Quark-Gluon Plasma Physics

5. Statistical Hadronization and Strangeness

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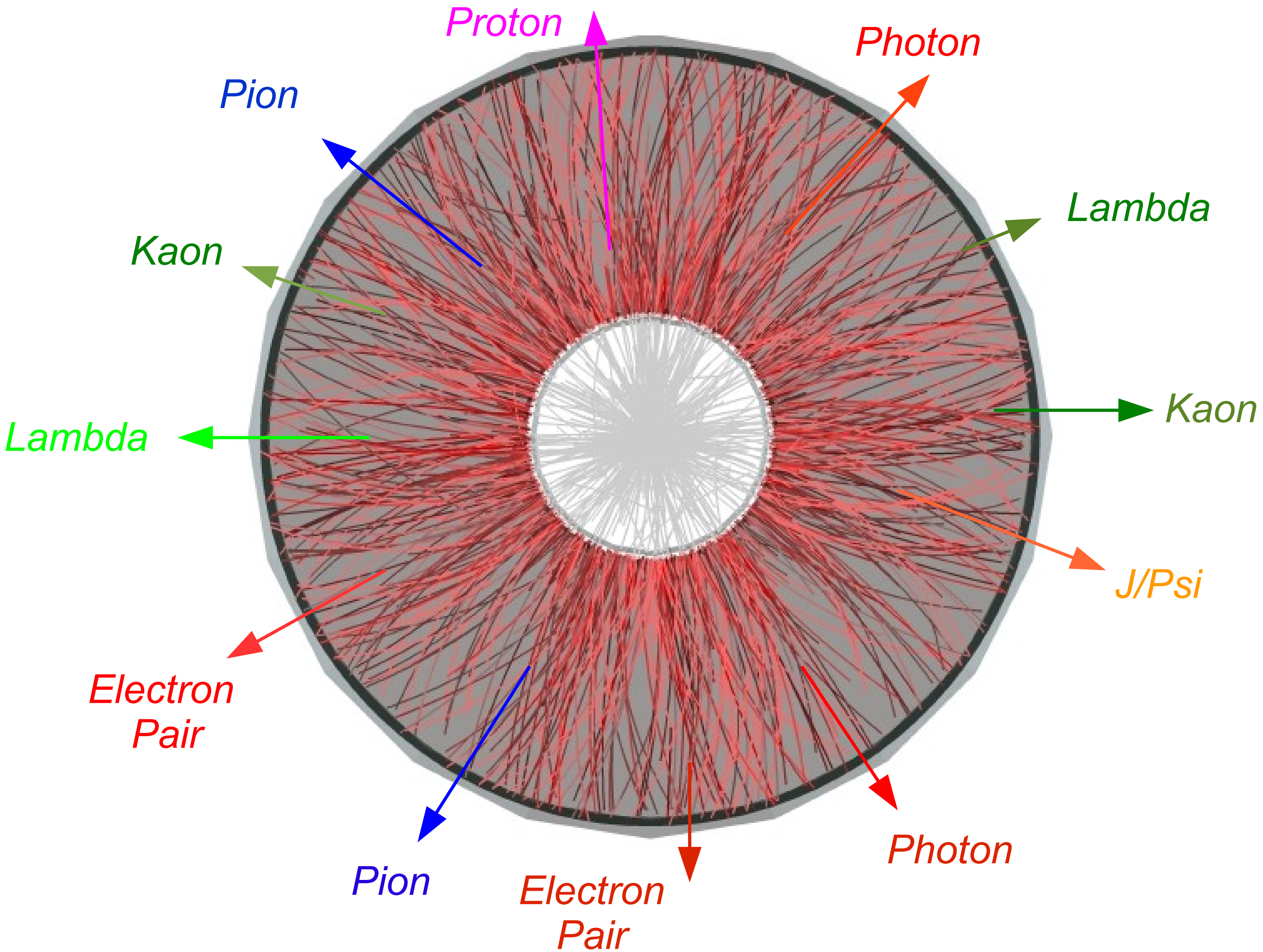
Hadronization of the nuclear fireball



the fireball properties can be determined by measurement of the emitted particles
in this chapter: hadrons with up,down,strange constituent quarks

5.1 Strangeness production in hadronic interactions

particles with strange quarks:

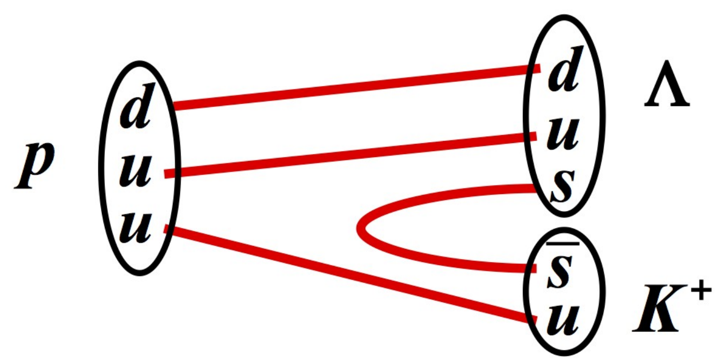
"hidden strangeness" (points to $\phi = (s\bar{s})$)

$$K^+ = (u\bar{s}),\ K^- = (\bar{u}s),\ K^0 = (d\bar{s}),\ \bar{K}^0 = (\bar{d}s),\ \phi = (s\bar{s}),$$

$$\Lambda = (uds),\ \Sigma = (qqs),\ \Xi = (qss),\ \Omega^- = (sss)$$

creation in collisions of hadrons:

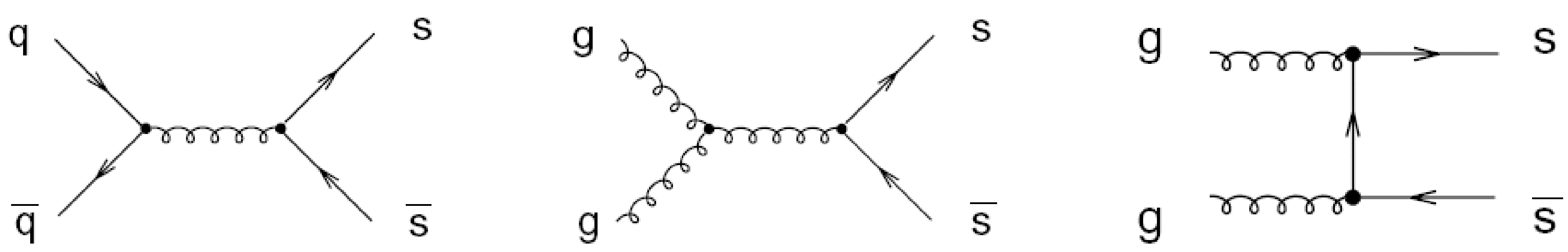
example 1: $p + p \to p + K^+ + \Lambda, \quad Q = m_\Lambda + m_{K+} - m_p \approx 670\,\text{MeV}$



associated production of strangeness

example 2: $p + p \to p + p + \Lambda + \bar{\Lambda}, \quad Q = 2m_\Lambda \approx 2230\,\text{MeV}$

Strangeness production in the QGP



$$Q_{\rm QGP} \approx 2m_s \approx 200\,{\rm MeV}$$

Q value in the QGP significantly lower than in hadronic interactions

this reflects the difference between the current quarks mass (QGP) and the constituent quark mass (chiral symmetry breaking)

Strangeness production in QGP

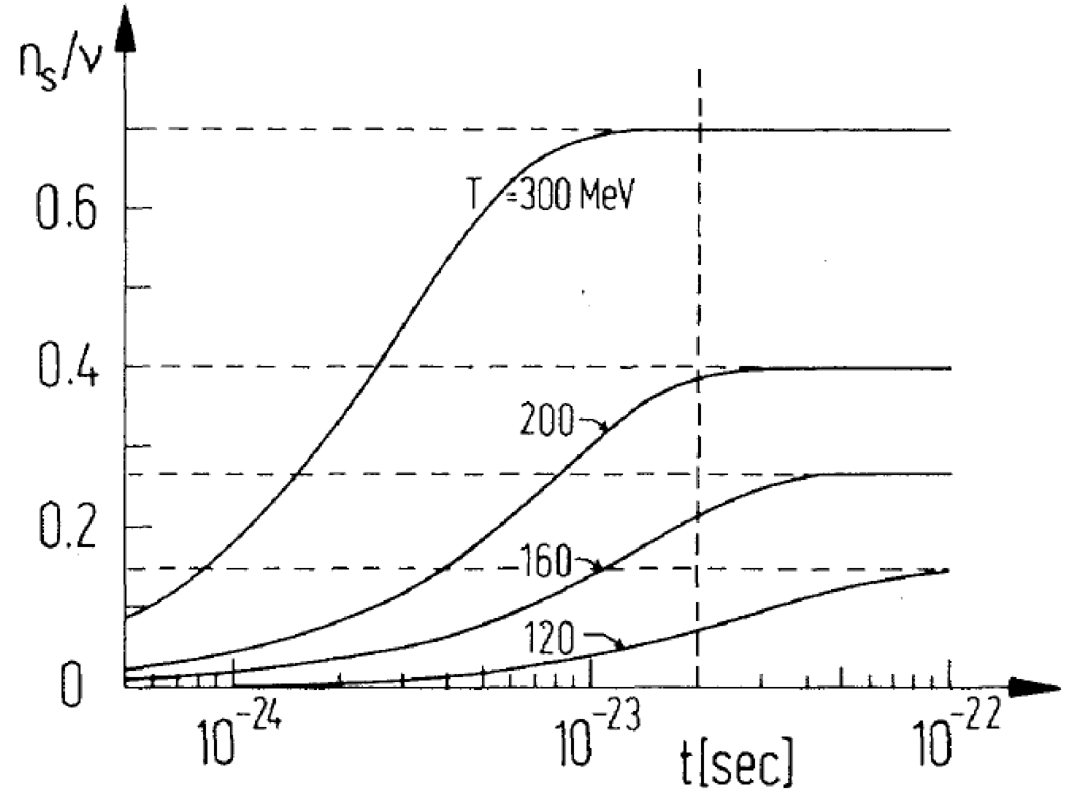
Expectation for strangeness production in heavy ion collisions where QGP is produced:

in QGP strangeness gets into equilibrium on a fast time scale

J. Rafelski, B. Müller, Phys. Rev. Lett. 48 (1982) 1066

there should be more strangeness in heavy ion collisions than in elementary collisions if a QGP is formed

enhanced production of strange hadrons one of the earliest predicted signature of QGP



ratio of strange quark to baryon number abundance in a QGP for various temperatures

Quark composition of the ideal QGP

"Boltzmann approximation": first term of the sum for m » T

particle densities for a non-interacting massive gas of fermions or bosons:

$$n_i = g_i \frac{4\pi}{(2\pi)^3} \int_0^\infty \frac{p^2\, dp}{\exp\left(\frac{\sqrt{p^2+m^2}-\mu}{T}\right) \pm 1} = \frac{g_i}{2\pi^2} m^2 T \sum_{k=1}^{\infty} \frac{(\mp 1)^{k+1}}{k} \lambda^k K_2\left(\frac{km}{T}\right)$$

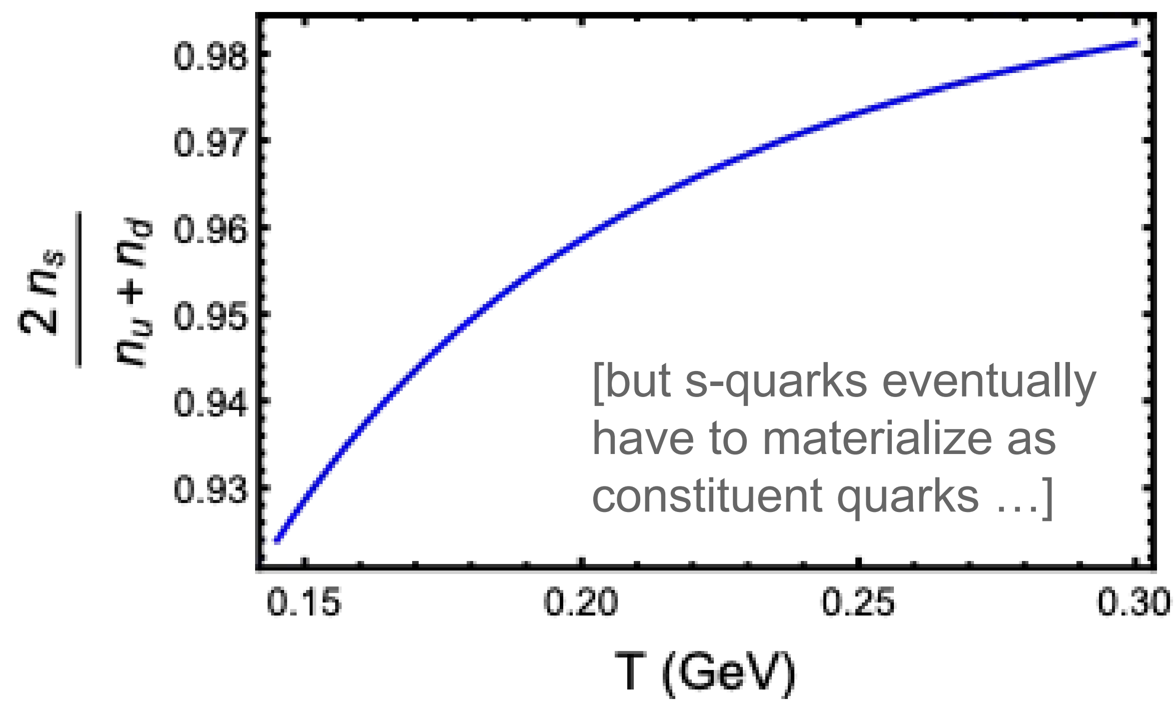
$$\lambda = e^{\mu/T}$$

upper sign: fermions, lower sign: bosons

quarks: fermions ("upper sign"),
m_u = 2.2 MeV, m_d = 4.7 MeV,
m_s = 96 MeV,

in a QGP with $\mu = 0$ and
150 < T < 300 MeV:

$$\frac{2(n_s + n_{\bar{s}})}{n_u + n_{\bar{u}} + n_d + n_{\bar{d}}} \approx 0.92\text{–}0.98$$

