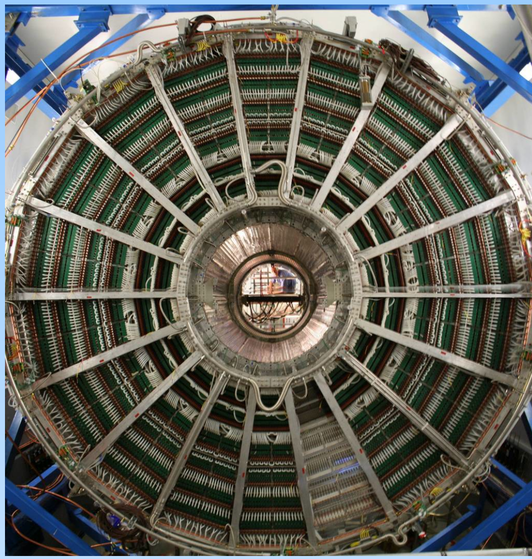

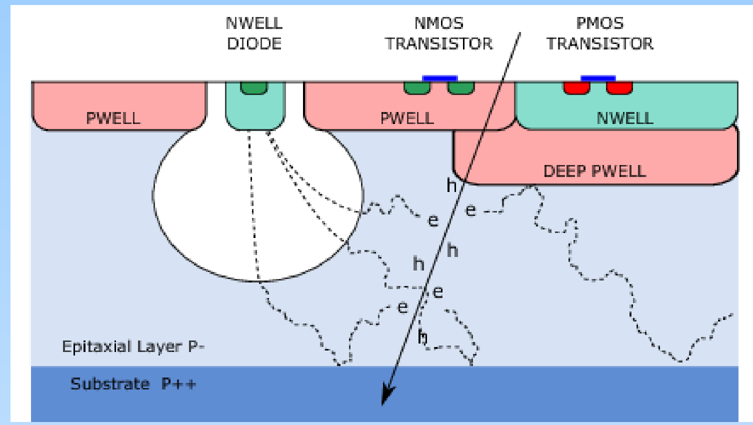

Interaction of particles with matter - 1

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GSI and University of Heidelberg

SS2017, Heidelberg
April 27, 2017

Particle detection

Absolute basic principles:

- Particle must INTERACT with the material of the detector

Examples !

- It has to transfer energy / momentum in some way

- Knowing the interaction of the particle with the detector material in detail allows us to deduce extended, precise and quantitative information about the particle properties

Particle detection happens via the energy the particle deposits in the material it traverses

Interactions with matter

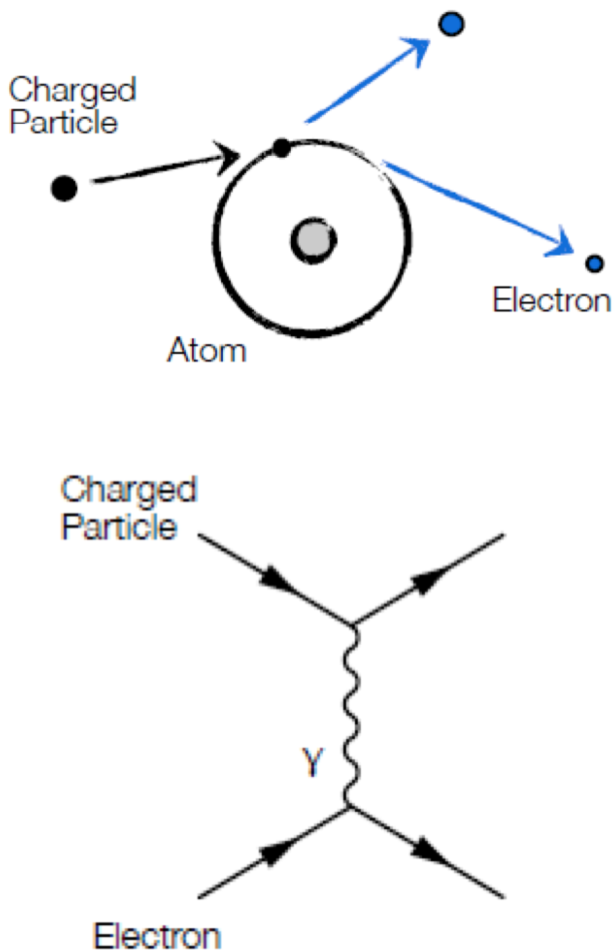
Mechanisms through which a particle interacts with the material it traverses, in a detector:

- **Charged particles:**
  - Ionization
  - Excitation
  - Bremsstrahlung
  - Cherenkov radiation
  - Transition radiation
- **Photons:**
  - Photo effect
  - Compton effect
  - Pair production
- **Neutrinos**: weak interaction
- **Hadrons:** nuclear interactions

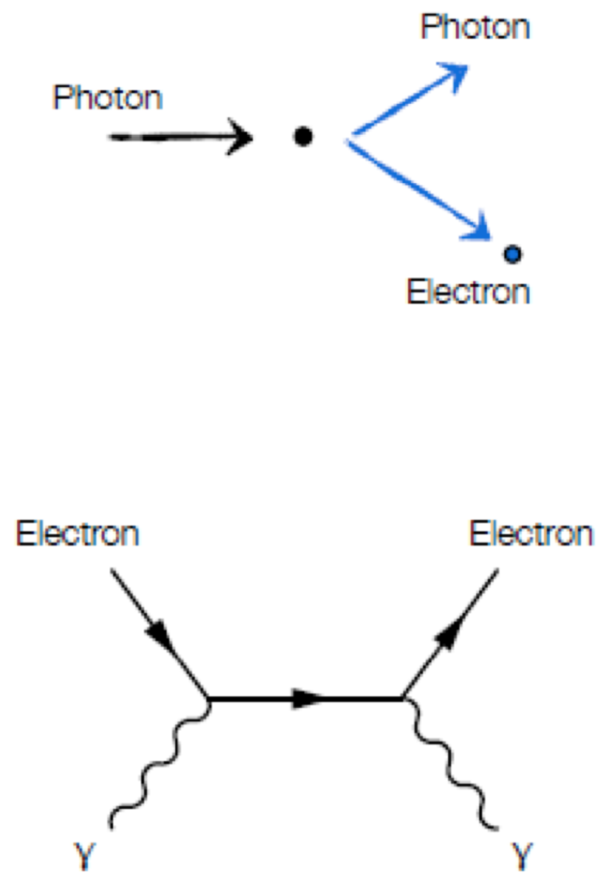
Distinguish between energy loss via multiple interactions and total energy loss in a single interaction (e.g. pair production)

Interactions with matter - examples

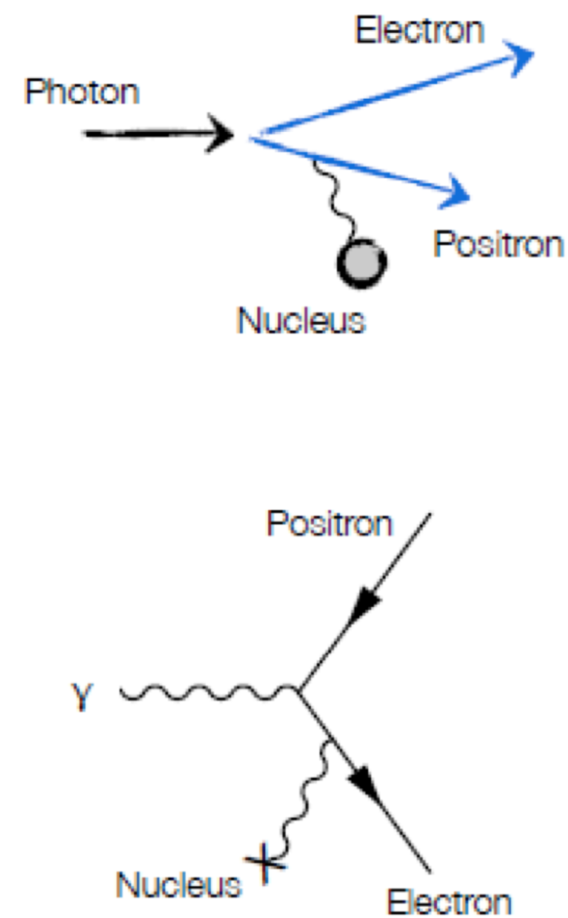
Ionization



Compton scattering



Pair production



Lectures

- Energy loss by ionization (by “heavy” particles)
- Interaction of electrons with matter:
  - Energy loss by ionization
  - Bremsstrahlung

TODAY

- Interaction of photons
- Cherenkov effect
- Transition radiation

NEXT WEEK

Interaction of charged particles

Charged particle X, with $Mc^2 \gg m_e c^2$ (electrons are discussed later)

Dominant: Coulomb interaction between the particle X and the atom →
2 electromagnetic processes:

1) elastic scattering from nuclei

$$\text{atom} + X \rightarrow \text{atom}^* + X \qquad \text{excitation}$$

$$\hookrightarrow \text{atom} + \gamma \qquad \text{de-excitation}$$

2) inelastic collisions with the atomic electrons of the material

$$\text{atom} + X \rightarrow \text{atom}^+ + e^- + X \qquad \text{ionization}$$

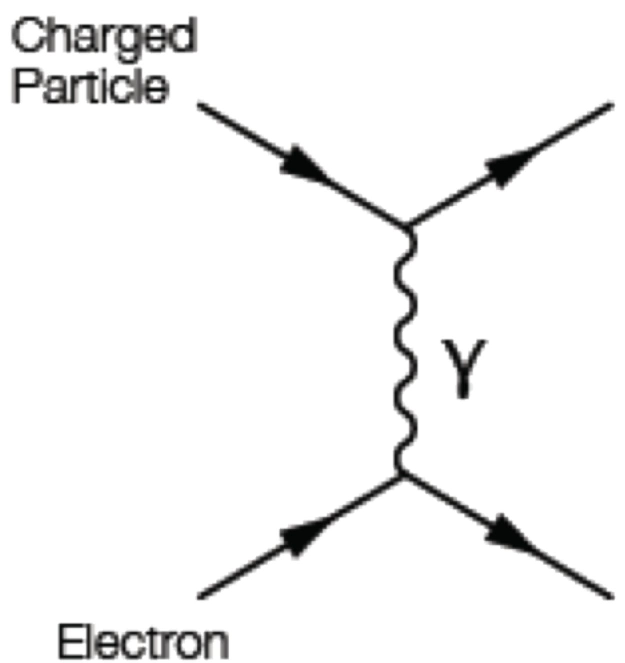
Energy loss by ionization dE/dx

- Charged particle: ze
- "heavy" particle: $Mc^2 \gg m_e c^2$ (electrons are discussed later)
- Energy high enough to "resolve" the inside of the atom: from the uncertainty principle

$$\lambda = \hbar / p \qquad \text{e.g. } 1\ \text{GeV/c} \rightarrow 1\ \text{fm}$$

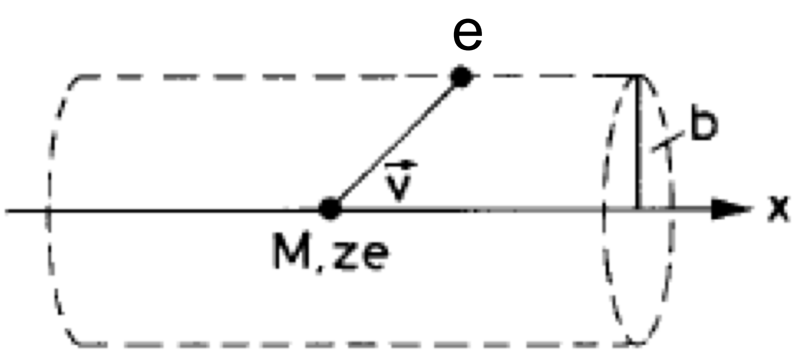
Interaction is dominated by elastic collisions with electrons:

- Classical derivation by N. Bohr (1913)
- Quantum mechanical derivation by H. Bethe (1930) and F. Bloch (1933)



dE/dx: classical derivation by Bohr (1913)

- Particle with charge ze moves with velocity v through a medium with electron density n
- Electrons are considered free and initially at rest



- Momentum transfer to a single electron, transverse distance b

because of symmetry

$$\Delta p_\perp = \int F_\perp dt = \int F_\perp \frac{dt}{dx} dx = \int F_\perp \frac{dx}{v}$$

$$\Delta p_\parallel \ : \ \text{averages to zero}$$

$$= \int_{-\infty}^{\infty} \frac{ze^2}{(x^2+b^2)} \cdot \frac{b}{\sqrt{x^2+b^2}} \cdot \frac{1}{v} \, dx = \frac{ze^2 b}{v} \left[\frac{x}{b^2\sqrt{x^2+b^2}} \right]_{-\infty}^{\infty} = \frac{2ze^2}{bv}$$

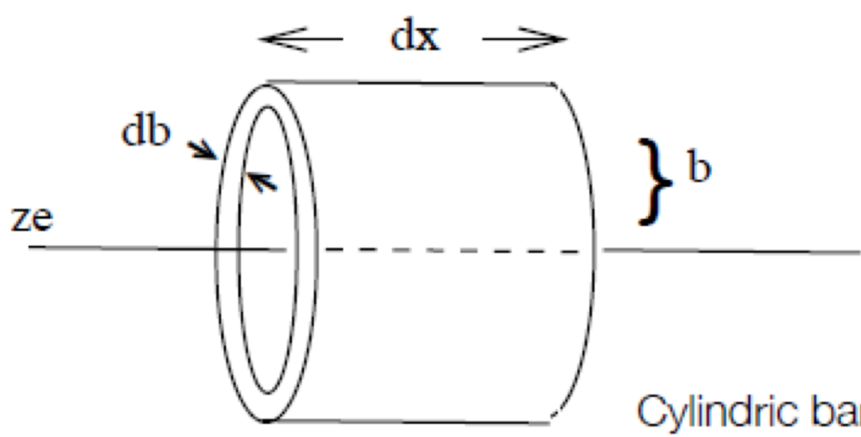
- Energy transfer to a single electron, at transverse distance b

$$\Delta E(b) = \frac{\Delta p^2}{2m_e}$$

dE/dx: classical derivation by Bohr (1913)

- To integrate over electrons present in the medium, consider a cylindrical barrel with N_e electrons:

  $N_e = n\,(2\pi b)\,db\,dx$



Cylindric barrel with N_e electrons

- Energy loss per path length dx for distance between b and b+db in medium with electron density n:

$$-dE(b) = \frac{\Delta p^2}{2m_e} \cdot 2\pi n b\, db\, dx = \frac{4z^2e^4}{2b^2v^2m_e} \cdot 2\pi n b\, db\, dx = \frac{4\pi\, n\, z^2 e^4}{m_e v^2}\frac{db}{b}dx$$

- Diverges for b → 0. Integrate over relevant range $b_{min} - b_{max}$:

$$-\frac{dE}{dx} = \frac{4\pi\, n\, z^2 e^4}{m_e v^2} \cdot \int_{b_{\min}}^{b_{\max}} \frac{db}{b} = \frac{4\pi\, n\, z^2 e^4}{m_e v^2} \ln \frac{b_{\max}}{b_{\min}}$$

dE/dx: classical derivation by Bohr (1913)

Determine the relevant range:

- **b_{min}** > λ: uncertainty principle → impact parameters below the electron de Broglie wavelength are not relevant:

$$b_{\rm min} = \lambda_{\rm e} = \frac{h}{p} = \frac{2\pi\hbar}{\gamma m_{\rm e} v}$$

- **b_{max}**: interaction time must be shorter than "period" of the electron to guarantee relevant energy transfer:

$$b_{\rm max} = \frac{\gamma v}{\langle \nu_{\rm e} \rangle}\,; \quad \left[\, \gamma = \frac{1}{\sqrt{1-\beta^2}} \,\right]$$

