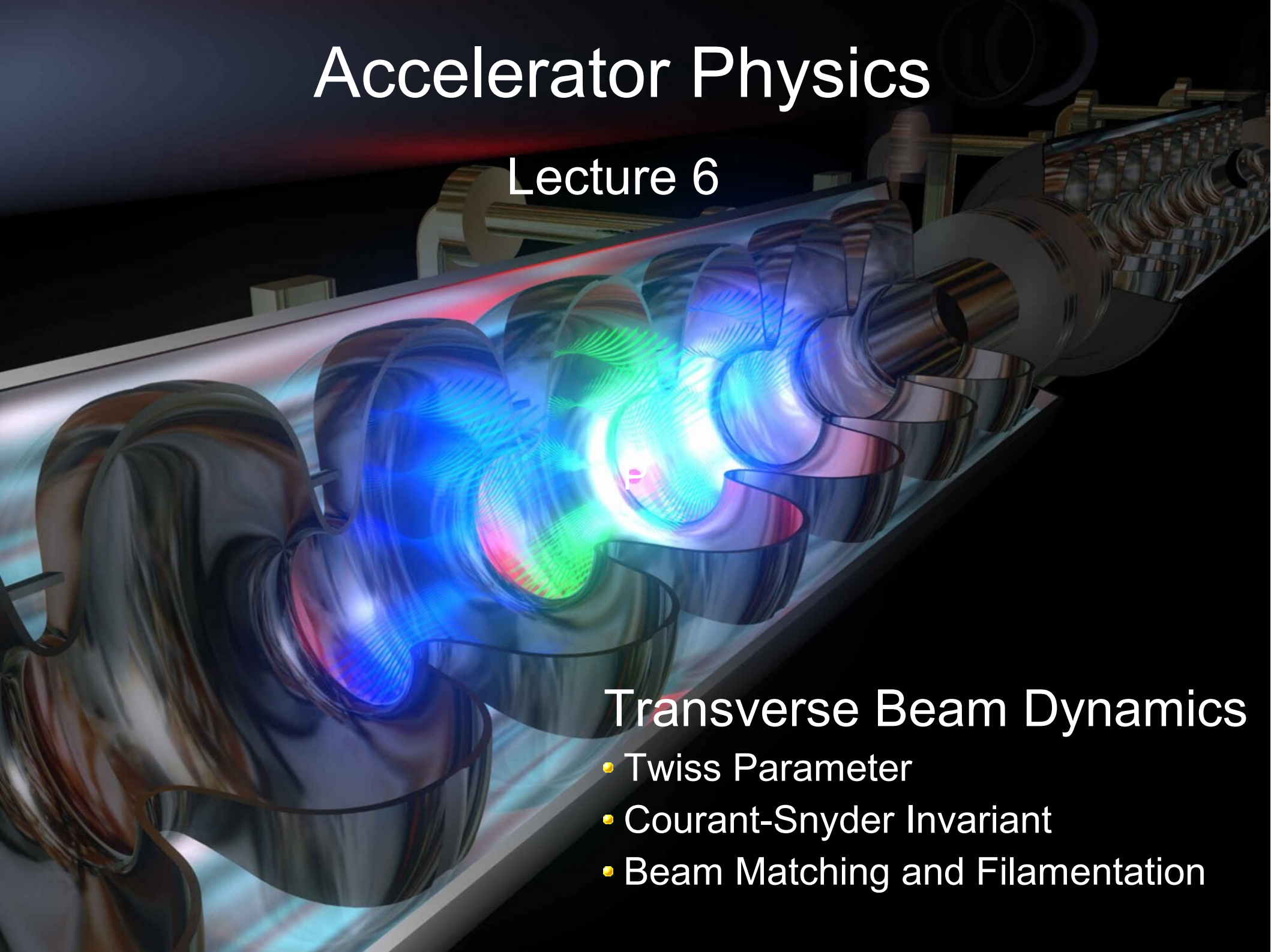


Accelerator Physics

Lecture 6