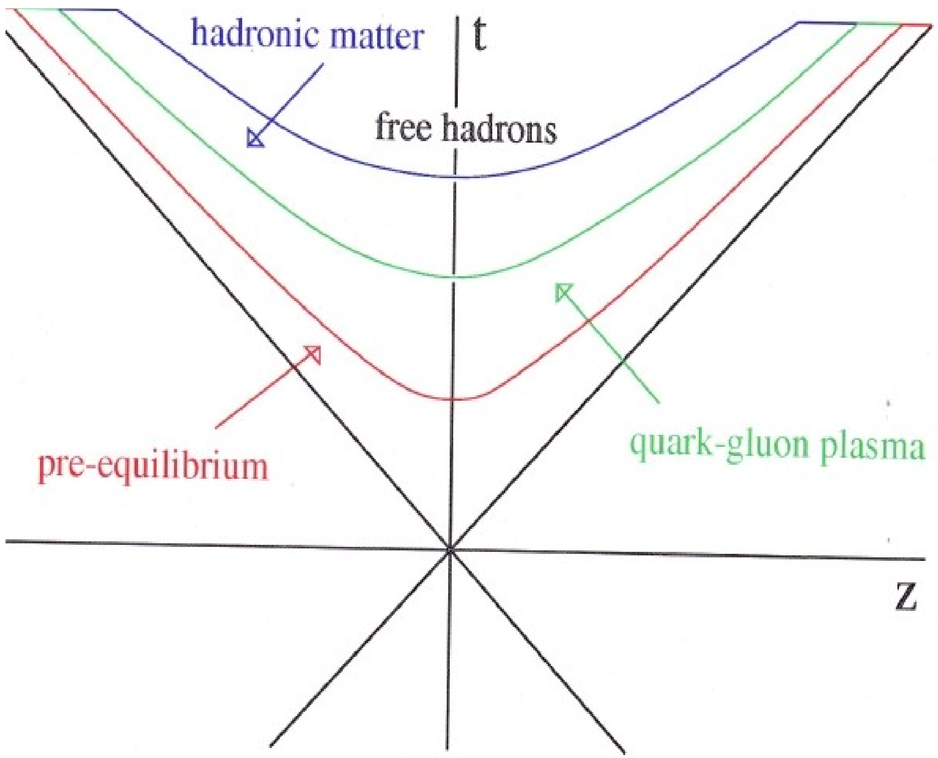


[but s-quarks eventually have to materialize as constituent quarks …]

The concept of hadro-chemical freeze-out

nuclear fireball evolves
(as sketched in lecture 1)
- it cools and expands,
- when it hits T_c, it hadronizes,
- maybe cools and expands further

- and finally falls apart when mean free path large as compared to interparticle distance



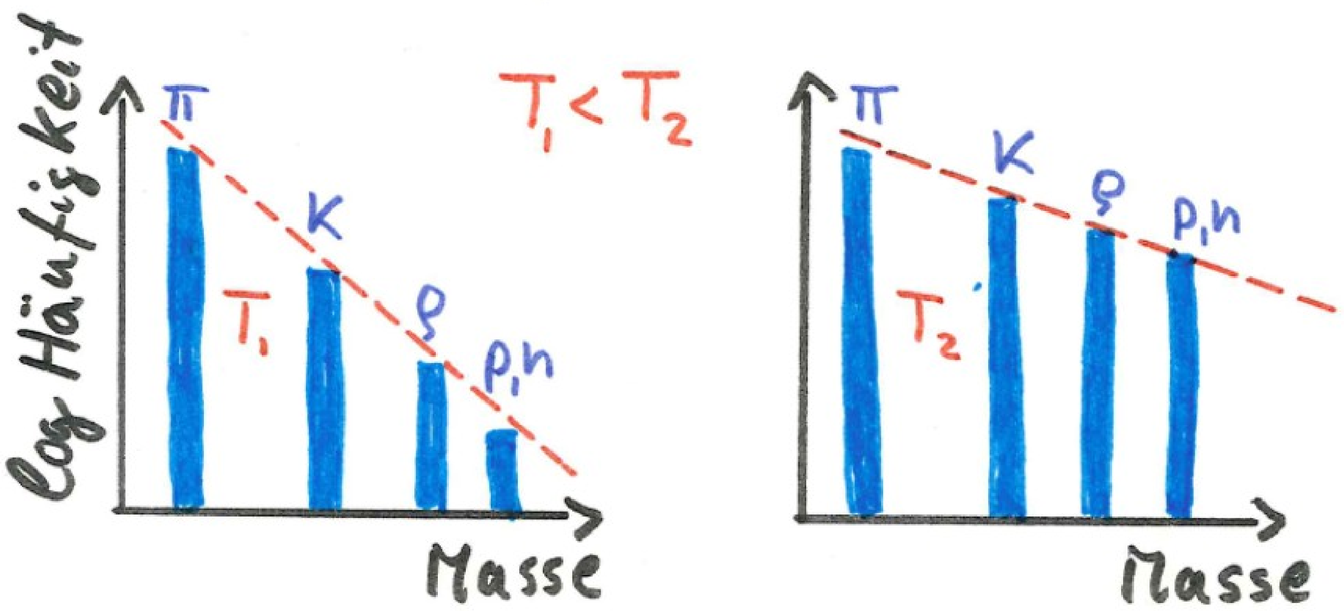
"kinetic freeze-out": momentum distributions are frozen in
- no more elastic scattering: T_{kin}

"chemical" or **"hadro-chemical freeze-out"**: abundancies of hadrons are frozen in
– no more inelastic scattering: T_{ch}

natural ordering: $T_c \geq T_{ch} \geq T_{kin}$

Thermal energy leads to population of hadronic states

equivalence of energy and mass



assume phase space is filled thermally (Boltzmann) at hadronization:

abundance of hadron species $\propto \mathrm{m}^{3/2}\exp(-\mathrm{m}/\mathrm{T})$

determined by temperature (and density) at time of production of hadrons
i.e. hadronization

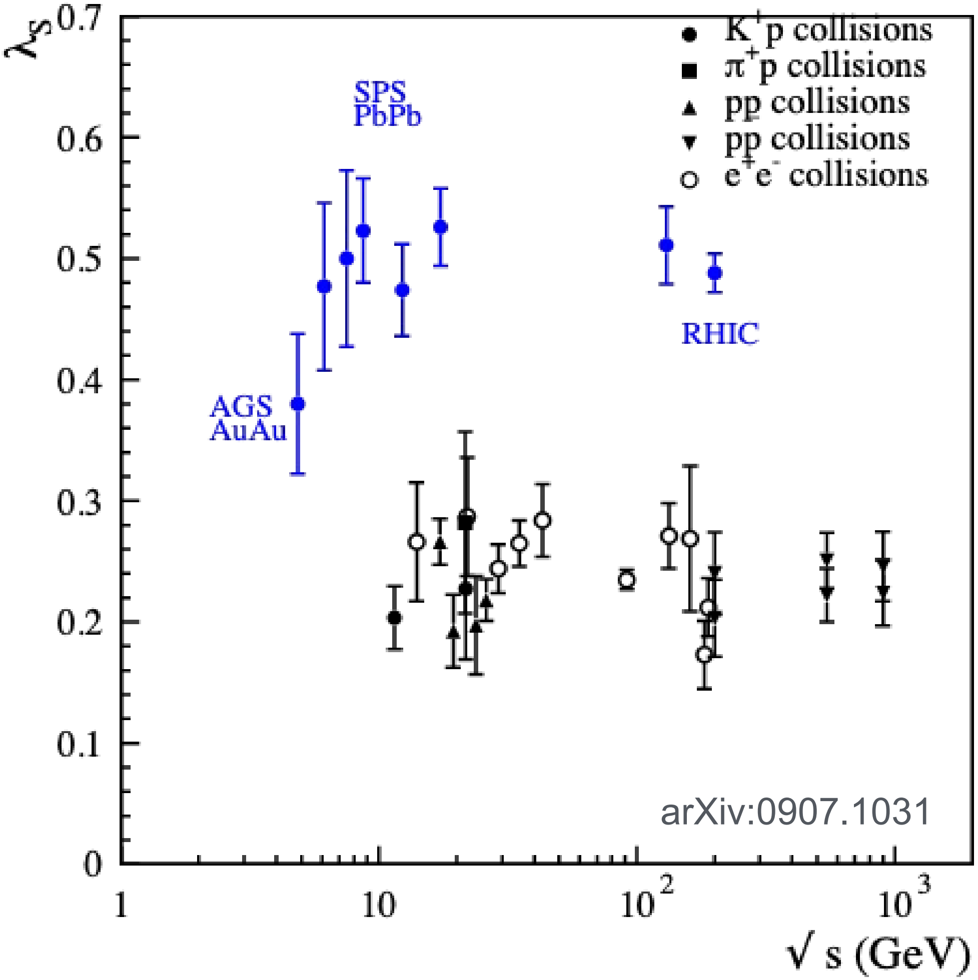
Fraction of valence strange quarks: A+A vs. e+e–, πp, pp

$$\lambda_s = \frac{2\langle s\bar{s}\rangle}{\langle u\bar{u}\rangle + \langle d\bar{d}\rangle}$$

ratio of newly created valence quark pairs before strong decays (ρ, Δ, …)

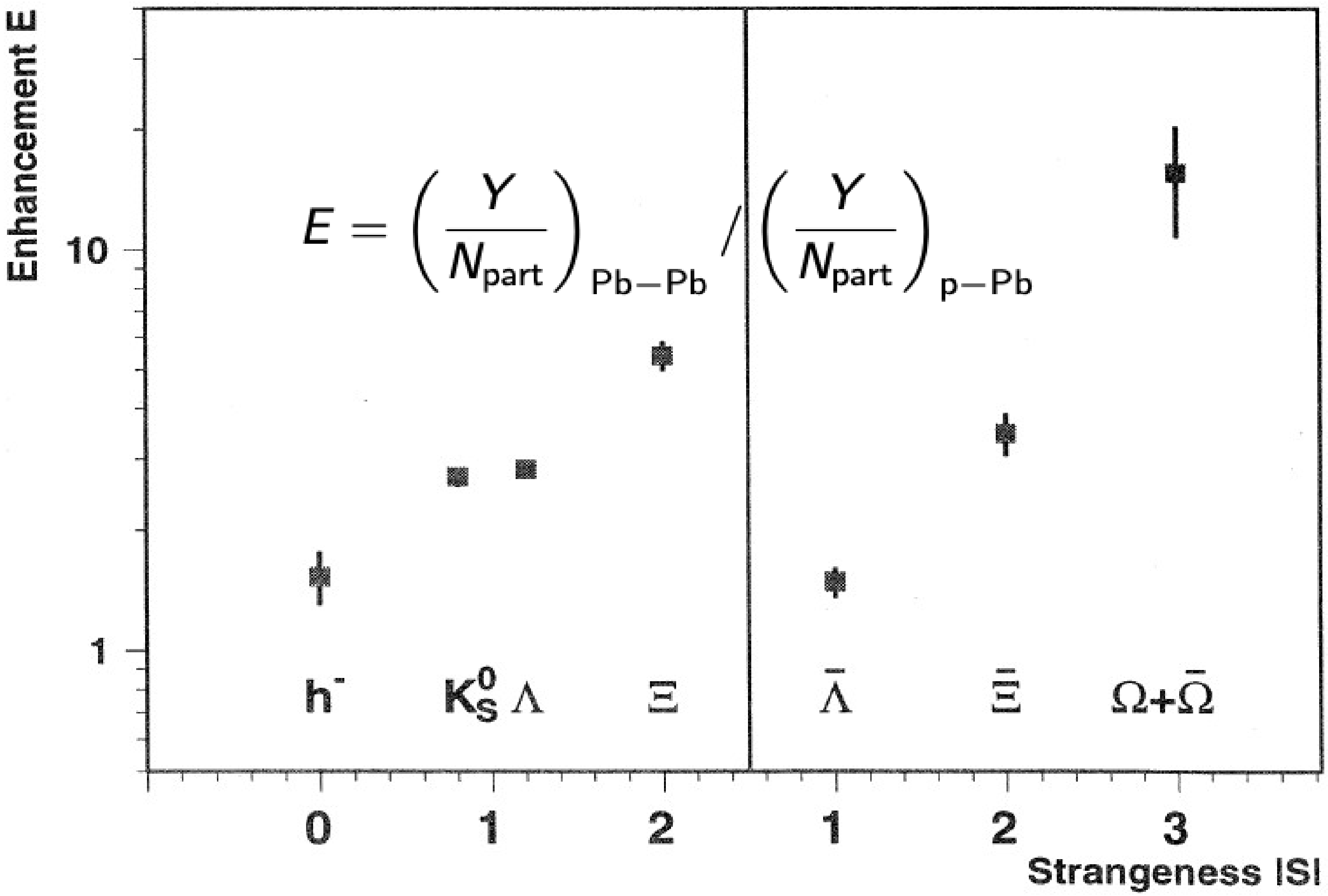
observation in elementary collisons: in addition to the exponentially falling trend with mass, hadrons with strange quarks show extra suppression
quantified by “Wroblewski factor” λ_s (Acta Phys. Pol. B16 (1985) 379

in nucleus-nucleus collisions this suppression is reduced relative to e+e–, πp, and pp collisions



Strangeness enhancement in Pb-Pb relative to p-Pb coll.