- After integration over b:

$$\boxed{-\frac{dE}{dx} = \frac{4\pi z^2 e^4}{m_{\rm e}\, c^2 \beta^2}\, n \cdot \ln \frac{m_{\rm e}\, c^2 \beta^2 \gamma^2}{2\pi\hbar \,\langle \nu_{\rm e} \rangle}}$$

Bethe-Bloch equation

Considering quantum mechanical effects:

$$-\left\langle \frac{dE}{dx} \right\rangle = K z^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \ln \frac{2 m_e c^2 \beta^2 \gamma^2 T_{\max}}{I^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2} \right] \quad [\cdot \rho]$$

density

$K = 4\pi N_A r_e^2 m_e c^2 = 0.307$ MeV g^{-1} cm^2

$T_{max} = 2 m_e c^2 \beta^2 \gamma^2 / (1 + 2\gamma m_e/M + (m_e/M)^2)$

[Max. energy transfer in single collision]

z : Charge of incident particle
M : Mass of incident particle

Z : Charge number of medium
A : Atomic mass of medium

I : Mean excitation energy of medium
δ : Density correction [transv. extension of electric field]

$N_A = 6.022 \cdot 10^{23}$
[Avogardo's number]

$r_e = e^2/4\pi\varepsilon_0 m_e c^2 = 2.8$ fm
[Classical electron radius]

$m_e = 511$ keV
[Electron mass]

$\beta = v/c$
[Velocity]

$\gamma = (1-\beta^2)^{-2}$
[Lorentz factor]

Validity:

$.05 < \beta\gamma < 500$
$M > m_\mu$

Bethe-Bloch equation

Considering quantum mechanical effects:

$$-\left\langle \frac{dE}{dx} \right\rangle = K z^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \ln \frac{2 m_e c^2 \beta^2 \gamma^2 T_{\max}}{I^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2} \right] \quad [\cdot \rho]$$

density

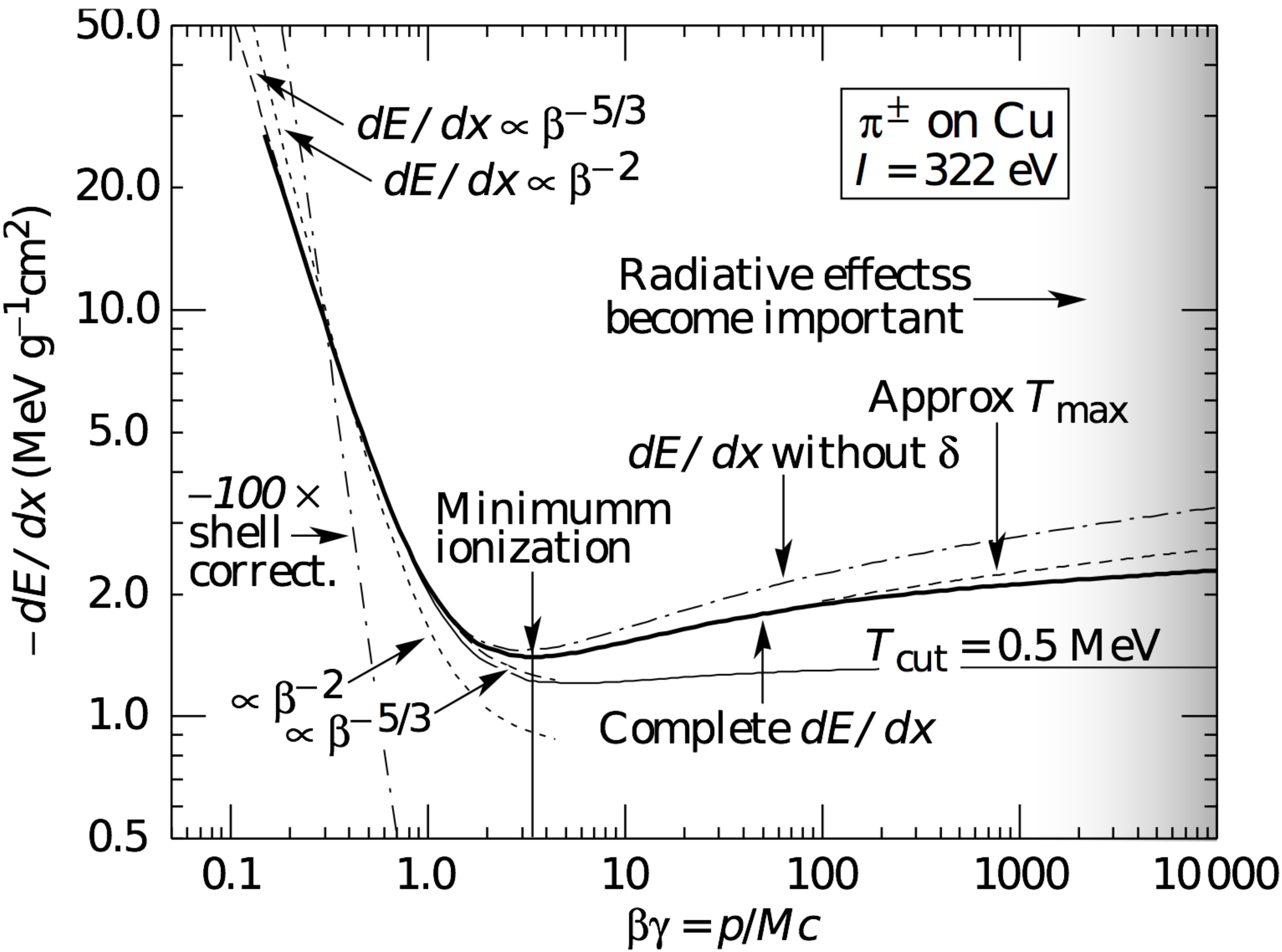
I = $\hbar$ <v> = effective ionization potential

Or mean excitation energy of the medium

<v> = average revolution frequency of electron

$T_{max} \approx 2\, m_e c^2 \beta^2 \gamma^2$ maximum energy transfer in a single collision, for $M \gg m_e$

dE/dx of pions in copper

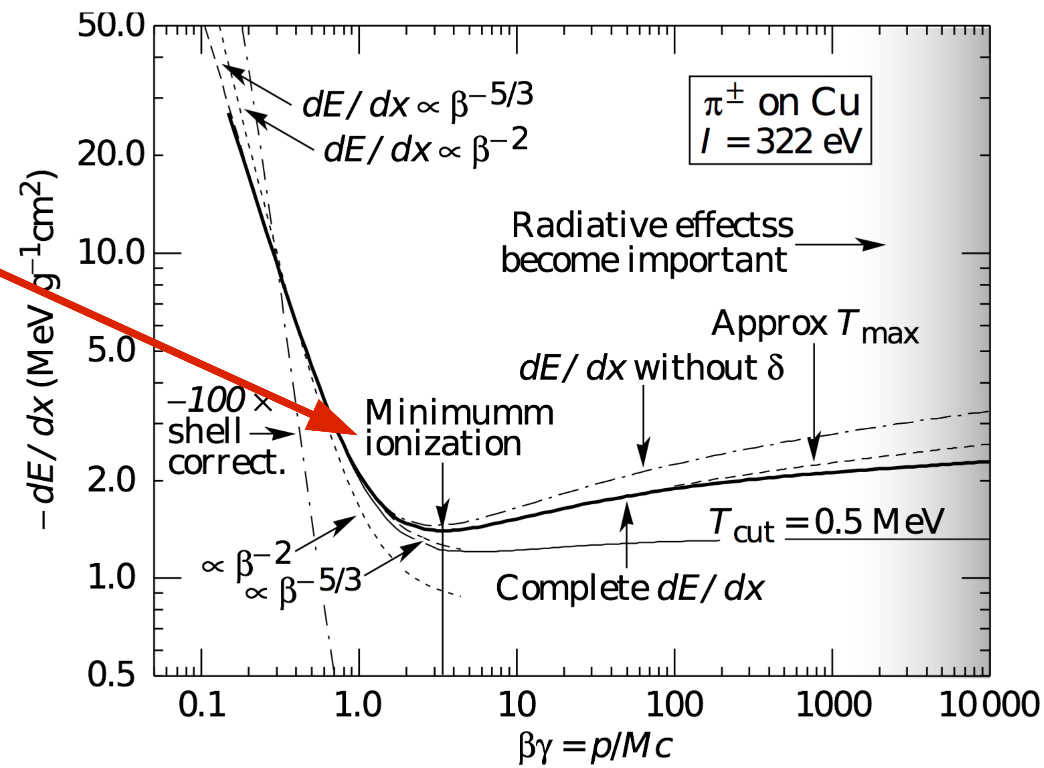


Bethe-Bloch: main features

- Minimum ionization:
  MIP = minimum ionizing particles
  for $\beta\gamma \approx 3\text{-}4$

- Units: MeV g^{-1} cm^2
  dE/dx_{min} ~ 1-2 MeV g^{-1} cm^2

- Density of copper: ρ=9.94 g/cm^2
  → MIP looses ~ 13 MeV/cm



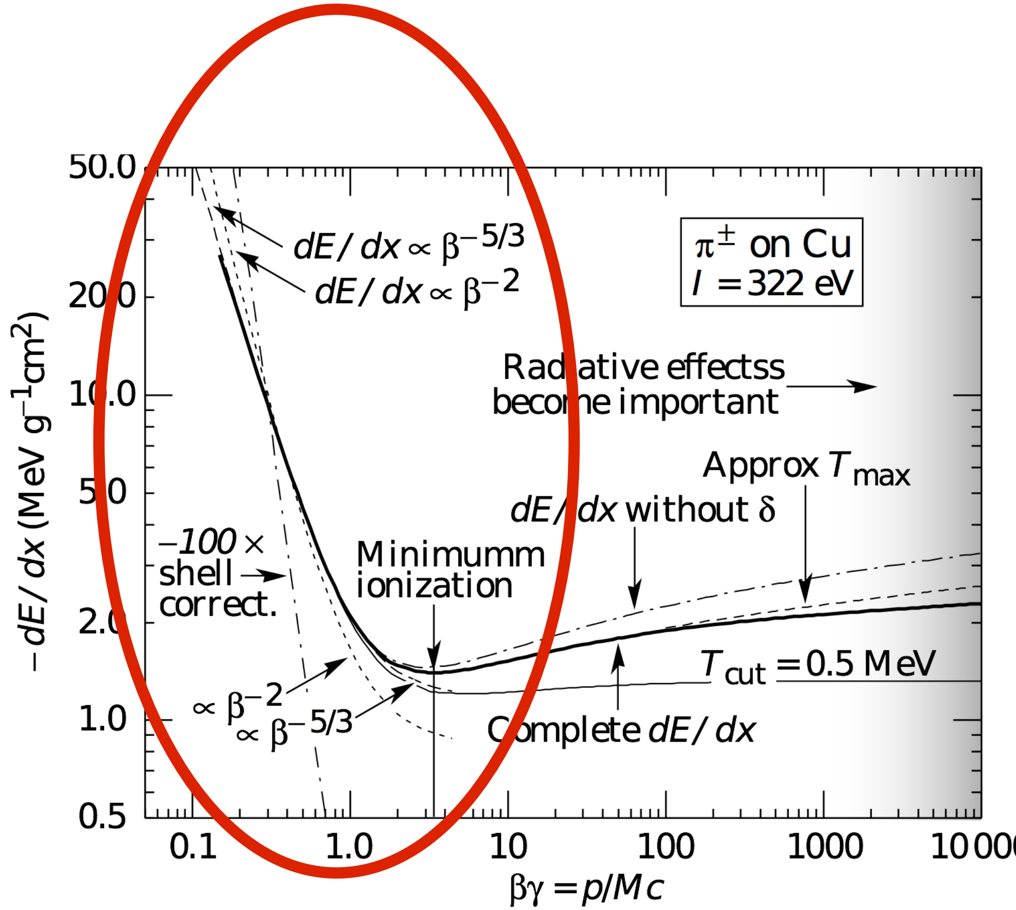
Bethe-Bloch: main features

Small βγ

- quick fall of dE/dx as β^{-2} (Bohr): kinematic factor from

$$\Delta p_\perp = \int F_\perp dt = \int F_\perp \frac{dx}{v}$$

- Precisely it is $\beta^{-5/3}$: slower particles experience the electric field for a longer time → stronger energy loss!



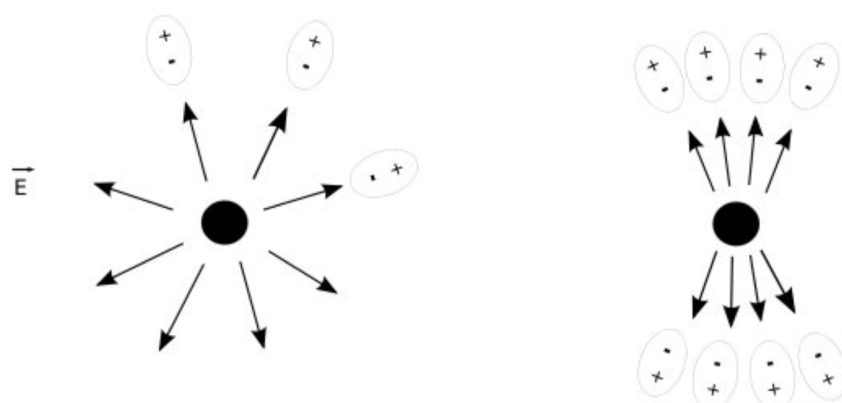
- **Shell corrections**: particle velocity can get close to the electron orbital velocity ($\beta c \sim v_e$):
  - Assumption of electron to be at rest is no longer valid
  - Capture processes become possible

Bethe-Bloch: main features

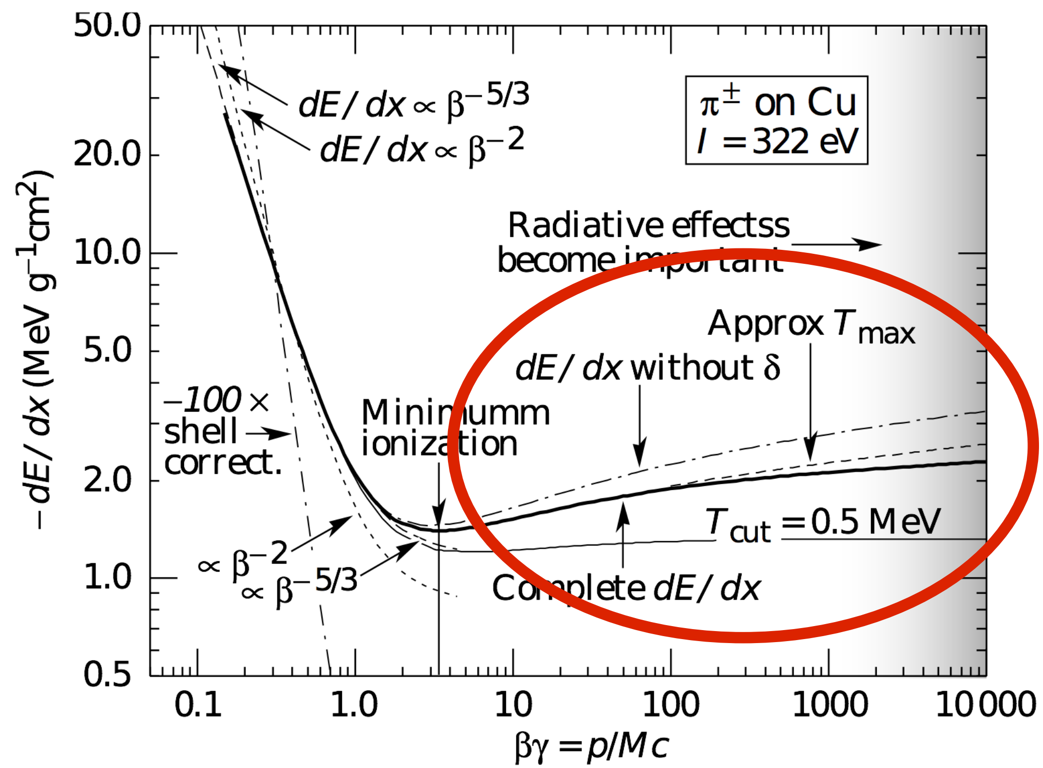
Large βγ

- Relativistic rise ~ ln $\beta^2\gamma^2$

  The transverse electric field increases due to Lorentz transformation → increase of contribution from larger b



