# Quark-Gluon Plasma Physics

## 4. Basics of Nucleon-Nucleon and Nucleus-Nucleus Collisions

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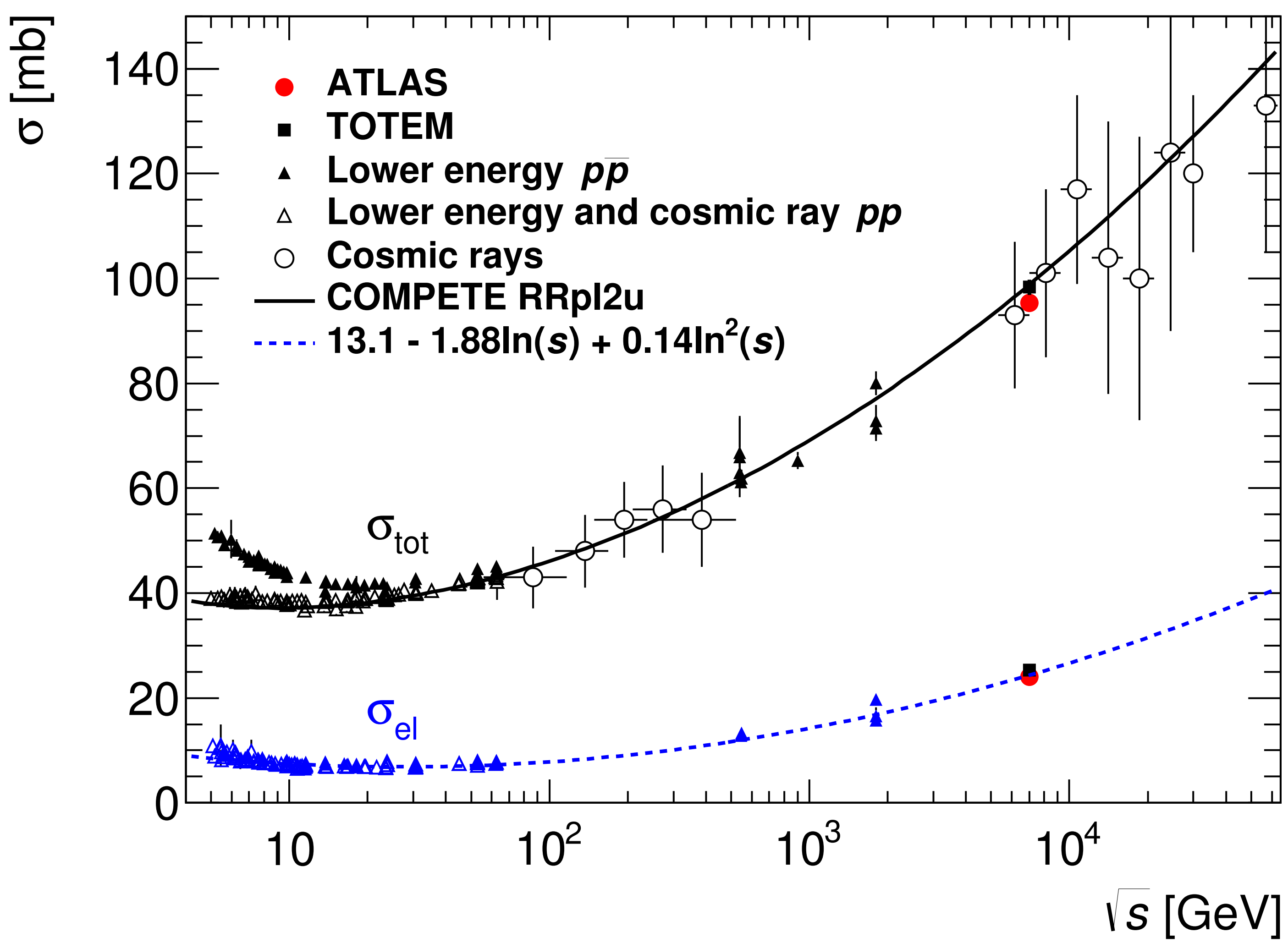
SS 2023

# Part I: proton-proton collisions

# Total p+p(pbar) Cross Section

ATLAS, arXiv:1408.5778




Above ~ $\sqrt{s}$ = 20 GeV all hadronic cross sections rise with increasing $\sqrt{s}$

Data show that

$$\sigma_{\text{tot}}(h + X) = \sigma_{\text{tot}}(\bar{h} + X)$$

(in line with Pomeranchuk's theorem)

Soft processes:
hard to calculate $\sigma_{tot}(\sqrt{s})$ in QCD

Modeling based on Regge theory: exchange of color-neutral object called *pomeron*

parameterization from Regge theory:

$$\sigma_{\text{tot}} = X s^{\epsilon} + Y s^{\epsilon'}$$

$$\epsilon = 0.08 - 0.1, \quad \epsilon' \approx -0.45$$

# Diffractive collisions (I)

(Single) diffraction in p+p:
"Projectile" proton is excited to a hadronic state *X* with mass *M*

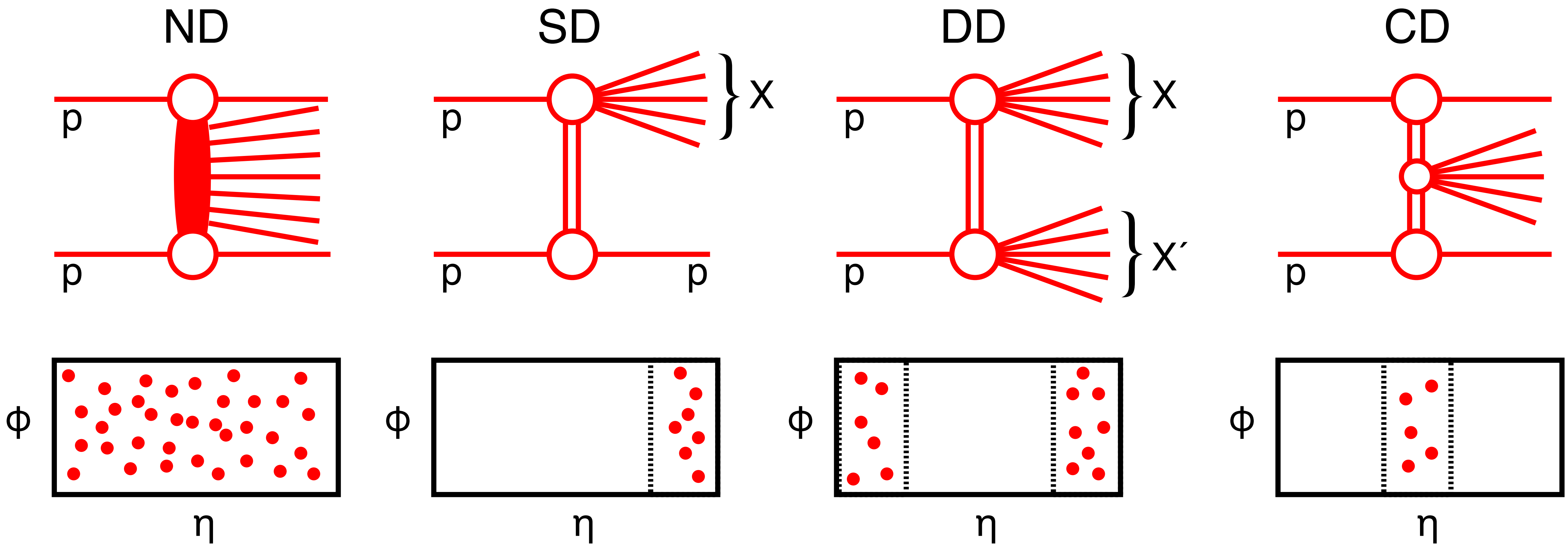
$$p_{\mathrm{proj}} + p_{\mathrm{targ}} \rightarrow X + p_{\mathrm{targ}}$$

The excited state X fragments, giving rise to the production of (a small number) of particles in the forward direction

Theoretical view:

- Diffractive events correspond to the exchange of a Pomeron
- The Pomeron carries the quantum numbers of the vacuum ($J^{PC} = 0^{++}$)
- Thus, there is no exchange of quantum numbers like color or charge
- In a QCD picture the Pomeron can be considered as a two- or multi-gluon state, see, e.g., O. Nachtmann (→ link)

# Diffractive collisions (II)




non-diffractive collisions

single-diffractive dissociation

double-diffractive dissociation

central diffraction

$$\sigma_{\text{tot}} = \sigma_{\text{el}} + \sigma_{\text{inel}}, \quad \sigma_{\text{inel}} = \sigma_{\text{SD}} + \sigma_{\text{DD}} + \sigma_{\text{CD}} + \sigma_{\text{ND}}$$

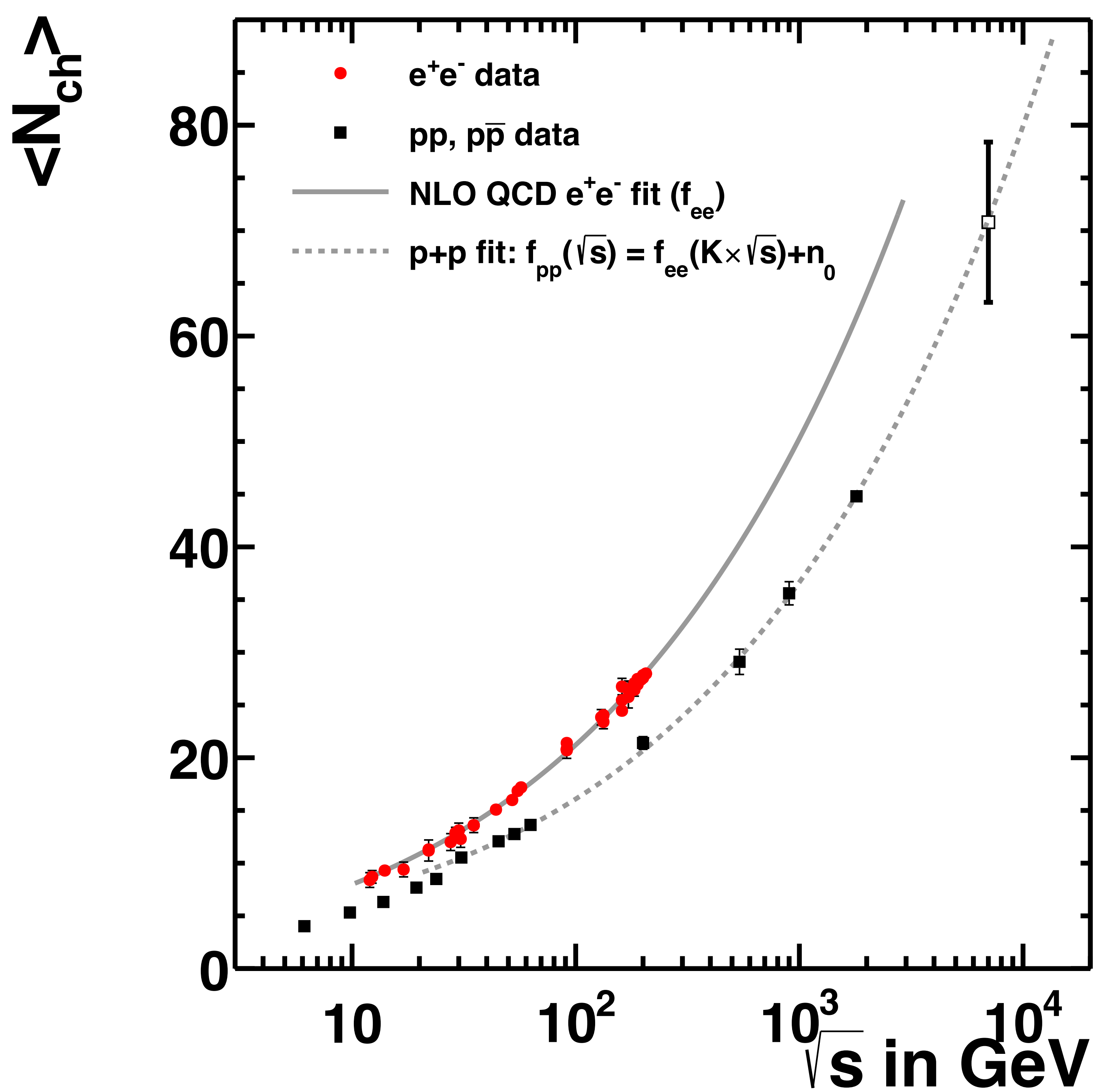
# Diffractive collisions (III)

UA5, Z. Phys. C33, 175, 1986

| $p+\bar{p}$ | $\sqrt{s}$ = 200 GeV | $\sqrt{s}$ = 900 GeV |
|---|---|---|
| Total inelastic | (41.8 ± 0.6) mb | (50.3 ± 0.4 ± 1.0) mb |
| Single-diffractive | (4.8 ± 0.5 ± 0.8 ) mb | (7.8 ± 0.5 ± 1.8 ) mb |
| Double-diffractive | (3.5 ± 2.2) mb | (4.0 ± 2.5) mb |
| Non-diffractive | ≈ 33.5 mb | ≈ 38.5 mb |

Fraction of diffractive dissociation events with respect to all inelastic collisions is about 20–30% (rather independent of $\sqrt{s}$)
See also ATLAS, arXiv:1201.2808

# Charged-particle Multiplicity as a fct. of $\sqrt{s}$: Similarities between pp and $e^+e^-$




The increase of $N_{ch}$ with $\sqrt{s}$ looks rather similar in p+p and $e^+e^-$

