# Scattering Matrix and Resonances

**Resonances**
- Partial Wave Decomposition
- Unitarity Limit
- Breit Wigner Distribution
- Helicities
- Rotations and Spin
- Helicity Amplitudes
- Klebsch Gordon Coefficients

**Decays**
- 2-body
- 3-body
- Dalitz Plot

# Legendre Polynomial

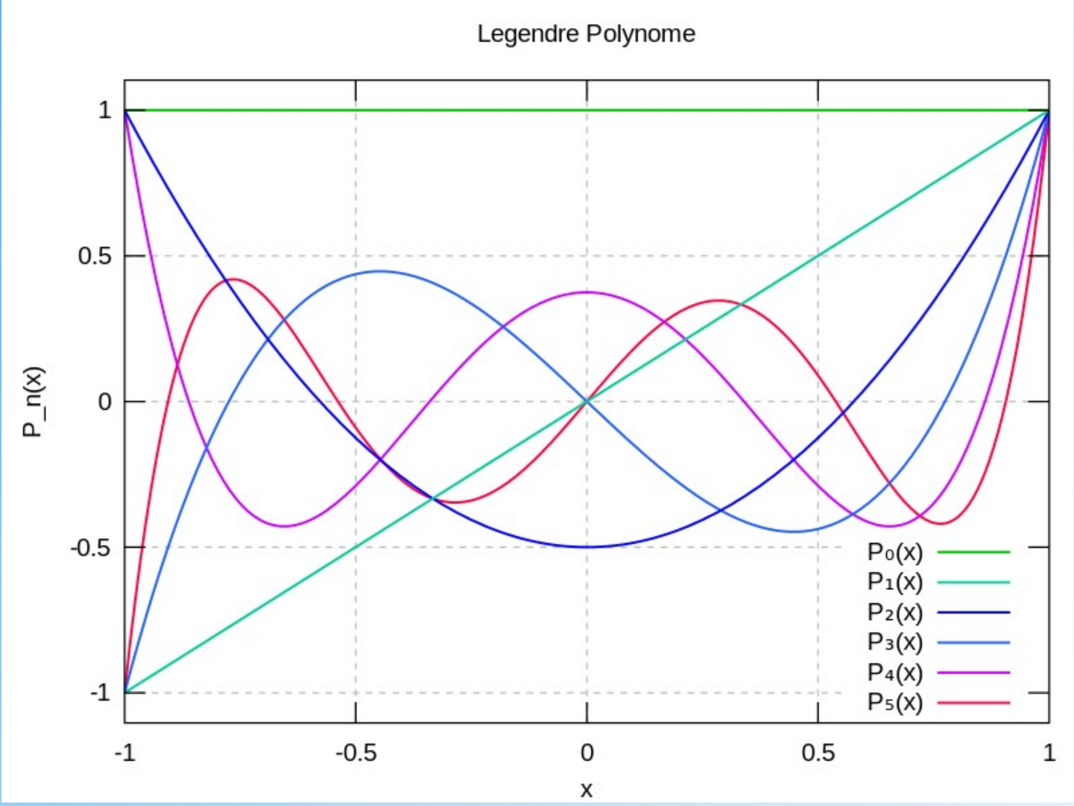
Legendre polynomial:

$$P_n(x) = \sum_{k=0}^{\lfloor n/2 \rfloor} (-1)^k \frac{(2n-2k)!}{(n-k)!\;(n-2k)!\;k!\;2^n} x^{n-2k}$$

relation to spherical harmonics

$$Y_{lm}(\vartheta, \varphi) := \frac{1}{\sqrt{2\pi}} N_{lm} P_{lm}(\cos\vartheta) e^{im\varphi}$$

$$N_{lm} := \sqrt{\frac{2l+1}{2} \cdot \frac{(l-m)!}{(l+m)!}}$$

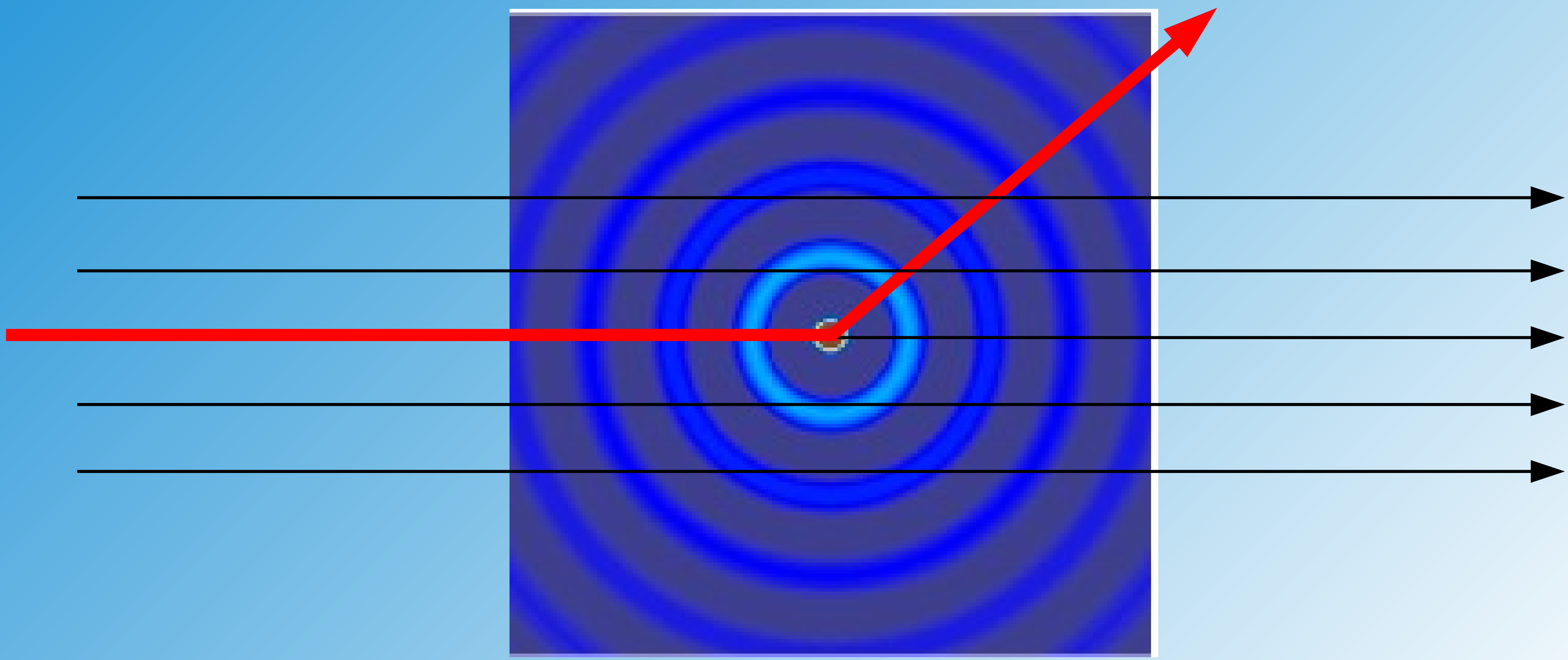


# Spherical Waves

scattering at disc of size a

$$a = \lim_{k \to \infty} \frac{\sin \delta}{k}$$

resonance if a ~ k



# Optical Theorem and Partial Waves

partial wave decomposition:

$$f^{el}(\theta, \sqrt{s}) = \frac{1}{|\vec{p}|} \sum_l (2l+1)\, t_l(\sqrt{s})\, P_l(\cos\theta)$$

partial wave amplitude and scattering phase:

$$t_l = e^{i\delta_l} \sin\delta_l \qquad \left|t_l^{\,2}\right| = \sin^2\delta_l$$

$$\Im(t_l) = \Im\left(e^{i\delta_l}\sin\delta_l\right) = \Im\left(\cos\delta_l \sin\delta_l + i\sin^2\delta_l\right) = \sin^2\delta_l$$

forward amplitude:

$$P_l(\cos\theta = 0) = 1$$

→ Plot

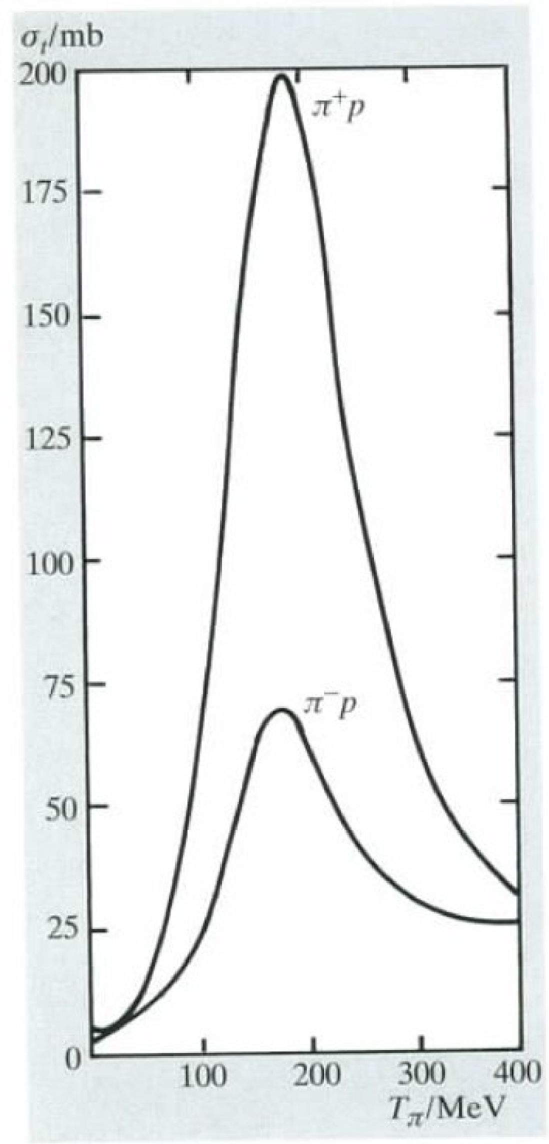
$$\sin^2\delta = \frac{p\,\Im(f_{el}(\theta=0))}{\sum_l (2l+1)}$$

finally:

$$\sigma_{tot} = \frac{4\pi}{|\vec{p}|^2} \sum_l (2l+1) \sin^2\delta \quad = \quad \frac{4\pi}{|\vec{p}|}\, \Im(f^{el}(\theta=0))$$

optical theorem

# Resonance Peaks



# Rotations I

General: $x_k' = R_{kl} x_l$ (orthonormal matrix) $R^{-1} = R^T$

$$R^{(3)} = \begin{pmatrix} \cos\theta_3 & -\sin\theta_3 & 0 \\ \sin\theta_3 & \cos\theta_3 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$R^{(1)}$ and $R^{(2)}$ similarly $\quad det\, R = \pm 1$

Infinitesimal Rotations:

$$R^{(3)}(d\theta) = 1 - i\, d\theta\, D_3$$

All orientations:

$$R^{(n)}(d\theta) = 1 - i\, d\theta\, n_i D_i$$
$$= 1 - i\, d\theta\, \vec{n}\, \vec{D}$$

Arbitrary Rotation:

$$\lim_{m\to\infty} = \left(1 - i\frac{\theta}{m}\vec{n}\,\vec{D}\right)^m = e^{-i\theta\vec{n}\,\vec{D}}$$

Generator $D_3$ :

$$D_3 = i\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$[D_i, D_j] = i\,\epsilon_{ijk}\, D_k$ (non-abelian)

- define rotation group SO(3)

# Rotations II

More general transformation *U*:

$$U(\vec{n}, d\theta) = 1 - i\, d\theta\, n_i \hat{J}_i$$ (generator of transformation *U*)

$\hat{J}$ can be identified as spin operator (Noether theorem)

Group properties (ref: Messiah, Greiner):

$$\hat{J}^2 = \hat{J}_1{}^2 + \hat{J}_2{}^2 + \hat{J}_3{}^2$$

$$\hat{J}^2 |j, j_3\rangle = j(j+1)\,|j, j_3\rangle$$

$$\hat{j}_3 |j, j_3\rangle = j_3\,|j, j_3\rangle$$

$$\hat{j}_\pm |j, j_3\rangle = (\hat{j}_1 \pm i\,\hat{j}_2)|j, j_3\rangle$$

$$= \sqrt{(j \mp j_3)\,(j \pm j_3 + 1)}\,|j, j_3 \pm 1\rangle$$

# Rotations III

using: $U(\vec{n},\theta) = e^{-i\theta\vec{n}\vec{D}}$

$$\rightarrow U(R) = e^{-i\alpha\hat{j}_z}\, e^{-i\beta\hat{j}_u}\, e^{-i\gamma\hat{j}_z}$$ (3 rotations)

transformation: → Plot

$$\langle j;j_3'|\psi'\rangle = \sum_{j_3} \langle j;j_3'|U(R)|j;j_3\rangle \langle j;j_3|\psi\rangle$$

$$\langle j;j_3'|U(R)|j;j_3\rangle = D^j_{j_3,j_3'}(\alpha,\beta,\gamma)$$

with:

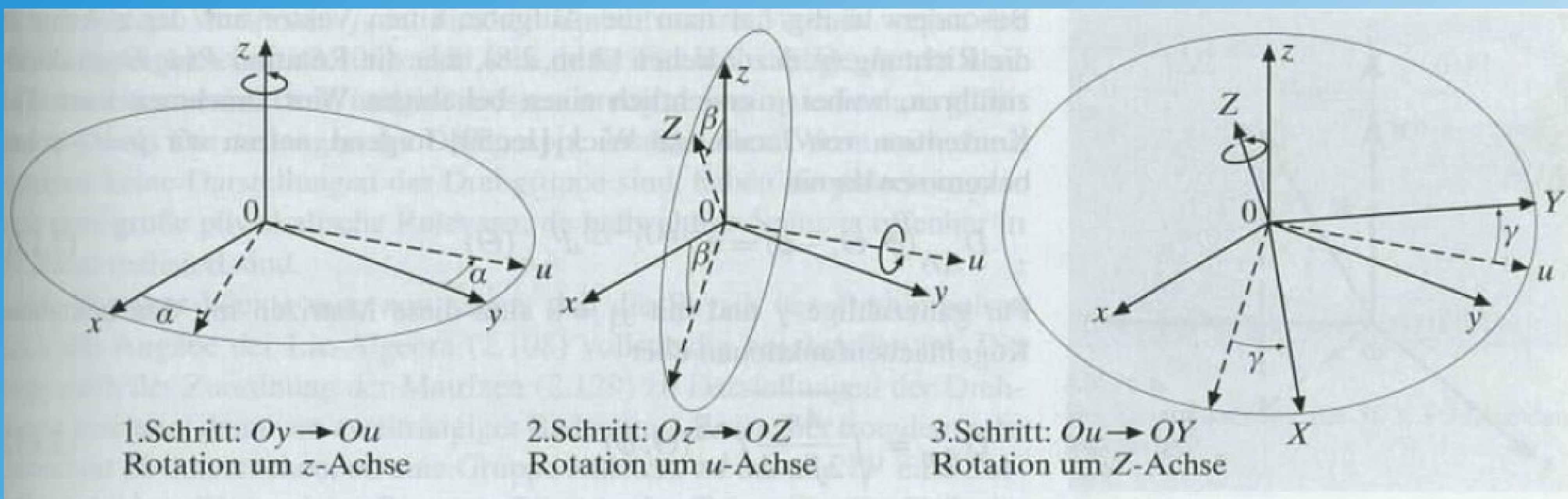
$$D^j_{j_3',j_3}(\alpha,\beta,\gamma) = e^{-i\alpha j_3'}\, d^j_{j_3',j_3}(\beta)\, e^{-i\gamma j_3}$$

**(d-functions)**

relation to spherical harmonics: → Plot

$$D^j_{j_3,0} = \sqrt{\frac{4\pi}{2j+1}\, Y^{*j_3}_j(\theta,\phi)}$$ → Plot

# Rotations in 3 dimensions



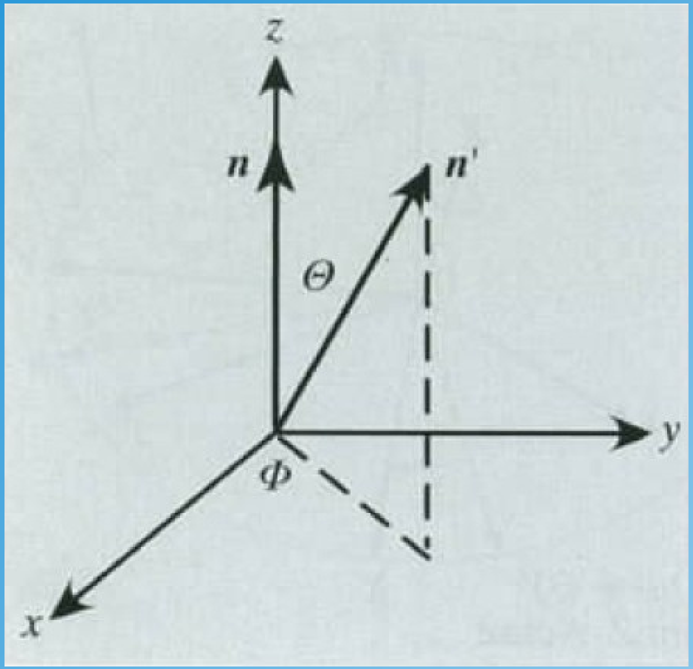
# d-Functions

**Tabelle 2.1**
Tabelle der $d^J_{M'M}(\Theta)$-Funktionen

| $J$ | $M'$ | $M$ | $d^J_{M'M}$ |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\cos\frac{\Theta}{2}$ |
| $\frac{1}{2}$ | $\frac{1}{2}$ | $-\frac{1}{2}$ | $-\sin\frac{\Theta}{2}$ |
| 1 | 1 | 1 | $\frac{1}{2}(1+\cos\Theta)$ |
| 1 | 1 | 0 | $-\frac{1}{\sqrt{2}}\sin\Theta$ |
| 1 | 1 | $-1$ | $\frac{1}{2}(1-\cos\Theta)$ |
| 1 | 0 | 0 | $\cos\Theta$ |
| $\frac{3}{2}$ | $\frac{3}{2}$ | $\frac{3}{2}$ | $\frac{1}{2}(1+\cos\Theta)\cos\frac{\Theta}{2}$ |
| $\frac{3}{2}$ | $\frac{3}{2}$ | $\frac{1}{2}$ | $-\frac{\sqrt{3}}{2}(1+\cos\Theta)\sin\frac{\Theta}{2}$ |
| $\frac{3}{2}$ | $\frac{3}{2}$ | $-\frac{1}{2}$ | $\frac{\sqrt{3}}{2}(1-\cos\Theta)\cos\frac{\Theta}{2}$ |
| $\frac{3}{2}$ | $\frac{3}{2}$ | $-\frac{3}{2}$ | $-\frac{1}{2}(1-\cos\Theta)\sin\frac{\Theta}{2}$ |
| $\frac{3}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}(3\cos\Theta-1)\cos\frac{\Theta}{2}$ |
| $\frac{3}{2}$ | $\frac{1}{2}$ | $-\frac{1}{2}$ | $-\frac{1}{2}(3\cos\Theta+1)\sin\frac{\Theta}{2}$ |