left: for small γ, right: for large γ



- Density correction must be considered: Fermi plateau →

Bethe-Bloch: main features

Large βγ: density correction $-\frac{\delta}{2}$

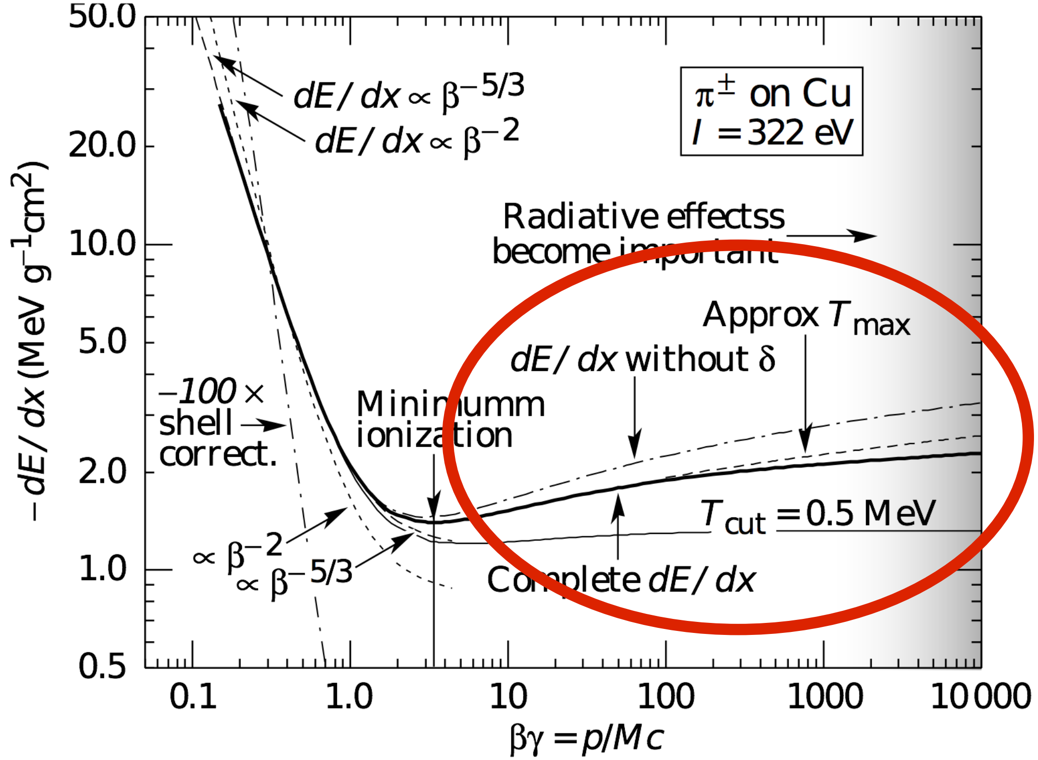
- Real media are polarized → effective shielding of electric field far from particle path → effectively reduces the long range contribution to relativistic rise
- High energy limit:

$$\frac{\delta}{2} \to \ln \frac{\hbar\omega_p}{I} + \ln \beta\gamma - \frac{1}{2}$$

with the plasma energy:

$$\hbar\omega_p = \sqrt{4\pi n r_e^3} m_e \frac{c^2}{\alpha}$$

effectively dE/dx grows like ln(βγ)



- Plasma energy ∝ √n → correction much larger for liquids and solids! Logarithmic rise ~20% in liquids and solids, ~50% in gases

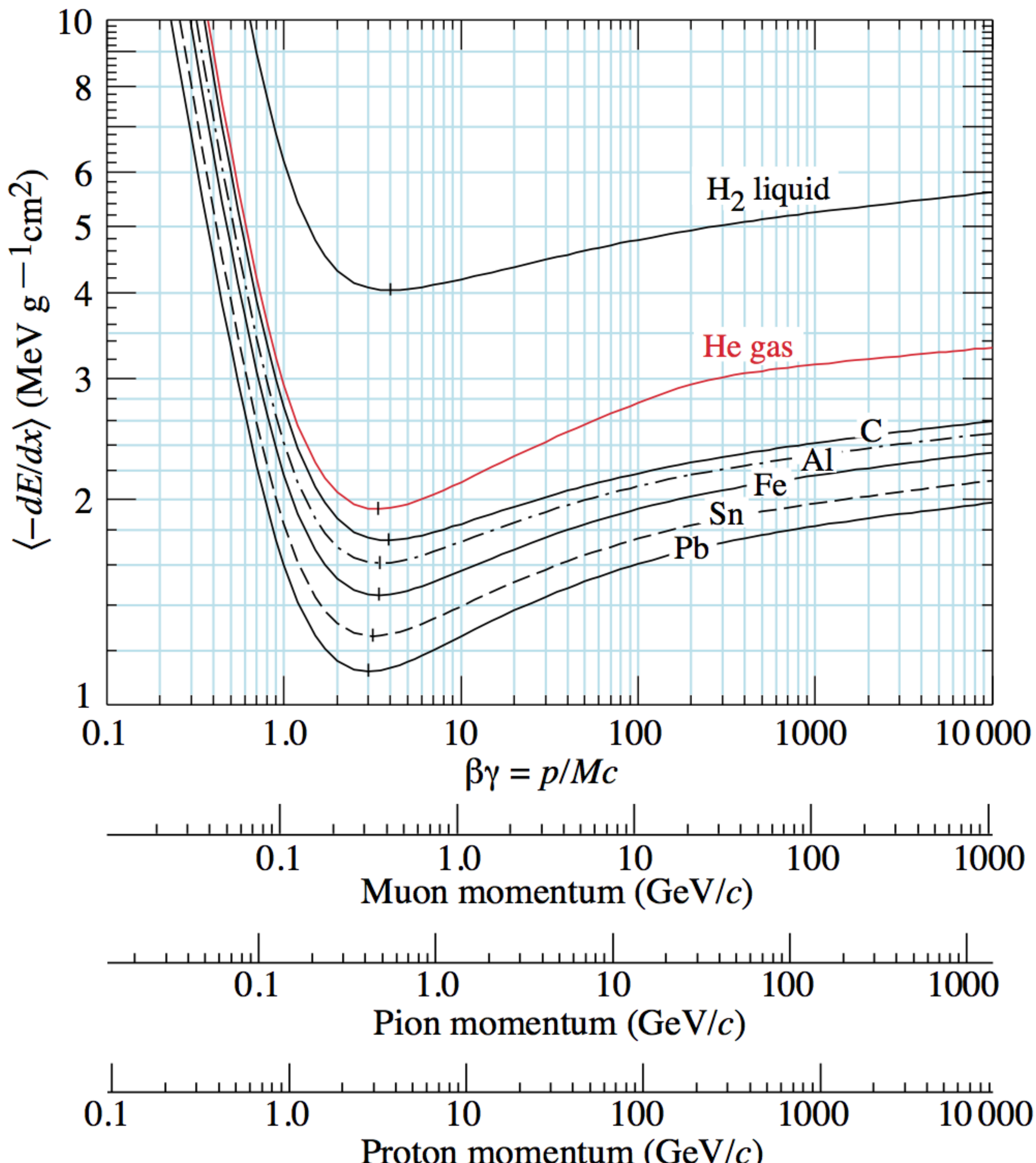
dE/dx

Particle Data Group:
pdg.lbl.gov/2016/reviews/rpp2016-rev-passage-particles-matter.pdf

Different detector materials

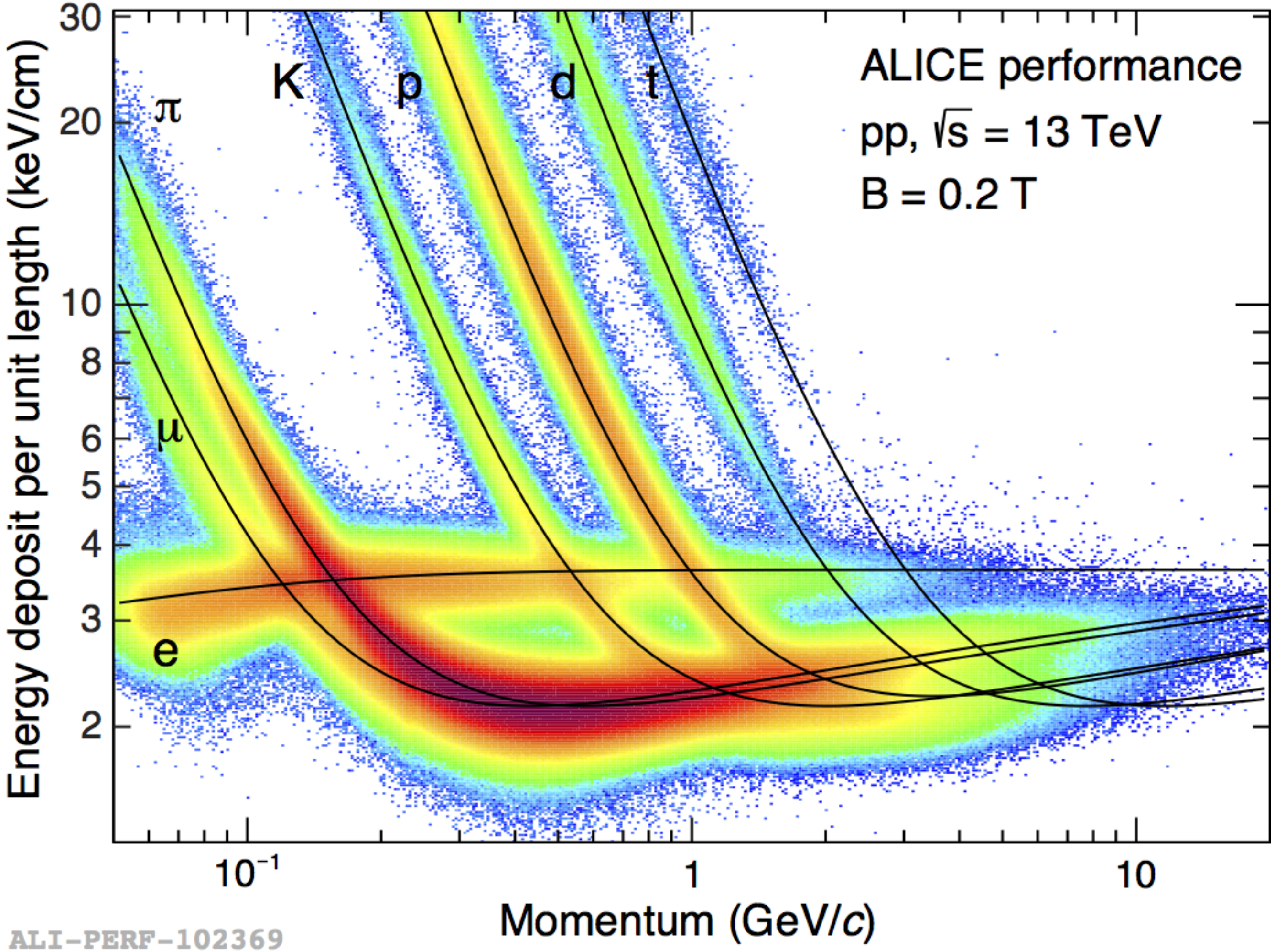
$$\frac{dE}{dx} \approx \frac{Z}{A}$$

(remember density!)

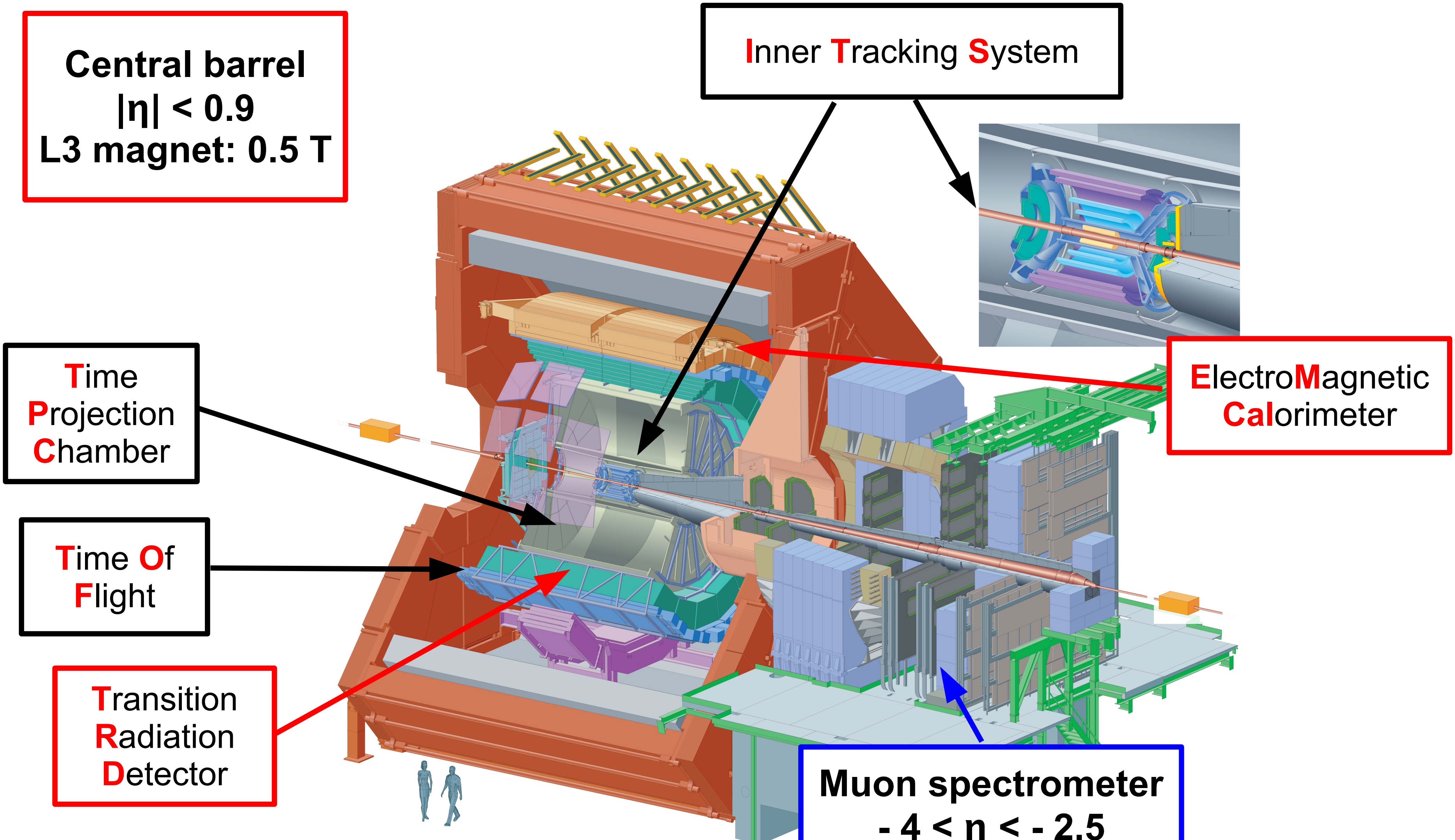


dE/dx depends on
$\beta\gamma = p/(Mc)$
→ at a given p, dE/dx is different for particles with different mass M

