

# Quark Mixing + CKM Matrix

- Quark Mixing
- GIM hypothesis
- CKM Matrix
- Wolfenstein Parameterisation

# Flavor Changing Neutral Currents

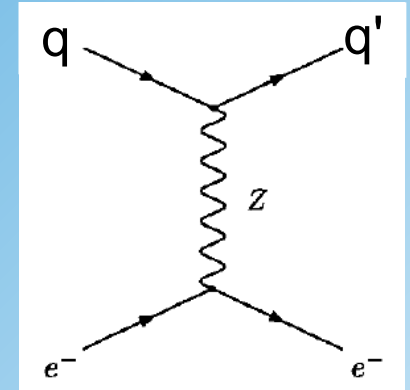
FCNC processes are highly suppressed in the Standard Model

Examples:

$$BR(K_S^0 \rightarrow e^+ e^+) < 9 \cdot 10^{-9}$$

$$BR(K^- \rightarrow \pi^- e^+ e^+) = 2.88 \cdot 10^{-7}$$

$$BR(K^- \rightarrow \pi^- \nu^+ \nu^+) = 1.5 \cdot 10^{-10}$$



The non-observation of FCNC lead to the prediction by Glashow, Iliopoulos and Maiani of the charm quark (GIM hypothesis)

$$\begin{pmatrix} u \\ d' \end{pmatrix} = \begin{pmatrix} u \\ d \cos \Theta_C + s \sin \Theta_C \end{pmatrix} \quad \begin{pmatrix} c \\ s' \end{pmatrix} = \begin{pmatrix} c \\ s \cos \Theta_C - d \sin \Theta_C \end{pmatrix}$$

Neutral Current:

$$\begin{aligned} J_{NC} = & \underbrace{\bar{u} \Gamma u + c \Gamma c + (\bar{d} \Gamma d + \bar{s} \Gamma s) \cos^2 \Theta_C + (\bar{d} \Gamma d + \bar{s} \Gamma s) \sin^2 \Theta_C}_{\Delta S=0} \\ & + \underbrace{(\bar{d} \Gamma s + \bar{s} \Gamma d - \bar{d} \Gamma s - \bar{s} \Gamma d) \sin \Theta_C \cos \Theta_C}_{|\Delta S|=1} \end{aligned}$$

flavor changing terms cancel out!