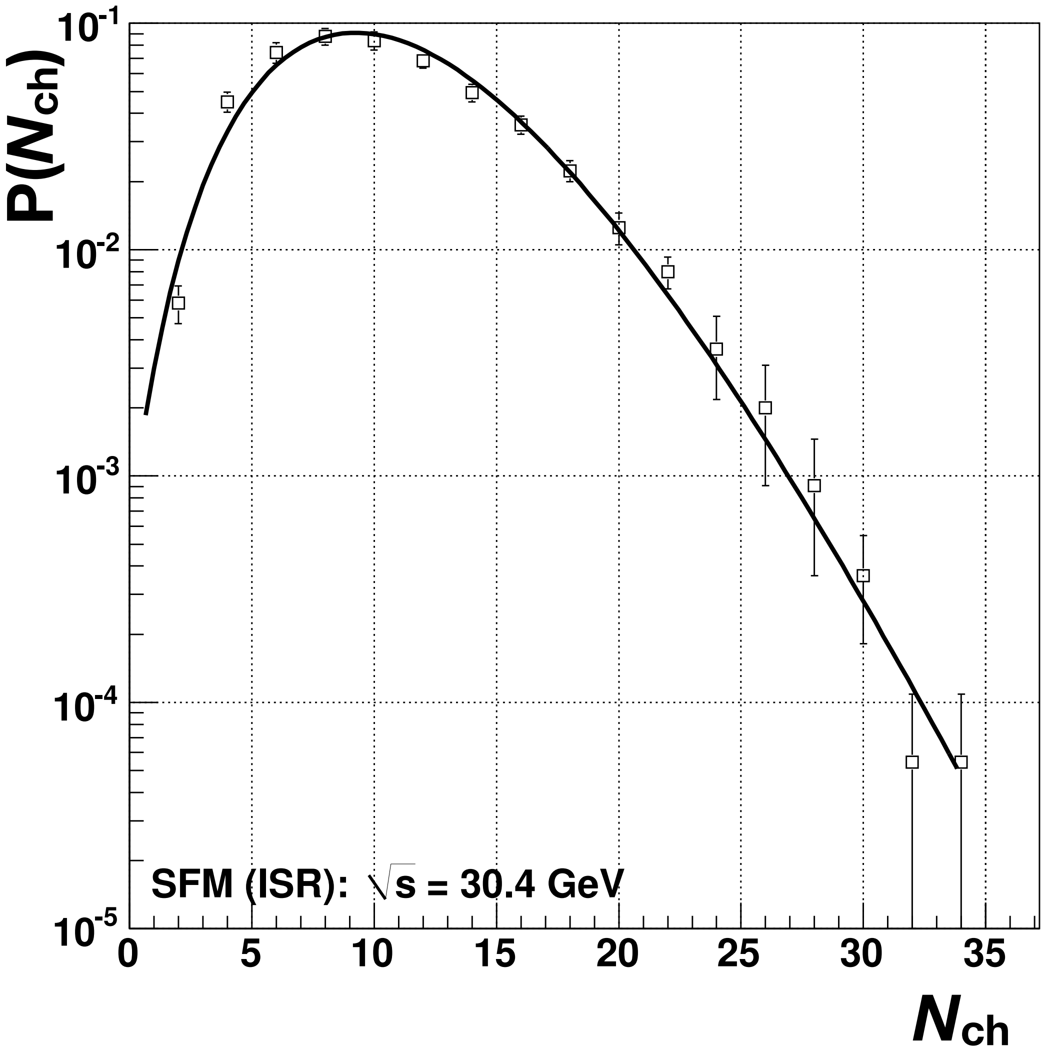
Roughly speaking, the energy available for particle production in p+p seems to be ~ 30–50%:

$$f(\sqrt{s}) := N_{ch}^{e+e-}(\sqrt{s})$$

$$\rightarrow N_{ch}^{p+p} = f(K\sqrt{s_{pp}}) + n_0$$

A fit yields: $K \approx 0.35, \quad n_0 \approx 2.2$

# What is the distribution of the number of produced particles per collision?




Independent sources: Poisson distribution

Observation:
Multiplicity distributions in pp, $e^+e^-$, and lepton-hadron collisions well described by a Negative Binomial Distribution (NBD)

Deviations from the NBD were discovered by UA5 at $\sqrt{s}$ = 900 GeV and later confirmed at the Tevatron at $\sqrt{s}$ = 1800 GeV (shoulder structure at $n \approx 2 \langle n \rangle$)

$$P_{\mu,k}^{\mathrm{NBD}}(n) = \frac{(n+k-1)\cdot(n+k-2)\cdot \ldots \cdot k}{\Gamma(n+1)} \left(\frac{\mu/k}{1+\mu/k}\right)^n \frac{1}{(1+\mu/k)^k}$$

$$\langle n \rangle = \mu, \;\; D := \sqrt{\langle n^2 \rangle - \langle n \rangle^2} = \sqrt{\mu \left(1 + \frac{\mu}{k}\right)}$$

Limits of the NBD:
$k \rightarrow \infty$ : Poisson distribution
integer $k$, $k$<0: Binomial distribution
($N = -k$, $p = -\langle n \rangle / k$)

# $\pi^0$ transverse momentum distributions at different $\sqrt{s}$

Low $p_T$ (< ~2 GeV/c ):
"soft processes"

$$E\frac{d^3\sigma}{d^3p} = A(\sqrt{s}) \cdot e^{-\alpha p_T}, \;\; \alpha \approx 6/(\mathrm{GeV}/c)$$

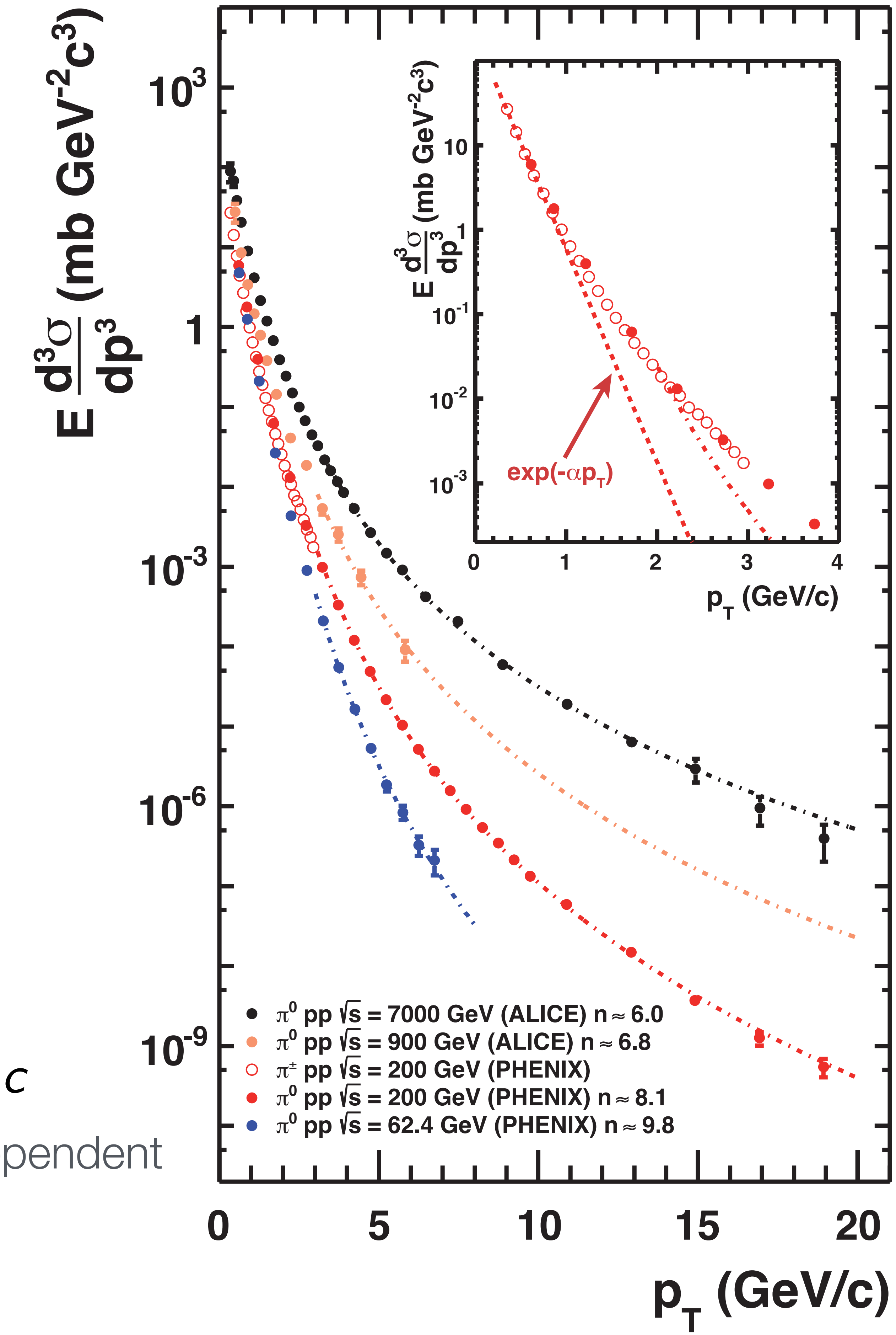
High $p_T$ ("hard scattering"):

$$E\frac{d^3\sigma}{d^3p} = B(\sqrt{s}) \cdot \frac{1}{p_T^{n(\sqrt{s})}}$$

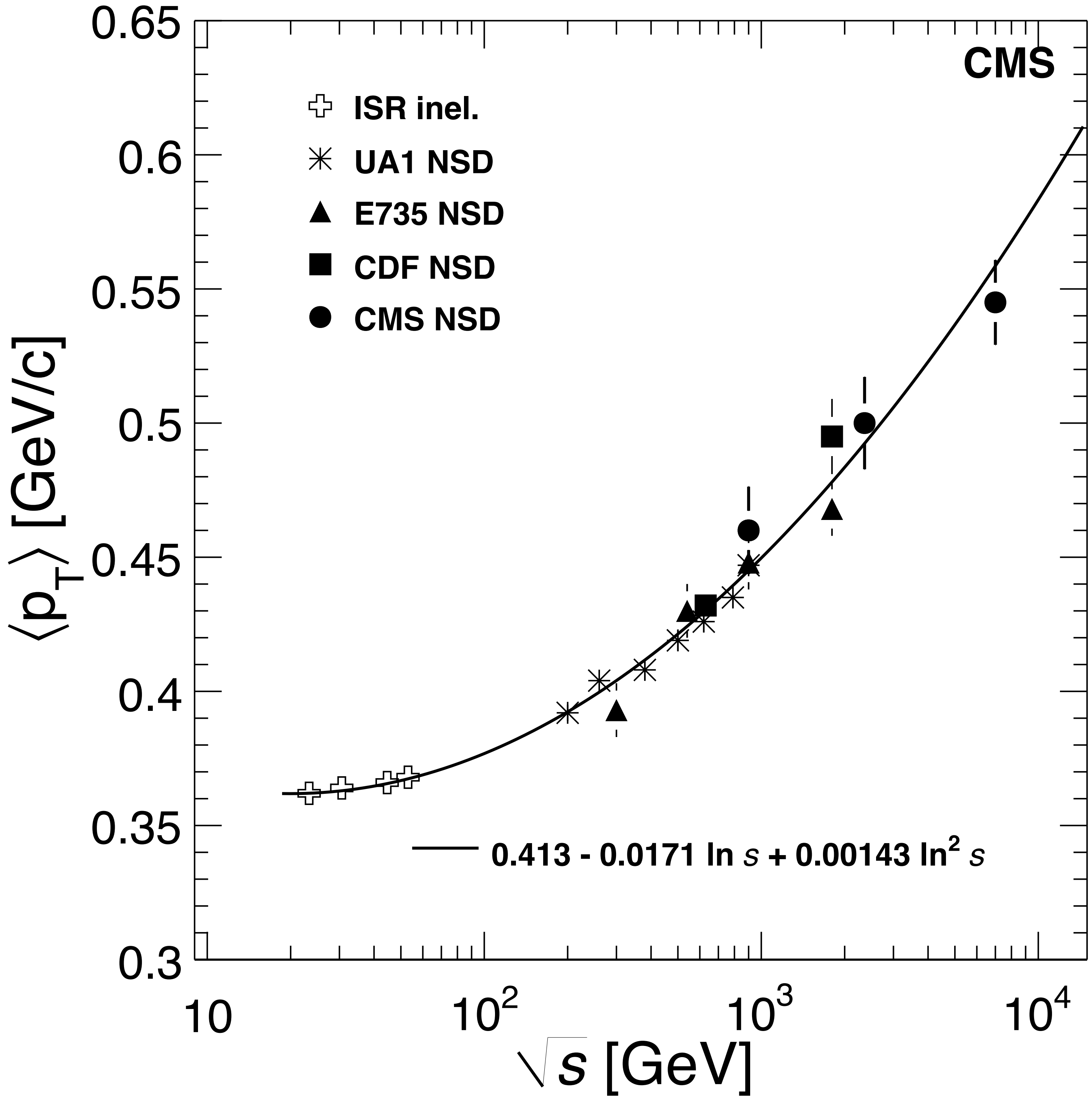
Average $p_T$:

$$\langle p_T \rangle = \frac{\int_0^\infty p_T \frac{dN_x}{dp_T} dp_T}{\int_0^\infty \frac{dN_x}{dp_T} dp_T} \approx 300 - 400 \mathrm{MeV}/c$$

pretty energy-independent
for $\sqrt{s}$ < 100 GeV



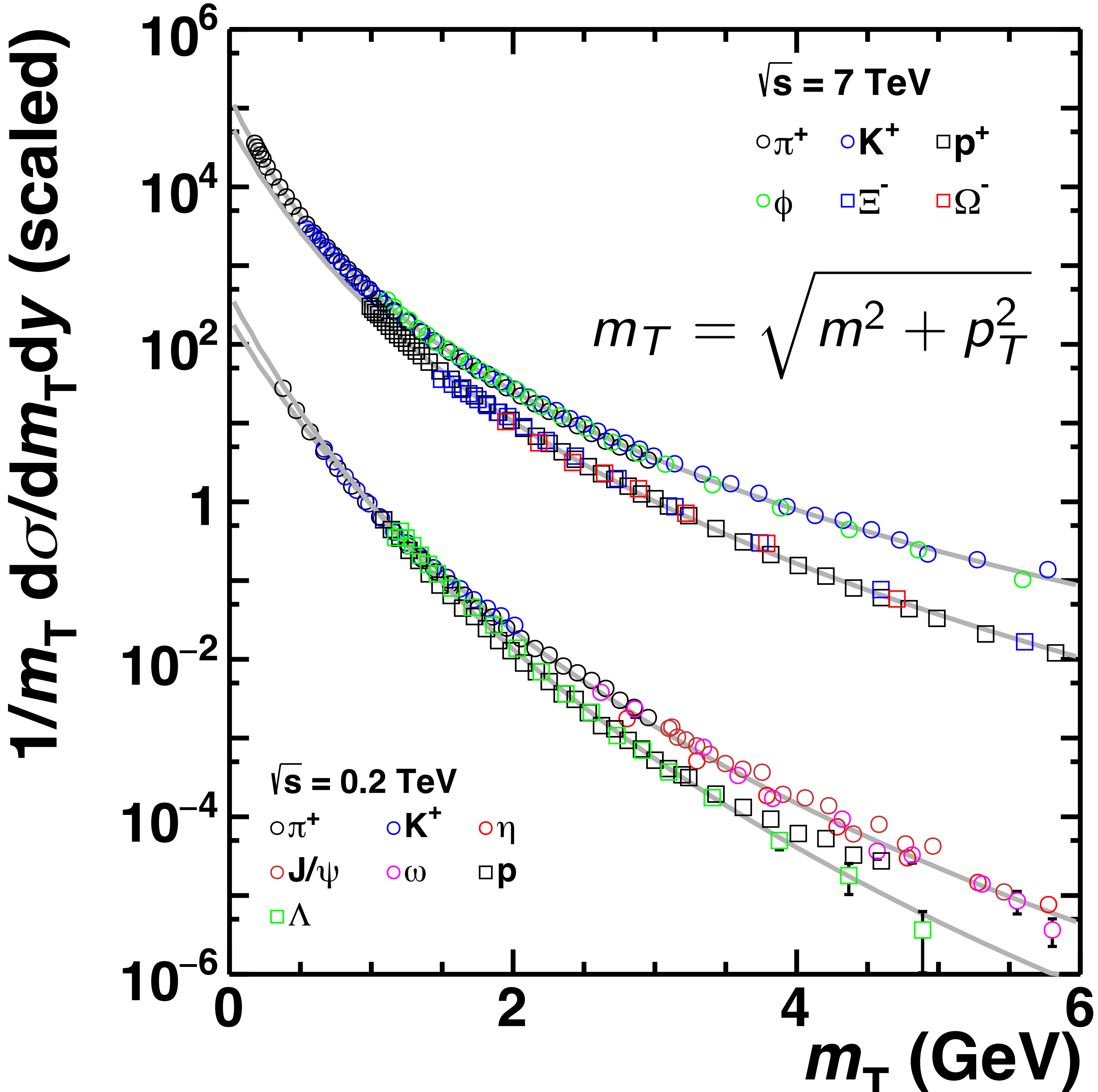
# Mean $p_T$ increases with $\sqrt{s}$




Increase of $\langle p_T \rangle$ with $\sqrt{s}$ (most likely) reflects increase in particle production from hard parton-parton scattering

CMS, PRL 105, 022002 (2010)
CDF, PRL 61, 1819 (1988)