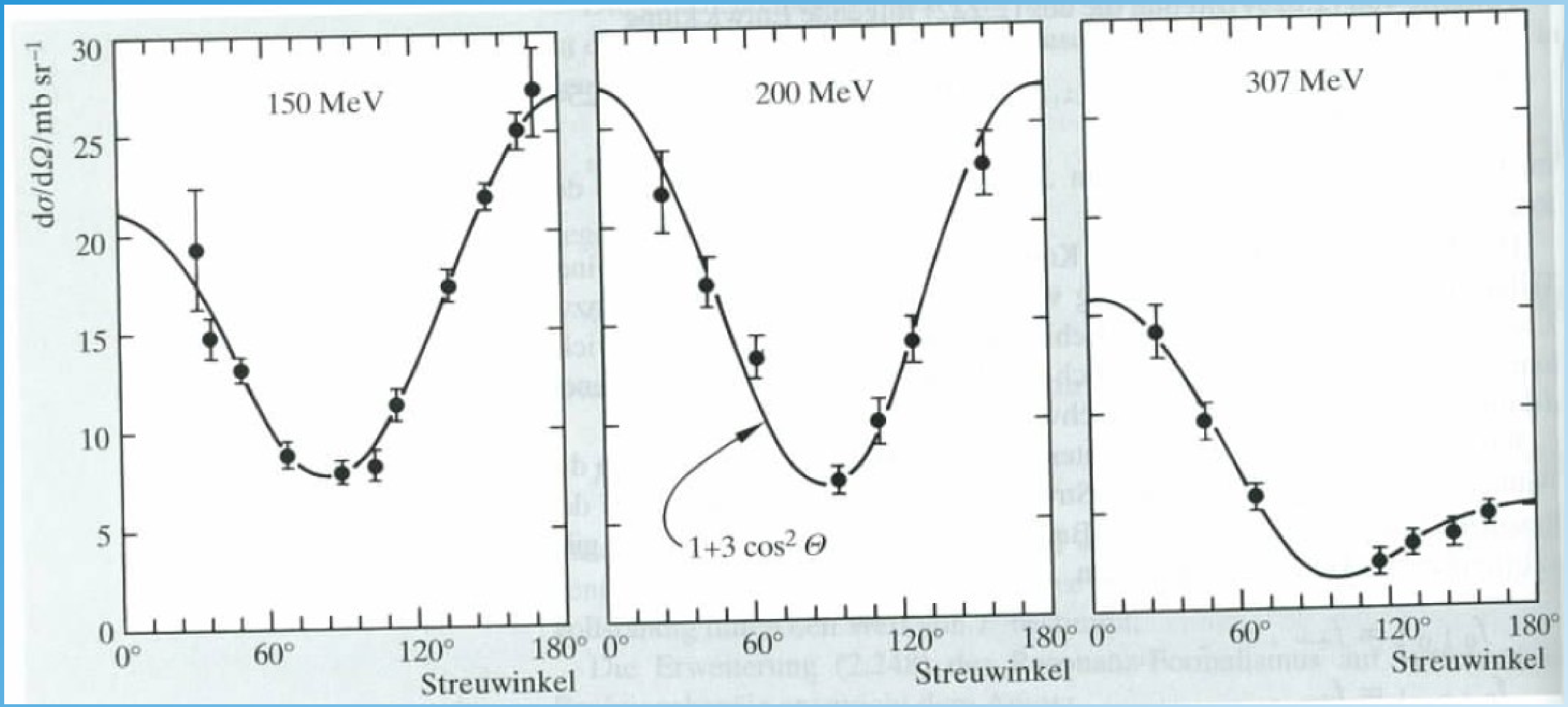
| | | | |
|---|---|---|---|
| 2 | 2 | 2 | $\frac{1}{4}(1+\cos\Theta)^2$ |
| 2 | 2 | 1 | $-\frac{1}{2}(1+\cos\Theta)\sin\Theta$ |
| 2 | 2 | 0 | $\frac{\sqrt{6}}{4}\sin^2\Theta$ |
| 2 | 2 | $-1$ | $-\frac{1}{2}(1-\cos\Theta)\sin\Theta$ |
| 2 | 2 | $-2$ | $\frac{1}{4}(1-\cos\Theta)^2$ |
| 2 | 1 | 1 | $\frac{1}{2}(1+\cos\Theta)(2\cos\Theta-1)$ |
| 2 | 1 | 0 | $-\sqrt{\frac{3}{2}}\sin\Theta\cos\Theta$ |
| 2 | 1 | $-1$ | $\frac{1}{2}(1-\cos\Theta)(2\cos\Theta+1)$ |
| 2 | 0 | 0 | $\frac{3}{2}\cos^2\Theta-\frac{1}{2}$ |

# Rotations and Spherical Harmonics




| $l$ | $m$ | $Y_l^m$ |
|---|---|---|
| 0 | 0 | $\frac{1}{\sqrt{4\pi}}$ |
| 1 | 0 | $\sqrt{\frac{3}{4\pi}}\cos\Theta$ |
| 1 | 1 | $-\sqrt{\frac{3}{8\pi}}\sin\Theta e^{i\phi}$ |
| 2 | 0 | $\sqrt{\frac{5}{4\pi}}\left(\frac{3}{2}\cos^2\Theta-\frac{1}{2}\right)$ |
| 2 | 1 | $-\sqrt{\frac{15}{8\pi}}\sin\Theta\cos\Theta e^{i\phi}$ |
| 2 | 2 | $\sqrt{\frac{15}{32\pi}}\sin^2\Theta e^{i2\phi}$ |

# Decay Angle Distributions




Pion - Proton Scattering

# 43. CLEBSCH-GORDAN COEFFICIENTS, SPHERICAL HARMONICS, AND $d$ FUNCTIONS

Note: A square-root sign is to be understood over *every* coefficient, *e.g.*, for $-8/15$ read $-\sqrt{8/15}$.

Notation:

| | | $J$ | $J$ | ... |
|---|---|---|---|---|
| | | $M$ | $M$ | ... |
| $m_1$ | $m_2$ | Coefficients | | |
| $m_1$ | $m_2$ | | | |
| . | . | | | |

$$Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta$$

$$Y_1^1 = -\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{i\phi}$$

$$Y_2^0 = \sqrt{\frac{5}{4\pi}}\left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)$$

$$Y_2^1 = -\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta\, e^{i\phi}$$

$$Y_2^2 = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2\theta\, e^{2i\phi}$$

$$Y_\ell^{-m} = (-1)^m Y_\ell^{m*}$$

$$d^\ell_{m,0} = \sqrt{\frac{4\pi}{2\ell+1}}\, Y_\ell^m\, e^{-im\phi}$$

$$\langle j_1 j_2 m_1 m_2 | j_1 j_2 J M\rangle = (-1)^{J-j_1-j_2}\langle j_2 j_1 m_2 m_1 | j_2 j_1 J M\rangle$$

