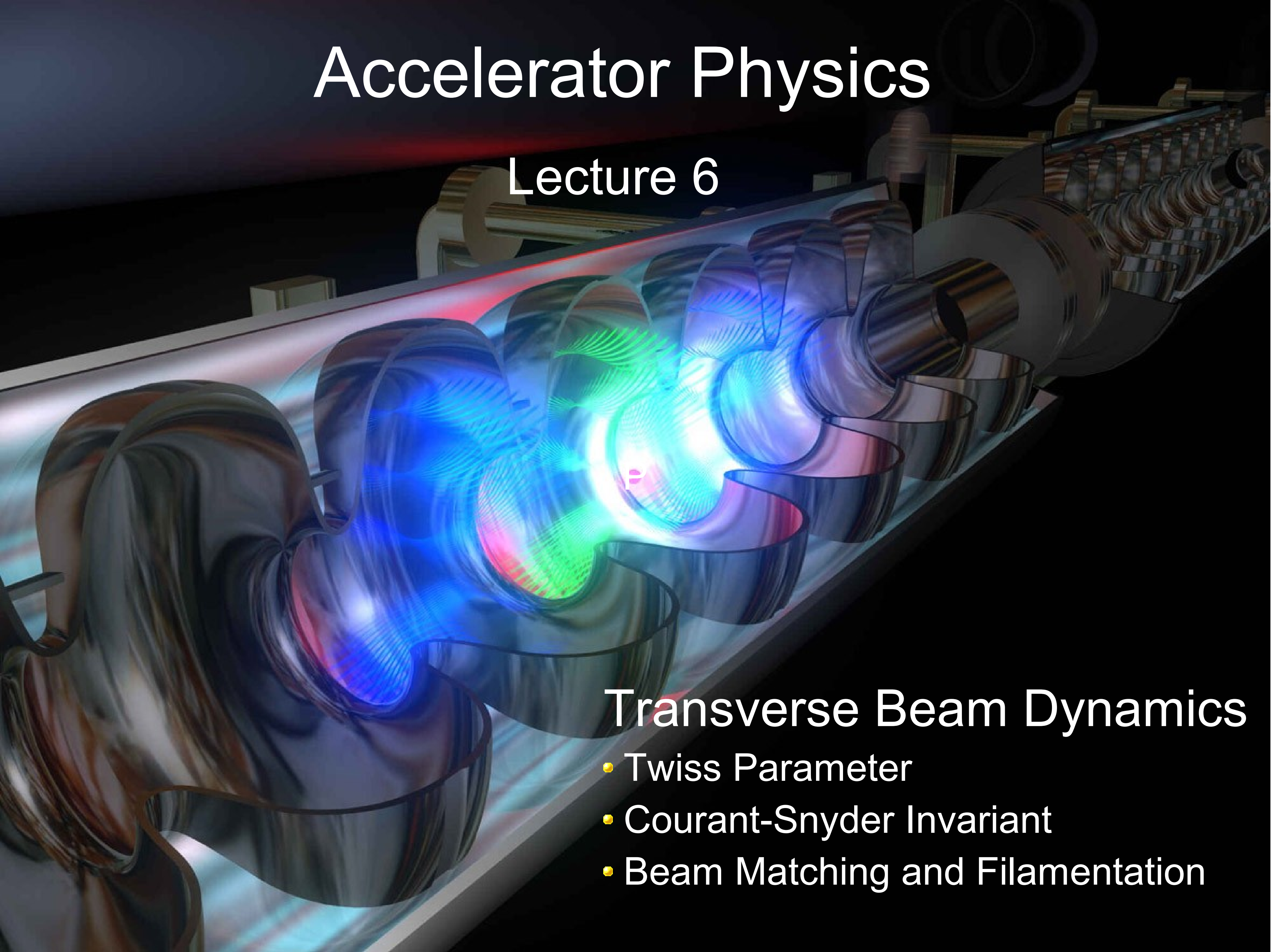
Accelerator Physics

Lecture 6



