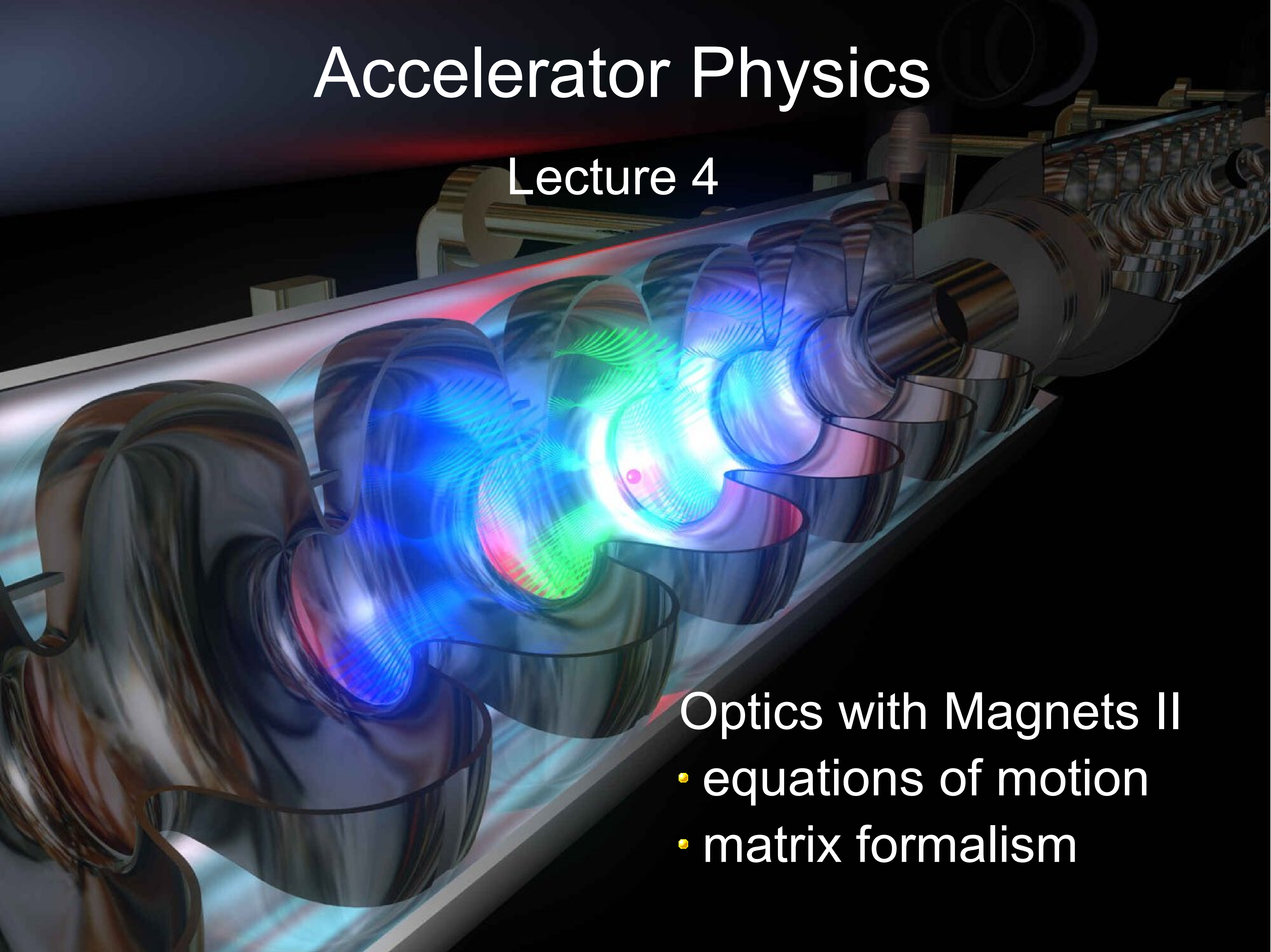


Accelerator Physics

Lecture 4

Optics with Magnets II

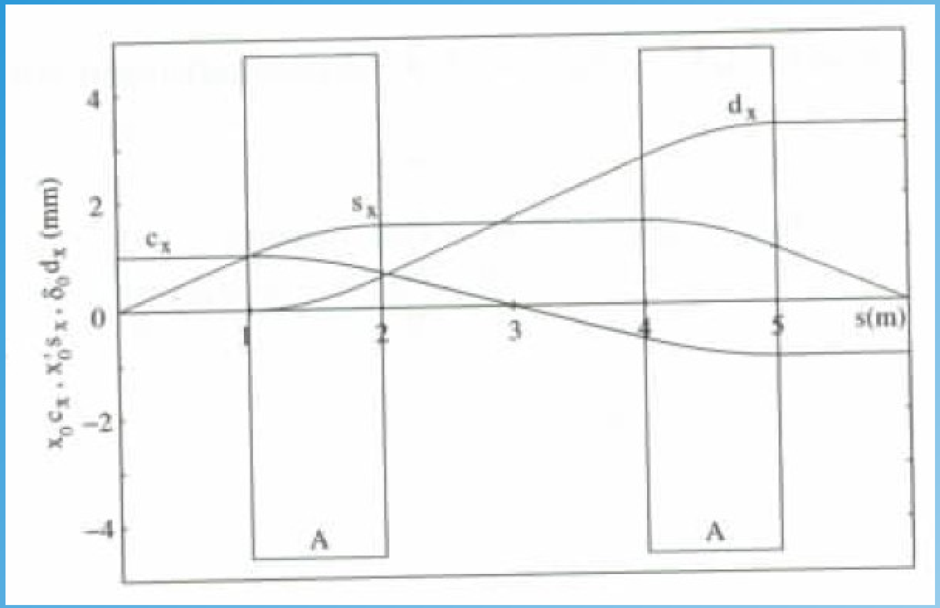
- equations of motion
- matrix formalism

Characteristic Solutions

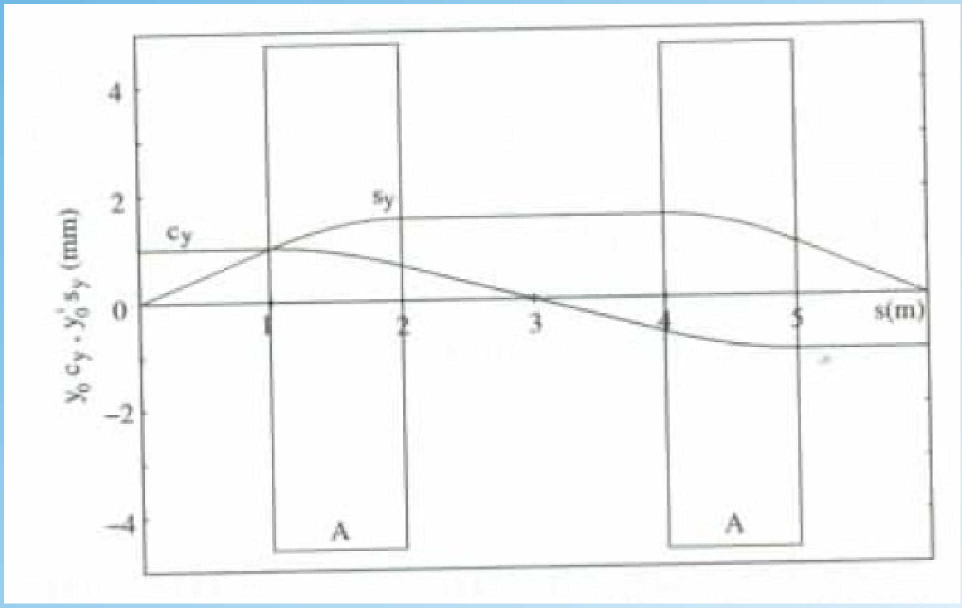
$$x(s) = x_0 c(s) + x_0' s(s)$$

$$y(s) = y_0 c(s) + y_0' s(s)$$

horizontal plane



vertical plane



drift dipol drift dipol drift

drift dipol drift dipol drift

field index n=0.5

Transfer Matrix

(transfer matrix, transport matrix, R matrix)

$$R = \begin{pmatrix} (x|x) & (x|x') & 0 & 0 & 0 & (x|\delta) \\ (x'|x) & (x'|x') & 0 & 0 & 0 & (x'|\delta) \\ 0 & 0 & (y|y) & (y|y') & 0 & 0 \\ 0 & 0 & (y'|y) & (y'|y') & 0 & 0 \\ (l|x) & (l|x') & 0 & 0 & (l|l) & (l|\delta) \\ 0 & 0 & 0 & 0 & 0 & (\delta|\delta) \end{pmatrix}$$

