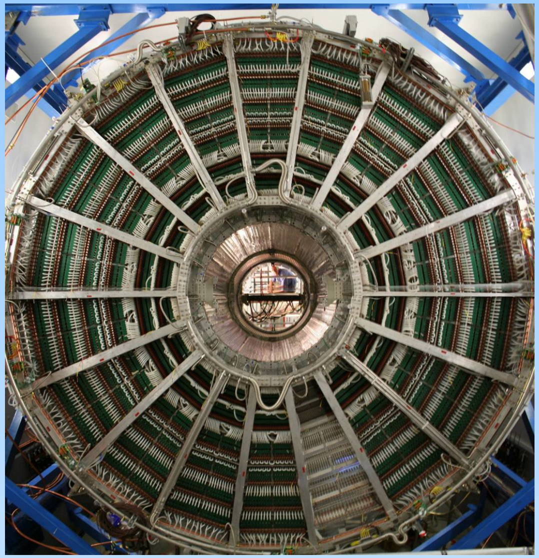
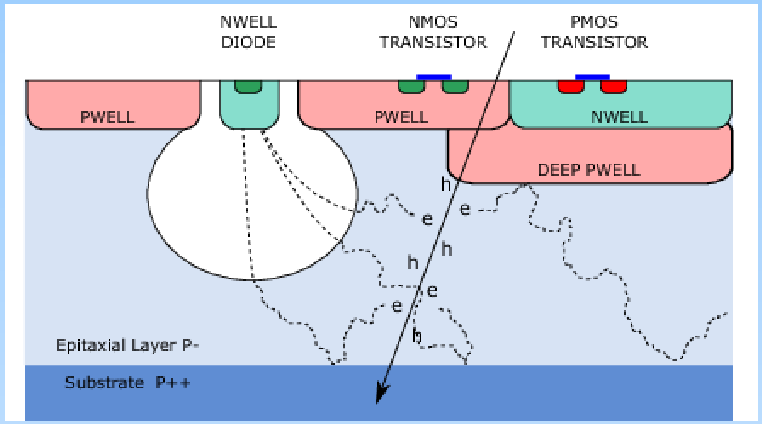




Interaction of particles with matter - 2

Silvia Masciocchi,
GSI and University of Heidelberg

SS2017, Heidelberg
May 3, 2017

Lectures

- Energy loss by ionization (by “heavy” particles)
- Interaction of electrons with matter:
  - Energy loss by ionization
  - Bremsstrahlung

LAST WEEK

- Cherenkov effect
- Transition radiation
- Interaction of photons
  - Photoelectric effect
  - Compton scattering
  - Pair production

TODAY

Charged particles

Cherenkov effect

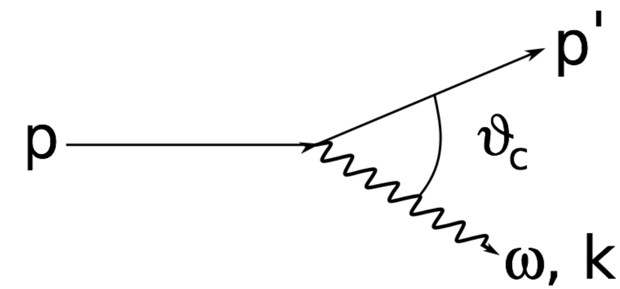
Cherenkov 1934

A charged particle with mass M and velocity $\beta = v/c$ travels in a medium with refractive index n:

$$n^2 = \varepsilon_1 = (c/c_m)^2$$

ε_1 = real part of the medium dielectric constant

c_m = speed of light in medium = c/n



If $v > c_m$, namely $\beta > \beta_{thr} = 1/n \rightarrow$ real photons are emitted:

- Photons are "soft"

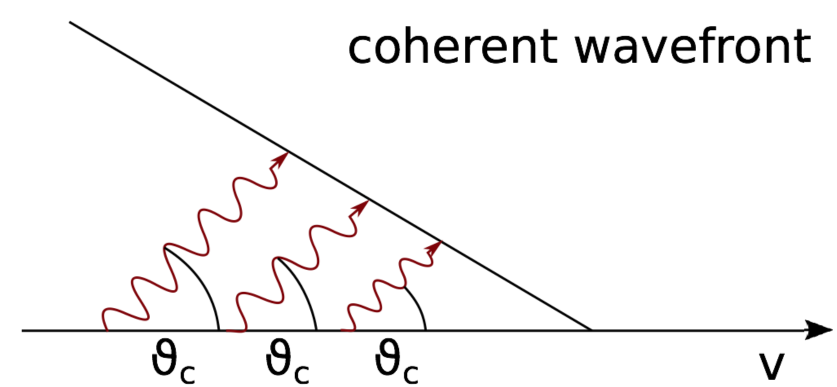
  $|p| \approx |p'|$

  $\omega \ll \gamma M c^2$

- Characteristic emission angle

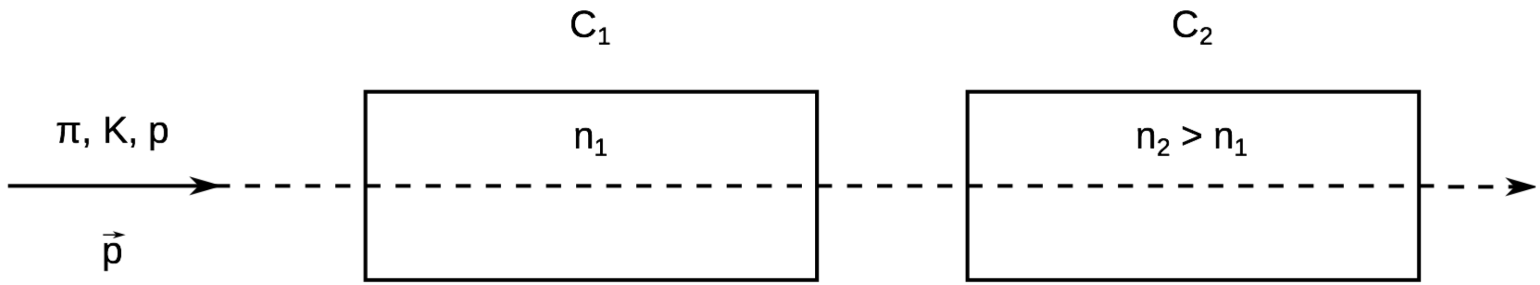
$$\cos\theta_c = \frac{\omega}{k \cdot v} = \frac{1}{n\beta}$$

Cherenkov angle



Cherenkov: first application

Threshold detector: use different materials (refractive indices) such that particles of different masses, at equal momentum p, produce Cherenkov radiation of not (pass the threshold or not):



Choose n_1, n_2 such that for a given p ($\beta = p/E$):

$$\beta_\pi > \frac{1}{n_1} \qquad \beta_K, \beta_p < \frac{1}{n_1}$$

$$\beta_\pi, \beta_K > \frac{1}{n_2} \qquad \beta_p < \frac{1}{n_2}$$

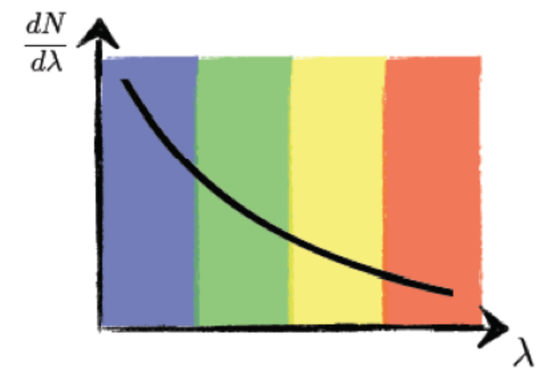
Particle identification: light in C_1 and C_2 → pion
light in C_1 and not in C_2 → kaon
no light in both C_1 and C_2 → proton

Cherenkov radiation: spectrum

Consider the spectrum of emitted photons per unit length versus:

- **Wavelength**: short wavelengths dominate (blue)

$$\frac{d^2N}{d\lambda dx} = \frac{2\pi\alpha z^2}{\lambda^2}\left(1 - \frac{1}{\beta^2 n^2(\lambda)}\right) = \frac{2\pi\alpha z^2}{\lambda^2}\sin^2\theta_C$$

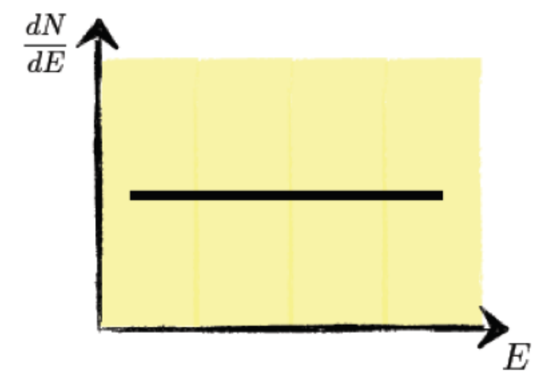


e.g. integrate over the typical sensitivity range of a good/typical photomultiplier (300-600 nm):

$$\frac{dN}{dx} = \int_{300\text{ nm}}^{600\text{ nm}} d\lambda \frac{d^2N}{d\lambda dx} = 750 \quad z^2 \sin^2\theta_C \text{ photons/cm}$$

- **Energy**

$$\frac{d^2N}{dE dx} = \frac{z^2\alpha}{\hbar c}\left(1 - \frac{1}{\beta^2 n^2(\lambda)}\right) = \frac{z^2\alpha}{\hbar c}\sin^2\theta_C$$



Cherenkov effect: typical numbers

- Number of photons per cm of radiator:

  assume n(ω) constant (e.g. true for visible light produced in gases 300 < λ < 600 nm)

	$(n-1)$	$(\beta\gamma)_{thr}$	θ_c^∞(deg)	$N_\gamma^\infty(cm^{-1})$
H_2	$0.14 \cdot 10^{-3}$	59.8	0.96	0.21
N_2	$0.3 \cdot 10^{-3}$	40.8	1.4	0.45
Freon 13	$0.72 \cdot 10^{-3}$	26.3	2.2	1.1
H_2O	0.33	1.13	41.2	165
lucite	0.49	0.91	47.8	412

- Energy loss: the energy loss by Cherenkov radiation is negligible wrt the one by ionization!

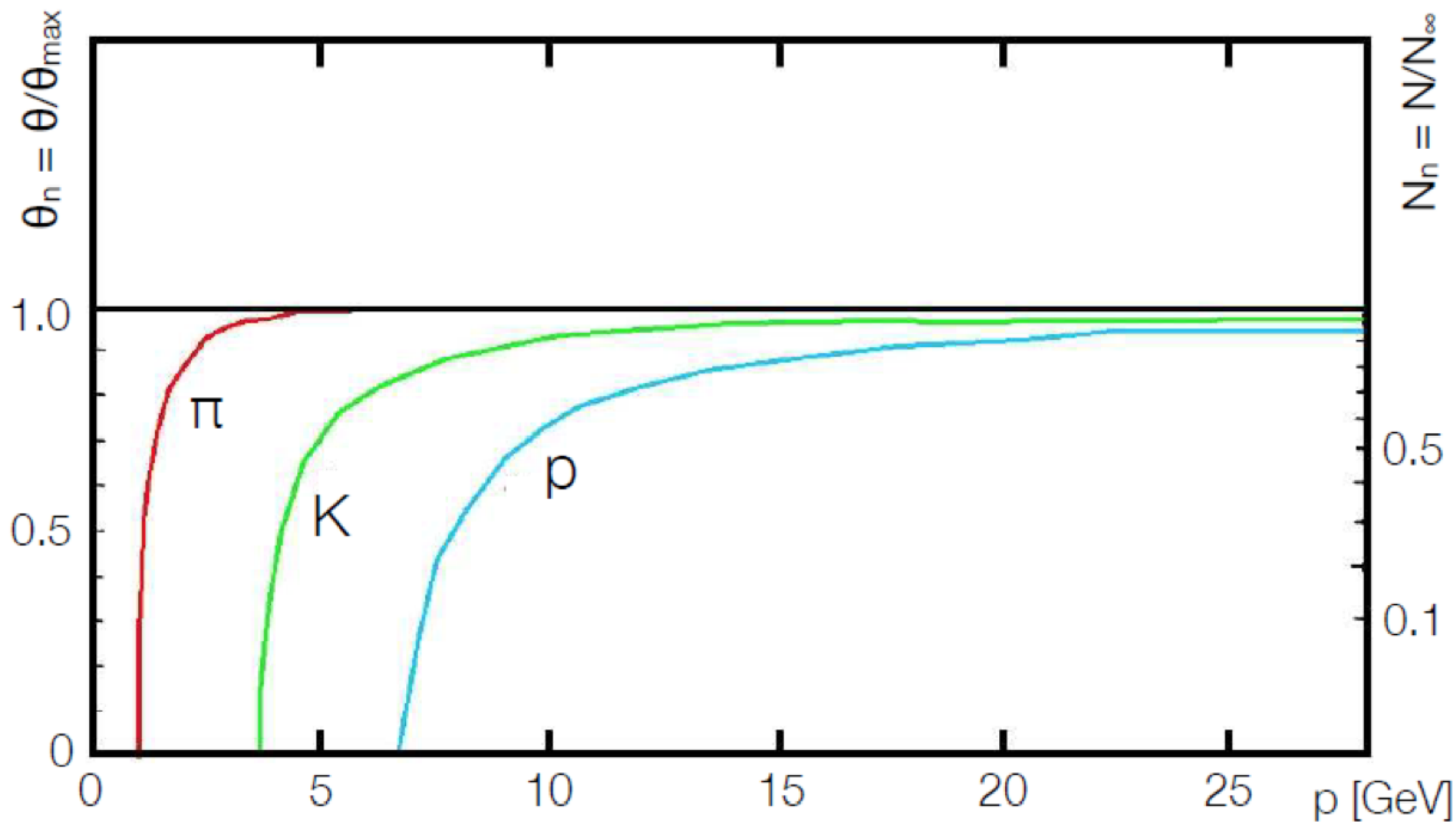
typical photon energy: $\simeq 3$ eV

in water $\left.\frac{dE}{dx}\right|_{\text{cher}} = 0.5\ \text{keV/cm} = 0.5\ \text{keV/g/cm}^2$

cf. ionization $\left.\frac{dE}{dx}\right|_{\text{ion}} \geq 2\ \text{MeV/g/cm}^2$

Cherenkov effect: momentum dependence

Asymptotic behavior of the Cherenkov angle and the number of produced photons, as a function of the particle momentum p (for β → 1):



$$\cos\theta_C \rightarrow \cos\theta_C^\infty = \frac{1}{n}$$

$$N_\gamma = x \cdot 370/\mathrm{cm}\left(1-\frac{1}{\beta^2 n^2}\right) \rightarrow x \cdot 370/\mathrm{cm}\left(1-\frac{1}{n^2}\right)$$