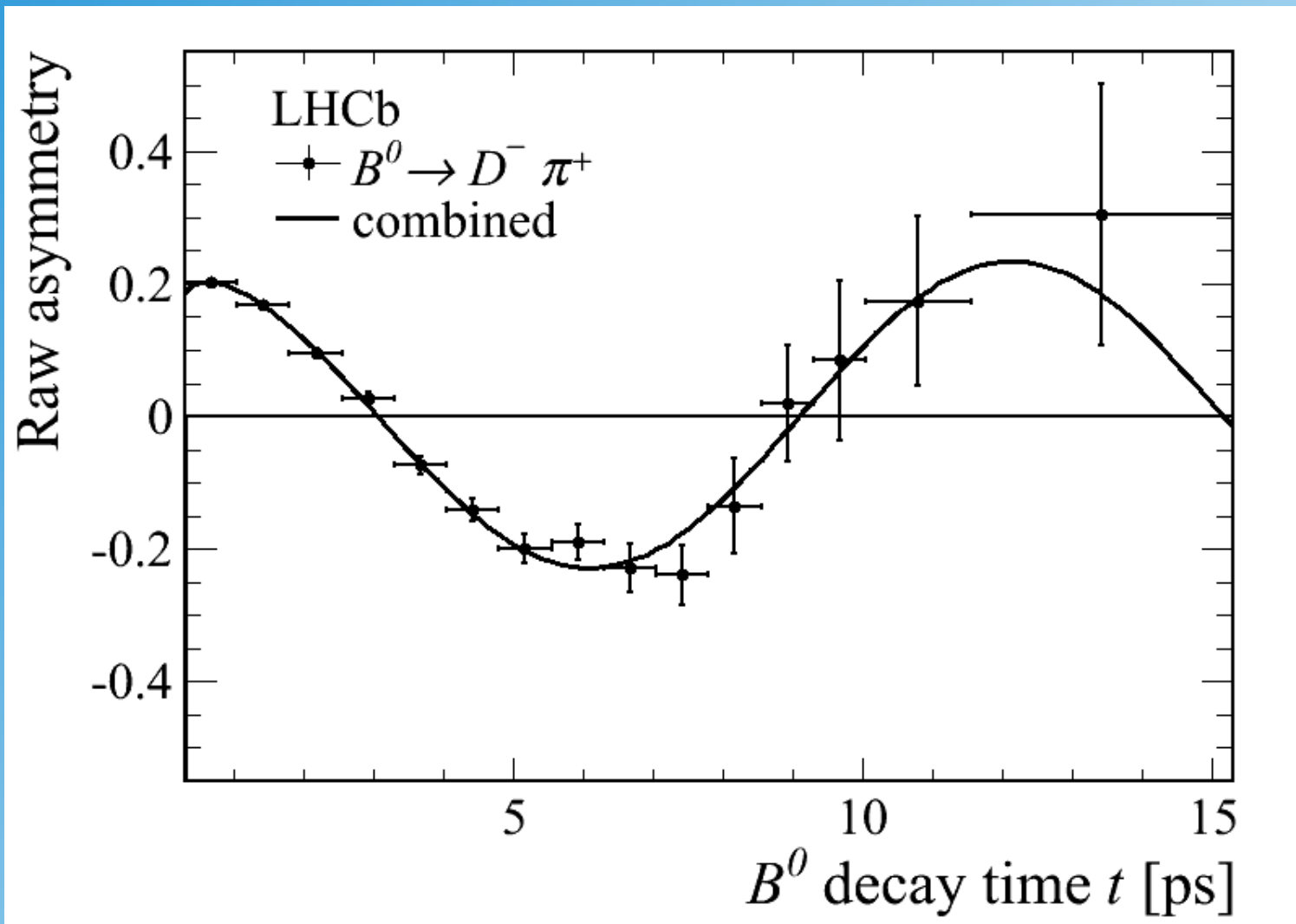
# CKM Matrix

$$V_{\text{CKM}} \equiv V_L^u V_L^{d\dagger} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}.$$