Abbildung	radial	axial
Punkt-zu-Punkt	$R_{12} = (x\|x') = 0$	$R_{34} = (y\|y') = 0$
Punkt-zu-Parallel	$R_{22} = (x'\|x') = 0$	$R_{44} = (y'\|y') = 0$
Parallel-zu-Punkt	$R_{11} = (x\|x) = 0$	$R_{33} = (y\|y) = 0$
Parallel-zu-Parallel	$R_{21} = (x'\|x) = 0$	$R_{43} = (y'\|y) = 0$
Ortsdispersion = 0	$R_{16} = (x\|\delta) = 0$	
Winkeldispersion = 0	$R_{26} = (x'\|\delta) = 0$	

Drift

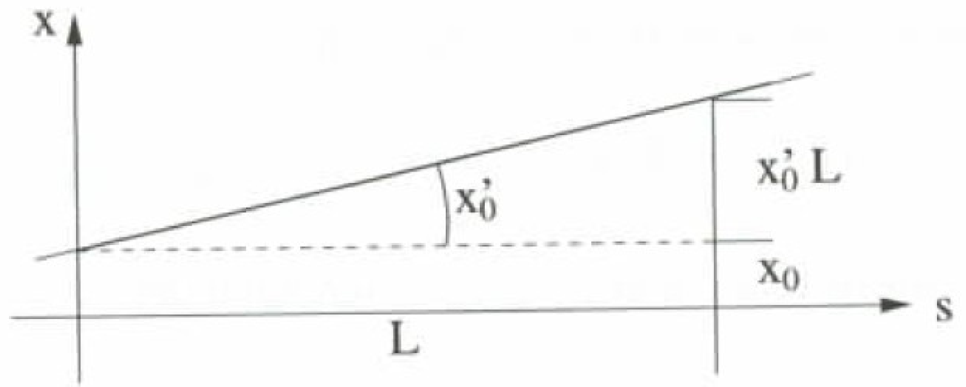


Abb. 4.8. Illustration zur Wirkung einer Driftstrecke

$$x = x_0 + L\, x_0'$$
$$x' = x_0'$$
$$y = y_0 + L\, y_0'$$
$$y' = y_0'$$
$$l = l_0 + L/\gamma^2\, \delta_0'$$
$$\delta' = \delta_0'$$

$$R = \begin{pmatrix} 1 & L & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & L & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & L/\gamma^2 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} .$$

$$R = \begin{pmatrix} (x|x) & (x|x') & 0 & 0 & 0 & (x|\delta) \\ (x'|x) & (x'|x') & 0 & 0 & 0 & (x'|\delta) \\ 0 & 0 & (y|y) & (y|y') & 0 & 0 \\ 0 & 0 & (y'|y) & (y'|y') & 0 & 0 \\ (l|x) & (l|x') & 0 & 0 & (l|l) & (l|\delta) \\ 0 & 0 & 0 & 0 & 0 & (\delta|\delta) \end{pmatrix}$$

Quadrupoles

Quadrupol (radial fokussierend und axial defokussierend)

$$R = \begin{pmatrix} \cos\sqrt{k}L & \frac{\sin\sqrt{k}L}{\sqrt{k}} & 0 & 0 & 0 & 0 \\ -\sqrt{k}\sin\sqrt{k}L & \cos\sqrt{k}L & 0 & 0 & 0 & 0 \\ 0 & 0 & \cosh\sqrt{k}L & \frac{\sinh\sqrt{k}L}{\sqrt{k}} & 0 & 0 \\ 0 & 0 & \sqrt{k}\sinh\sqrt{k}L & \cosh\sqrt{k}L & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \frac{L}{\gamma^2} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix},$$