Transverse Beam Dynamics

- Twiss Parameter
- Courant-Snyder Invariant
- Beam Matching and Filamentation

SPS Lattice

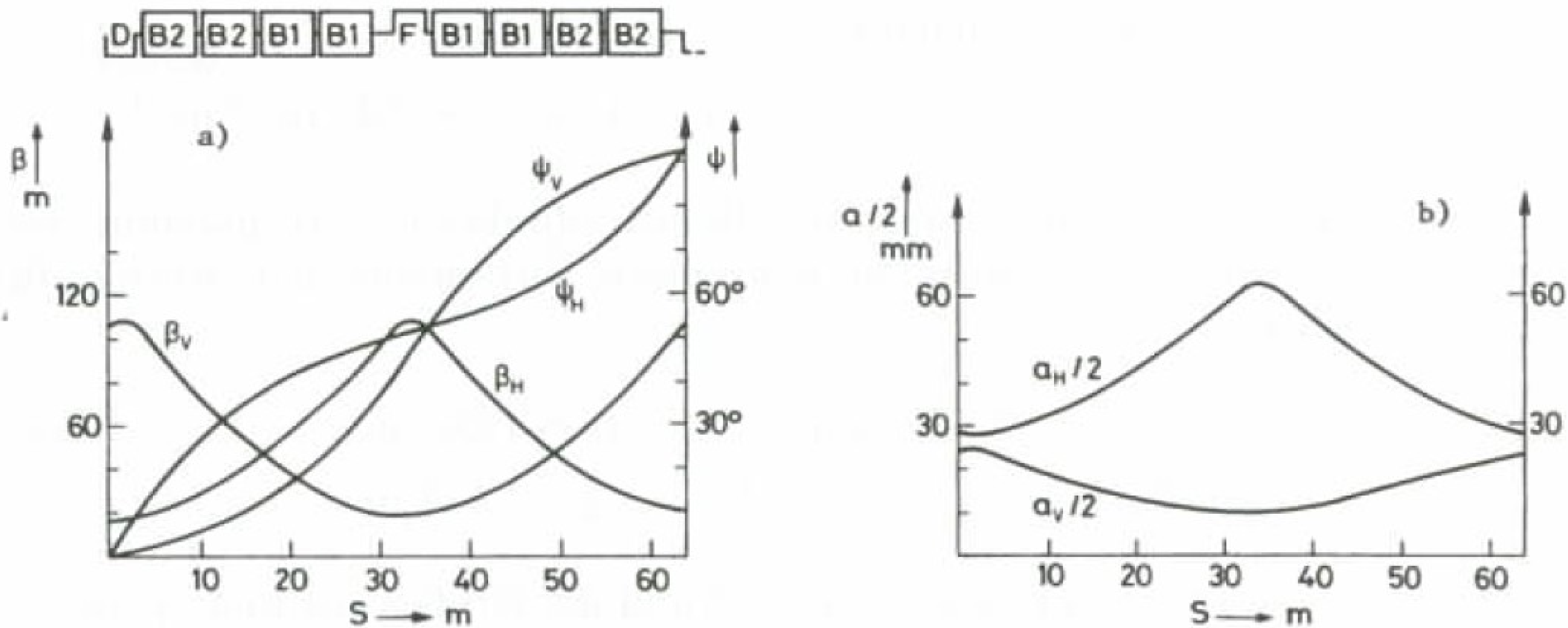