WA97, PLB 449 (1999) 401



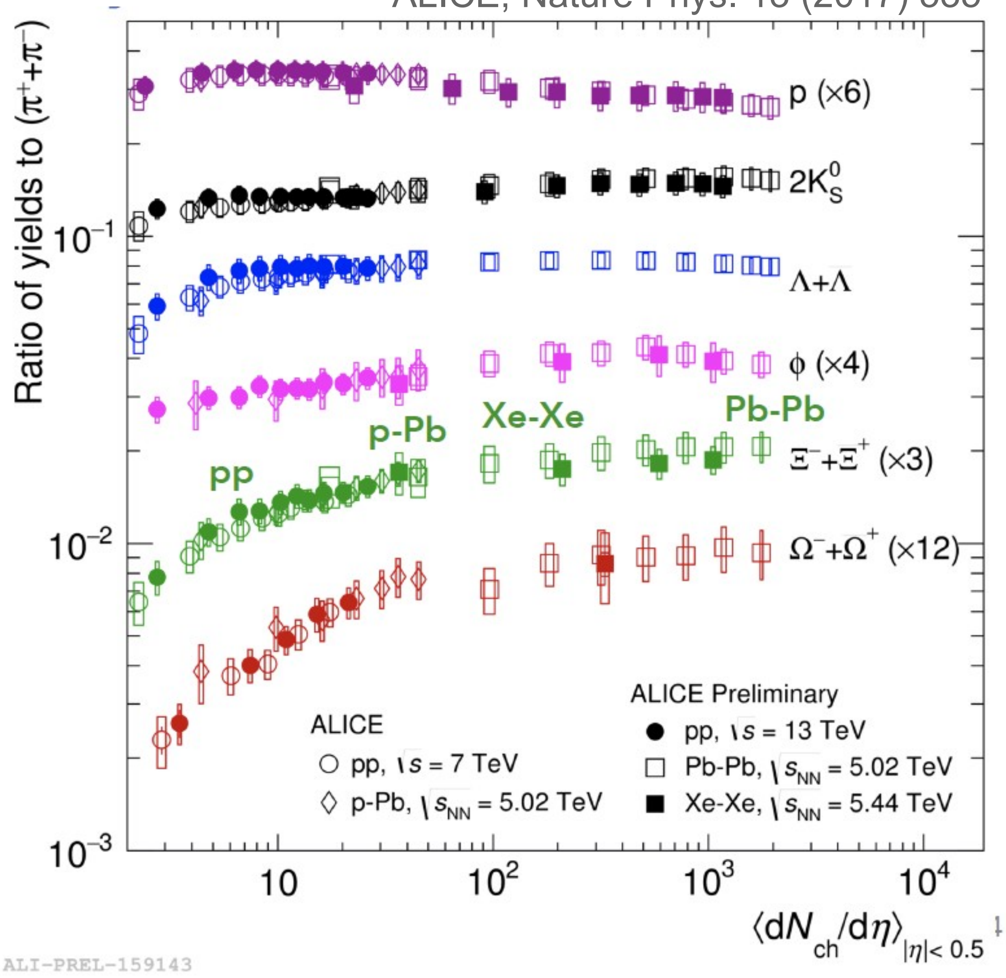
0-40% Pb-Pb
at $\sqrt{s_{NN}}$ = 17.3 GeV

p-Be reference
instead of p-Pb:
similar behavior
(NA57)

Strangeness enhancement increases with valence s quark content
(up to factor 17 for the Ω baryon)

Strangeness enhancement in Pb-Pb relative to pp and p-Pb at the LHC

ALICE, Nature Phys. 13 (2017) 535

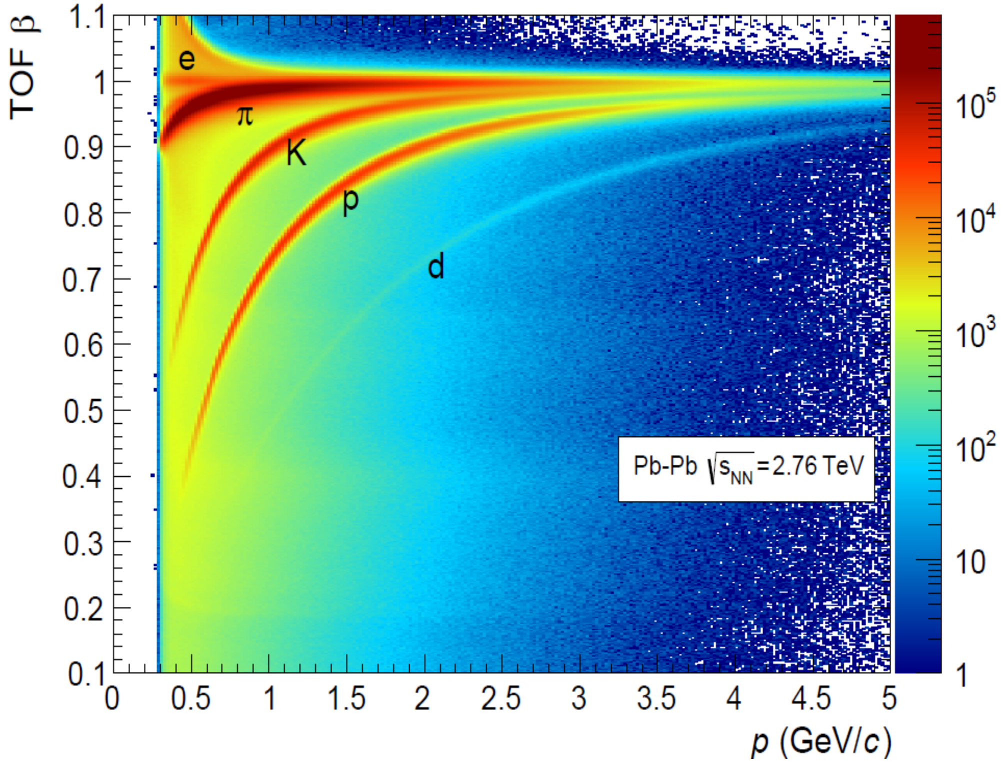


note: enhancement reduced compared to SPS
e.g. for Ω factor 4.5 vs 17

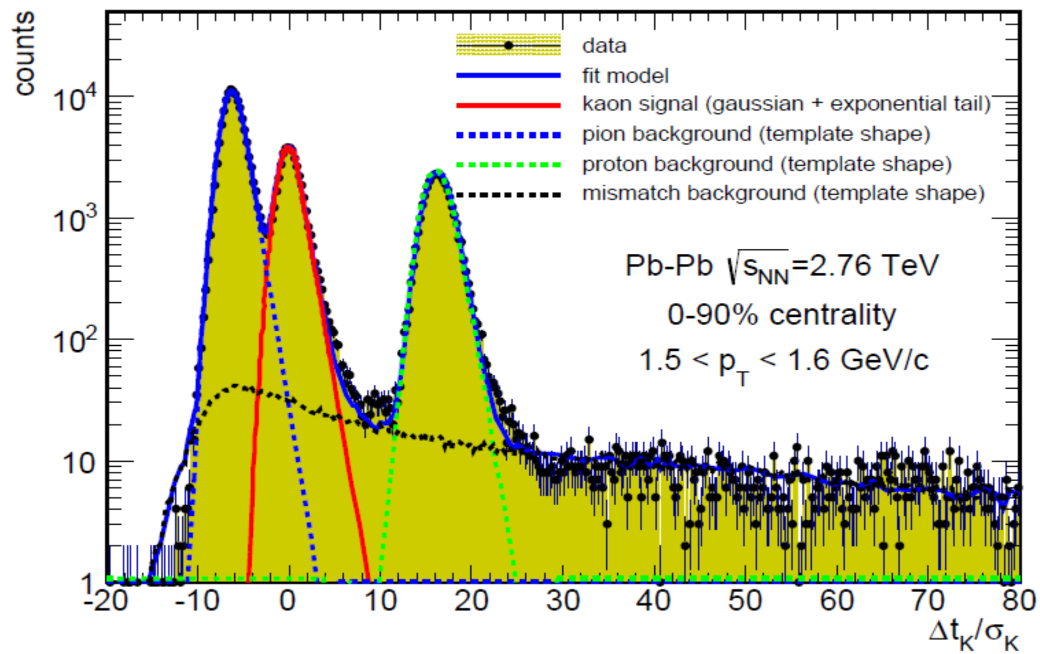
will see below: to coin what is happening in terms of ‘enhancement’ is misleading

How to measure production yields of identified hadrons

identification via time-of-flight plus momentum measurement:



works well as long at β/c < 1
but then fades quickly

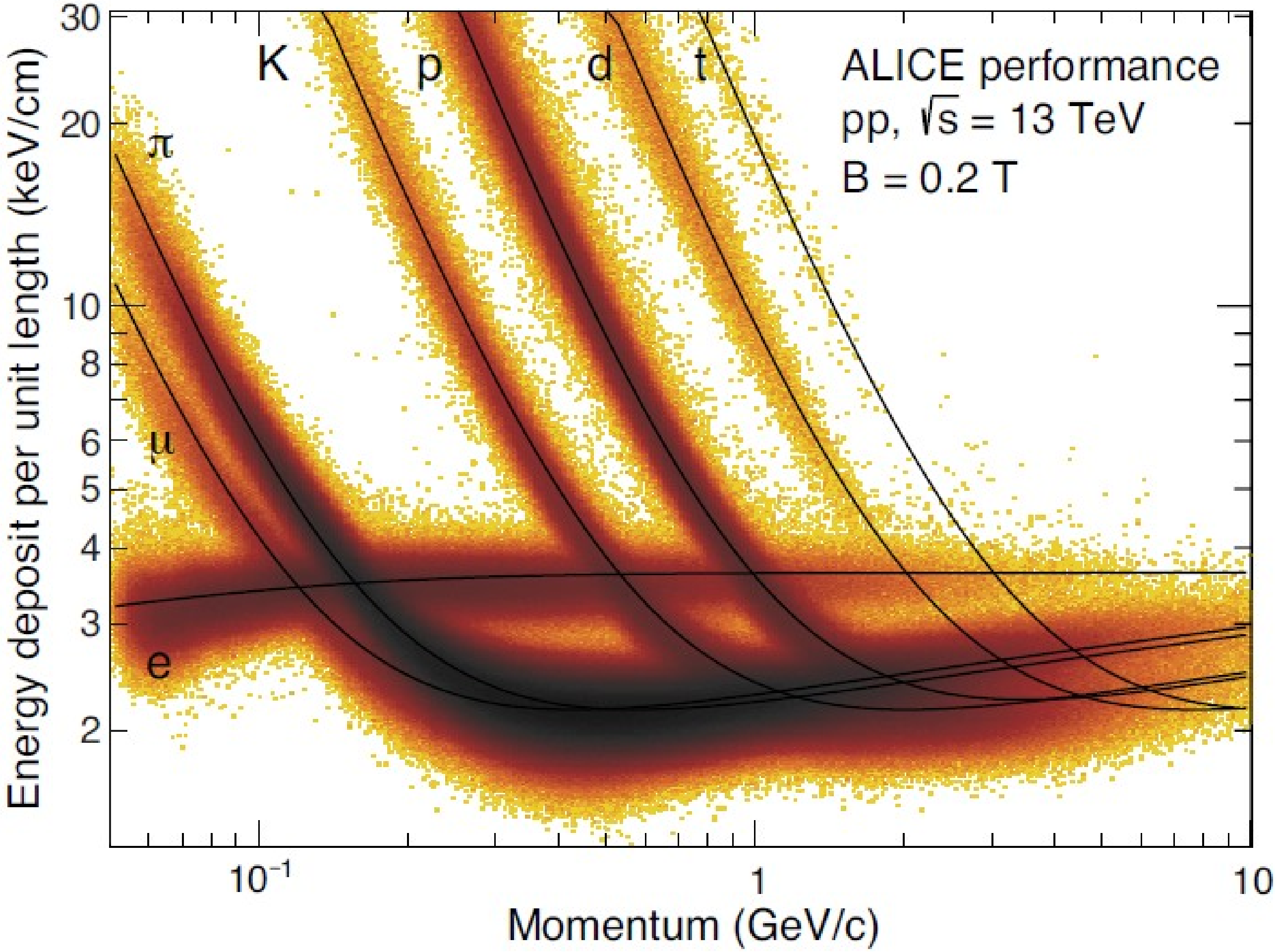


How to measure production yields of identified hadrons

identification via specific energy loss:

example: ALICE TPC - 150 space points per track

M. Tanabashi et al. (Particle Data Group), Phys. Rev. D 98, 030001, Fig. 34.15



How to measure production yields of identified hadrons

Identification via invariant mass of decay products

$$
\begin{aligned}
M^2 &= \left[\begin{pmatrix} E_1 \\ \vec{p}_1 \end{pmatrix} + \begin{pmatrix} E_2 \\ \vec{p}_2 \end{pmatrix}\right]^2 = (E_1 + E_2)^2 - (\vec{p}_1 + \vec{p}_2)^2 \\
&= m_1^2 + m_2^2 + 2E_1 E_2 - 2\vec{p}_1 \cdot \vec{p}_2 \\
&= m_1^1 + m_2^2 + 2E_1 E_2 - 2p_1 p_2 \cos\vartheta
\end{aligned}
$$