Quadrupol (radial defokussierend und axial fokussierend)

$$R = \begin{pmatrix} \cosh\sqrt{k}L & \frac{\sinh\sqrt{k}L}{\sqrt{k}} & 0 & 0 & 0 & 0 \\ \sqrt{k}\sinh\sqrt{k}L & \cosh\sqrt{k}L & 0 & 0 & 0 & 0 \\ 0 & 0 & \cos\sqrt{k}L & \frac{\sin\sqrt{k}L}{\sqrt{k}} & 0 & 0 \\ 0 & 0 & -\sqrt{k}\sin\sqrt{k}L & \cos\sqrt{k}L & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \frac{L}{\gamma^2} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

$$g = \frac{\partial B_y}{\partial x} = \frac{\partial B_x}{\partial y}$$

$$k = \frac{|g|}{(B\rho_0)} = \frac{|B_0|}{a}\frac{1}{(B\rho_0)}$$

$$R = \begin{pmatrix} (x|x) & (x|x') & 0 & 0 & 0 & (x|\delta) \\ (x'|x) & (x'|x') & 0 & 0 & 0 & (x'|\delta) \\ 0 & 0 & (y|y) & (y|y') & 0 & 0 \\ 0 & 0 & (y'|y) & (y'|y') & 0 & 0 \\ (l|x) & (l|x') & 0 & 0 & (l|l) & (l|\delta) \\ 0 & 0 & 0 & 0 & 0 & (\delta|\delta) \end{pmatrix}$$

Dipole Magnet

dispersion

$$k_x = \frac{1}{\rho_0^2} \qquad k_y = 0$$

deflection angle:

$\alpha = L / \rho_0$

Homogener Ablenkmagnet (Dipolmagnet, Sektormagnet)

$$R = \begin{pmatrix} \cos\alpha & \rho_0 \sin\alpha & 0 & 0 & 0 & \rho_0(1-\cos\alpha) \\ -\frac{\sin\alpha}{\rho_0} & \cos\alpha & 0 & 0 & 0 & \sin\alpha \\ 0 & 0 & 1 & \rho_0\alpha & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ -\sin\alpha & -\rho_0(1-\cos\alpha) & 0 & 0 & 1 & \rho_0\frac{\alpha}{\gamma^2} - \rho_0(\alpha - \sin\alpha) \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

orbit effects

drift

$$R = \begin{pmatrix} (x|x) & (x|x') & 0 & 0 & 0 & (x|\delta) \\ (x'|x) & (x'|x') & 0 & 0 & 0 & (x'|\delta) \\ 0 & 0 & (y|y) & (y|y') & 0 & 0 \\ 0 & 0 & (y'|y) & (y'|y') & 0 & 0 \\ (l|x) & (l|x') & 0 & 0 & (l|l) & (l|\delta) \\ 0 & 0 & 0 & 0 & 0 & (\delta|\delta) \end{pmatrix}$$

Weakly Focussing Dipole

$$R = \begin{pmatrix} (x|x) & (x|x') & 0 & 0 & 0 & (x|\delta) \\ (x'|x) & (x'|x') & 0 & 0 & 0 & (x'|\delta) \\ 0 & 0 & (y|y) & (y|y') & 0 & 0 \\ 0 & 0 & (y'|y) & (y'|y') & 0 & 0 \\ (l|x) & (l|x') & 0 & 0 & (l|l) & (l|\delta) \\ 0 & 0 & 0 & 0 & 0 & (\delta|\delta) \end{pmatrix}$$

weakly focussing $0 < n < 1$

$$k_x = \frac{1-n}{\rho_0^2} \qquad k_y = \frac{n}{\rho^2}$$

deflection angle:
$\alpha = L / \rho_0$

focussing in both planes (x-y)