## 1/2×1/2

| $m_1$ | $m_2$ | J=1, M=+1 |
|---|---|---|
| +1/2 | +1/2 | 1 |

| $m_1$ | $m_2$ | J=1, M=0 | J=0, M=0 |
|---|---|---|---|
| +1/2 | −1/2 | 1/2 | 1/2 |
| −1/2 | +1/2 | 1/2 | −1/2 |

| $m_1$ | $m_2$ | J=1, M=−1 |
|---|---|---|
| −1/2 | −1/2 | 1 |

## 1×1/2

| $m_1$ | $m_2$ | J=3/2, M=+3/2 |
|---|---|---|
| +1 | +1/2 | 1 |

| $m_1$ | $m_2$ | J=3/2, M=+1/2 | J=1/2, M=+1/2 |
|---|---|---|---|
| +1 | −1/2 | 1/3 | 2/3 |
| 0 | +1/2 | 2/3 | −1/3 |

| $m_1$ | $m_2$ | J=3/2, M=−1/2 | J=1/2, M=−1/2 |
|---|---|---|---|
| 0 | −1/2 | 2/3 | 1/3 |
| −1 | +1/2 | 1/3 | −2/3 |

| $m_1$ | $m_2$ | J=3/2, M=−3/2 |
|---|---|---|
| −1 | −1/2 | 1 |

## 2×1

| $m_1$ | $m_2$ | J=3, M=+3 |
|---|---|---|
| +2 | +1 | 1 |

| $m_1$ | $m_2$ | J=3, M=+2 | J=2, M=+2 |
|---|---|---|---|
| +2 | 0 | 1/3 | 2/3 |
| +1 | +1 | 2/3 | −1/3 |

| $m_1$ | $m_2$ | J=3, M=+1 | J=2, M=+1 | J=1, M=+1 |
|---|---|---|---|---|
| +2 | −1 | 1/15 | 1/3 | 3/5 |
| +1 | 0 | 8/15 | 1/6 | −3/10 |
| 0 | +1 | 2/5 | −1/2 | 1/10 |

| $m_1$ | $m_2$ | J=3, M=0 | J=2, M=0 | J=1, M=0 |
|---|---|---|---|---|
| +1 | −1 | 1/5 | 1/2 | 3/10 |
| 0 | 0 | 3/5 | 0 | −2/5 |
| −1 | +1 | 1/5 | −1/2 | 3/10 |

| $m_1$ | $m_2$ | J=3, M=−1 | J=2, M=−1 | J=1, M=−1 |
|---|---|---|---|---|
| 0 | −1 | 2/5 | 1/2 | 1/10 |
| −1 | 0 | 8/15 | −1/6 | −3/10 |
| −2 | +1 | 1/15 | −1/3 | 3/5 |

| $m_1$ | $m_2$ | J=3, M=−2 | J=2, M=−2 |
|---|---|---|---|
| −1 | −1 | 2/3 | 1/3 |
| −2 | 0 | 1/3 | −2/3 |

| $m_1$ | $m_2$ | J=3, M=−3 |
|---|---|---|
| −2 | −1 | 1 |

## 1×1

| $m_1$ | $m_2$ | J=2, M=+2 |
|---|---|---|
| +1 | +1 | 1 |

| $m_1$ | $m_2$ | J=2, M=+1 | J=1, M=+1 |
|---|---|---|---|
| +1 | 0 | 1/2 | 1/2 |
| 0 | +1 | 1/2 | −1/2 |

| $m_1$ | $m_2$ | J=2, M=0 | J=1, M=0 | J=0, M=0 |
|---|---|---|---|---|
| +1 | −1 | 1/6 | 1/2 | 1/3 |
| 0 | 0 | 2/3 | 0 | −1/3 |
| −1 | +1 | 1/6 | −1/2 | 1/3 |

| $m_1$ | $m_2$ | J=2, M=−1 | J=1, M=−1 |
|---|---|---|---|
| 0 | −1 | 1/2 | 1/2 |
| −1 | 0 | 1/2 | −1/2 |

| $m_1$ | $m_2$ | J=2, M=−2 |
|---|---|---|
| −1 | −1 | 1 |

## 2×1/2

| $m_1$ | $m_2$ | J=5/2, M=+5/2 |
|---|---|---|
| +2 | +1/2 | 1 |

| $m_1$ | $m_2$ | J=5/2, M=+3/2 | J=3/2, M=+3/2 |
|---|---|---|---|
| +2 | −1/2 | 1/5 | 4/5 |
| +1 | +1/2 | 4/5 | −1/5 |

| $m_1$ | $m_2$ | J=5/2, M=+1/2 | J=3/2, M=+1/2 |
|---|---|---|---|
| +1 | −1/2 | 2/5 | 3/5 |
| 0 | +1/2 | 3/5 | −2/5 |

| $m_1$ | $m_2$ | J=5/2, M=−1/2 | J=3/2, M=−1/2 |
|---|---|---|---|
| 0 | −1/2 | 3/5 | 2/5 |
| −1 | +1/2 | 2/5 | −3/5 |

| $m_1$ | $m_2$ | J=5/2, M=−3/2 | J=3/2, M=−3/2 |
|---|---|---|---|
| −1 | −1/2 | 4/5 | 1/5 |
| −2 | +1/2 | 1/5 | −4/5 |

| $m_1$ | $m_2$ | J=5/2, M=−5/2 |
|---|---|---|
| −2 | −1/2 | 1 |

## 3/2×1/2

| $m_1$ | $m_2$ | J=2, M=+2 |
|---|---|---|
| +3/2 | +1/2 | 1 |

| $m_1$ | $m_2$ | J=2, M=+1 | J=1, M=+1 |
|---|---|---|---|
| +3/2 | −1/2 | 1/4 | 3/4 |
| +1/2 | +1/2 | 3/4 | −1/4 |

| $m_1$ | $m_2$ | J=2, M=0 | J=1, M=0 |
|---|---|---|---|
| +1/2 | −1/2 | 1/2 | 1/2 |
| −1/2 | +1/2 | 1/2 | −1/2 |

| $m_1$ | $m_2$ | J=2, M=−1 | J=1, M=−1 |
|---|---|---|---|
| −1/2 | −1/2 | 3/4 | 1/4 |
| −3/2 | +1/2 | 1/4 | −3/4 |

| $m_1$ | $m_2$ | J=2, M=−2 |
|---|---|---|
| −3/2 | −1/2 | 1 |

## 3/2×1

| $m_1$ | $m_2$ | J=5/2, M=+5/2 |
|---|---|---|
| +3/2 | +1 | 1 |

| $m_1$ | $m_2$ | J=5/2, M=+3/2 | J=3/2, M=+3/2 |
|---|---|---|---|
| +3/2 | 0 | 2/5 | 3/5 |
| +1/2 | +1 | 3/5 | −2/5 |

| $m_1$ | $m_2$ | J=5/2, M=+1/2 | J=3/2, M=+1/2 | J=1/2, M=+1/2 |
|---|---|---|---|---|
| +3/2 | −1 | 1/10 | 2/5 | 1/2 |
| +1/2 | 0 | 3/5 | 1/15 | −1/3 |
| −1/2 | +1 | 3/10 | −8/15 | 1/6 |

| $m_1$ | $m_2$ | J=5/2, M=−1/2 | J=3/2, M=−1/2 | J=1/2, M=−1/2 |
|---|---|---|---|---|
| +1/2 | −1 | 3/10 | 8/15 | 1/6 |
| −1/2 | 0 | 3/5 | −1/15 | −1/3 |
| −3/2 | +1 | 1/10 | −2/5 | 1/2 |

