# Quark-Gluon Plasma Physics

## 6. Space-time evolution of the QGP

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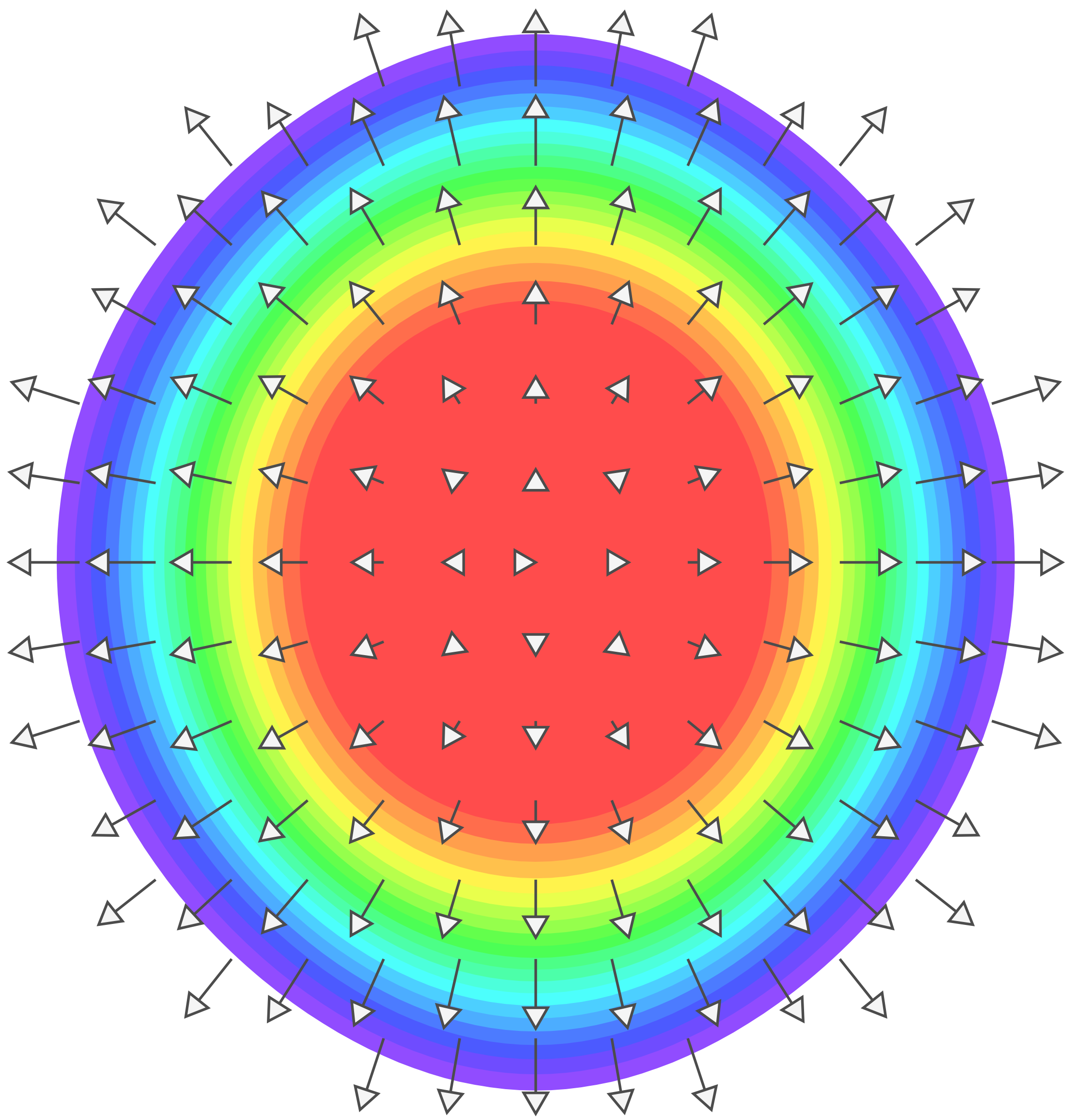
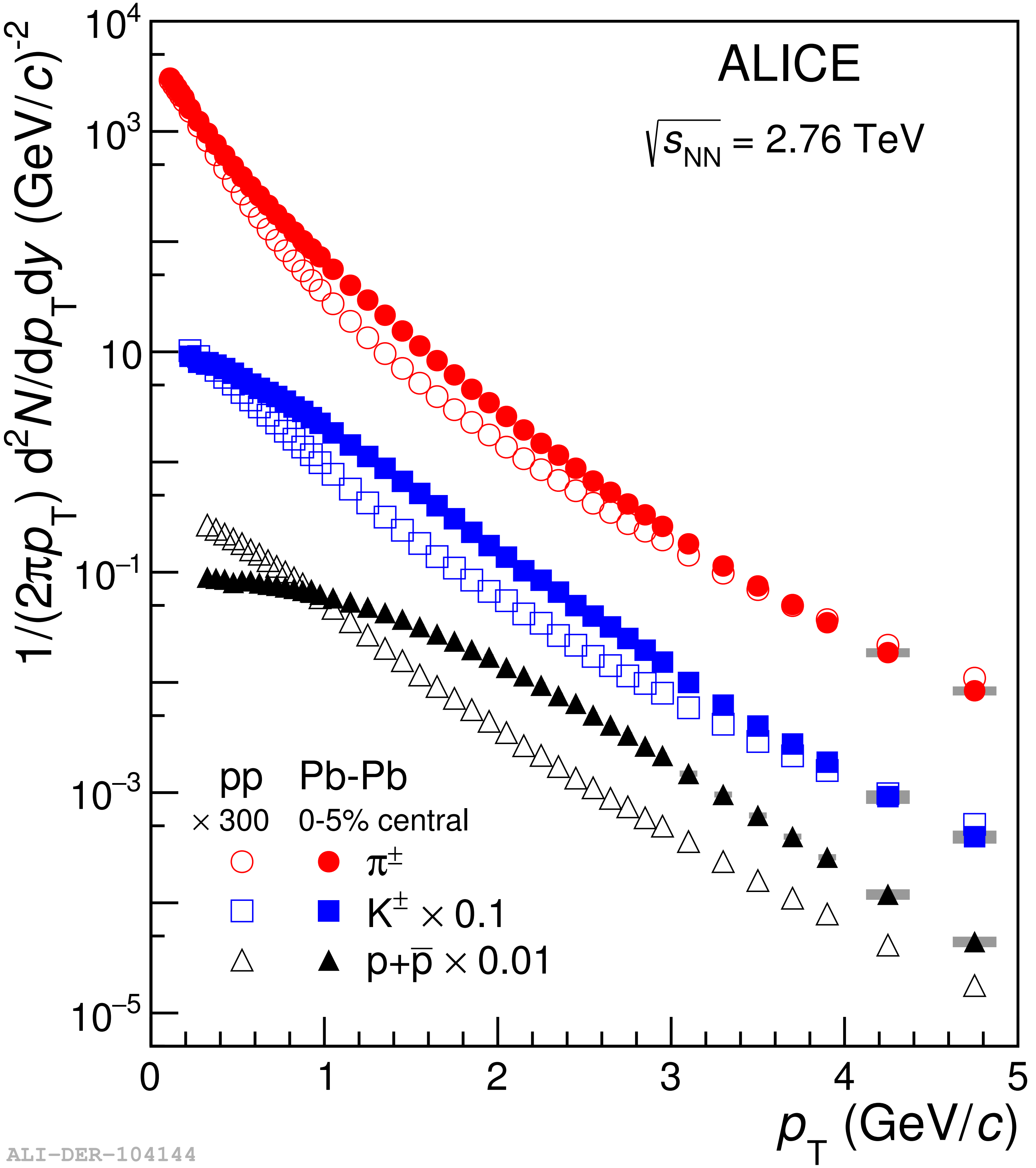
# Basics of relativistic hydrodynamics

# Evidence for collective behavior in heavy-ion collisions

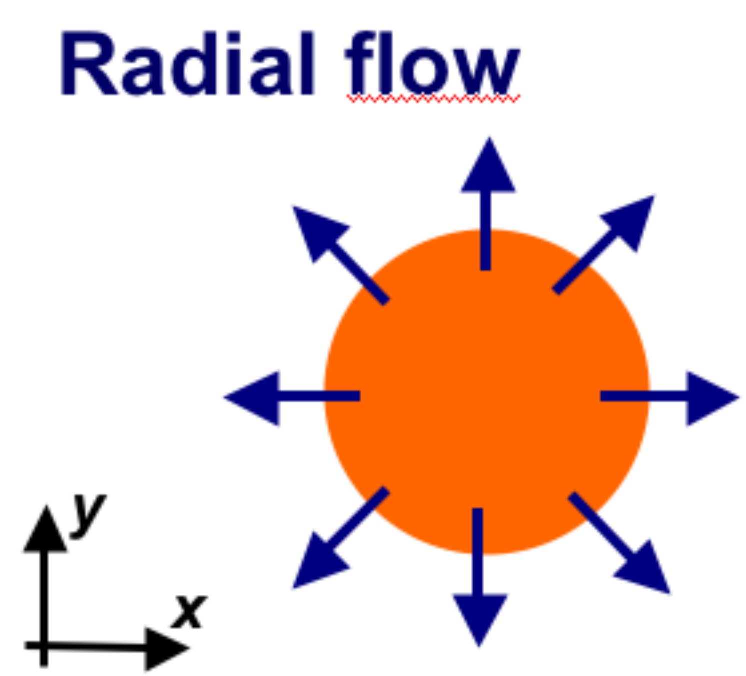


- Shape of low-$p_T$ transverse momentum spectra for particles with different masses
- Azimuthal anisotropy of produced particles
- Source sizes from Hanbury Brown-Twiss correlations
- …

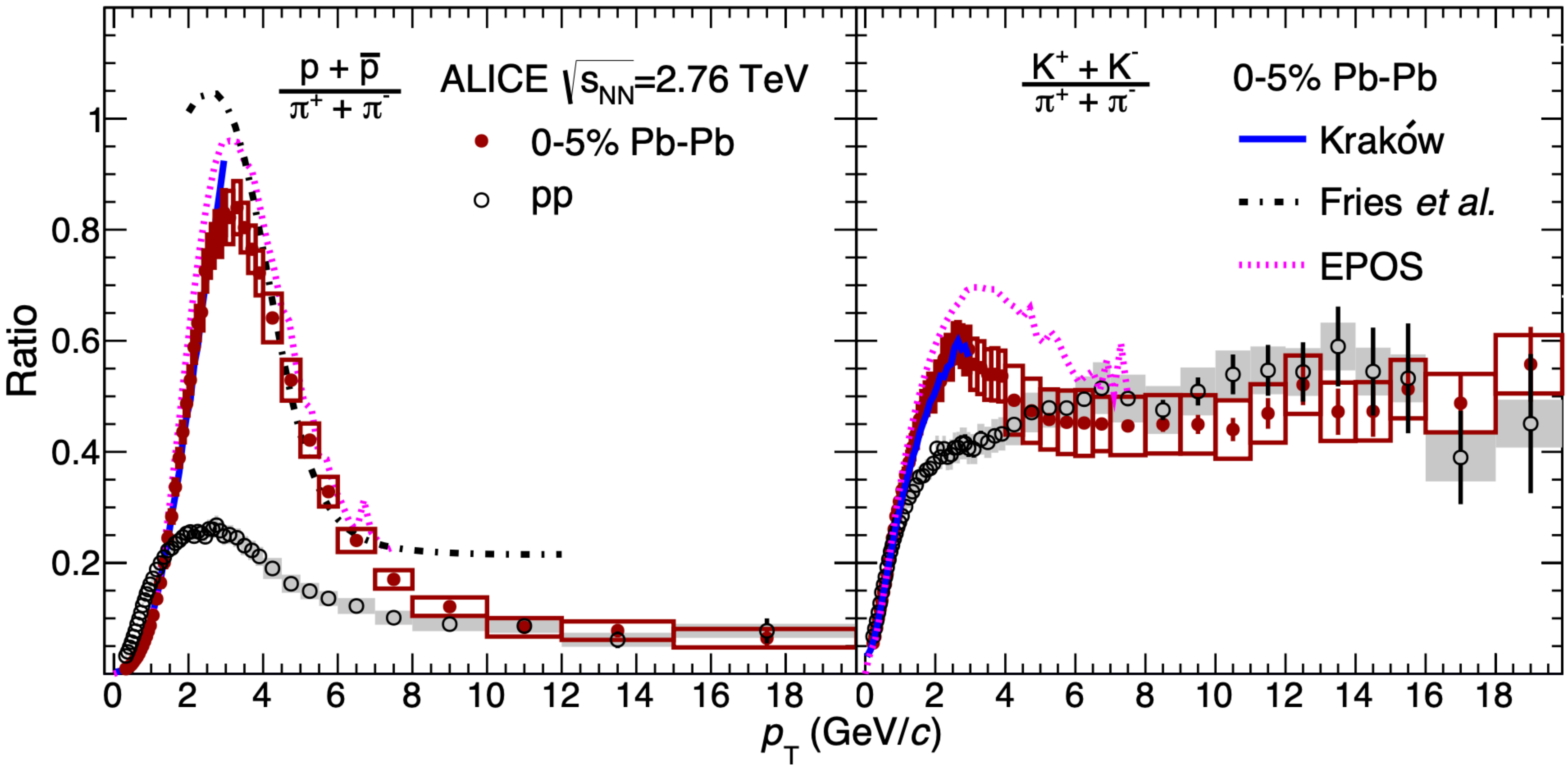
# Evidence for radial flow (1)



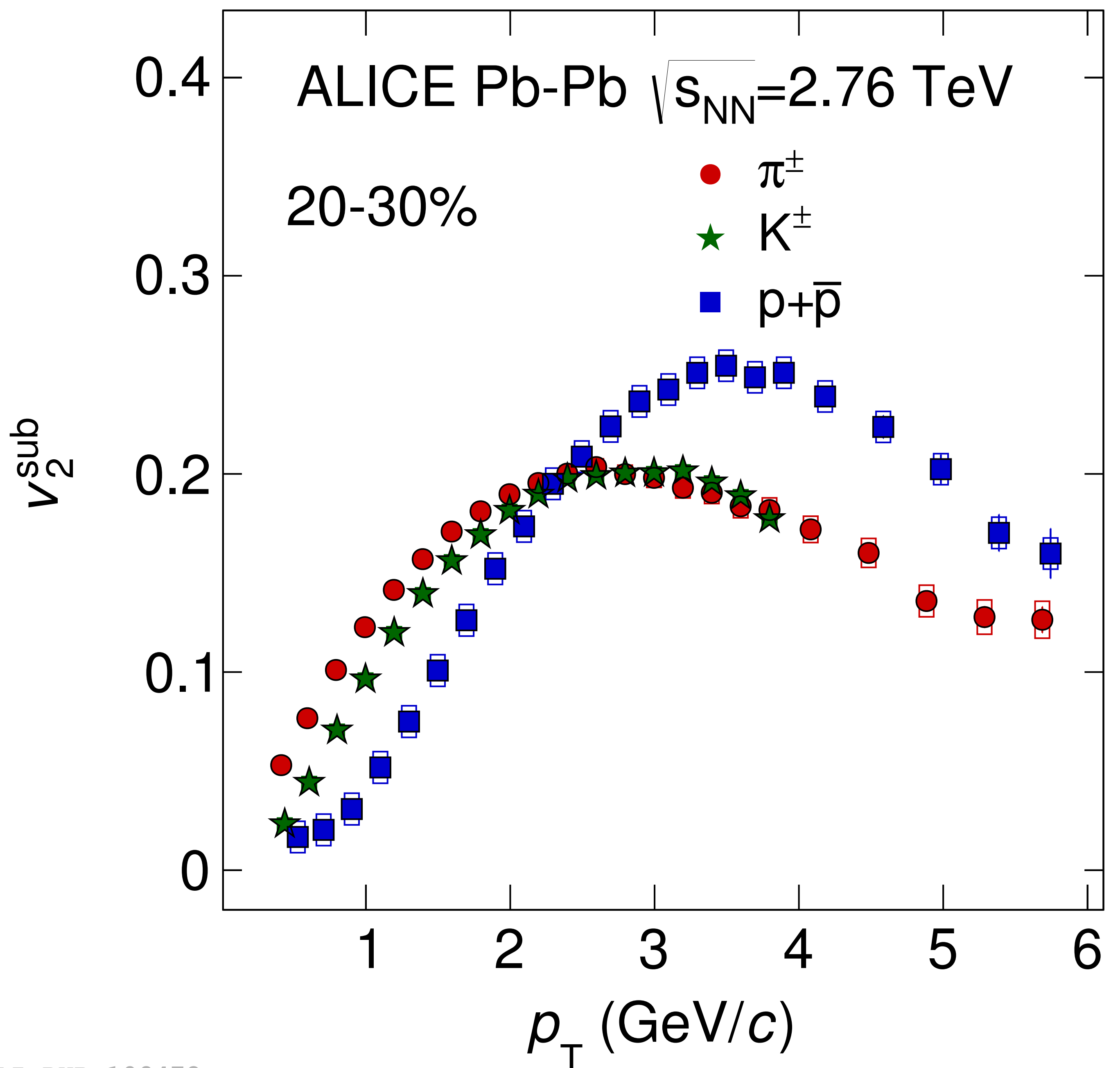

- Shape is different in pp and A-A
- Stronger effect for heavier particles



# Evidence for radial flow (2)

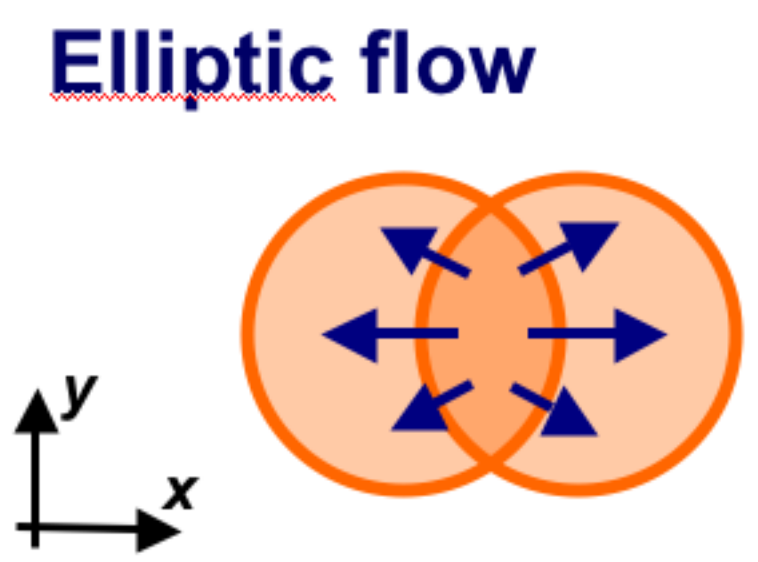


# Evidence for elliptic flow




ALI-PUB-109472

Good explanation:
Azimuthal variation of the flow velocity



# Basics of relativistic hydrodynamics

See e.g. Ollitrault, arXiv:0708.2433

Standard thermodynamics: $P$, $T$, μ constant over the entire volume

Hydrodynamics assumes *local* thermodynamic equilibrium: $P(x^\mu)$, $T(x^\mu)$, $\mu(x^\mu)$

Local thermodynamic equilibrium only possible if mean free path between two collisions much shorter than all characteristic scales of the system:

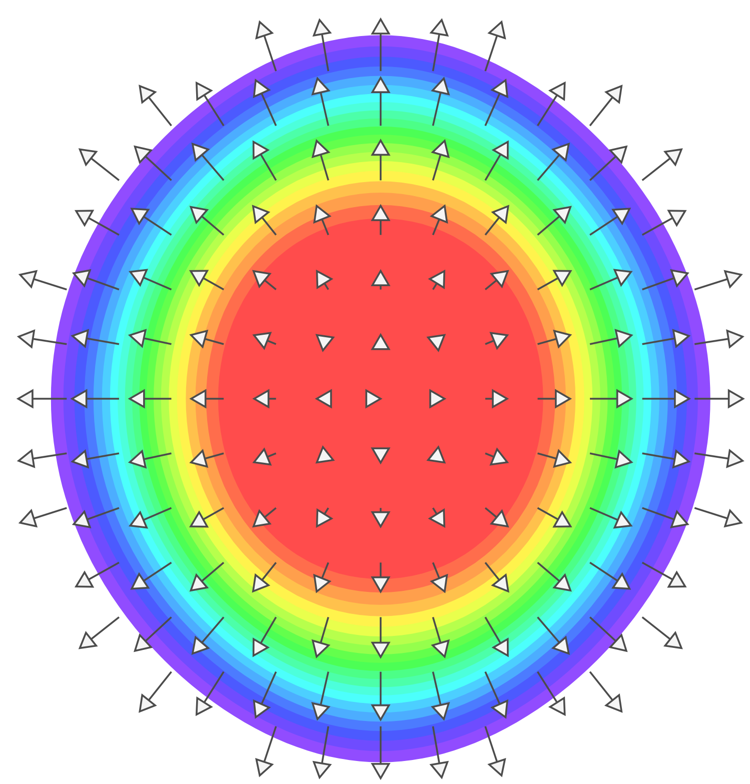
$$\lambda_{\mathrm{mfp}} \ll L$$

This is the limit of non-viscous hydrodynamics.