Application: measurement of β

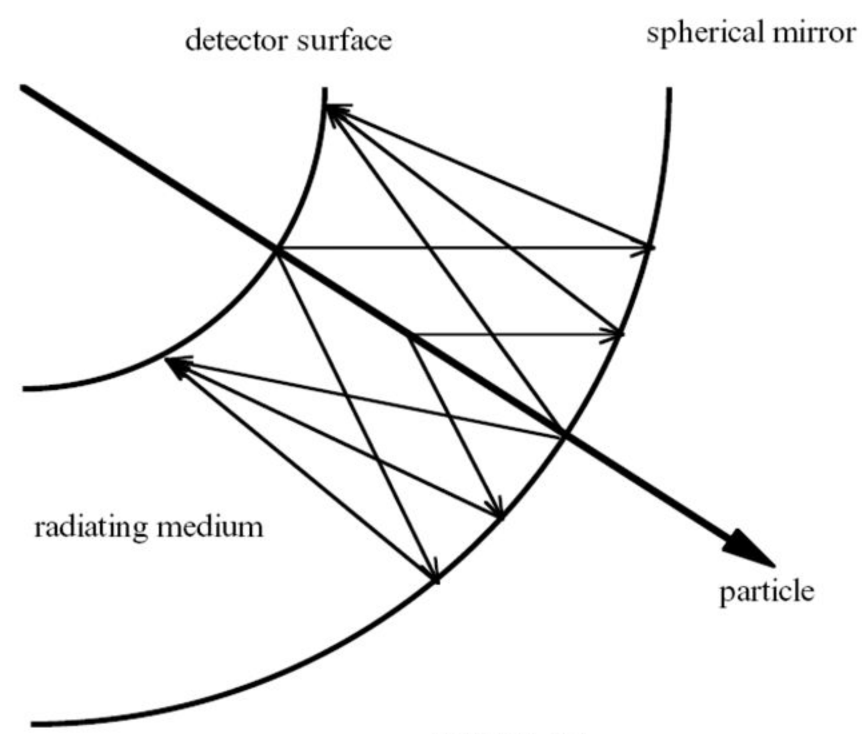
In a medium of known refractive index n, measure the Cherenkov angle and therefore determine the particle $\beta = p/E$ (→ identity)

Principle of:

RICH – Ring Imaging CHerenkov Detector

DIRC – Detection of Internally Reflected Cherenkov light

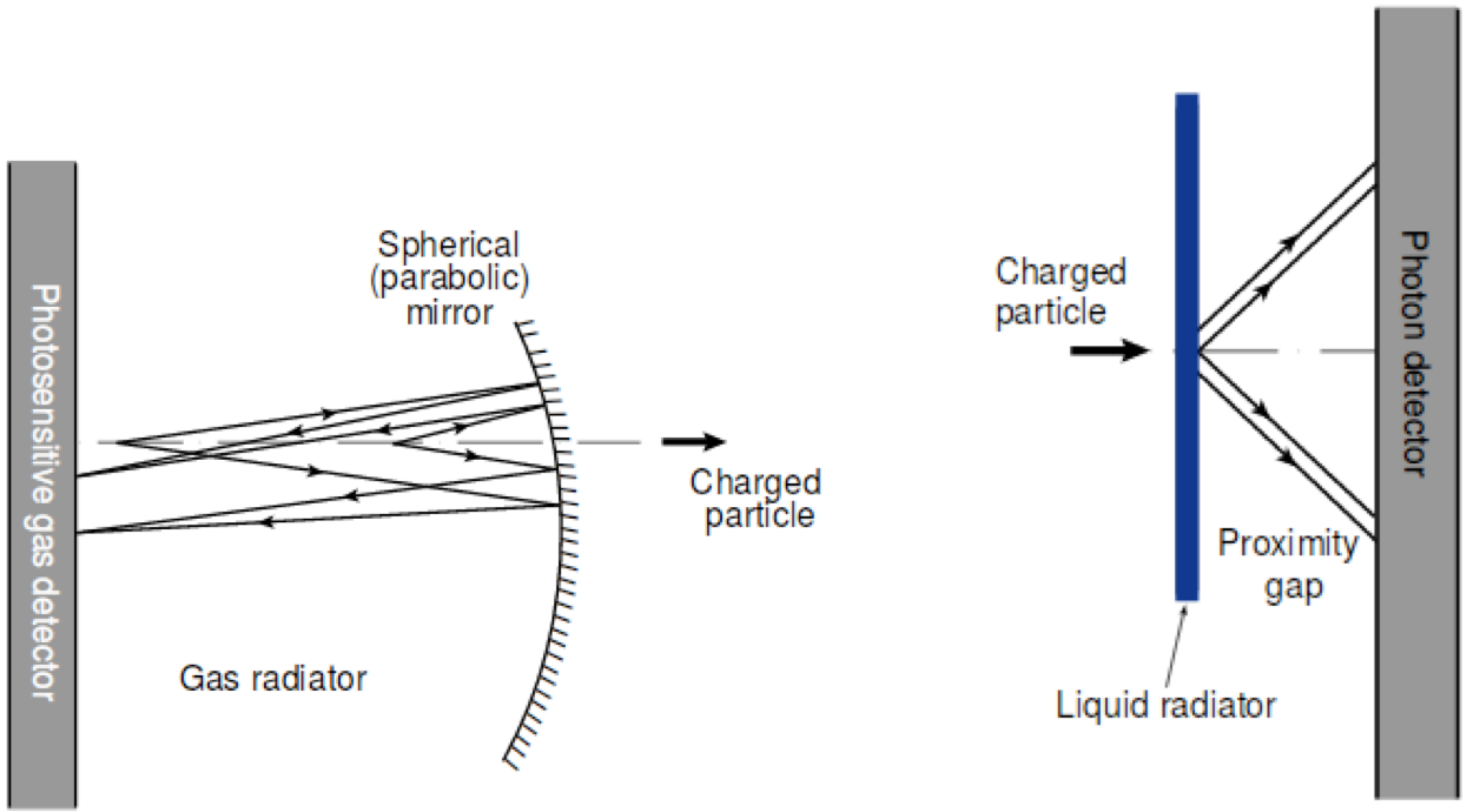
DISC – special variant of DIRC



RICH detectors

Principle: image the Cherenkov cone into a ring, of which measure the radius. Particle momentum provided by other detectors

Components: radiator (+ mirror) + photon detector

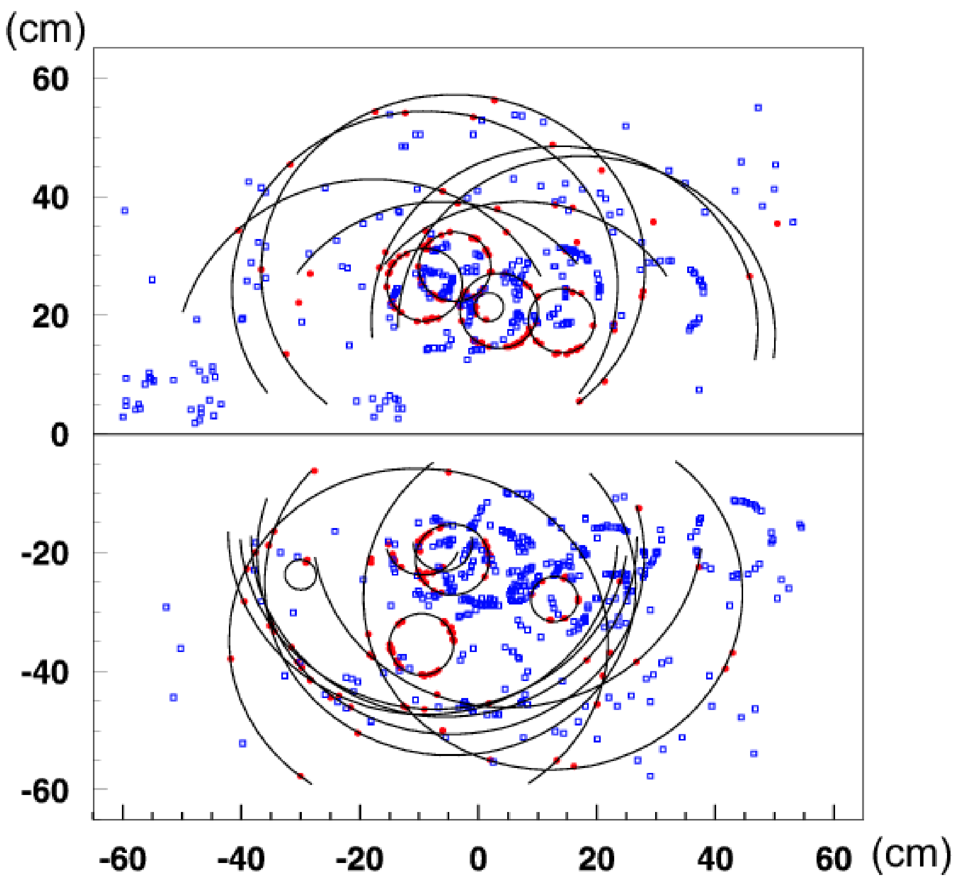


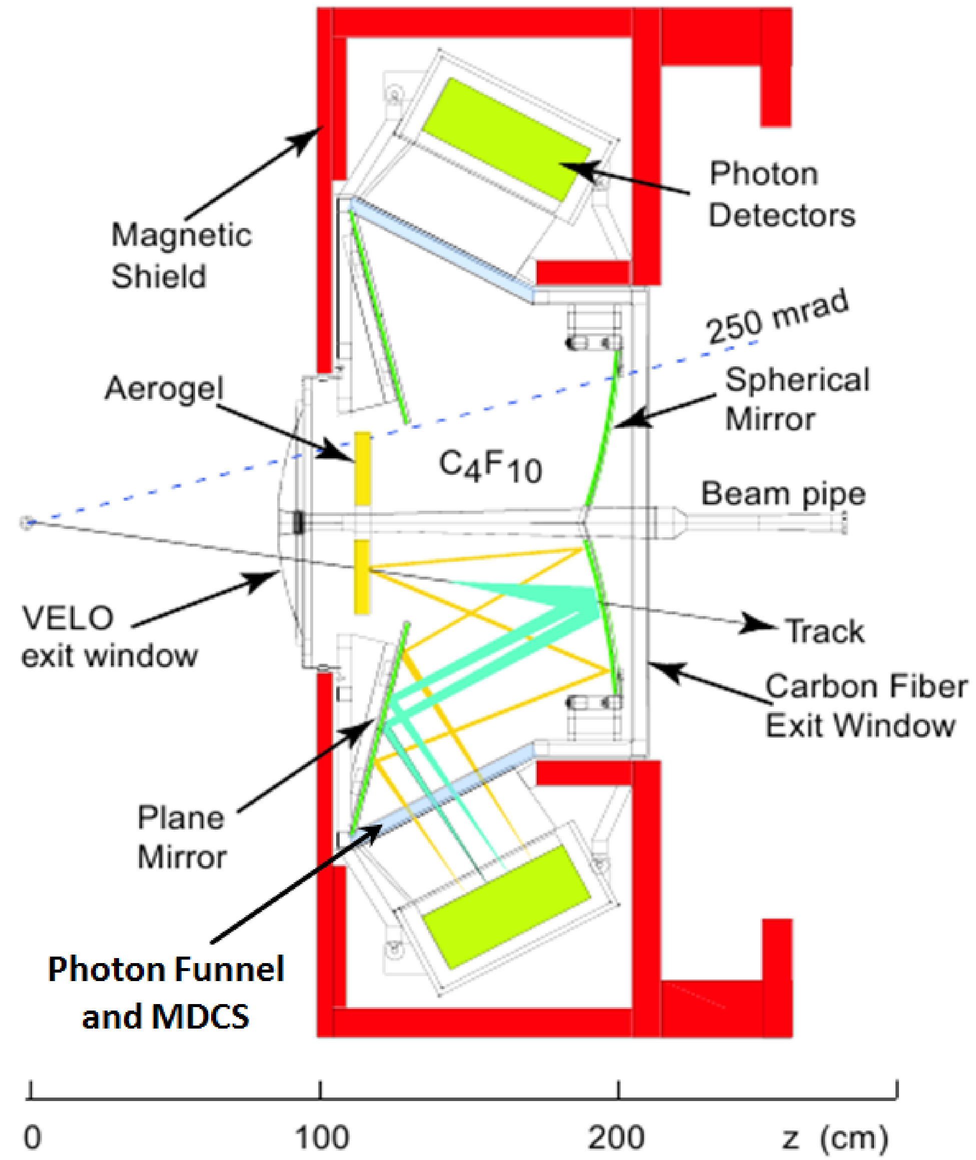
The LHCb RICH-1 detector

Two radiators: aerogel + C_4F_{10}

Spherical + flat mirrors

Hybrid Photon Detectors





Necessary radiator thickness

For a good measurement of the ring (and consequently of β), a sufficient number of photons must be produced and a sufficient number of photoelectrons must be detected!

$n_e = n_\gamma$ (Cherenkov) x $\varepsilon_{\text{light collection}}$ x $\eta_{\text{quantum efficiency}}$

$\varepsilon_{\text{light collection}} \approx 0.8$

$\eta_{\text{quantum efficiency}} \approx 0.2$

Example: ask for $n_e \geq 4$ to reconstruct a good ring

The efficiency must be ≥ 90%

n_e follows a Poisson distribution: $P(4)+P(5)+P(6)+\ldots > 0.9$

$\rightarrow < n_e > = 7$

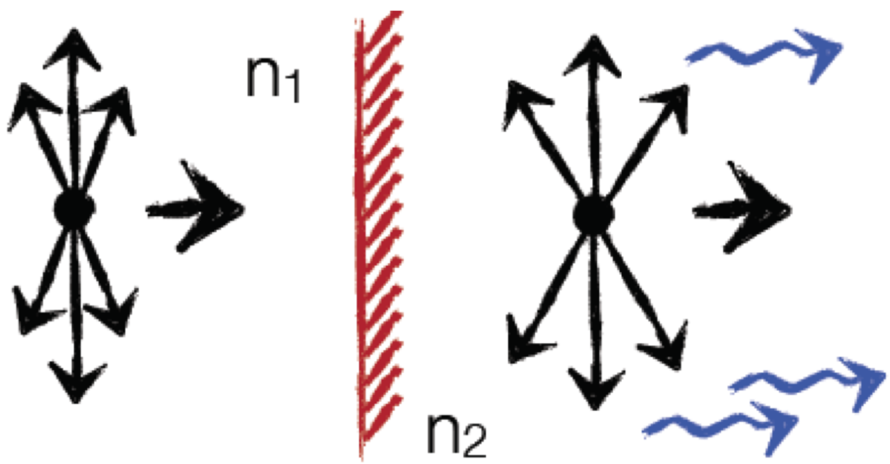
$\rightarrow$ need n_γ (Cherenkov) ~ 44 photons

$\rightarrow$ e.g. 0.4 m freon

Spherical mirror array: LHCb

Transition radiation

A particle at high energy (= large γ) crossing the boundary between two different dielectrics, having different indices of refraction, can produce “transition radiation” → can emit real photons



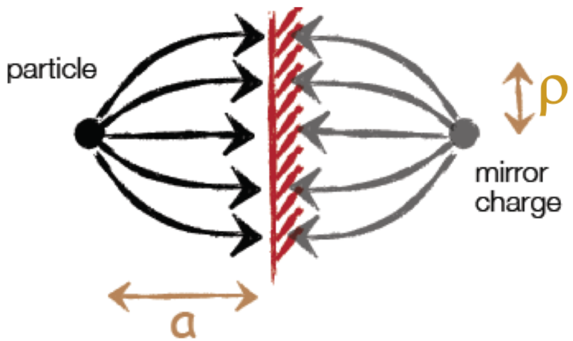
- Predicted by Ginzburg and Frank (1946)
- Observed (optically) by Goldsmith and Jelley (1959)
- Experimental confirmation with X-ray measurement (1970s)

Explanation: re-arrangement of electric field

Transition radiation: classical model

Simple classical model:

- electron moves in vacuum towards a conducting plate
- E-field described by method of mirror charges
- → as generated by dipole:



$$\vec{p} = 2e\vec{a} \qquad \left|\vec{E}_n\right| \propto \frac{\vec{p}}{\left(a^2+\rho^2\right)^{\frac{3}{2}}}$$

- Dipole moment changes in time → induces the radiation of photons
- Radiated power:

$$\frac{dP}{d(\hbar\omega)} \propto \frac{\alpha \cdot E}{mc^2}$$

  - ω-independent → white spectrum
  - dP ~ γ (but check the relativistic generalization
  - dP ~ α → one α per boundary

Transition radiation: full calculation

Full quantum mechanical calculation:

- Interference: coherent superposition of radiation from neighboring points in vicinity of the track
  → angular distribution strongly peaked forward
- Depth from boundary up to which contributions add coherently → formation length D
- Volume element producing coherent radiation V
- Photon energy: X-rays $E_\gamma^{max} \simeq \gamma \hbar \omega_p$

$$\theta \simeq \frac{1}{\gamma}$$

$$D \simeq \frac{\gamma \cdot c}{\omega_p}$$

$$V = \pi \rho_{max} D$$

Relevant parameter: plasma frequency ω_p:

$$\sqrt{\epsilon_1} = n(\omega) \simeq 1 - \frac{\omega_p^2}{\omega^2} \quad \text{with} \quad \omega_p = \sqrt{\frac{4\pi\alpha n_e}{m_e c^2}} = 28.8\sqrt{\varrho\frac{Z}{A}}\ \text{eV}$$

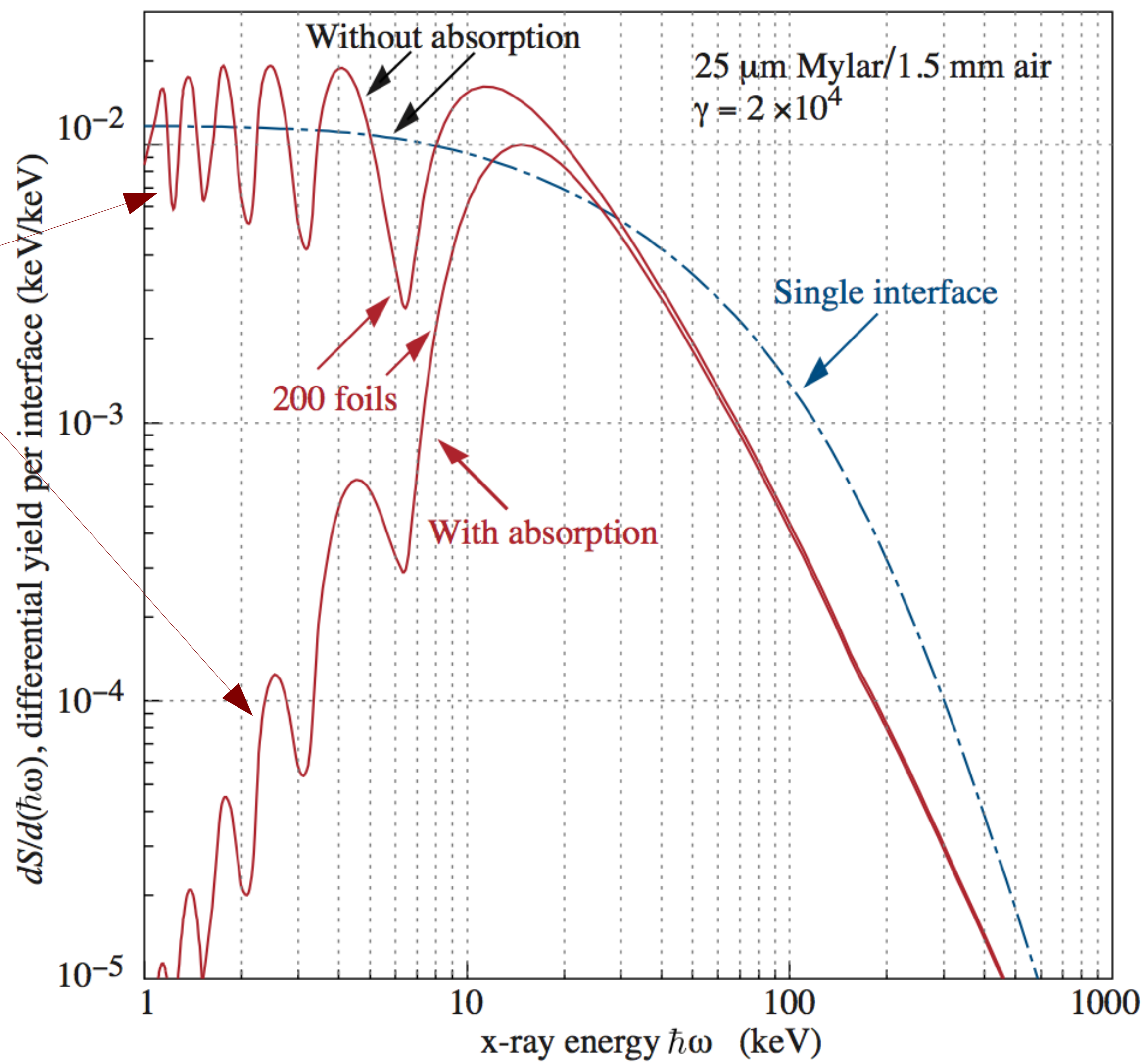
Typical values: polyethylene CH_2 ω_p= 20 eV, ρ=1 g/cm^3 → D ≈ 10 μm

For d > D → absorption effects important! Consider foils of thickness D!

Per boundary: ~ α photons → many boundaries !! O(100 foils) → <n_γ> ~ 1-2

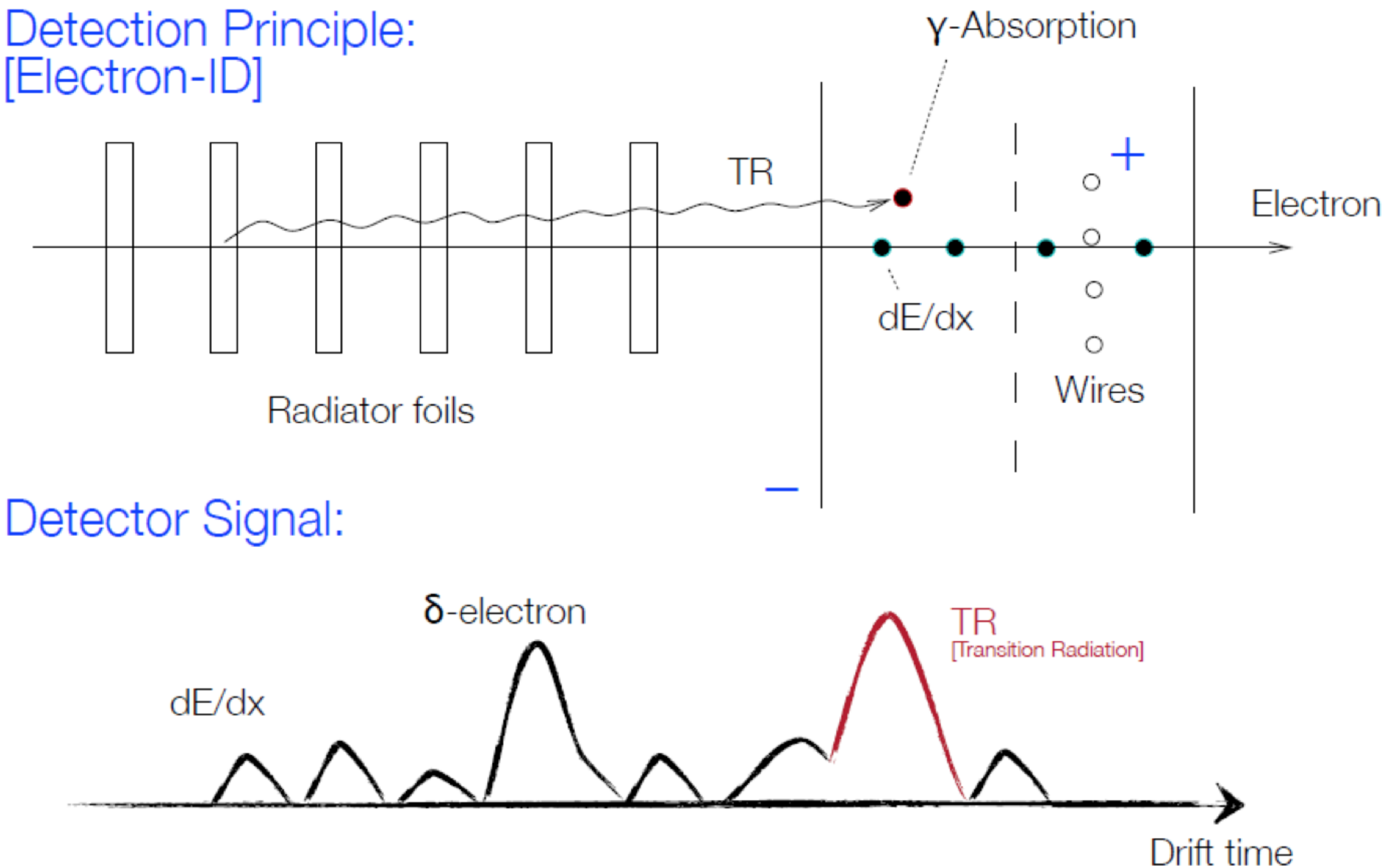
Transition radiation spectrum

Oscillations are due to interference between successive boundaries



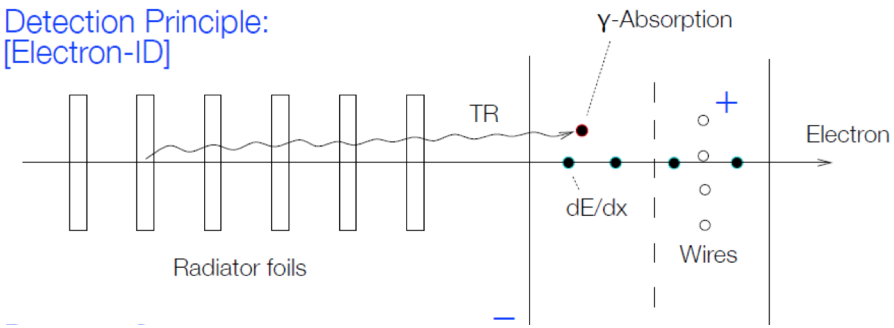
X-ray photon energy spectra for a radiator consisting of 200 25 μm thick foils of Mylar and for a single surface

Principle of a transition radiation detector

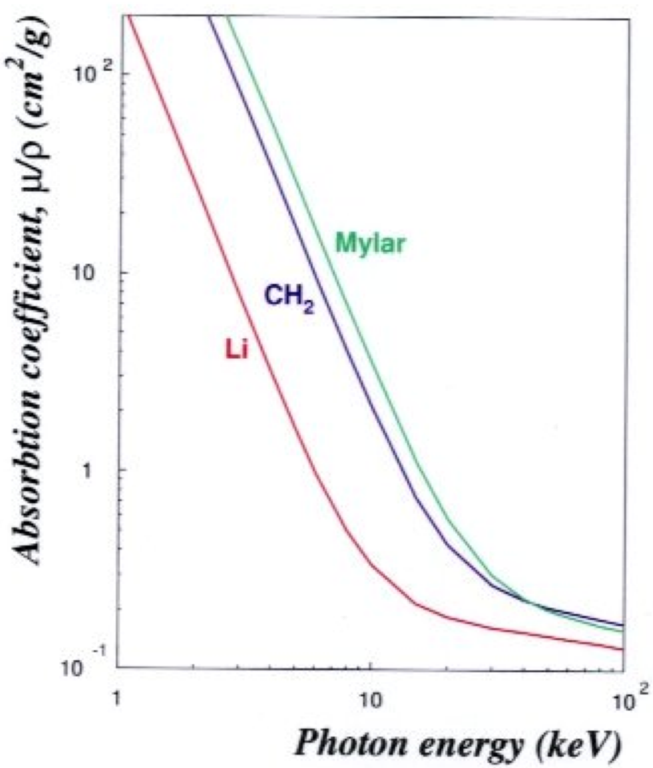


Principle of a transition radiation detector

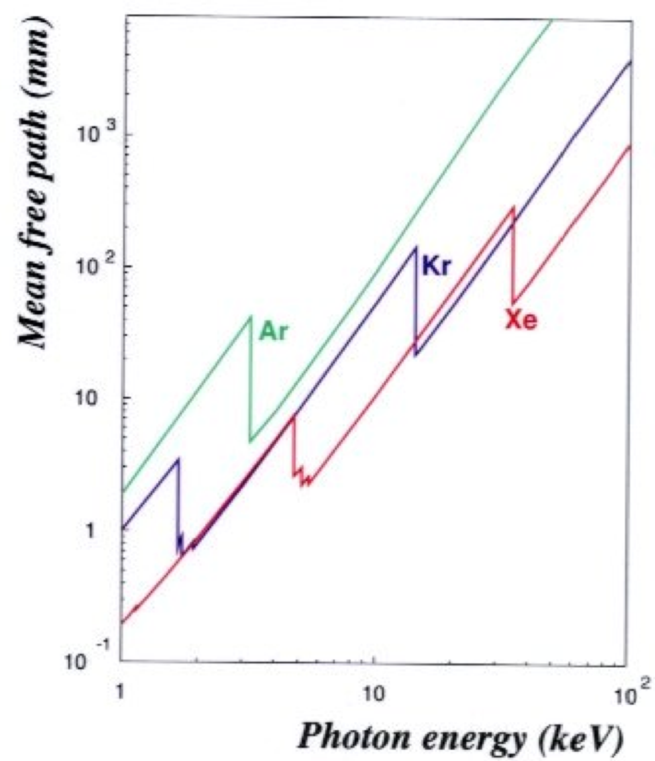
Choice of
1) radiator material and structure
2) photon detector



Minimize photon absorption in the radiator

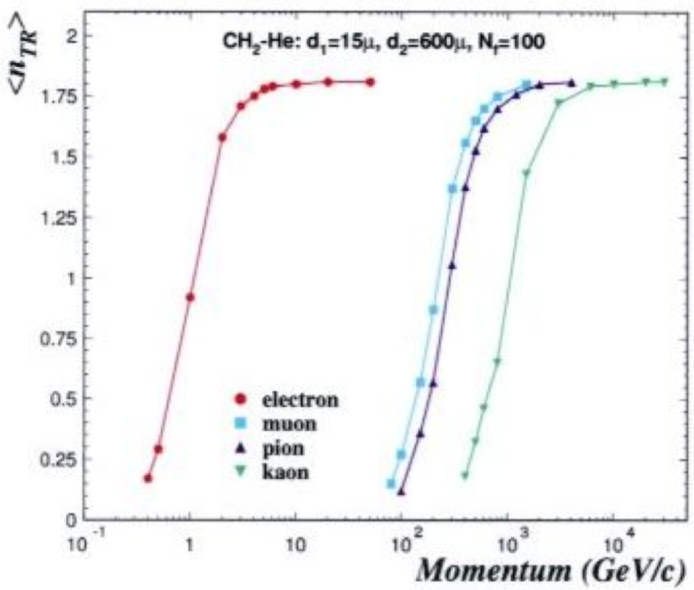


Maximize photon absorption in the detector

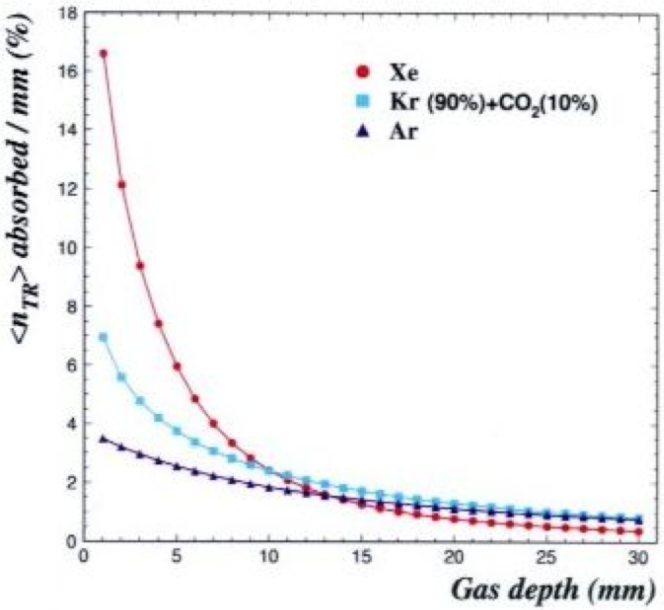


Principle of a transition radiation detector

Expected performance for polyethylene radiator foil stacks and various detector gases:

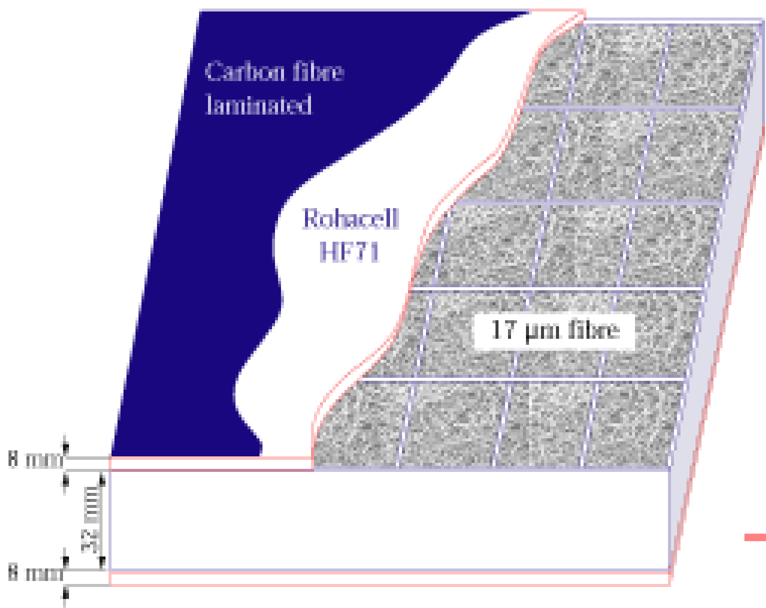


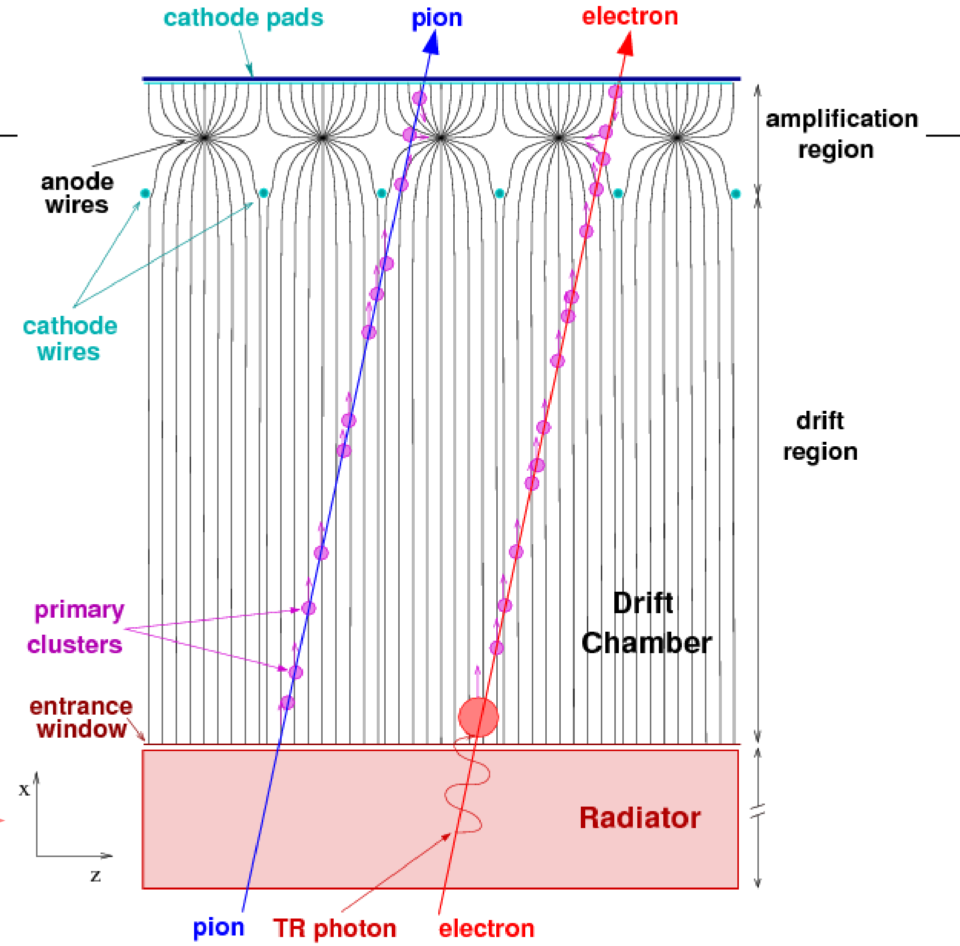
onset of TR production for electrons, muons, pions and kaons. Radiator of 100 foils, thickness d1, spacing d2

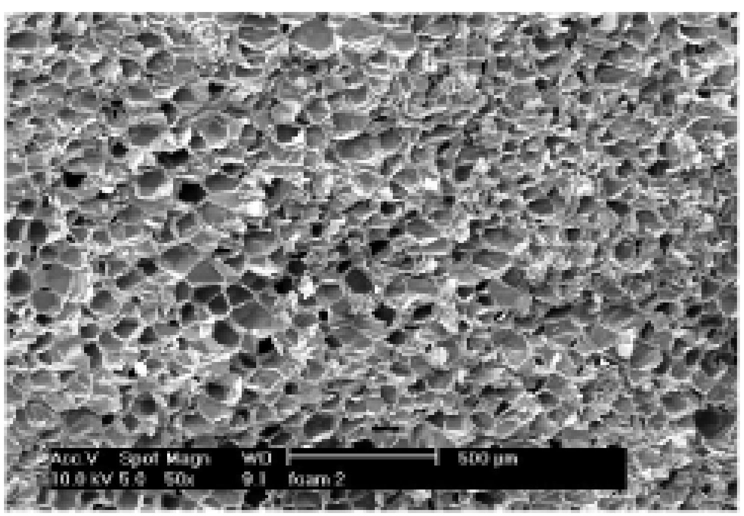


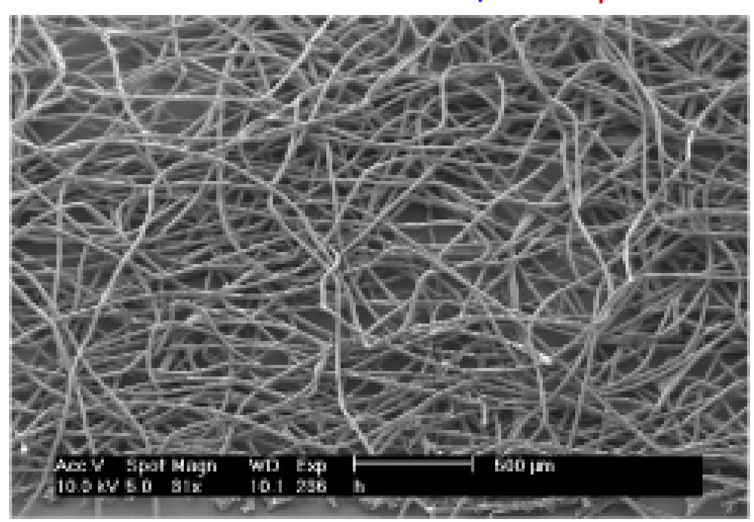
fraction of absorbed TR photons as a function of detector depth. For good absorption probability preferential use of Xe gas, typical dimension cm

ALICE TRD

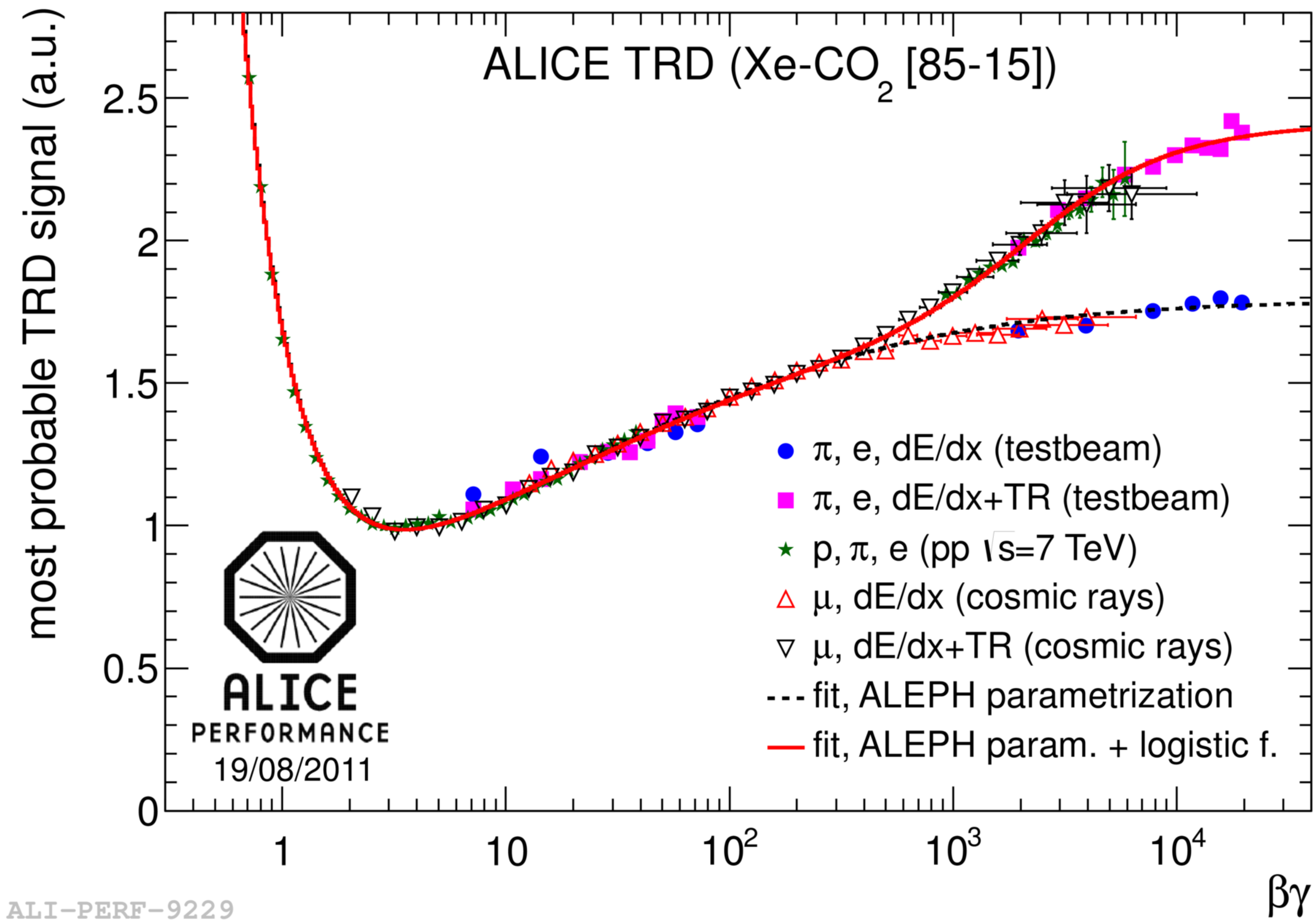

Carbon fibre
laminated
Rohacell
HF71
17 µm fibre
8 mm
32 mm
8 mm


cathode pads
pion
electron
anode
wires
amplification
region
cathode
wires
drift
region
primary
clusters
entrance
window
Drift
Chamber
Radiator
x
z
pion
TR photon
electron


500 µm

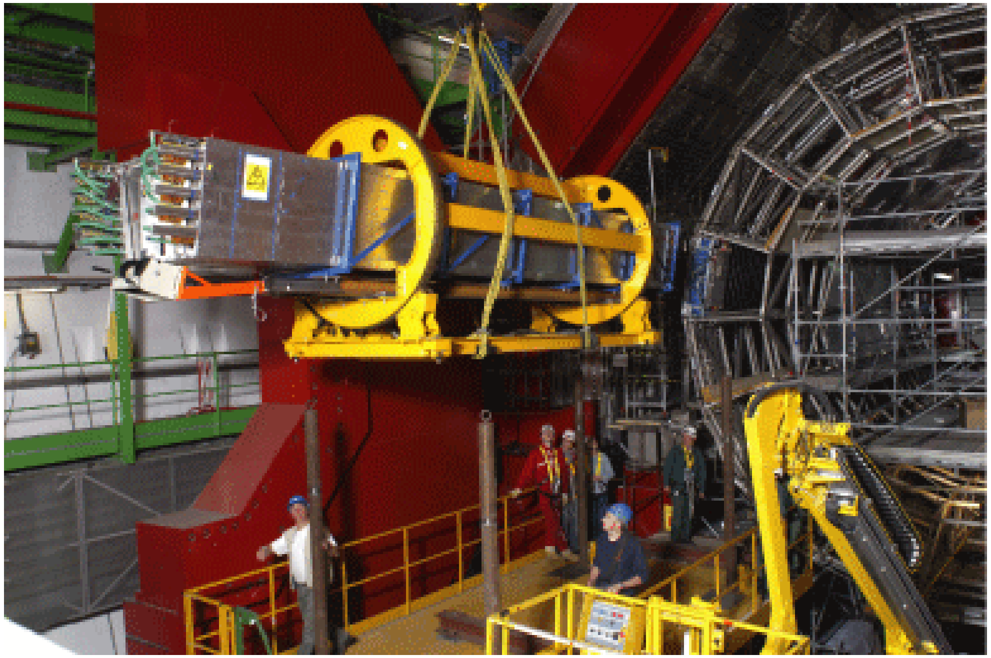

500 µm

ALICE TRD



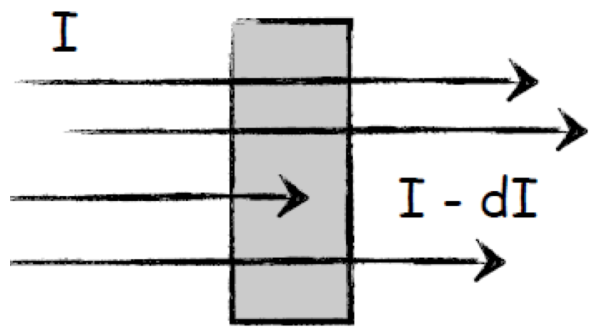
Demonstration of the onset of TR at βγ ≈ 500 (X. Lu, Hd)