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

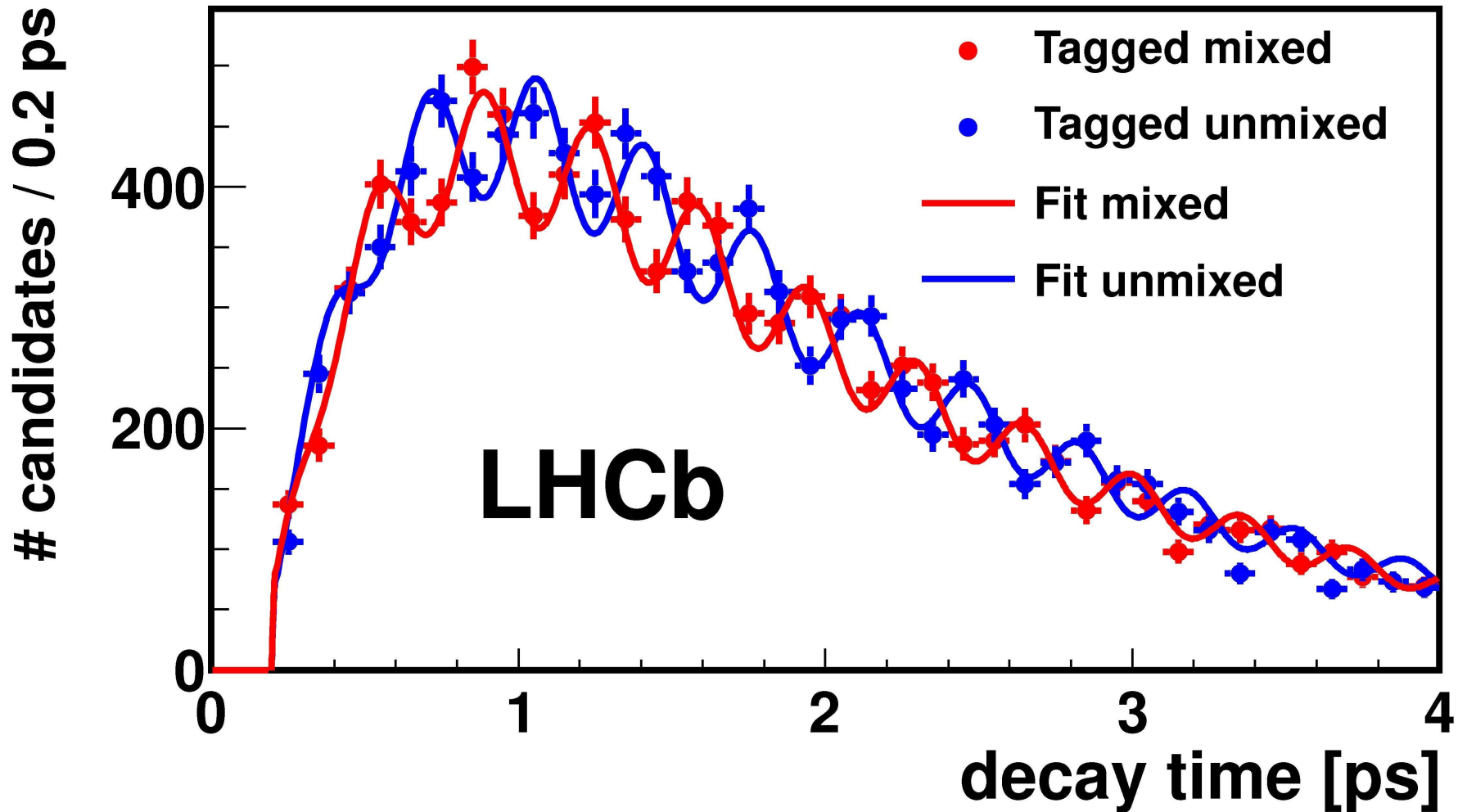
# $B_0$ Oscillations

asymmetry: 
$$A = \frac{N(B_0) - N(\bar{B}_0)}{N(B_0) + N(\bar{B}_0)}$$

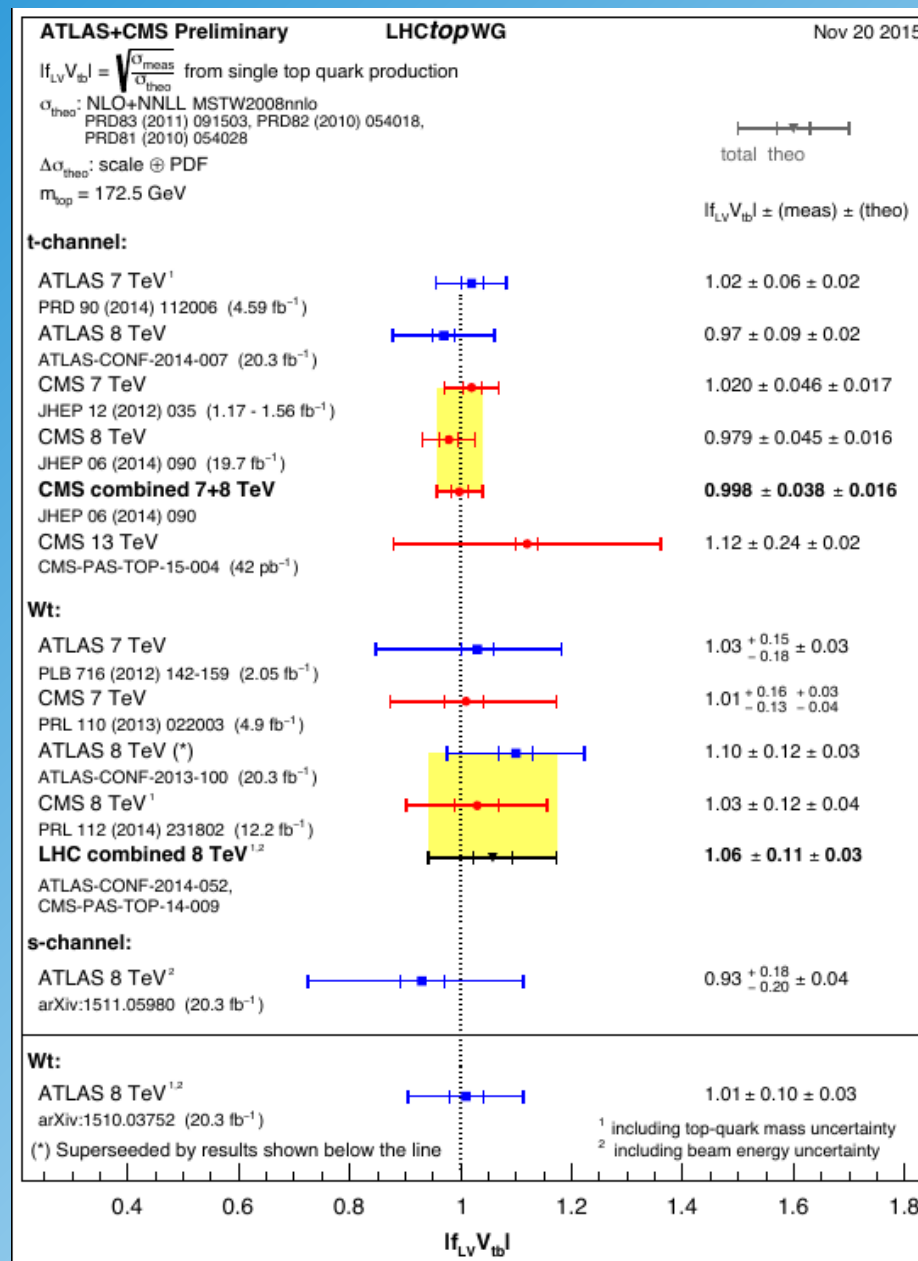


# $B_s$ Oscillations

Rate of:  $B_s \rightarrow D_s^+ \pi^-$  and  $\bar{B}_s \rightarrow D_s^- \pi^+$



# Single Top Production and $V_{tb}$





# Wolfenstein Parameterisation

$$V_{\text{CKM}} = \begin{pmatrix} 0.97419 \pm 0.00022 & 0.2257 \pm 0.0010 & 0.00359 \pm 0.00016 \\ 0.2256 \pm 0.0010 & 0.97334 \pm 0.00023 & 0.0415^{+0.0010}_{-0.0011} \\ 0.00874^{+0.00026}_{-0.00037} & 0.0407 \pm 0.0010 & 0.999133^{+0.000044}_{-0.000043} \end{pmatrix}$$

$$V = \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4).$$

$$s_{12} = \lambda = \frac{|V_{us}|}{\sqrt{|V_{ud}|^2 + |V_{us}|^2}}, \quad s_{23} = A\lambda^2 = \lambda \left| \frac{V_{cb}}{V_{us}} \right|$$

$$s_{13}e^{i\delta} = V_{ub}^* = A\lambda^3(\rho + i\eta) = \frac{A\lambda^3(\bar{\rho} + i\bar{\eta})\sqrt{1 - A^2\lambda^4}}{\sqrt{1 - \lambda^2}[1 - A^2\lambda^4(\bar{\rho} + i\bar{\eta})]}$$