# $m_T$ scaling in pp collisions




$m_T$ scaling (early ref's):
Nucl. Phys. B70, 189–204 (1974)
Nucl.Phys. B120 (1977) 14-22

$m_T$ scaling:
shape of $m_T$ spectra the same for different hadron species

example: $$\frac{dN/dm_T|_\eta}{dN/dm_T|_{\pi^0}} \approx 0.45$$

possible interpretation:
thermodynamic models

$$E\frac{d^3n}{d^3p} \propto Ee^{-E/T}$$
$$\rightarrow \frac{1}{m_T}\frac{dn}{dm_T} \propto K_1\left(\frac{m_T}{T}\right)$$

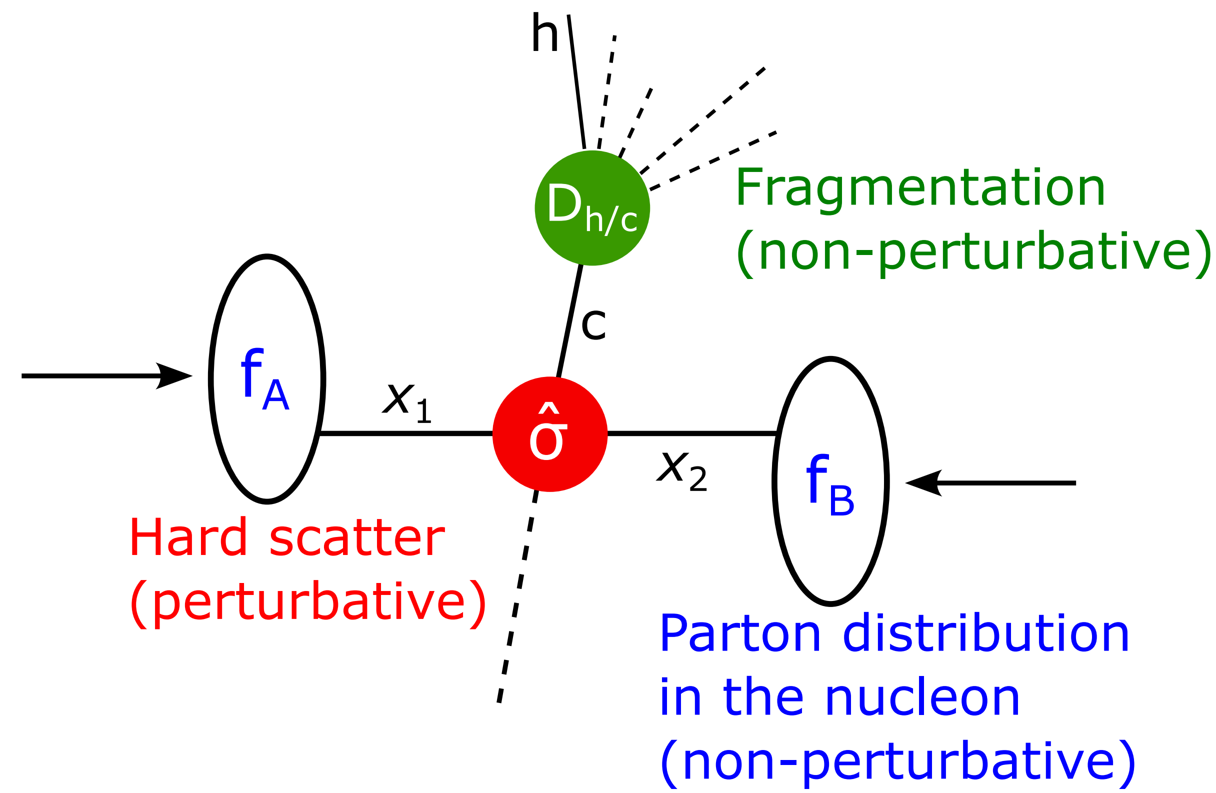
RHIC/LHC:
$m_T$ scaling (approximately) satisfied, different universal function for mesons and baryons

Do deviations from $m_T$ scaling in pp at low $p_T$ indicate onset of radial flow? (1312.4230)

# Theoretical modeling: General considerations

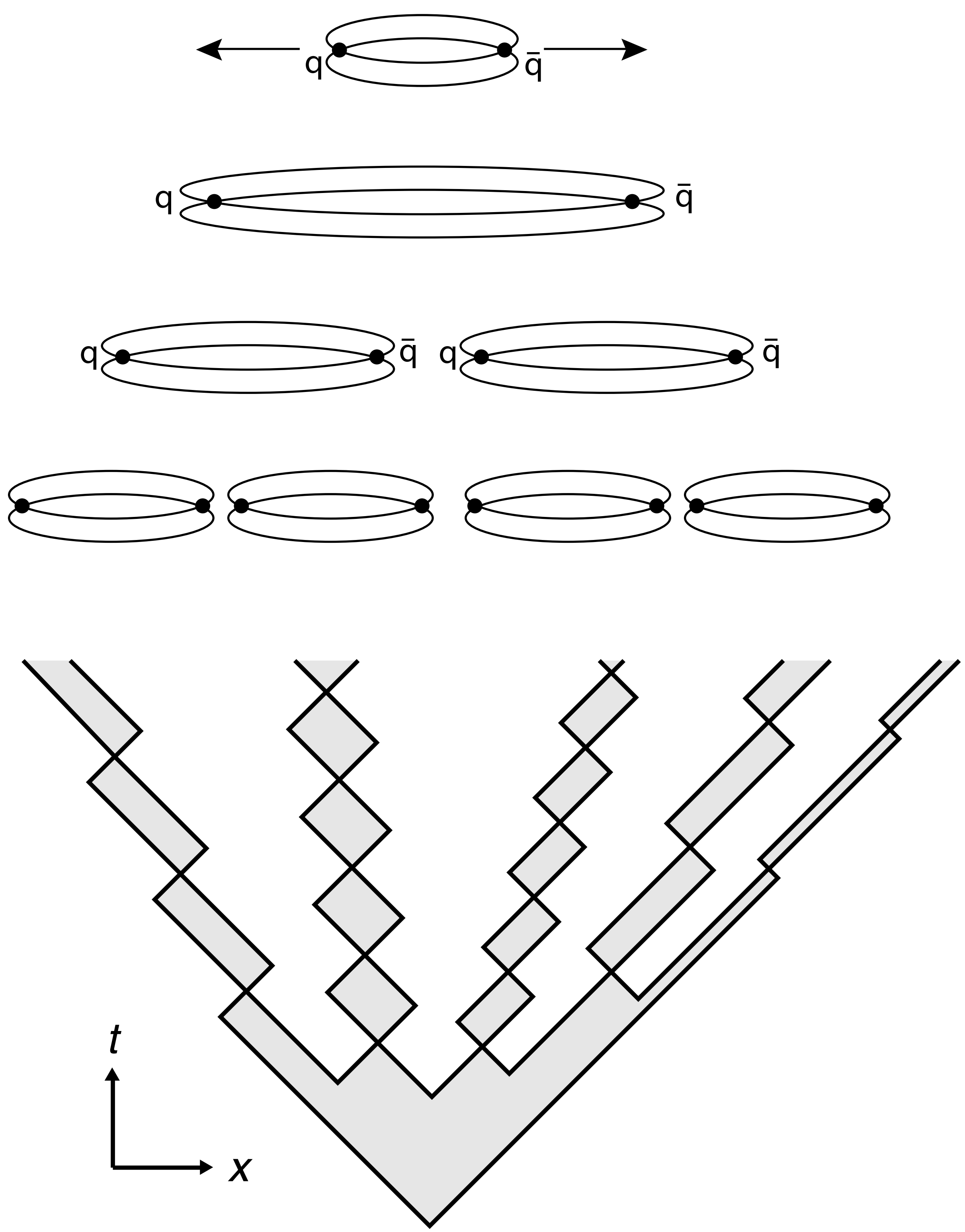
- Description of particle production amenable to perturbative methods only at sufficiently large $p_T$ (so that $\alpha_s$ becomes sufficiently small)
  - parton distributions (PDF)
  - parton-parton cross section from perturbative QCD (pQCD)
  - fragmentation functions (FF)

$$E\frac{\mathrm{d}^3\sigma}{\mathrm{d}^3 p} = \int \mathrm{PDF} \otimes \mathrm{pQCD} \otimes \mathrm{FF}$$



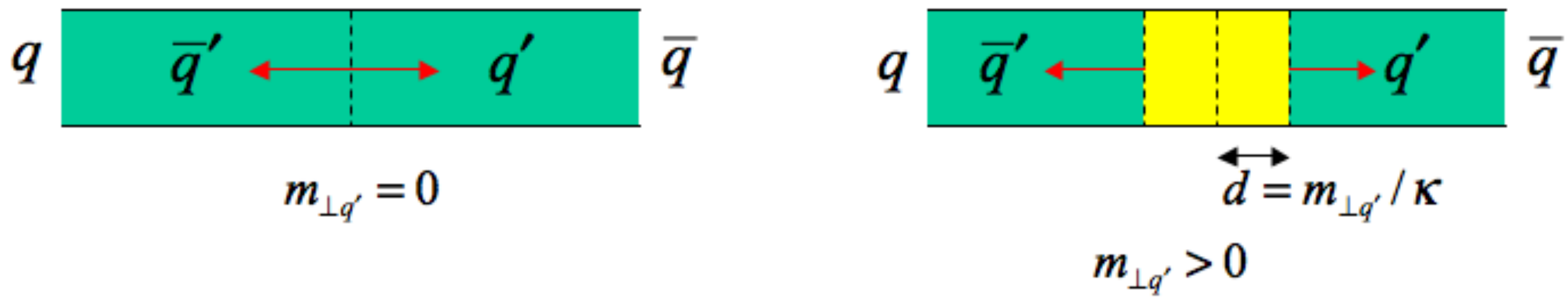

- Low-$p_T$:
  Need to work with (QCD inspired) models, and confront them with data
  - e.g. Lund string model

# Modeling particle production as string breaking (I)




- Color flux tube between two quarks breaks due to quark-antiquark pair production in the intense color field
- Lund model:
  The basic assumption of the symmetric Lund model is that the vertices at which the quark and the antiquark are produced lie approximately on a curve on constant proper time
- Result: flat rapidity distribution of the produced particles

# Modeling particle production as string breaking (II)




In terms of the transverse mass of the produced quark ($m_{\mathrm{T,q'}} = m_{\mathrm{T,q'bar}}$) the probability that the break-up occurs is:

$$P \propto \exp\left(-\frac{\pi m_{\perp q'}^2}{k}\right) = \exp\left(-\frac{\pi p_{\perp q'}^2}{k}\right) \exp\left(-\frac{\pi m_{q'}^2}{k}\right)$$

This leads to a transverse momentum distribution for the quarks of the form:

$$\frac{1}{p_T}\frac{\mathrm{d}N_{\mathrm{quark}}}{\mathrm{d}p_T} = \mathrm{const.} \cdot \exp\left(-\pi p_T^2/k\right) \quad \rightsquigarrow \quad \sqrt{\langle p_T^2 \rangle_{\mathrm{quark}}} = \sqrt{k/\pi}$$

For pions (two quarks) one obtains: $\sqrt{\langle p_T^2 \rangle_{\mathrm{pion}}} = \sqrt{2k/\pi}$

With a string tension of 1 GeV/fm this yields $\langle p_T \rangle_{\mathrm{pion}} \approx 0.37$ GeV/$c$, in approximate agreement with data

# Modeling particle production as string breaking (III)

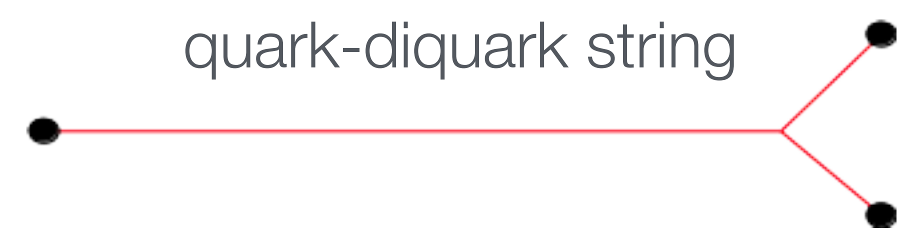
Convolution of the string breaking mechanism with fluctuations of the string tension described by a Gaussian give rise to exponential $p_T$ spectra

Phys. Lett. B466, 301–304 (1999)

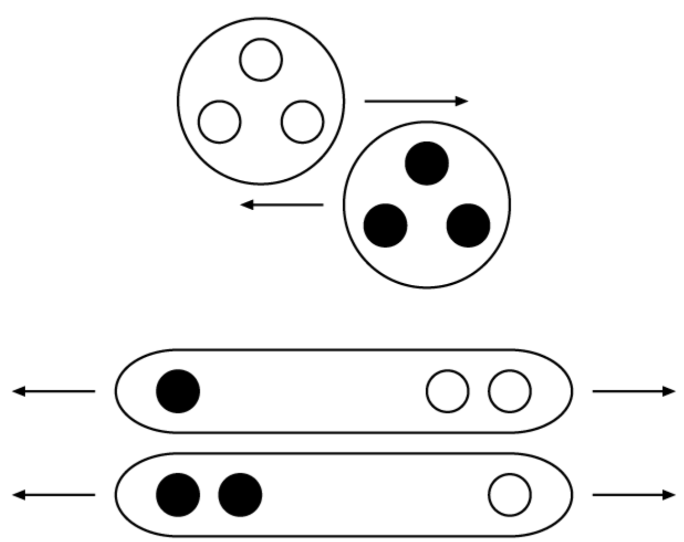
The tunneling process implies heavy-quark suppression:

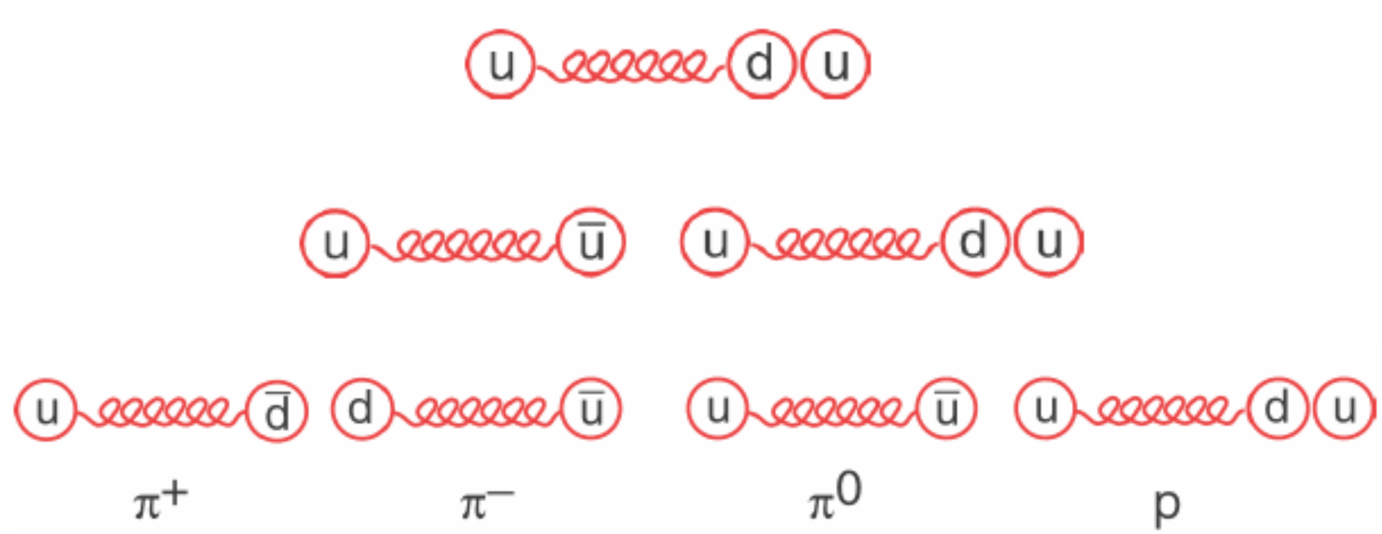
$$u\bar{u} : d\bar{d} : s\bar{s} : c\bar{c} \approx 1 : 1 : 0.3 : 10^{-11}$$