ALICE TDR



Interaction of photons with matter

Characteristic of photons: can be removed from incoming beam of intensity "I", with one single interaction:



$dI = -I\,\mu\,dx$

$\mu\,(E, Z, \rho)$: absorption coefficient

Lambert-Beer law of attenuation:

$I(x) = I_0 \exp(-\mu x)$

- Mean free path of photon in matter: $$\lambda = \frac{1}{n\sigma} = \frac{1}{\mu}$$
- To become independent from state (liquid, gaseous): mass absorption coefficient: $$\tau = \frac{\mu}{\rho} = N_A \frac{\sigma}{A}$$

Example: E_γ = 100 keV, Z=26 (iron), λ= 3 g/cm^2 or 0.4 cm

Interaction processes

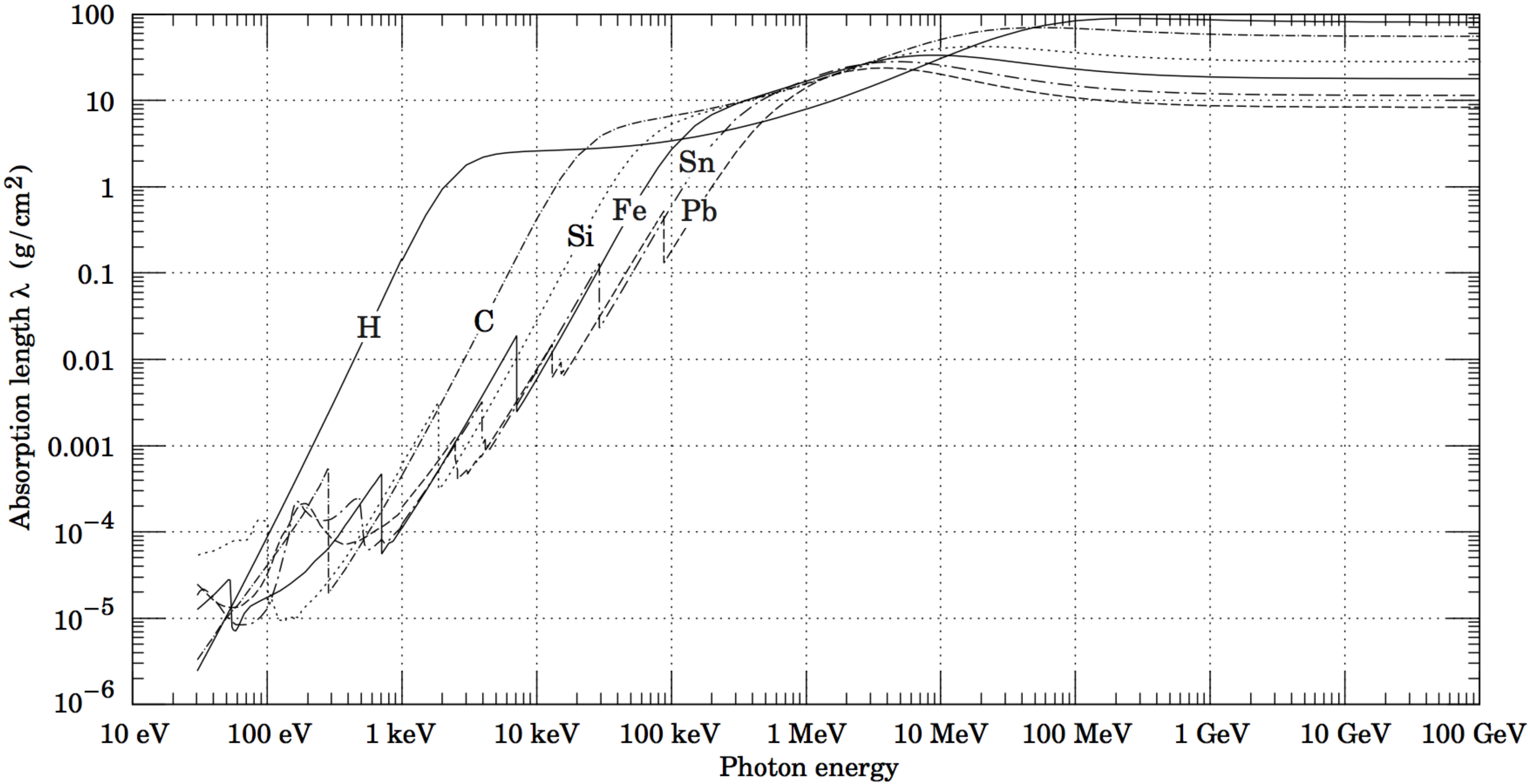
The most important processes of interaction of photons with matter, in order of growing importance with increasing photon energy E, are:

- Photoelectric effect
- Compton scattering: incoherent scattering off an electron
- pair production: interaction in nuclear field

Other processes, not as important for energy loss:

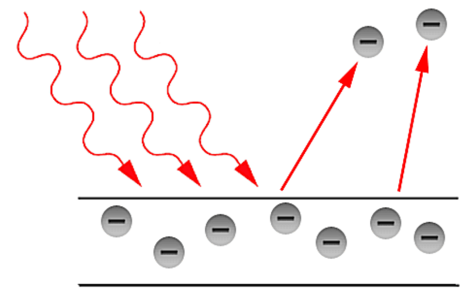
- Rayleigh scattering: coherent $\gamma + A \rightarrow \gamma + A$: atom neither ionized nor excited
- Thomson scattering: elastic scattering $\gamma + e \rightarrow \gamma + e$
- Photo nuclear absorption: γ + nucleus $\rightarrow$ (p or n) + nucleus
- Hadron pair production: $\gamma + A \rightarrow h^{+} + h^{-} + A$

Absorption length



Particle Data Group, 2016

Photoelectric effect



$$\gamma + \text{atom} \rightarrow \text{atom}^{+} + e^{-}$$

$$E_e = h\nu - I_b$$

Where: $h\nu = E\gamma$ = photon energy,

I_b = binding energy of the electron (K, L, M absorption edges)

Binding energy depends strongly on Z → the cross section will depend strongly on Z:

- $I \ll E_\gamma \ll mc^2$: $\sigma_{Ph} = \alpha \pi a_b Z^5 \left(\frac{I_0}{E_\gamma}\right)^{\frac{7}{2}}$

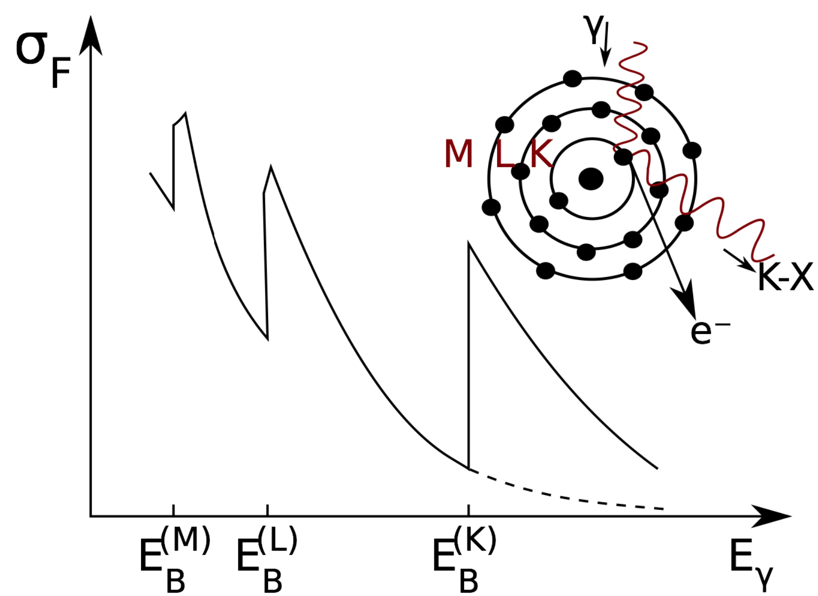
  where:

  $a_b = 0.53 \times 10^{-10}$ m

  $I_0 = 13.6$ eV

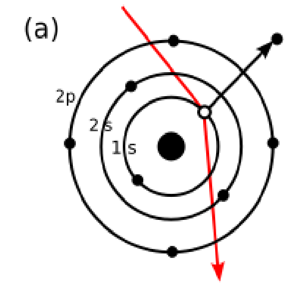
  e.g. = 0.1 MeV → $\sigma_{Ph}(\text{Fe}) = 29$ b

  $\sigma_{Ph}(\text{Pb}) = 5$ kb



- $E_\gamma \gg mc^2$: $\sigma_{Ph} \propto \frac{Z^5}{E_\gamma}$

Photoelectric effect - 2

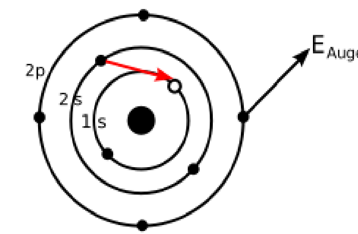


Electron collision

The de-excitation of the excited atom can happen via two main processes:

- Auger electrons: atom** $^{+}$ → atom* $^{+}$ + e^{-}

  Auger electrons deposit their energy locally due to their very small energy (<10 keV)



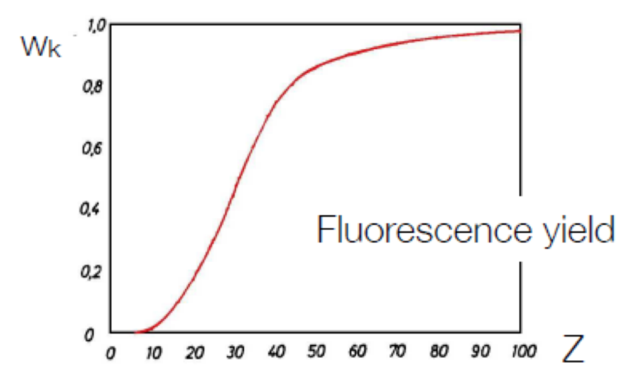
Auger electron emission

- Fluorescence: atom** $^{+}$ → atom* $^{+}$ + γ

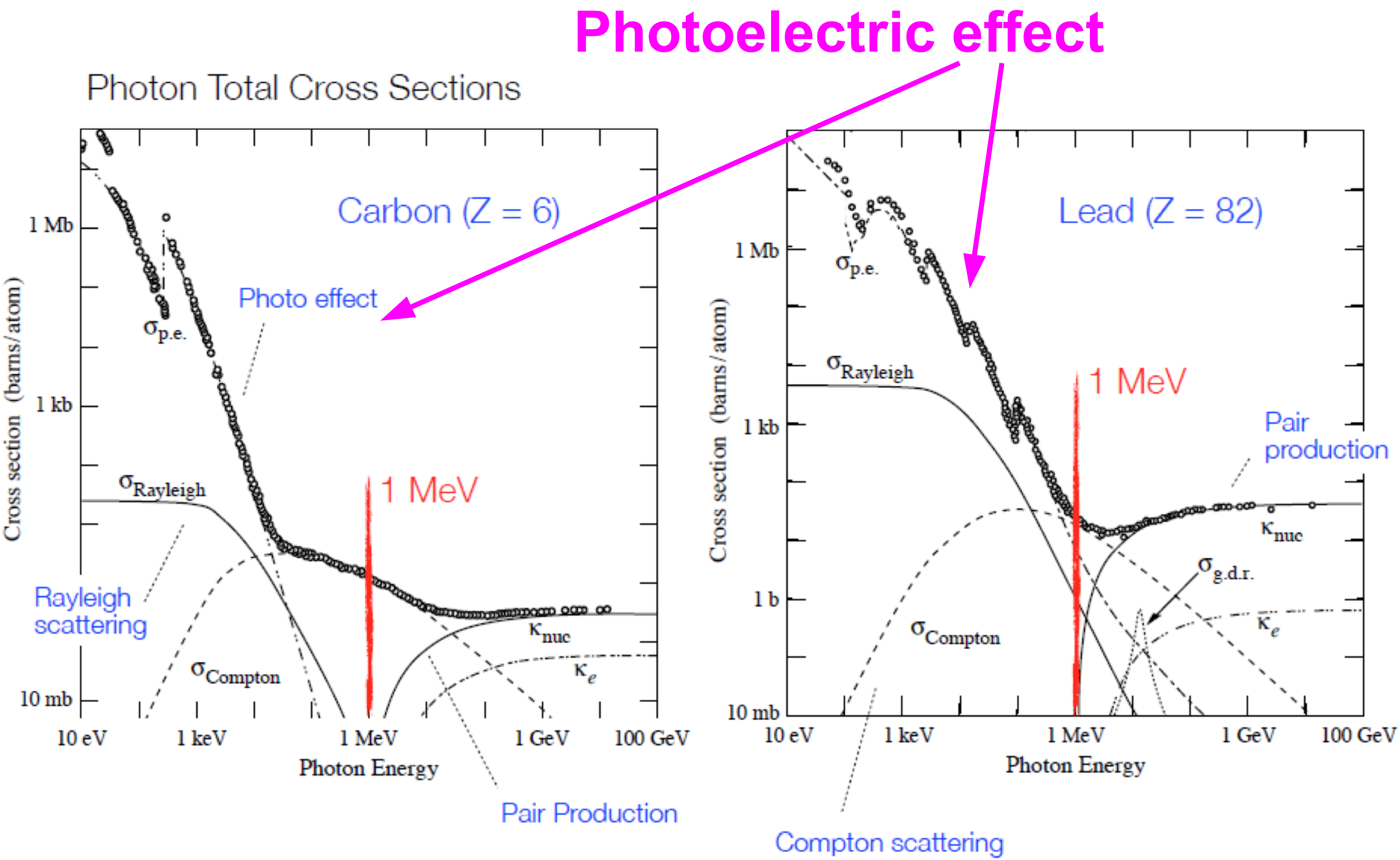
  Fluorescence photons (X-rays) must interact via the photoelectric effect → much longer range

  The relative fluorescence yield increases with Z

  w_K = P(fluor.) / [P(fluor.) + P(Auger)]



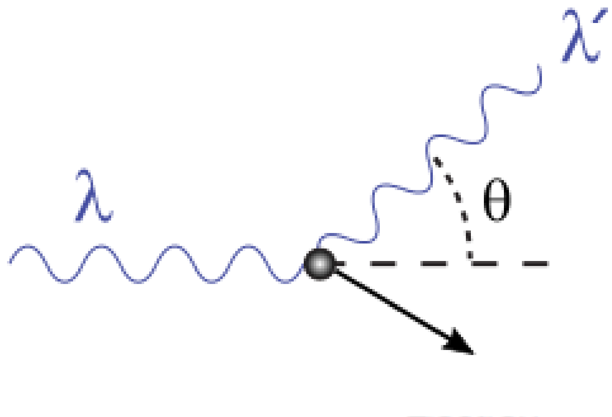
Photon total cross section



Compton scattering

Incoherent scattering of photon off an electron:

γ + e⁻ → (γ)' + (e⁻)'