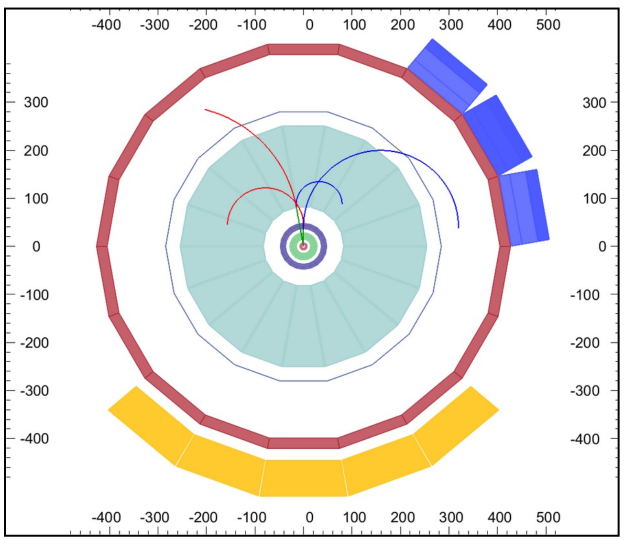
How to measure production yields of identified hadrons

electromagnetic decays:

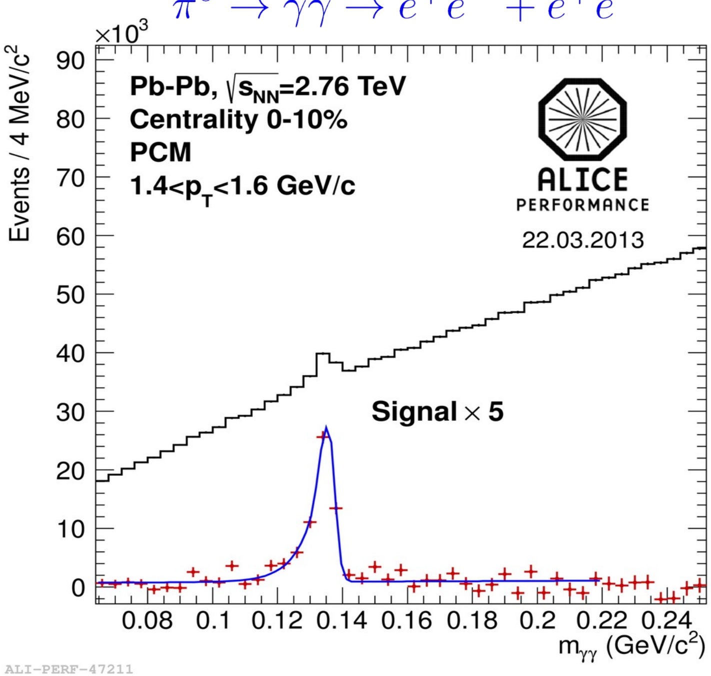
$$\pi^0 \to \gamma\gamma \qquad m_{\pi^0} = 0.135\text{GeV},\ \text{BR} = 0.988,\ c\tau = 25.1\,\text{nm}$$
$$\eta \to \gamma\gamma \qquad m_{\eta} = 0.548\text{GeV},\ \text{BR} = 0.393,\ c\tau = 0.2\,\text{nm}$$

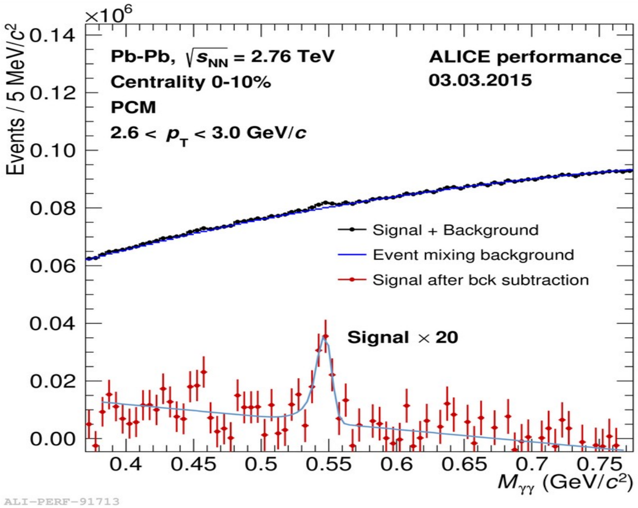
happen practically in the interaction point/target

detect photons in calorimeter
or via e^+e^- from conversion in detector material



$\pi^0 \to \gamma\gamma \to e^+e^- + e^+e^-$





$\eta \to \gamma\gamma \to e^+e^- + e^+e^-$

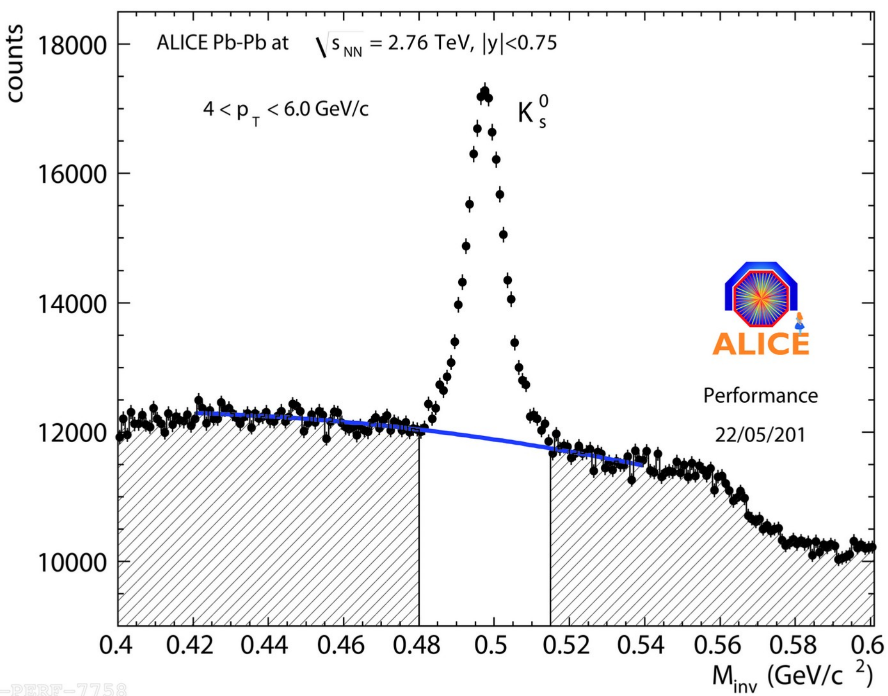
How to measure production yields of identified hadrons

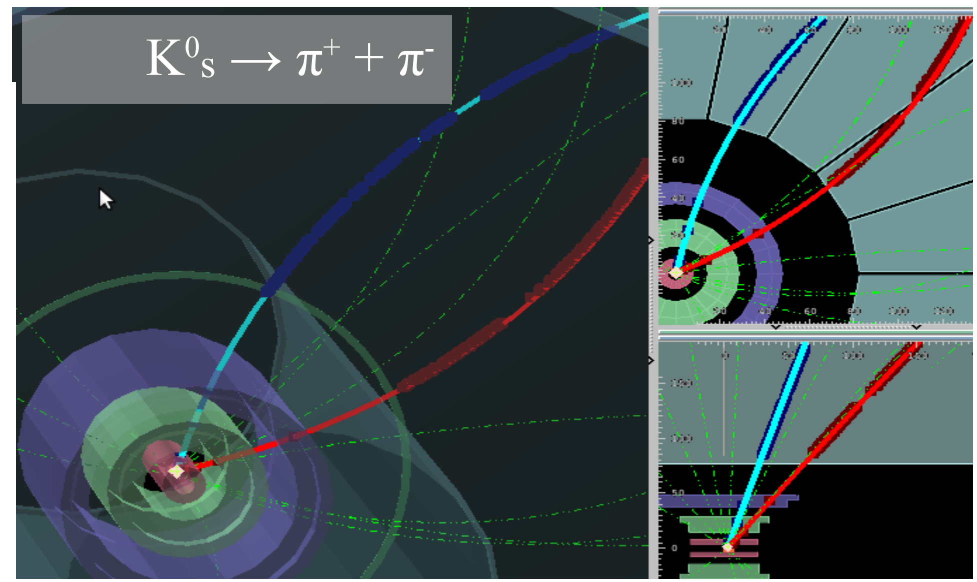
Identification via invariant mass of weak decay products

$$\mathrm{K^0_s} \rightarrow \pi^+ + \pi^- \qquad (\mathrm{B.R.}68\%) \qquad c\tau = 2.68\,\mathrm{cm}$$
$$\Lambda \rightarrow \mathrm{p} + \pi^- \qquad (\mathrm{B.R.}64\%) \qquad c\tau = 7.89\,\mathrm{cm}$$

look for secondary decay vertex of a neutral object a few 10 cm away from interaction point

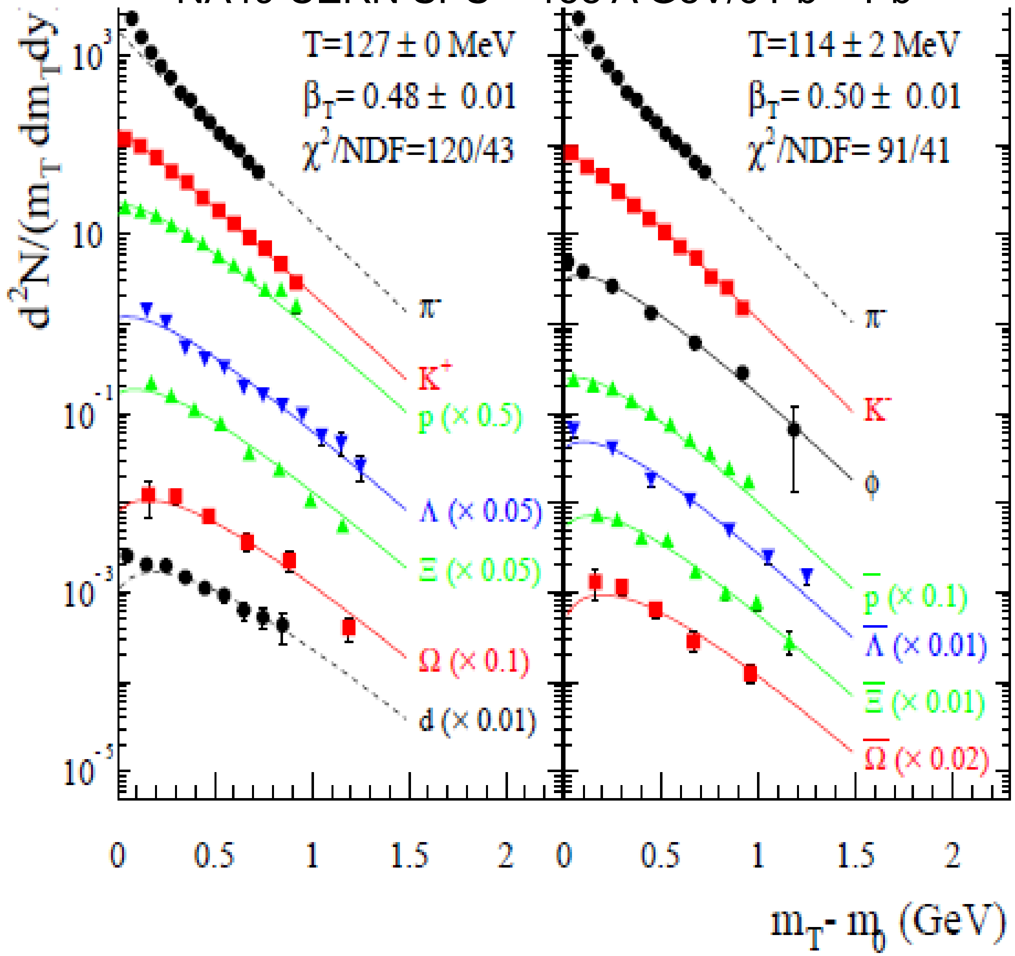
works up to very high momentum!





Hadron production in central PbPb collisions at the CERN SPS

NA49 CERN SPS – 158 A GeV/c Pb + Pb

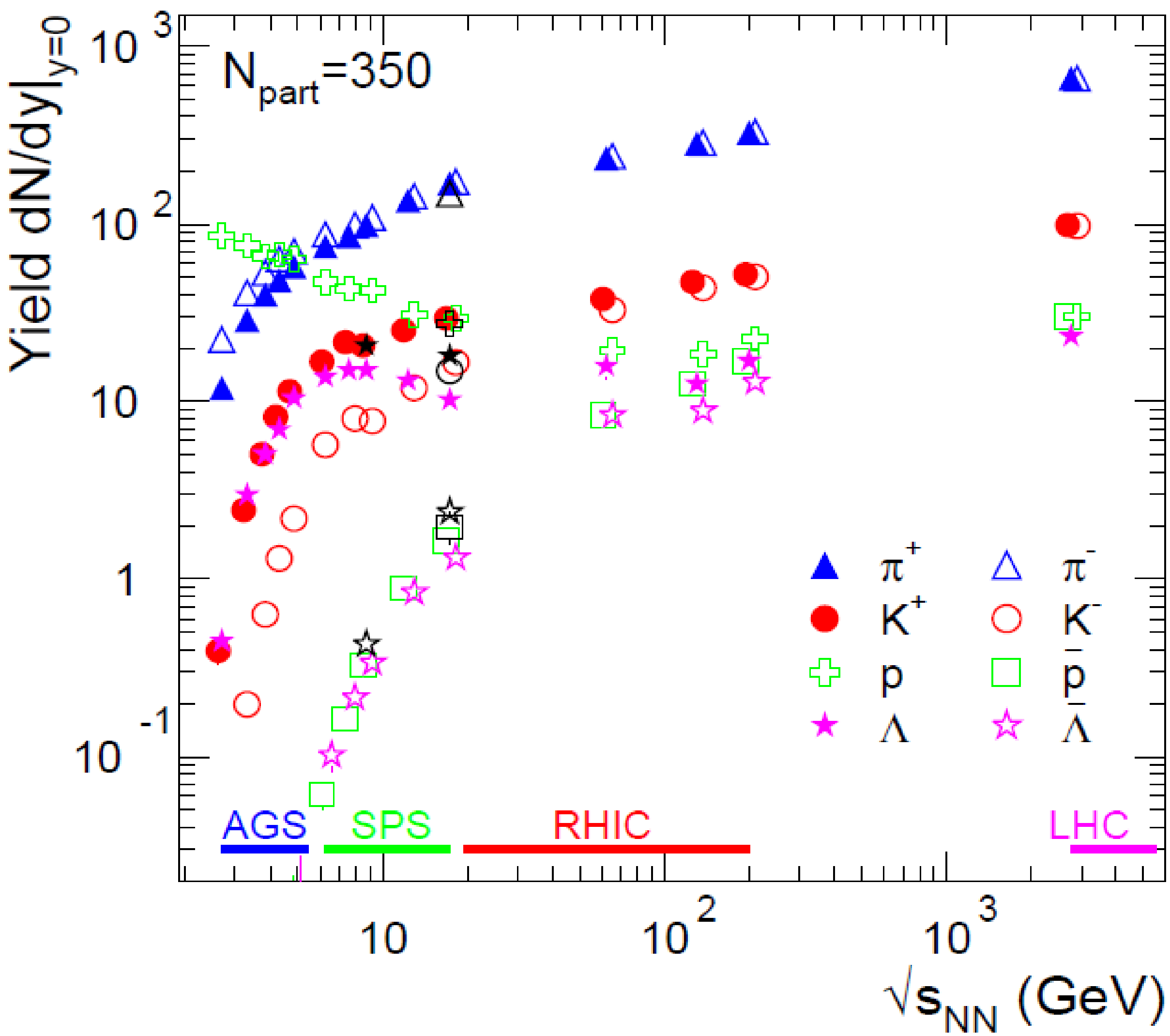


measure p_t spectra and integrate/extrapolate over all values of p_t from 0 to ∞ in order to obtain particle yield

between 5 different experiments a comprehensive data set for 158 A GeV PbPb collisions