The production of baryons can be modeled by replacing the q-qbar pair by an quark-diquark pair




Collisions of hadrons described as excitation of quark-diquarks strings:

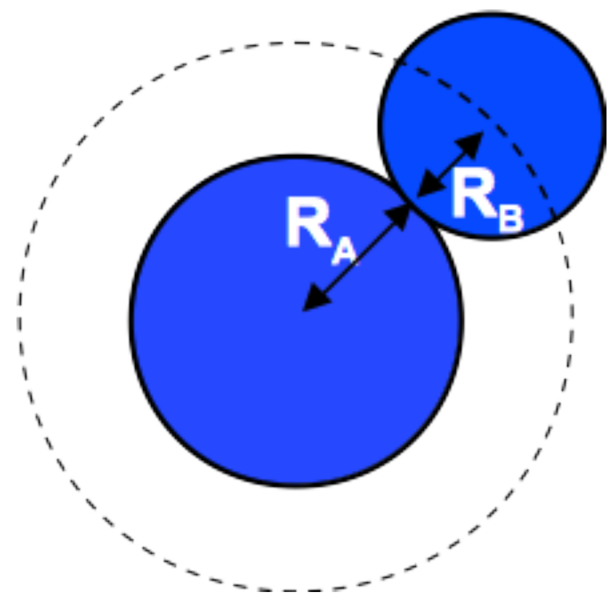




# Part II: nucleus-nucleus collisions

# Ultra-Relativistic Nucleus-Nucleus Collisions: Importance of Nuclear Geometry

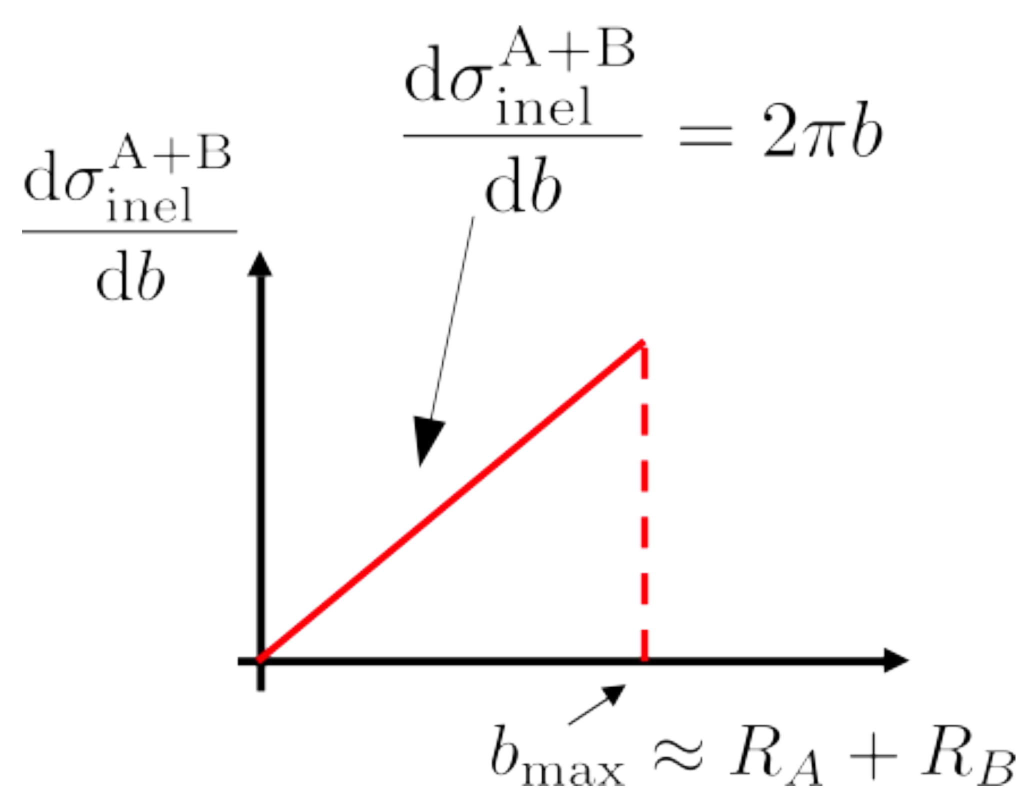
- Ultra-relativistic energies
  - De Broglie wave length much smaller than size of the nucleon
  - Wave character of the nucleon can be neglected for the estimation of the total cross section



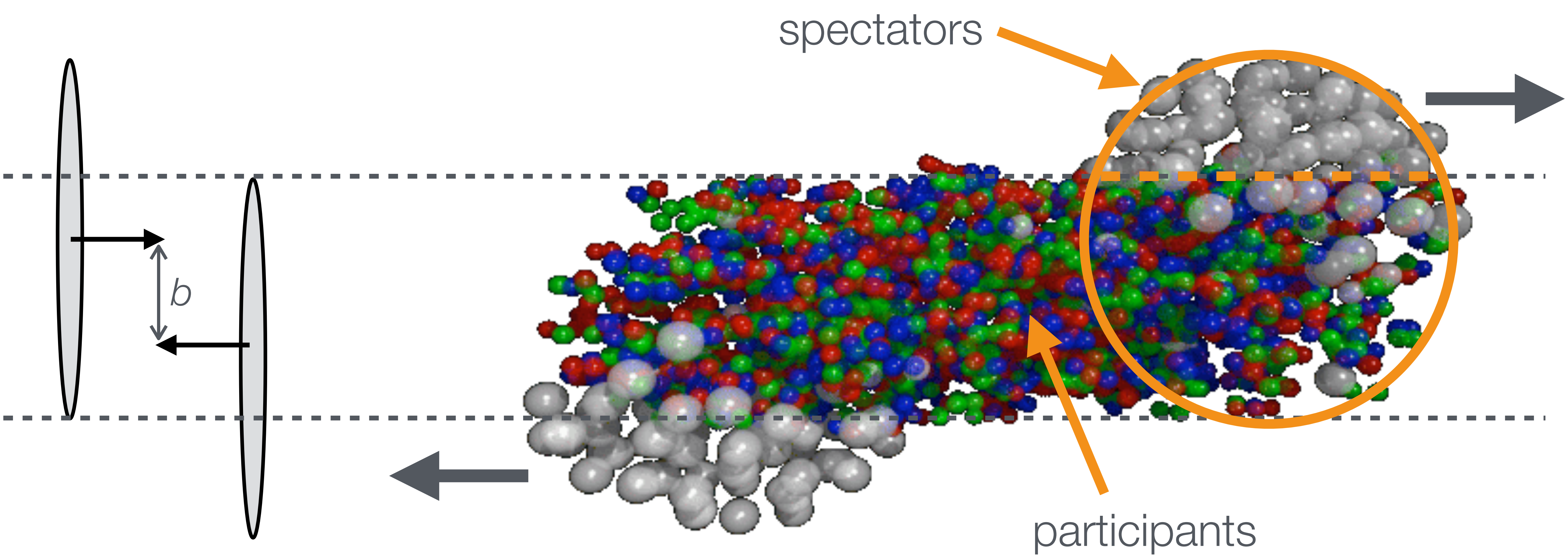

- Nucleus-Nucleus collision can be considered as a collision of two black disks

$$R_A \approx r_0 \cdot A^{1/3}, \quad r_0 = 1.2\,\mathrm{fm}$$

$$\sigma_{\mathrm{inel}}^{\mathrm{A+B}} \approx \sigma_{\mathrm{geo}} \approx \pi r_0^2 (A^{1/3} + B^{1/3})^2$$

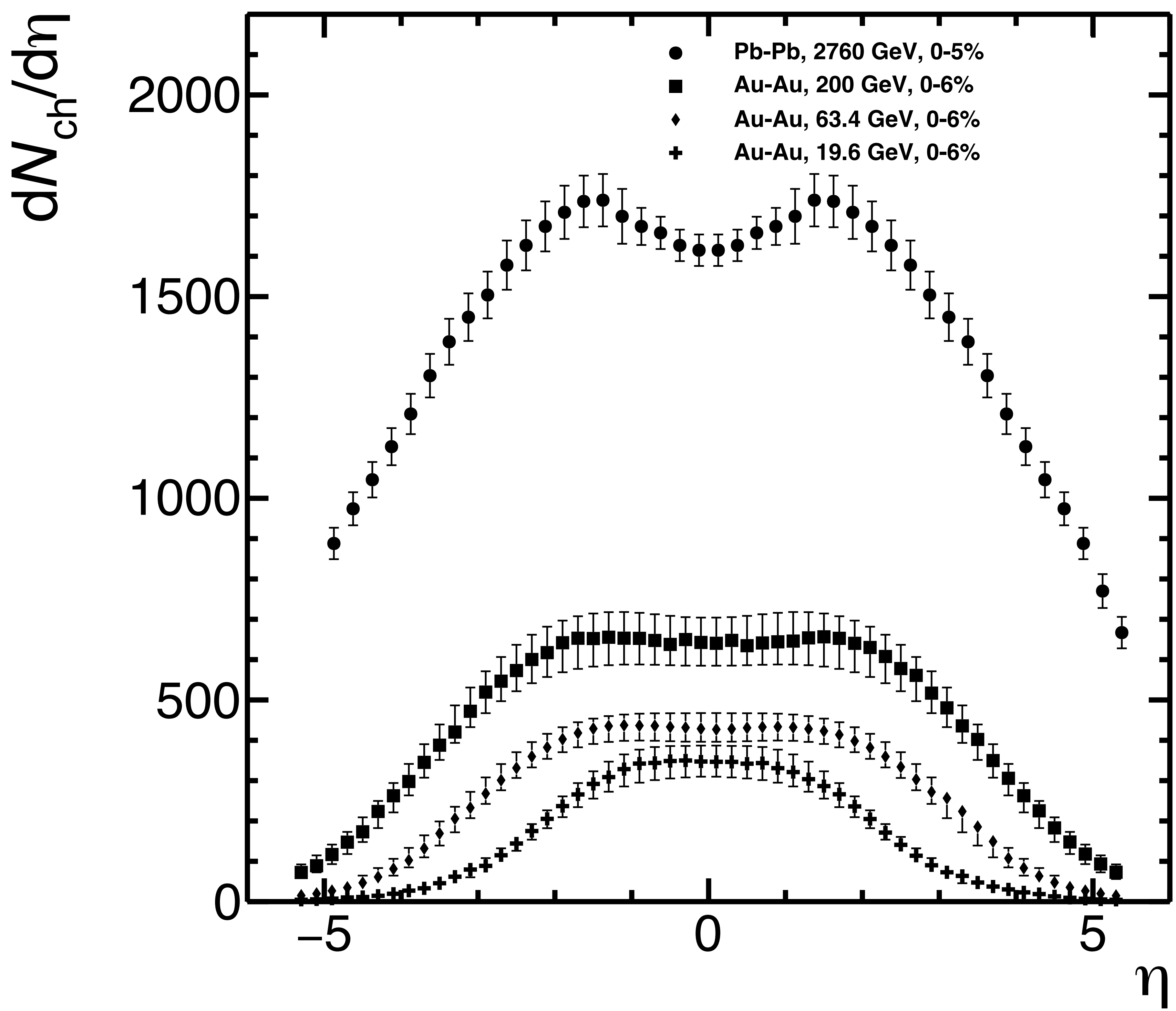


# Participants and spectators

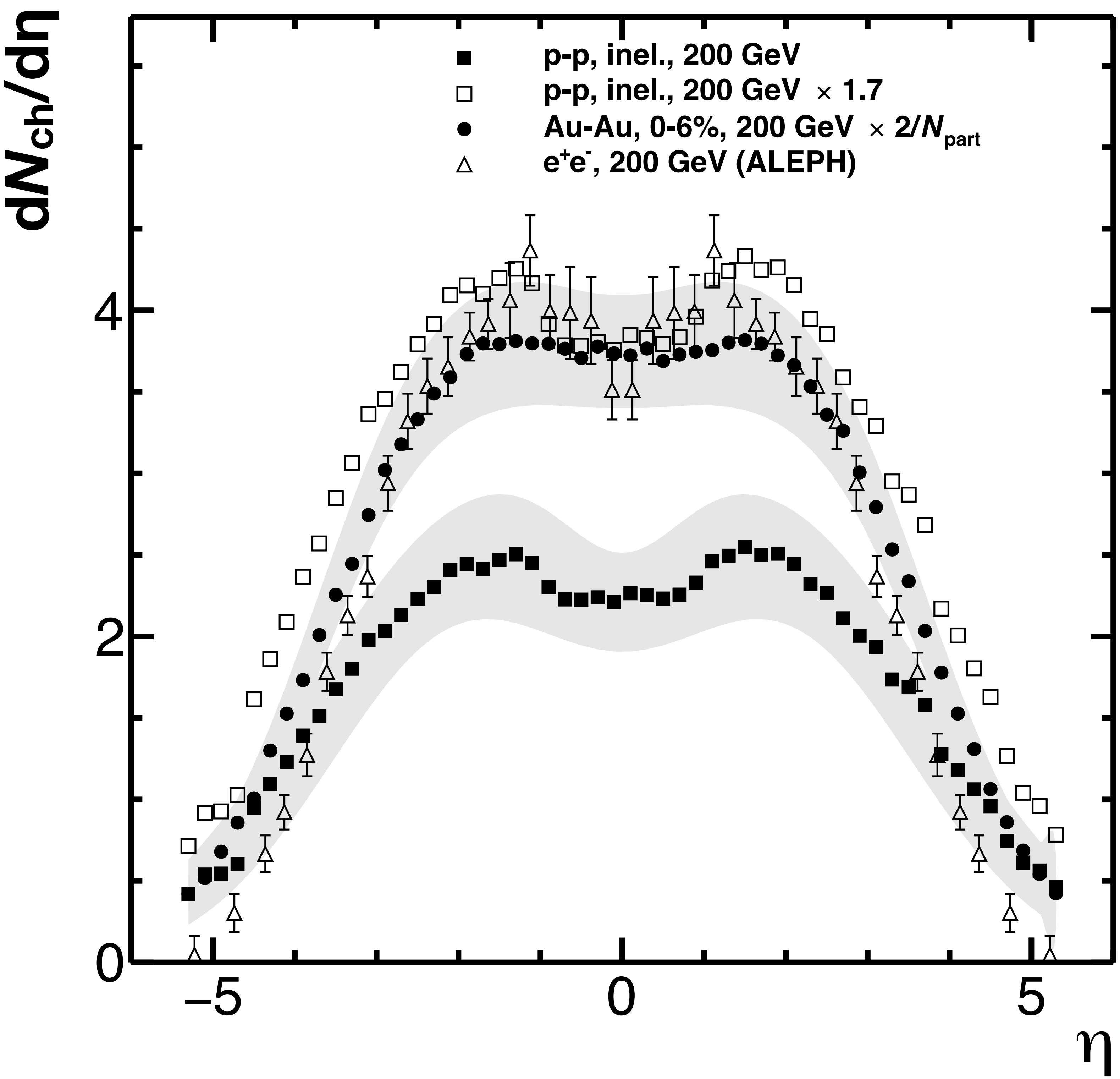



- $N_{coll}$: number of inelastic nucleon-nucleon collisions
- $N_{part}$: number of nucleons which underwent at least one inelastic nucleon-nucleon collisions