Schwach fokussierender Ablenkmagnet $(0 < n < 1)$

$$R_x = \begin{pmatrix} \cos\sqrt{1-n}\,\alpha & \frac{\rho_0 \sin\sqrt{1-n}\,\alpha}{\sqrt{1-n}} & \frac{\rho_0(1-\cos\sqrt{1-n}\,\alpha)}{1-n} \\ -\frac{\sqrt{1-n}\sin\sqrt{1-n}\,\alpha}{\rho_0} & \cos\sqrt{1-n}\,\alpha & \frac{\sin\sqrt{1-n}\,\alpha}{\sqrt{1-n}} \\ 0 & 0 & 1 \end{pmatrix}$$

$$R_y = \begin{pmatrix} \cos\sqrt{n}\,\alpha & \frac{\rho_0 \sin\sqrt{n}\,\alpha}{\sqrt{n}} \\ -\frac{\sqrt{n}\sin\sqrt{n}\,\alpha}{\rho_0} & \cos\sqrt{n}\,\alpha \end{pmatrix} .$$

Strongly Focussing Dipole

$$R = \begin{pmatrix} (x|x) & (x|x') & 0 & 0 & 0 & (x|\delta) \\ (x'|x) & (x'|x') & 0 & 0 & 0 & (x'|\delta) \\ 0 & 0 & (y|y) & (y|y') & 0 & 0 \\ 0 & 0 & (y'|y) & (y'|y') & 0 & 0 \\ (l|x) & (l|x') & 0 & 0 & (l|l) & (l|\delta) \\ 0 & 0 & 0 & 0 & 0 & (\delta|\delta) \end{pmatrix}$$

Stark fokussierender Ablenkmagnet ($n < 0$)

strongly focussing

$$R_x = \begin{pmatrix} \cos\sqrt{1-n}\alpha & \frac{\rho_0 \sin\sqrt{1-n}\alpha}{\sqrt{1-n}} & \frac{\rho_0(1-\cos\sqrt{1-n}\alpha)}{1-n} \\ -\frac{\sqrt{1-n}\sin\sqrt{1-n}\alpha}{\rho_0} & \cos\sqrt{1-n}\alpha & \frac{\sin\sqrt{1-n}\alpha}{\sqrt{1-n}} \\ 0 & 0 & 1 \end{pmatrix}$$

defocussing

$$R_y = \begin{pmatrix} \cosh\sqrt{|n|}\alpha & \frac{\rho_0 \sinh\sqrt{|n|}\alpha}{\sqrt{|n|}} \\ \frac{\sqrt{|n|}\sinh\sqrt{|n|}\alpha}{\rho_0} & \cosh\sqrt{|n|}\alpha \end{pmatrix} .$$

Stark fokussierender Ablenkmagnet ($n > 1$)