4-velocity of a fluid element:

$$u = \gamma(1, \vec{\beta}), \quad u^\mu u_\mu = 1$$

$$\gamma = \frac{1}{\sqrt{1 - \vec{\beta}^2}}$$



# Number conservation

Mass conservation in nonrelativistic hydrodynamics:

mass flux density:
$\rho\vec{v} \equiv \rho(t,\vec{r})\vec{v}(t,\vec{r})$

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}(\rho\vec{v}) = 0 \qquad \text{[continuity equation]}$$

Lorentz contraction in the relativistic case: $\rho \quad \rightarrow \quad n\gamma = nu^0$

conserved quantity, e.g. baryon number

The continuity equation then reads: $$\frac{\partial(nu^0)}{\partial t} + \vec{\nabla}(n\vec{u}) = 0$$

$N^\mu = nu^\mu$
$nu^0$ : baryon density
$n\vec{u}$ : baryon flux

The conservation of *n* can be written more elegantly as

$$\partial_\mu(nu^\mu) = 0$$

For a general 4-vector a we have:

$$\partial_\mu \equiv \frac{\partial}{\partial x^\mu} = (\frac{\partial}{\partial t}, \vec{\nabla}), \quad \partial^\mu \equiv \frac{\partial}{\partial x_\mu} = (\frac{\partial}{\partial t}, -\vec{\nabla}), \qquad \partial_\mu a^\mu = (\frac{\partial}{\partial t}, \vec{\nabla}) \cdot (a^0, \vec{a}) = \frac{\partial a^0}{\partial t} + \vec{\nabla}\vec{a}$$

covariant derivative ($\partial_\mu$), contravariant derivative ($\partial^\mu$)

# Energy and momentum conservation

Analogous to the contravariant 4-vector $J^\mu = nu^\mu$ one can define conserved currents for the energy and the three moments components. These can be written as a contravariant tensor:

$$T^{\mu\nu}$$

energy-momentum tensor

$\nu$ : component of the 4-momentum

$\mu$ : component of the associated current

$$T^{\mu\nu} = \begin{pmatrix} \text{energy density} & \text{momentum density} \\ \text{energy flux density} & \text{momentum flux density} \end{pmatrix}$$

$T^{00}$ : the energy density

$T^{0j}$ : density of the $j$-th component of the momentum, $j$ = 1, 2, 3

$T^{i0}$ : energy flux along axis $i$

$T^{ij}$ : flux along axis $i$ of the $j$-th component of the momentum

Examples: $T^{00} = \frac{\partial E}{\partial x \partial y \partial z} \equiv \varepsilon, \quad T^{11} = \frac{\partial p_x}{\partial t \partial y \partial z}$ — force in $x$ direction acting on an surface $\Delta y\, \Delta z$ perpendicular to the force → pressure

# Equations of non-viscous hydrodynamics

Energy-momentum tensor in the fluid rest frame:

$$T_{\mathrm{R}}^{\mu\nu} = \begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & P & 0 & 0 \\ 0 & 0 & P & 0 \\ 0 & 0 & 0 & P \end{pmatrix}$$

pressure

rest frame: pressure is the same in all direction, constant energy density and momentum

For moving fluid cell (Lorentz transformation):
(without derivation)

$$g^{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$$

$$T^{\mu\nu} = (\varepsilon + P) u^{\mu} u^{\nu} - P g^{\mu\nu}$$

Local conservation of energy, momentum and baryon number can then be written as

$$\partial_{\mu} T^{\mu\nu} = 0 \qquad \partial_{\mu}(n u^{\mu}) = 0$$

5 equations for 6 unknowns:
$(u_x, u_y, u_z, \varepsilon, P, n_{\mathrm{B}})$

System with several conserved charges:

$$\partial_{\mu}(n_i u^{\mu}) = 0$$

$i$ = baryon number, strangeness, charge, …

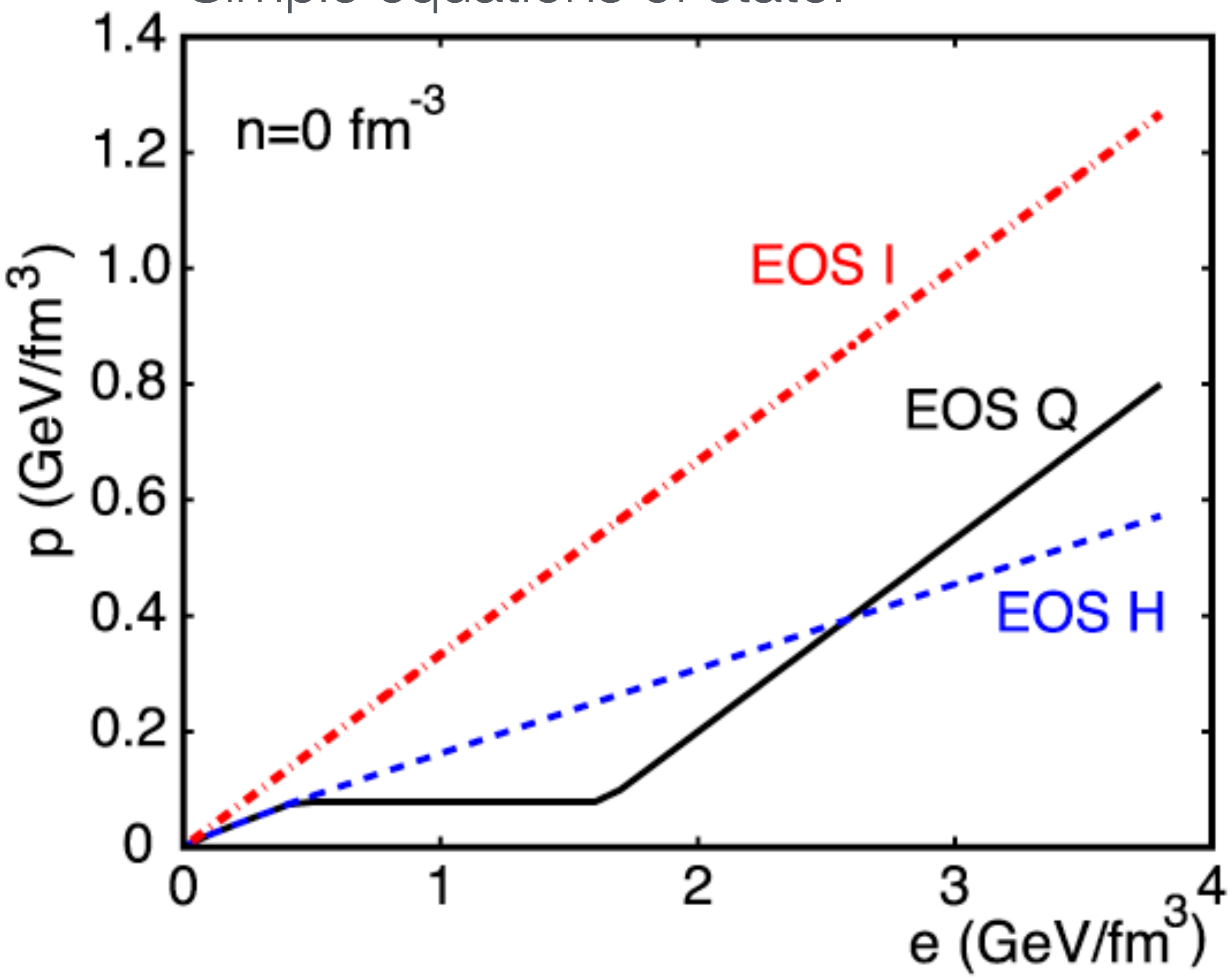
# Ingredients of hydrodynamic models

- Equation of state (EoS) needed to close the system:

$$P(\varepsilon, n_{\mathrm{B}})$$

- Via the EoS hydrodynamics allows one to relate observables with QCD thermodynamics
- Initial conditions ($\varepsilon(x, y, z)$)
  - Glauber MC
  - Color glass condensate
- Transition to free-streaming particles
  - E.g. at given local temperature

Simple equations of state:




EOS I: ultra-relativistic gas $P = \varepsilon/3$

EOS H: resonance gas, $P \approx 0.15\ \varepsilon$

EOS Q: phase transition,
QGP ↔ resonance gas

# Intermediate summary: What do we learn from hydro modeling of HICs?

| | |
|---|---|
| Initial state: | **unknown** (use another model) |
| Equation of state: | **want to study** |
| Transport coefficients: | **want to study** |
| Freeze-out: | **unknown** (use another model) |

Pasi Huovinen, THOR Winter School 2020

# Cooper-Frye freeze-out formula

See Ulrich W. Heinz, Concepts of heavy ion physics, hep-ph/0407360

## Count particles of species *i* crossing a 3-dimensional hyper-surface Σ:

$$N_i = \int_\Sigma d^3\sigma_\mu(x)\, N_i^\mu(x) \qquad N_i^\mu = \int \frac{d^3p}{E} p^\mu f_i(x,p) \qquad f_i(x,p) = \frac{g}{(2\pi)^3} \frac{1}{\exp\left(\frac{p_\mu u^\mu(x) - \mu(x)}{T(x)}\right) \pm 1}$$

any surface that encloses the future light cone emerging from the collision point

Single particle phase-space distribution.

In rest frame of the fluid cell

$u^\mu = (1,0,0,0) \rightsquigarrow p_\mu \cdot u^\mu = E^*$

i.e., $E^*$ = energy in fluid rest frame

## Particle spectra from fluid motion:

Cooper, Frye, Phys. Rev. D10 (1974) 186

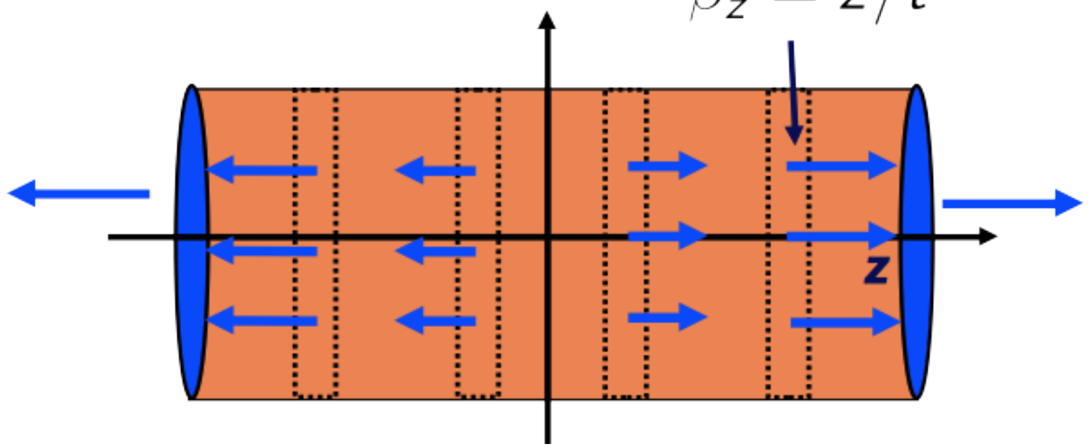
local thermal distribution

normal vector to the 3d freeze-out hyper surface Σ in space-time defined e.g. by $T = T_{\mathrm{fo}}$

$$E\frac{\mathrm{d}N}{\mathrm{d}^3p} = \frac{1}{2\pi p_T}\frac{\mathrm{d}^2N}{\mathrm{d}p_T\,\mathrm{d}y} = \int_{\Sigma_f} f(x,p) p^\mu\,\mathrm{d}\sigma_\mu$$

$$= \frac{g}{(2\pi)^3}\int_{\Sigma_f} \frac{p^\mu\,\mathrm{d}\sigma_\mu}{\exp\left(\frac{p_\mu \cdot u^\mu(x) - \mu(x)}{T(x)}\right) \pm 1}$$

# Longitudinal expansion: Bjorken's scaling solution (I)




proper time:

$$\tau = t/\gamma = t\sqrt{1-\beta_z^2} = \sqrt{t^2 - z^2}$$

The Bjorken model is a 1d hydrodynamic model (expansion only in *z* direction). The initial conditions correspond to the one which one would get from free streaming particles starting at (*t*, *z*) = (0, 0).

Initial conditions in the Bjorken model:

preserved during the hydro evolution, i.e., $u^\mu(\tau) = \frac{x^\mu}{\tau}$

$$\varepsilon(\tau_0) = \varepsilon_0, \quad u^\mu = \frac{1}{\tau_0}(t, 0, 0, z) = \frac{x^\mu}{\tau_0}$$

initial energy density

In this case the equations of ideal hydrodynamics simplify to

$$\frac{d\varepsilon}{d\tau} + \frac{\varepsilon + p}{\tau} = 0$$

# Longitudinal expansion: Bjorken's scaling solution (II)

For an ideal gas of quarks and gluons, i.e., for

$$\varepsilon = 3p, \quad \varepsilon \propto T^4$$

this gives

$$\varepsilon(\tau) = \varepsilon_0 \left(\frac{\tau}{\tau_0}\right)^{-4/3}, \quad T(\tau) = T_0 \left(\frac{\tau}{\tau_0}\right)^{-1/3}$$

The temperature drops to the critical temperature at the proper time

$$\tau_c = \tau_0 \left(\frac{T_0}{T_c}\right)^3$$

The QGP lifetime is therefore given by

$$\Delta\tau_{\mathrm{QGP}} = \tau_c - \tau_0 = \tau_0 \left[\left(\frac{T_0}{T_c}\right)^3 - 1\right]$$

# Mixed phase in the Bjorken model

Entropy conservation in ideal hydrodynamics leads in the case of the Bjorken model (independent of the equation of state) to

$$s(\tau) = \frac{s_0 \tau_0}{\tau}$$

In case of an the ideal QGP: $s = \frac{\varepsilon + p}{T} = \frac{4}{3}\frac{\varepsilon}{T} = \frac{4}{3}\frac{\varepsilon_0}{T_0}\frac{\tau_0}{\tau}$

If we consider a QGP/hadron gas phase transition we have a first oder phase transition and a mixed phase with temperature $T_c$. The **entropy in the mixed phase** is given by

$\xi(\tau)$: fraction of fireball in hadron gas phase

$$s(\tau) = s_{HG}(T_c)\xi(\tau) + s_{QGP}(T_c)(1 - \xi(\tau)) = \frac{s_{QGP}(T_c)\tau_c}{\tau}$$

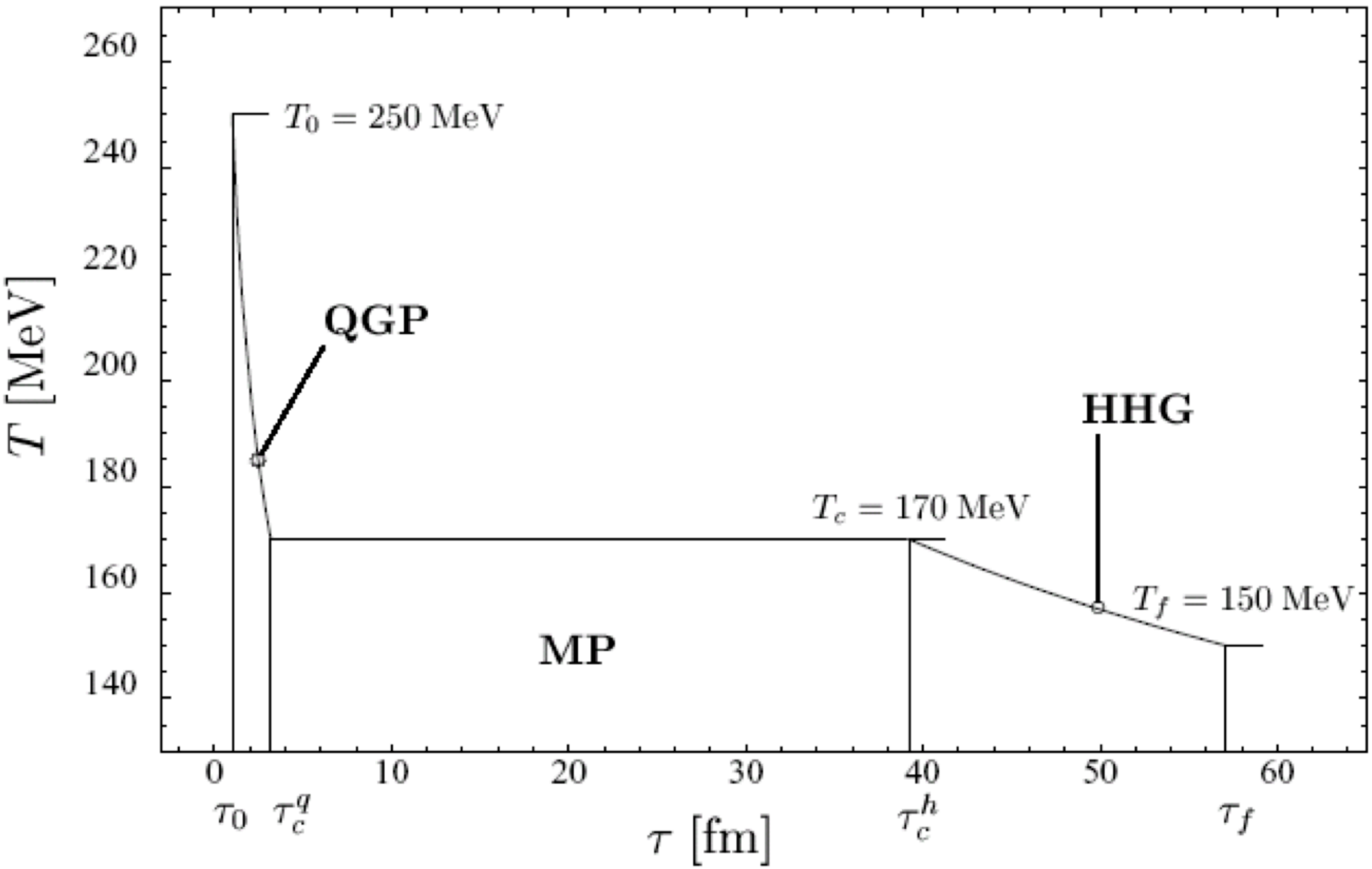
This equation determines the time dependence of $\xi(\tau)$ and the time $\tau_h$ at which the mixed phase vanishes:

end of mixed phase

$$\frac{s_{HG}}{s_{QGP}} = \frac{g_{HG}}{g_{QGP}} \rightsquigarrow \quad \xi(\tau) = \frac{1 - \tau_c/\tau}{1 - g_{HG}/g_{QGP}} \quad \rightsquigarrow \quad \tau_h = \tau_c \frac{g_{QGP}}{g_{HG}}$$

the hadron gas close to $T_c$ can be described with $g_{HG} \approx 12$

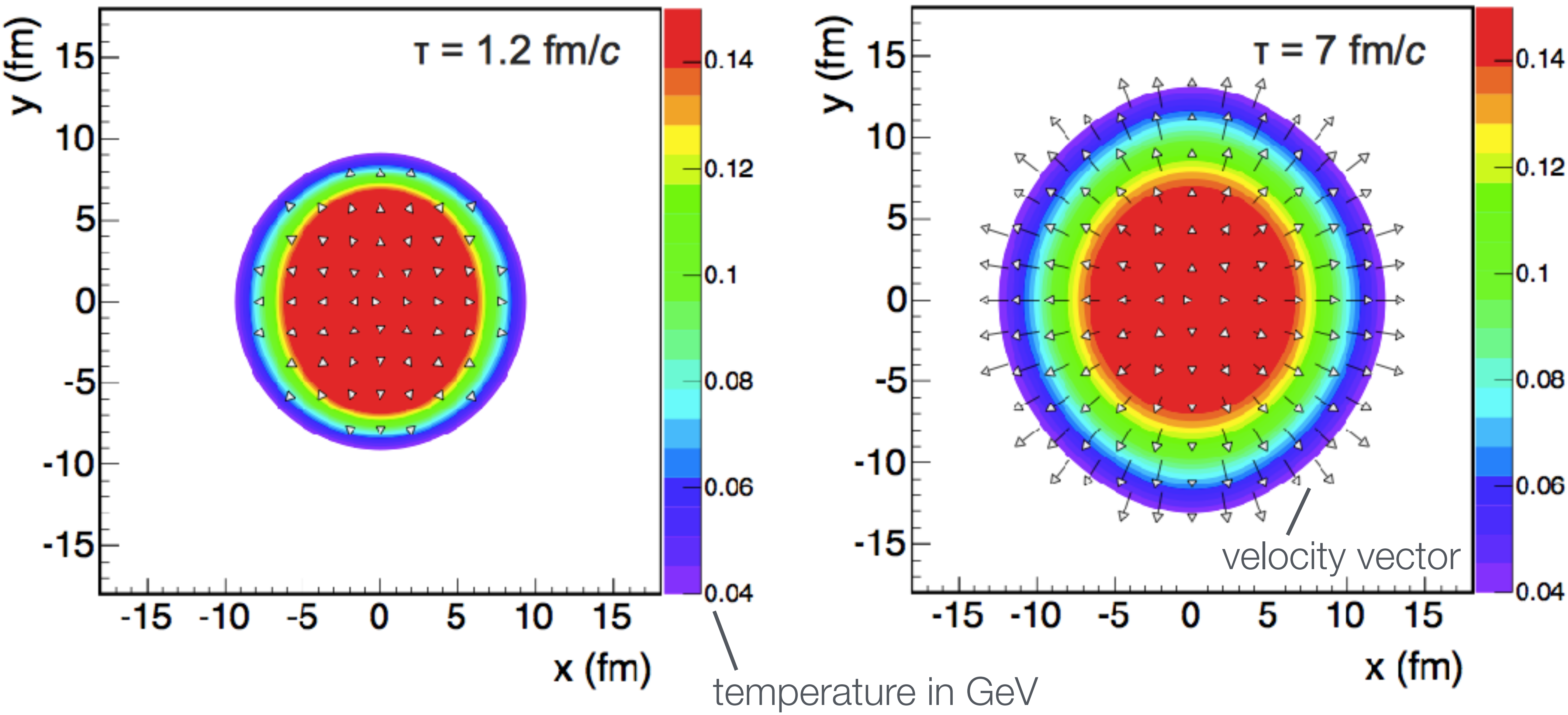
# Temperature evolution in the Bjorken model



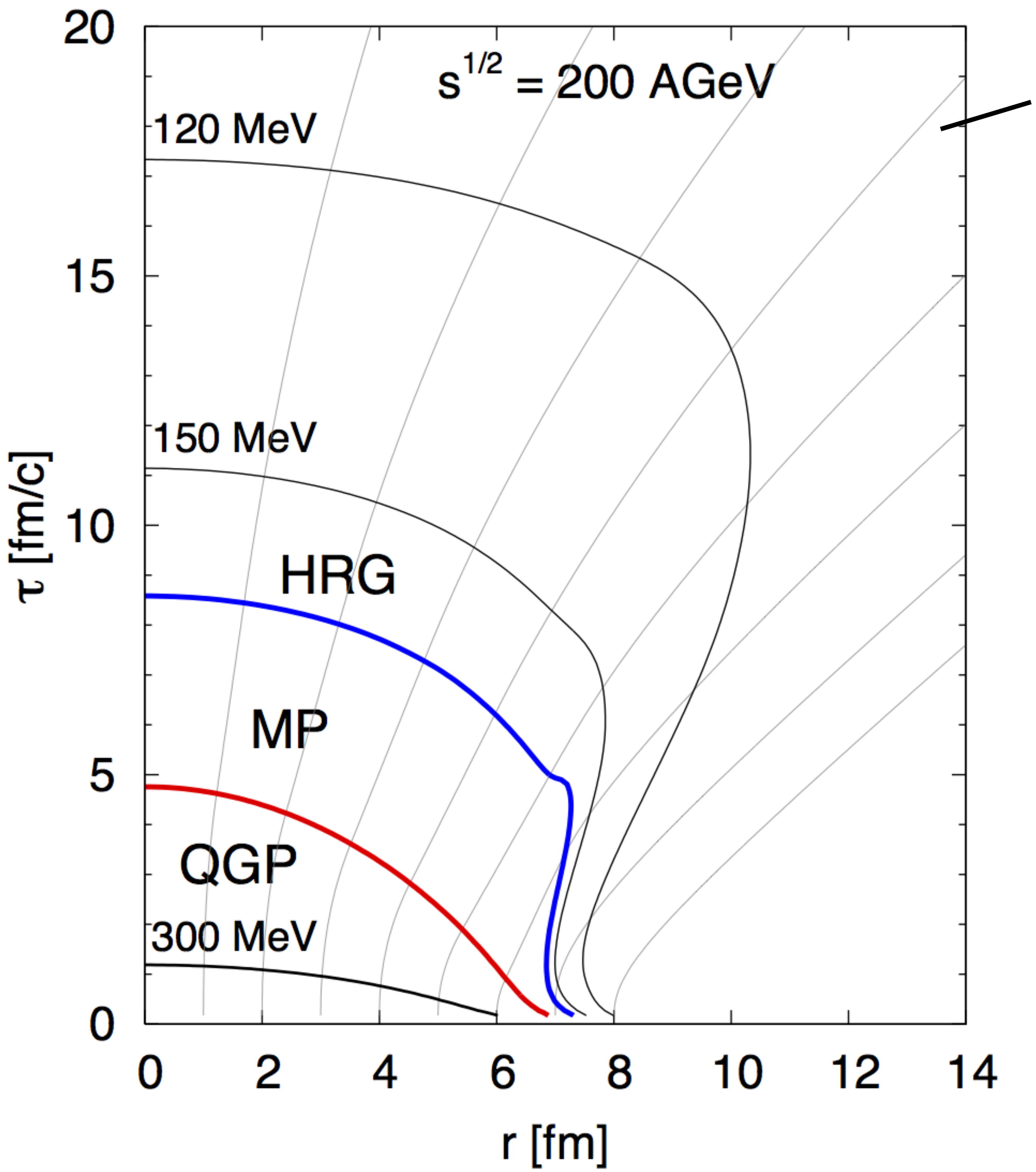
# Transverse expansion

Transverse expansion of the fireball in a hydro model (temperature profile)