| $m_1$ | $m_2$ | J=5/2, M=−3/2 | J=3/2, M=−3/2 |
|---|---|---|---|
| −1/2 | −1 | 3/5 | 2/5 |
| −3/2 | 0 | 2/5 | −3/5 |

| $m_1$ | $m_2$ | J=5/2, M=−5/2 |
|---|---|---|
| −3/2 | −1 | 1 |

$$d^{j}_{m',m} = (-1)^{m-m'} d^{j}_{m,m'} = d^{j}_{-m,-m'}$$

$$d^{1}_{0,0} = \cos\theta \qquad d^{1/2}_{1/2,1/2} = \cos\frac{\theta}{2} \qquad d^{1/2}_{1/2,-1/2} = -\sin\frac{\theta}{2}$$

$$d^{1}_{1,1} = \frac{1+\cos\theta}{2} \qquad d^{1}_{1,0} = -\frac{\sin\theta}{\sqrt{2}} \qquad d^{1}_{1,-1} = \frac{1-\cos\theta}{2}$$

## 3/2×3/2

| $m_1$ $m_2$ | J=3, M=+3 |
|---|---|
| +3/2 +3/2 | 1 |

| $m_1$ $m_2$ | J=3, M=+2 | J=2, M=+2 |
|---|---|---|
| +3/2 +1/2 | 1/2 | 1/2 |
| +1/2 +3/2 | 1/2 | −1/2 |

| $m_1$ $m_2$ | J=3, M=+1 | J=2, M=+1 | J=1, M=+1 |
|---|---|---|---|
| +3/2 −1/2 | 1/5 | 1/2 | 3/10 |
| +1/2 +1/2 | 3/5 | 0 | −2/5 |
| −1/2 +3/2 | 1/5 | −1/2 | 3/10 |

| $m_1$ $m_2$ | J=3, M=0 | J=2, M=0 | J=1, M=0 | J=0, M=0 |
|---|---|---|---|---|
| +3/2 −3/2 | 1/20 | 1/4 | 9/20 | 1/4 |
| +1/2 −1/2 | 9/20 | 1/4 | −1/20 | −1/4 |
| −1/2 +1/2 | 9/20 | −1/4 | −1/20 | 1/4 |
| −3/2 +3/2 | 1/20 | −1/4 | 9/20 | −1/4 |

| $m_1$ $m_2$ | J=3, M=−1 | J=2, M=−1 | J=1, M=−1 |
|---|---|---|---|
| +1/2 −3/2 | 1/5 | 1/2 | 3/10 |
| −1/2 −1/2 | 3/5 | 0 | −2/5 |
| −3/2 +1/2 | 1/5 | −1/2 | 3/10 |

| $m_1$ $m_2$ | J=3, M=−2 | J=2, M=−2 |
|---|---|---|
| −1/2 −3/2 | 1/2 | 1/2 |
| −3/2 −1/2 | 1/2 | −1/2 |

| $m_1$ $m_2$ | J=3, M=−3 |
|---|---|
| −3/2 −3/2 | 1 |

## 2×3/2

| $m_1$ $m_2$ | J=7/2, M=+7/2 |
|---|---|
| +2 +3/2 | 1 |

| $m_1$ $m_2$ | J=7/2, M=+5/2 | J=5/2, M=+5/2 |
|---|---|---|
| +2 +1/2 | 3/7 | 4/7 |
| +1 +3/2 | 4/7 | −3/7 |

| $m_1$ $m_2$ | J=7/2, M=+3/2 | J=5/2, M=+3/2 | J=3/2, M=+3/2 |
|---|---|---|---|
| +2 −1/2 | 1/7 | 16/35 | 2/5 |
| +1 +1/2 | 4/7 | 1/35 | −2/5 |
| 0 +3/2 | 2/7 | −18/35 | 1/5 |

| $m_1$ $m_2$ | J=7/2, M=+1/2 | J=5/2, M=+1/2 | J=3/2, M=+1/2 | J=1/2, M=+1/2 |
|---|---|---|---|---|
| +2 −3/2 | 1/35 | 6/35 | 2/5 | 2/5 |
| +1 −1/2 | 12/35 | 5/14 | 0 | −3/10 |
| 0 +1/2 | 18/35 | −3/35 | −1/5 | 1/5 |
| −1 +3/2 | 4/35 | −27/70 | 2/5 | −1/10 |

| $m_1$ $m_2$ | J=7/2, M=−1/2 | J=5/2, M=−1/2 | J=3/2, M=−1/2 | J=1/2, M=−1/2 |
|---|---|---|---|---|
| +1 −3/2 | 4/35 | 27/70 | 2/5 | 1/10 |
| 0 −1/2 | 18/35 | 3/35 | −1/5 | −1/5 |
| −1 +1/2 | 12/35 | −5/14 | 0 | 3/10 |
| −2 +3/2 | 1/35 | −6/35 | 2/5 | −2/5 |

| $m_1$ $m_2$ | J=7/2, M=−3/2 | J=5/2, M=−3/2 | J=3/2, M=−3/2 |
|---|---|---|---|
| 0 −3/2 | 2/7 | 18/35 | 1/5 |
| −1 −1/2 | 4/7 | −1/35 | −2/5 |
| −2 +1/2 | 1/7 | −16/35 | 2/5 |

| $m_1$ $m_2$ | J=7/2, M=−5/2 | J=5/2, M=−5/2 |
|---|---|---|
| −1 −3/2 | 4/7 | 3/7 |
| −2 −1/2 | 3/7 | −4/7 |

| $m_1$ $m_2$ | J=7/2, M=−7/2 |
|---|---|
| −2 −3/2 | 1 |

## 2×2

| $m_1$ $m_2$ | J=4, M=+4 |
|---|---|
| +2 +2 | 1 |

| $m_1$ $m_2$ | J=4, M=+3 | J=3, M=+3 |
|---|---|---|
| +2 +1 | 1/2 | 1/2 |
| +1 +2 | 1/2 | −1/2 |

| $m_1$ $m_2$ | J=4, M=+2 | J=3, M=+2 | J=2, M=+2 |
|---|---|---|---|
| +2 0 | 3/14 | 1/2 | 2/7 |
| +1 +1 | 4/7 | 0 | −3/7 |
| 0 +2 | 3/14 | −1/2 | 2/7 |

| $m_1$ $m_2$ | J=4, M=+1 | J=3, M=+1 | J=2, M=+1 | J=1, M=+1 |
|---|---|---|---|---|
| +2 −1 | 1/14 | 3/10 | 3/7 | 1/5 |
| +1 0 | 3/7 | 1/5 | −1/14 | −3/10 |
| 0 +1 | 3/7 | −1/5 | −1/14 | 3/10 |
| −1 +2 | 1/14 | −3/10 | 3/7 | −1/5 |