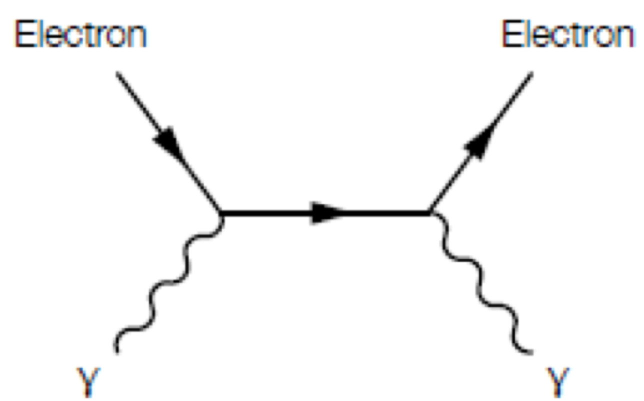
Energy of the outgoing photon:

$$E'_\gamma = \frac{E_\gamma}{1 + \frac{E_\gamma}{m_e c^2}(1 - \cos\theta)}$$

Kinetic energy of the outgoing electron:

$$T_e = \frac{\frac{E_\gamma^2}{m_e c^2}(1 - \cos\theta)}{1 + \frac{E_\gamma}{m_e c^2}(1 - \cos\theta)}$$

Feynman diagram



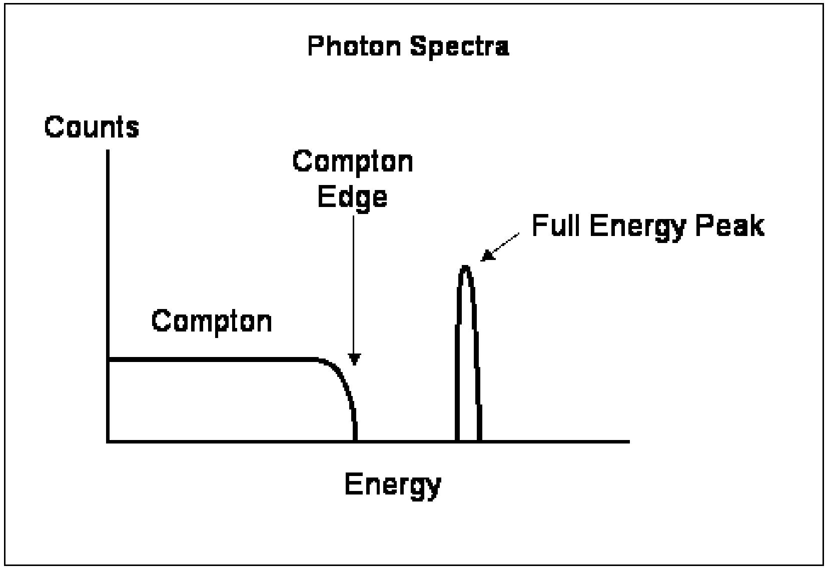
Max energy transfer in back scattering:

$$\left(\frac{T_e}{E_\gamma}\right)_{\text{max}} = \frac{E_\gamma}{m_e c^2} \frac{2}{1 + \frac{2E_\gamma}{m_e c^2}}$$

$$\Delta E = E_\gamma - T_{e,max} = \frac{E_\gamma}{1 + \frac{2E_\gamma}{m_e c^2}} \rightarrow \frac{m_e c^2}{2}$$

Compton edge

If the scattered photon is not absorbed in the detector material, there will be a small amount of energy "missing" from the Full Energy Peak (FEP) → Compton edge

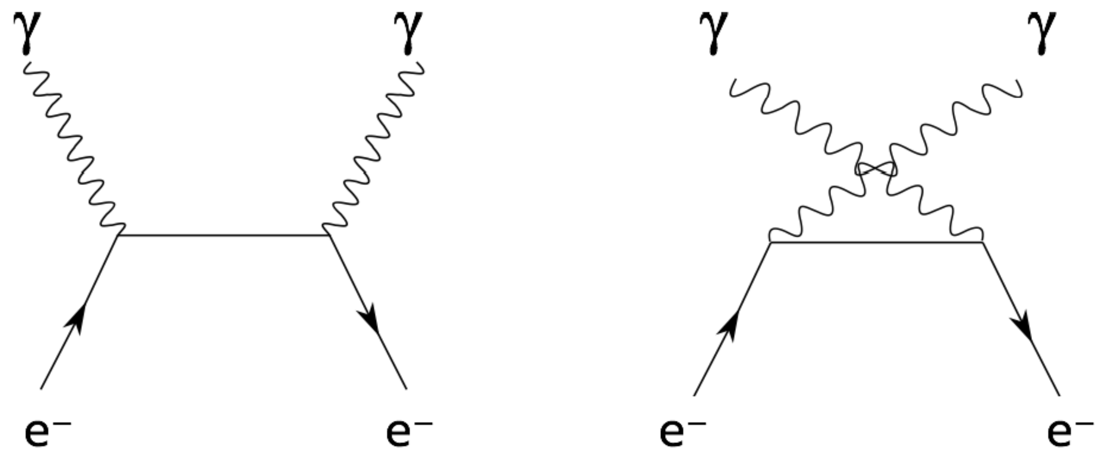


FEP: photoelectric effect and Compton effect when the scattered photon is absorbed in the detector. Intensity depends on detector volume, width depends on detector resolution.

Compton effect: cross section

Cross section: calculation in QED - *1929 O. Klein and Y. Nishina*

Go to page 26



Klein-Nishina diagrams

Thomson cross section ($\gamma e \rightarrow \gamma e$): $$\sigma_{\mathrm{Th}} = \frac{8\pi}{3} r_e^2 = 0.66 \text{ b}$$

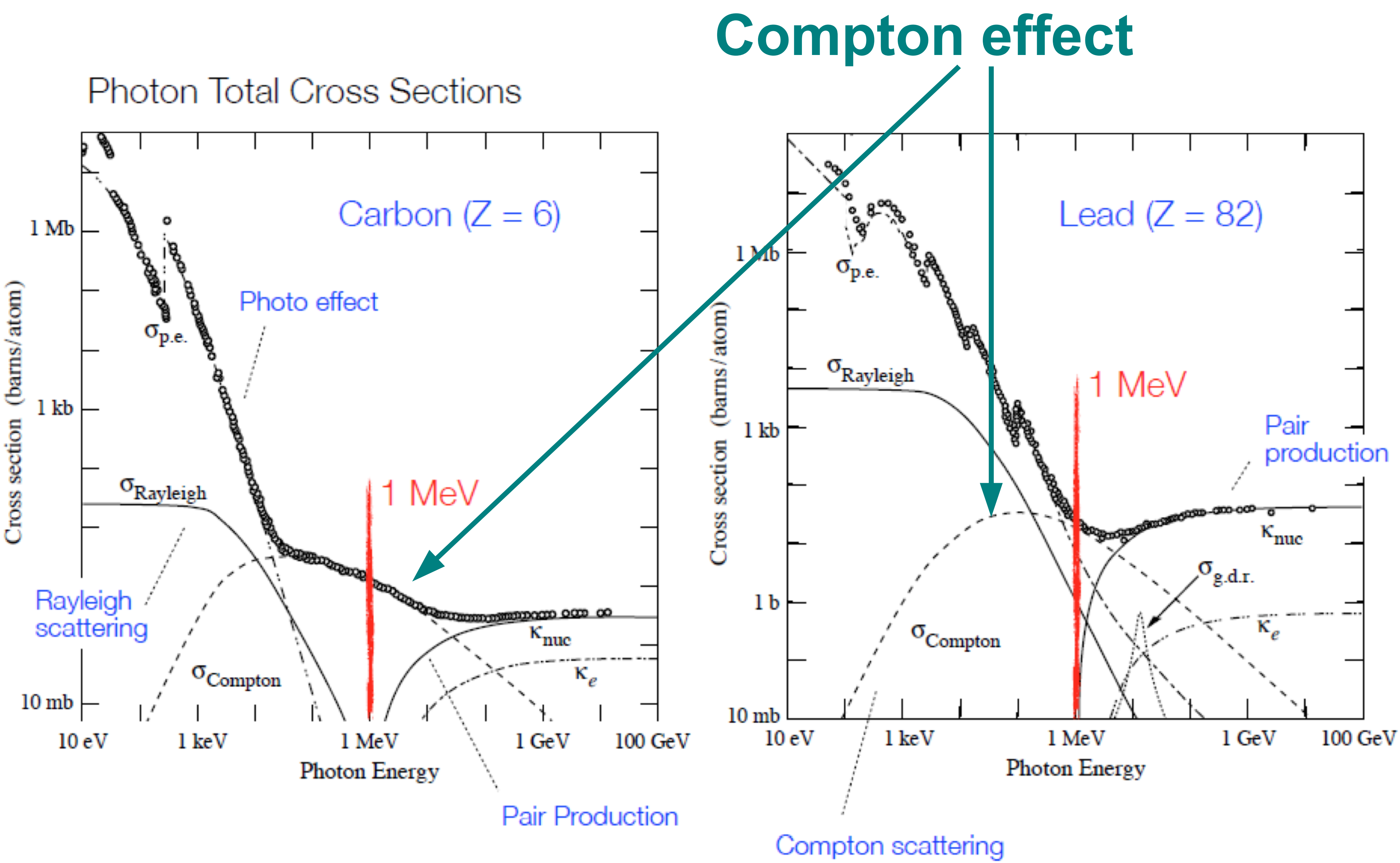
Integrating the differential cross section by Klein and Nishina:

$$E_\gamma \ll m_e c^2 \qquad \sigma_c = \sigma_{\mathrm{Th}} \left(1 - 2\mathcal{E}\right)$$

$$E_\gamma \gg m_e c^2 \qquad \sigma_c = \frac{3}{8} \sigma_{\mathrm{Th}} \frac{1}{\mathcal{E}} \left(\ln 2\mathcal{E} + \frac{1}{2}\right)$$

$$\mathcal{E} = \frac{E_\gamma}{m_e c^2}$$

Photon total cross section

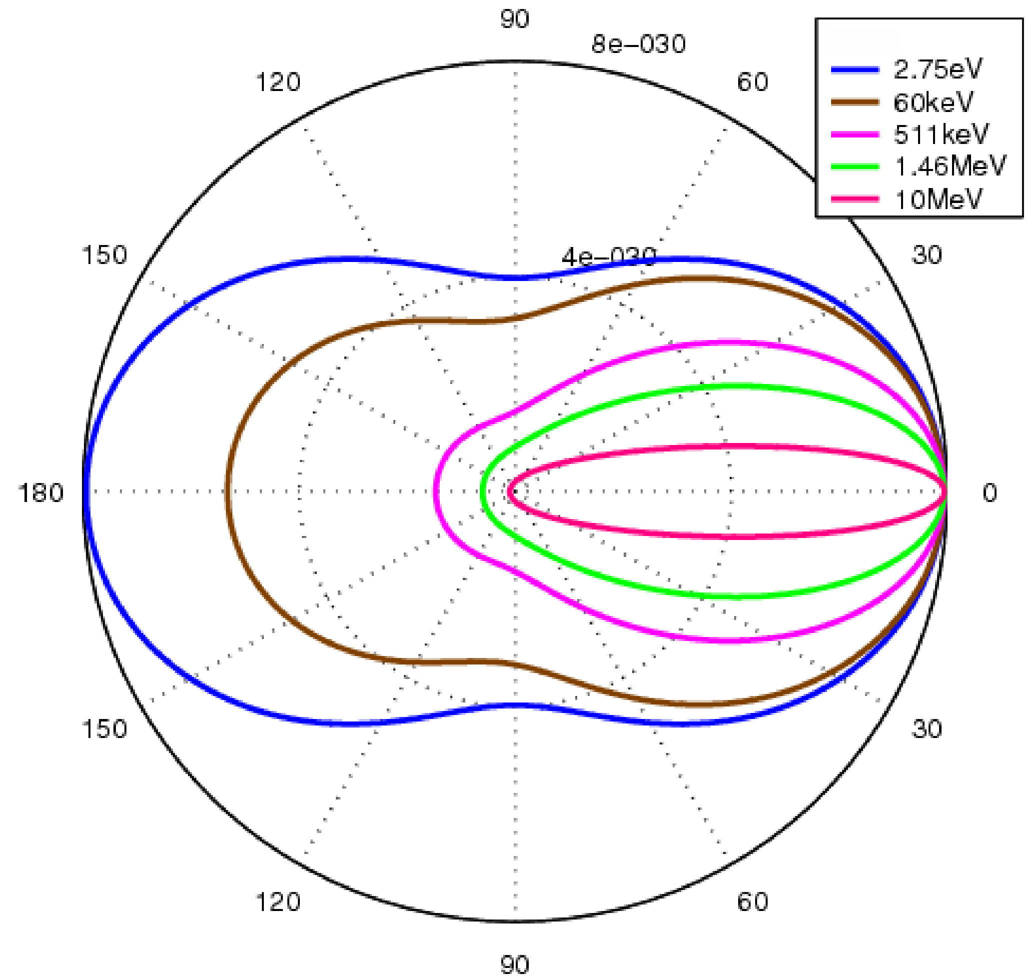


Compton: angular distribution

$$\frac{d\sigma_c}{d\Omega} = \frac{r_e^2}{2} \frac{1}{(1+\mathcal{E}(1-\cos\theta))^2}\left[1+\cos\theta+\frac{\mathcal{E}^2(1-\cos\theta)^2}{1+\mathcal{E}(1-\cos\theta)}\right] \qquad \mathcal{E} = \frac{E_\gamma}{m_e c^2}$$

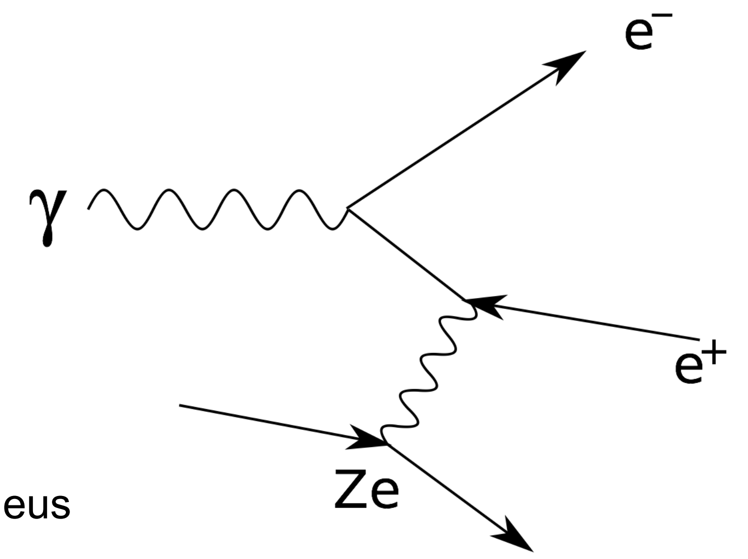
Klein-Nishina differential distribution

Photons with large energies peaked in forward direction



Pair production: Bethe-Heitler process

Interaction in the Coulomb field of the atomic nucleus (not possible in free space)



Energy threshold:

$$E_\gamma \geq 2 m_e c^2 \left(1 + \frac{m_e}{m_n}\right)$$

2x electron mass

kinetic energy transferred to nucleus

Angular distribution: the produced electrons are in a narrow forward cone, with opening angle of $\theta = m_e / E_\gamma$

