$$

$$m_\psi \bar\psi \psi \to m_\psi \bar\psi e^{-i(1+\gamma_5)/2\,\omega} e^{i(1-\gamma_5)/2\,\omega} \psi$$

$$\bar\psi \to \psi^\dagger e^{-i(1-\gamma_5)/2} \gamma^0 = \psi^\dagger \gamma^0 e^{-i(1+\gamma_5)/2}$$
$$= \bar\psi e^{-i(1+\gamma_5)/2}$$

inf. $\bar\psi\psi \to -i\bar\psi\gamma_5\psi$

$$= m_\psi \bar\psi e^{-i\gamma_5\omega} \psi$$

$$= m_\psi \left(\bar\psi_R e^{i\omega} \psi_L + \bar\psi_L e^{-i\omega} \psi_R\right)$$

Assume we have scalar field ϕ with $\langle \phi \rangle = v/\sqrt{2} \neq 0$

⇒ "Massterms":

Yukawa term

$$\mathcal{L}_Y(x) = -h_\psi\left(\bar\psi_R\, \phi^\dagger \psi_L + \bar\psi_L\, \phi\, \psi_R\right)$$

with doublet ϕ:

$$\phi(x) = \begin{pmatrix} \phi_1(x) \\ \phi_2(x) \end{pmatrix}, \quad \phi(x) \to \mathcal{U}(x) \begin{pmatrix} \phi_1(x) \\ \phi_2(x) \end{pmatrix}, \quad \mathcal{U} \in SU(2)$$

$\Rightarrow \quad \phi^\dagger(x)\, \Psi_L(x) \rightarrow \phi^\dagger(x)\, \mathcal{U}^\dagger(x)\, \mathcal{U}(x)\, \Psi_L(x) = \phi^\dagger(x)\, \Psi_L(x)$

$\Rightarrow$ Yukawa term $\mathcal{L}_Y(x) \xrightarrow{\mathcal{U}} \mathcal{L}_Y(x)$ gauge invariant under $SU(2)$!

Hypercharge: $\phi(x) \rightarrow e^{iY_H\omega}\, \phi(x)$

with $\boxed{Y_H = Y_L - Y_R = 1/2}$

$\Rightarrow \quad \mathcal{L}_Y(x) \xrightarrow{e^{iY\omega}} \mathcal{L}_Y(x)$

$$\left[\ \overline{\Psi}_R\, \phi^\dagger\, \Psi_L = \overline{\Psi}_R\, e^{-iY_R\omega}\, \phi^\dagger\, e^{-i(Y_L - Y_R)\omega}\, e^{iY_L\omega}\, \Psi_L = \overline{\Psi}_R\, \phi^\dagger\, \Psi_L \ \right]$$

Electric charge: $\phi_1(x)$: $Y_H + I_3 = 1$

$\phi_2(x)$: $Y_H + I_3 = 0$ $\quad \langle\phi\rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$

In summary $\mathcal{L}(x)$ is gauge invariant,

ϕ couples to W_μ and to B_μ!

Mass term for ~~for~~ fermions: (| $\phi_0 = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$)

$$-h_e\left(\overline{\Psi}_{eR}\, \phi_0^\dagger\, \Psi_L + h.c.\right) = -h_e v/\sqrt{2}\left(\overline{\Psi}_{eR}\, \Psi_L + h.c.\right) : \boxed{m_e = h_e v/\sqrt{2}}$$