| $m_1$ $m_2$ | J=4, M=0 | J=3, M=0 | J=2, M=0 | J=1, M=0 | J=0, M=0 |
|---|---|---|---|---|---|
| +2 −2 | 1/70 | 1/10 | 2/7 | 2/5 | 1/5 |
| +1 −1 | 8/35 | 2/5 | 1/14 | −1/10 | −1/5 |
| 0 0 | 18/35 | 0 | −2/7 | 0 | 1/5 |
| −1 +1 | 8/35 | −2/5 | 1/14 | 1/10 | −1/5 |
| −2 +2 | 1/70 | −1/10 | 2/7 | −2/5 | 1/5 |

| $m_1$ $m_2$ | J=4, M=−1 | J=3, M=−1 | J=2, M=−1 | J=1, M=−1 |
|---|---|---|---|---|
| +1 −2 | 1/14 | 3/10 | 3/7 | 1/5 |
| 0 −1 | 3/7 | 1/5 | −1/14 | −3/10 |
| −1 0 | 3/7 | −1/5 | −1/14 | 3/10 |
| −2 +1 | 1/14 | −3/10 | 3/7 | −1/5 |

| $m_1$ $m_2$ | J=4, M=−2 | J=3, M=−2 | J=2, M=−2 |
|---|---|---|---|
| 0 −2 | 3/14 | 1/2 | 2/7 |
| −1 −1 | 4/7 | 0 | −3/7 |
| −2 0 | 3/14 | −1/2 | 2/7 |

| $m_1$ $m_2$ | J=4, M=−3 | J=3, M=−3 |
|---|---|---|
| −1 −2 | 1/2 | 1/2 |
| −2 −1 | 1/2 | −1/2 |

| $m_1$ $m_2$ | J=4, M=−4 |
|---|---|
| −2 −2 | 1 |

$$d^{3/2}_{3/2,3/2} = \frac{1+\cos\theta}{2}\cos\frac{\theta}{2}$$

$$d^{3/2}_{3/2,1/2} = -\sqrt{3}\frac{1+\cos\theta}{2}\sin\frac{\theta}{2}$$

$$d^{3/2}_{3/2,-1/2} = \sqrt{3}\frac{1-\cos\theta}{2}\cos\frac{\theta}{2}$$

$$d^{3/2}_{3/2,-3/2} = -\frac{1-\cos\theta}{2}\sin\frac{\theta}{2}$$

$$d^{3/2}_{1/2,1/2} = \frac{3\cos\theta-1}{2}\cos\frac{\theta}{2}$$

$$d^{3/2}_{1/2,-1/2} = -\frac{3\cos\theta+1}{2}\sin\frac{\theta}{2}$$

$$d^{2}_{2,2} = \left(\frac{1+\cos\theta}{2}\right)^2$$

$$d^{2}_{2,1} = -\frac{1+\cos\theta}{2}\sin\theta$$

$$d^{2}_{2,0} = \frac{\sqrt{6}}{4}\sin^2\theta$$

$$d^{2}_{2,-1} = -\frac{1-\cos\theta}{2}\sin\theta$$

$$d^{2}_{2,-2} = \left(\frac{1-\cos\theta}{2}\right)^2$$

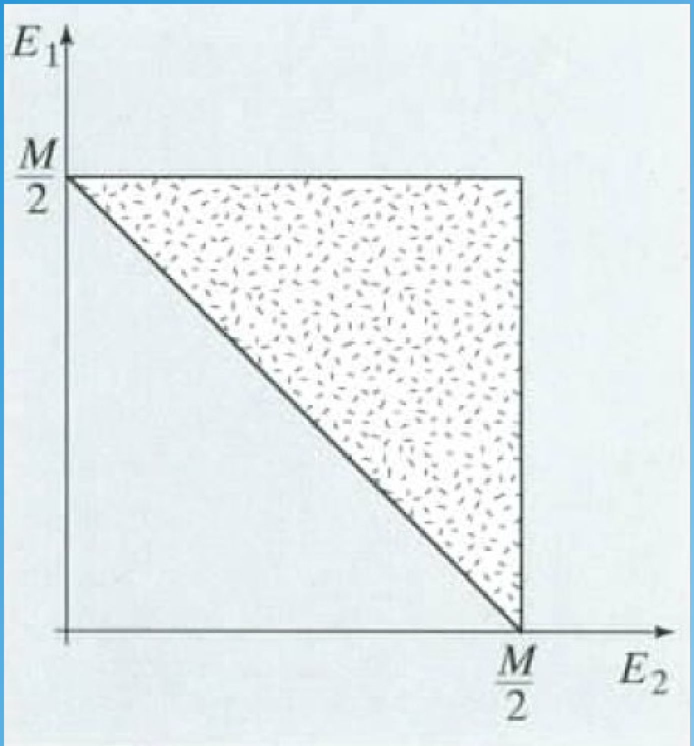
$$d^{2}_{1,1} = \frac{1+\cos\theta}{2}(2\cos\theta-1)$$