Particle production in central AA collisions

A. Andronic arXiv:1407.5003



a summary of 25 years of experimental research

systematic trends with beam energy:
mesons rise and level off
baryons drop
antibaryons rise steeply

can we understand all of these?

5.2 Statistical model description of hadron yields

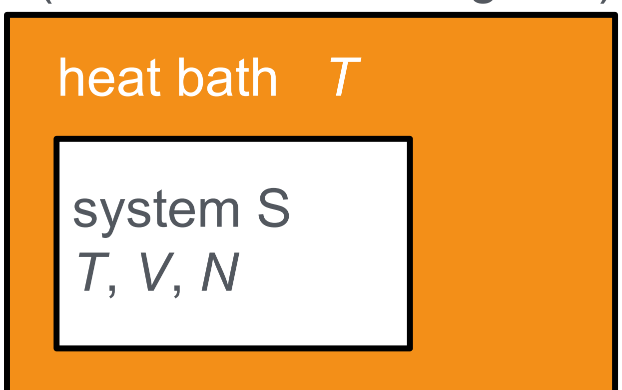
idea: freeze-out of the QGP creates an equilibrated hadron resonance gas (HRG)

the HRG then freezes out at a characteristic temperature T_{ch} which determines the yields of different particle species

what is the appropriate statistical ensemble for the theoretical treatment?

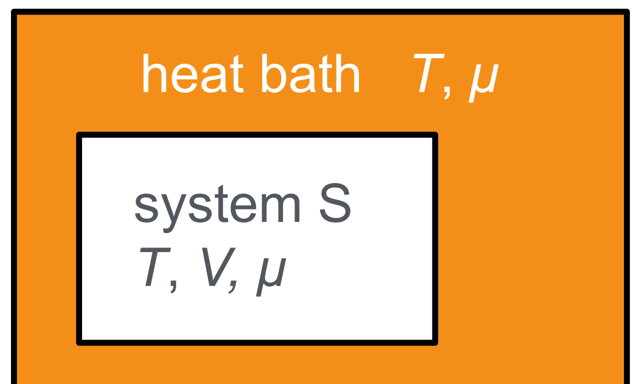
canonical ensemble:
N and *V* fixed, energy *E* of the system fluctuates

($E_s + E_b = E$, T is given)



when multiplicity is low, conservation laws must be implemented locally event-by-event (Hagedorn 1971)

grand-canonical ensemble:
V fixed, energy *E* and particle number *N* fluctuate
(T, μ given)

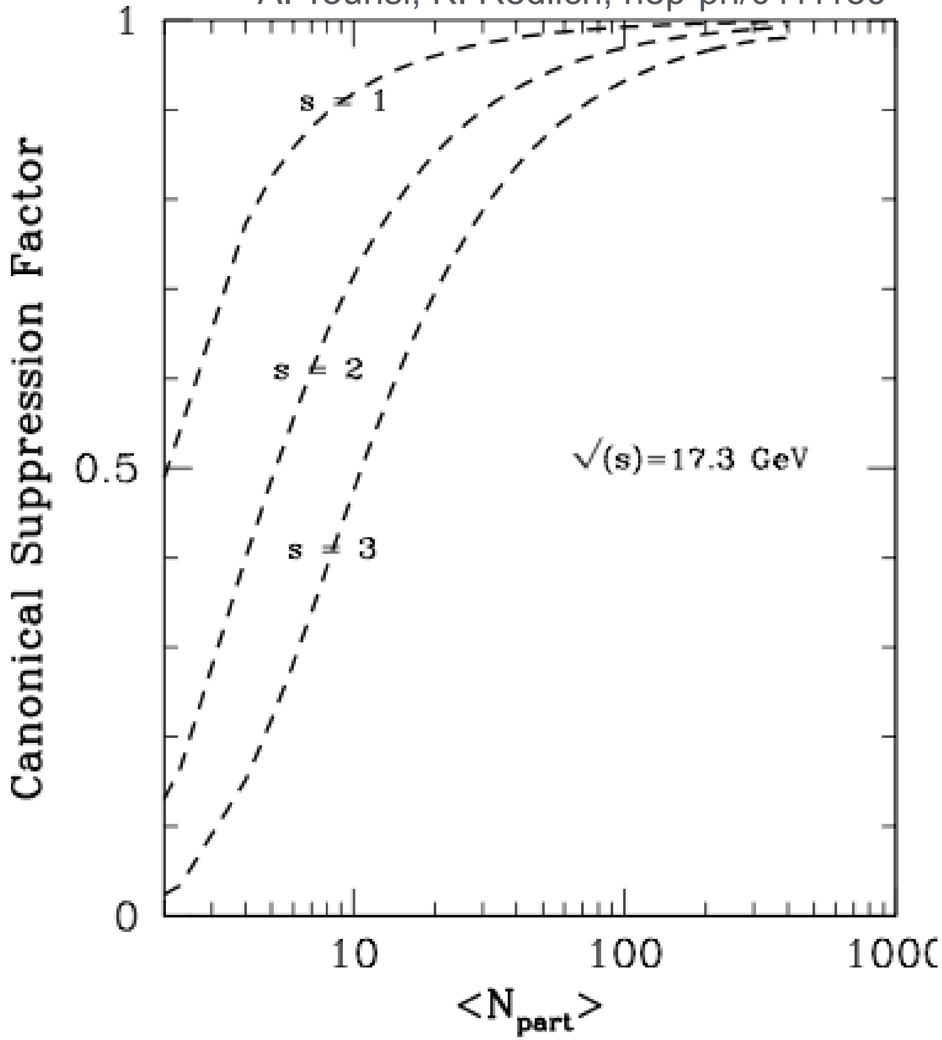


when number of produced particles large, conservation of additive quantum numbers can be implemented on average (use of chemical potential)

Braun-Munzinger, Redlich, Stachel, nucl-th/030401

Grand canonical ensemble: large volume limit of the canonical treatment

A. Tounsi, K. Redlich, hep-ph/0111159



Canonical suppression factor F_S:

$$n_K^C = n_K^{GC} \cdot F_S$$

$$F_S = \frac{I_K(2n_K^{GC} V)}{I_0(2n_K^{GC} V)}$$

n_K : density of particles with strangeness K = |S|, S =-1, -2, -3ddd

I_n : modified Bessel function of the first kind

already at moderately central PbPb collisions at SPS energy the grand canonical ansatz is justified

Grand canonical ensemble and application to data from high energy heavy ion collisions

partition function: $\ln Z_i = \frac{V g_i}{2\pi^2} \int_0^\infty \pm p^2 dp \ln(1 \pm \exp(-(E_i - \mu_i)/T))$

particle densities: $n_i = N/V = -\frac{T}{V}\frac{\partial \ln Z_i}{\partial \mu} = \frac{g_i}{2\pi^2} \int_0^\infty \frac{p^2\,\mathrm{d}p}{\exp((E_i - \mu_i)/T) \pm 1}$

for every conserved quantum number there is a chemical potential:

$$\mu_i = \mu_B B_i + \mu_S S_i + \mu_{I_3} I_i^3$$

but can use conservation laws to constrain V, μ_S, μ_{I_3}

baryon number: $\mathrm{V}\sum_{\mathrm{i}} \mathrm{n_i B_i} = \mathrm{Z} + \mathrm{N}$ $\rightarrow \mathrm{V}$

strangeness: $V\sum_i n_i S_i = 0$ $\rightarrow \mu_S$

charge: $V\sum_i n_i I_i^3 = \frac{Z - N}{2}$ $\rightarrow \mu_{I_3}$

only 2 free parameters left →

fit at each energy provides values for T and μ_b

Comparison to experimental data

compute primary thermal occupation probability for each particle species

spectrum of hadrons involves all confirmed hadronic states as of PDG compilation

implement all strong decays according to PDG
(example: for T=160 MeV, 80% of all pions come from strong decays)

compute for a grid of (T,μ_b) χ^2 between statistical ensemble calculation and data

minimize χ^2 to obtain for each beam energy and collision system best set (T,μ_b)

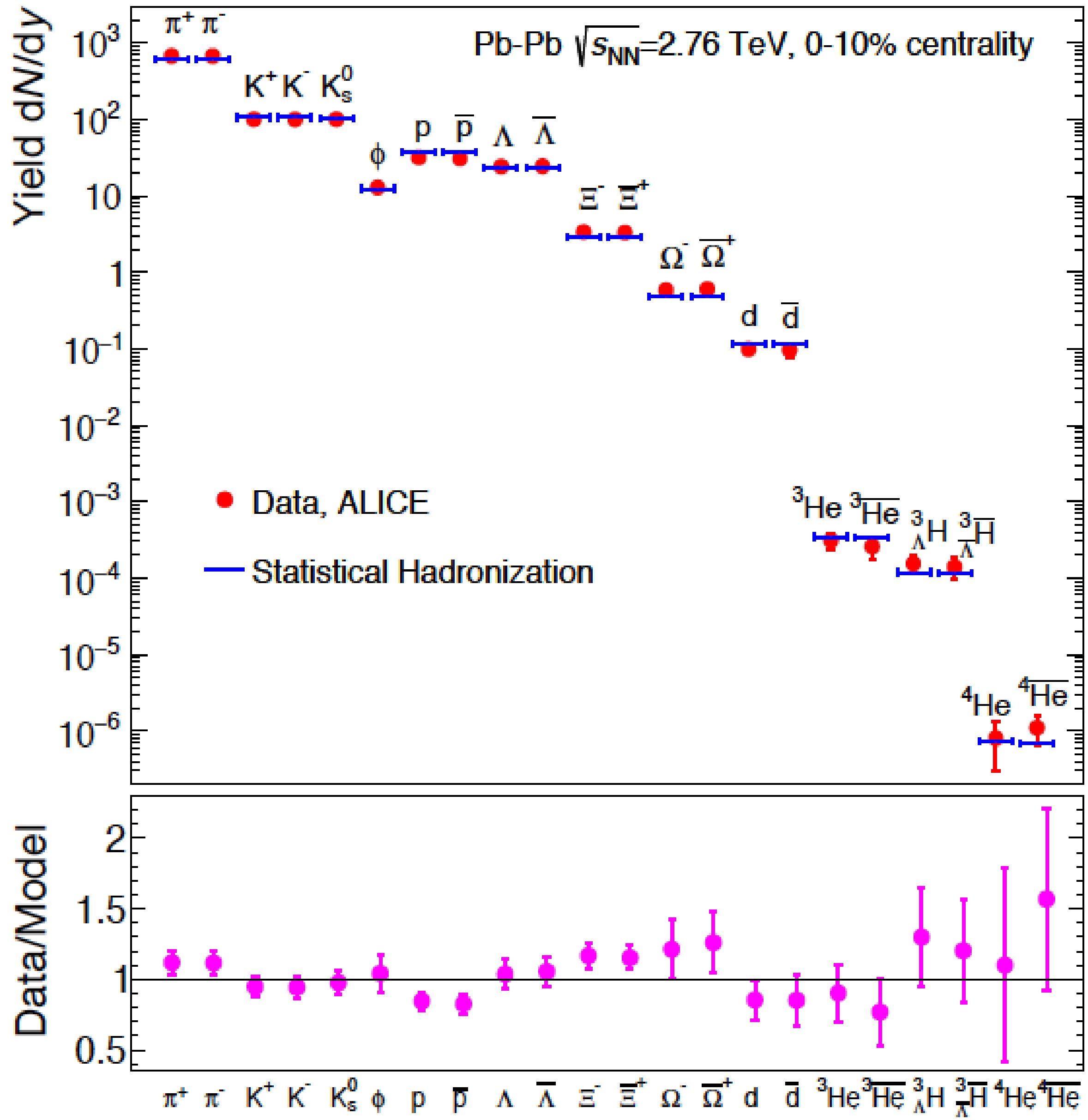
Hadron yields at the LHC compared to statistical model

data very well reproduced except 2.7 sigma deviation for protons
- understood in the mean time