dE/dx used in practice
the ALICE Time Projection Chamber



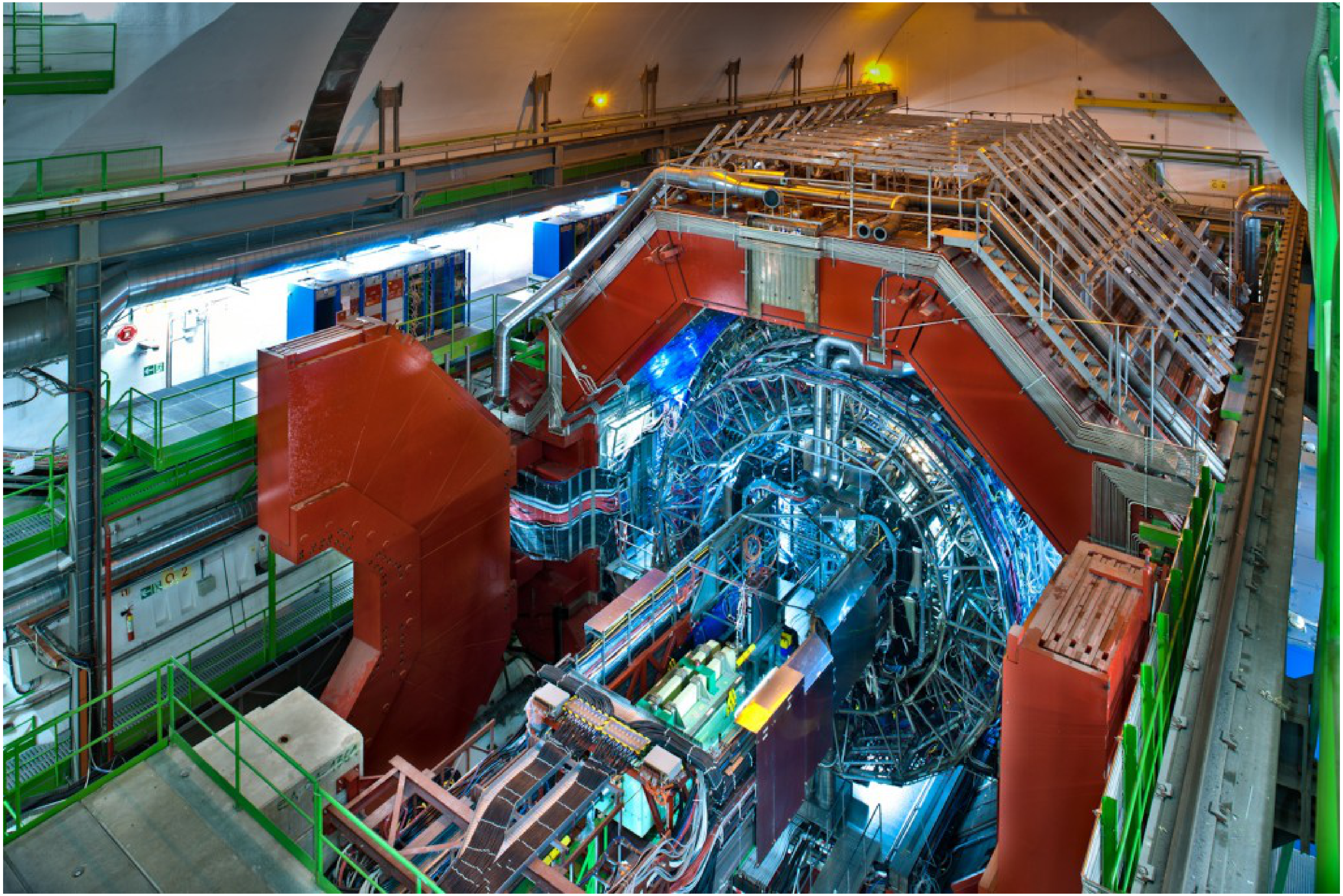
ALICE: A Large Ion Collider Experiment



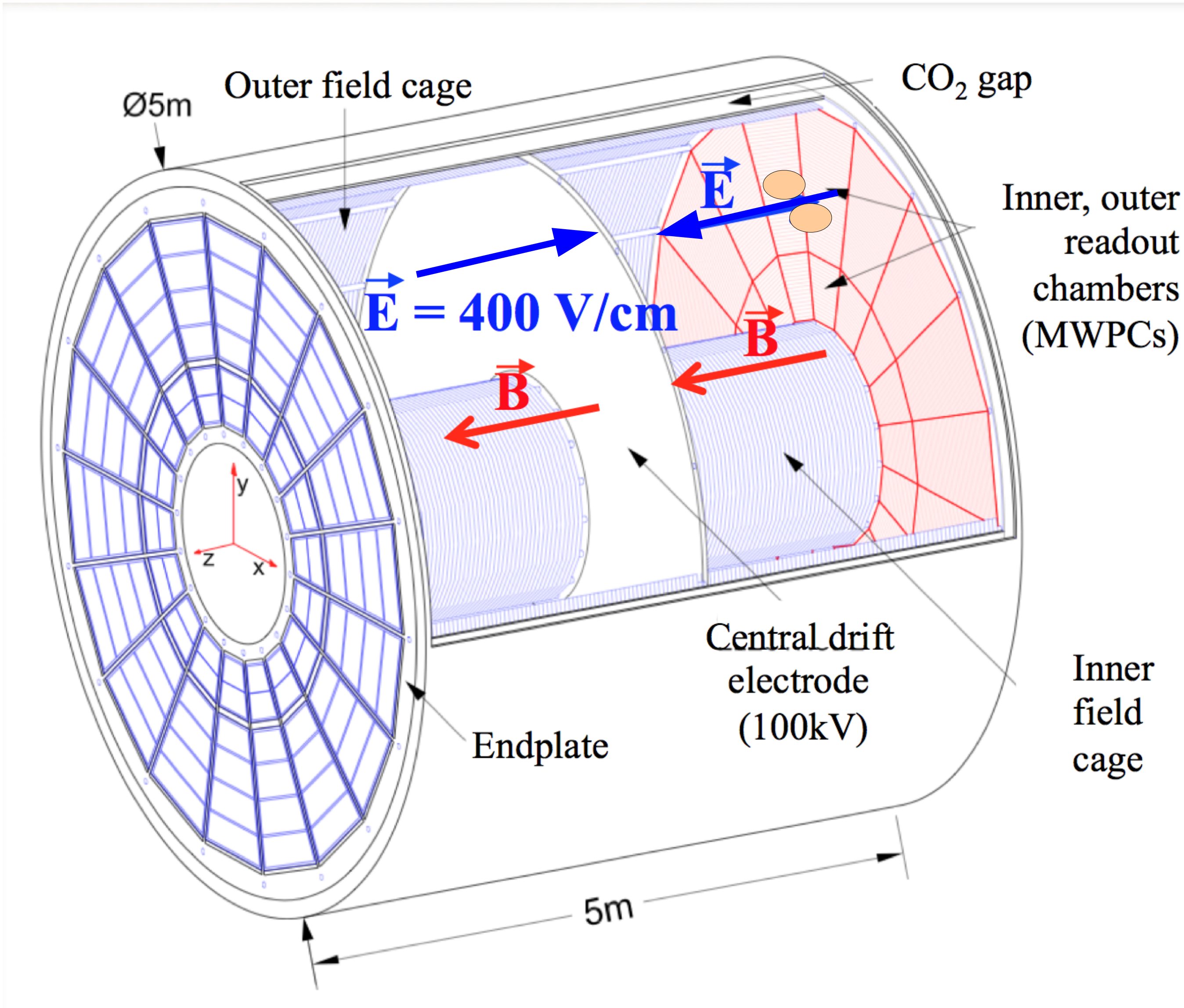
ALICE TPC

ALICE TPC

ALICE TPC

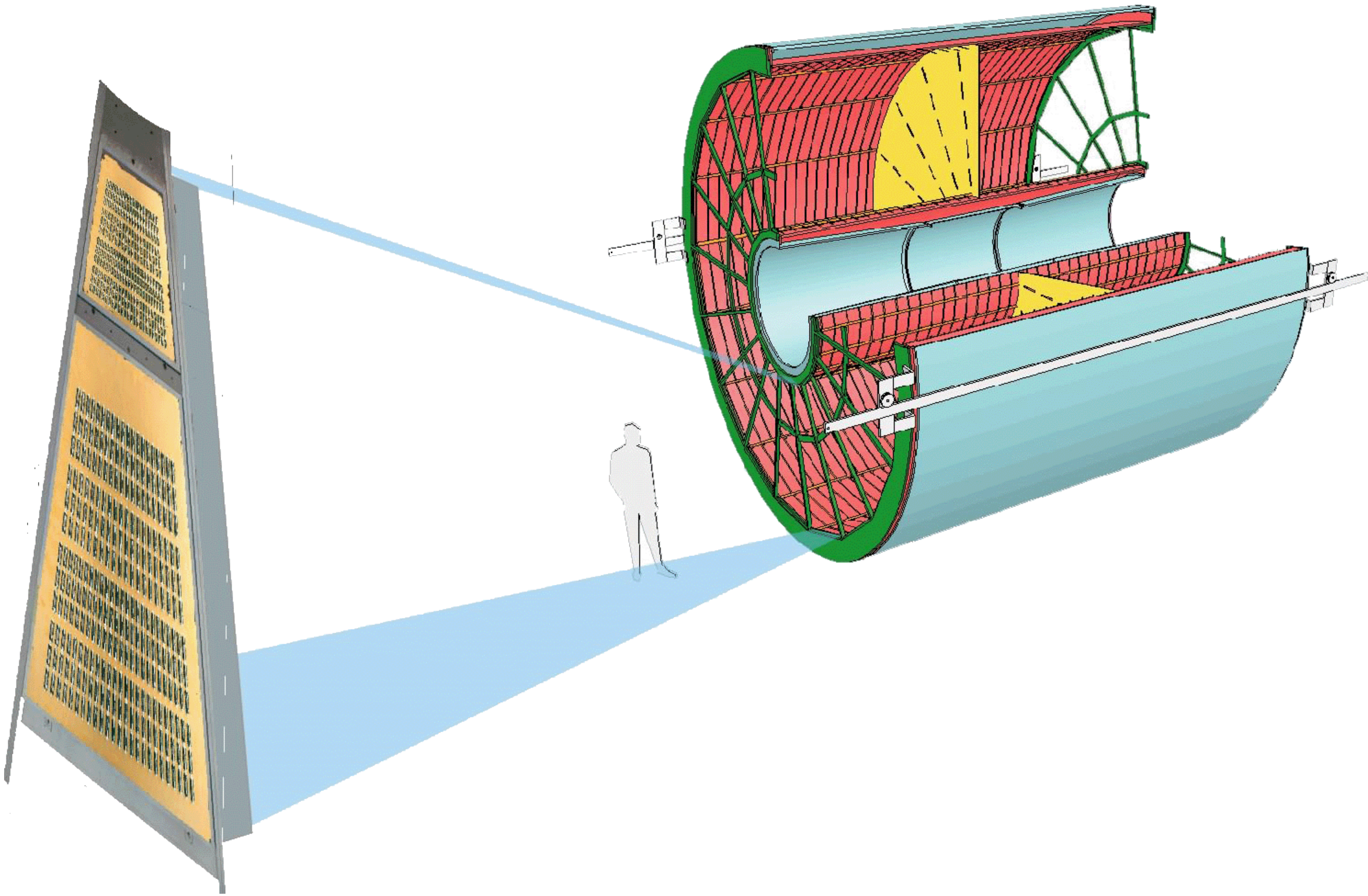


ALICE TPC

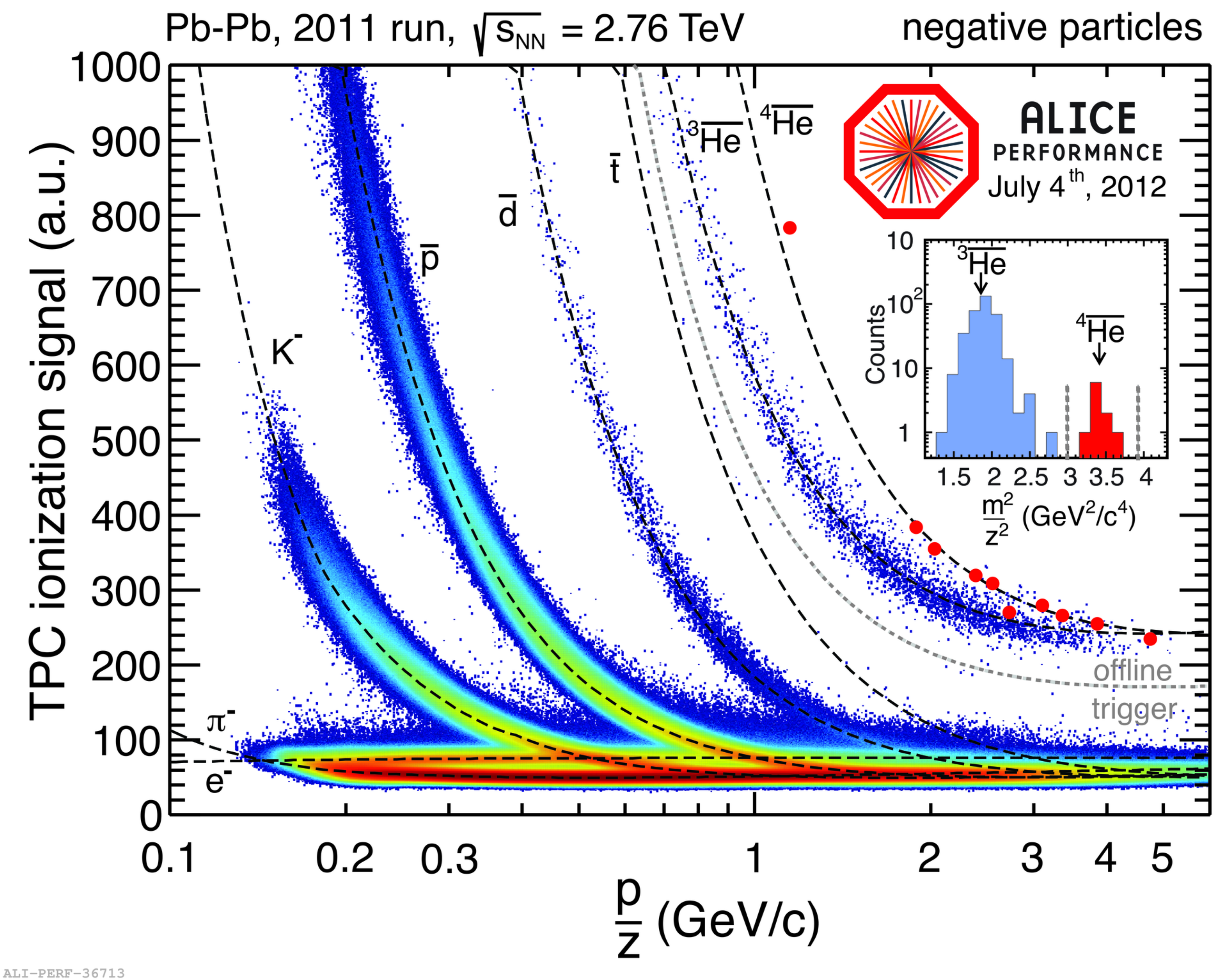


Ø5m
Outer field cage
CO_2 gap
Inner, outer readout chambers (MWPCs)
$\vec{E}$
$\vec{E}$ = 400 V/cm
$\vec{B}$
$\vec{B}$
y
z
x
Central drift electrode (100kV)
Inner field cage
Endplate
5m