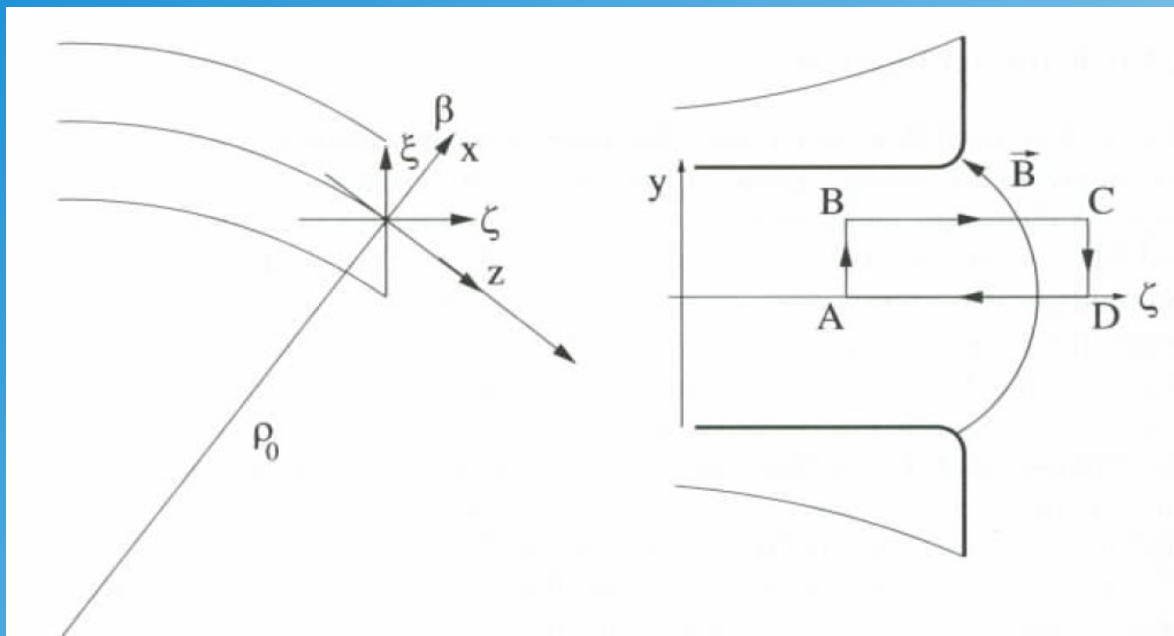
strongly focussing

$$R_x = \begin{pmatrix} \cosh\sqrt{|1-n|}\alpha & \frac{\rho_0 \sinh\sqrt{|1-n|}\alpha}{\sqrt{|1-n|}} & \frac{\rho_0\left(1-\cosh\sqrt{|1-n|}\alpha\right)}{1-n} \\ \frac{\sqrt{|1-n|}\sinh\sqrt{|1-n|}\alpha}{\rho_0} & \cosh\sqrt{|1-n|}\alpha & \frac{\sinh\sqrt{|1-n|}\alpha}{\sqrt{|1-n|}} \\ 0 & 0 & 1 \end{pmatrix} .$$

defocussing

$$R_y = \begin{pmatrix} \cos\sqrt{n}\alpha & \frac{\rho_0 \sin\sqrt{n}\alpha}{\sqrt{n}} \\ -\frac{\sqrt{n}\sin\sqrt{n}\alpha}{\rho_0} & \cos\sqrt{n}\alpha \end{pmatrix} .$$

Edge Focussing



for calculation
see Hinterberger

$$R = \begin{pmatrix} (x|x) & (x|x') & 0 & 0 & 0 & (x|\delta) \\ (x'|x) & (x'|x') & 0 & 0 & 0 & (x'|\delta) \\ 0 & 0 & (y|y) & (y|y') & 0 & 0 \\ 0 & 0 & (y'|y) & (y'|y') & 0 & 0 \\ (l|x) & (l|x') & 0 & 0 & (l|l) & (l|\delta) \\ 0 & 0 & 0 & 0 & 0 & (\delta|\delta) \end{pmatrix}$$

Kantenfokussierung

$$R = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{\tan\beta}{\rho_0} & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -\frac{\tan\beta_{\mathrm{eff}}}{\rho_0} & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