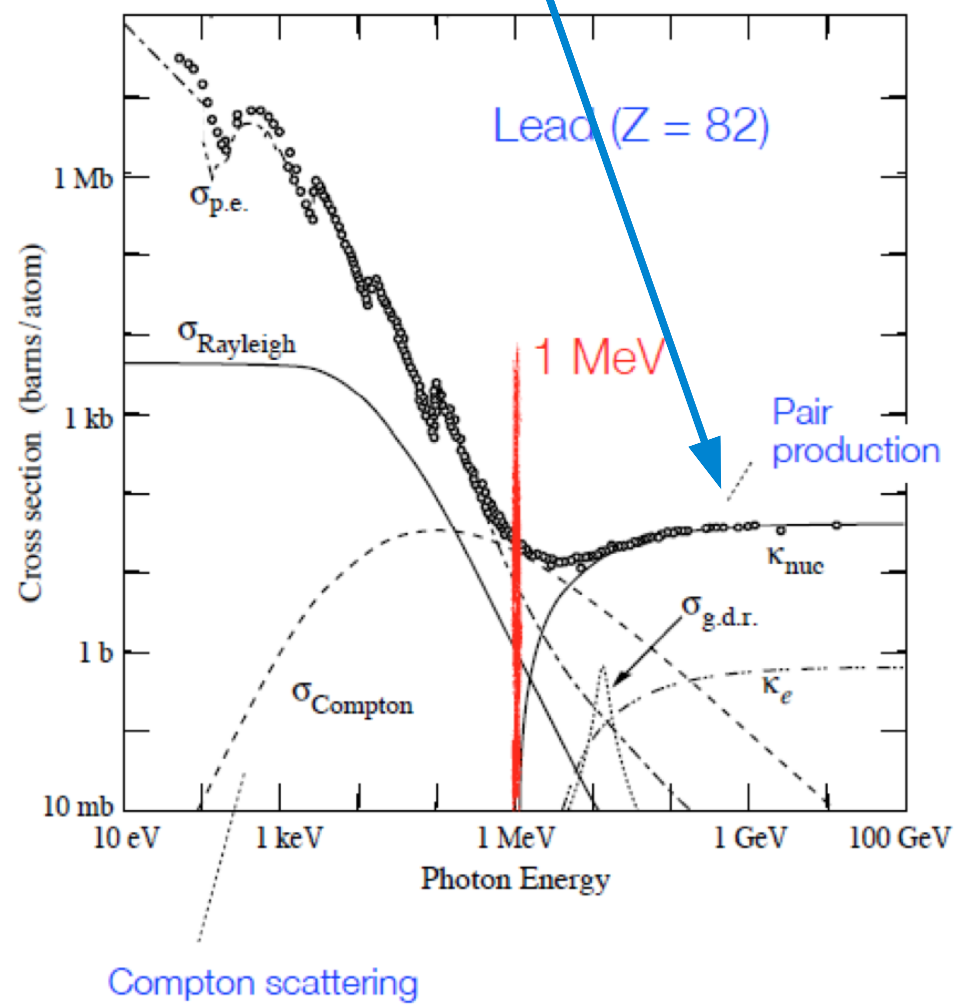
Pair production: Bethe-Heitler process

Cross section: raises above threshold, but eventually saturates at large E_γ because of screening effects of the nuclear charge

$E_\gamma \gg m_e c^2$

$$\sigma_{Pair} = 4 Z^2 \alpha r_e^2 \left(\frac{7}{9} \ln \frac{183}{Z^{1/3}} - \frac{1}{54}\right)$$

$$\approx 4 Z^2 \alpha r_e^2 \left(\frac{7}{9} \ln \frac{183}{Z^{1/3}}\right)$$



Pair production: Bethe-Heitler process

Pair production cross section

$$\sigma_{\text{Pair}} \approx \frac{7}{9} 4 \alpha r_e^2 Z^2 \ln \frac{183}{Z^{1/3}} = \frac{7}{9} \frac{A}{N_A} X_0$$

X_0: radiation length (in cm or g/cm^2)

Absorption coefficient:

(μ = nσ n=particle density)

$$\mu_{\text{Pair}} = \rho \cdot \frac{N_A}{A} \sigma_{\text{Pair}}$$

$$\approx \frac{7}{9} \frac{1}{X_0}$$

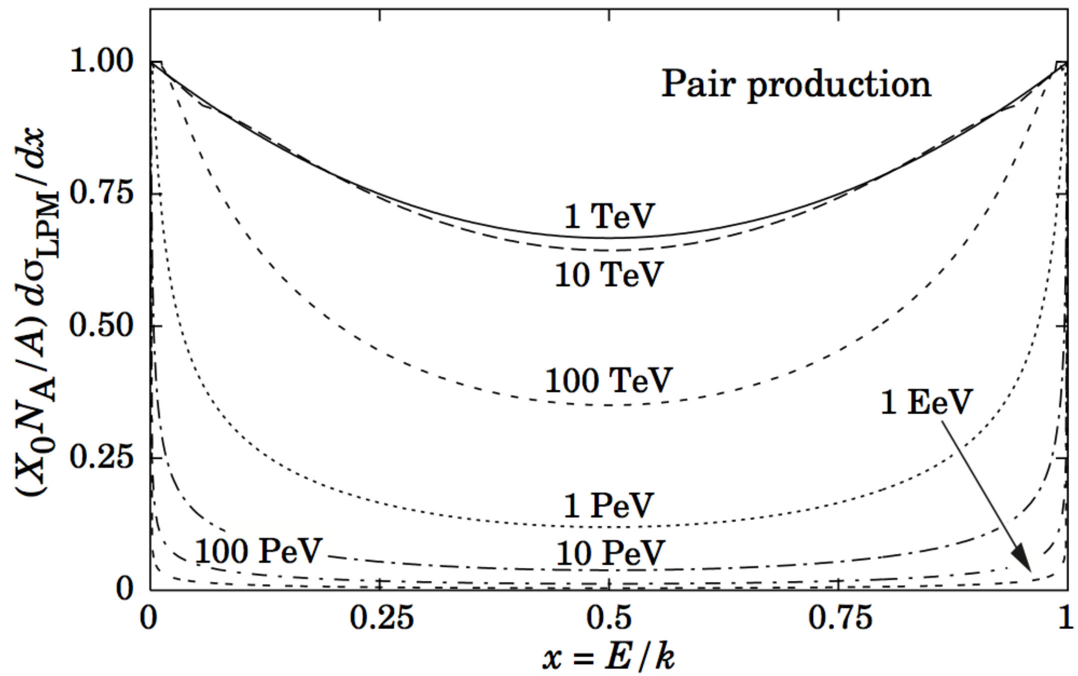
	ρ (g/cm^3)	X_0 (cm)
liq H_2	0.071	865
C	2.27	18.8
Fe	7.87	1.76
Pb	11.35	0.56
air	0.0012	30 420

Pair production: fractional e^-,e^+ energy

k = incident photon energy

x = fractional energy transfer to the pair-produced electron (or positron) = E/k

Normalized pair-production cross section versus the fractional electron energy: necessarily symmetric



At ultra-high energies (TeV) → Landau-Pomeranchuk-Migdal effect: quantum mechanical interference between amplitudes from different scattering centers.

Relevant scale: formation length = length over which highly relativistic electron and photon split apart

Negative interference: reduction of cross section

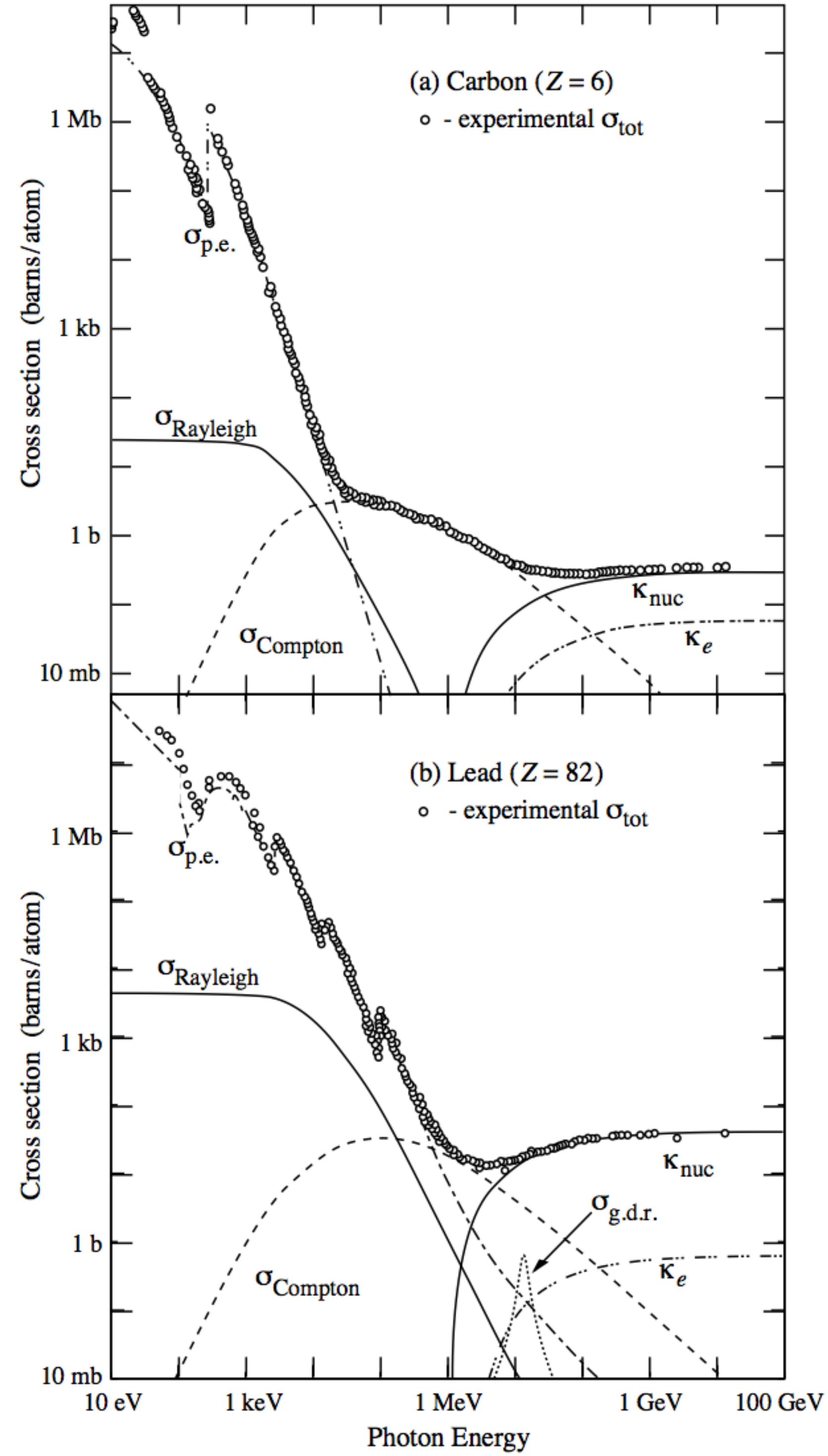
Total photon cross section and absorption length

$$
\begin{aligned}
\sigma_{tot} &= \sigma_{Ph} + \sigma_c + \sigma_p \\
\mu &= \mu_{Ph} + \mu_c + \mu_p \\
\mu_i &= n\sigma_i = \frac{N_A \rho}{A}\sigma_i
\end{aligned}
$$

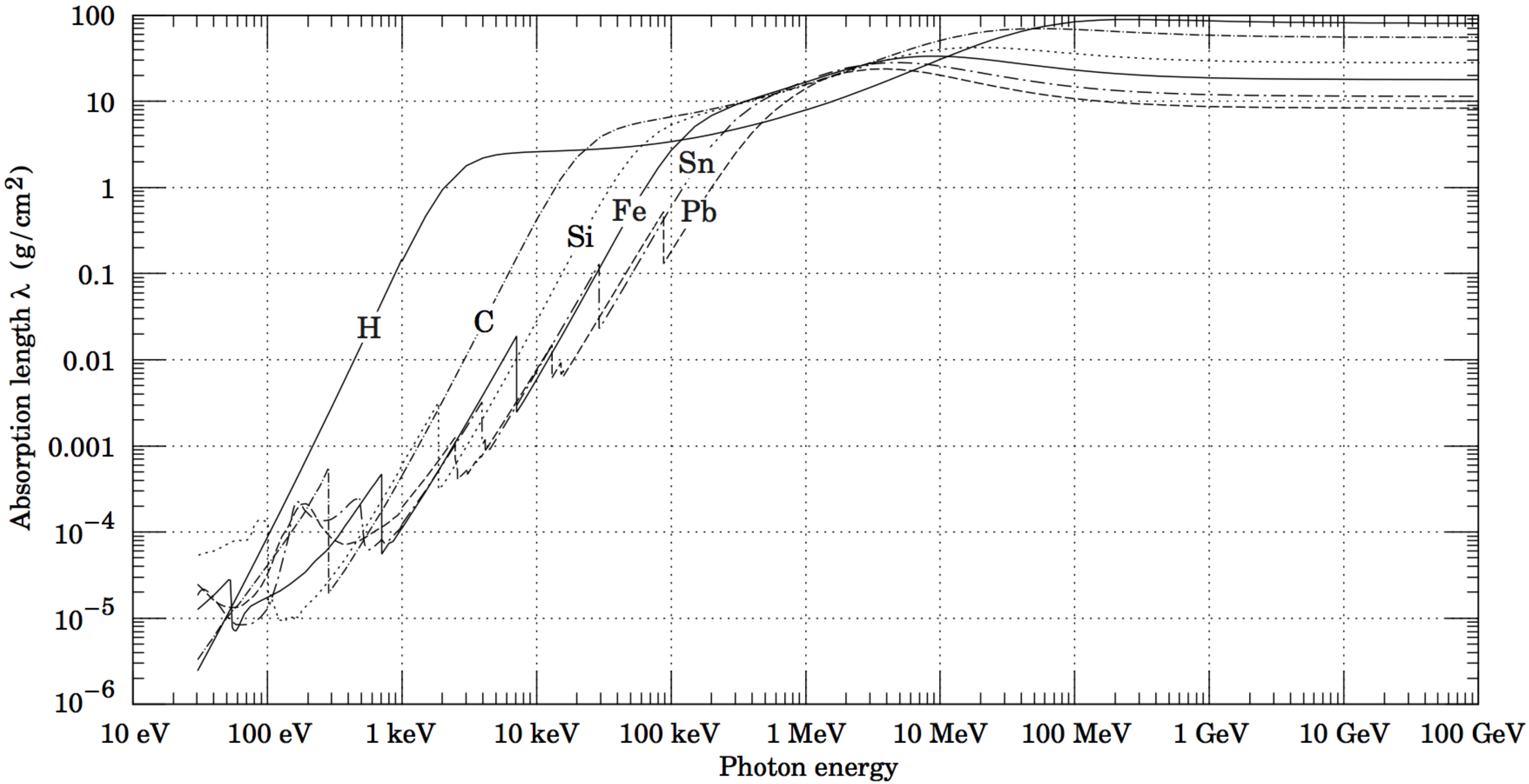
Figure 33.15: Photon total cross sections as a function of energy in carbon and lead, showing the contributions of different processes [51]:

$\sigma_{\text{p.e.}}$ = Atomic photoelectric effect (electron ejection, photon absorption)
σ_{Rayleigh} = Rayleigh (coherent) scattering–atom neither ionized nor excited
σ_{Compton} = Incoherent scattering (Compton scattering off an electron)
κ_{nuc} = Pair production, nuclear field
κ_e = Pair production, electron field
$\sigma_{\text{g.d.r.}}$ = Photonuclear interactions, most notably the Giant Dipole Resonance [52]. In these interactions, the target nucleus is broken up.

Original figures through the courtesy of John H. Hubbell (NIST).