ALICE TPC



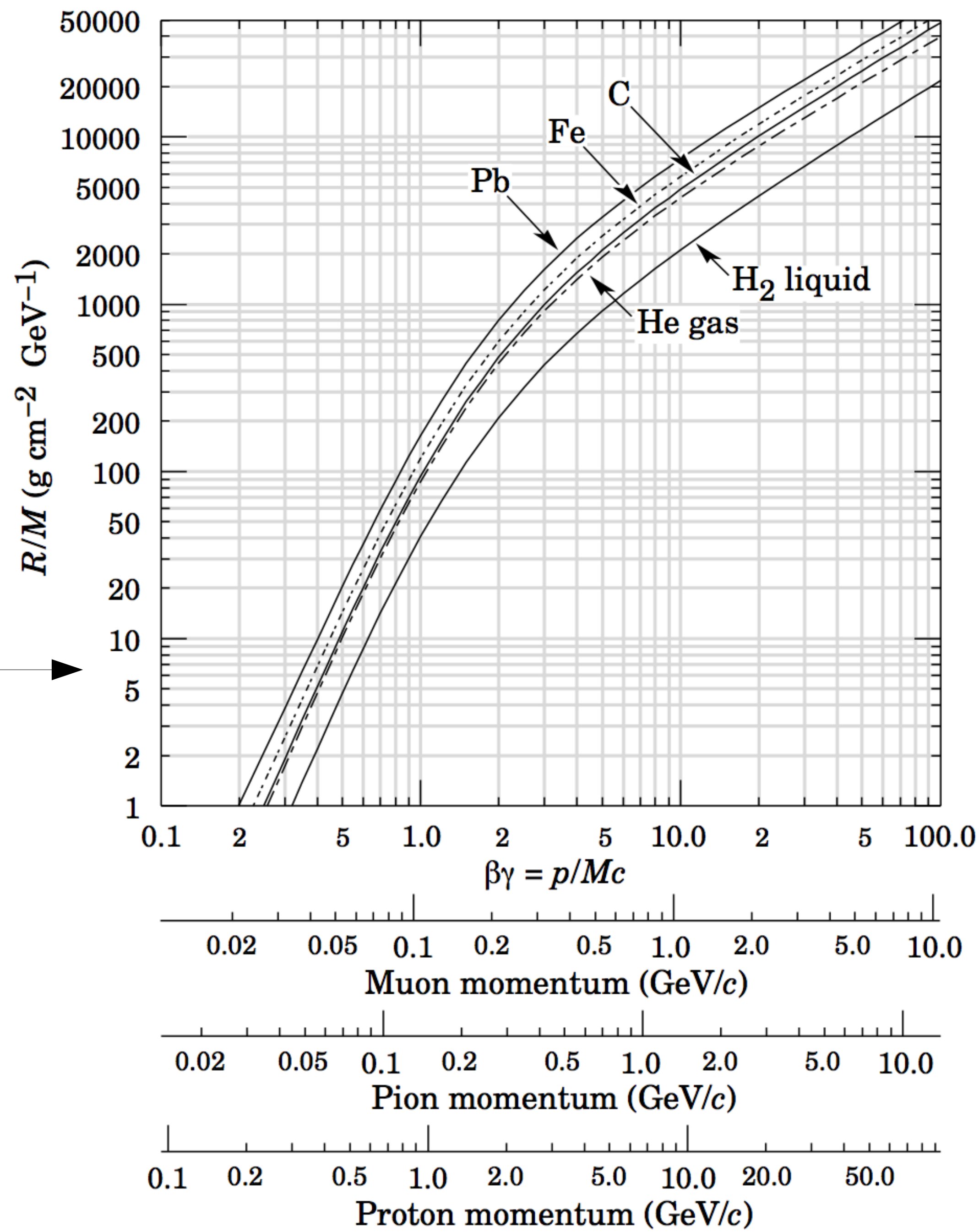
Measuring the heaviest anti-particles



Range of particles

Integrate over (changing!!) energy loss from initial energy E to 0, to calculate the range:

$$R = \int_E^0 \frac{\mathrm{d}E}{\mathrm{d}E/\mathrm{d}x}$$



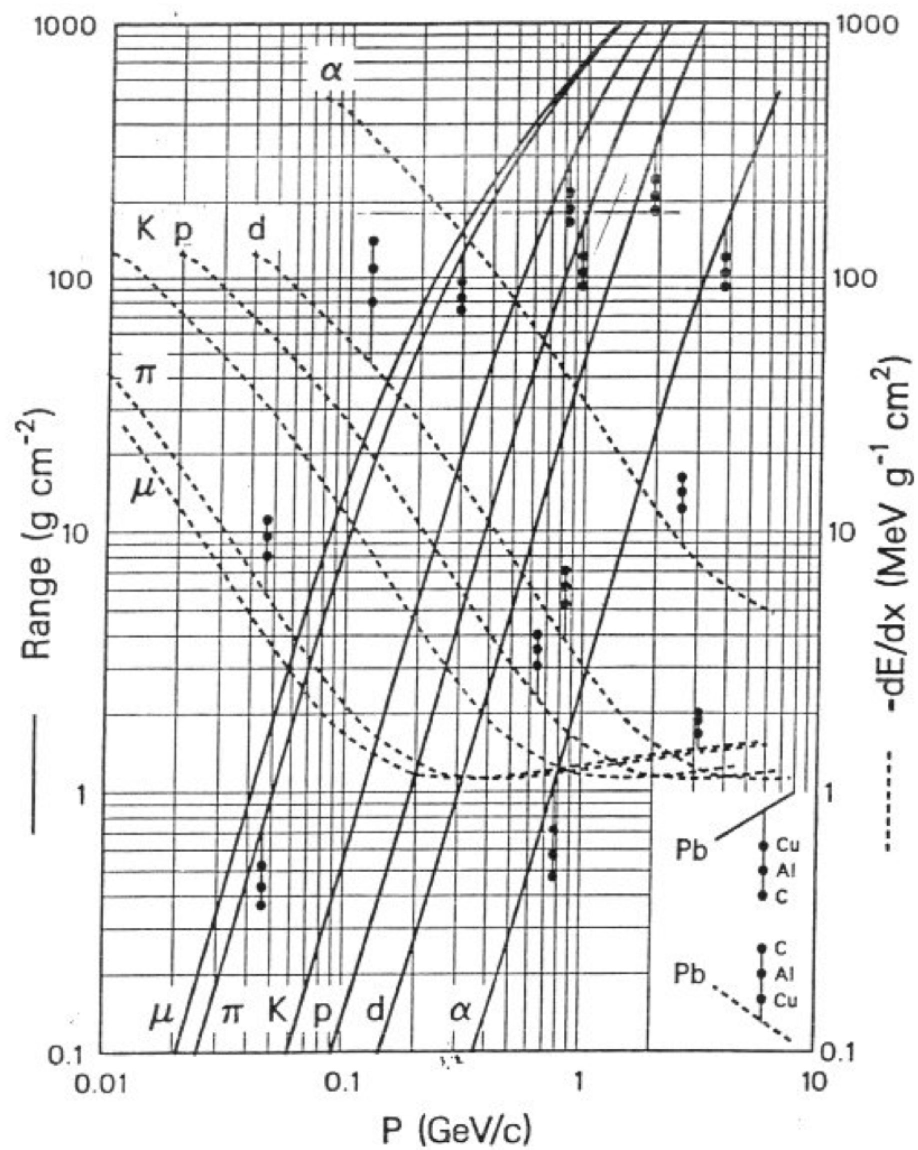
Here: Range of heavy charged particles in liquid (bubble chamber) hydrogen, helium gas, carbon, iron, and lead. For example:

- For a K^+ whose momentum is 700 MeV/c, $\beta\gamma = 1.42$. For lead we read $R/M \approx 396$, and so the range is 195 g cm^{-2} (17 cm).

Range of particles

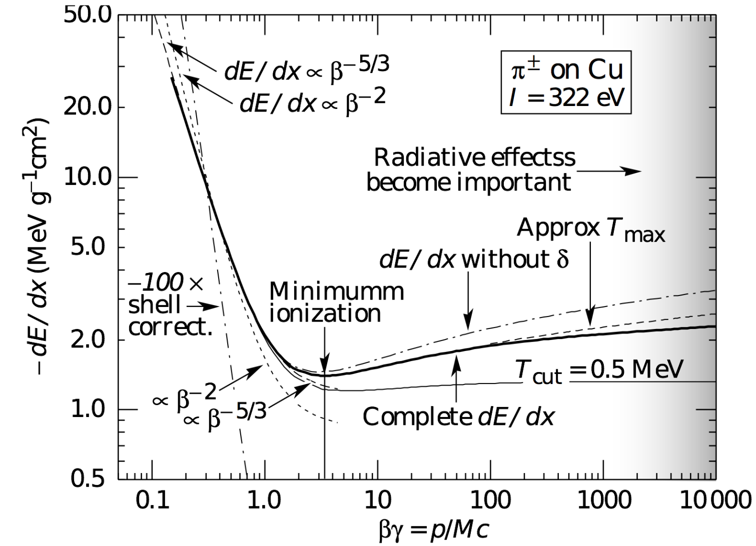
Integrate over (changing!!) energy loss from initial energy E to 0, to calculate the range:

$$R = \int_E^0 \frac{\mathrm{d}E}{\mathrm{d}E/\mathrm{d}x}$$



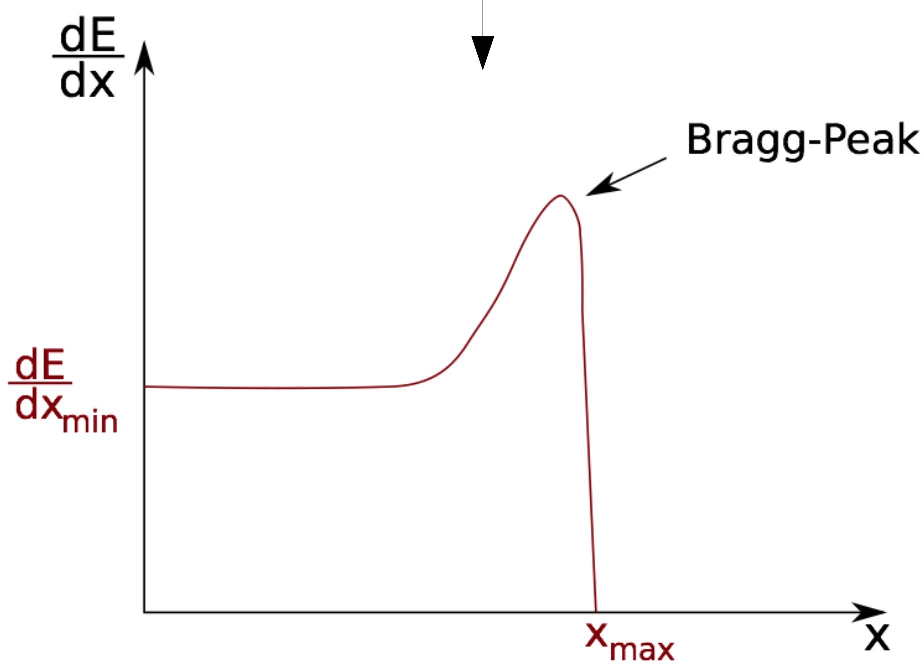
Mean range and energy loss due to ionization in lead, copper, aluminum and carbon

Particles stopped in medium



for $\beta\gamma \simeq 3.5$ $\quad \langle \frac{dE}{dx} \rangle \simeq \frac{dE}{dx}_{min}$

for $\beta\gamma \lesssim 3.5$ $\quad$ steep rise $\quad \langle \frac{dE}{dx} \rangle \gg \frac{dE}{dx}_{min}$



Energy loss curve vs depth showing Bragg peak

Application: tumor therapy

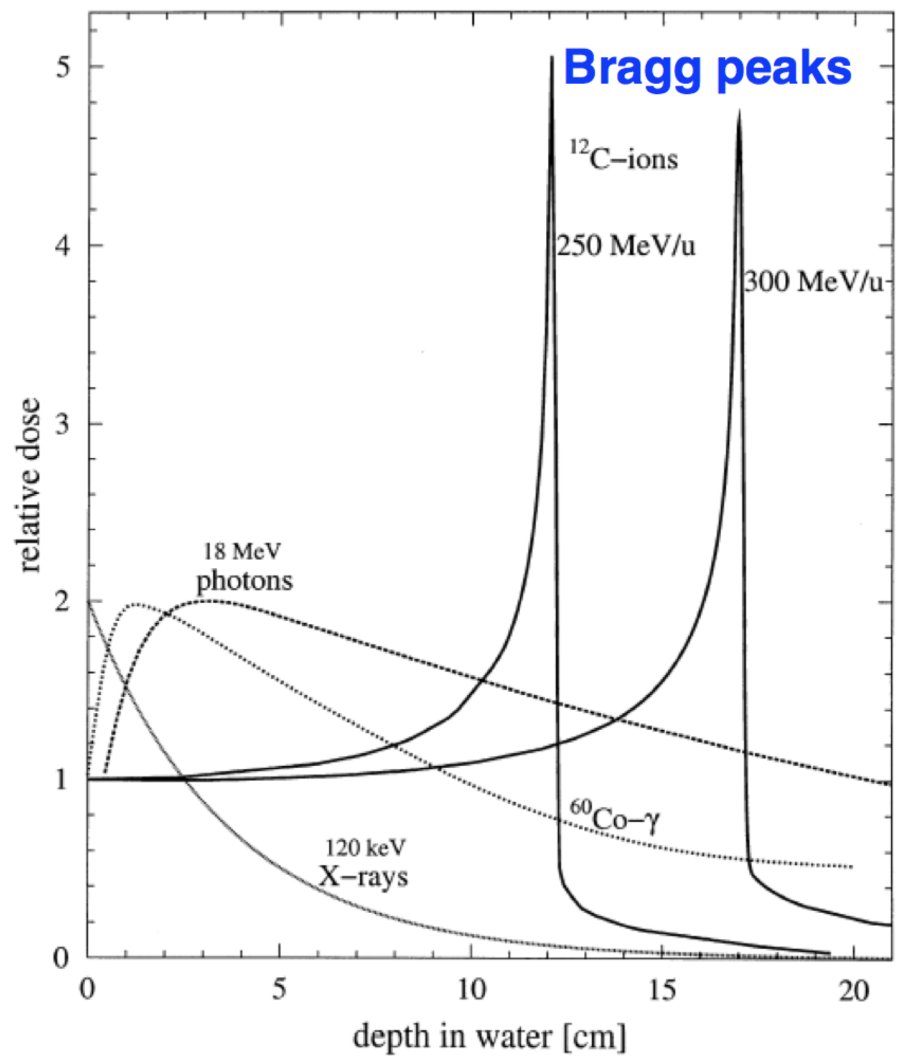
Possibility to deposit a rather precise dose at a well defined depth (body), by variation of the beam energy

Initially with protons, later also with heavier ions such as ^{12}C.