# Charged particle pseudorapidity distributions for different $\sqrt{s_{NN}}$

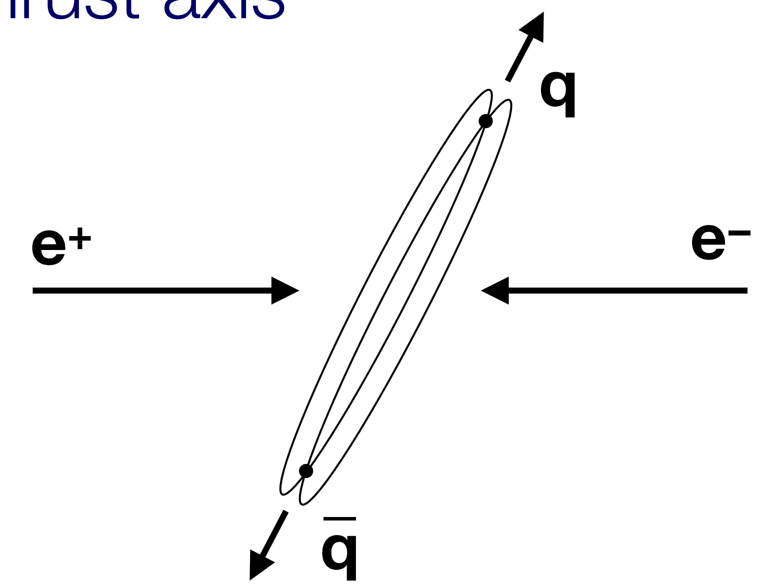


# Charged-particle Pseudorapidity Distributions: Comparison e+e-, pp, and AA



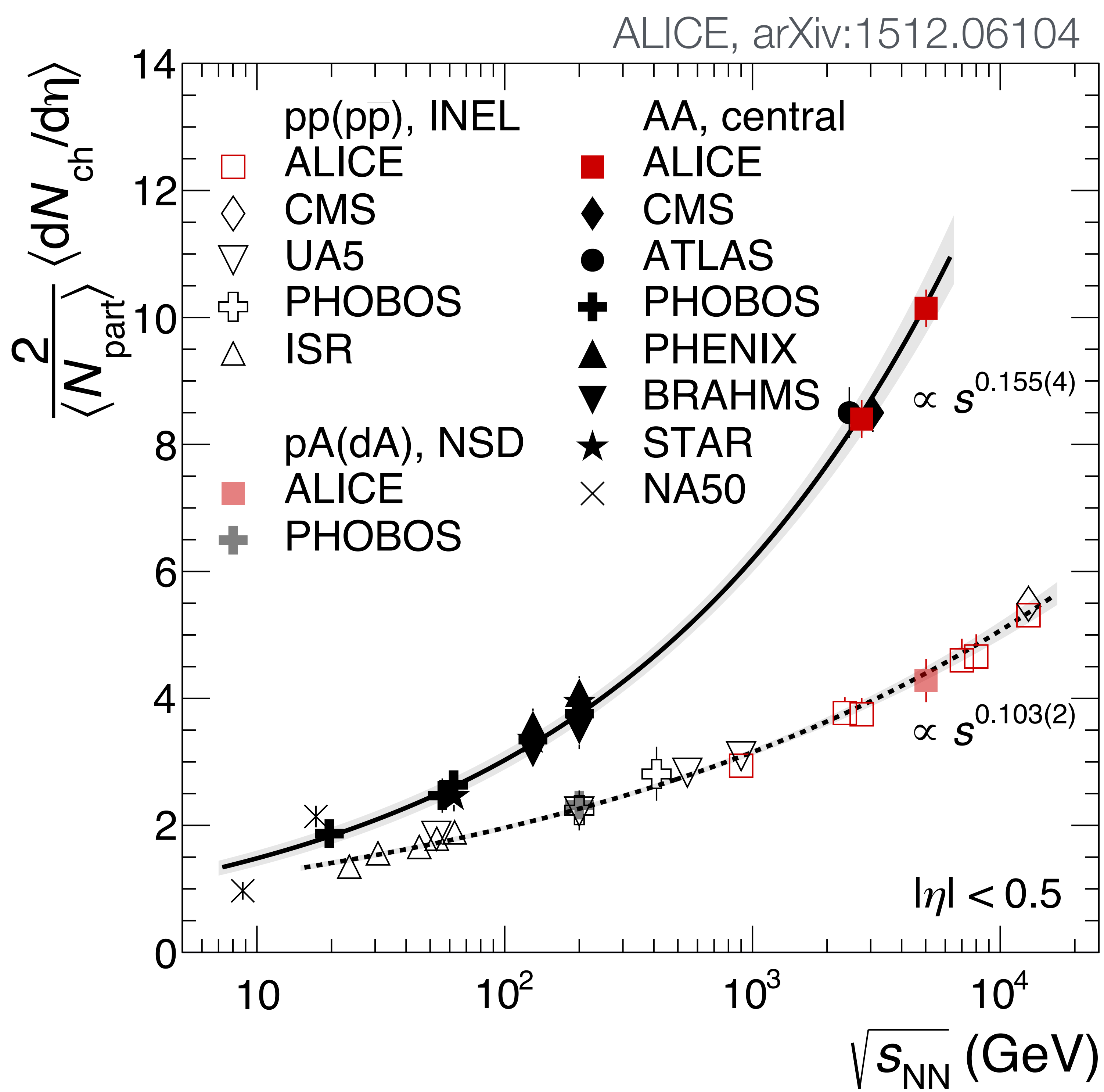

Multiplicity per participant higher in AA than in pp

$e^+e^-$:
pseudorapidity along the thrust axis




AA and $e^+e^-$ $\eta$ distributions strikingly similar

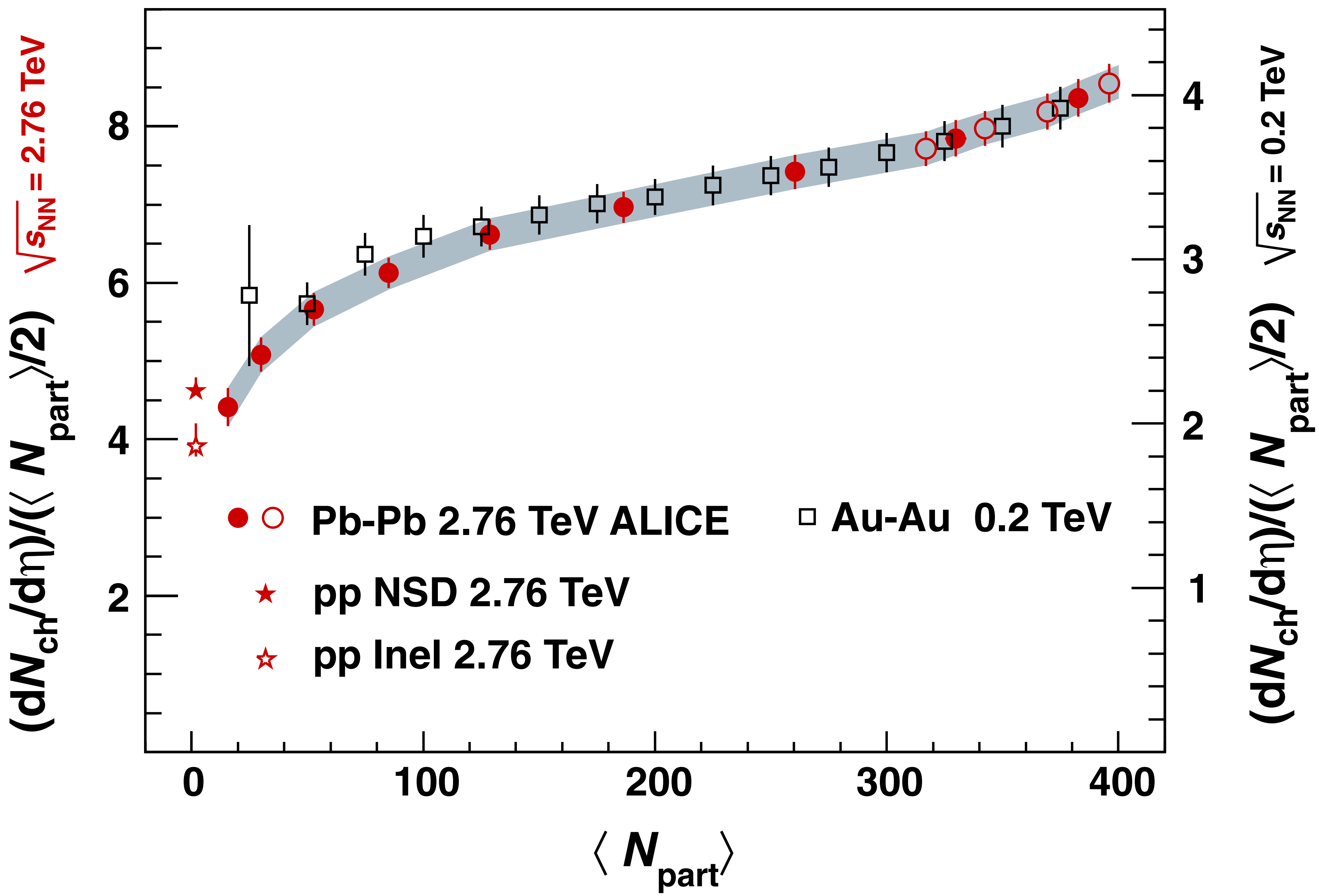
# $dN_{ch}/d\eta$ vs $\sqrt{s_{NN}}$ in pp and central A-A collisions




- $dN_{ch}/d\eta$ scales with $s^{\alpha}$
- Increase in central A+A stronger than in p+p

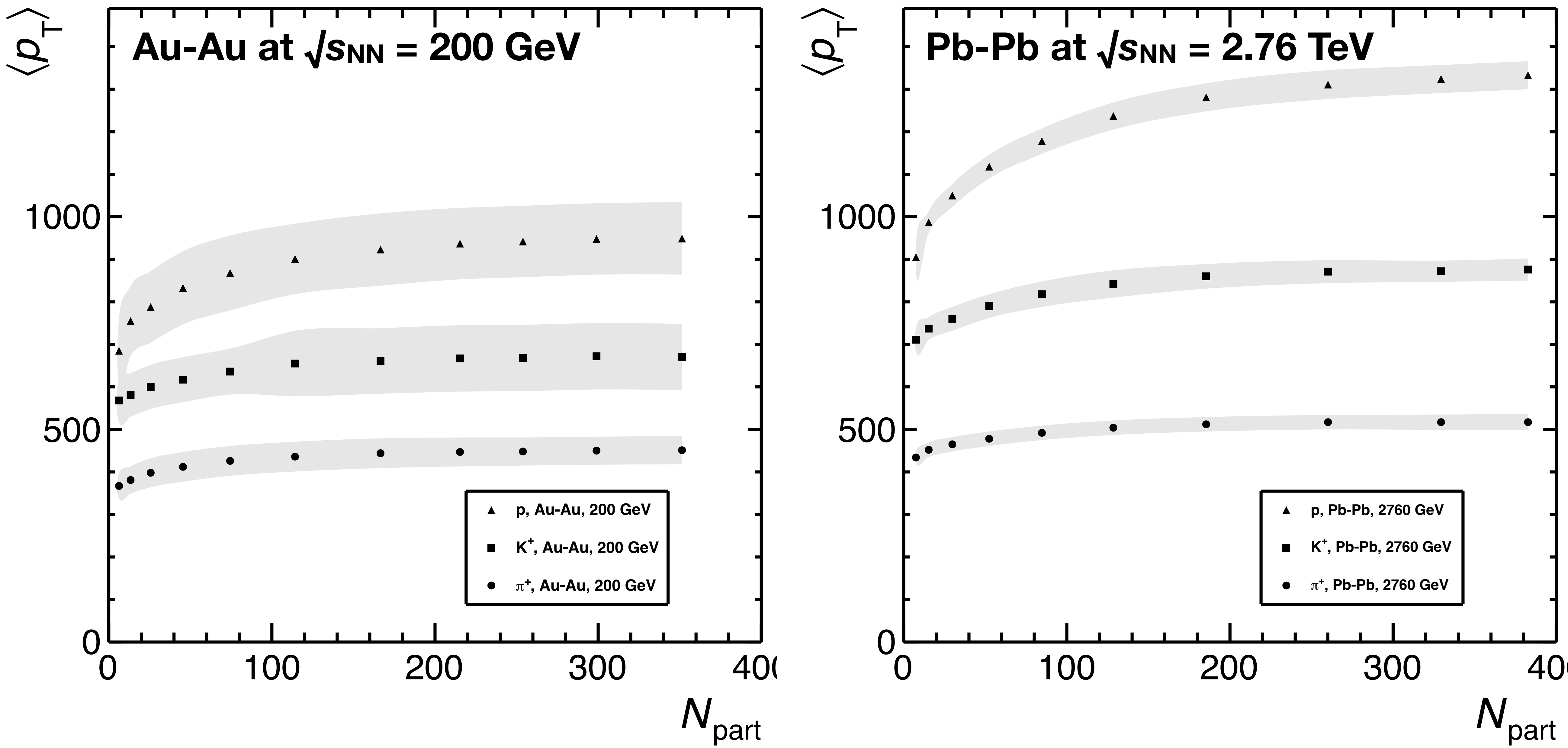
# Centrality dependence of d$N_{ch}$/dη

ALICE, arXiv:1012.1657



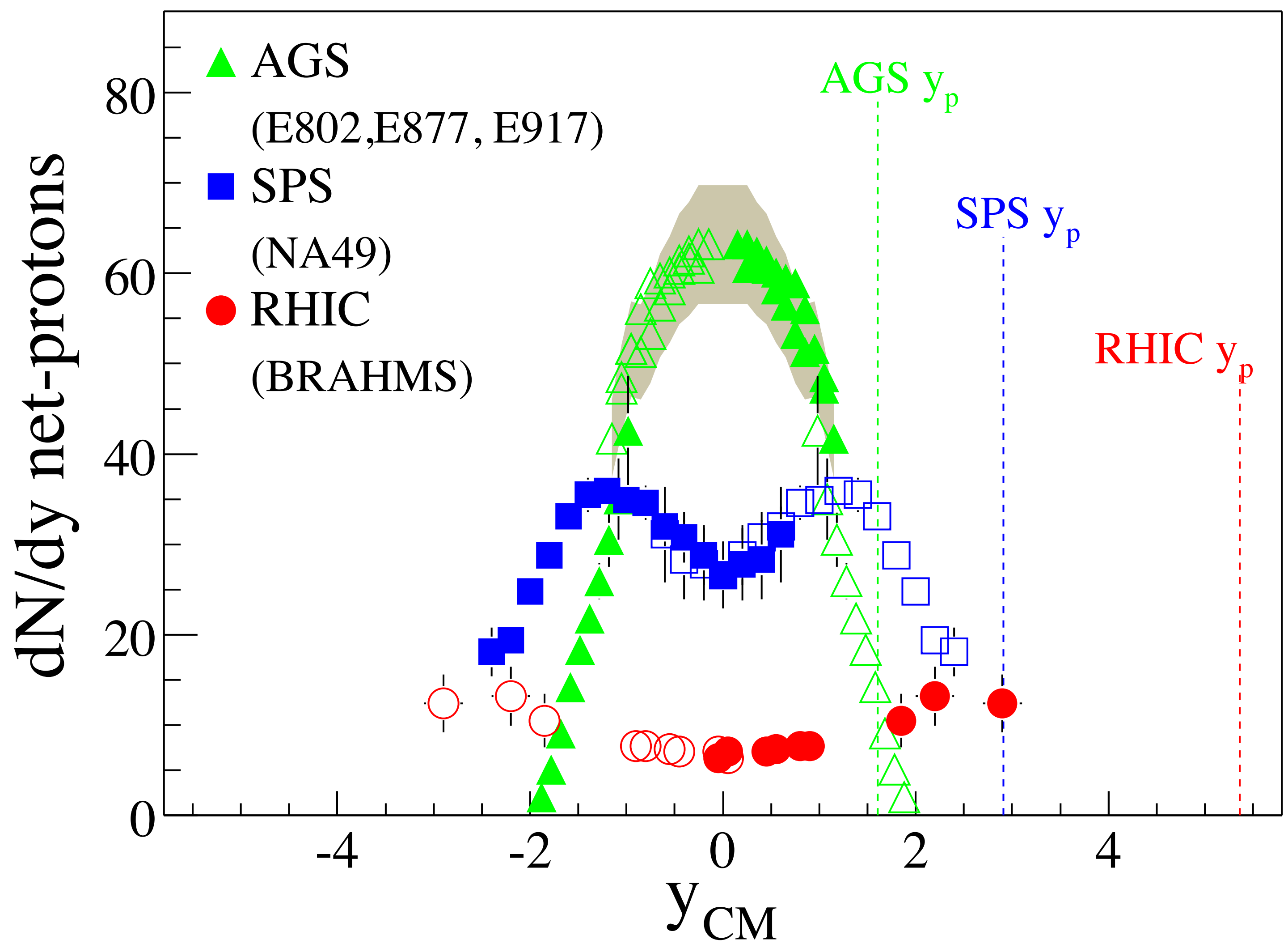

- d$N_{ch}$/dη / $N_{part}$ increases with centrality
- Relative increase similar at RHIC and the LHC: Importance of geometry!