$$d^{2}_{1,0} = -\sqrt{\frac{3}{2}}\sin\theta\cos\theta$$

$$d^{2}_{1,-1} = \frac{1-\cos\theta}{2}(2\cos\theta+1)$$

$$d^{2}_{0,0} = \left(\frac{3}{2}\cos^2\theta-\frac{1}{2}\right)$$

# Dalitz Plot



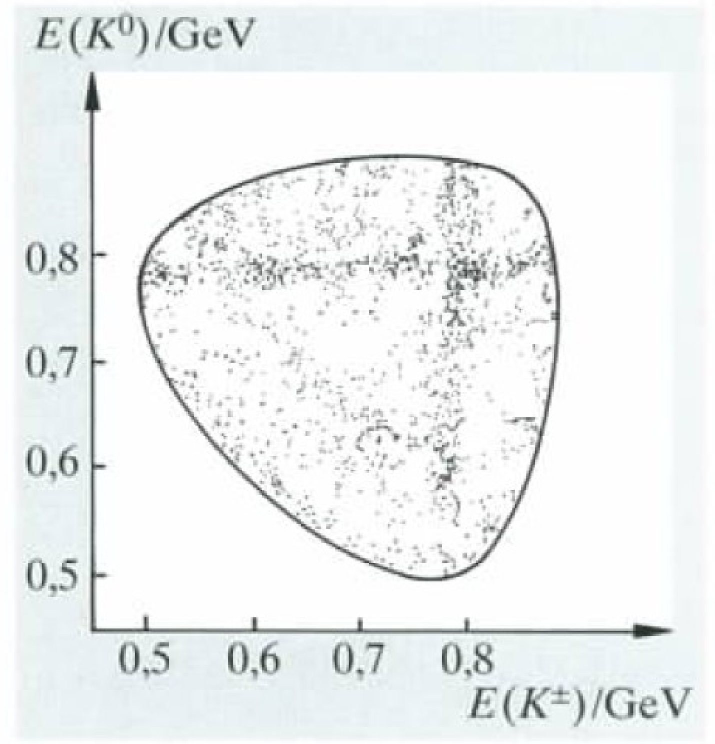
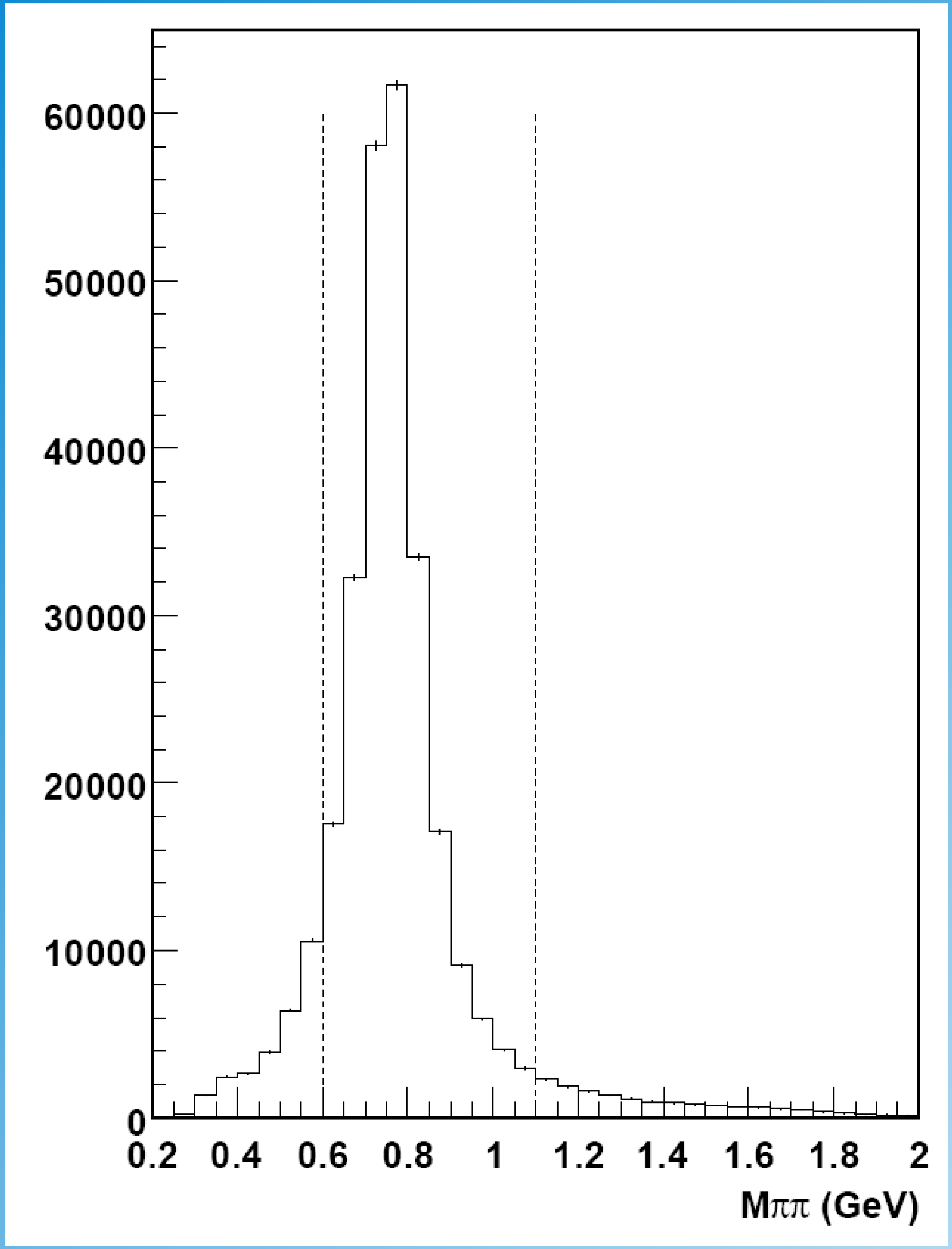




**Abb. 2.2**
Beispiel für eine Dalitz-Auftragung. Untersucht wurde die Reaktion $p + \bar{p} \rightarrow \overset{(-)}{K}{}^0 + K^{\mp} + \pi^{\pm}$. Die erhöhte Punktdichte beweist die Bildung von Kaon-Resonanzen

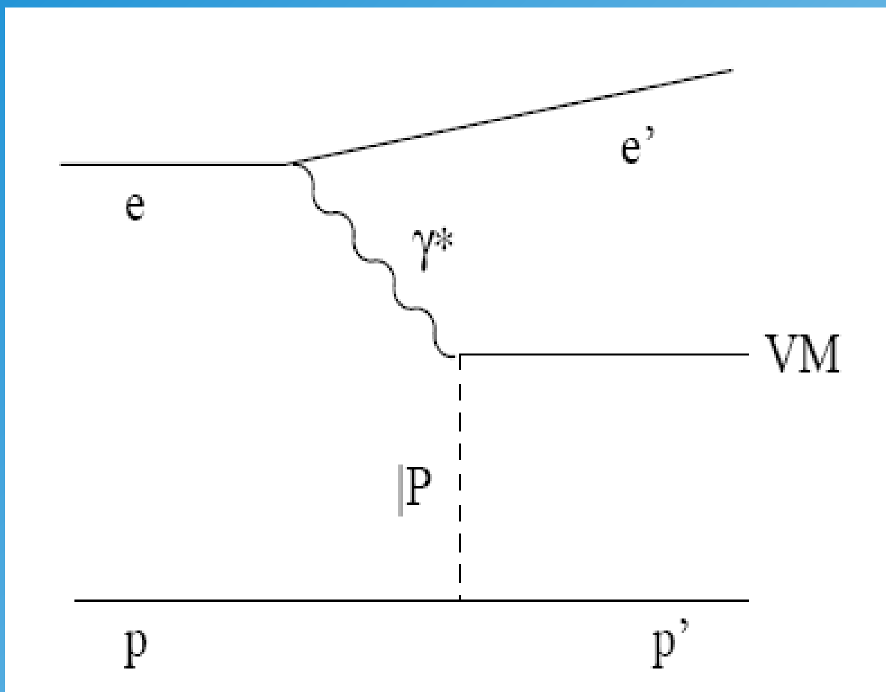
# Rho Mass Peak




measured at HERA:

$\rho^0 \longrightarrow \pi^+ \pi^-$

# Decays of Photoproduced Rho




$$ep \rightarrow ep\, \rho^0$$

$$\rho^0 \rightarrow \pi^+ \pi^-$$