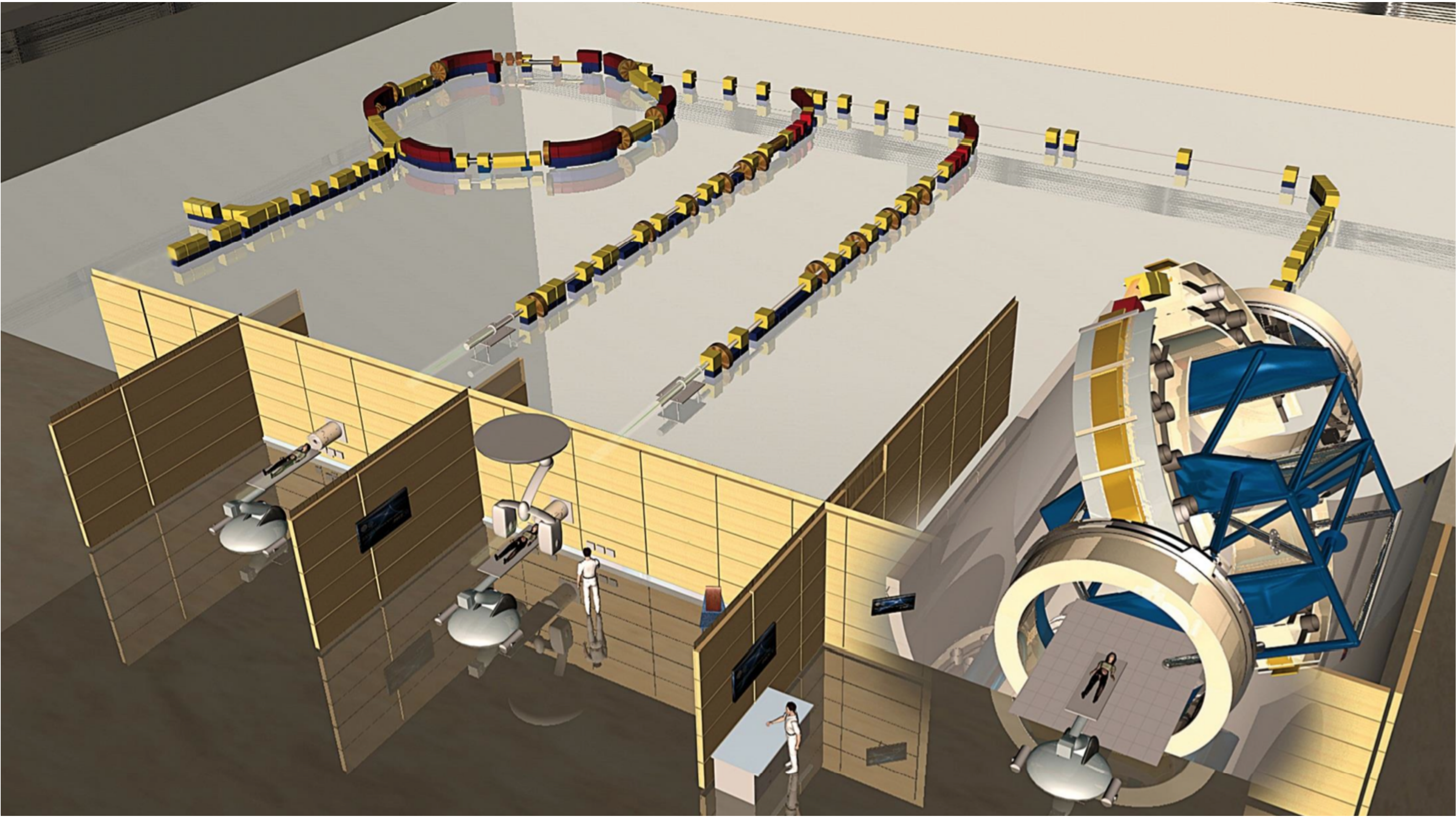
Precise 3D irradiation profile, also with suitably shaped absorbers (custom made for patient).

High precision beam scanning.

Tumor treatment at HIT (Heidelberg Ion-Beam Therapy center) in collaboration between DKFZ and GSI



Heidelberg Ion-beam Therapy Center (HIT)



Delta electrons

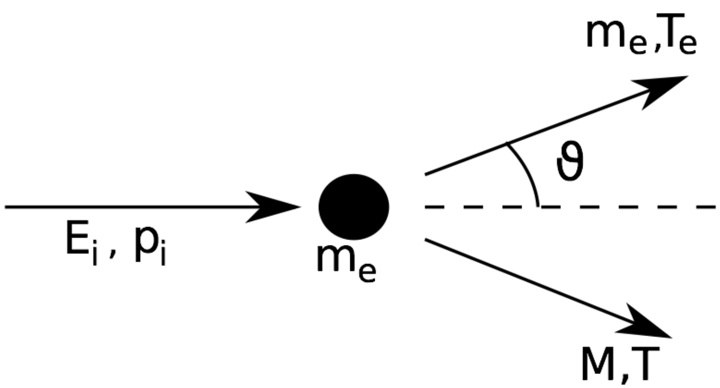
Electrons liberated by ionization can have large energies. Above a certain threshold (e.g. T_{cut}) they are called **δ electrons**.

Early observation in emulsions.

$$T_e = 2m_e \frac{\vec{p}_i^{\,2}\cos^2\theta}{(E_i + m_e)^2 - \vec{p}_i^{\,2}\cos^2\theta}$$

$$\Rightarrow \quad T_e^{max} = \frac{2m_e\vec{p}_i^{\,2}}{(E_i + m_e)^2 - \vec{p}_i^{\,2}}$$

$$\cong \frac{2m_e c^2\beta^2\gamma^2}{1 + 2\frac{m_e\gamma}{M} + \left(\frac{m_e}{M}\right)^2} \quad \text{for} |\vec{p}_i| \gg M, m_e$$



Massive highly relativistic particle can transfer practically all its energy to a single electron!

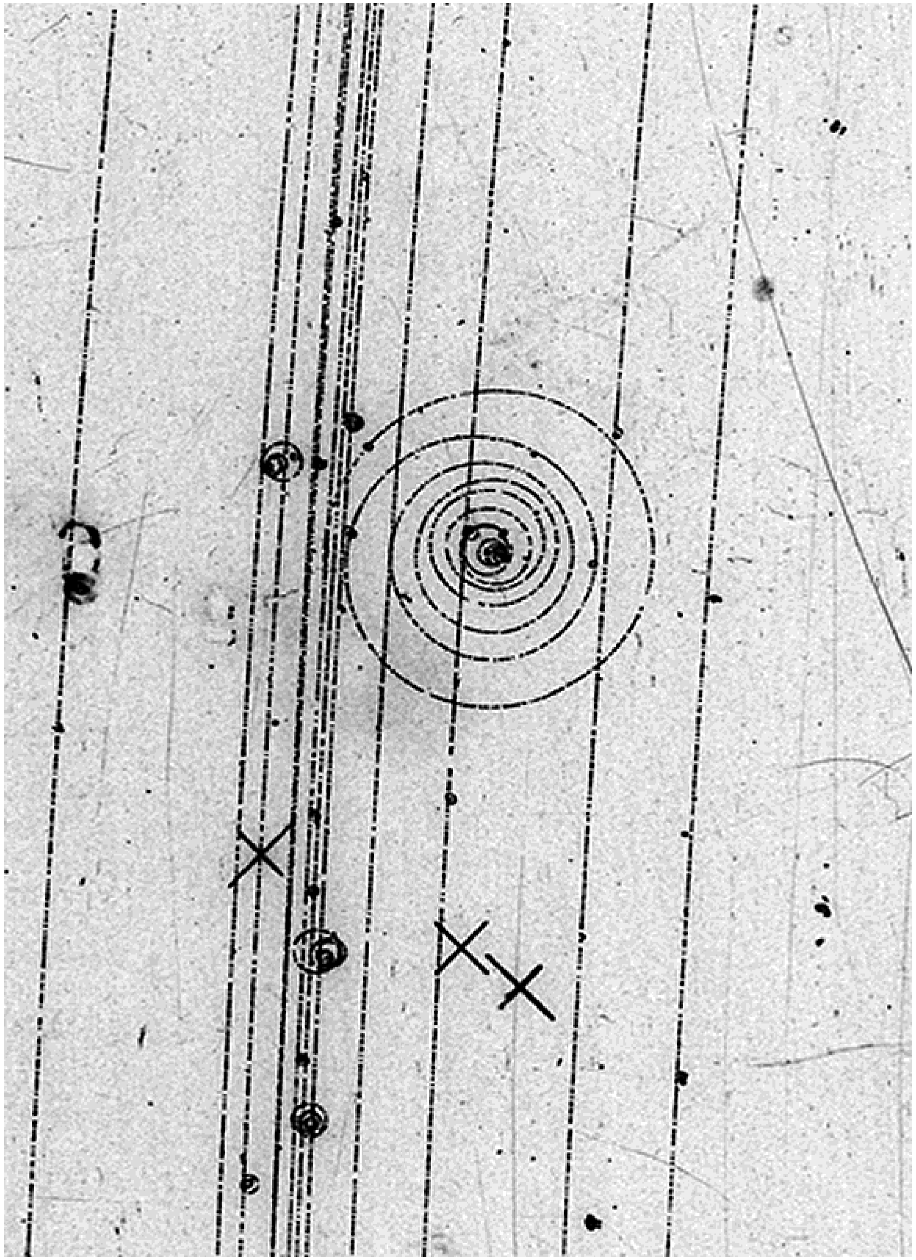
Probability distribution for energy transfer to a single electron:

$$\frac{d^2W}{dx\,dE} = 2m_e c^2 \pi r_e^2 \frac{z^2}{\beta^2} \cdot \frac{Z}{A} N_A \cdot \rho \cdot \frac{1}{E^2}$$

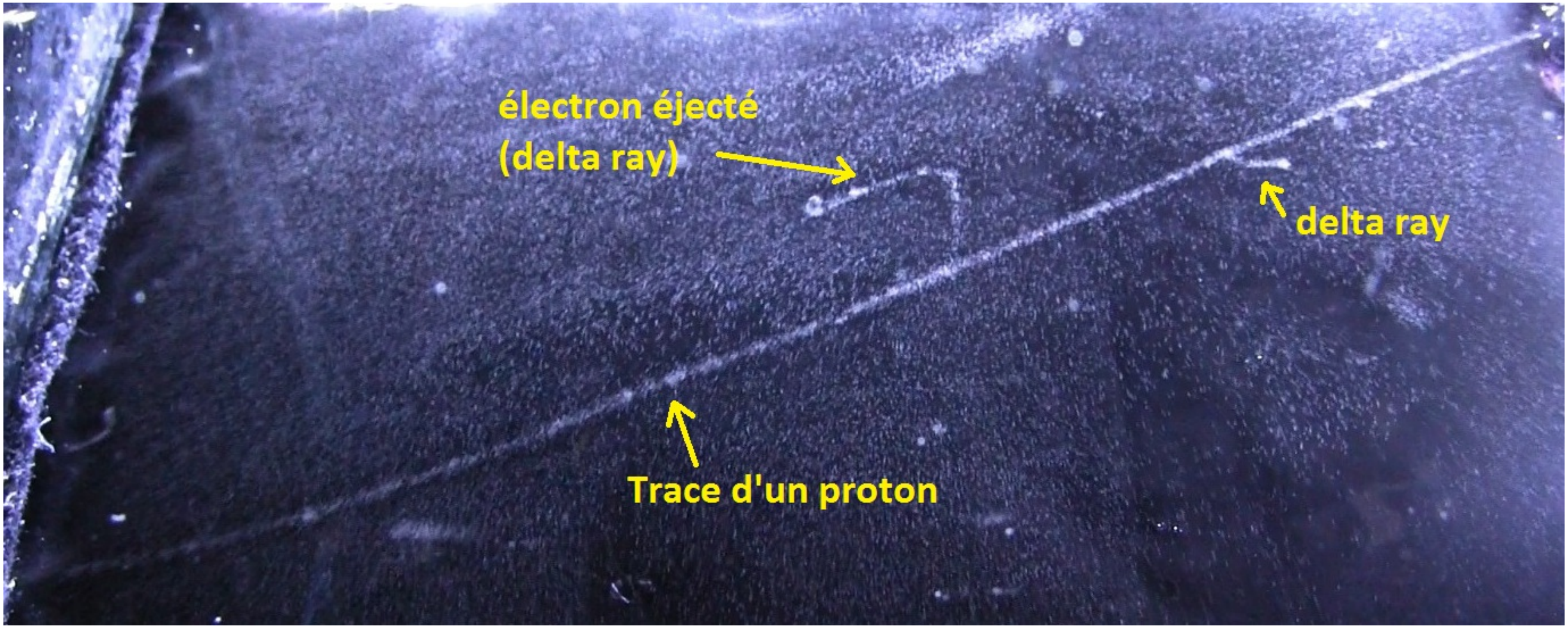
Delta electrons

Picture from CERN 2-meter hydrogen bubble chamber exposed to a beam of negative kaons K^- ,with energy 4.2 GeV. This piece corresponds to about 70 cm in the bubble chamber.

The 12 parallel lines are trails of bubbles – initiated by the ionization of hydrogen by the beam particles, which enter at the bottom of the picture.



Delta electrons



Cloud chamber

Limitation to the measurement of the incoming particle: most often the δ electron is NOT detected as part of the ionization trail

→ broadening of track

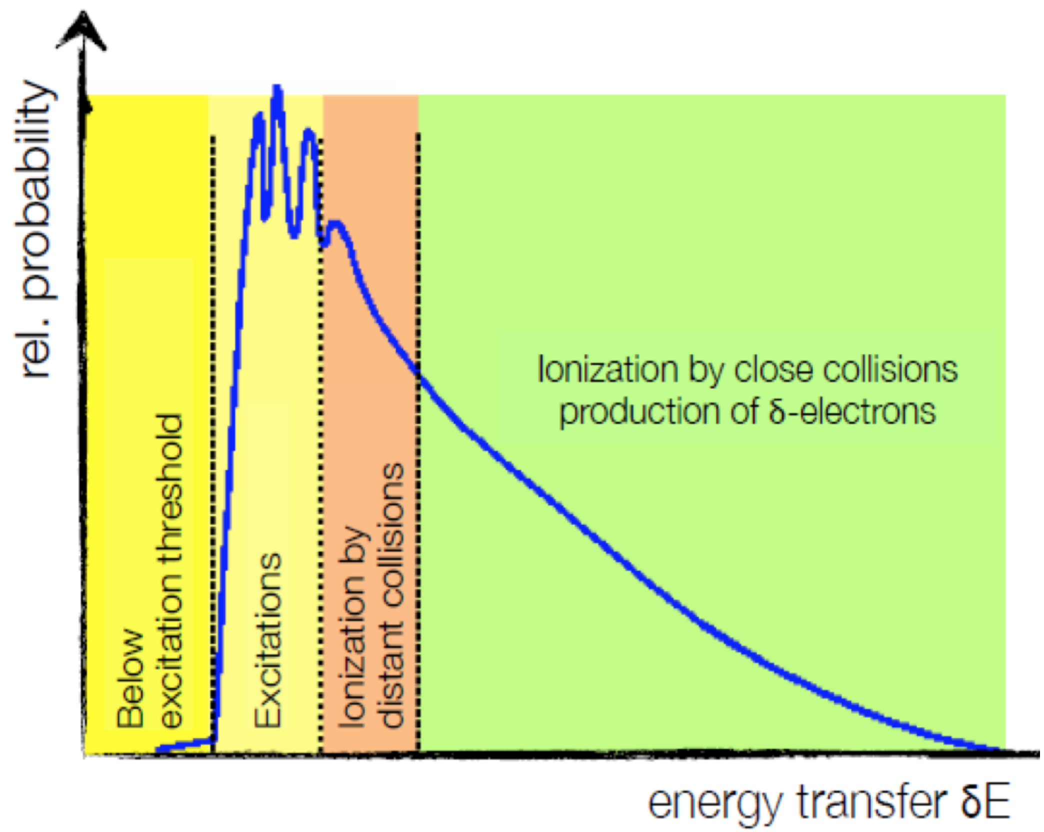
→ broadening of energy loss distribution

dE/dx fluctuations

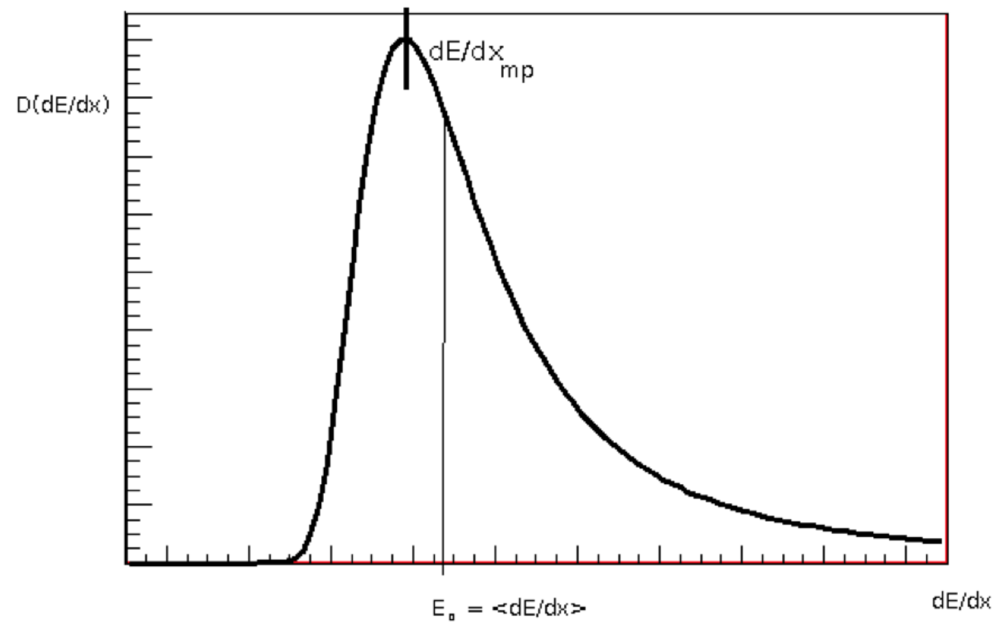
- The Bethe-Bloch formula describes the MEAN energy loss
  The energy loss is measured in a detector of finite thickness Δx with
  - N = number of collisions
  - δE = energy loss in a single collision

$$\Delta E = \sum_{n=1}^{N} \delta E_n$$

- The single energy loss is a statistical process, δE is distributed statistically → energy loss “straggling”
  (strong fluctuations, complex problem)
- For this absorbers: Landau distribution (see next)



dE/dx Landau distribution



Distribution of energy loss in single collisions: Gauss plus tail towards high losses due to the δ electrons