# Average $p_T$ of pions, kaons, and protons in Au-Au@200 GeV and Pb-Pb@2.76 TeV



# Nuclear stopping power (Au-Au at $\sqrt{s_{NN}}$ = 200 GeV)

Brahms, PRL 93:102301, 2004




## Average rapidity loss:

Initial rapidity:

$$y_\mathrm{p} = 5.36$$

Net baryons after the collision:

$$\langle y \rangle = \frac{2}{N_\mathrm{part}} \int_0^{y_p} y \frac{dN_{B-\bar{B}}}{dy}\, dy$$

Average rapidity loss:

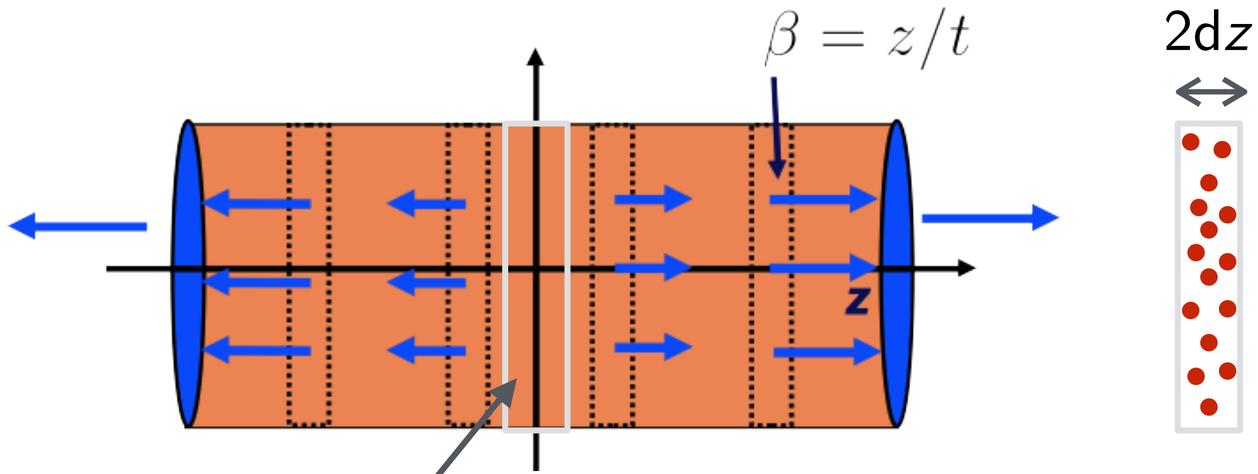
$$\langle \delta y \rangle = y_p - \langle y \rangle \approx 2$$

## Average energy per (net) baryon:

$$E_\mathrm{p} = 100\,\mathrm{GeV}, \qquad \langle E \rangle = \frac{1}{N_\mathrm{part}} \int_{-y_p}^{y_p} \underbrace{\langle m_T \rangle \cosh y}_{E} \frac{\mathrm{d}N_{B-\bar{B}}}{\mathrm{d}y}\, \mathrm{d}y \approx 27 \pm 6\,\mathrm{GeV}$$

Average energy loss of a nucleon in central Au+Au@200GeV is 73 ± 6 GeV

# Bjorken's formula for the initial energy density




Consider total energy in slice at $z = 0$ at time $\tau_0$

Assumptions:

- Particles (quarks and gluons) materialize at proper time $\tau_0$
- Position $z$ and longitudinal velocity (i.e. rapidity) are correlated
  - As if particles streamed freely from the origin

$z = \tau \sinh y$

$$\varepsilon = \frac{E}{V} = \frac{1}{A}\frac{dE}{dz}\bigg|_{z=0} = \frac{1}{A}\frac{dE}{dy}\bigg|_{y=0}\frac{dy}{dz}\bigg|_{z=0} = \frac{1}{A}\frac{dE}{dy}\bigg|_{y=0}\frac{1}{\tau} = \frac{\langle m_T \rangle}{A \cdot \tau}\frac{dN}{dy}\bigg|_{y=0}$$

$A$ = transverse area

$$\boxed{\varepsilon = \frac{1}{A \cdot \tau_0}\frac{dE_T}{dy}\bigg|_{y=0}, \quad \tau_0 \approx 1\,\mathrm{fm}/c}$$

However, this formula neglects longitudinal work:

- $dE/dy$ drops as a fct. of time
- Bjorken formula underestimates ε

J.D. Bjorken, Phys.Rev. D27 (1983) 140-151, 3494 citations on inspirehep.net on May 11, 2023

# Energy density in central Pb-Pb collisions at the LHC

$$\varepsilon = \frac{1}{A \cdot \tau_0} \left. \frac{\mathrm{d}E_\mathrm{T}}{\mathrm{d}y} \right|_{y=0}$$

$$= \frac{1}{A \cdot \tau_0} J(y,\eta) \left. \frac{\mathrm{d}E_\mathrm{T}}{\mathrm{d}\eta} \right|_{\eta=0}$$

$$\text{with } J(y,\eta) \approx 1.09$$

Transverse area:

$$A = \pi R_\mathrm{Pb}^2 \quad \text{with } R_\mathrm{Pb} \approx 7\,\mathrm{fm}$$

Central Pb-Pb at $\sqrt{s_{NN}} = 2.76$ TeV:

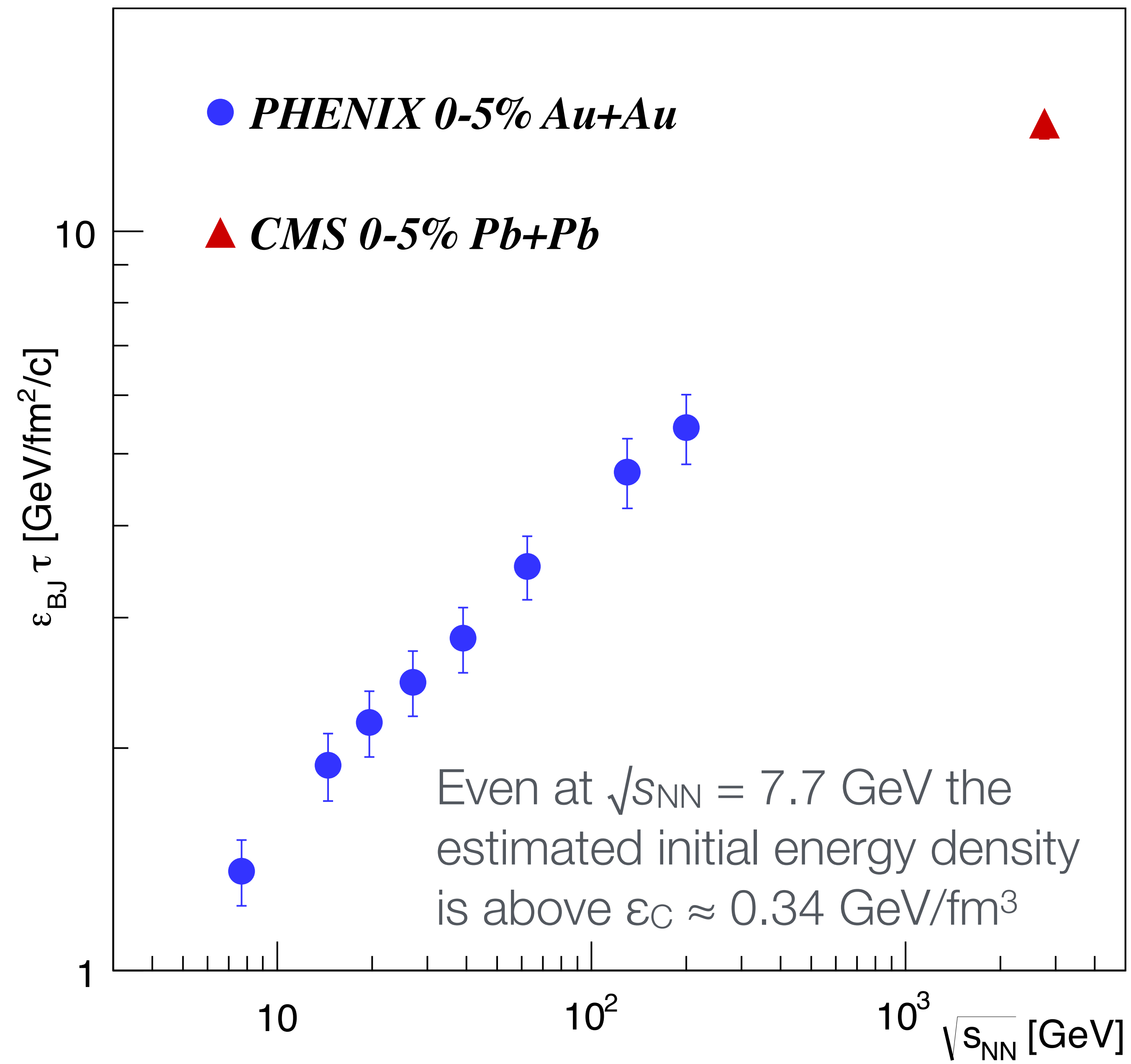
$$dE_T/d\eta = 2000\,\mathrm{GeV}$$

Energy density:

$$\varepsilon_\mathrm{LHC} = 14\,\mathrm{GeV/fm^3}$$

$$\approx 2.6 \times \varepsilon_\mathrm{RHIC} \text{ for } \tau_0 = 1\,\mathrm{fm}/c$$