2+1 d hydro: Bjorken flow in longitudinal direction



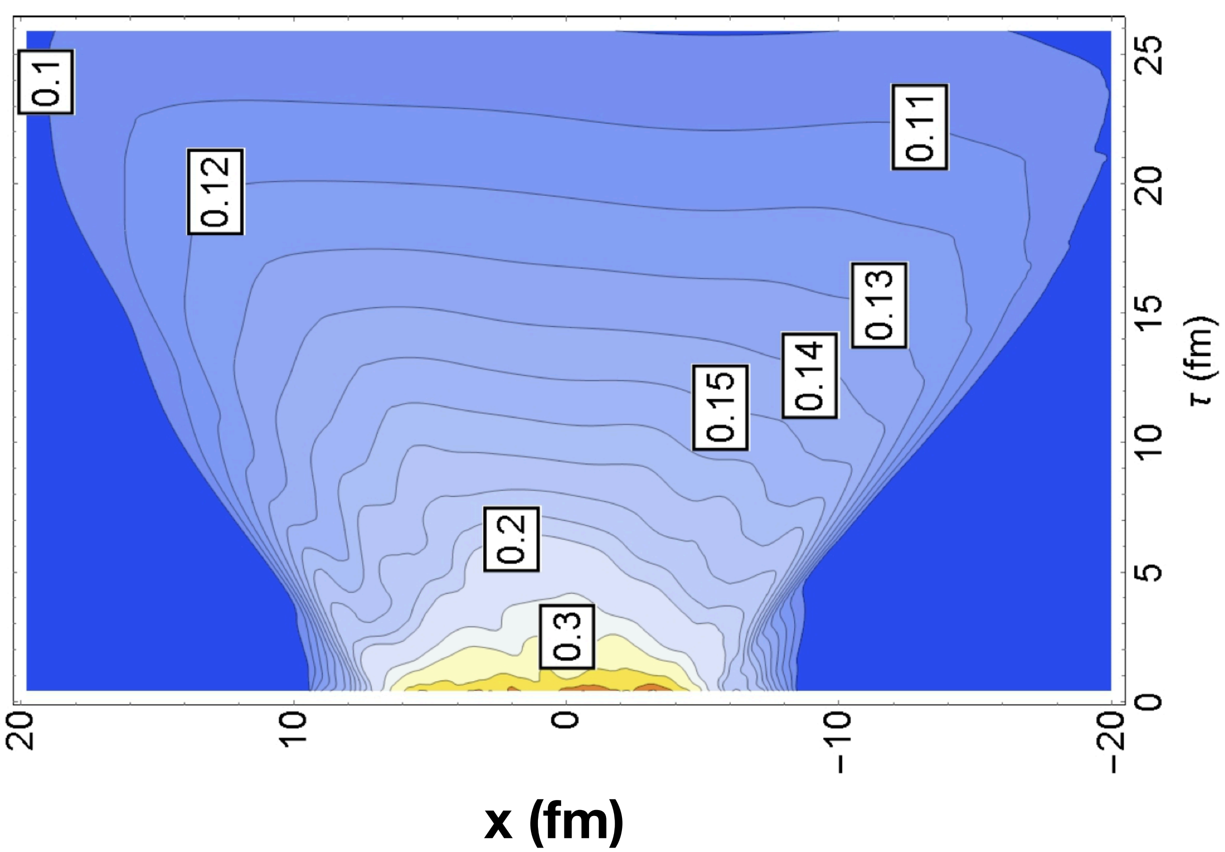
# Temperature Contours and Flow lines



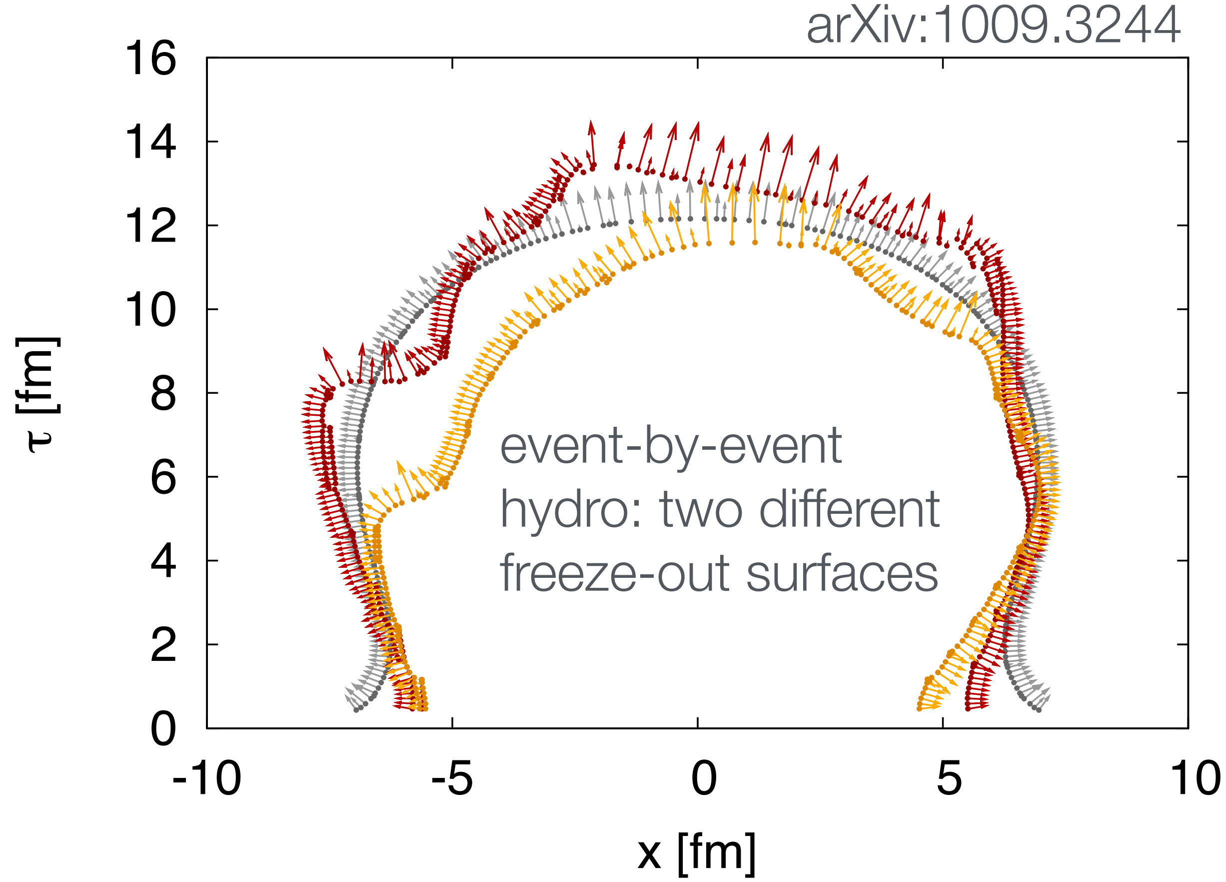

flow lines indicate motion of fluid cells

# Hydrodynamic modeling of heavy-ion collisions: State of the art

- Equation of state from lattice QCD
- (2+1)D or (3+1)D viscous hydrodynamics
- Fluctuating initial conditions (event-by-event hydro)
- Hydrodynamic evolution followed by hadronic cascade

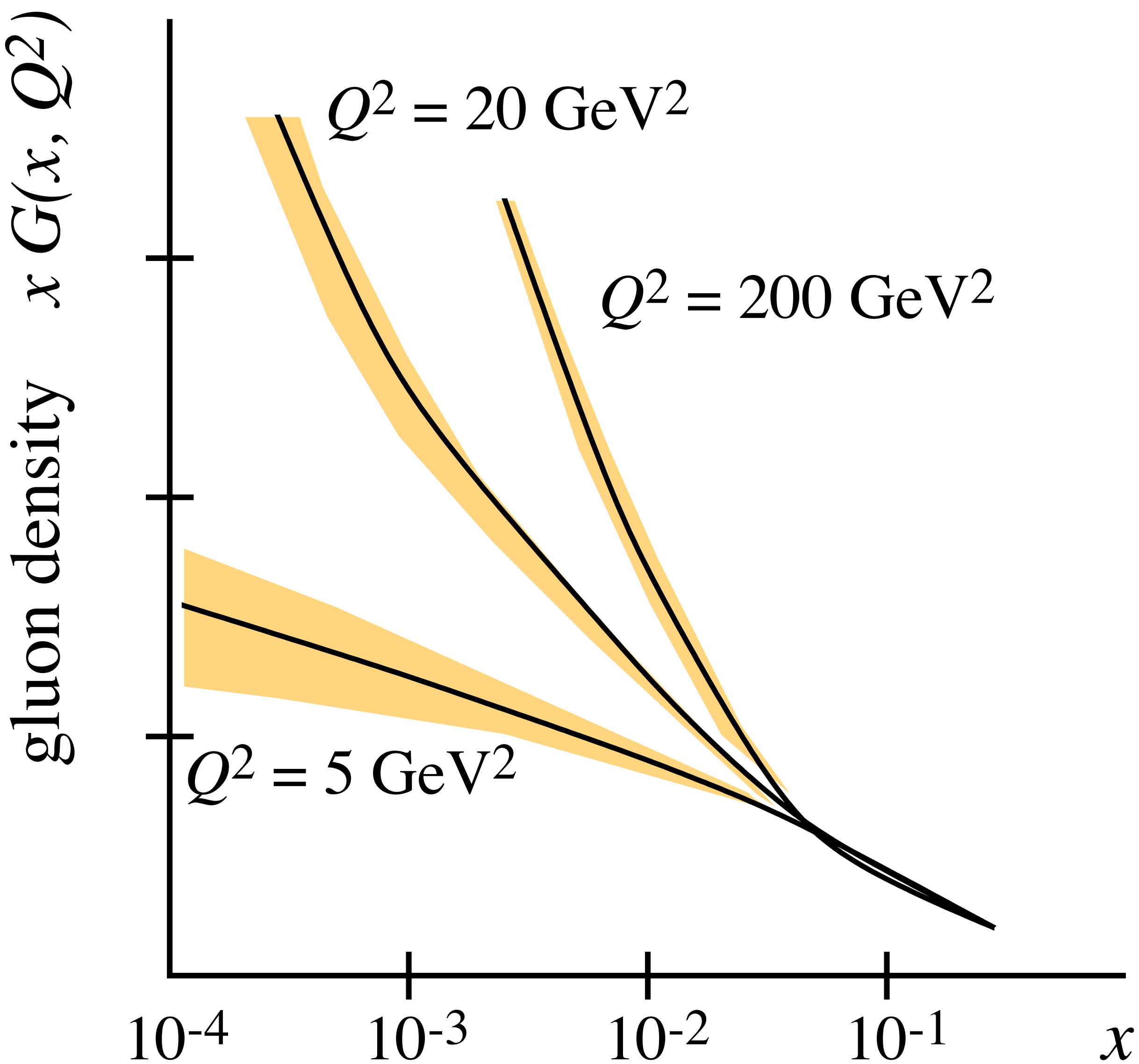


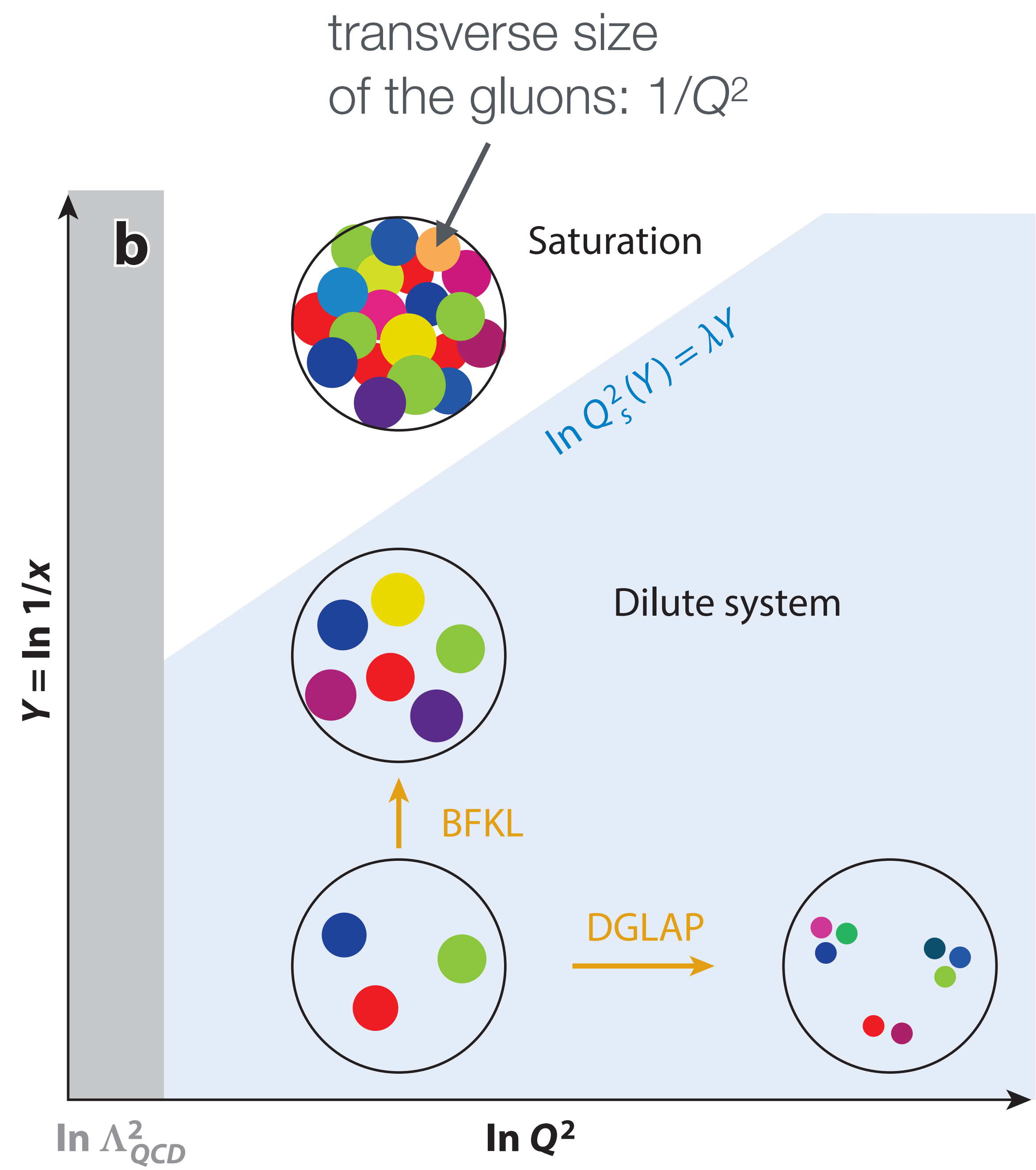

arXiv:1009.3244



# Initial conditions from gluon saturation models (I)

Annu. Rev. Nucl. Part. Sci. 2010.60:463



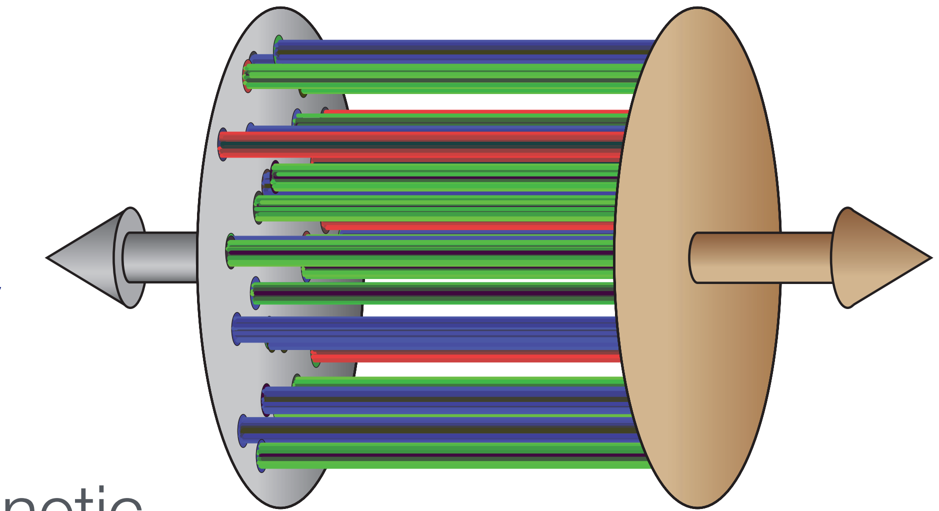




Growth of gluons saturates at an occupation number $1/\alpha_s$. This defines a (semihard) scale $Q_s(x)$, i.e., a typical gluon transverse momentum.

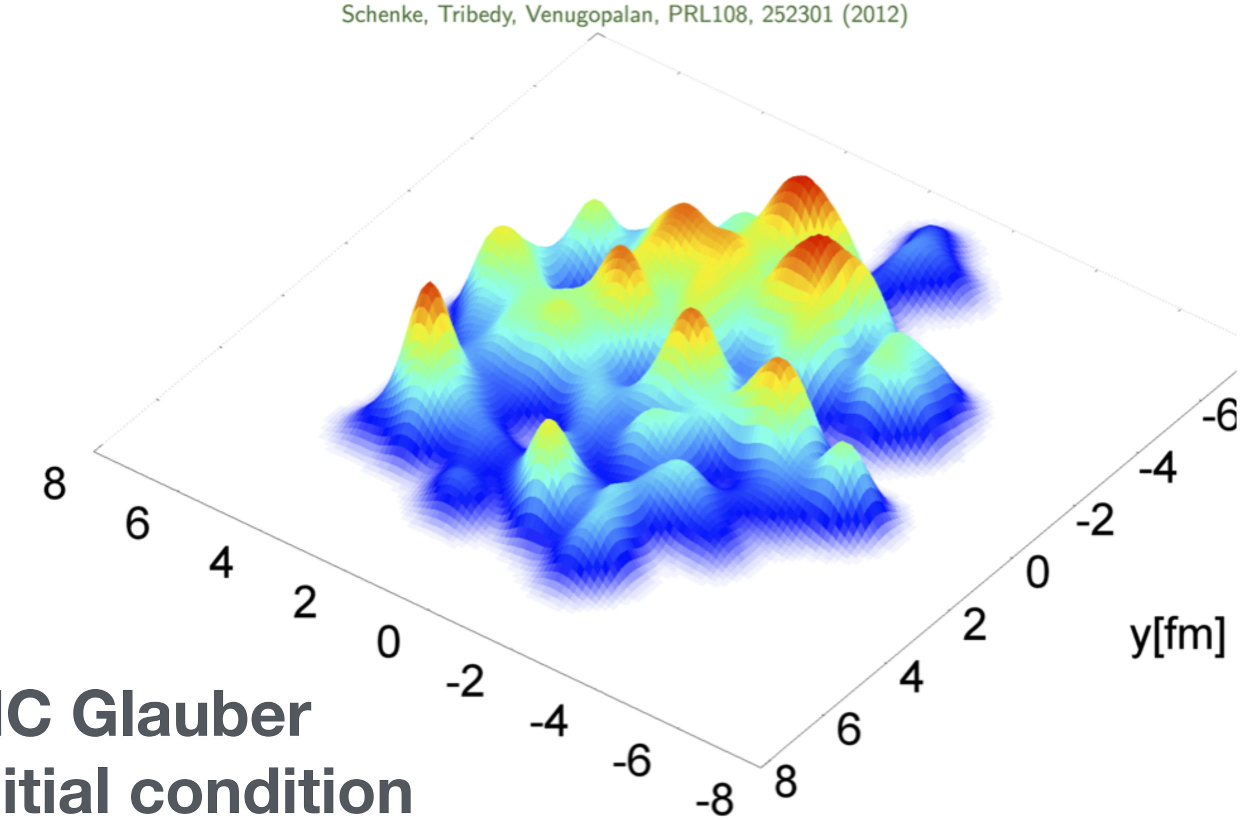
$$\frac{1}{2(N_c^2-1)}\,\frac{xG(x,Q_s^2)}{\pi R^2 Q_s^2} = \frac{1}{\alpha_s(Q_s^2)}$$

# Initial conditions from gluon saturation models (II)

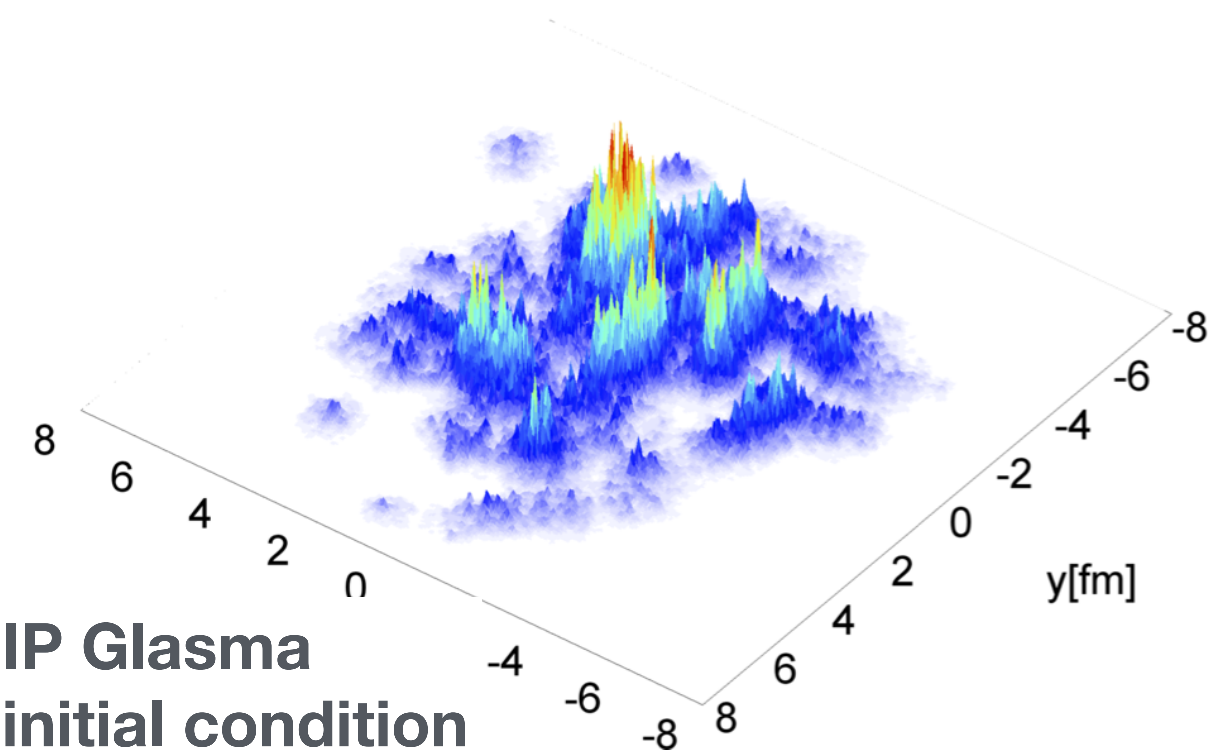
Annu. Rev. Nucl. Part. Sci. 2010.60:463
Rev.Mod.Phys. 93 (2021) 3, 035003



- Color glass condensate:
  Effective field theory, which describes universal properties of saturated gluons in hadron wave functions
- CGC dynamics defines field configurations at early times
  - Strong longitudinal chromoelectric and chromomagnetic fields screened on transverse distance scales $1/Q_S$.