Transverse Beam Dynamics

- Twiss Parameter
- Courant-Snyder Invariant
- Beam Matching and Filamentation

SPS Lattice

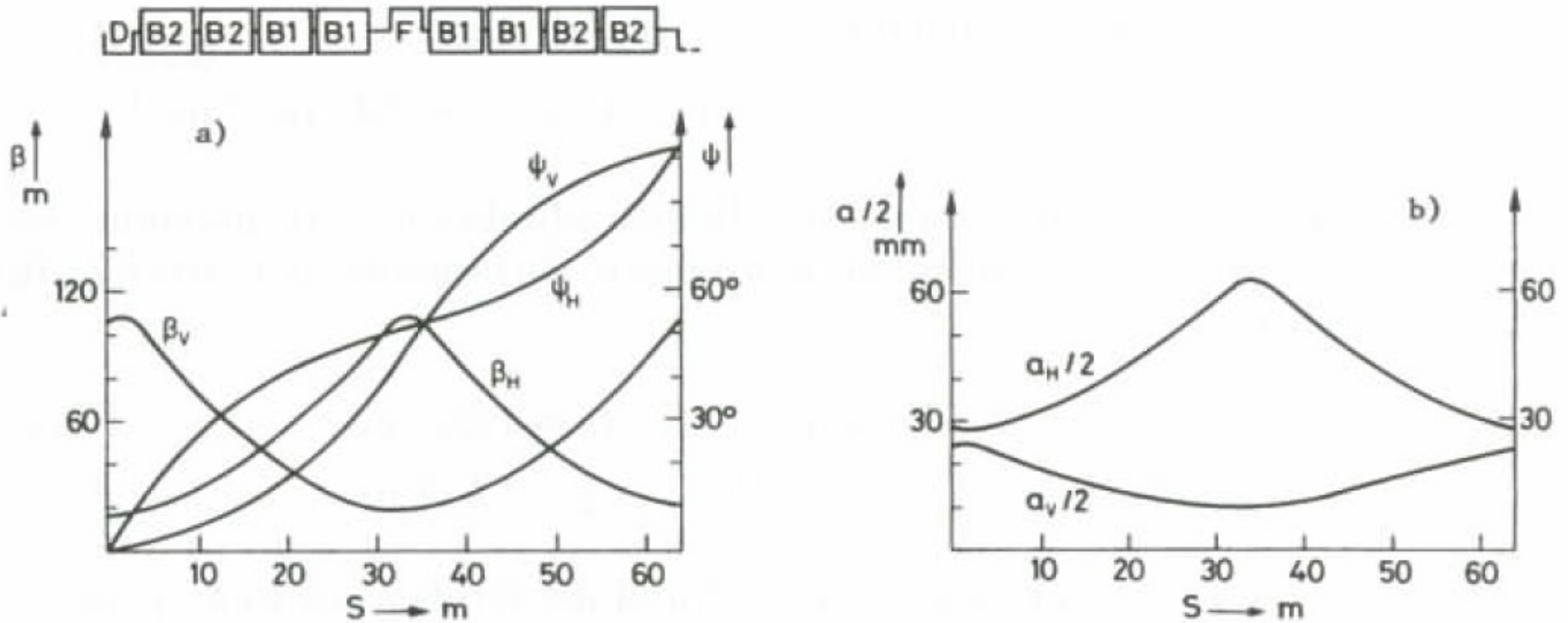