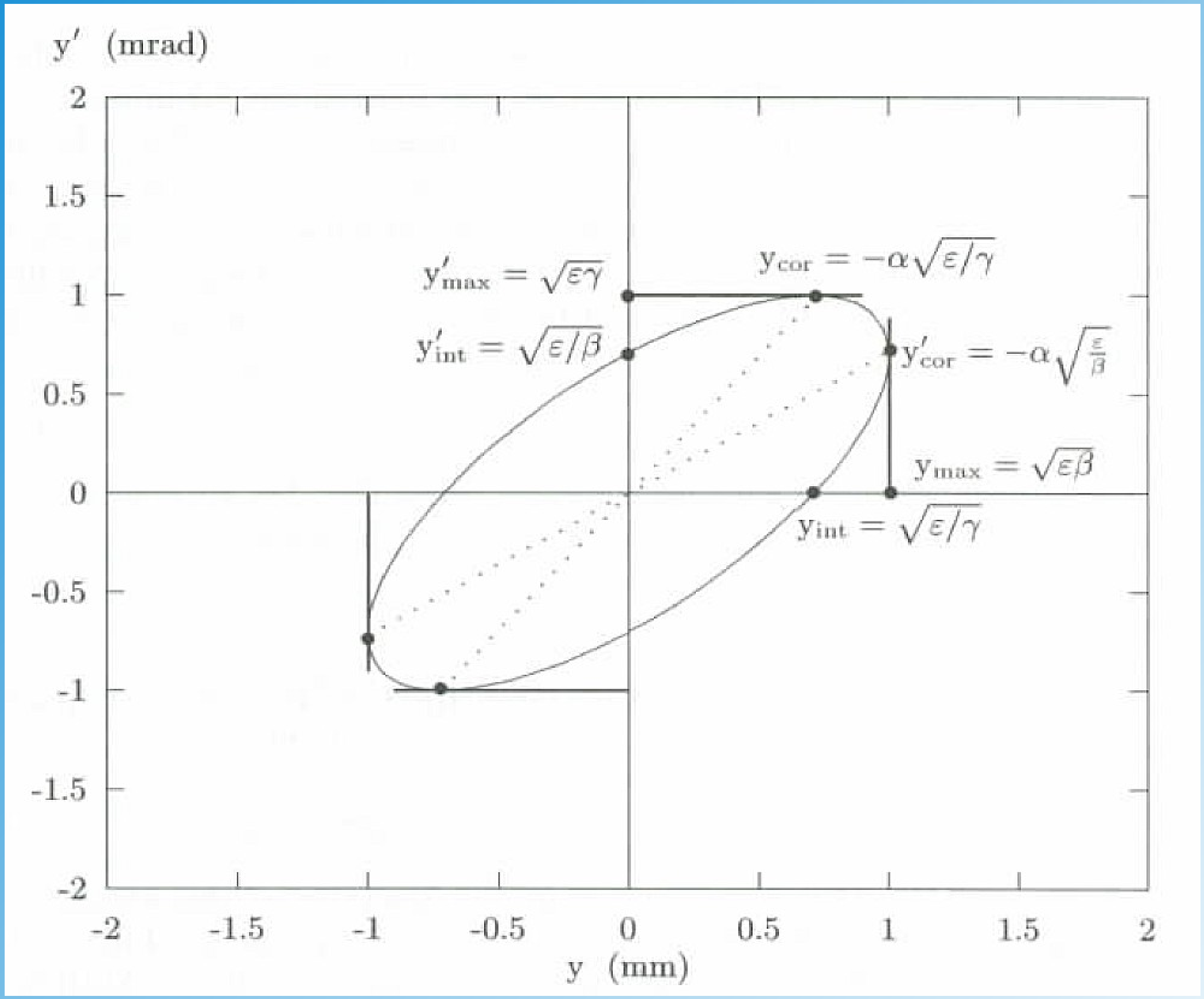
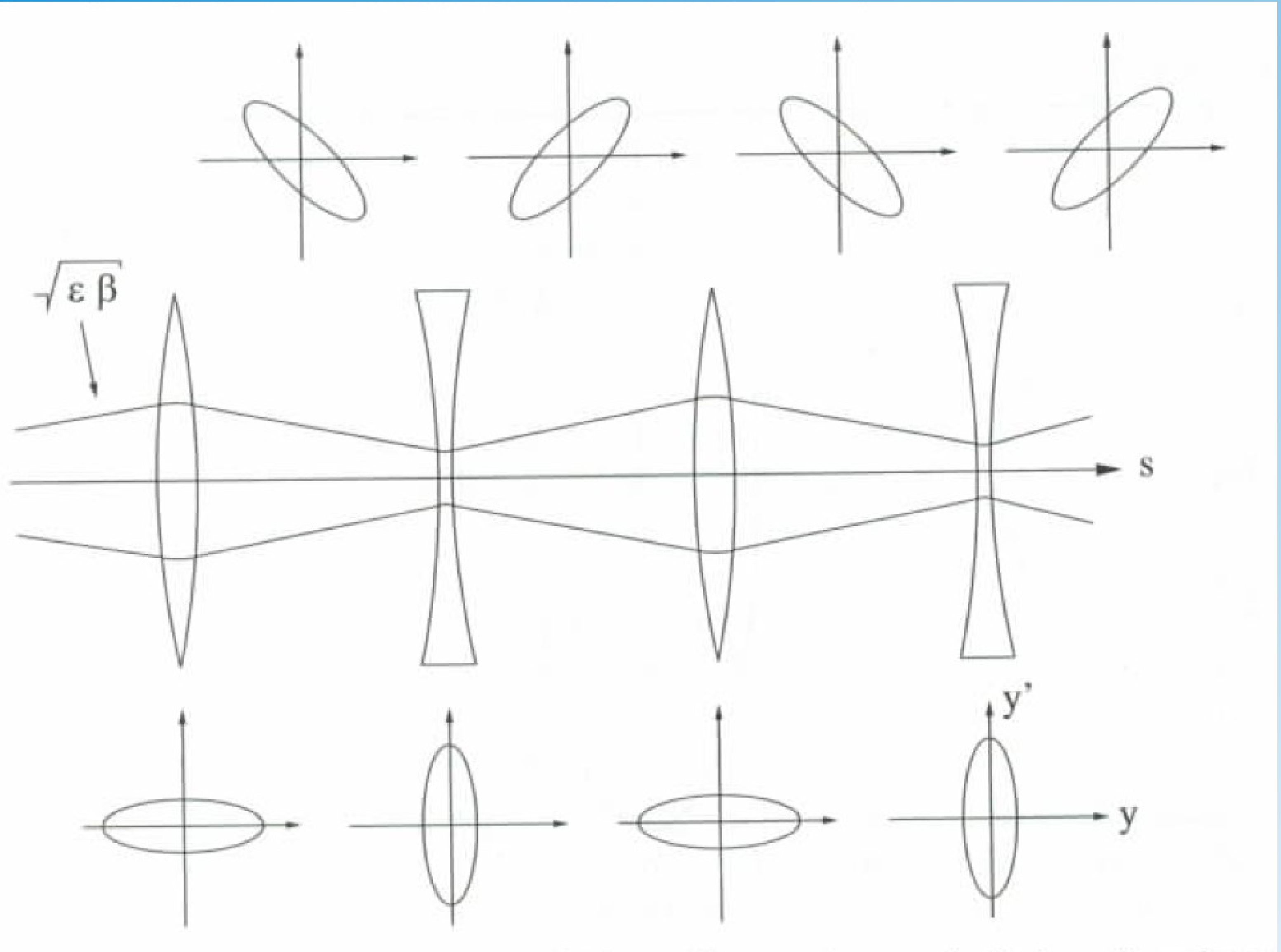