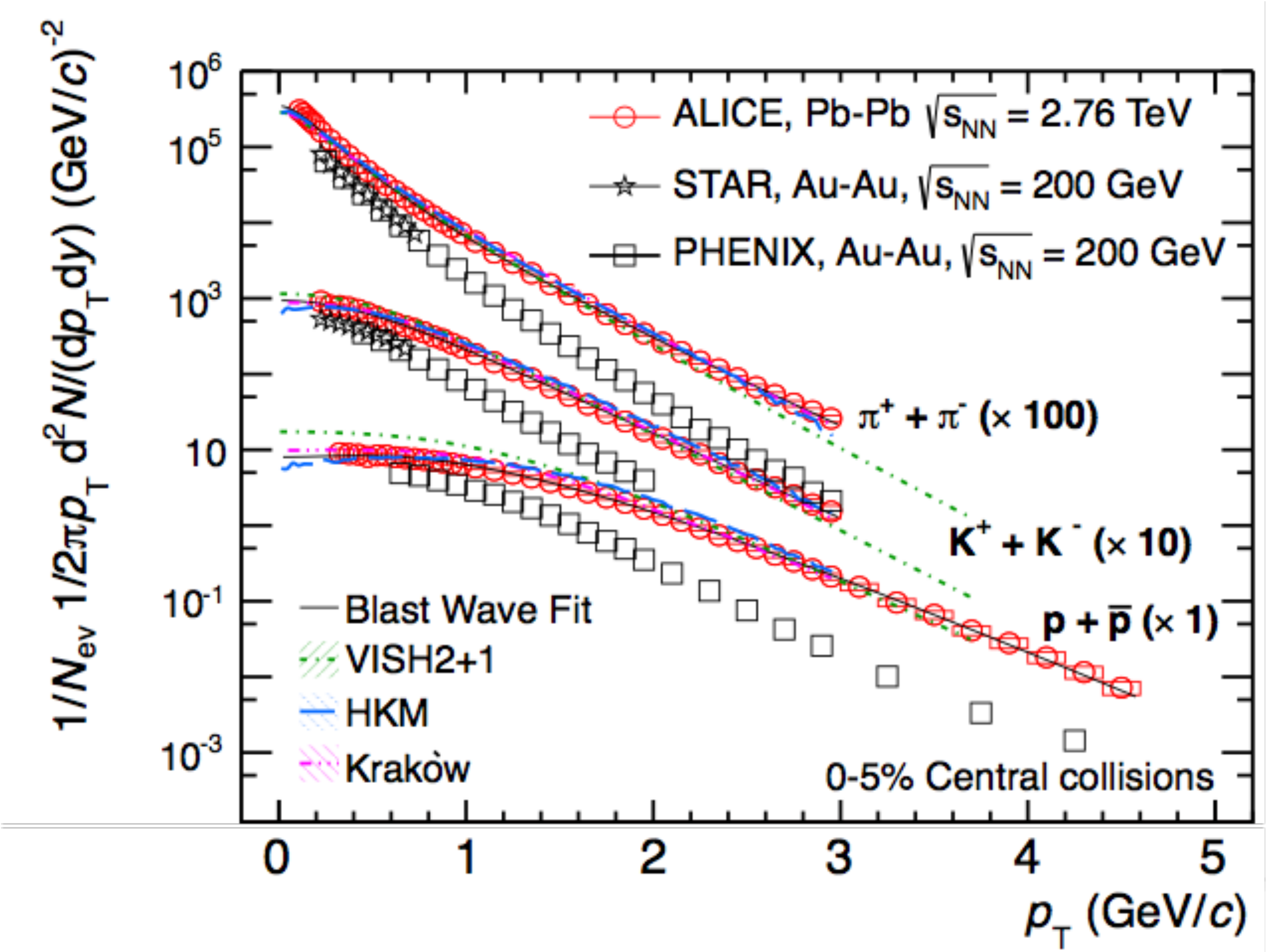

**MC Glauber initial condition**




**IP Glasma initial condition**

# Spectra and Radial flow

# Comparison of π, K, p spectra with hydro models



# The blast-wave model: A Simple model to describe the effect of radial flow on particle spectra

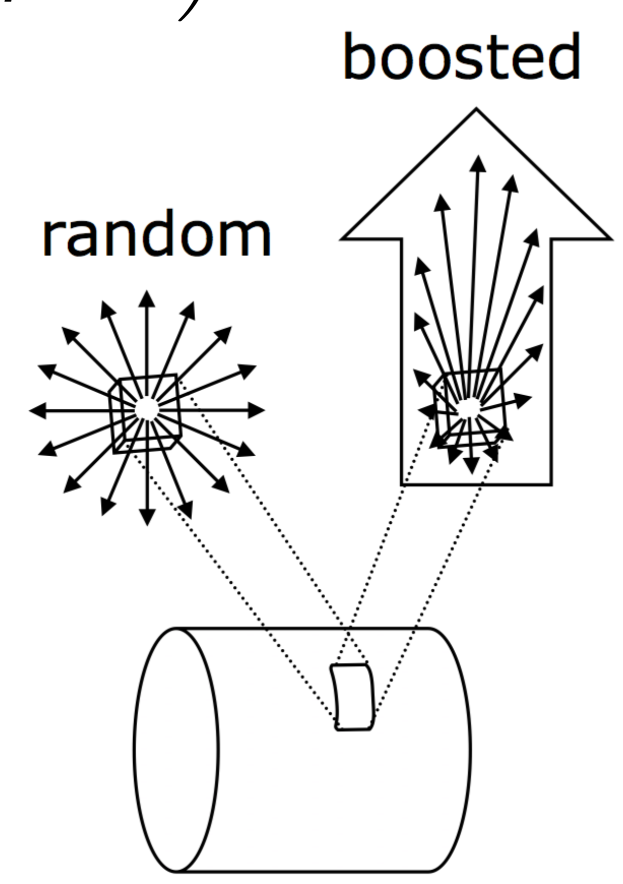
Transverse velocity profile: $$\beta_T(r) = \beta_s \left(\frac{r}{R}\right)^n$$

Superposition of thermal sources with different radial velocities:

$$\frac{1}{m_T}\frac{dn}{dm_T} \propto \int_0^R r\,dr\, m_T I_0\left(\frac{p_T \sinh\rho}{T}\right) K_1\left(\frac{m_T \cosh\rho}{T}\right)$$

$$\rho := \mathrm{arctanh}(\beta_T) \quad \text{"transverse rapidity"}$$

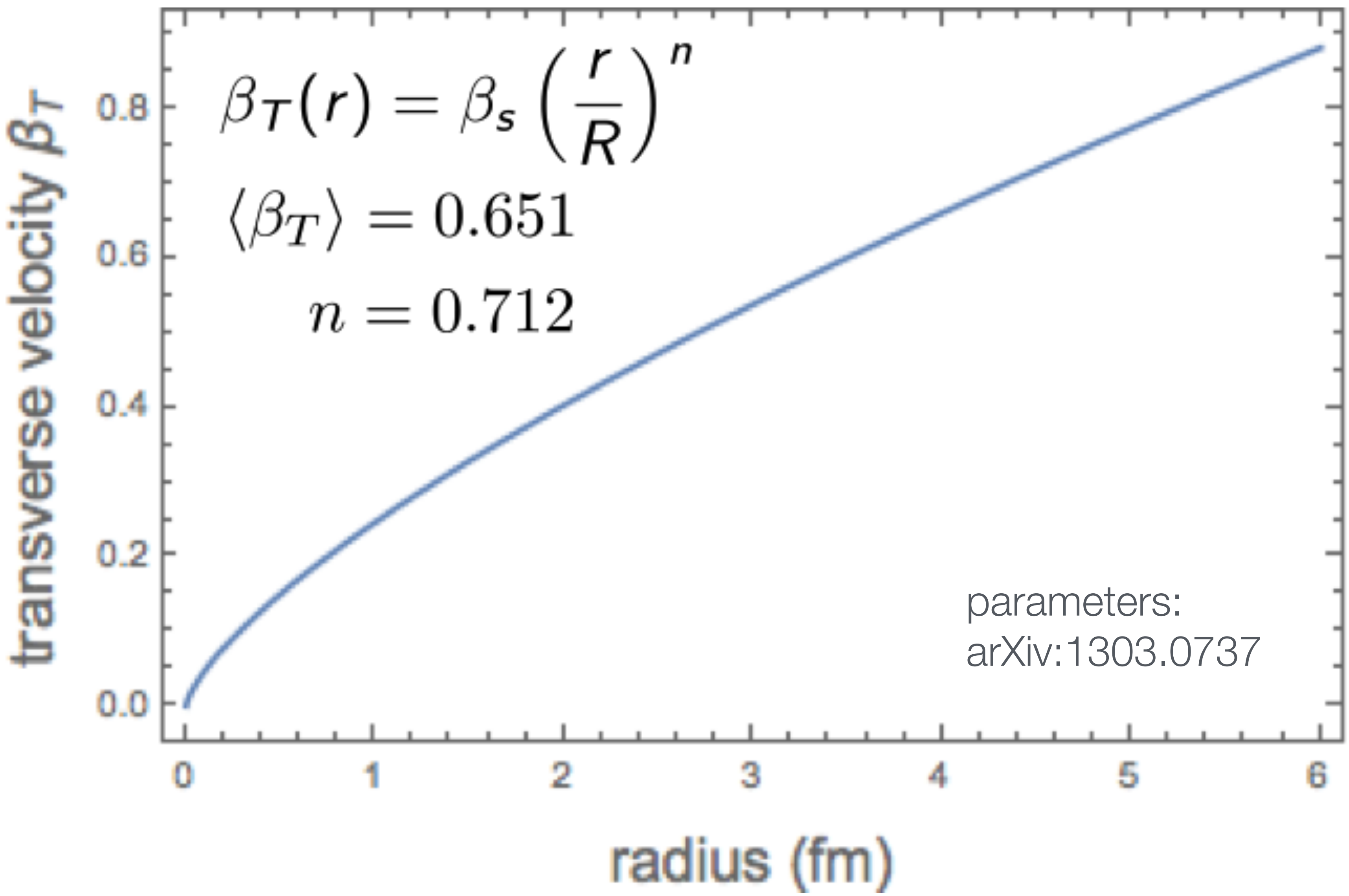
$$I_0,\ K_1 : \ \text{modified Bessel functions}$$




Schnedermann, Sollfrank, Heinz, Phys.Rev.C48:2462-2475,1993

Freeze-out at a 3d hyper-surface, typically instantaneous, e.g.: $$t_f(r,z) = \sqrt{\tau_f^2 + z^2}$$

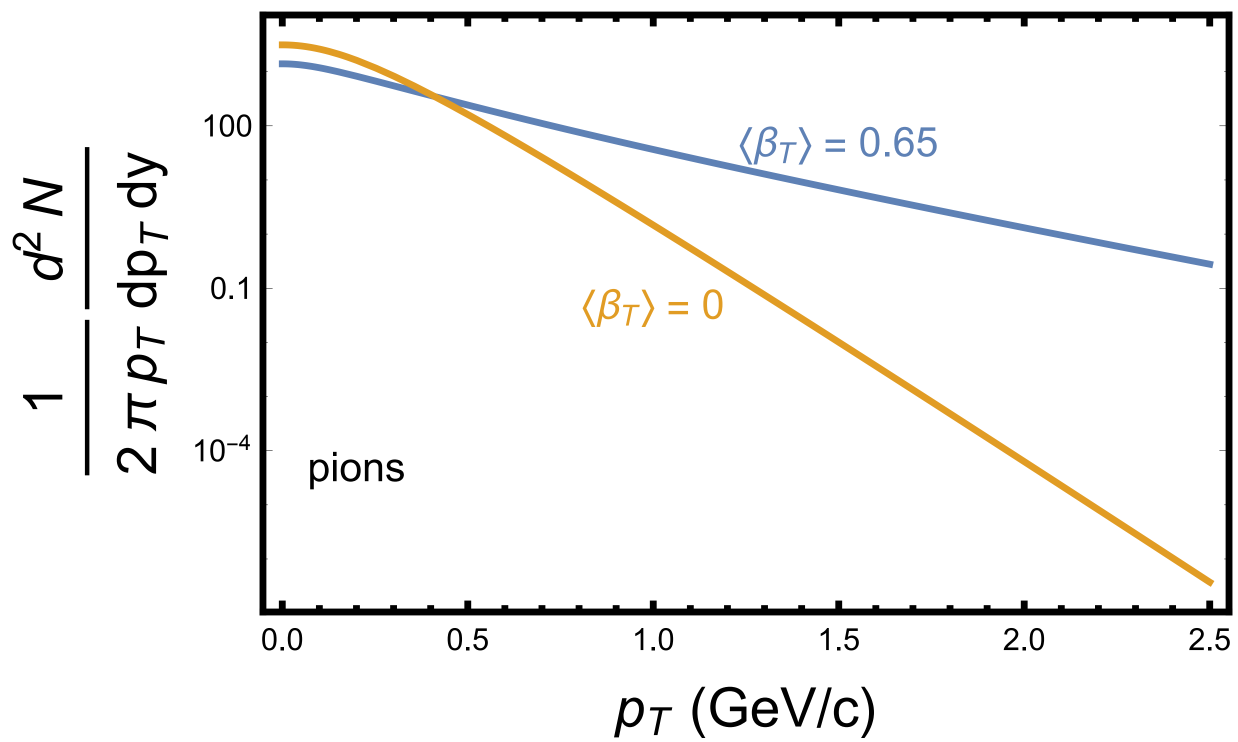
# Example: Radial Flow Velocity Profile from Blast-wave Fit to 2.76 TeV Pb-Pb Spectra (0-5%)



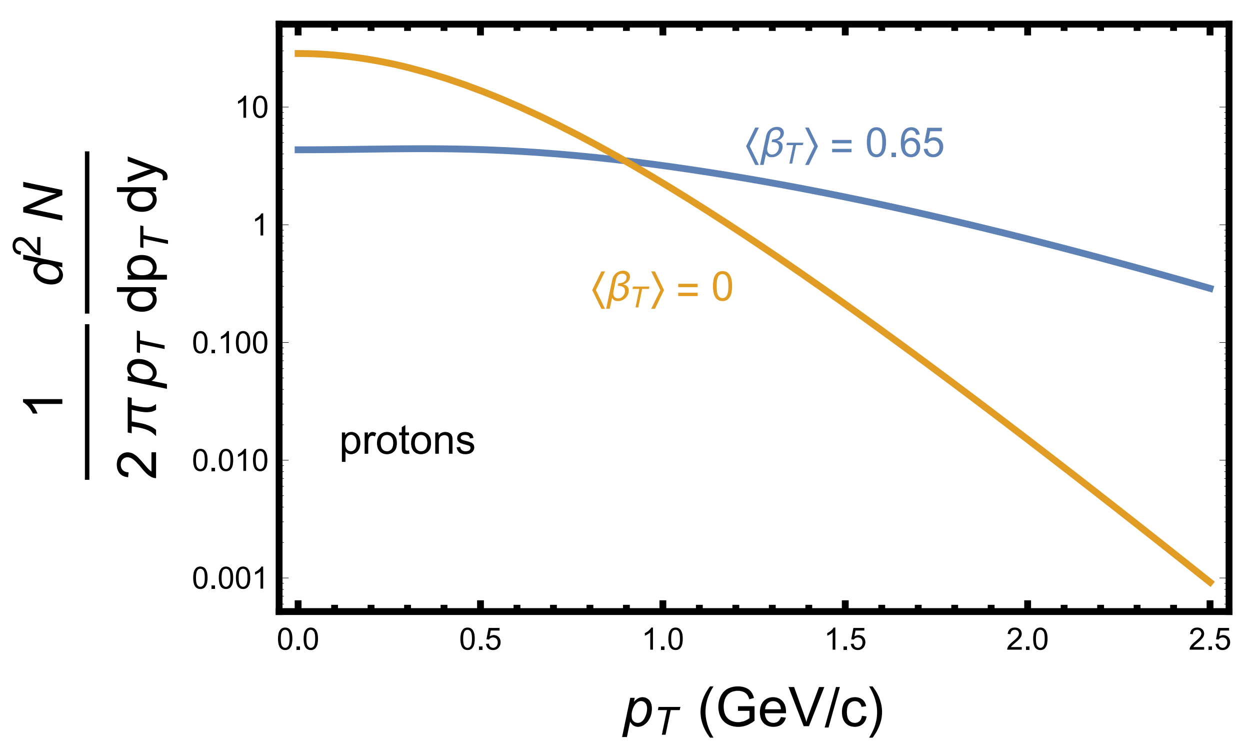

$$\langle \beta_T \rangle = \frac{\int_0^R \int_0^{2\pi} r dr d\varphi \beta_T(r)}{\int_0^R \int_0^{2\pi} r dr d\varphi} = \frac{2}{n+2} \beta_s$$

$$\langle \beta_T \rangle = 0.651, n = 0.712$$
$$\rightarrow \quad \beta_s = 0.8$$

# Example: Pion and Proton $p_T$ Spectra from blast-wave model



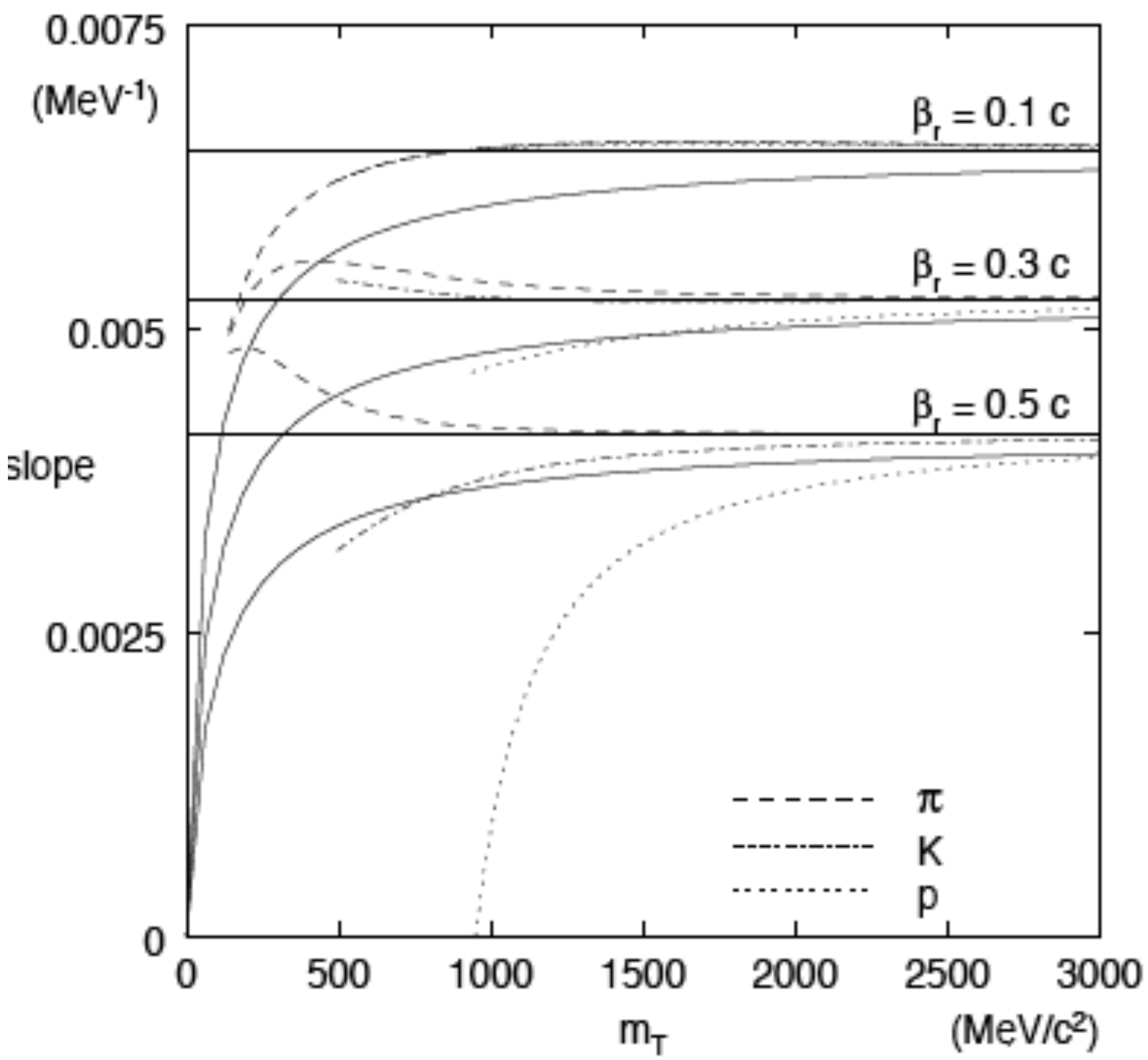

Parameters for 0-5% most central Pb-Pb collisions at 2.76 TeV, arXiv:1303.0737




Larger $p_T$ kick for particles with higher mass:

$$p = \beta_{\text{source}} \gamma_{\text{source}} m + \text{"thermal"}$$

# Local slope of $m_T$ spectra with radial flow



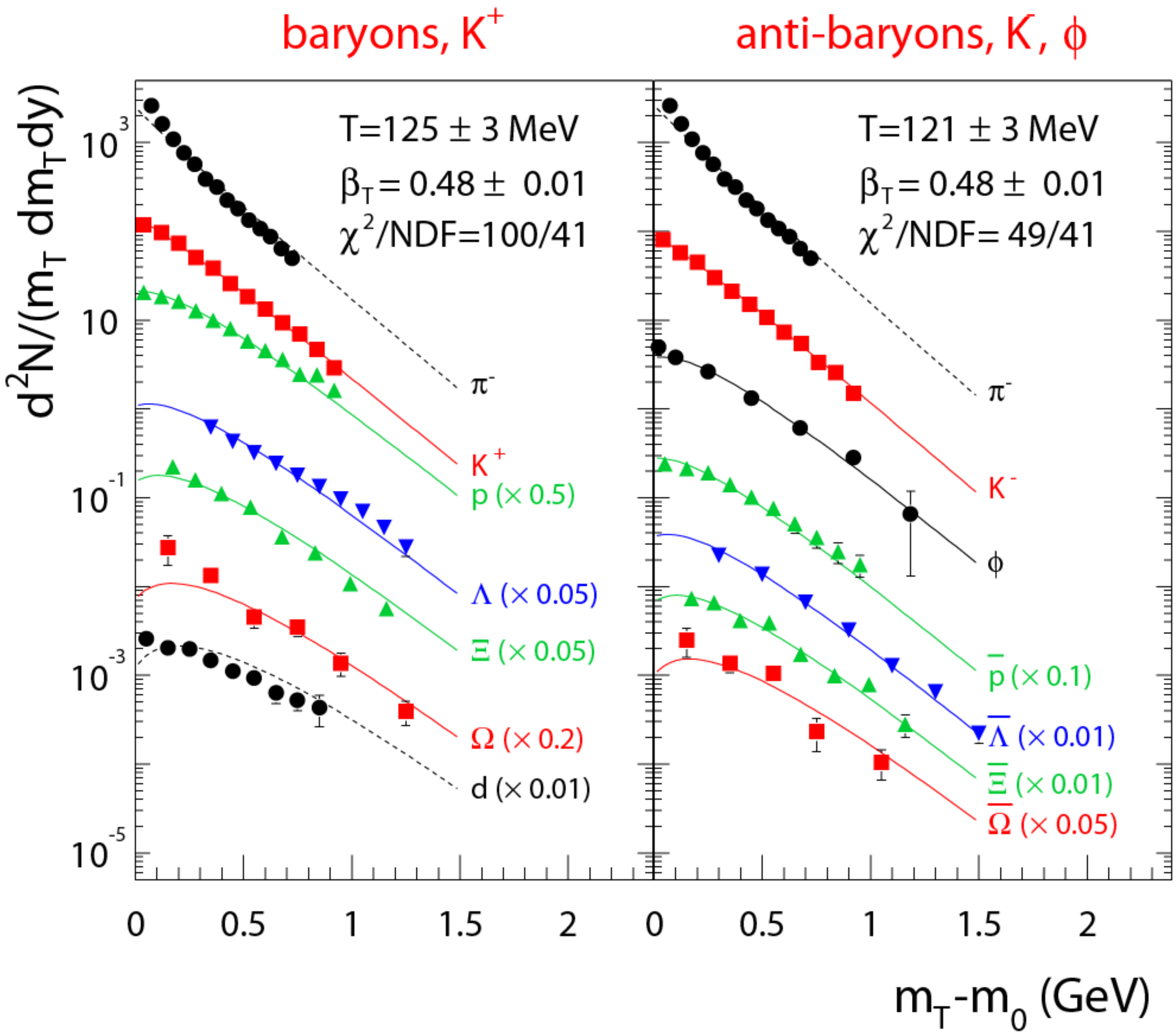

$m_T$ slopes with transverse flow for pions for fixed transverse expansion velocity $\beta_r$

$$\lim_{m_T \to \infty} \frac{d}{dm_T} \ln \left( \frac{1}{m_T} \frac{dn}{dm_T} \right) = -\frac{1}{T} \sqrt{\frac{1-\beta_r}{1+\beta_r}}$$