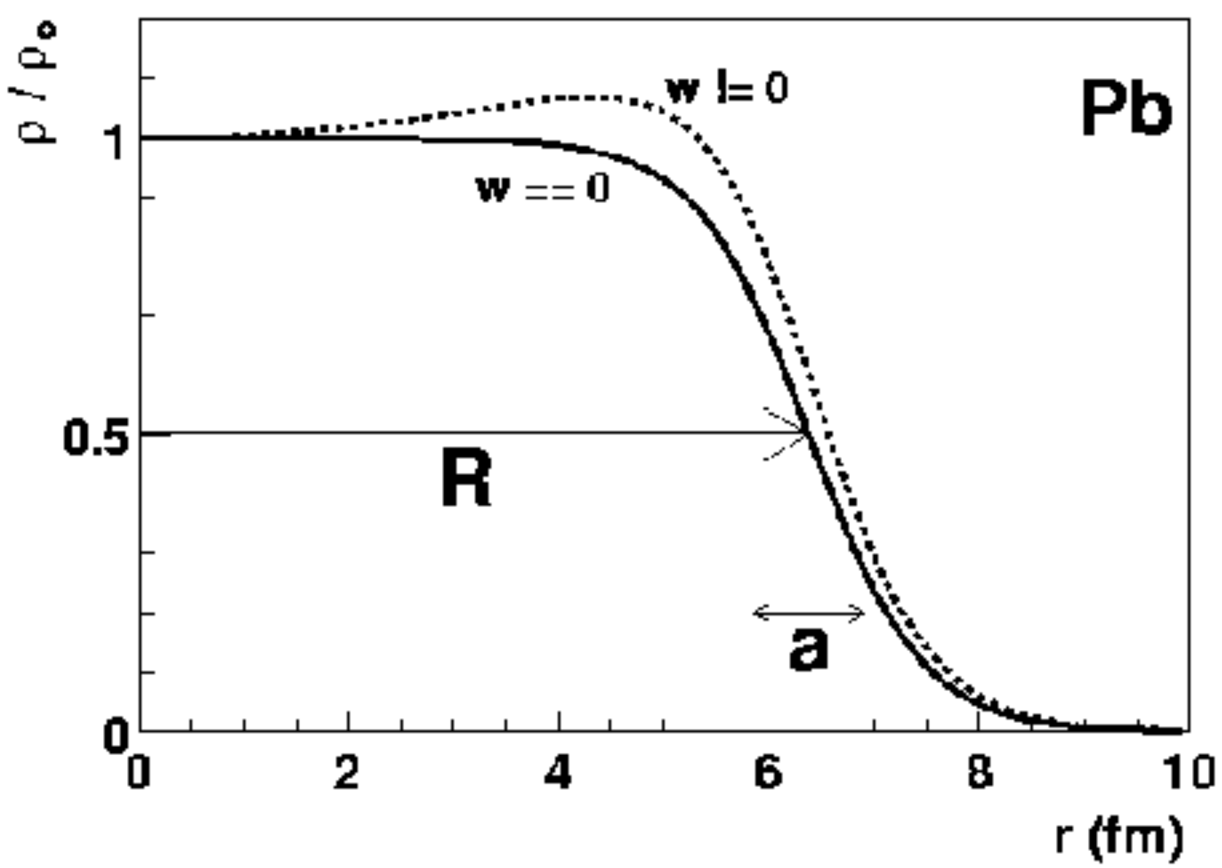
PHENIX, arXiv:1509.06727



# Glauber modeling:
# An interface between theory and experiment

Starting point: nucleon density

$$\rho(r) = \frac{\rho_0 \left(1 + wr^2/R^2\right)}{1 + \exp((r - R)/a)}$$



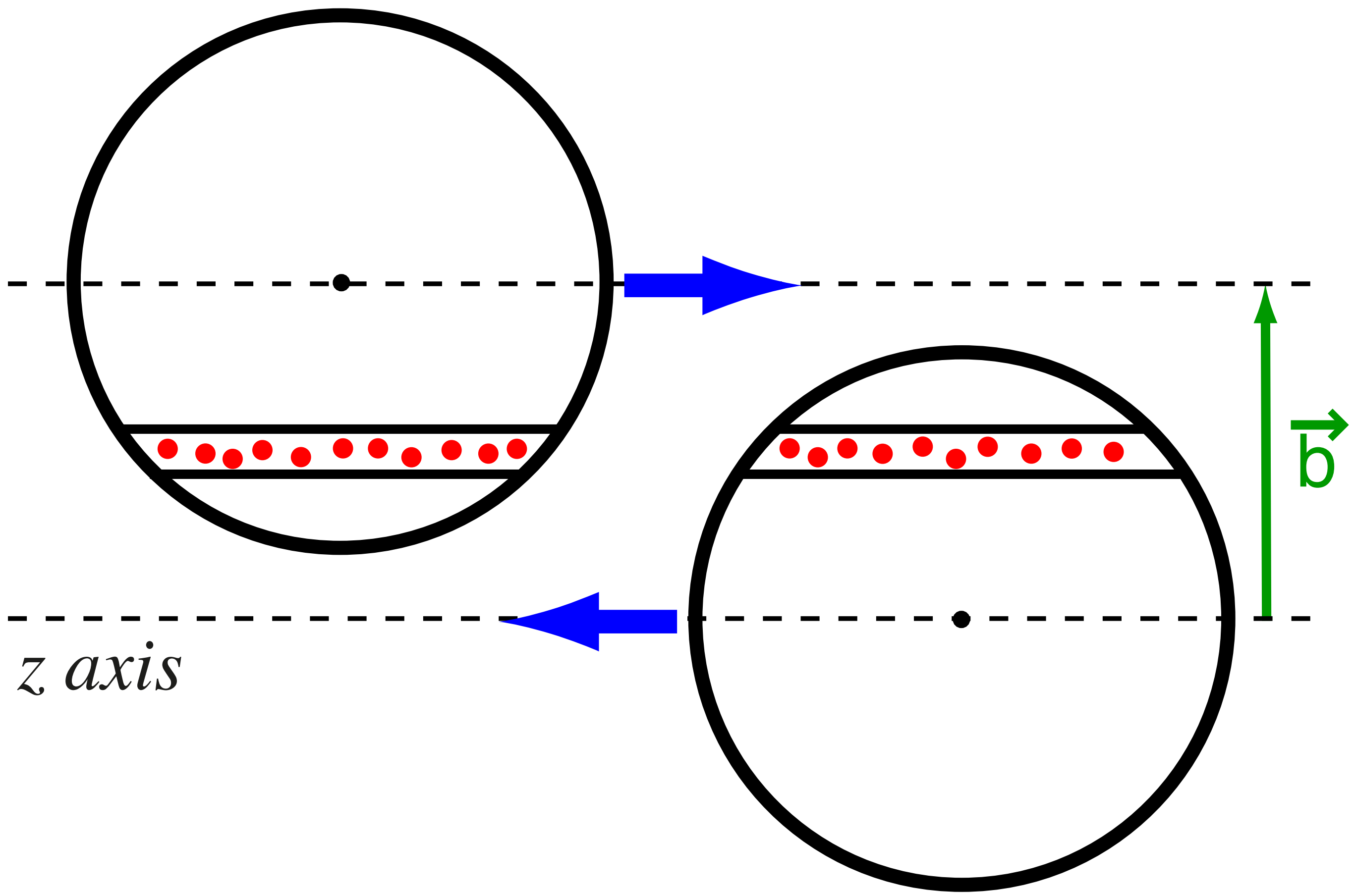

| Nucleus | A | R (fm) | a (fm) | w |
|---|---|---|---|---|
| ***C*** | 12 | 2.47 | 0 | 0 |
| ***O*** | 16 | 2.608 | 0.513 | -0.051 |
| ***Al*** | 27 | 3.07 | 0.519 | 0 |
| ***S*** | 32 | 3.458 | 0.61 | 0 |
| ***Ca*** | 40 | 3.76 | 0.586 | -0.161 |
| ***Ni*** | 58 | 4.309 | 0.516 | -0.1308 |
| ***Cu*** | 63 | 4.2 | 0.596 | 0 |
| ***W*** | 186 | 6.51 | 0.535 | 0 |
| ***Au*** | **197** | **6.38** | **0.535** | **0** |
| ***Pb*** | 208 | 6.68 | 0.546 | 0 |
| ***U*** | 238 | 6.68 | 0.6 | 0 |

Woods-Saxon parameters typically from $e^-$-nucleus scattering (sensitive to charge distribution only)

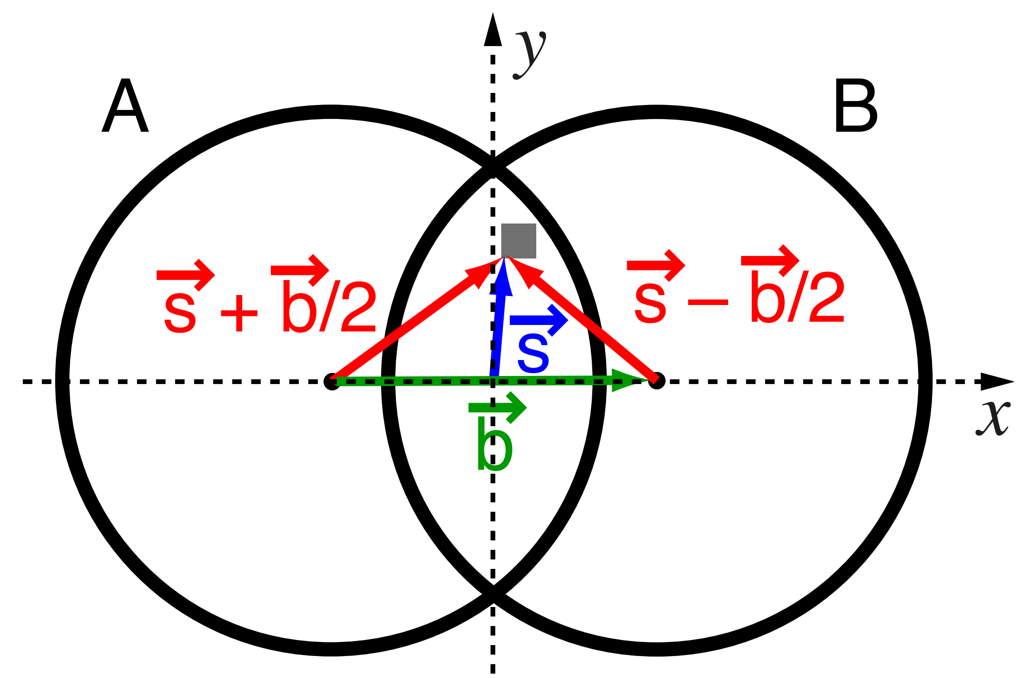
Difference between neutron and proton distribution small and typically neglected

# Nuclear Thickness Function

side view:




transverse plane:




Projection of nucleon density on the transverse plane ("nuclear thickness fct."):

$$T_{\mathrm{A}}(\vec{s}\,') = \int \mathrm{d}z \; \rho_{\mathrm{A}}(z, \vec{s}\,')$$

(analogous for nucleus B)

Number of nucleon-nucleon encounters per transverse area element:

$$\mathrm{d}\,T_{\mathrm{AB}} = T_{\mathrm{A}}(\vec{s} + \vec{b}/2) \cdot T_{\mathrm{B}}(\vec{s} - \vec{b}/2)\; \mathrm{d}^2 s$$

# Nuclear Overlap function and the number of nucleon-nucleon collisions

Nuclear overlap function:

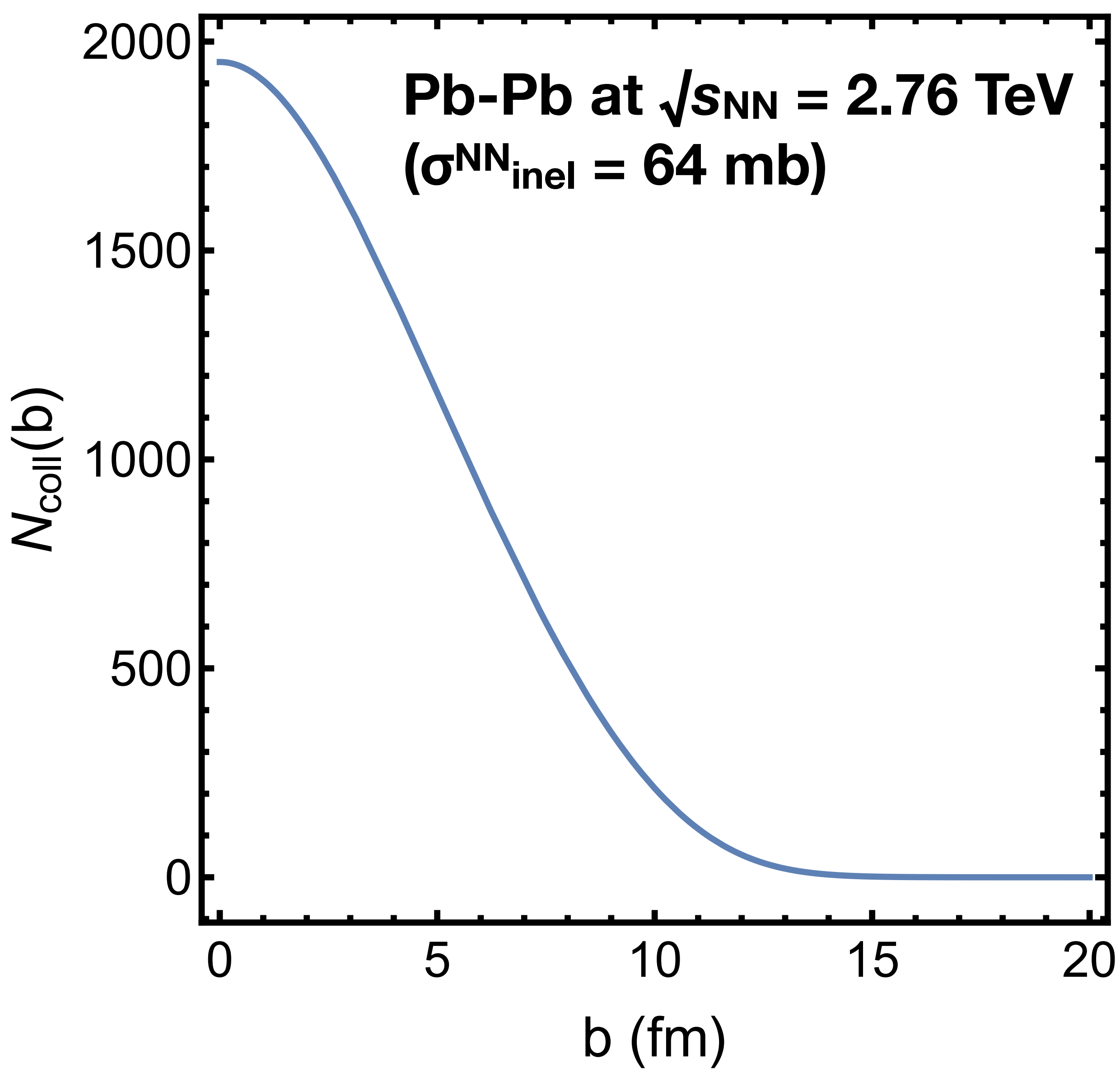
$$T_{AB}(\vec{b}) = \int T_A(\vec{s} + \vec{b}/2) \cdot T_B(\vec{s} - \vec{b}/2)\, \mathrm{d}^2 s$$

Nuclear overlap function resembles integrated luminosity of a collider:

$$N_{coll}(b) = T_{AB}(b) \cdot \sigma_{inel}^{NN}$$

Or, more generally, for a process with cross section $\sigma_{int}$:

$$N_{int}(b) = T_{AB}(b) \cdot \sigma_{int}$$



# Probability for an Inelastic A+B collision

Def's (different normalization of the thickness functions):

$$\hat{T}_\mathrm{A}(\vec{s}') = T_\mathrm{A}(\vec{s}')/A \qquad \hat{T}_\mathrm{B}(\vec{s}') = T_\mathrm{B}(\vec{s}')/B \qquad \hat{T}_\mathrm{AB}(\vec{b}) = T_\mathrm{AB}(\vec{b})/(AB)$$

We can then write:

$$N_\mathrm{coll}(b) = AB\,\hat{T}_\mathrm{AB}(b)\cdot\sigma_\mathrm{inel}^\mathrm{NN} \qquad p_\mathrm{NN} = \hat{T}_\mathrm{AB}(\vec{b})\cdot\sigma_\mathrm{inel}^\mathrm{NN}$$

probability for a certain nucleon from nucleus A to collide with a certain nucleon from nucleus B

Probability for $k$ nucleon-nucleon coll.:

$$P(k,\vec{b}) = \binom{AB}{k} p_\mathrm{NN}^k (1-p_\mathrm{NN})^{AB-k}$$

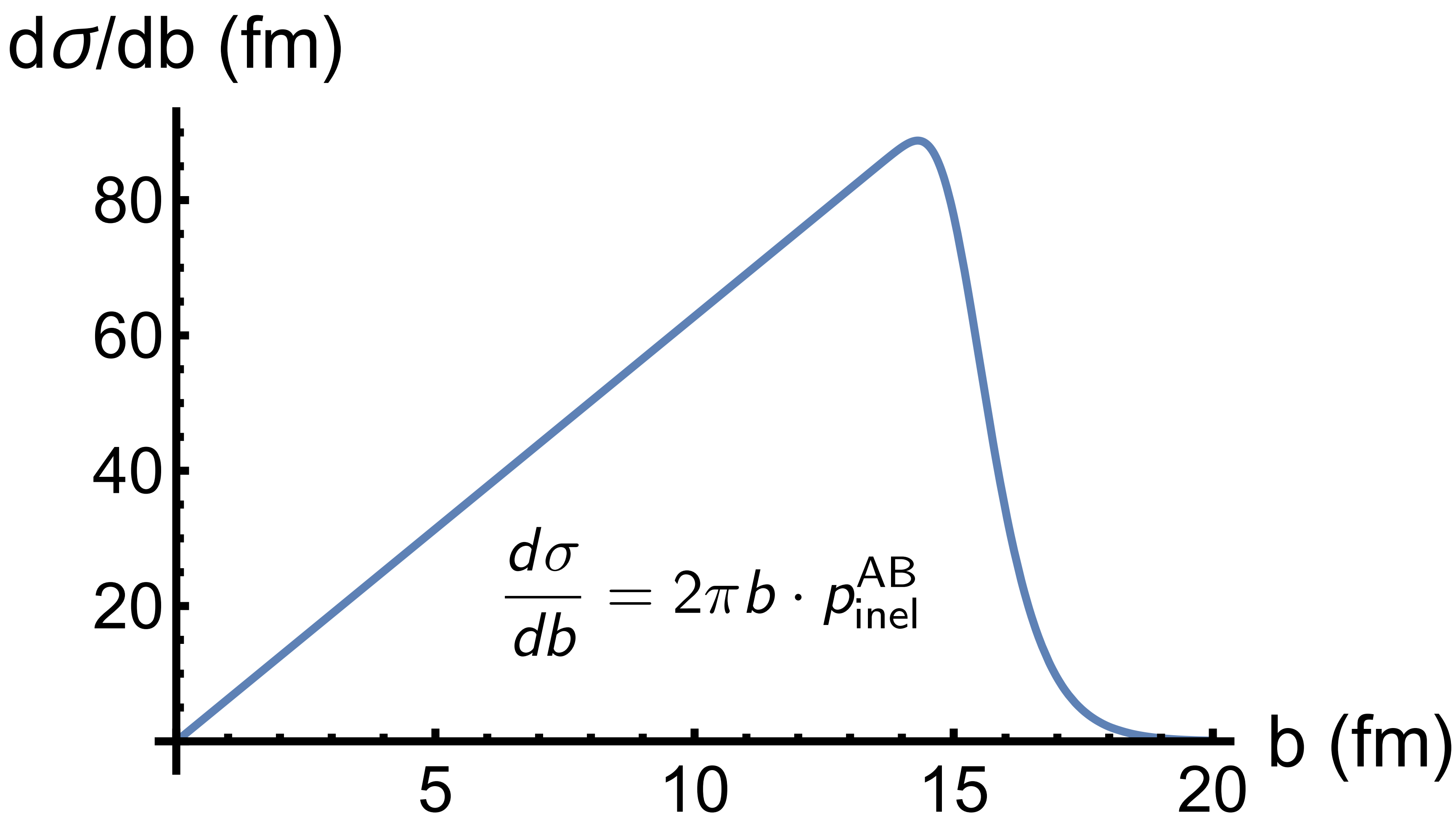
$$(1-x)^n = \exp(n\ln(1-x)) \overset{x\to 0}{\approx} \exp(-nx)$$