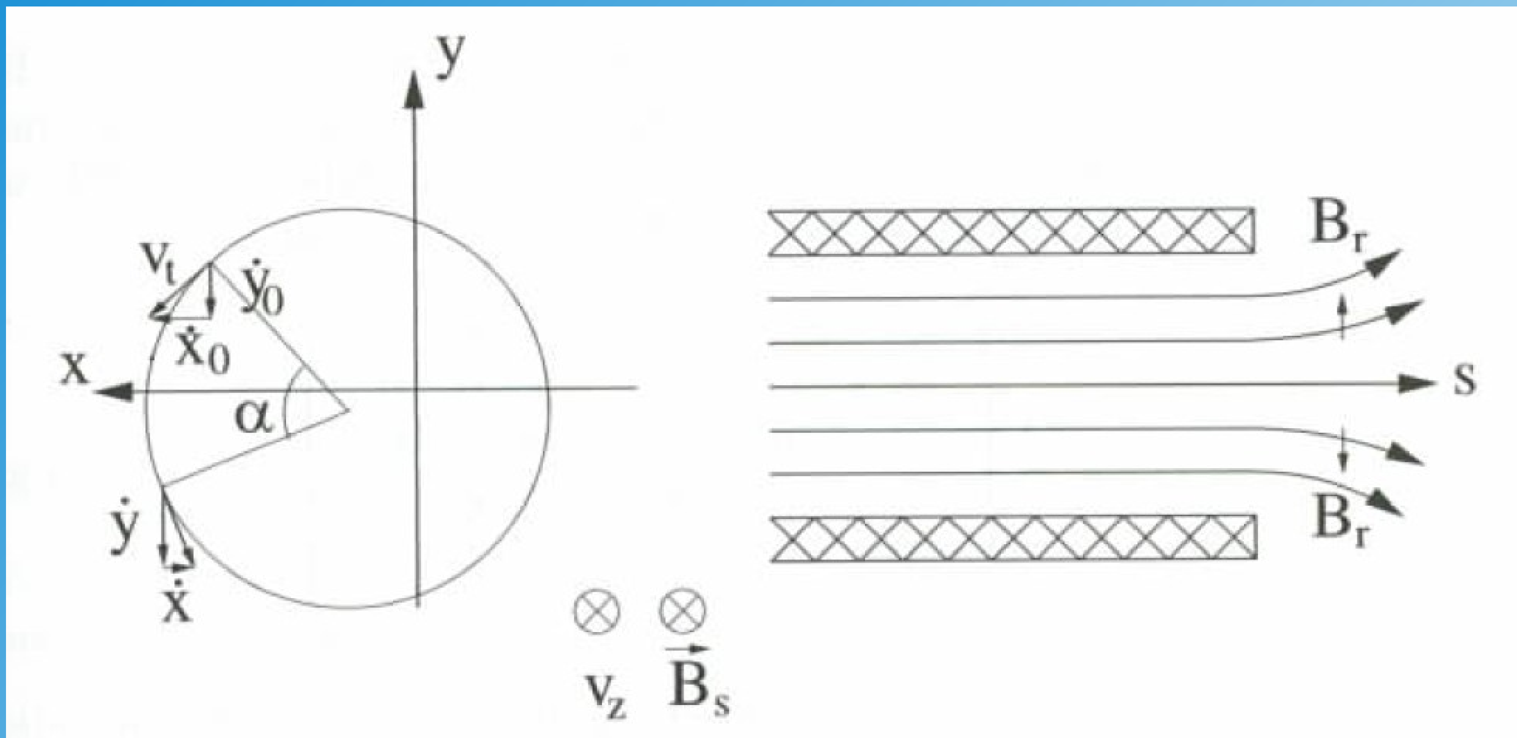
Rotations

Rotation um den Winkel α

$$R(\alpha) = \begin{pmatrix} \cos\alpha & 0 & \sin\alpha & 0 & 0 & 0 \\ 0 & \cos\alpha & 0 & \sin\alpha & 0 & 0 \\ -\sin\alpha & 0 & \cos\alpha & 0 & 0 & 0 \\ 0 & -\sin\alpha & 0 & \cos\alpha & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$R = \begin{pmatrix} (x|x) & (x|x') & 0 & 0 & 0 & (x|\delta) \\ (x'|x) & (x'|x') & 0 & 0 & 0 & (x'|\delta) \\ 0 & 0 & (y|y) & (y|y') & 0 & 0 \\ 0 & 0 & (y'|y) & (y'|y') & 0 & 0 \\ (l|x) & (l|x') & 0 & 0 & (l|l) & (l|\delta) \\ 0 & 0 & 0 & 0 & 0 & (\delta|\delta) \end{pmatrix}$$

Solenoid I



$$R = \begin{pmatrix} (x|x) & (x|x') & 0 & 0 & 0 & (x|\delta) \\ (x'|x) & (x'|x') & 0 & 0 & 0 & (x'|\delta) \\ 0 & 0 & (y|y) & (y|y') & 0 & 0 \\ 0 & 0 & (y'|y) & (y'|y') & 0 & 0 \\ (l|x) & (l|x') & 0 & 0 & (l|l) & (l|\delta) \\ 0 & 0 & 0 & 0 & 0 & (\delta|\delta) \end{pmatrix}$$

Solenoid II

transfer matrix

$$R = \begin{pmatrix} C^2 & SC/K & SC & S^2/K & 0 & 0 \\ -KSC & C^2 & -KS^2 & SC & 0 & 0 \\ -SC & -S^2/K & C^2 & SC/K & 0 & 0 \\ KS^2 & -SC & -KSC & C^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & L/\gamma^2 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

α = -2 KL

C = cos(KL)

S = sin(KL)