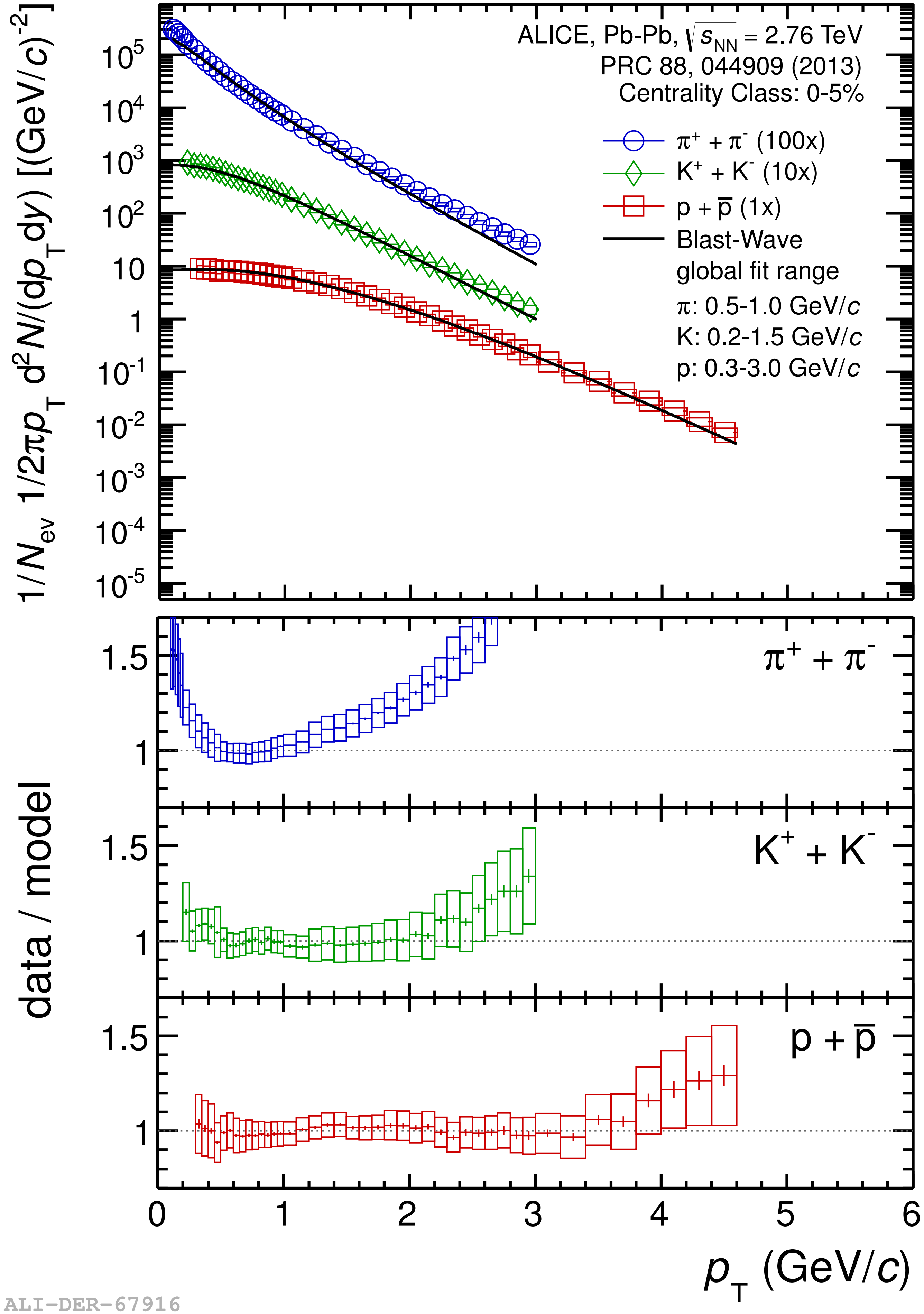
The apparent temperature, i.e., the inverse slope at high $m_T$, is larger than the original temperature by a blue shift factor:

$$T_{\text{eff}} = T \sqrt{\frac{1+\beta_r}{1-\beta_r}}$$

# Blast-wave fit for CERN SPS data (NA49)



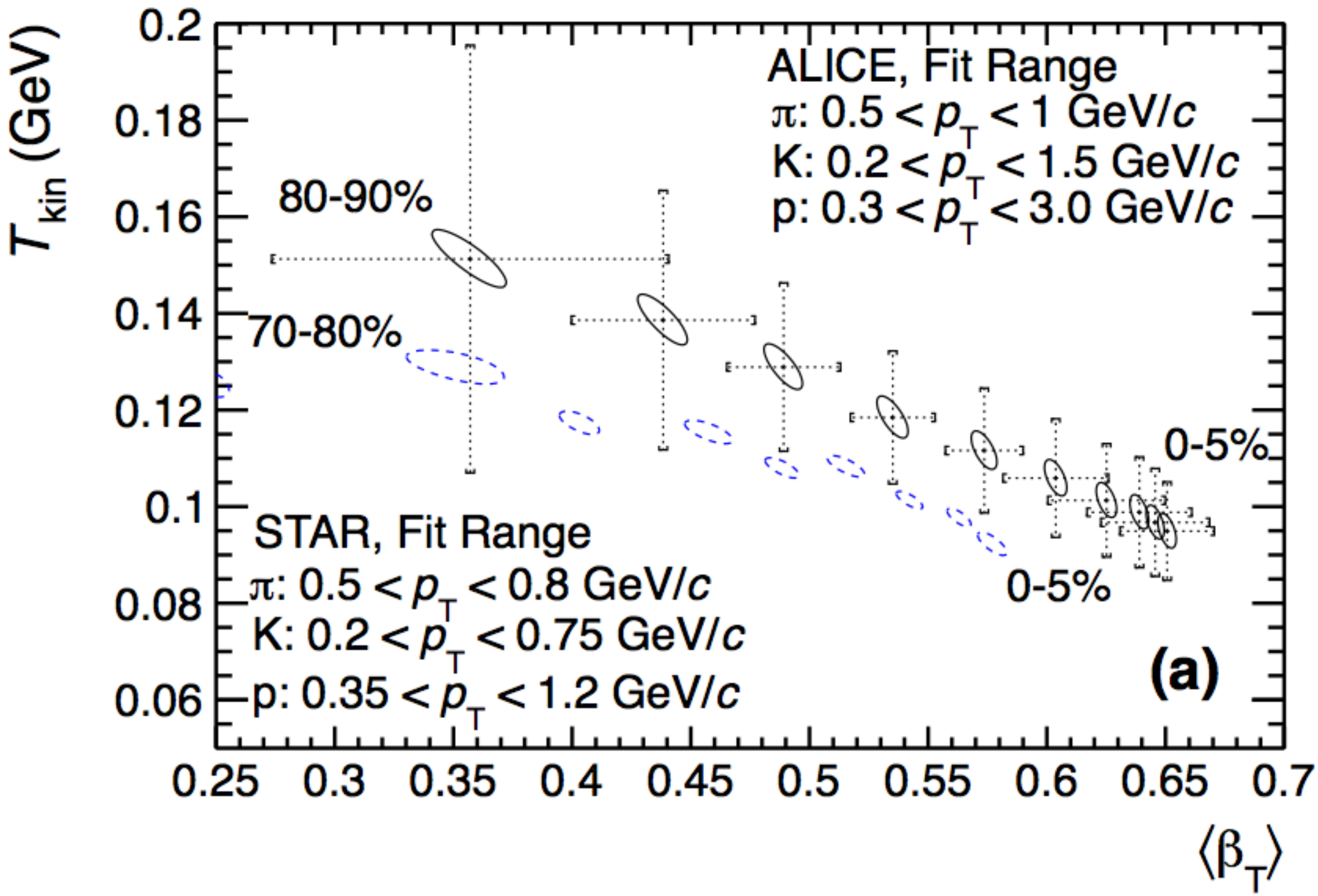
# Blast-wave fit LHC




Works well for K and p

For pions, the contribution from resonance decays at low $p_T$ and hard scattering at high $p_T$ probably explains the discrepancy

# $T$ und ⟨β⟩ for different centralities at RHIC and the LHC

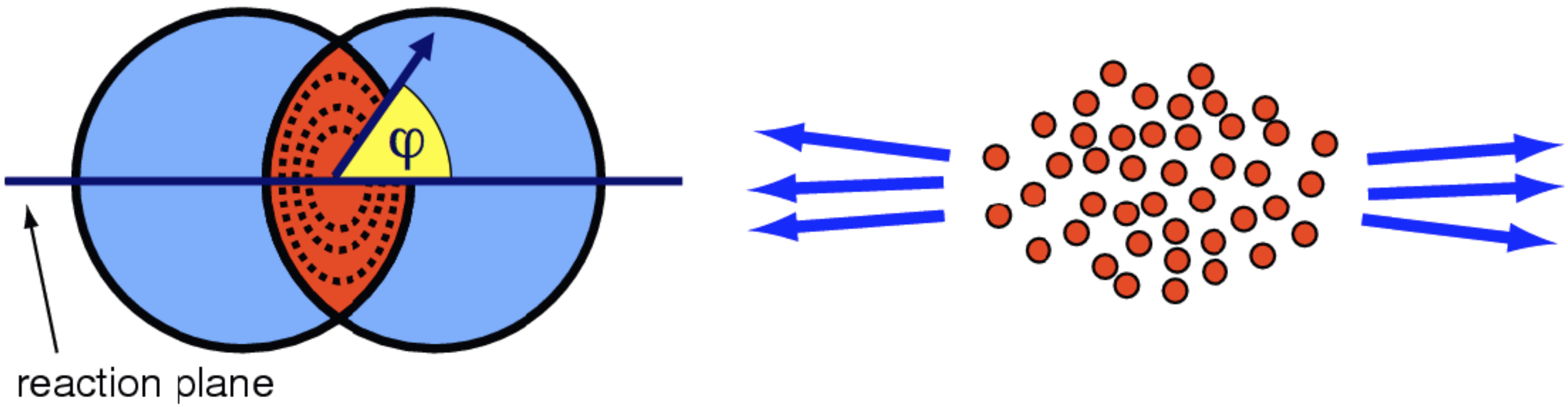



10% larger flow velocities in central collisions at the LHC than at RHIC

# Elliptic flow and higher flow harmonics

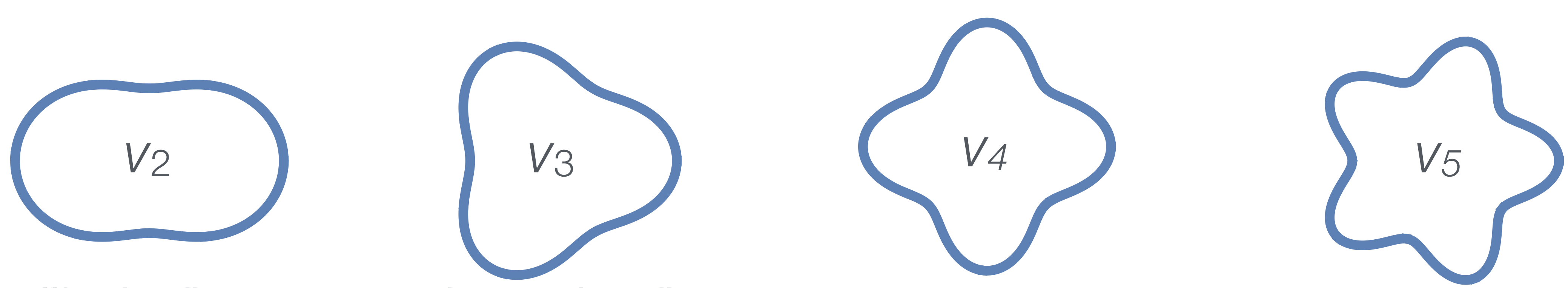
# Azimuthal distribution of produced particles

spatial anisotropy → momentum anisotropy



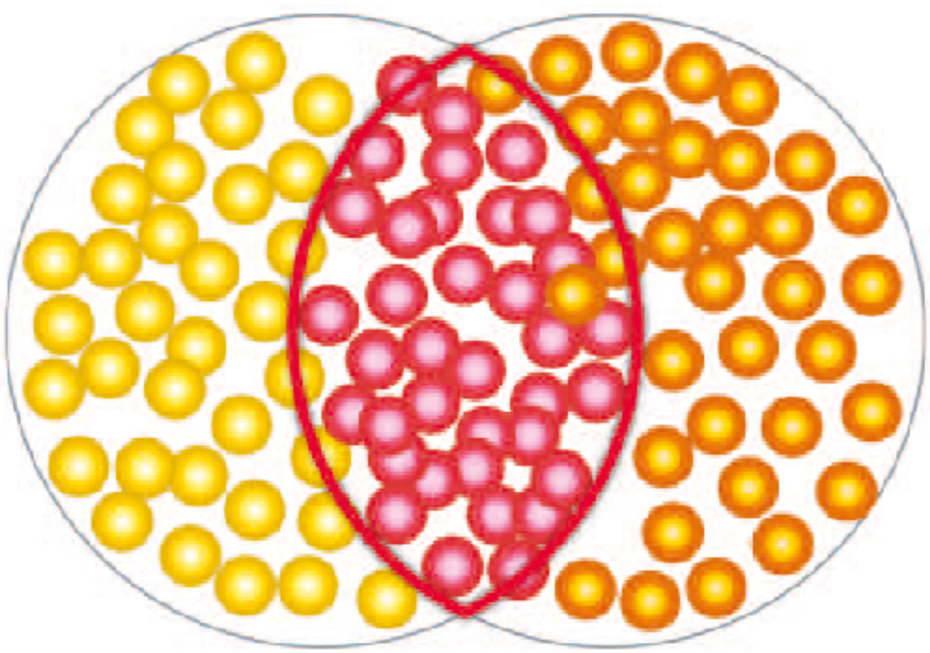

$$\frac{dN}{d\varphi} \propto 1 + 2\sum_{n=1}^{\infty} v_n \cos[n(\varphi - \Psi_n)]$$

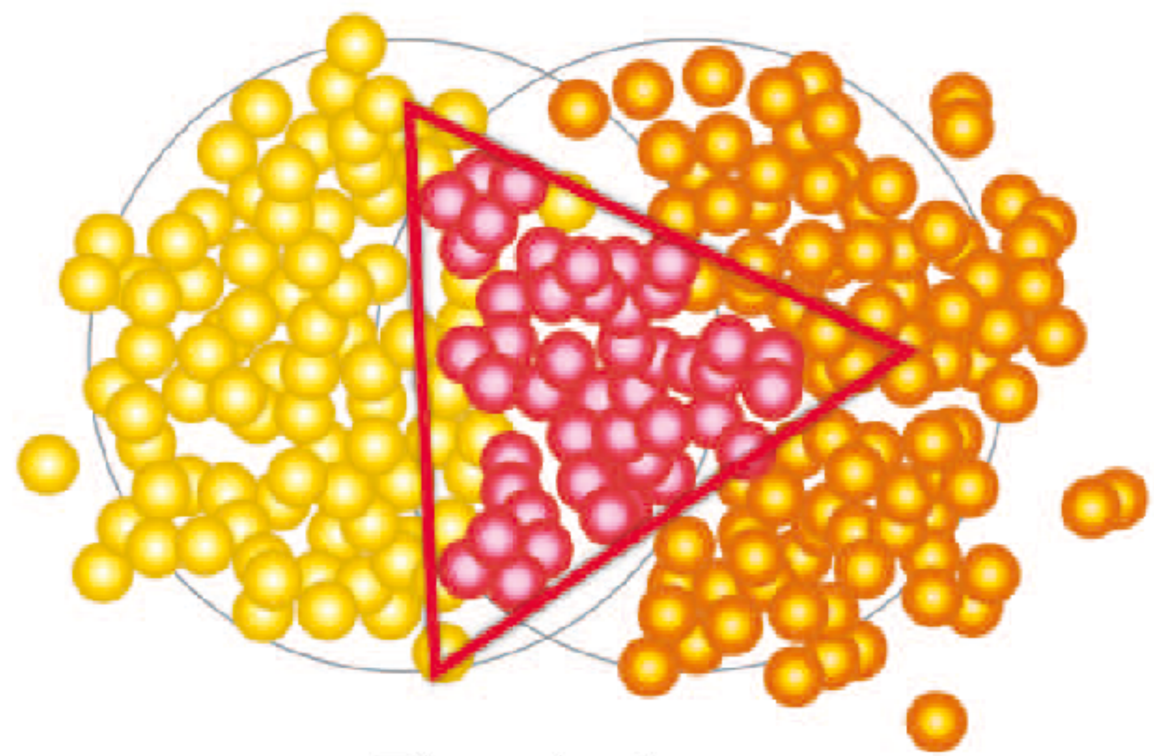
Fourier coefficients: $v_n(p_T, y) = \langle \cos[n(\varphi - \Psi_n)] \rangle$



# Origin of odd flow components ($v_3$, $v_5$, …)

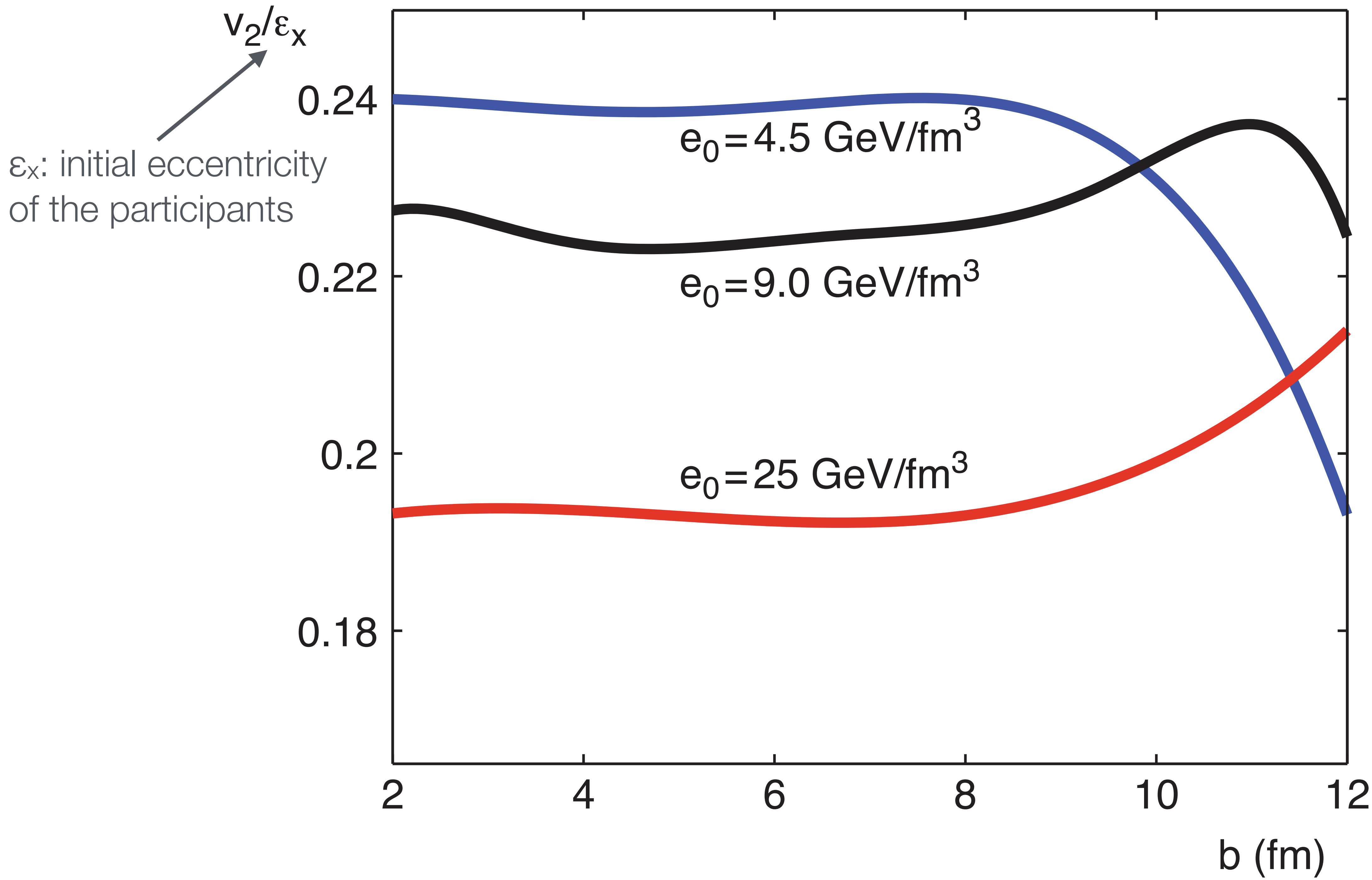
- $v_2$ is related to the geometry of the overlap zone
- Higher moments result from fluctuations of the initial energy distribution





Müller, Jacak, http://dx.doi.org/10.1126/science.1215901

# Hydrodynamic models: $v_2/\varepsilon$ approx. constant




Ideal hydrodynamics gives $v_2 \approx 0.2 – 0\text{-}25\ \varepsilon$

# How the $v_n$ are measured (1): Event-plane method (more or less obsolete by now)

Event flow vector $Q_n$ e.g., measured at forward rapidities:

$$Q_n = \sum_k e^{in\varphi_k} = |Q_n| e^{in\Psi_{n,rec}} = Q_{n,x} + iQ_{n,y}$$

often with weights:

$$Q_{n,x} = \sum_k w_k \cos(n\varphi_k), \quad Q_{n,y} = \sum_k w_k \sin(n\varphi_k)$$

Event plane angle reconstructed in a given event:

$$\Psi_{n,rec} = \frac{1}{n} \mathrm{atan2}(Q_{n,y}, Q_{n,x})$$

Reconstructed event plane angle fluctuates around "true" reaction plane angle. The reconstructed $v_n$ is therefore corrected for the event plane resolution:

$$v_n = \frac{v_n^{\mathrm{rec}}}{R_n}, \qquad v_n^{\mathrm{rec}} = \langle \cos[n(\varphi - \Psi_n^{\mathrm{rec}})] \rangle, \qquad R_n = \text{"resolution correction"}$$

What the event plane method measures depends on the resolution which depends on the number of particles used in the event plane determination:

$$\langle v^\alpha \rangle^{1/\alpha} \quad \text{where} \quad 1 \le \alpha \le 2$$

J.-Y.Ollitrault, nucl-ex/9711003
S. A. Voloshin, A. M. Poskanzer, and R. Snellings, Landolt-Bornstein 23 (2010), 293

Therefore other methods are used today where possible.