Kinetic term : $\partial_\mu \phi^+ \partial^\mu \phi$

$g = e / \sin\Theta_W$

$g' = e / \cos\Theta_W$

$\rightarrow \mathcal{D}_\mu \phi^+ \mathcal{D}^\mu \phi$

with $\mathcal{D}_\mu = \partial_\mu + i g W_\mu + i g' B_\mu Y_H$

$W_\mu = W_\mu^a \sigma^a/2$

Higgs Lagrangian: (for electron)

$$\mathcal{L}_H(x) = \mathcal{D}_\mu \phi^+ \mathcal{D}^\mu \phi - h_e(\overline{\Psi}_{e_R} \phi^+ \Psi_L + h.c.)$$

$$- V(\phi^+\phi)$$

V is gauge invariant, as

$$\phi^+\phi \xrightarrow{U} \phi^+ U^+ U \phi = \phi^+\phi$$

$$\phi^+\phi \xrightarrow{e^{iY_H\omega}} \phi^+ e^{-iY_H\omega} e^{iY_H\omega} \phi$$

mass of Z^0, $W^\pm$: Take $\phi_0 = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$

$$\mathcal{D}^\mu \phi_0 = \frac{iv}{2\sqrt{2}} \begin{pmatrix} \sqrt{2}\, g\, W^{+\mu} \\ g' B^\mu - g W_3^\mu \end{pmatrix}$$

with $g' B^\mu - g W_3^\mu = \frac{g}{\cos\theta_W}\left(\sin\theta_W B^\mu - \cos\theta_W W_3^\mu\right)$

$$= -\frac{g}{\cos\theta_W} Z^{0\mu}$$

$$\Rightarrow \quad \mathcal{D}_\mu \phi_0^\dagger \mathcal{D}^\mu \phi_0 = \frac{v^2 g^2}{8}\left(2 W^-_\mu W^{+\mu} + \frac{1}{\cos^2\theta_W} Z_\mu Z^{0\mu}\right)$$

This provides mass terms for $W^\pm$, Z^0:

$$\boxed{\begin{aligned} m_Z &= \tfrac{1}{2} v g / \cos\theta_W \\ m_W &= \tfrac{1}{2} v g \end{aligned}}$$

and $\sin^2\theta_W = 1 - \frac{m_W^2}{m_Z^2}$

Full Lagrange density

$$\mathcal{L}(x) = \overset{\text{p. 4-8}}{\mathcal{L}_{EW}(x)} + \overset{\text{p. 4-10}}{\mathcal{L}_H(x)}$$

$$= -\frac{1}{4} W^a_{\mu\nu} W^{a\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

$$+ \bar{\Psi} i\gamma^\mu D_\mu \Psi - h_\psi(\bar{\psi}_R \phi^+ \psi_L + \bar{\psi}_L \phi \psi_R)$$
$$+ D_\mu \phi^+ D^\mu \phi - V(\phi^+\phi)$$

with

$$Z^0_\mu = \cos\Theta_W W^3_\mu - \sin\Theta_W B_\mu$$

$$A_\mu = \sin\Theta_W W^3_\mu + \cos\Theta_W B_\mu$$

and currents $\frac{2e}{\sin 2\Theta_W} Z^0_\mu j^\mu_{nc}$, $-e A_\mu j^\mu_{em}$

$$j^\mu_{em} = \text{p. } (4-8)$$

$$j^\mu_{nc} = \text{p. } (4-9)$$

Parameters:

$$g, \sin\Theta_W, v, h_\varphi$$

Measurements:

(i) fine structure constant

$$\alpha = \frac{e^2}{4\pi} = \frac{g^2 \sin^2\Theta_W}{4\pi}$$

137.035 9.. see QED slides

(ii) Fermi coupling constant

$$G_F = \frac{g^2\sqrt{2}}{8\, m_W^2} = \frac{1}{\sqrt{2}\, v^2} = 1.16639(1)\cdot 10^{-5}\,\mathrm{GeV}$$

(iii) the Z^0-boson mass

$$m_Z = \frac{g v}{2\cos\Theta_W} = 91.1882(22)\ \mathrm{GeV}$$

(iv) Fermion masses

$$m_f = h_f\, v/\sqrt{2}$$

[mass hierarchy not understood]