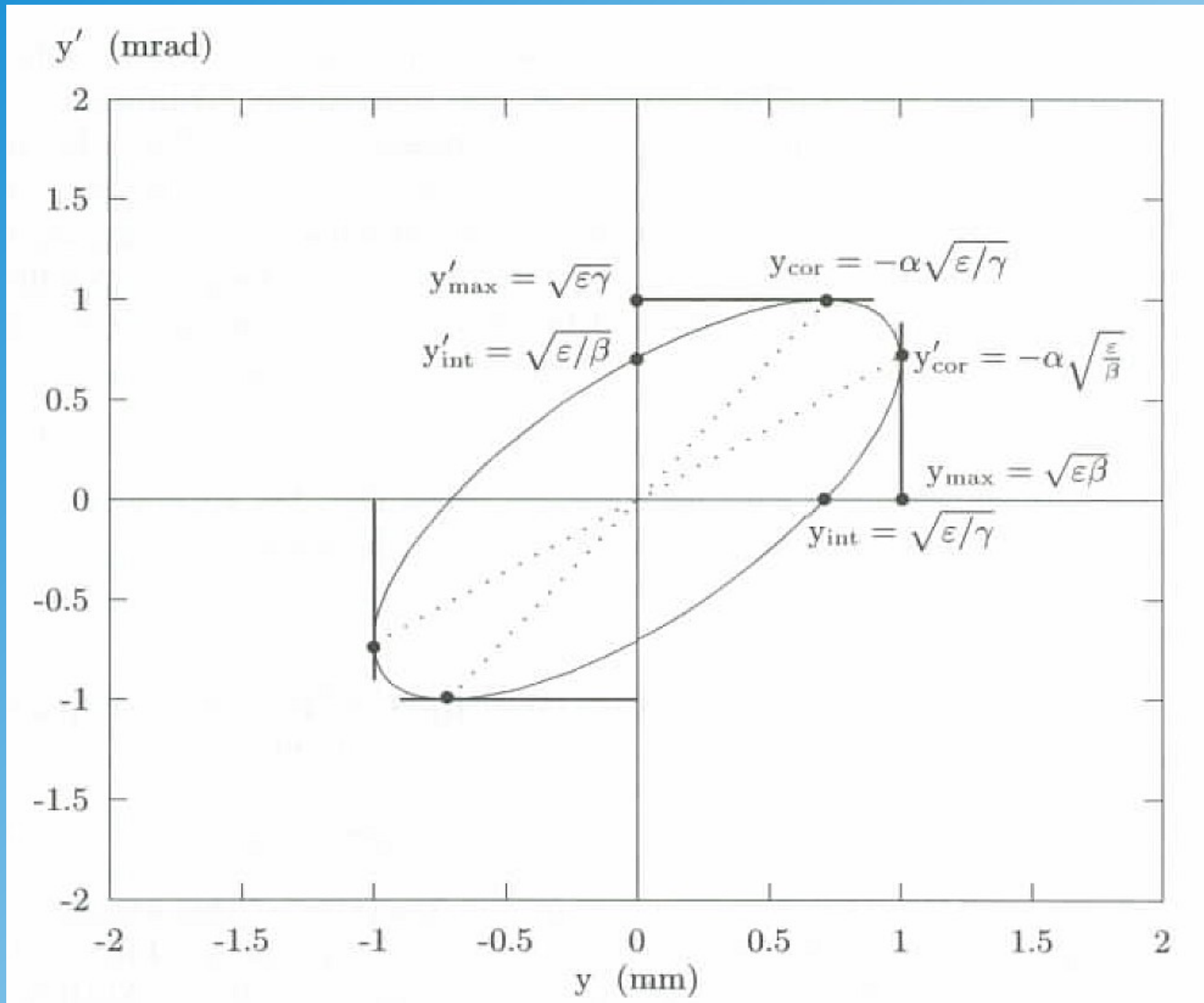
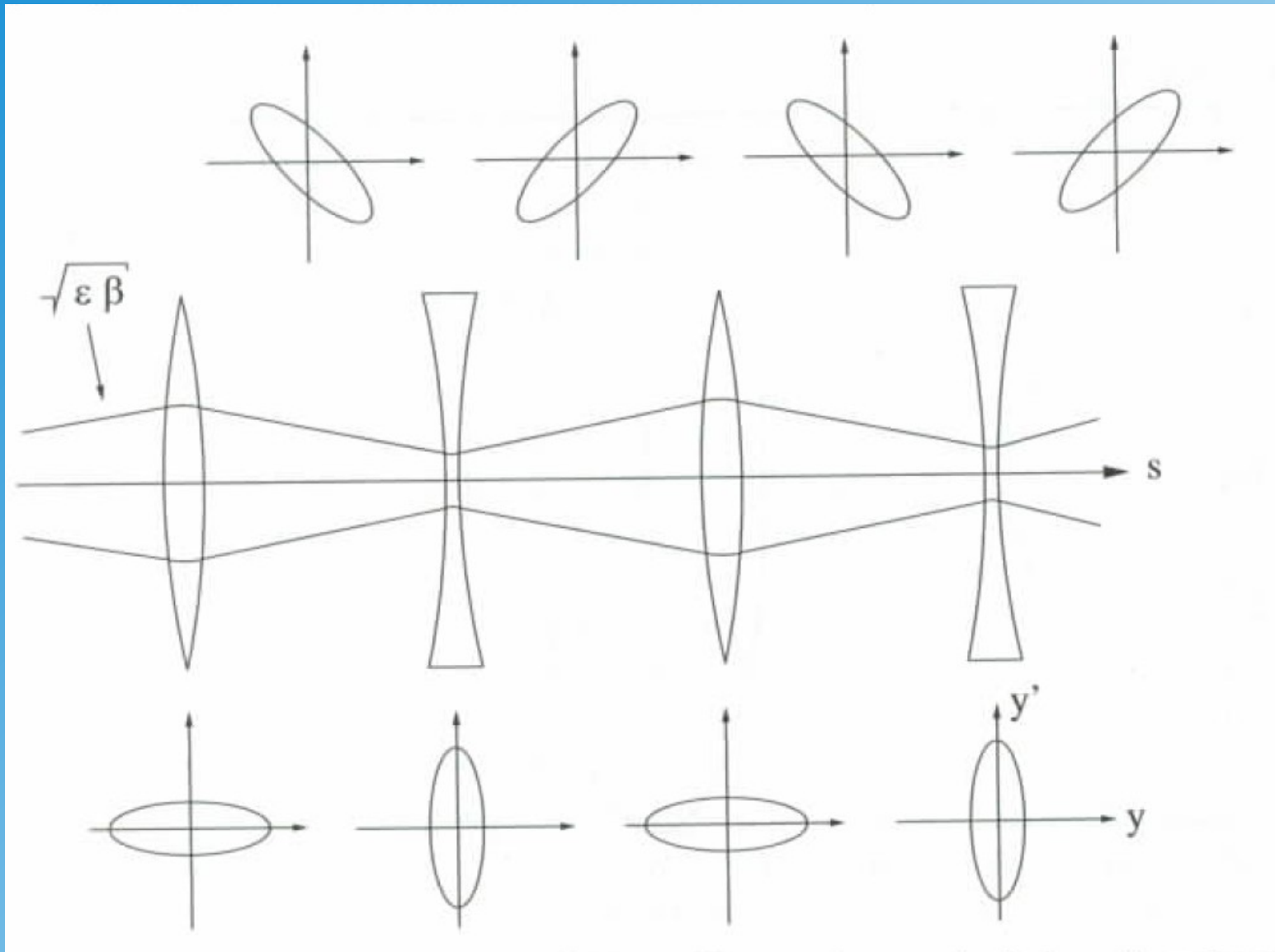