Landau for thin absorbers, approximation for thicker ones:
(Vavilov 1957)

$$D\left(\frac{\mathrm{d}E}{\mathrm{d}x}\right) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\underbrace{\frac{\frac{\mathrm{d}E}{\mathrm{d}x} - \frac{\mathrm{d}E}{\mathrm{d}x}_{\mathrm{mp}}}{\xi}}_{\lambda} + e^{-\lambda}\right)\right)$$

ξ is a material constant

more precise: Allison & Cobb (using measurements and numerical solution)
Ann. Rev. Nuclear Sci. 30 (1980) 253

Straggling functions

Energy loss distribution normalized to thickness x
For increasing x:

- Most probable value Δp/x shifts to larger values
- Relative width shrinks
- Asymmetry of distribution decreases

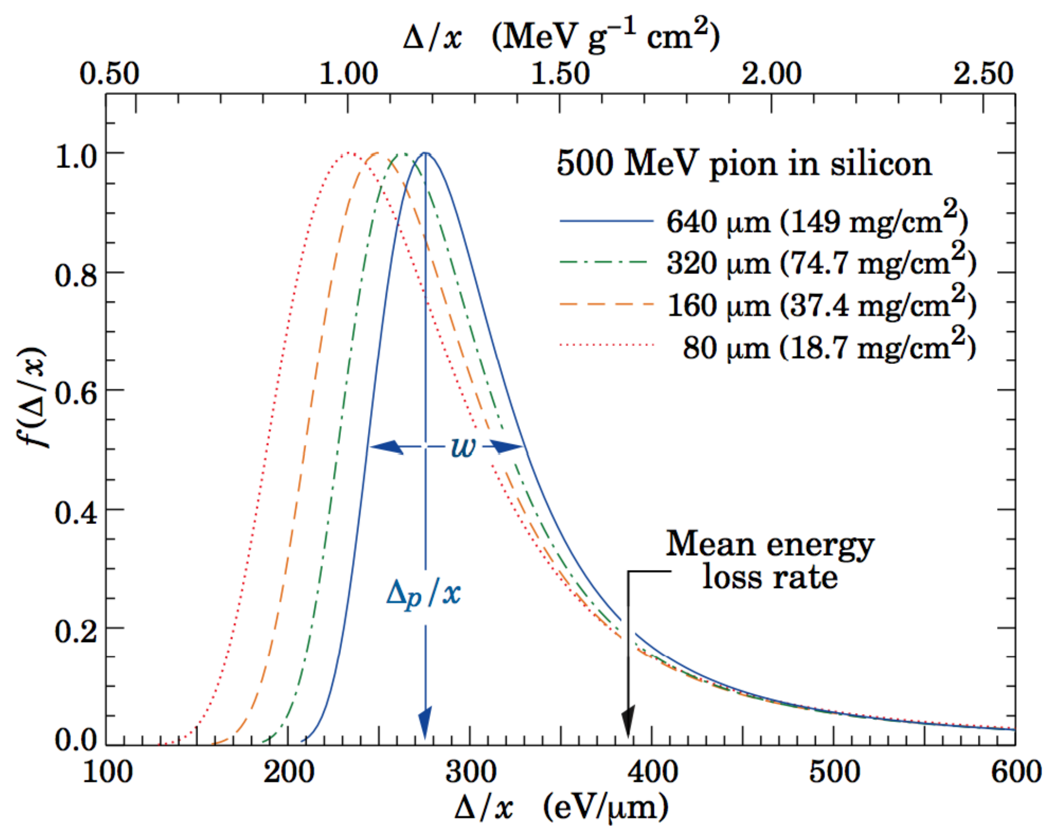
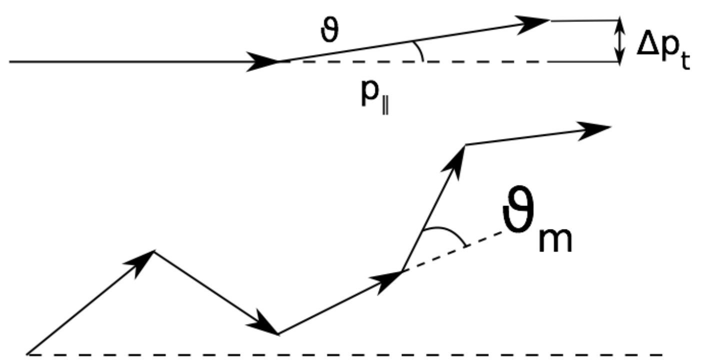


Figure 33.8: Straggling functions in silicon for 500 MeV pions, normalized to unity at the most probable value δ_p/x. The width w is the full width at half maximum.

Multiple (Coulomb) scattering

Incident particle can also scatter in the Coulomb field of the NUCLEUS !

Deflection of trajectory will be more significant because of the factor Z !



$$\theta \simeq \frac{\Delta p_\perp}{p_{||}} \simeq \frac{\Delta p_\perp}{p} = \frac{2Zze^2}{b}\frac{1}{pv}$$

after k collisions

$$\langle \theta_k^2 \rangle = \sum_{m=1}^{k} \theta_m^2 = k\langle \theta^2 \rangle$$

- Single collision (thin absorber): Rutherford scattering $d\sigma/d\Omega \propto \sin^{-4}\theta/2$
- Few collisions: difficult problem
- Many (>20) collisions: statistical treatment “Molière theory”

Multiple (Coulomb) scattering: Molière theory

Obtain the mean deflection angle in a plane by averaging over many collisions and integrating over b:

$$\sqrt{\langle \theta^2(x) \rangle} = \theta_{rms}^{plane} = \frac{13.6\ \mathrm{MeV}}{\beta p c} z \sqrt{\frac{x}{X_0}} \left(1 + 0.038 \ln \frac{x}{X_0}\right)$$

- Material constant X_0: radiation length
- $\propto \sqrt{x} \rightarrow$ use thin detectors
- $\propto 1/\sqrt{X_0} \rightarrow$ use light detectors
- $\propto 1/\beta p \rightarrow$ serious problem at low momenta

In 3 dimensions: $\theta_{rms}^{space} = \sqrt{2}\ \theta_{rms}^{plane}$ $\quad 13.6 \rightarrow 19.2$

Multiple scattering limits the momentum and tracking resolution, particularly at low momenta!

Ionization yield

Mean number of electron-ion pairs produced along the track of the ionizing particle:

- Total ionization = primary ionization + secondary ionization due to energetic primary electrons $n_t = n_p + n_s$
- Consider also the energy loss by excitation (smaller)

→ mean energy W to produce an electron-ion pair: $n_t = \dfrac{\Delta E}{W}$

W > ionization potential I_0 since:

- Also ionization of inner shells
- Excitation that may not lead to ionization

$n_t \approx 2\text{-}6\ n_p$

Ionization yield

typical values

	I_0 (eV)	W (eV)	n_p (cm^{-1})	n_t (cm^{-1})
H_2	15.4	37	5.2	9.2
N_2	15.5	35	10	56
0_2	12.2	31	22	73
Ne	21.6	36	12	39
Ar	15.8	26	29	94
Kr	14.0	24	22	192
Xe	12.1	22	44	307
CO_2	13.7	33	34	91
CH_4	13.1	28	16	53
		in gases $\approx$ 30 eV	diff. due to ρ and Z	diff. due to electronic struct.

In comparison, in semiconductors: W = 3.6 eV in Si, 2.85 eV in Ge
Additional factor 103 due to density → many more electron-ion pairs!!

Streamer chamber image

Interaction of electrons with matter

Energy loss by ionization

Bethe-Bloch equation must be modified to account for:

- Small mass of electron → deflections become more important
- Incident and target electron have the same mass m_e (T_{max} = T/2)
- Quantum mechanics: after the scattering, the incoming electron and the one from ionization are indistinguishable

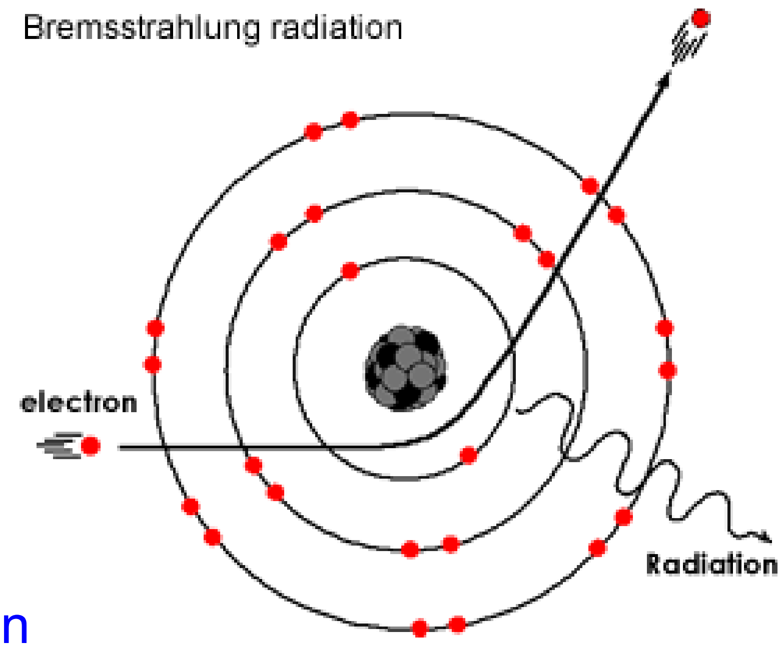
$$\boxed{-\left\langle \frac{dE}{dx} \right\rangle_{\mathrm{el.}} = K \frac{Z}{A} \frac{1}{\beta^2} \left[\ln \frac{m_e \beta^2 c^2 \gamma^2 T}{2 I^2} + F(\gamma) \right]}$$

Energy loss for electrons and positrons is DIFFERENT:

- positron is not indistinguishable from electron in atom
- Low energy positrons have larger energy loss because of annihilation
- At same β, the difference is within 10%

Bremsstrahlung

Acceleration of charged particles in the Coulomb field of the nucleus:



QED process (Fermi 1924, Weizsäcker-Williams 1938)

Emission of a real photon

$$\frac{dE}{dx} = 4\alpha N_A \frac{z^2 Z^2}{A} \left(\frac{1}{4\pi\epsilon_0} \frac{e^2}{mc^2} \right)^2 E \ \ln \frac{183}{Z^{\frac{1}{3}}} \propto \frac{E}{m^2}$$

Relevant for electron and positrons in the range up to few hundred GeV/c

$$\frac{dE}{dx} = 4\alpha N_A \frac{Z^2}{A} r_e^2 \cdot E \ \ln \frac{183}{Z^{\frac{1}{3}}}$$

$$\frac{dE}{dx} = \frac{E}{X_0} \quad \text{with} \quad X_0 = \frac{A}{4\alpha N_A \ Z^2 r_e^2 \ \ln \frac{183}{Z^{\frac{1}{3}}}}$$

[Radiation length in g/cm^2]

Radiation length X_0

Bremsstrahlung: radiation length

$$-\frac{dE}{dx} = \frac{E}{X_0} \qquad \rightarrow \qquad E(x) = E_0 \exp\left(-\frac{x}{X_0}\right)$$

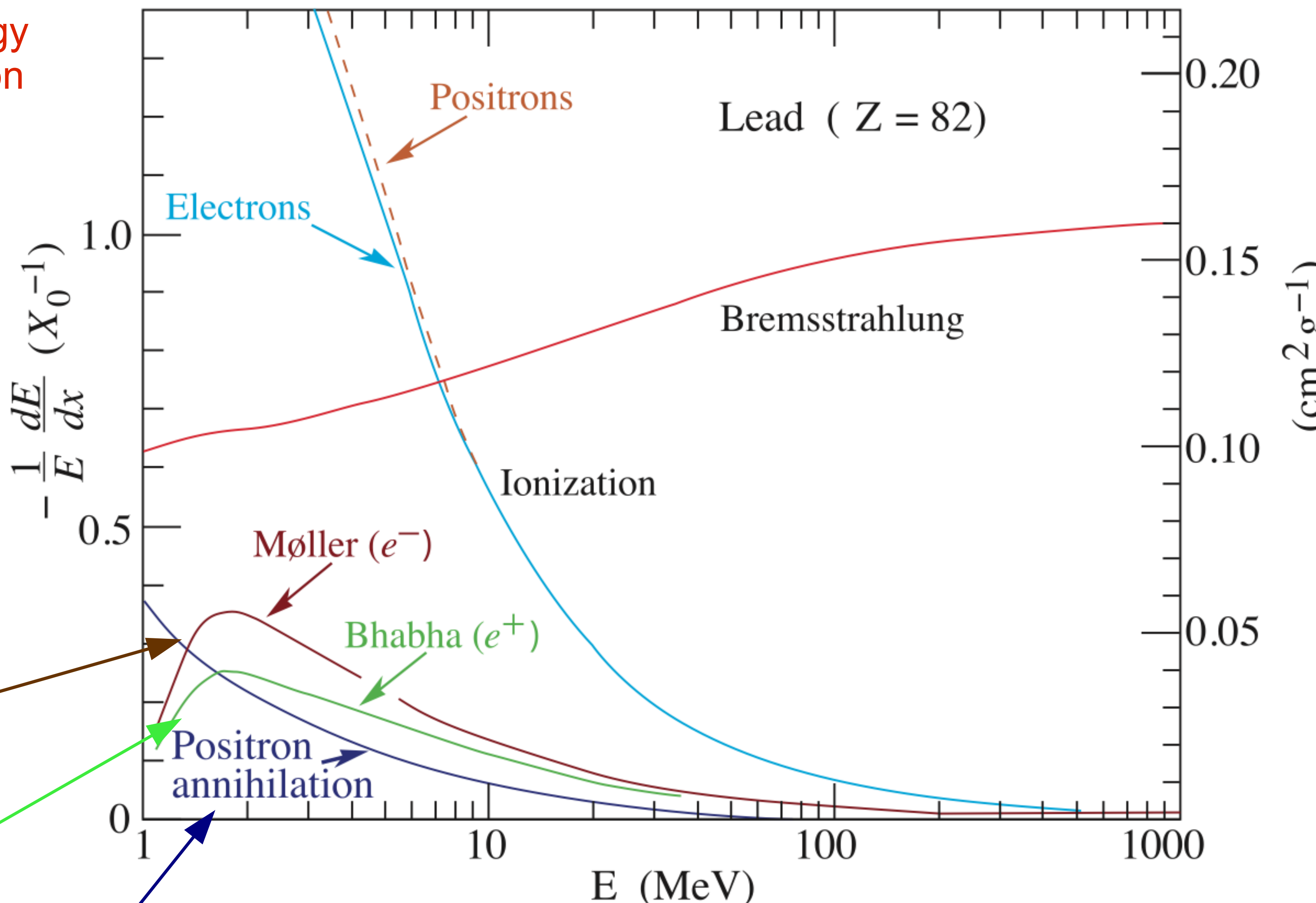
X_0: distance after which the energy of the electron is reduced to E_0/e