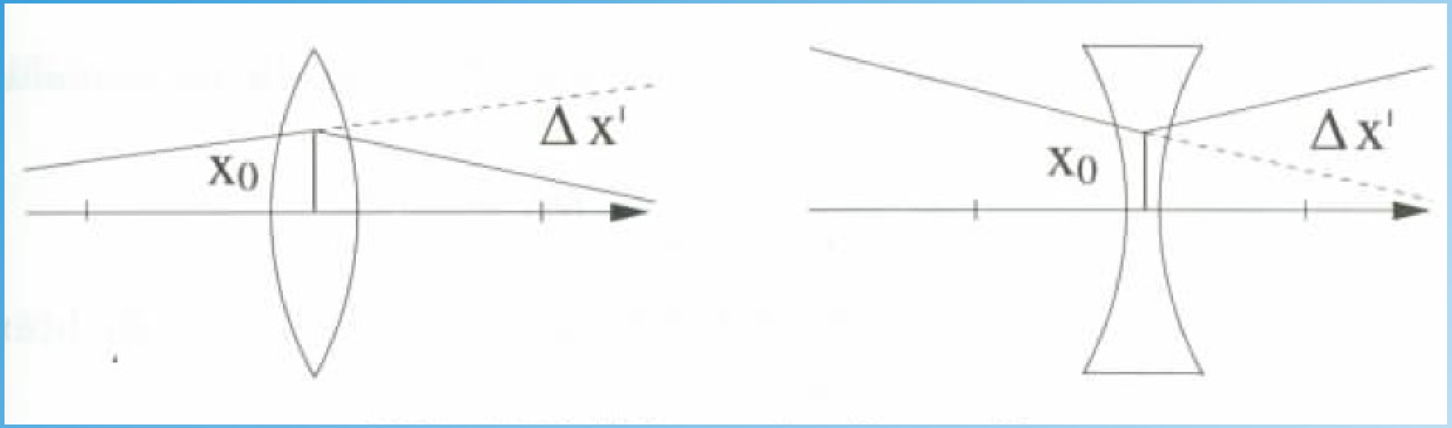
$K = B_s / 2 / (B\rho)_0$

transfer matrix without rotation

$$R(\alpha/2)R = \begin{pmatrix} C & S/K & 0 & 0 & 0 & 0 \\ -KS & C & 0 & 0 & 0 & 0 \\ 0 & 0 & C & S/K & 0 & 0 \\ 0 & 0 & -KS & C & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & L/\gamma^2 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

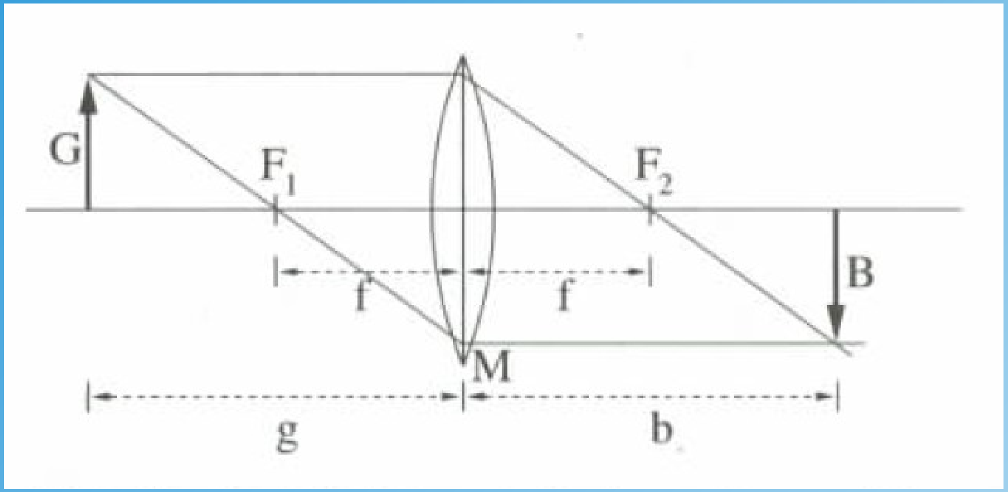
Geometrical Optics I

(de-)focussing elements represented by lenses



Geometrical Optics II

ideal lense



real lense