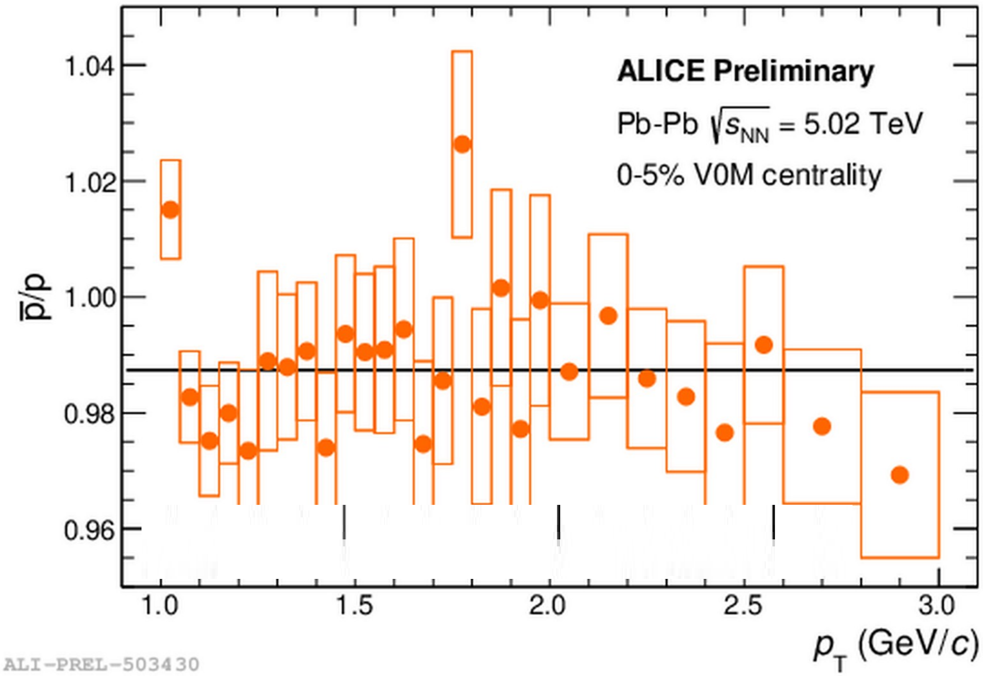
S-matrix correction to take into account π-n interaction via measured phase shifts → perfect fit of protons

Andronic, Braun-Munzinger, Friman, Lo, Redlich, Stachel, arXiv:1808.03102

Andronic, Braun-Munzinger, Redlich, Stachel, Nature 561 (2018) 321

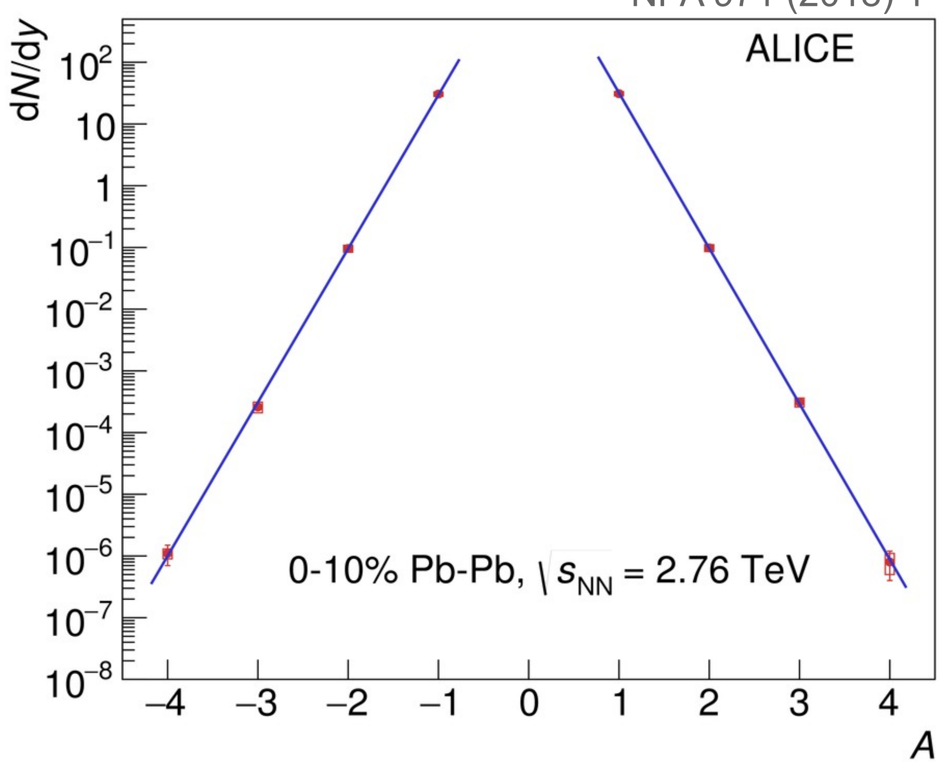


Hadron yields at the LHC – PbPb at 2.76 TeV/nucleon pair



- matter and anti-matter produced in equal proportions at LHC
- consistent with net-baryon free central region, (μ_b = 0.6 ± 0.1 MeV) similar to early universe

NPA 971 (2018) 1



consequence: penalty factor $\exp(-m_n/T) \approx 300$ for nuclei and anti-nuclei as $\mu_b = 0$ at LHC

Production of hadrons and (anti-)nuclei at LHC

1 free parameter: temperature T

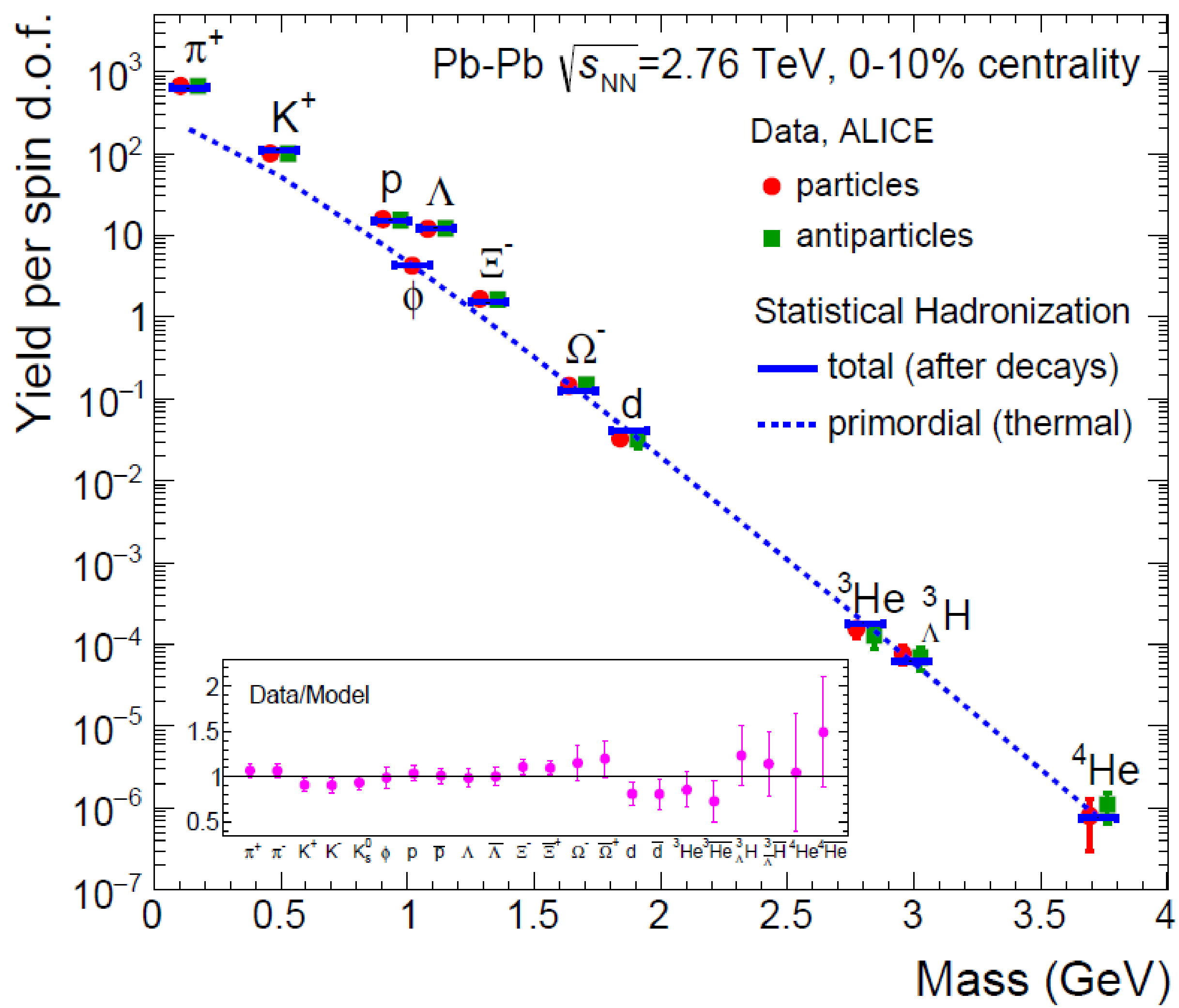
$T = 156.5 \pm 1.5$ MeV

$\chi^2 = 16.9$ for 19 dof

agreement over 9 orders of magnitude with QCD statistical operator prediction (- strong decays need to be added)

- even large very fragile hypernuclei follow the same systematics

A. Andronic, P. Braun-Munzinger, K. Redlich, J. Stachel, Nature 561 (2018) 321



Beam energy dependence of hadron yields from AGS to LHC

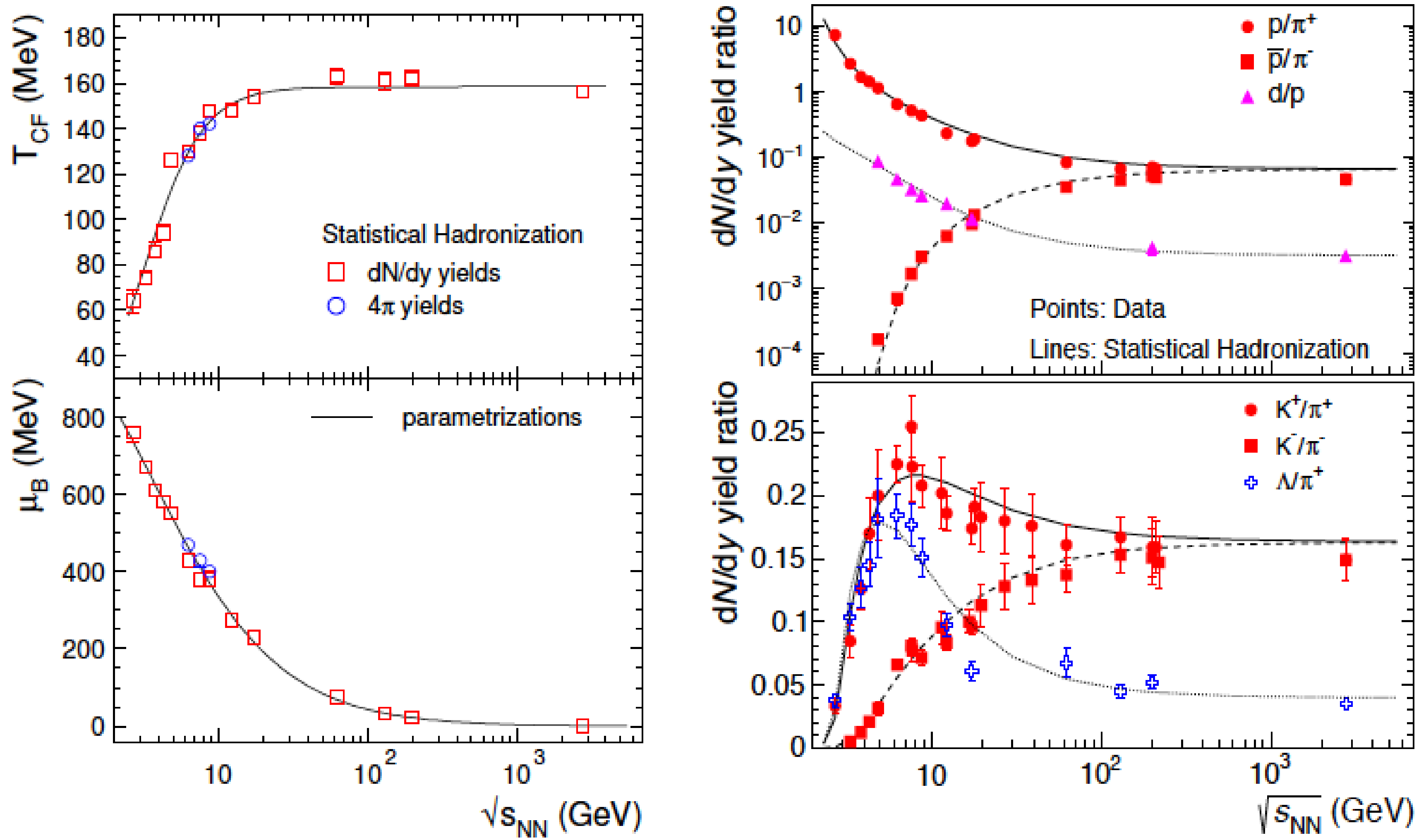
fits work equally well at lower beam energies
following the obtained T and m_b evolution, features of proton/pion, kaon/pion, deuteron/proton and Lambda/pion ratios reproduced in detail