Abb. 6.11. SPS-Einheitszelle: Horizontale und vertikale Betatronfunktion $\beta_x = \beta_H$ und $\beta_y = \beta_V$, Betatronphasen $\psi_x = \phi_H$ und $\psi_y = \phi_V$ und Enveloppen $a_H/2 = \sqrt{\beta_x \epsilon_x}$, $a_V/2 = \sqrt{\beta_y \epsilon_y}$. Aus [Wi85] entnommen

Phase Space Ellipse



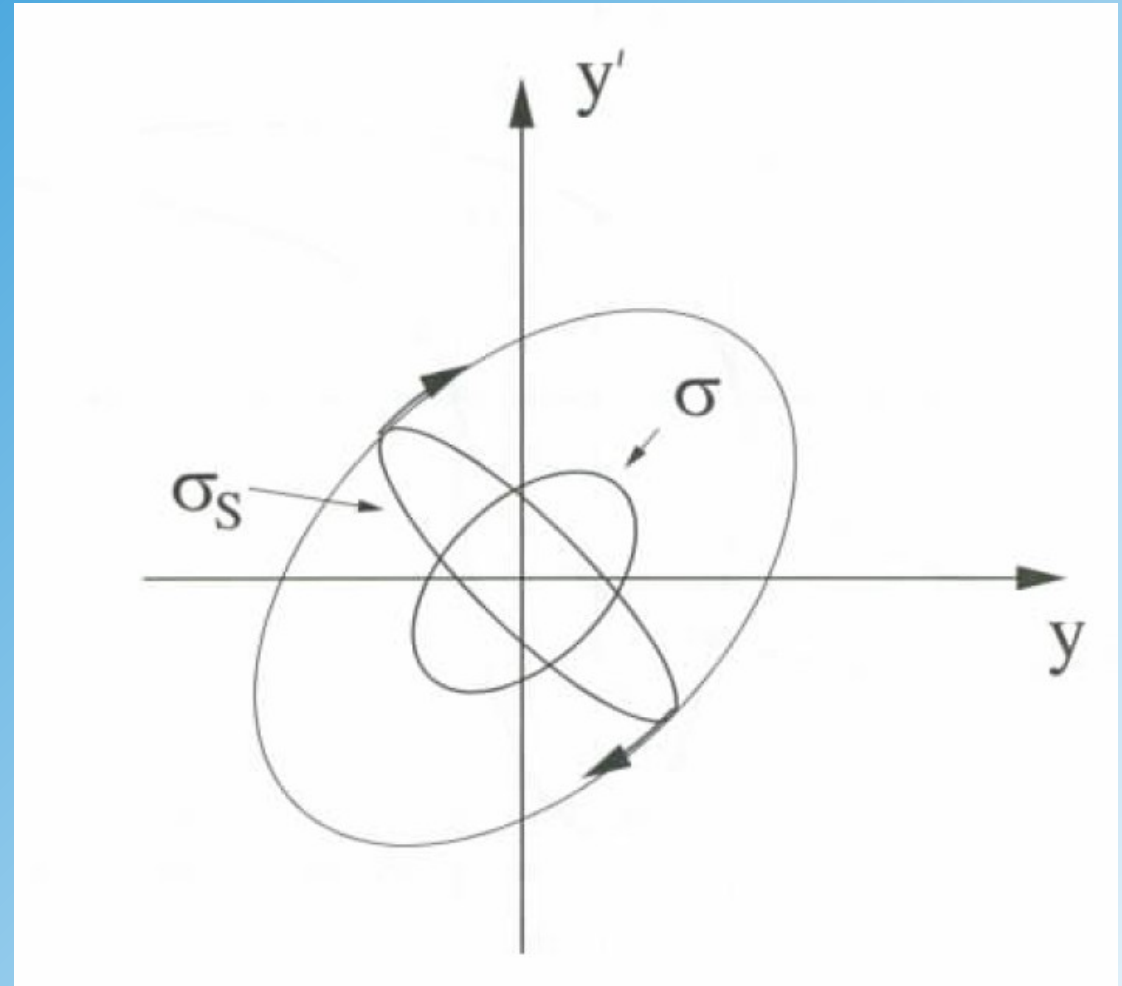
Beam Envelopes in Periodic Structure



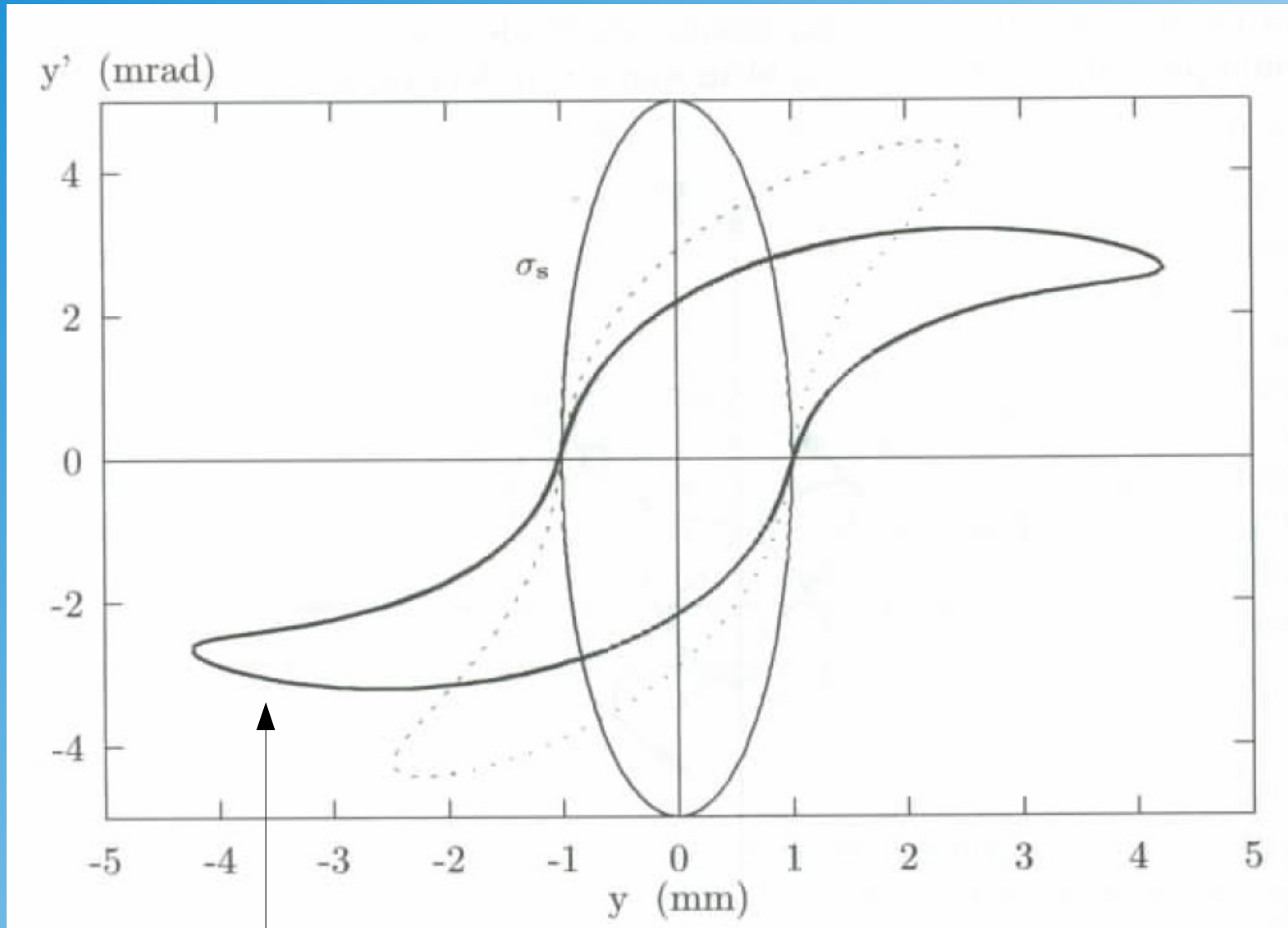
Beam-Machine Matching

σ_s = beam ellipse

σ = machine ellipse

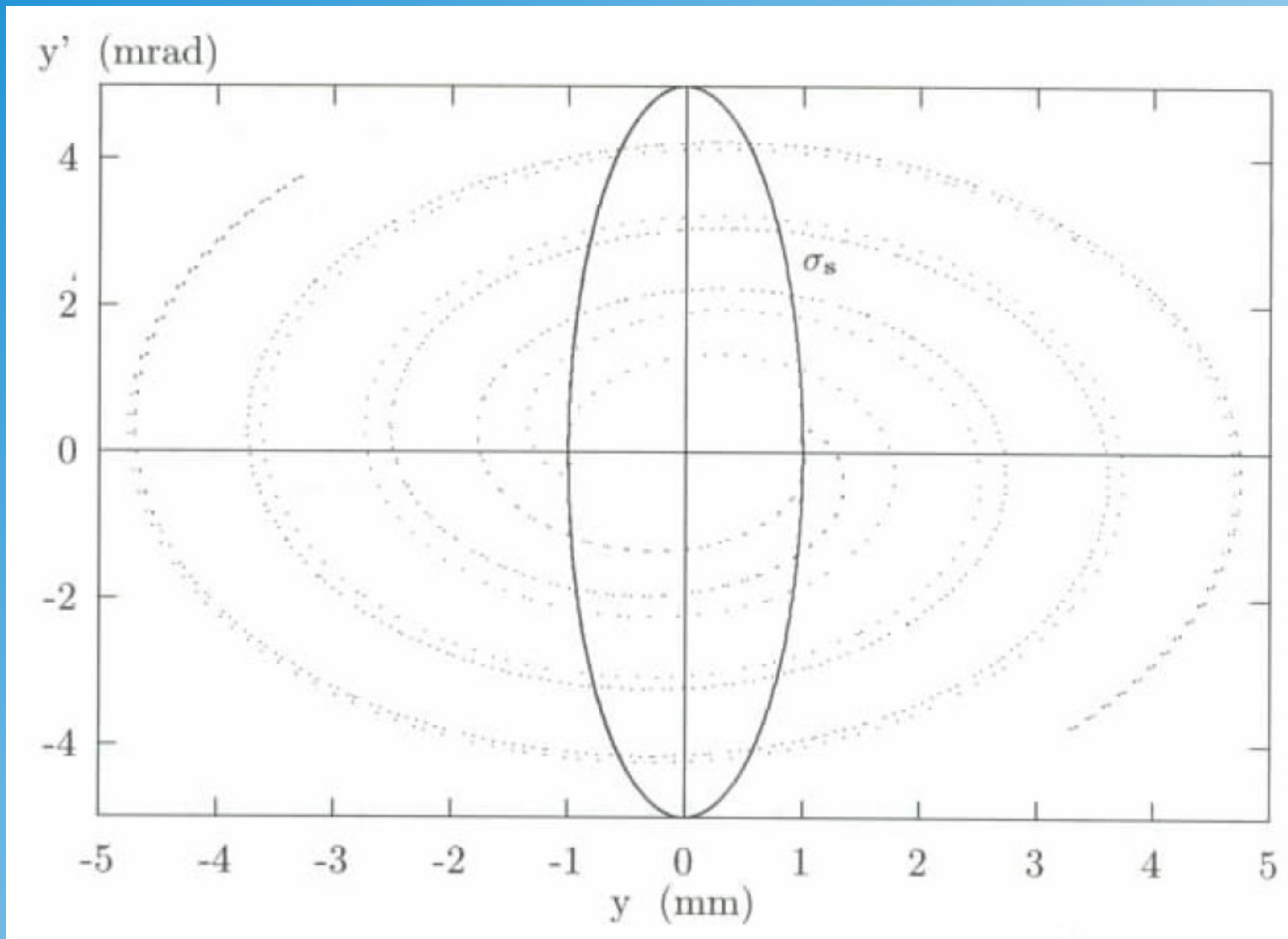


Filamentation I



non-linear effects!

Filamentation II



spiraling!

Summary (Hinterberger)

$$M = \begin{pmatrix} \cos \mu + \alpha \sin \mu & \beta \sin \mu \\ -\gamma \sin \mu & \cos \mu - \alpha \sin \mu \end{pmatrix}$$