# How the $v_n$ are measured (2): Cumulants

Two-particle correlations:

$$\langle\langle e^{i2(\varphi_1-\varphi_2)}\rangle\rangle = \langle\langle e^{i2(\varphi_1-\Psi_{RP}-(\varphi_2-\Psi_{RP}))}\rangle\rangle,$$

$$= \langle\langle e^{i2(\varphi_1-\Psi_{RP})}\rangle\langle e^{-i2(\varphi_2-\Psi_{RP})}\rangle\rangle = \langle v_2^2\rangle$$

if correlations are only due to collective flow

Cumulants:

two-particle correlations

average over all particles within an event, followed by averaging over all events

if correlations are only due to collective flow

$$c_n\{2\} \equiv \left\langle\left\langle e^{in(\varphi_1-\varphi_2)}\right\rangle\right\rangle = \left\langle v_n^2\right\rangle$$

$$c_n\{4\} \equiv \left\langle\left\langle e^{in(\varphi_1+\varphi_2-\varphi_3-\varphi_4)}\right\rangle\right\rangle - 2\left\langle\left\langle e^{in(\varphi_1-\varphi_2)}\right\rangle\right\rangle^2 = \left\langle -v_n^4\right\rangle$$

$c_n\{4\}$ is a measure of genuine 4-particle correlations, i.e., it is insensitive to two-particle non-flow correlations. It can, however, still be influenced by higher-order non-flow contributions.

$$v_n\{2\}^2 := c_n\{2\}, \qquad v_n\{4\}^4 := -c_n\{4\}$$

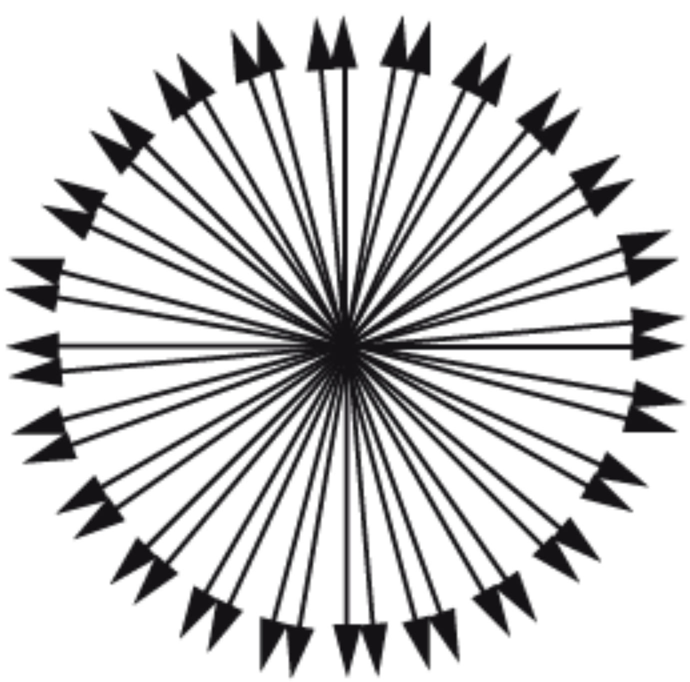
# Non-flow effects

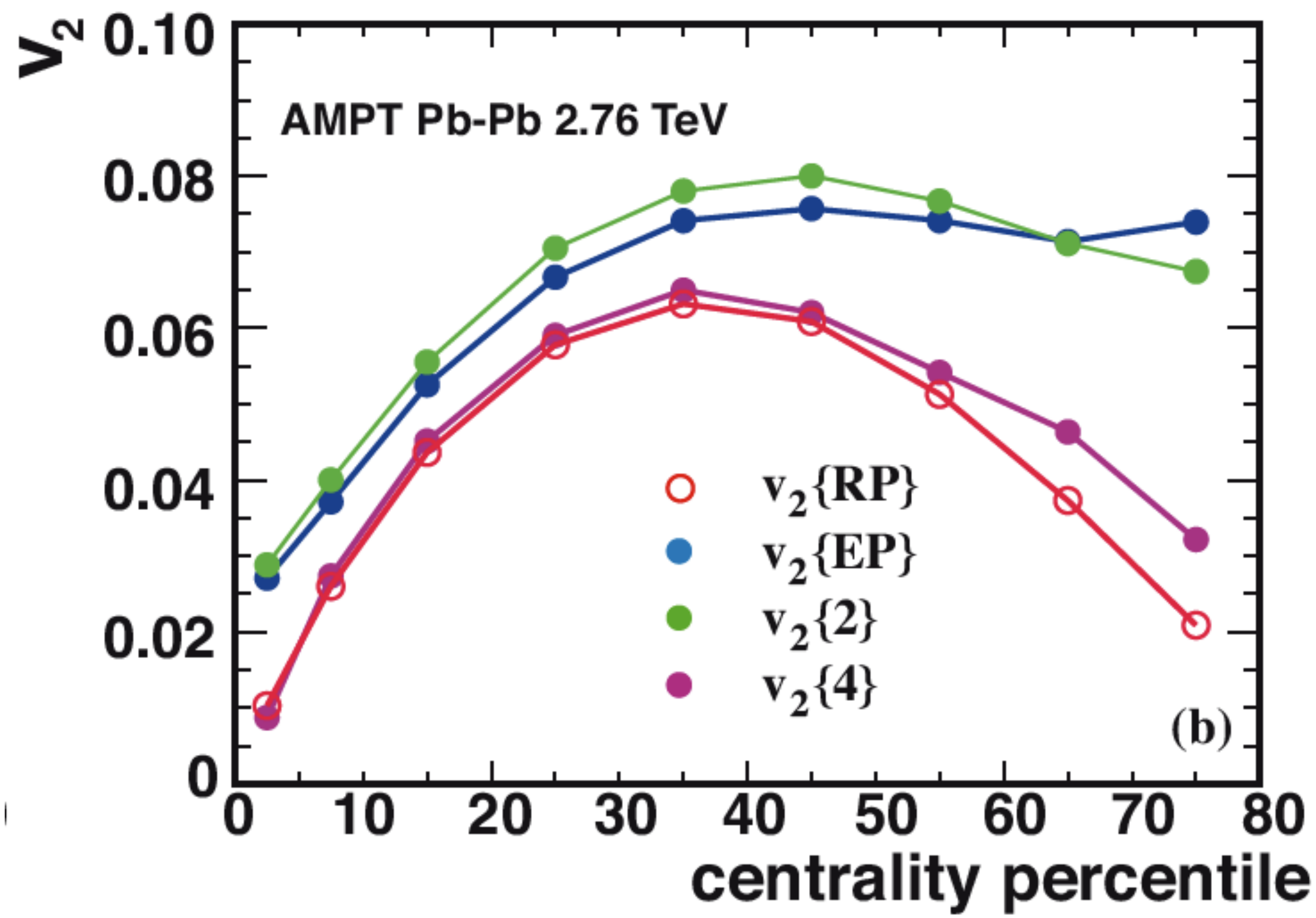
Not only flow leads to azimuthal correlations.
Examples: resonance decays, jets, …

$$v_n\{2\}^2 = \langle v_n^2 \rangle + \delta_n$$

Different methods have different sensitivities to nonflow effects. The 4-particle cumulant method is significantly less sensitive to nonflow effects than the 2-particle cumulant method

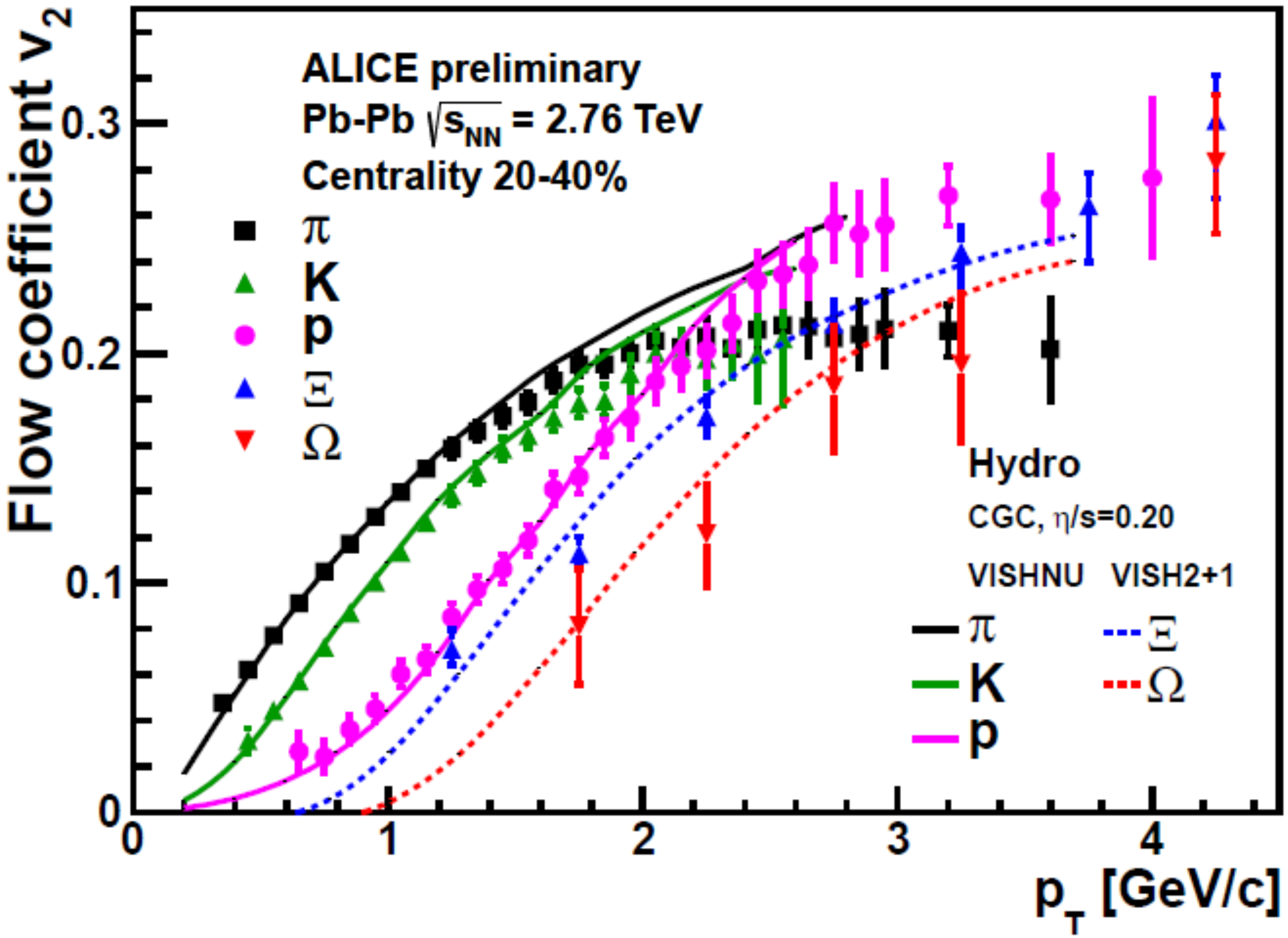
Example:
$v_2 = 0$, $v_2\{2\} > 0$





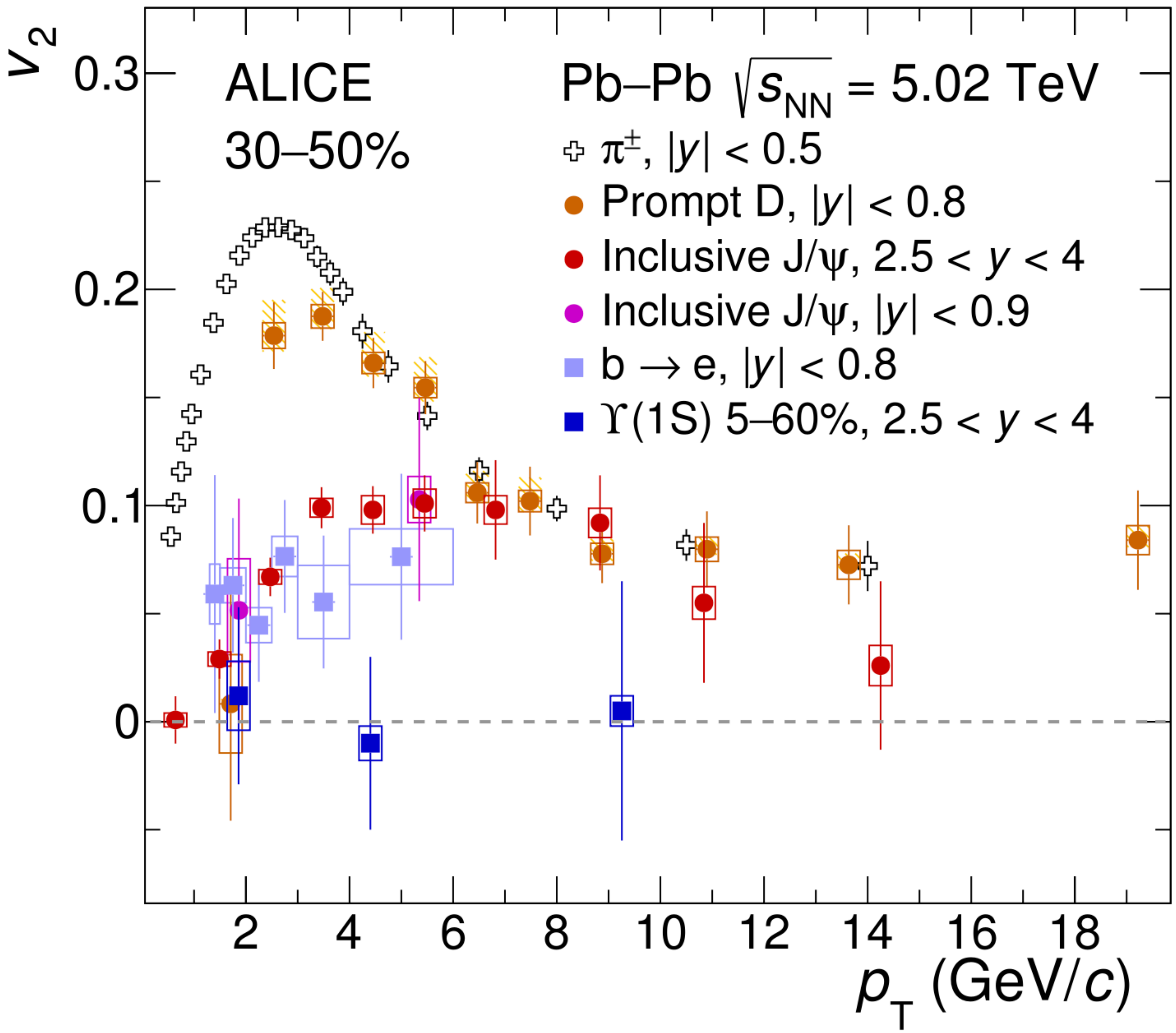
# Elliptic flow of identified hadrons: Reproduced by viscous hydro with η/s = 0.2

final results: arXiv:1405.4632




Dependence of $v_2$ on particle mass ("mass ordering") is considered as strong indication for hydrodynamic space-time evolution

# D meson $v_2$ in Pb-Pb: Charm quarks seem to flow, too!




Given their large mass, it is not obvious that charm quarks take part in the collective expansion of the medium.

$D^+$: $c\bar{d}$

$D^-$: $d\bar{c}$

$D^0$: $c\bar{u}$

$\bar{D}^0$: $u\bar{c}$

ALICE, arXiv:2211.04384

# Viscosity

Pitch drop experiment, started in Queensland, Australia in 1927

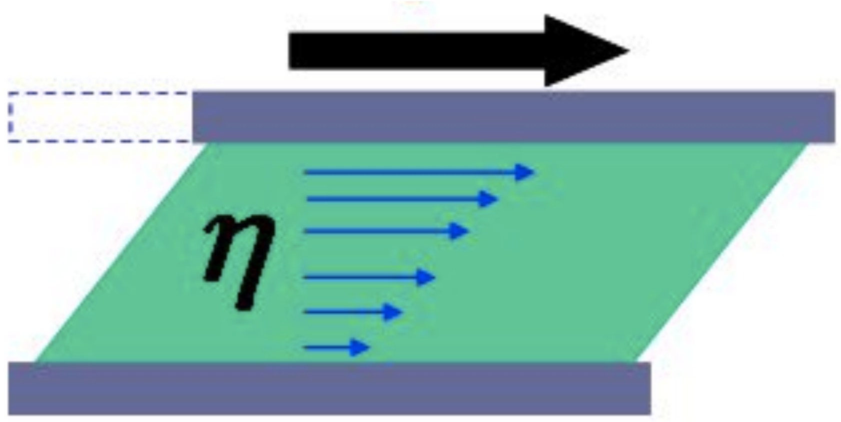
| Date | Event | Duration | | |
|---|---|---|---|---|
| | | Years | Months | |
| 1927 | Hot pitch poured | | | |
| October 1930 | Stem cut | | | |
| December 1938 | 1st drop fell | 8.1 | 98 | |
| February 1947 | 2nd drop fell | 8.2 | 99 | |
| April 1954 | 3rd drop fell | 7.2 | 86 | |
| May 1962 | 4th drop fell | 8.1 | 97 | |
| August 1970 | 5th drop fell | 8.3 | 99 | |
| April 1979 | 6th drop fell | 8.7 | 104 | |
| July 1988 | 7th drop fell | 9.2 | 111 | |
| November 2000 | 8th drop fell[A] | 12.3 | 148 | |
| April 2014 | 9th drop[B] | 13.4 | 156 | |


Meaningful comparison of different fluids: $\eta/s$

https://en.wikipedia.org/wiki/Pitch_drop_experiment

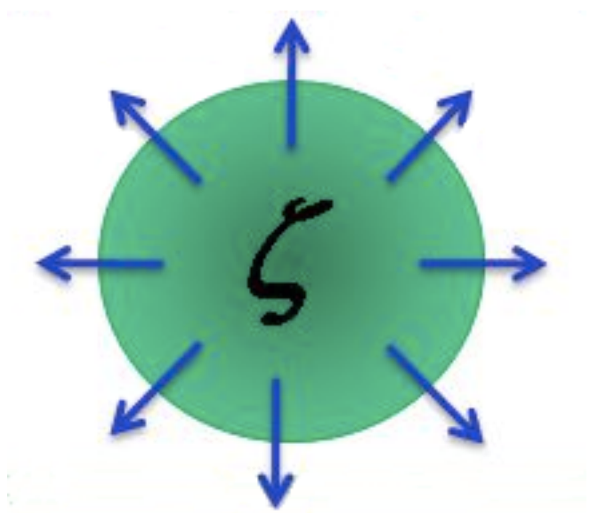
# Shear and bulk viscosity

## Shear viscosity




Acts against buildup of flow anisotropies ($v_2$, $v_3$, $v_4$, $v_5$, ,..)

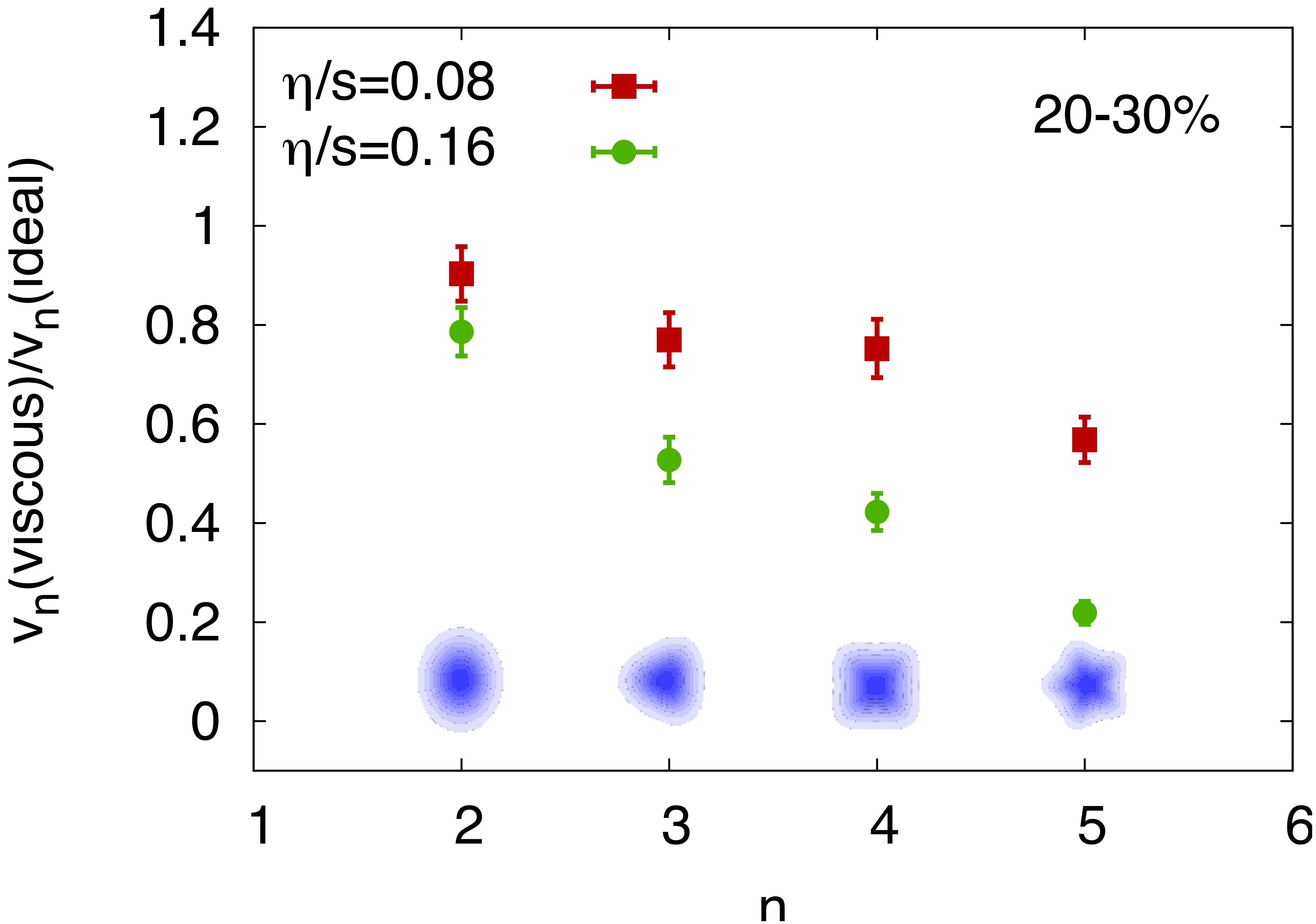
## Bulk viscosity




Acts against buildup of radial flow

# Higher flow harmonics are particularly sensitive to η/s

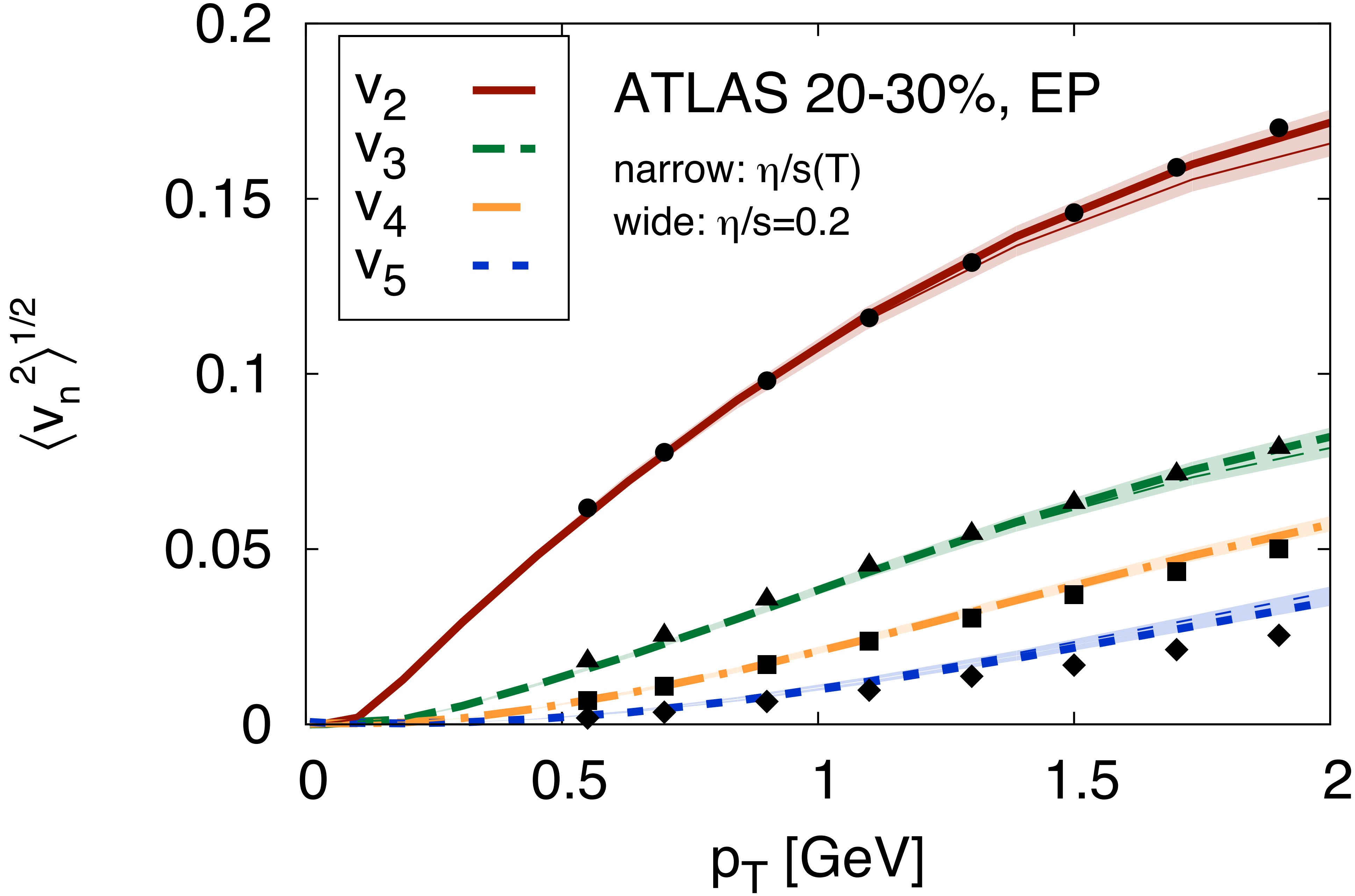
Schenke, Jeon Gale, arXiv:1109.6289




Major uncertainty in extracting η/s from data: uncertainty of initial conditions

# η/s from comparison to data

arXiv:1209.6330

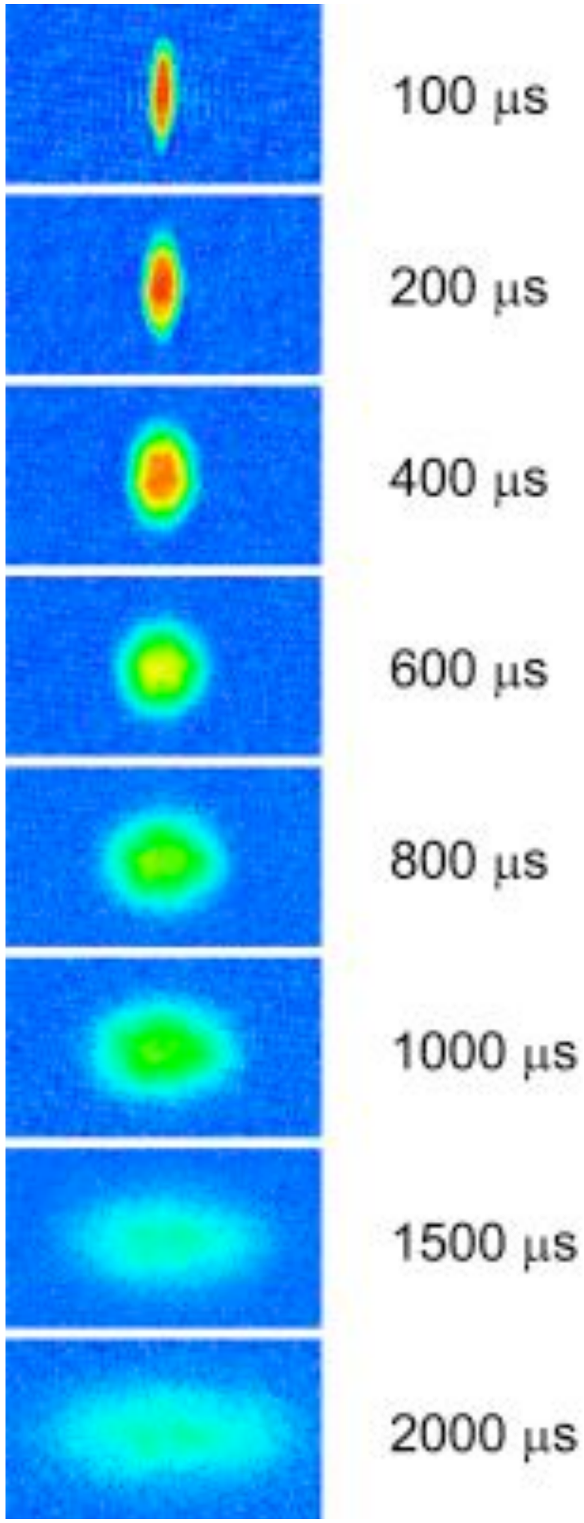



Pb-Pb at $\sqrt{s_{NN}}$ = 2.76 TeV:

arXiv:1301.2826

$$(\eta/s)_{QGP} \approx 0.2 = 2.5 \times \frac{1}{4\pi} \qquad \text{(20\% stat. err., 50\% syst. err.)}$$