<u>salient features:</u>

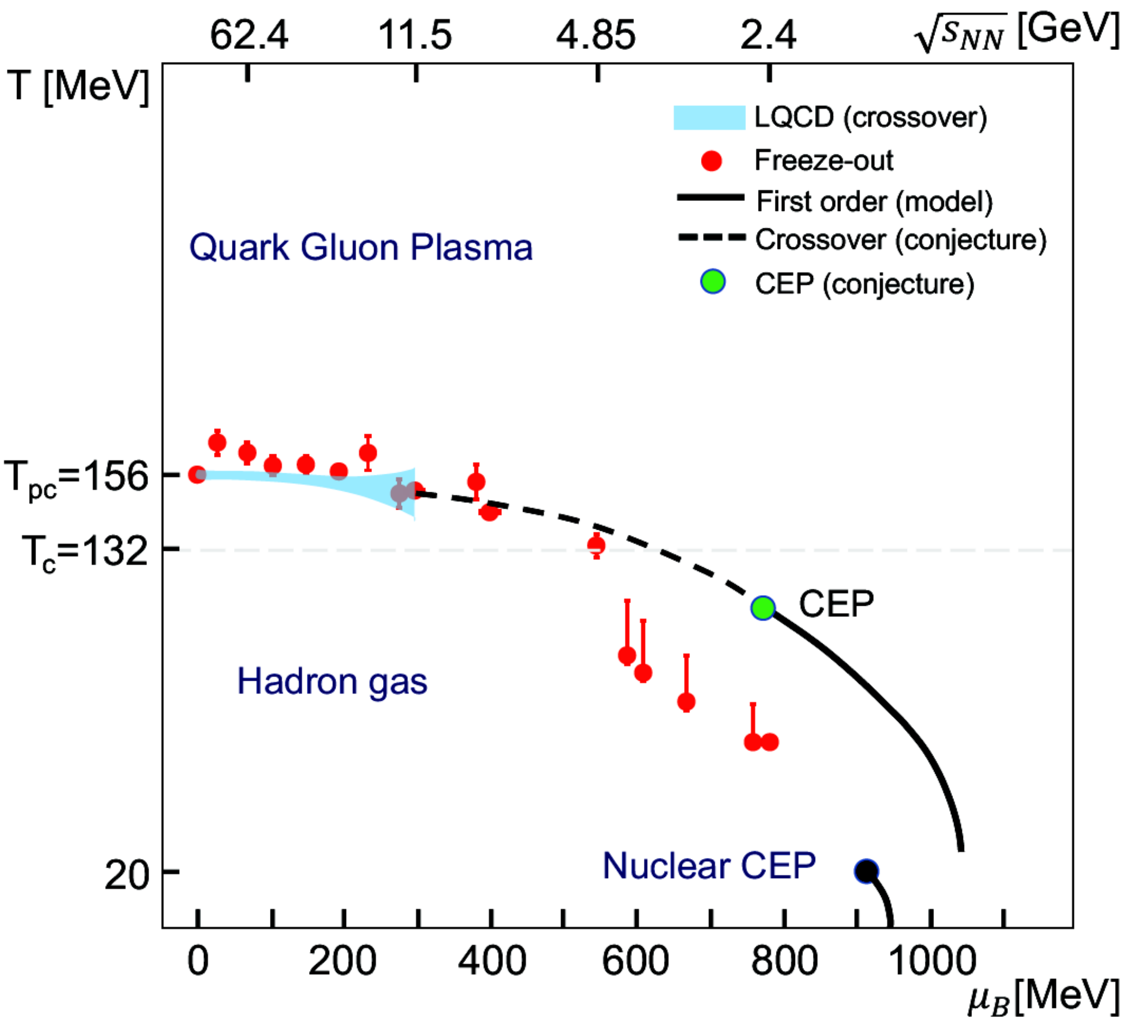
A. Andronic, P. Braun-Munzinger, K. Redlich, J. Stachel, Nature 561 (2018) 321

a limiting temperature is reached

nuclear matter becomes increasingly transparent



Freeze-out points and the QCD phase diagram



P. Braun-Munzinger, A. Rustamov, J. Stachel arXiv:2211.08819

- T_{ch} at LHC in exact agreement with the pseudo-critical temperature T_{pc} from lQCD

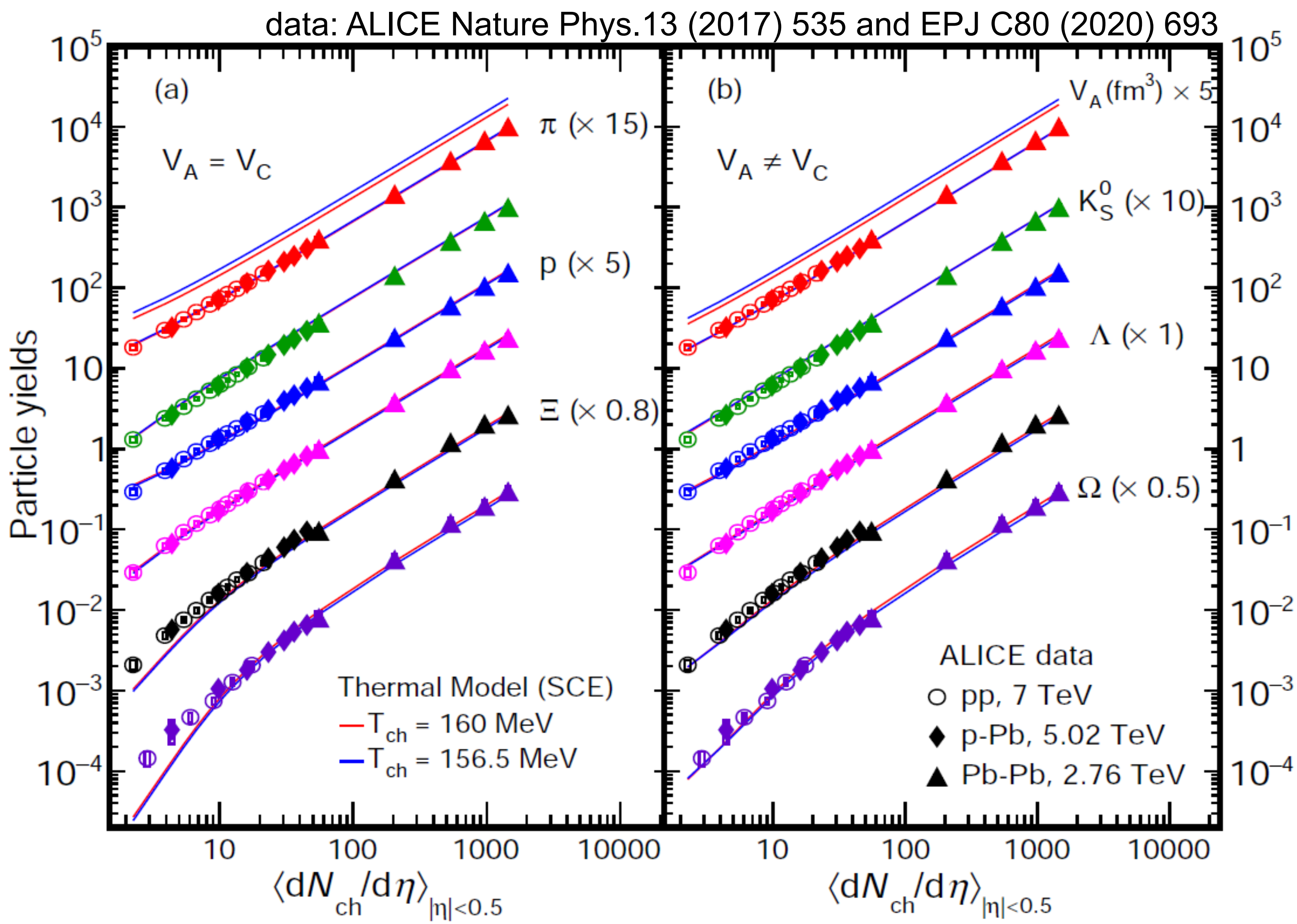
A. Bazavov et al. PLB 795 (2019) 15
S. Borsanyi et al. PRL 125 (2020) 052001

- why chemical freeze-out very close to T_{pc}?

close to T_{pc} rate for multi-particle reactions explodes

P. Braun-Munzinger, J. Stachel, C. Wetterich (2004)

From pp to PbPb collisions: smooth evolution

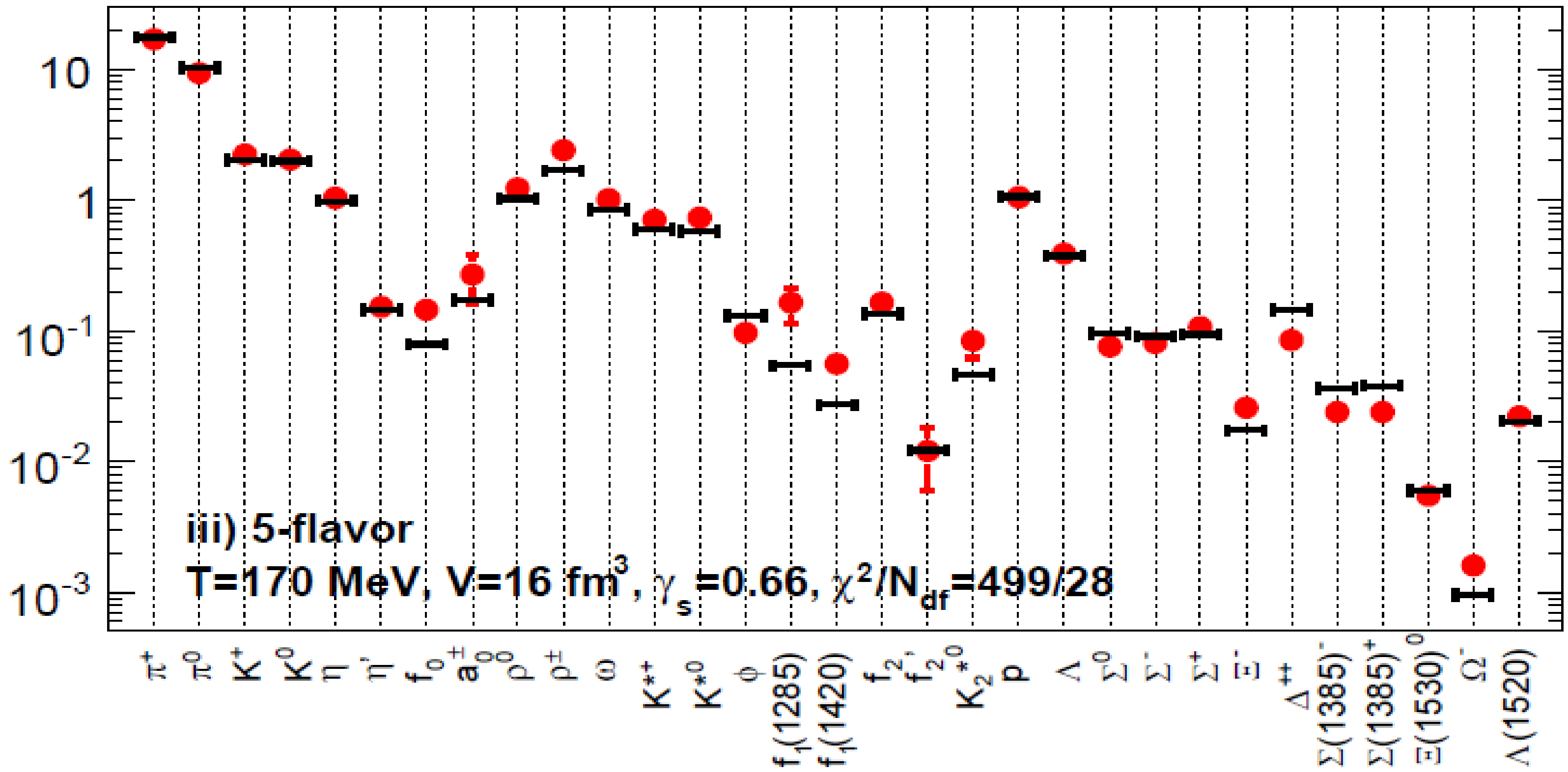


universal hadronization can be described with few parameters in addition to T, μ_B → transition from canonical to grand-canonical thermodynamics

J. Cleymans, P.M. Lo, K. Redlich, N. Sharma, PRC 103 (2021) 014904

Hadronization of jets in e^+e^- collisions also shows statistical features

Andronic, Beutler, Braun-Munzinger, Redlich, Stachel arXiv:0804.4132



data: LEP
$\sqrt{s}$ = 91 GeV

fit much worse than for heavy ion collision data
strangeness not equilibrated but suppressed significantly

5.3. How is chemical equilibration achieved?

2-particle collisions not enough – takes about one order of magnitude too long

even when system is initialized in equilibrium at T = 170 MeV, it falls out of equilibrium quickly

<u>simple example:</u>
use a data driven estimate of rate of cooling
near chemical freeze-out (can be explained later) $|\dot{T}/T| = \tau_T^{-1} = (13 \pm 1)\%/\mathrm{fm}$
typical densities at $T_{ch} : \rho_\pi = 0.174/\mathrm{fm}^3 (\mathrm{incl.res.}), \rho_K = 0.030/\mathrm{fm}^3 \rho_\Omega = 0.0003/\mathrm{fm}^3$
to maintain equilibrium during 5 MeV temperature drop need a relative rate of change of densities of $|\frac{\bar{r}_\Omega}{n_\Omega} - \frac{\bar{r}_K}{n_K}| = \tau_\Omega^{-1} - \tau_K^{-1} = 1.10 - 0.55/\mathrm{fm} = 0.55/\mathrm{fm}$
so W density needs to change by 100 % in 1 fm/c
typical reactions with large cross section (10 mb) and rel. velocities of 0.6 give

$$\Omega + \bar{K} \rightarrow \Xi + \pi \qquad \rightarrow \qquad \bar{r}_\Omega / n_\Omega = n_{\bar{K}} \langle v\sigma \rangle = 0.018/\mathrm{fm}$$
$$\pi + \pi \rightarrow K + \bar{K} \quad (\sigma = 3\mathrm{mb}) \qquad \bar{r}_K / n_K = 0.18/\mathrm{fm}$$

much too slow to maintain equlibrium even over drop of T of 5 MeV!
much harder to get into equilibrium!