Photon absorption length

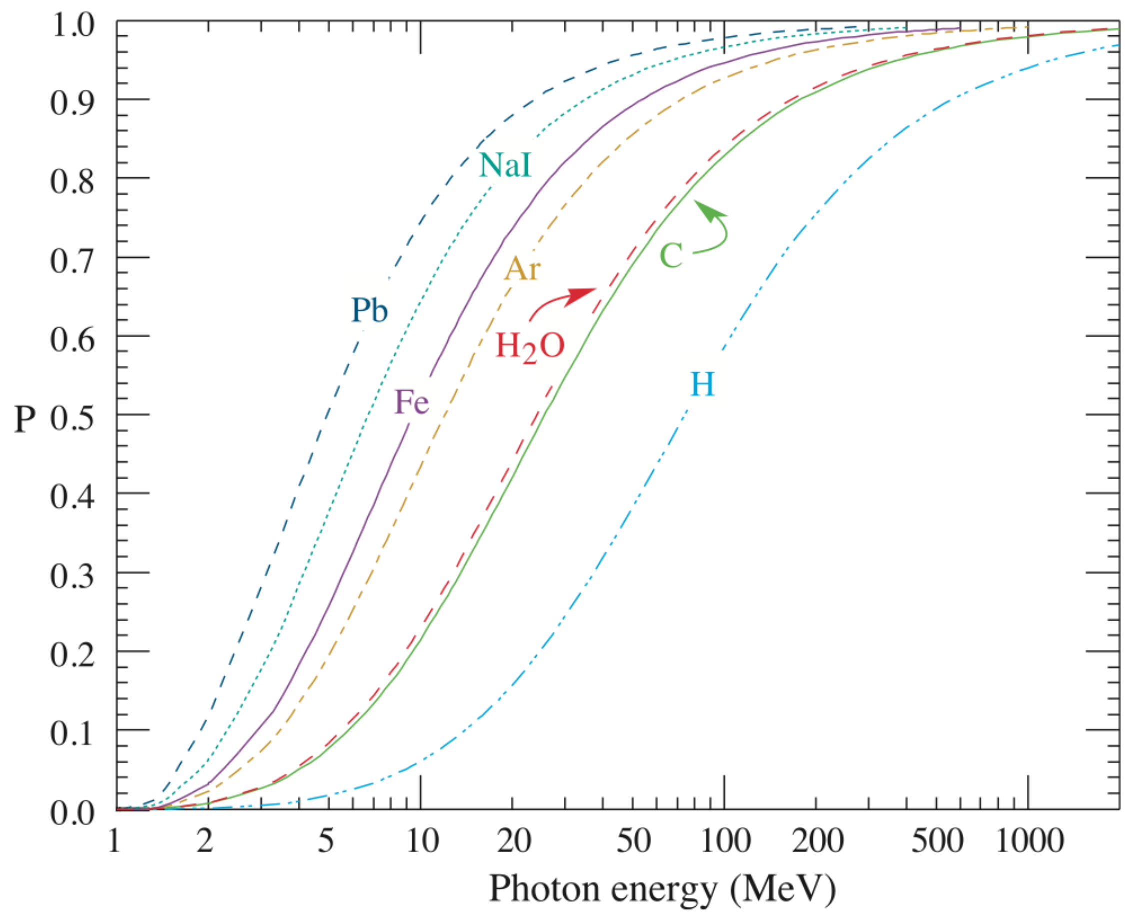


$$I(x) = I_0 \exp(-x/\lambda)$$

1 MeV photon travels about 1 cm in Pb, about 5 cm in C

Contribution by pair production

Probability P that a photon interaction will result in conversion to an e^+e^- pair



For increasing photon energy, pair production becomes dominant:

for Pb beyond 4 MeV

for H beyond 70 MeV

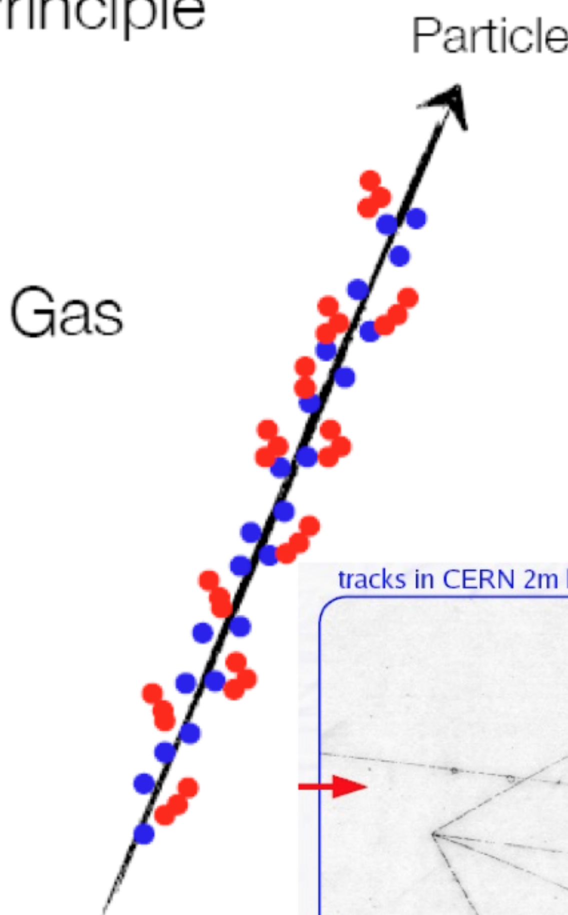
Next lectures

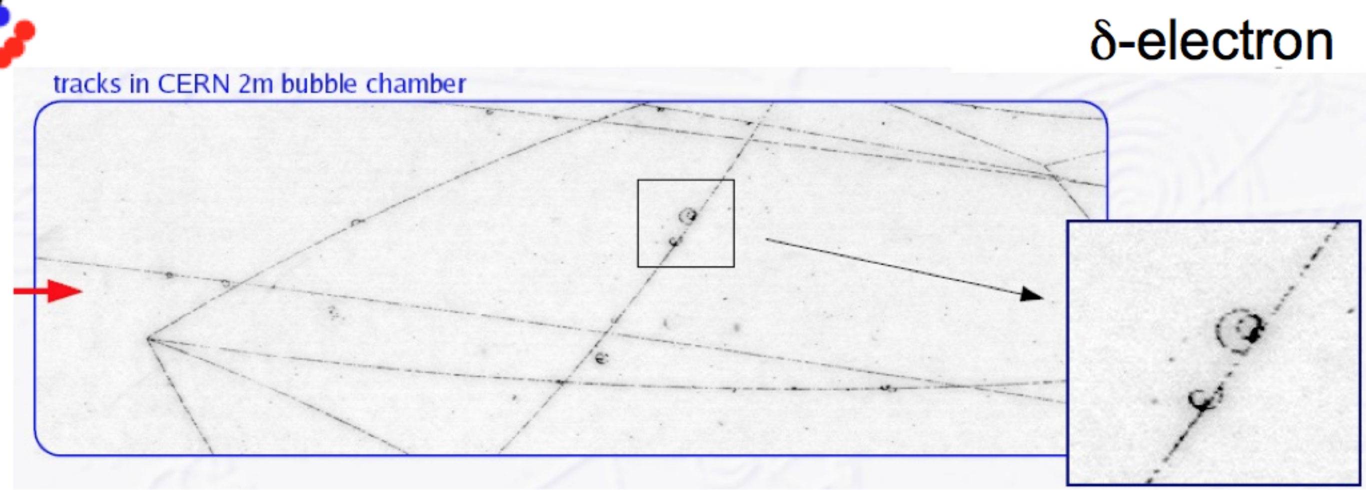
We review the most important type of particle detectors in use in particle and nuclear physics:

- Gas detectors
- Semiconductor detectors
- Scintillators
- Calorimeters

Gas detectors

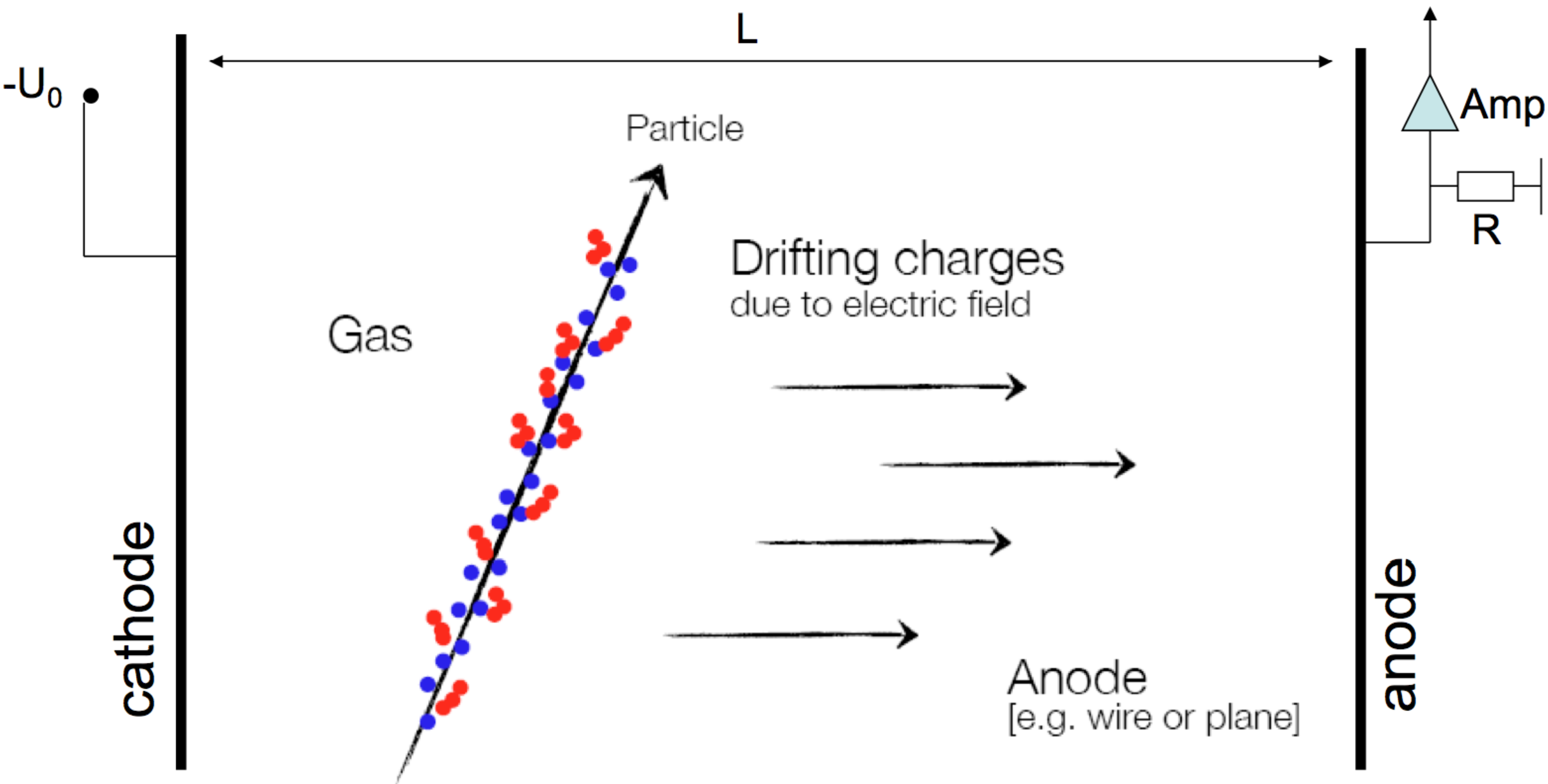
Schematic Principle
of gas detectors





- Primary Ionization
- Secondary Ionization (due to δ-electrons)

Gas detectors



- Primary Ionization
- Secondary Ionization (due to δ-electrons)

Gas detectors

Modes of operation
depending on the strength of the electric field applied

W. Price, “Nuclear Radiation Detection”, 1958

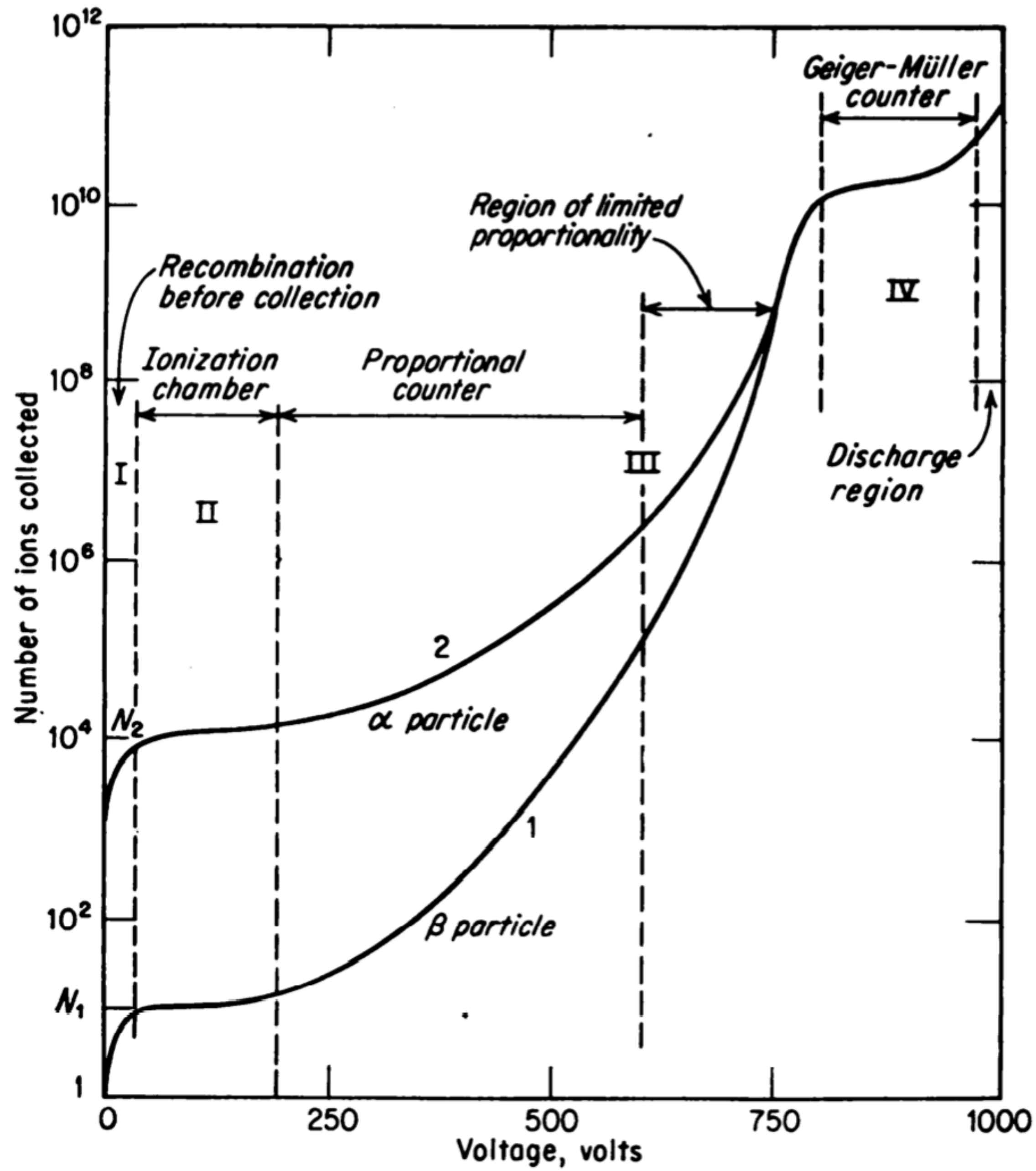


FIG. 2–2. Pulse-height versus applied-voltage curves to illustrate ionization, proportional, and Geiger-Müller regions of operation.