Abb. 6.11. SPS-Einheitszelle: Horizontale und vertikale Betatronfunktion $\beta_x = \beta_H$ und $\beta_y = \beta_V$, Betatronphasen $\psi_x = \phi_H$ und $\psi_y = \phi_V$ und Enveloppen $a_H/2 = \sqrt{\beta_x \epsilon_x}$, $a_V/2 = \sqrt{\beta_y \epsilon_y}$. Aus [Wi85] entnommen

Phase Space Ellipse



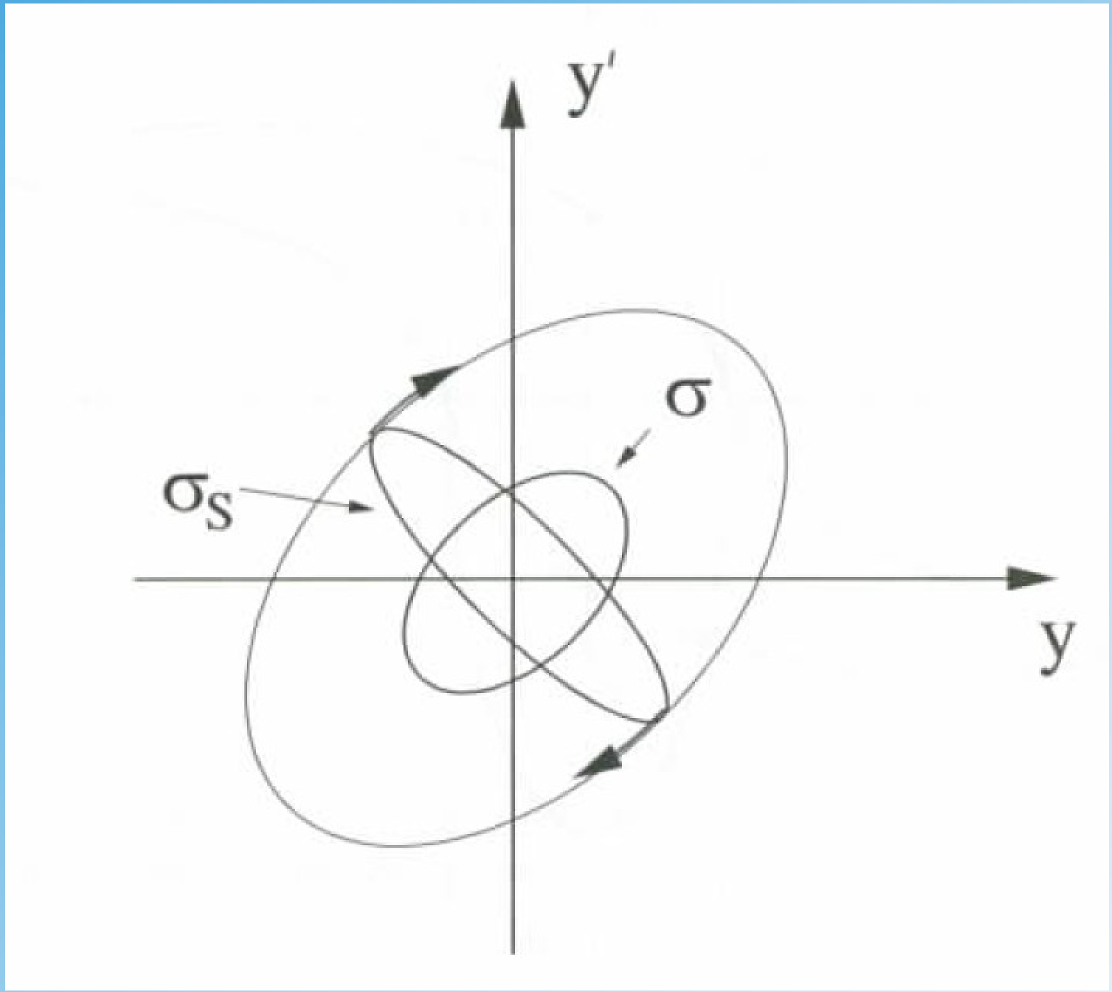
Beam Envelopes in Periodic Structure



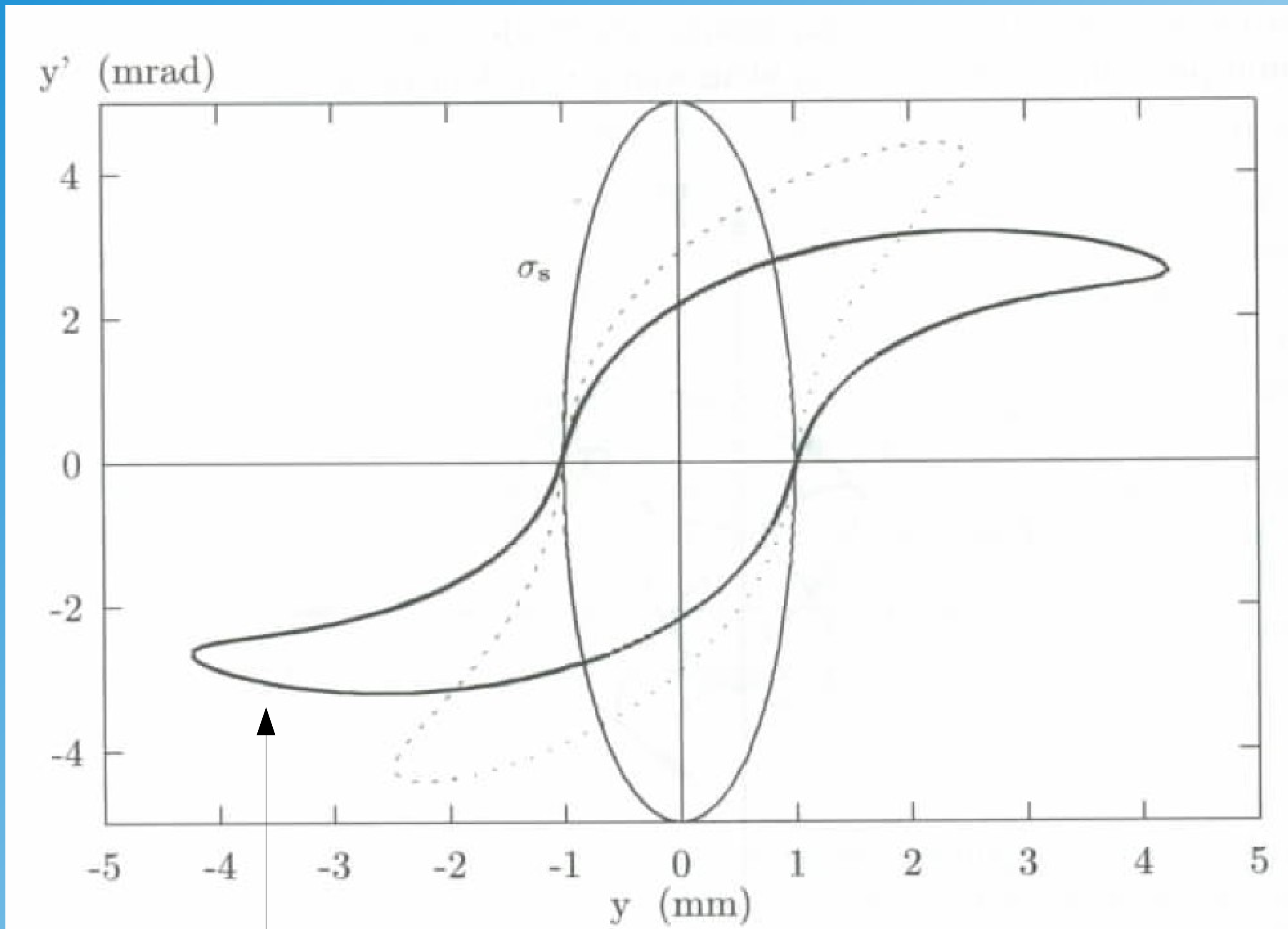
Beam-Machine Matching

σ_s= beam ellipse

σ = machine ellipse

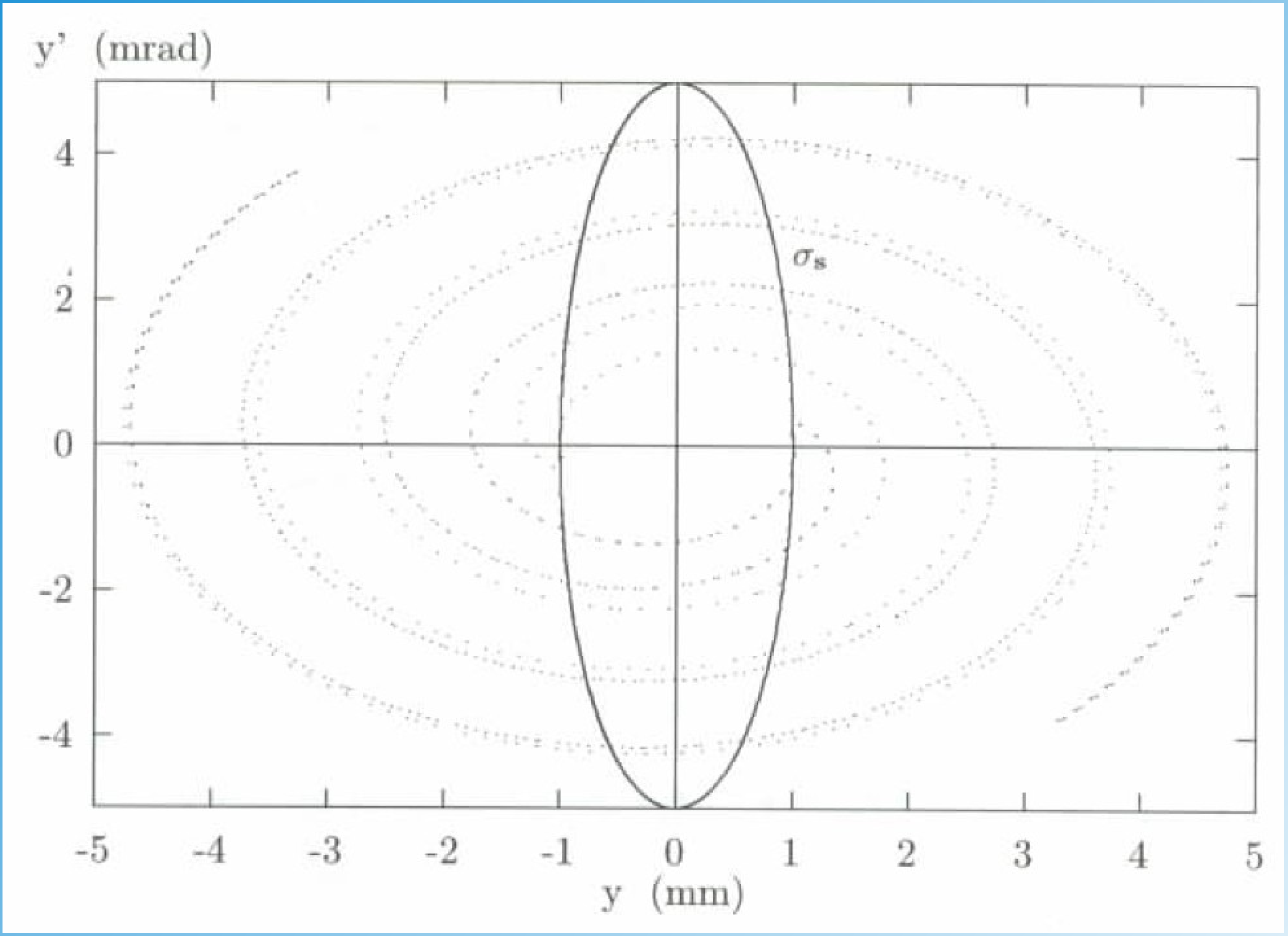


Filamentation I



non-linear effects!

Filamentation II



spiraling!

Summary (Hinterberger)

$$M = \begin{pmatrix} \cos\mu + \alpha\sin\mu & \beta\sin\mu \\ -\gamma\sin\mu & \cos\mu - \alpha\sin\mu \end{pmatrix}$$