A possible scenario for rapid equilibration

P. Braun-Munzinger, J. Stachel, C. Wetterich, Phys. Lett. B596 (2004) 61

near phase boundary multiparticle reactions become important
dynamics associated with collective excitations
(key word: critical opalescence at phase transition)
propagation and scattering of these collective excitations expressed in form of multi-hadron scattering

<u>will see:</u> this drives the system into equilibrium very rapidly

Evaluation of multi-strange baryon yield as most challenging test case

consider situation at T_{ch} = 176 MeV first

rate of change of density for n_{in} ingoing and n_{out} outgoing particles

$$r(n_{in}, n_{out}) = \bar{n}(\mathrm{T})^{n_{in}} |\mathcal{M}|^2 \phi$$

with

$$\phi = \prod_{k=1}^{n_{out}} \left(\int \frac{d^3 p_k}{(2\pi)^3 (2E_k)} \right) (2\pi)^4 \delta^4 \left(\sum_k p_k^\mu \right)$$

the phase space factor ϕ depends on √s

needs to be weighted by the probability *f*(s) that multi-particle scattering occurs at a given value of √s

evaluate numerically in Monte-Carlo using thermal momentum distribution

typical reaction $\Omega + \bar{N} \rightarrow 2\pi + 3K$

assume cross section equal to the measured one for $p + \bar{p} \rightarrow 5\pi$

at proper energy above threshold, i.e. √s = 3.25 GeV ≈ 6.4 mb

compute matrix element and use for rate of $2\pi + 3K \rightarrow \bar{N} + \Omega$

$$r_\Omega = n_\pi^5 (n_K/n_\pi)^3 |\mathcal{M}|^2 \phi$$

Evaluation of multi-strange baryon yield

reaction $2\pi + 3K \to \bar{N} + \Omega$ leads to

$$r_\Omega = 0.00014 \text{fm}^{-4} \quad \text{or} \quad r_\Omega / n_\Omega = 1/\tau_\Omega = 0.46/\text{fm}$$

can achieve final density starting from only pions and kaons at t=0 in 2.2 fm/c

similarly one obtains

for $3\pi + 2K \to \Xi + \bar{N} \quad \text{or} \quad \tau_\Xi = 0.71 \, \text{fm}$

and

for $4\pi + K \to \Lambda + \bar{N} \quad \text{or} \quad \tau_\Lambda = 0.66 \, \text{fm}$

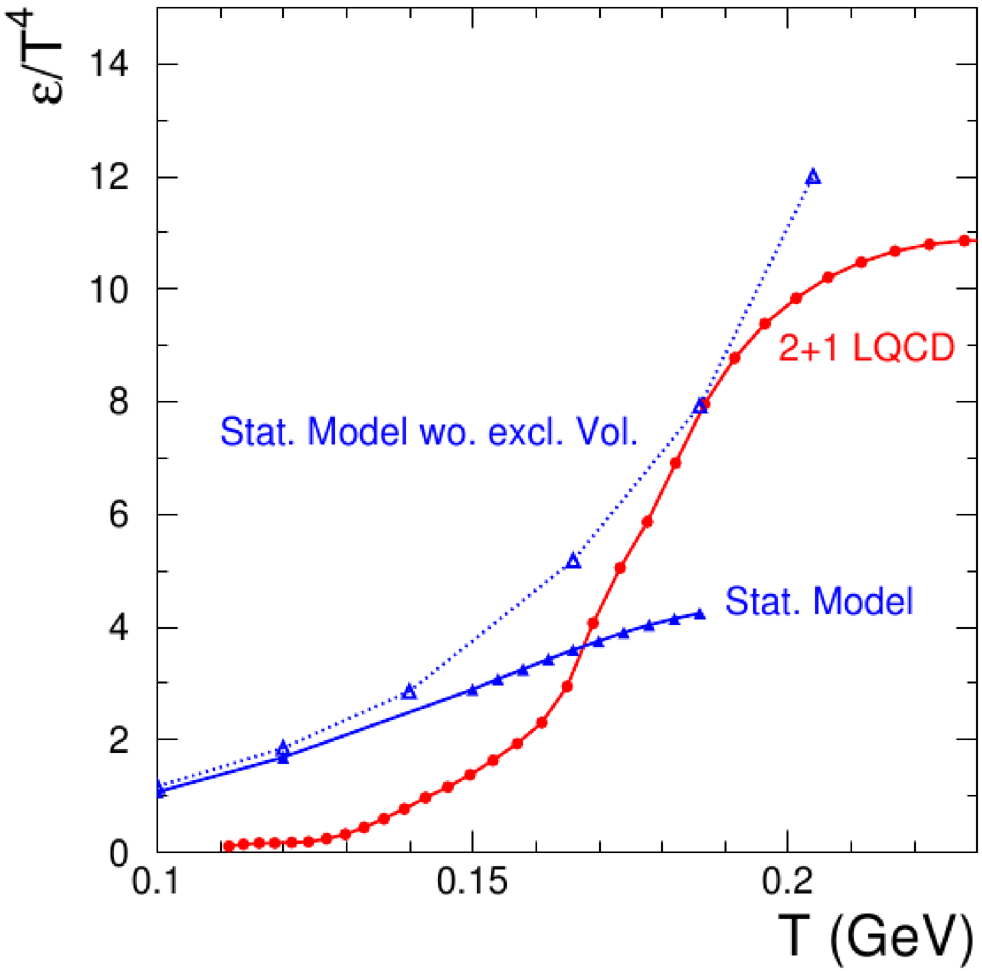
Why do all particle yields show one common freeze-out T?

density of particles varies rapidly (factor 2 within 8 MeV) with T near the phase transition due to increase in degrees of freedom.

also: system spends time at T_c -> volume has to triple (entropy cons.)

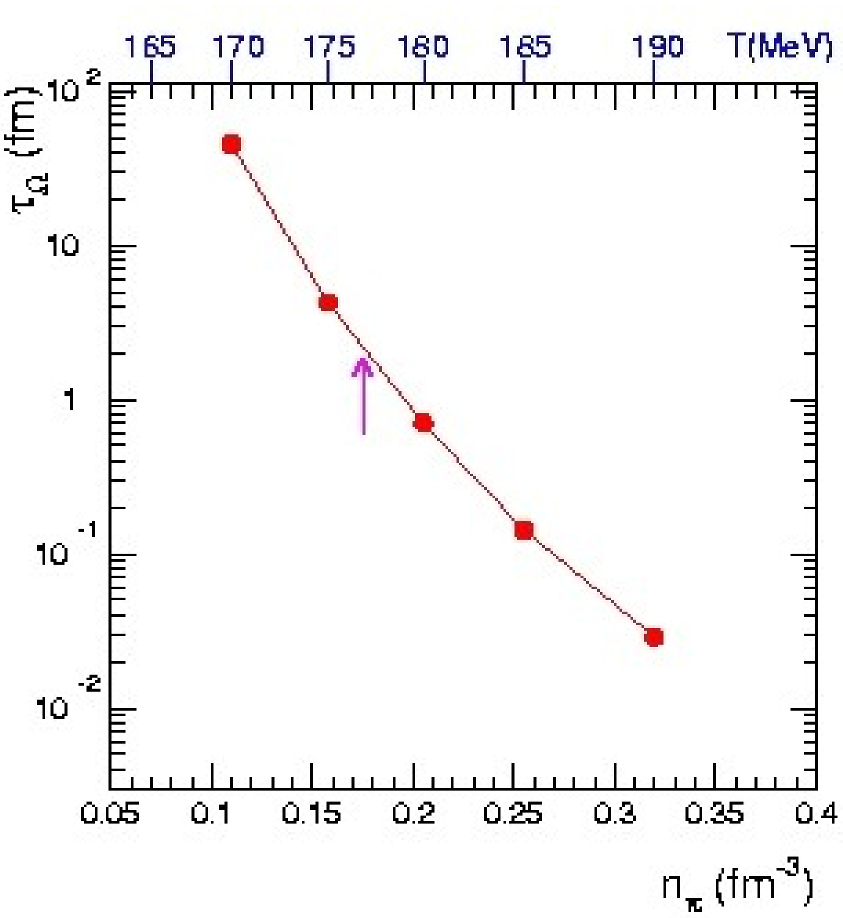
multi-particle collisions are strongly enhanced at high density and lead to chem. equilibrium very near to T_c independently of cross section

all particles can freeze out within narrow temperature interval



Lattice QCD by F. Karsch et al.

Density dependence of characteristic time for multi-strange baryon production

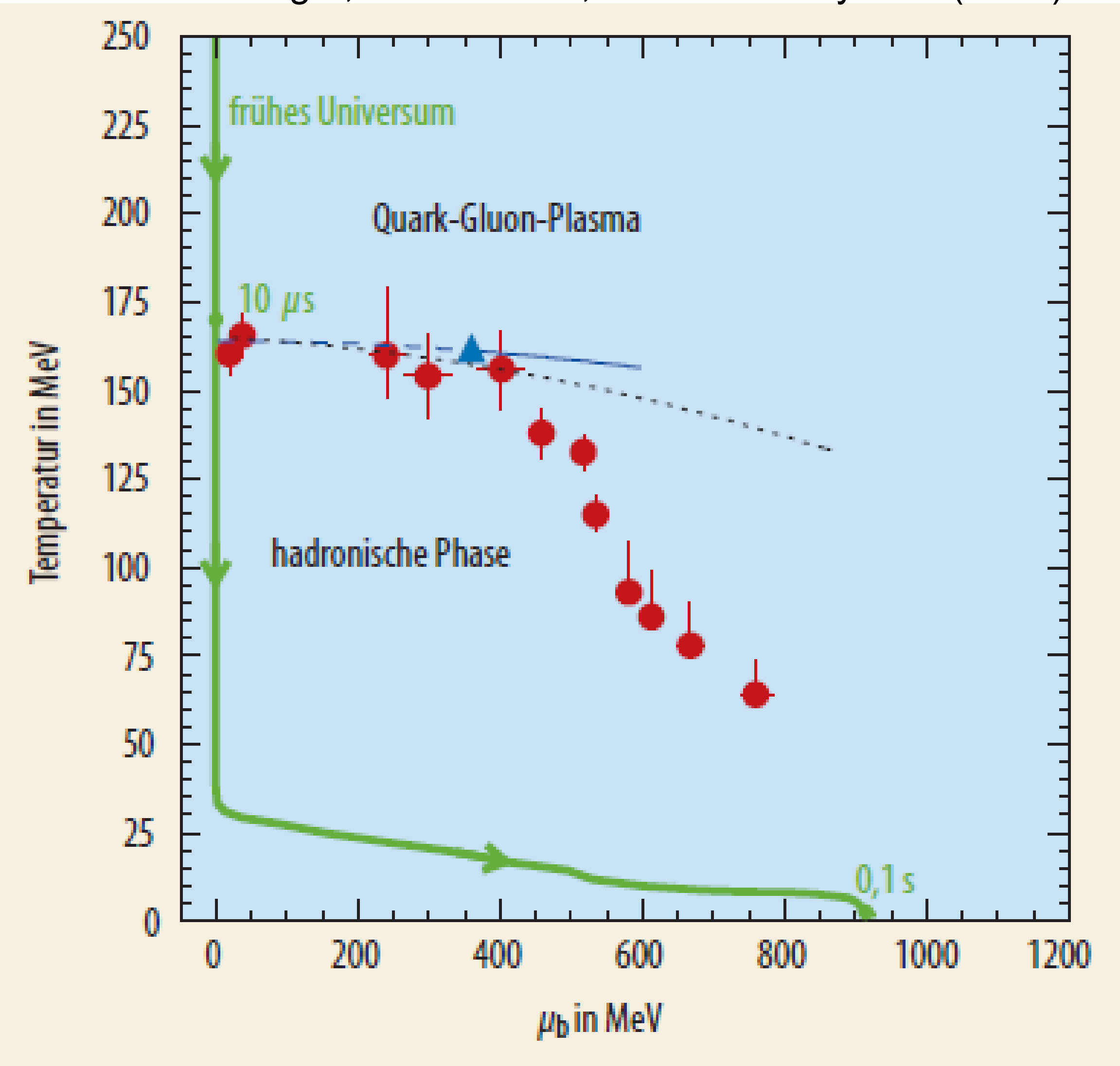


- near phase transition particle density varies rapidly with T (see previous slide)
- for SPS energies and above reaction such as 2□+KKK→ Ω Nbar bring multi-strange baryons close to equilibrium rapidly
- in region around T_c equilibration time $\tau_\Omega \propto T^{-60}$!
- increase n_p by 1/3: t = 0.2 fm/c (corresponds to increase in T by 8 MeV) decrease n_p by 1/3: t = 27 fm/c

® all particles freeze out within a very narrow temperature window due to the extreme temperature sensitivity of multi-particle reactions

In the early universe freeze-out happened after order of 0.1 s

P. Braun-Munzinger, J. Wambach, Rev. Mod. Phys. 81 (2009)1031



isentropic expansion and full chemical equilibrium between hadrons, leptons, photons

plus:
charge neutrality
net lepton number = net baryon number
constant entropy per baryon

hadro-chemical freeze-out quite different: at 0.1 s and $\mu_B \approx 0.9$ GeV

p 75%
$2\ 10^{-5}$ d
$8\ 10^{-5}$ ^{3}He
24.5% ^{4}He
$1.5\ 10^{-10}$ ^{7}Li

while at LHC factor 300 for every nucleon added