IV-3 Spontaneous Symmetry Breaking

i) Simple example: O(2) - model, ϕ complex field

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - \mu^2 \phi^* \phi - \lambda (\phi^* \phi)^2$$

with invariance (global)

$$\phi \to e^{i\omega} \phi \qquad (\partial_\mu \omega = 0)$$

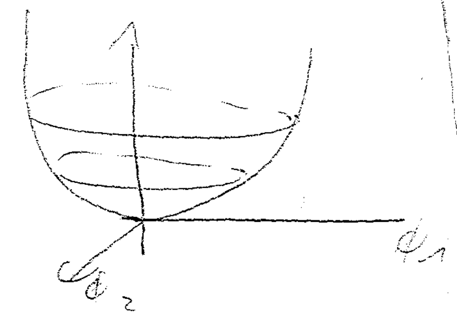
$$\to \phi^* \to \phi^* e^{-i\omega}$$

Hamiltonian density

$$\mathcal{H} = \underbrace{\partial_\mu \phi^* \partial_\mu \phi}_{\partial_t \phi^* \partial_t \phi + \vec{\nabla} \phi^* \vec{\nabla} \phi} + V(\phi^* \phi)$$

with $V(\phi \phi^*) = \mu^2 \phi^* \phi + \lambda (\phi^* \phi)^2$

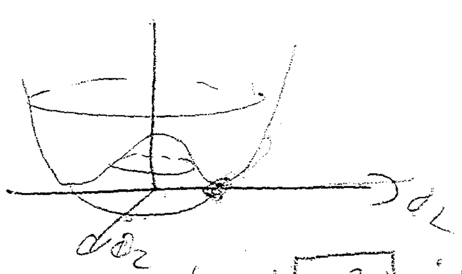
$\mu^2 > 0$:



Minimum: $\phi_0 = 0$

mass: $m^2 = \left.\frac{\partial^2 V}{\partial \phi \partial \phi^*}\right|_{\phi_0} = \mu^2$

$\mu^2 < 0$



Minimum: $\phi_0 = \sqrt{\frac{-\mu^2}{2\lambda}}\, e^{i\Theta}$

masses: $\Theta = 0$, $\phi = \phi_1 + i\phi_2$

$$\frac{1}{2} \left.\frac{\partial^2 V}{\partial \phi_1^2}\right|_{\phi_0} = -2\mu^2$$

$$\frac{1}{2} \left.\frac{\partial^2 V}{\partial \phi_2^2}\right|_{\phi_0} = 0$$

$\Rightarrow$ 1 massive boson (radial mode)

1 massless boson (Goldstone boson)

Spontaneous Symmetry breaking: theory rests in given minimum

Remark: In QM the ground state is <u>symmetric!</u> In QFT for $d > 2$ spont. sym. break (SSB) is possible, $d \leq 2$ no SSB for a cont. sym. can occur (Mermin-Wagner (-Coleman)), but discrete (SSB)

For $d \leq 2$ no SSB can occur: QM $\hat{=}$ $d = 0$

Lagrangian: $\phi(x) = \frac{1}{\sqrt{2}}\left(v + \sigma(x) + i\,\pi(x)\right)$, $v = \sqrt{-\mu^2/\lambda}$

radial mode $\quad$ Goldstone

$$\mathcal{L} = \frac{1}{2}\left[\partial_\mu \sigma\, \partial^\mu \sigma + \partial_\mu \pi\, \partial^\mu \pi\right]$$

$$- \frac{1}{2}\mu^2\left[(v+\sigma)^2 + \pi^2\right] - \frac{1}{4}\lambda\left[(v+\sigma)^2 + \pi^2\right]^2$$

$$= \frac{1}{2}\left[\partial_\mu \sigma\, \partial^\mu \sigma - m_\sigma^2 \sigma^2\right] + \frac{1}{2}\partial_\mu \pi\, \partial^\mu \pi$$

$+$ interaction-terms (+ const.)

with $\boxed{m_\sigma^2 = -2\mu^2}$