$$= \cos \mu \overbrace{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}^I + \sin \mu \overbrace{\begin{pmatrix} \alpha & \beta \\ -\gamma & -\alpha \end{pmatrix}}^J$$

- Periodizität:

$$\beta(s + C) = \beta(s)$$

- $\alpha(s)$ und $\gamma(s)$:

$$\alpha(s) = -\frac{1}{2}\beta'(s),$$

$$\gamma(s) = \frac{1 + \alpha^2(s)}{\beta(s)} = \frac{1 + [\beta'(s)/2]^2}{\beta(s)}$$

- Eigenellipse der Maschine (Maschinenellipse):

$$\sigma_e = \epsilon \begin{pmatrix} \beta(s) & -\alpha(s) \\ -\alpha(s) & \gamma(s) \end{pmatrix}$$

- Strahlenveloppe eines angepassten Strahls⁹:

$$y_{\max}(s) = \sqrt{\epsilon} \sqrt{\beta(s)}$$

- Bahn eines einzelnen Teilchens:

$$y(s) = a \sqrt{\beta(s)} \cos \left[\int_{s_0}^s \frac{d\bar{s}}{\beta(\bar{s})} + \psi_0 \right]$$

$$\sigma_e = \epsilon \begin{pmatrix} \beta(s) & -\alpha(s) \\ -\alpha(s) & \gamma(s) \end{pmatrix}$$

- Strahlenveloppe eines angepassten Strahls⁹:

$$y_{\max}(s) = \sqrt{\epsilon} \sqrt{\beta(s)}$$

- Bahn eines einzelnen Teilchens:

$$y(s) = a \sqrt{\beta(s)} \cos[\psi(s) + \psi_0] = a \sqrt{\beta(s)} \cos \left[\int_{s_0}^s \frac{d\bar{s}}{\beta(\bar{s})} + \psi_0 \right]$$

- Lokale Wellenzahl der Betatronschiwingung:

$$k(s) = \frac{2\pi}{\lambda(s)} = \frac{d\psi}{ds} = \frac{1}{\beta(s)}$$

- Lokale Wellenlänge der Betatronschiwingung:

$$\lambda(s) = 2\pi\beta(s)$$

- Betatronphasenvorschub:

$$\Delta\psi = \int_{s_0}^s \frac{d\bar{s}}{\beta(\bar{s})}$$

- Betatronphasenvorschub pro Umlauf:

$$\mu = \oint \frac{d\bar{s}}{\beta(\bar{s})}$$

- Betatronschiwingungszahl (Arbeitspunkt):

$$Q = \frac{1}{2\pi} \oint \frac{d\bar{s}}{\beta(\bar{s})}$$