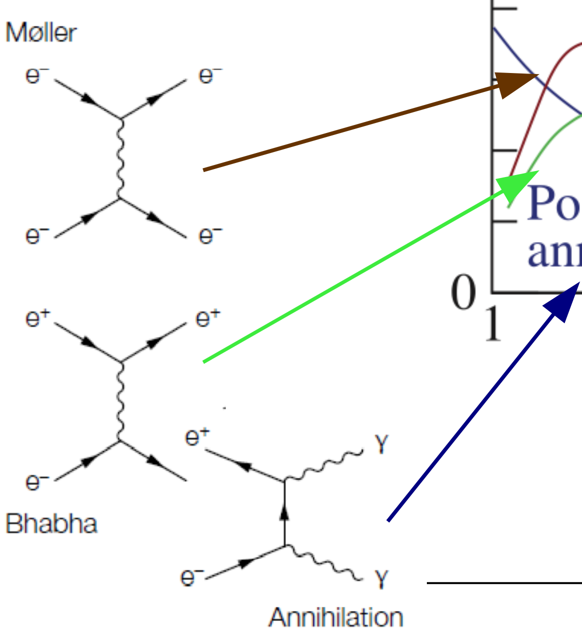
For materials which are mixtures of more components:

$$\frac{1}{X_0} = \sum_i \frac{w_i}{X_{0i}} \qquad w_i \text{ weight fraction of substance } i$$

Overview: energy loss by electrons

Fractional energy loss per radiation length





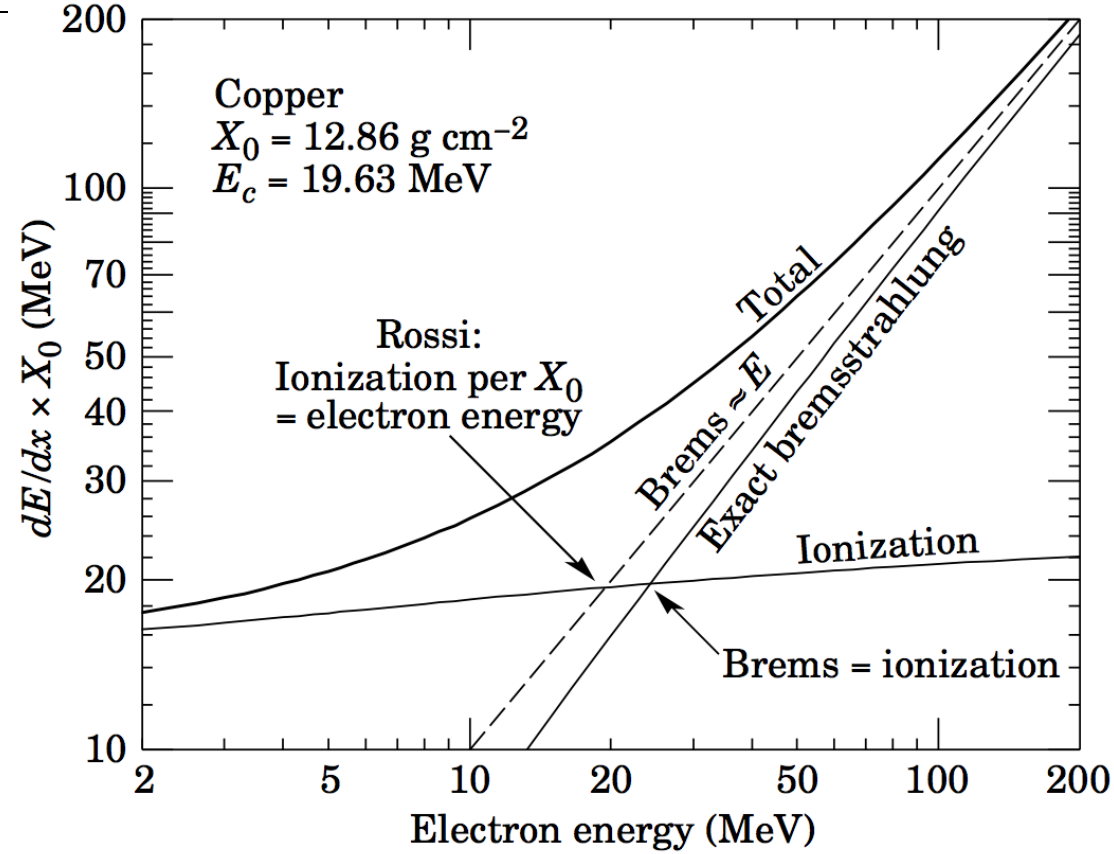
Critical energy

Total energy loss of electrons:

$$\left(\frac{dE}{dx}\right)_{Tot} = \left(\frac{dE}{dx}\right)_{Ion} + \left(\frac{dE}{dx}\right)_{Brems}$$

Critical energy:

$$\left(\frac{dE}{dx}(E_c)\right)_{Brems} = \left(\frac{dE}{dx}(E_c)\right)_{Ion}$$



Approximation:

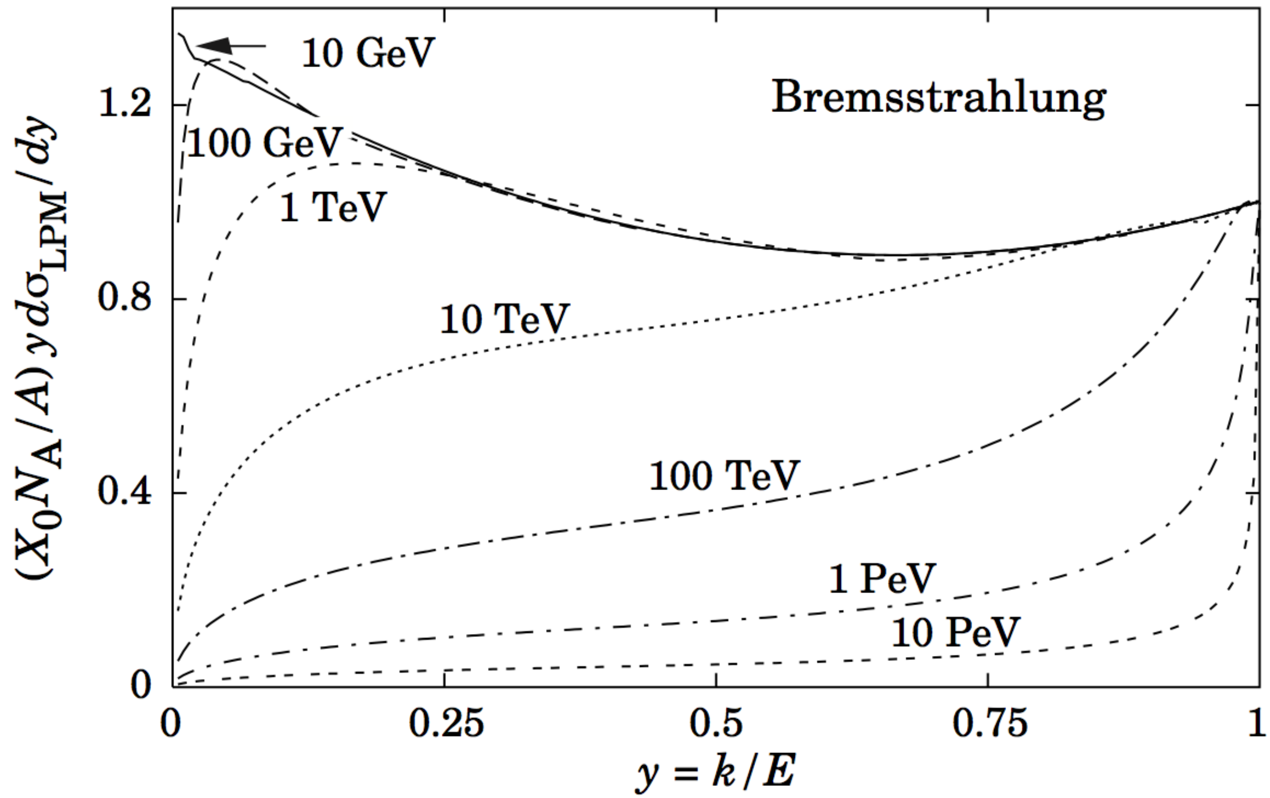
$$E_c^{Gas} = \frac{710\,\text{MeV}}{Z+0.92} \qquad E_c^{Sol,Liq} = \frac{610\,\text{MeV}}{Z+1.24}$$

Example: Cu   Ec ≈ 610/30 MeV ≈ 20 MeV

Bremsstrahlung: energy spectrum

Normalized bremsstrahlung cross section $y d\sigma/dy$ as a function of the fractional photon energy $y=k/E$

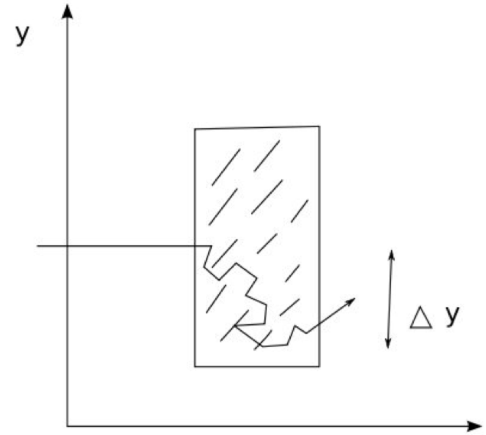
Units: photons per radiation length



Cross section suppressed at low y: for small photon energies, successive radiations interfere (Landau-Pomeranchuk-Migdal effect)

Stronger suppression for larger electron energy E

Multiple scattering



Difference between heavy particles and electrons:

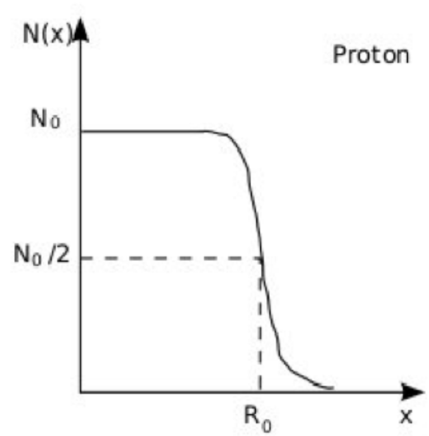
- Heavy particle: the track is more or less straight
- Electron: can be scattered to large angles!

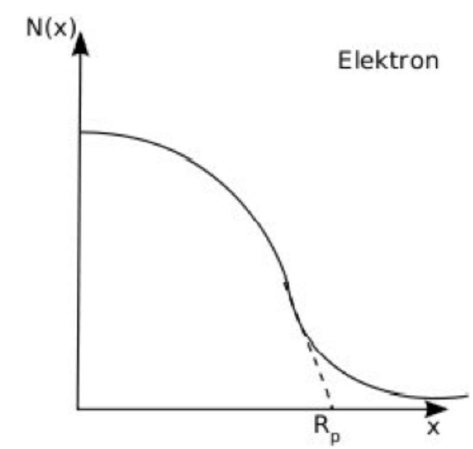
Transverse deflection of an electron of energy $E=E_c$, after traversing a distance X_0 (= one radiation length):

	E_c (MeV)	R_M (cm)	X_0 (cm)
Pb	7.2	1.6	0.56
scint.	80	9.1	42
NaI	12.5	4.4	2.6

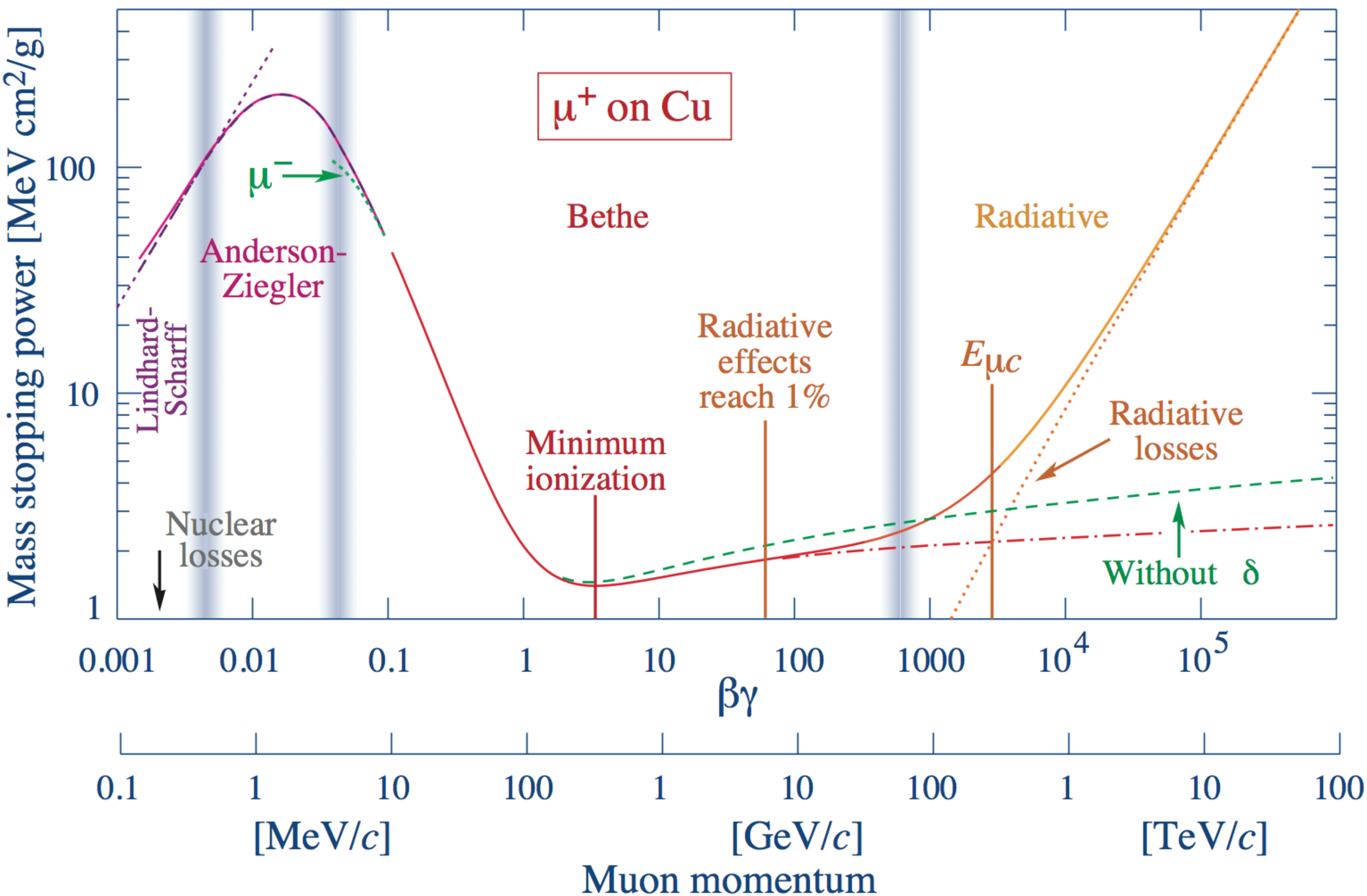
Molière radius:

$$\Delta y = R_M = \frac{21\,\mathrm{MeV}}{E_c} X_0$$





Bethe-Bloch curve for muons



Lectures

- Energy loss by ionization (by "heavy" particles)
- Interaction of electrons with matter:
  - Energy loss by ionization
  - Bremsstrahlung

TODAY

- Interaction of photons
- Cherenkov effect
- Transition radiation

NEXT WEEK