# Universal aspects of the underlying physics



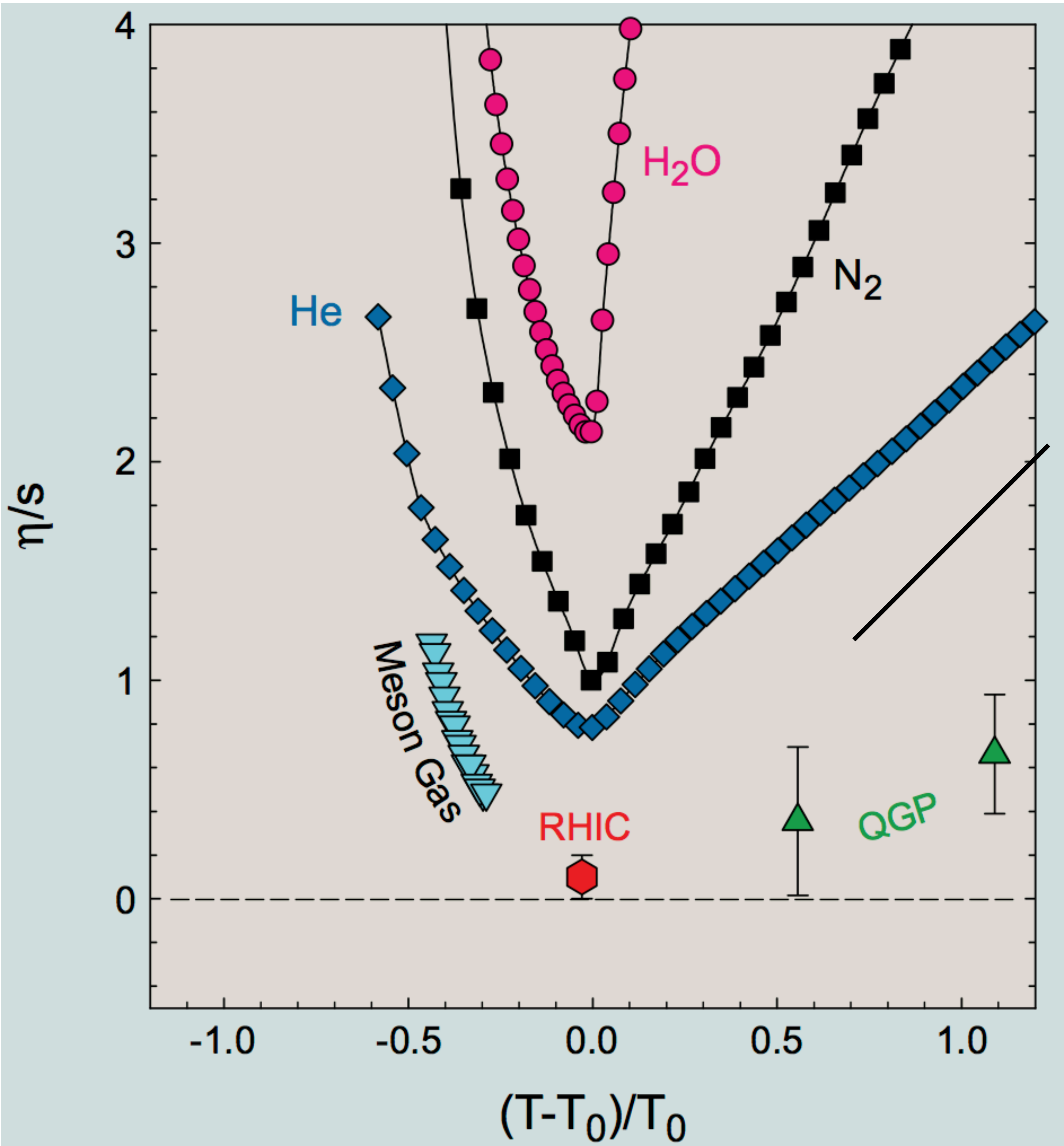

- Strongly-interacting degenerate gas of fermionic $^{6}$Li atoms at 0.1 μK
- Cigar-shaped cloud initially trapped by a laser field
- Anisotropic expansion upon abruptly turning off the trap: Elliptic flow!
- η/s can be extracted: [PhD thesis Chenglin Cao]

$$(\eta/s)_{^6\text{Li gas}} \approx 0.4 = 5 \times \frac{1}{4\pi}$$

The ultimate goal is to unveil the universal physical laws governing seemingly different physical systems (with temperature scales differing by 19 order of magnitude)

John Thomas, https://www.physics.ncsu.edu/jet/research/stronginter/index.html

# Temperature-dependence of η/s for different gases




η/s appears to be minimal at a phase transition

QGP is a candidate for being the most perfect fluid

Conjectured lower bound from string theory

$$\eta/s|_{\mathrm{KSS}} = \frac{1}{4\pi} \approx 0.08$$

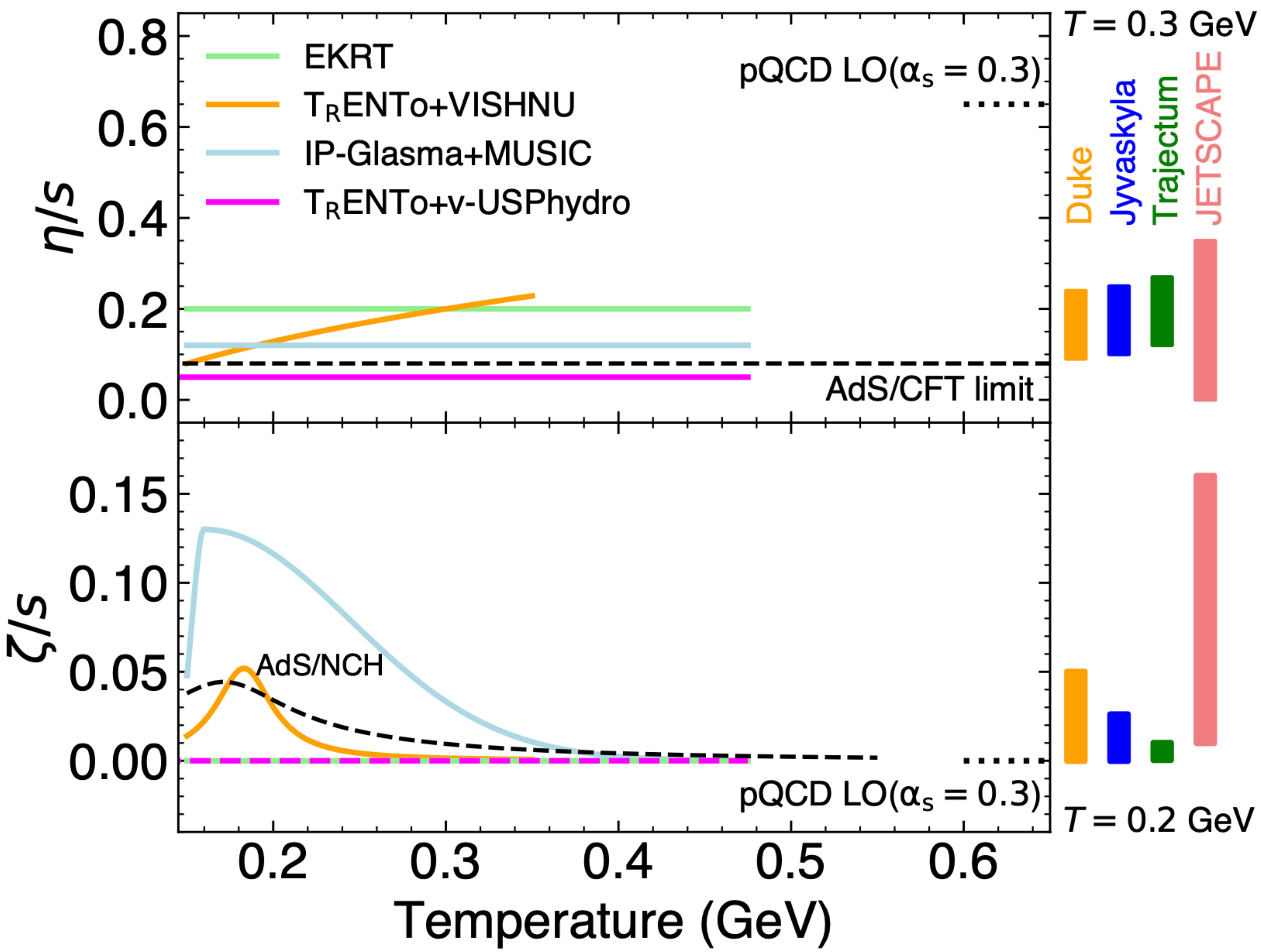
in natural units

SI units: $\eta/s|_{\mathrm{KSS}} = \frac{\hbar}{4\pi k_B}$

Kovtun, Son, Starinets, Phys.Rev.Lett. 94 (2005) 111601

# Current constraints on shear and bulk viscosity

ALICE, arXiv:2211.04384

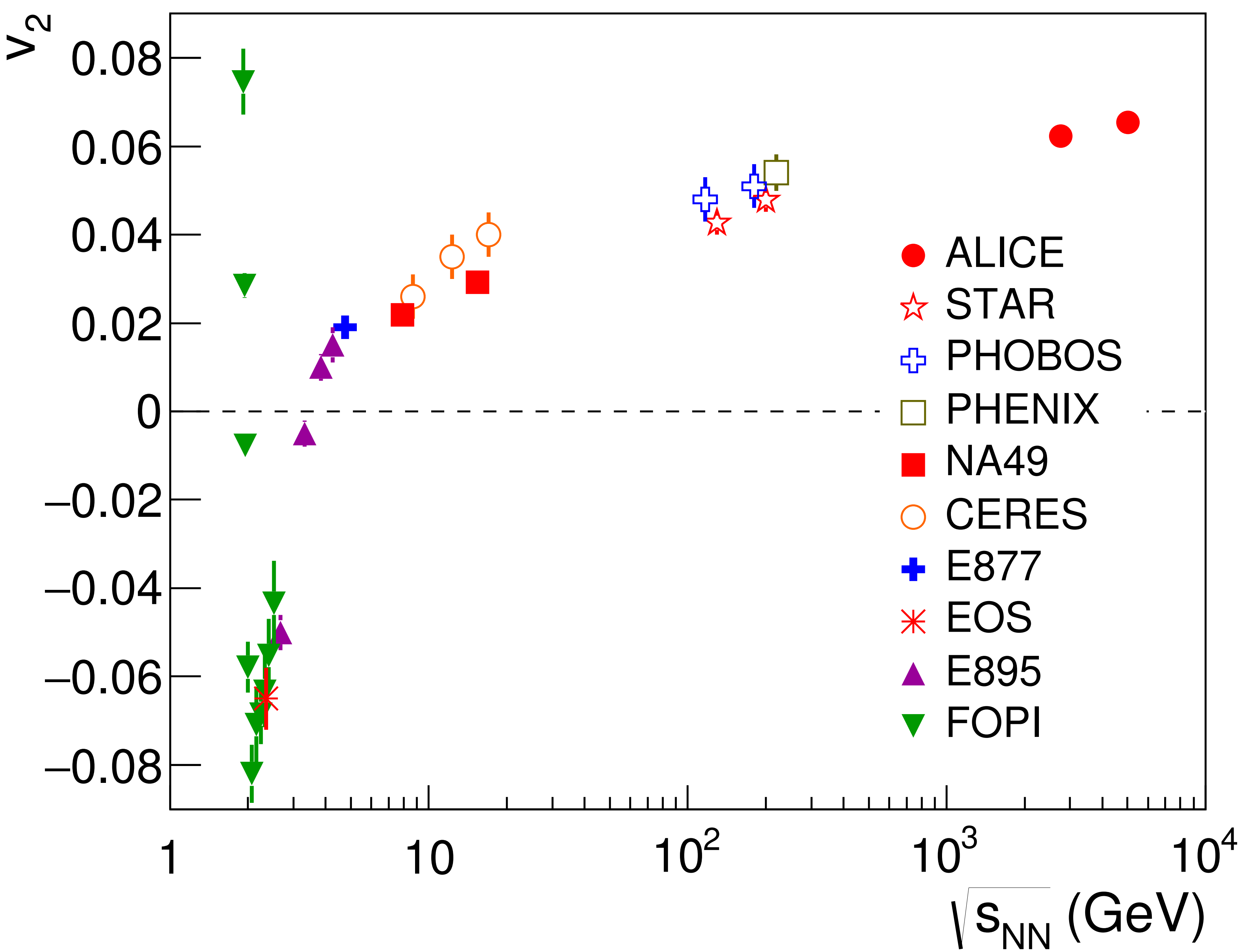



Model parameters like η/s constrained by applying a **Bayesian analysis**.

Credible intervals for model parameters from posterior distribution:

$$P_{\text{posterior}}(\theta|\text{data}) \propto L(\text{data}|\theta) P_{\text{prior}}(\theta)$$

# Elliptic flow as a function of $\sqrt{s_{NN}}$




**$\sqrt{s_{NN}} < 2$ GeV**

rotation of the collision system leads to fragments being emitted in-plane

**$\sqrt{s_{NN}} \approx 2$–4 GeV**

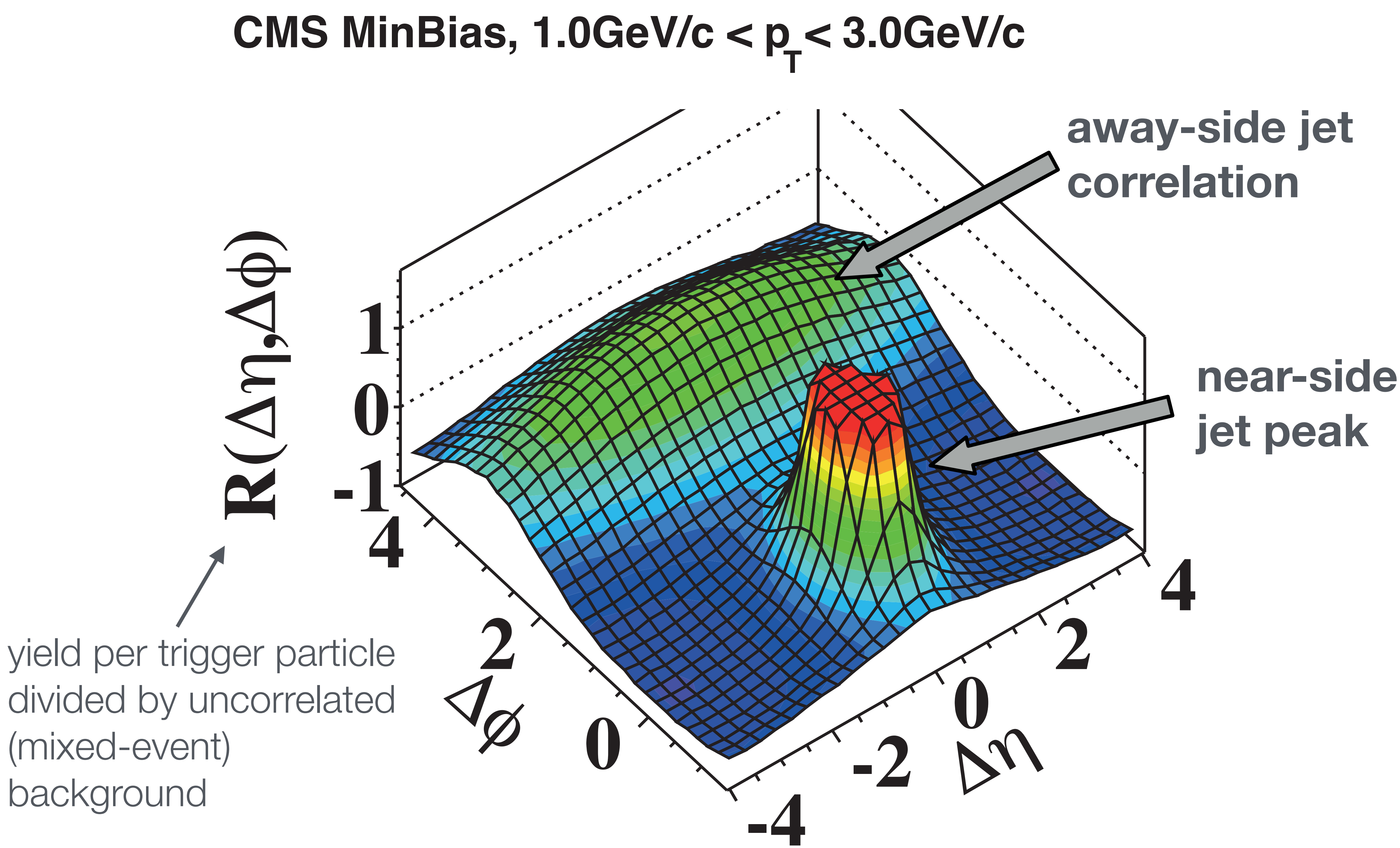
velocity of the nuclei is small so that presence of spectator matter inhibits in-plane particle emission ("**squeeze-out**")

**$\sqrt{s_{NN}} > 4$ GeV**

initial eccentricity leads to pressure gradients that cause positive $v_2$ ("**hydro in-plane flow**")

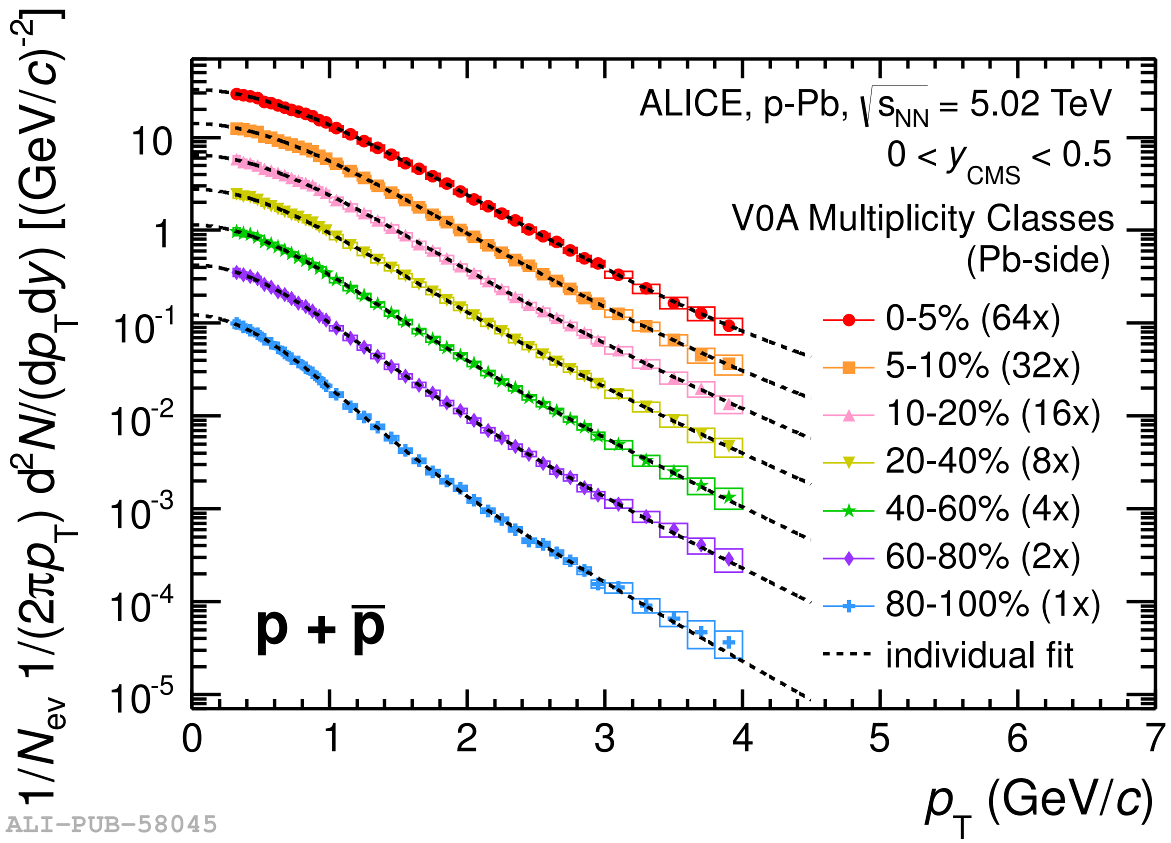
# Collective flow in small systems?