$$= \cos\mu \overbrace{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}^{I} + \sin\mu \overbrace{\begin{pmatrix} \alpha & \beta \\ -\gamma & -\alpha \end{pmatrix}}^{J}$$

- Periodizität:

$$\beta(s + C) = \beta(s)$$

- $\alpha(s)$ und $\gamma(s)$:

$$\alpha(s) = -\frac{1}{2}\beta'(s)\,,$$

$$\gamma(s) = \frac{1 + \alpha^2(s)}{\beta(s)} = \frac{1 + [\beta'(s)/2]^2}{\beta(s)}$$

- Eigenellipse der Maschine (Maschinenellipse):

$$\sigma_e = \epsilon \begin{pmatrix} \beta(s) & -\alpha(s) \\ -\alpha(s) & \gamma(s) \end{pmatrix}$$

- Strahlenveloppe eines angepassten Strahls[9]:

$$y_{\max}(s) = \sqrt{\epsilon}\sqrt{\beta(s)}$$

- Bahn eines einzelnen Teilchens:

$$\sigma_e = \epsilon \begin{pmatrix} \beta(s) & -\alpha(s) \\ -\alpha(s) & \gamma(s) \end{pmatrix}$$

- Strahlenveloppe eines angepassten Strahls[9]:

$$y_{\max}(s) = \sqrt{\epsilon}\sqrt{\beta(s)}$$

- Bahn eines einzelnen Teilchens:

$$y(s) = a\sqrt{\beta(s)}\cos[\psi(s) + \psi_0] = a\sqrt{\beta(s)}\cos\left[\int_{s_0}^{s} \frac{\mathrm{d}\bar{s}}{\beta(\bar{s})} + \psi_0\right]$$

- Lokale Wellenzahl der Betatronschwingung:

$$k(s) = \frac{2\pi}{\lambda(s)} = \frac{\mathrm{d}\psi}{\mathrm{d}s} = \frac{1}{\beta(s)}$$

- Lokale Wellenlänge der Betatronschwingung:

$$\lambda(s) = 2\pi\beta(s)$$

- Betatronphasenvorschub:

$$\Delta\psi = \int_{s_0}^{s} \frac{\mathrm{d}\bar{s}}{\beta(\bar{s})}$$

- Betatronphasenvorschub pro Umlauf:

$$\mu = \oint \frac{\mathrm{d}\bar{s}}{\beta(\bar{s})}$$

- Betatronschwingungszahl (Arbeitspunkt):

$$Q = \frac{1}{2\pi}\oint \frac{\mathrm{d}\bar{s}}{\beta(\bar{s})}$$