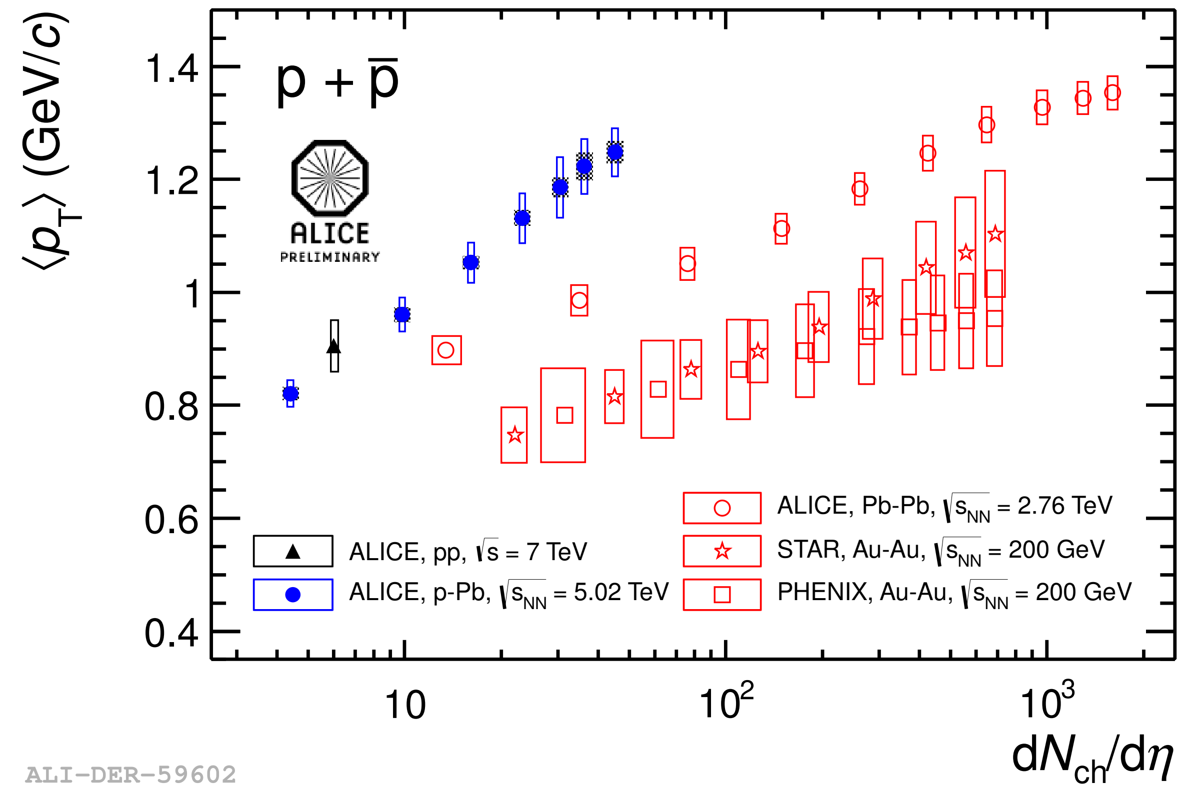
# Collectivity in small systems: 2-particle correlation in pp at $\sqrt{s}$ = 7 TeV

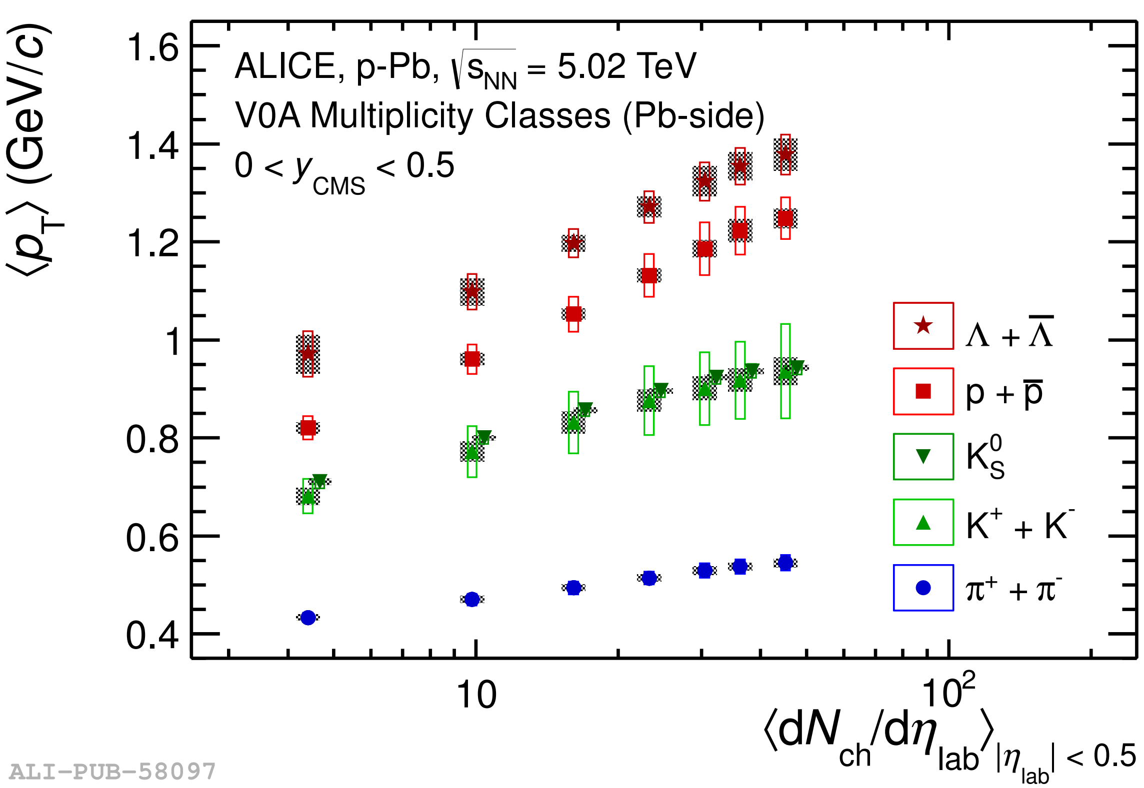



No indication for collective effects in minimum bias pp collisions at 7 TeV

# Radial flow in p-Pb?



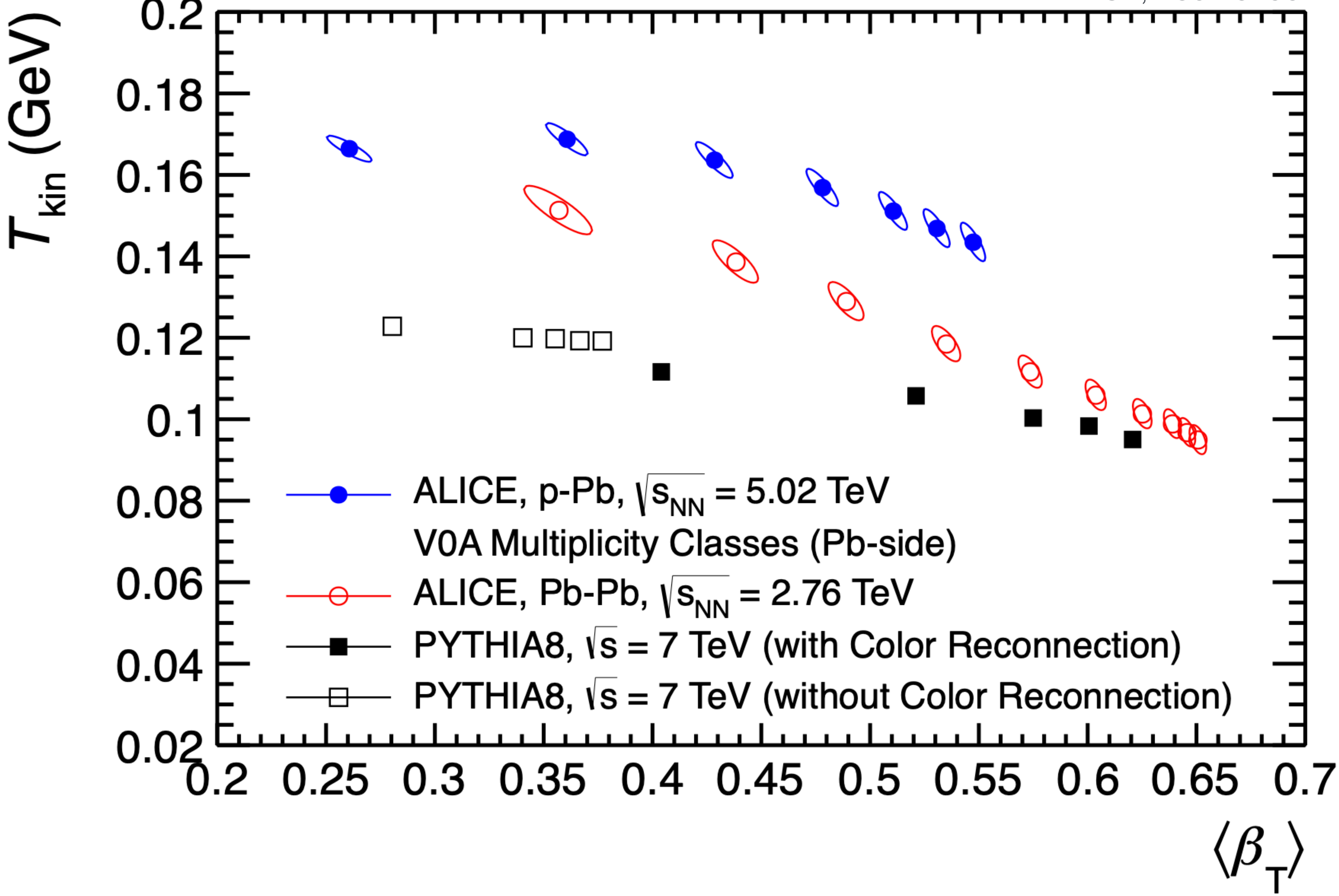

Shape of spectra changes with $dN_{ch}/d\eta$

Increase of $\langle p_T \rangle$ with $dN_{ch}/d\eta$

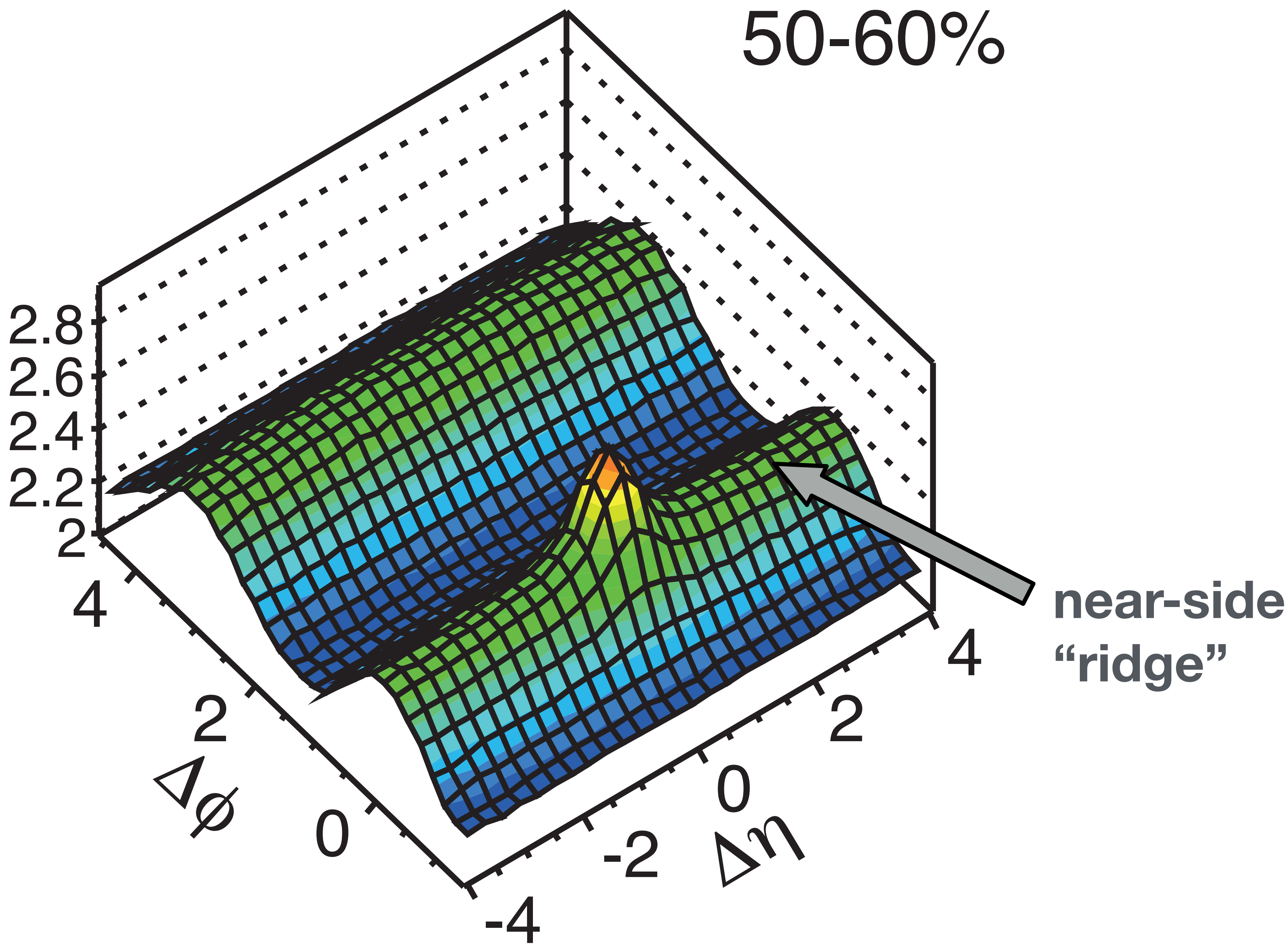
Effects which can be explained as resulting from radial flow

# Results of blast-wave fits in p-Pb:
# Radial flow also in high-mult. p-Pb collisions?

ALICE, 1307.6796

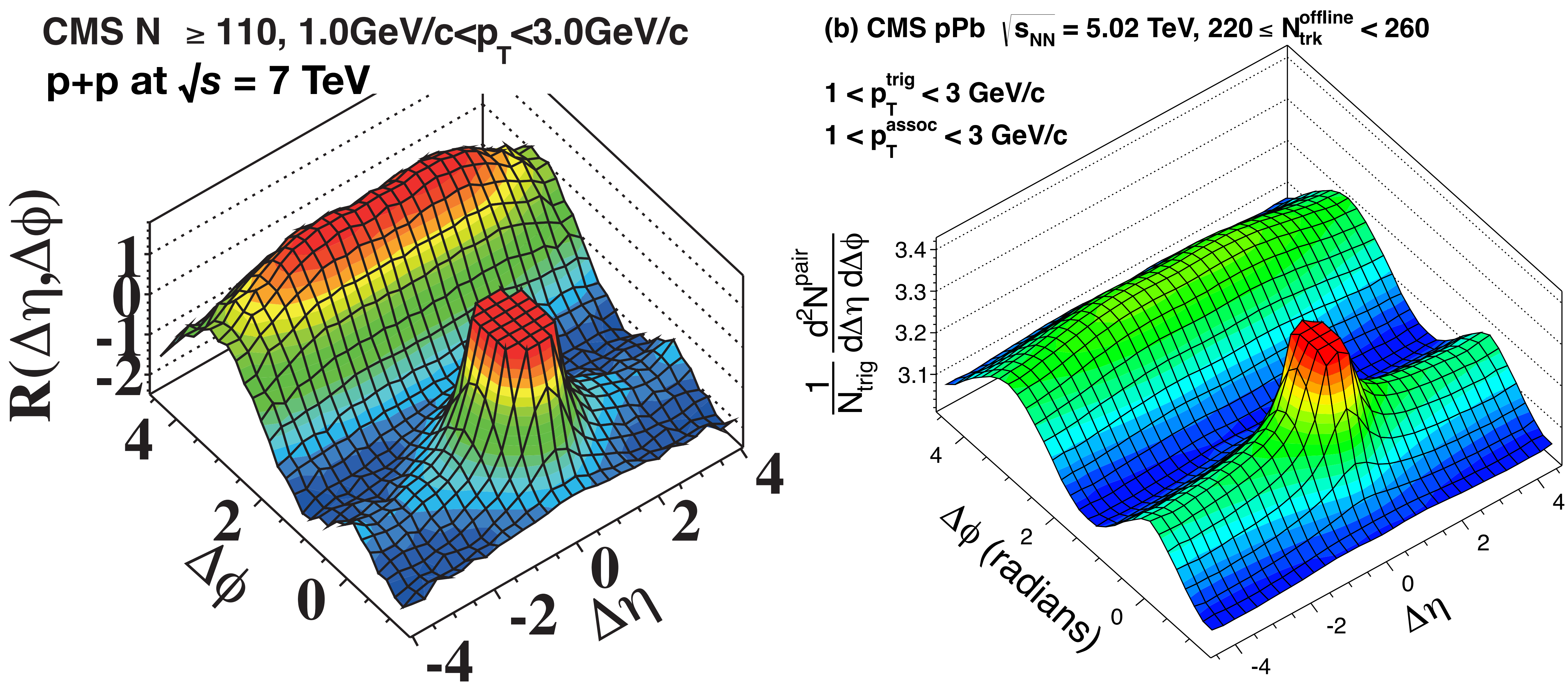


# Collectivity in small systems:
# Two-particle correlations in Pb-Pb collisions




collective flow + jet correlations

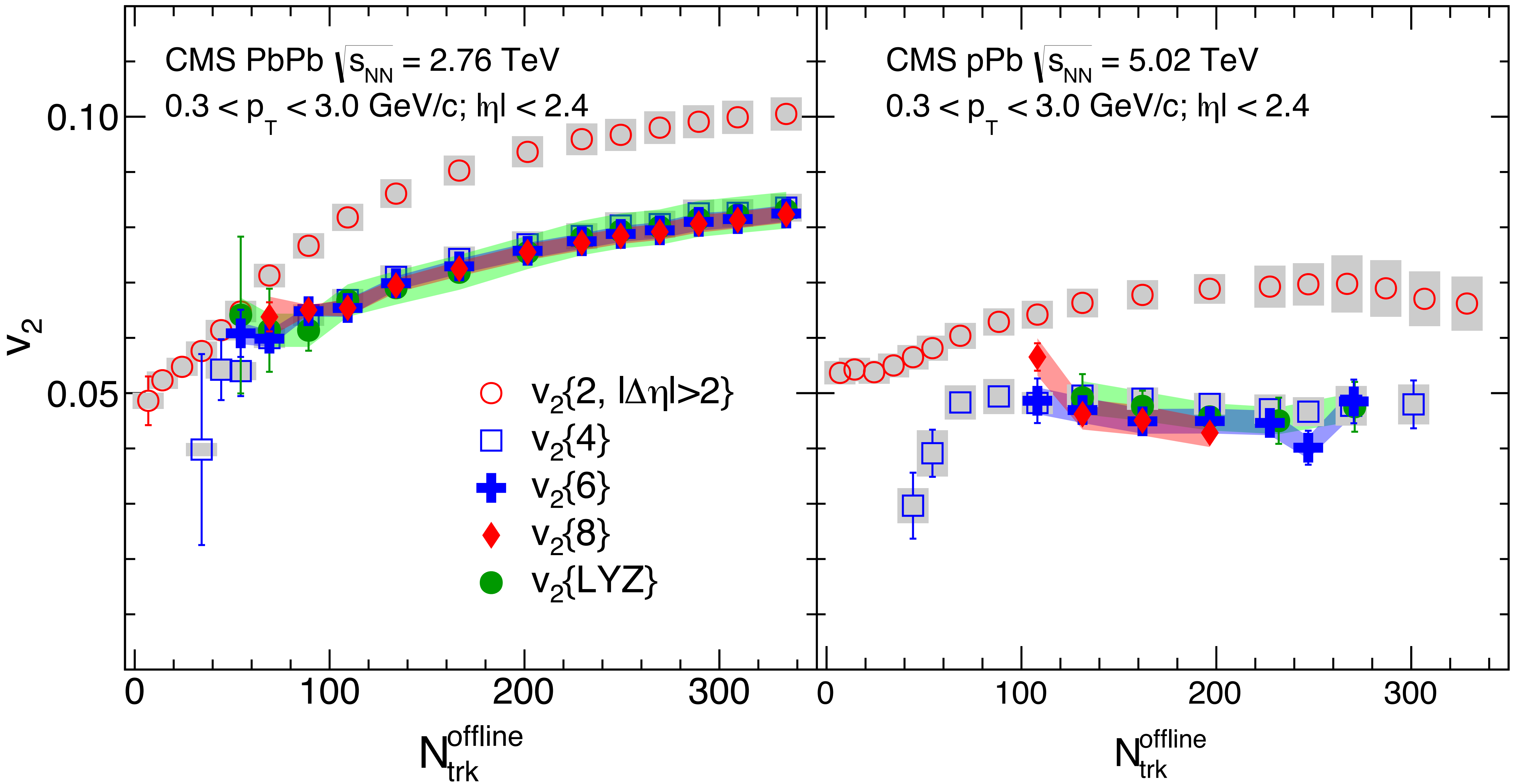
# Collectivity in small systems: Two-particle correlations in high-multiplicity pp and p-Pb




Flow-like two-particle correlation become visible in high-multiplicity pp and p-Pb collisions at the LHC

# Comparison of $v_2$ in Pb-Pb and p-Pb for the same track multiplicity

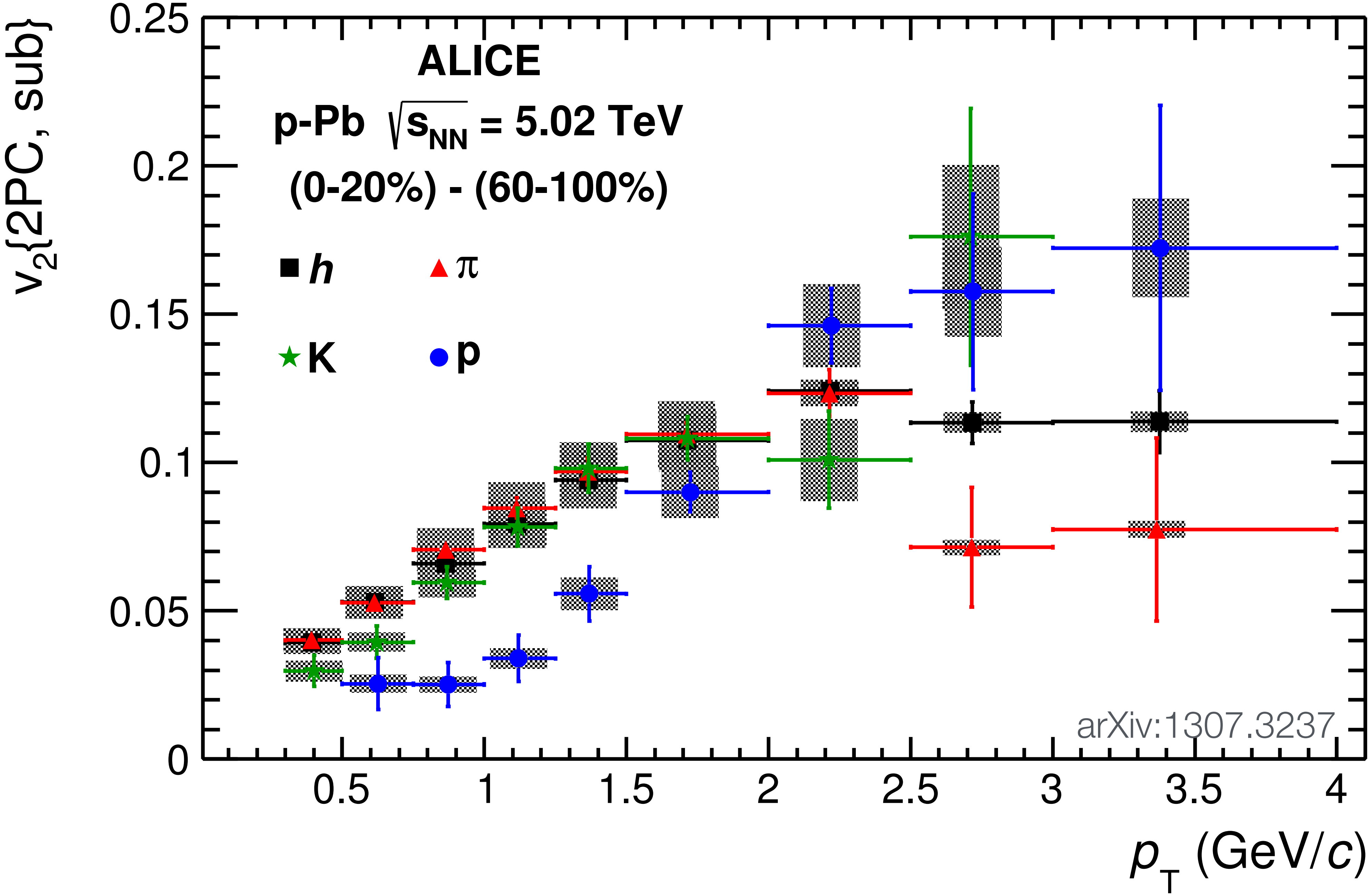
CMS, arXiv:1502.05382v2




- $v_2\{8\}$ measured: $v_2$ in p-Pb is a genuine multi-particle effect
- $v_2$ in p-Pb only slightly smaller than in Pb-Pb

# Collectivity in small systems: Mass ordering in p-Pb collisions

$v_2$ from fit of two-particle correlation, jet-like correlation removed by taking the difference between central and peripheral p-Pb collisions




Consistent with hydrodynamic expansion of the medium als in p-Pb

# Summary/questions space-time evolution

- Hydrodynamic models provide an economic description of many observables (spectra, flow)
- Shear viscosity / entropy density ratio in Pb-Pb at $\sqrt{s_{NN}} = 2.76$ TeV from comparing hydrodynamic models to data:

$$(\eta/s)_{QGP} \approx 0.2 = 2.5 \times \left.\frac{\eta}{s}\right|_{\mathrm{min,KSS}} = 2.5 \times \frac{1}{4\pi}$$

- Appropriate theoretical treatment of thermalization and matching to hydrodynamics?
  - Strong coupling or weak coupling approach?
  - Weak coupling: Applicable at asymptotic energies, but still useful at current $\sqrt{s_{NN}}$
  - Strong coupling (string/gauge theory duality), see e.g. arXiv:1501.04952: Fast thermalization of the order of $1/T$, but too much stopping?
- Does one need hydrodynamics to explain collective effects in small system (pp, p-Pb)?