Probability for $k = 0$ is $(1 - p_\mathrm{NN})^{AB}$. Thus:

$$p_\mathrm{inel}^\mathrm{AB}(\vec{b}) = 1 - (1 - \hat{T}_\mathrm{AB}(\vec{b})\cdot\sigma_\mathrm{inel}^\mathrm{NN})^{AB} \approx 1 - \exp(-AB\,\hat{T}_\mathrm{AB}(\vec{b})\cdot\sigma_\mathrm{inel}^\mathrm{NN})$$

Poisson limit of the binomial distribution

# dσ/db for Pb-Pb




Total cross section: $$\sigma_{\text{inel}}^{\text{AB}} = \int_0^\infty \frac{d\sigma}{db}\, db \approx 784\ \text{fm}^2 = 7.84\ \text{b}$$

# Number of Participants

Probability that a test nucleon of nucleus A interacts with a certain nucleon of nucleus B:

$$p_{\mathrm{NN,A}}(\vec{s}) = \hat{T}_{\mathrm{B}}(\vec{s} - \vec{b}/2)\sigma_{\mathrm{inel}}^{\mathrm{NN}}$$

Probability that the test nucleon does not interact with any of the *B* nucleons of nucleus B:

$$(1 - p_{\mathrm{NN,A}}(\vec{s}))^{B}$$

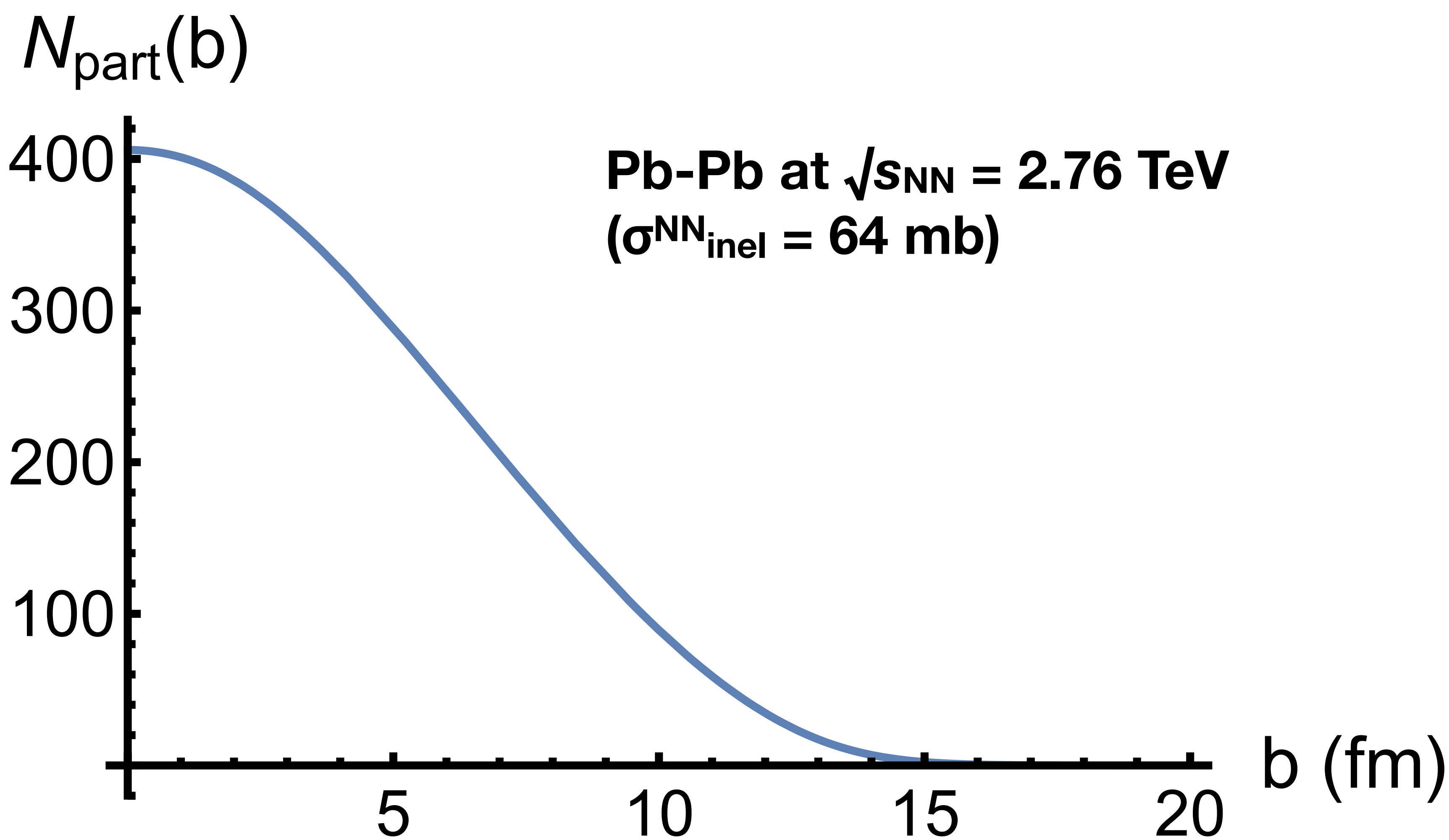
Probability that the test nucleon makes at least one interaction:

$$1 - (1 - p_{\mathrm{NN,A}}(\vec{s}))^{B} \approx 1 - \exp(-B p_{\mathrm{NN,A}}(\vec{s}))$$

Number of participants:

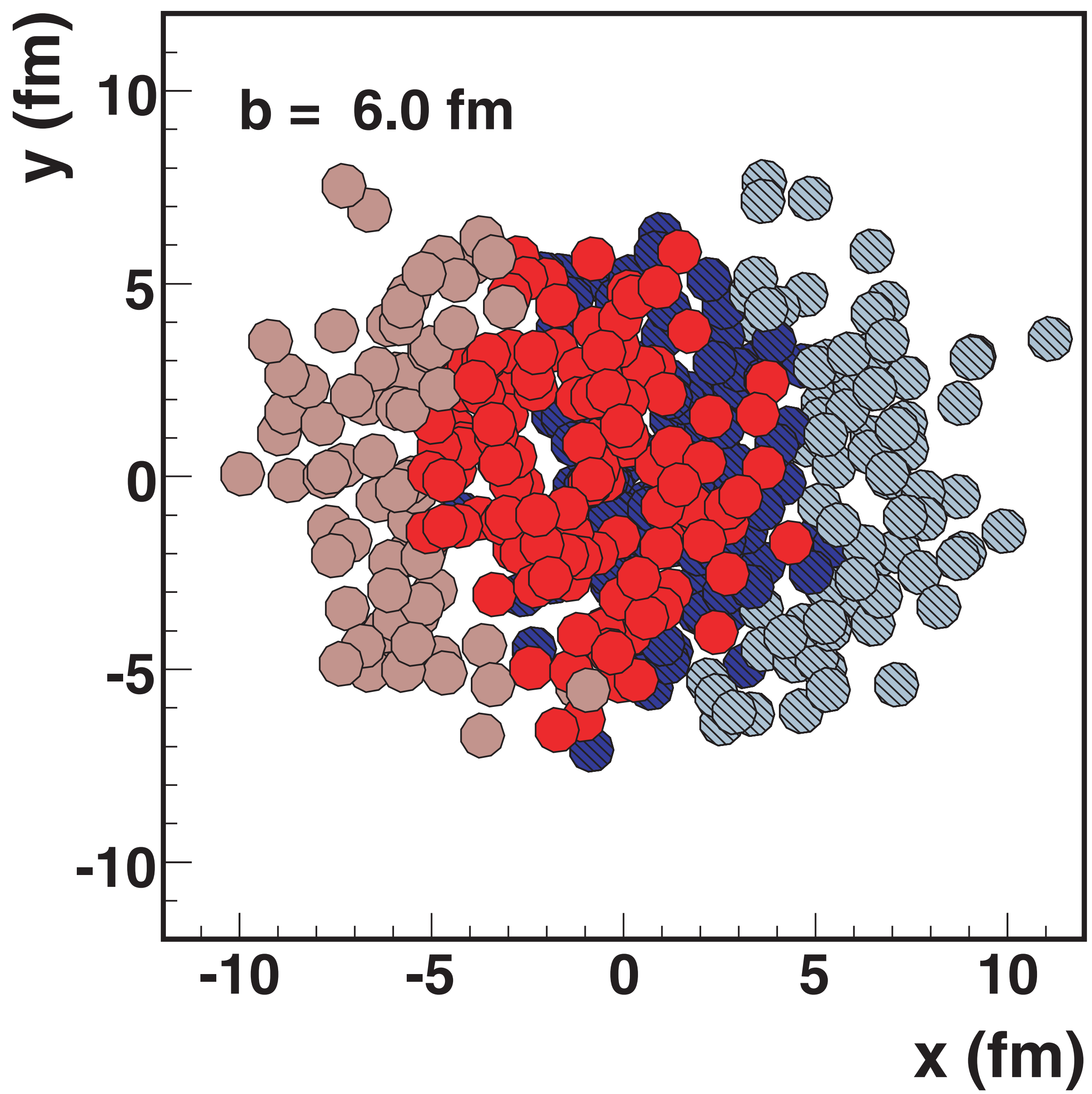
$$\begin{aligned} N_{\mathrm{part}}(\vec{b}) &= N_{\mathrm{part}}^{\mathrm{A}}(\vec{b}) + N_{\mathrm{part}}^{\mathrm{B}}(\vec{b}) \\ &= \int T_{\mathrm{A}}(\vec{s} + \vec{b}/2) \cdot \left[1 - \exp(-T_{\mathrm{B}}(\vec{s} - \vec{b}/2)\sigma_{\mathrm{inel}}^{\mathrm{NN}})\right] \mathrm{d}^2 s \\ &\quad + \int T_{\mathrm{B}}(\vec{s} - \vec{b}/2) \cdot \left[1 - \exp(-T_{\mathrm{A}}(\vec{s} + \vec{b}/2)\sigma_{\mathrm{inel}}^{\mathrm{NN}})\right] \mathrm{d}^2 s \end{aligned}$$

# $N_{part}$ vs Impact Parameter $b$



# Glauber Monte Carlo Approach

Ann.Rev.Nucl.Part.Sci. 57 (2007) 205-243



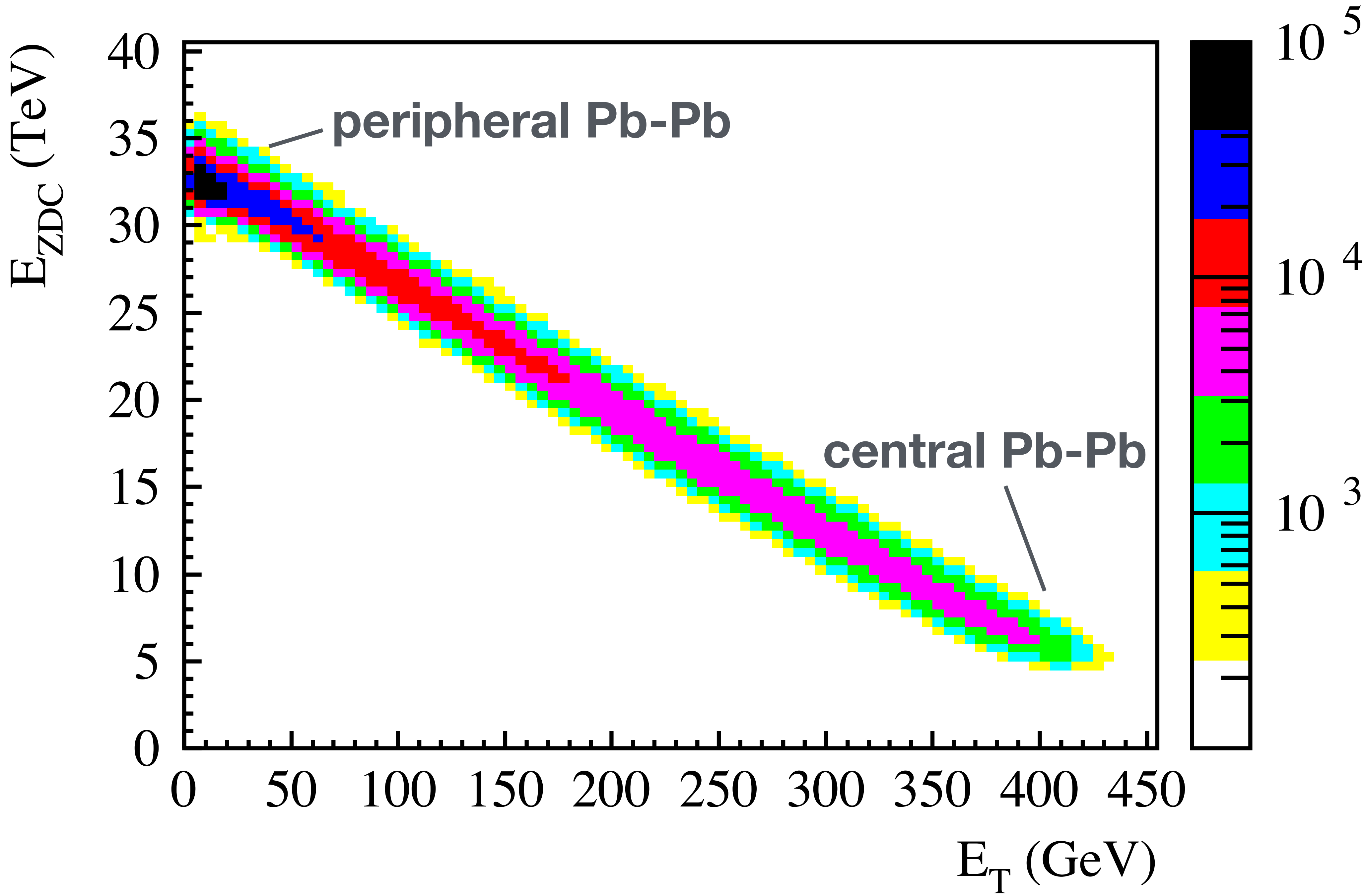

- Randomly select impact parameter $b$
- Distribute nucleons of two nuclei according to nuclear density distribution
- Consider all pairs with one nucleon from nucleus A and the other from B
- Count pair as inel. n-n collision if distance d in $x$-$y$ plane satisfies:

$$d < \sqrt{\sigma_{\text{inel}}^{\text{NN}} / \pi}$$

- Repeat many times: $\langle N_{\text{part}} \rangle(b)$ $\langle N_{\text{coll}} \rangle(b)$

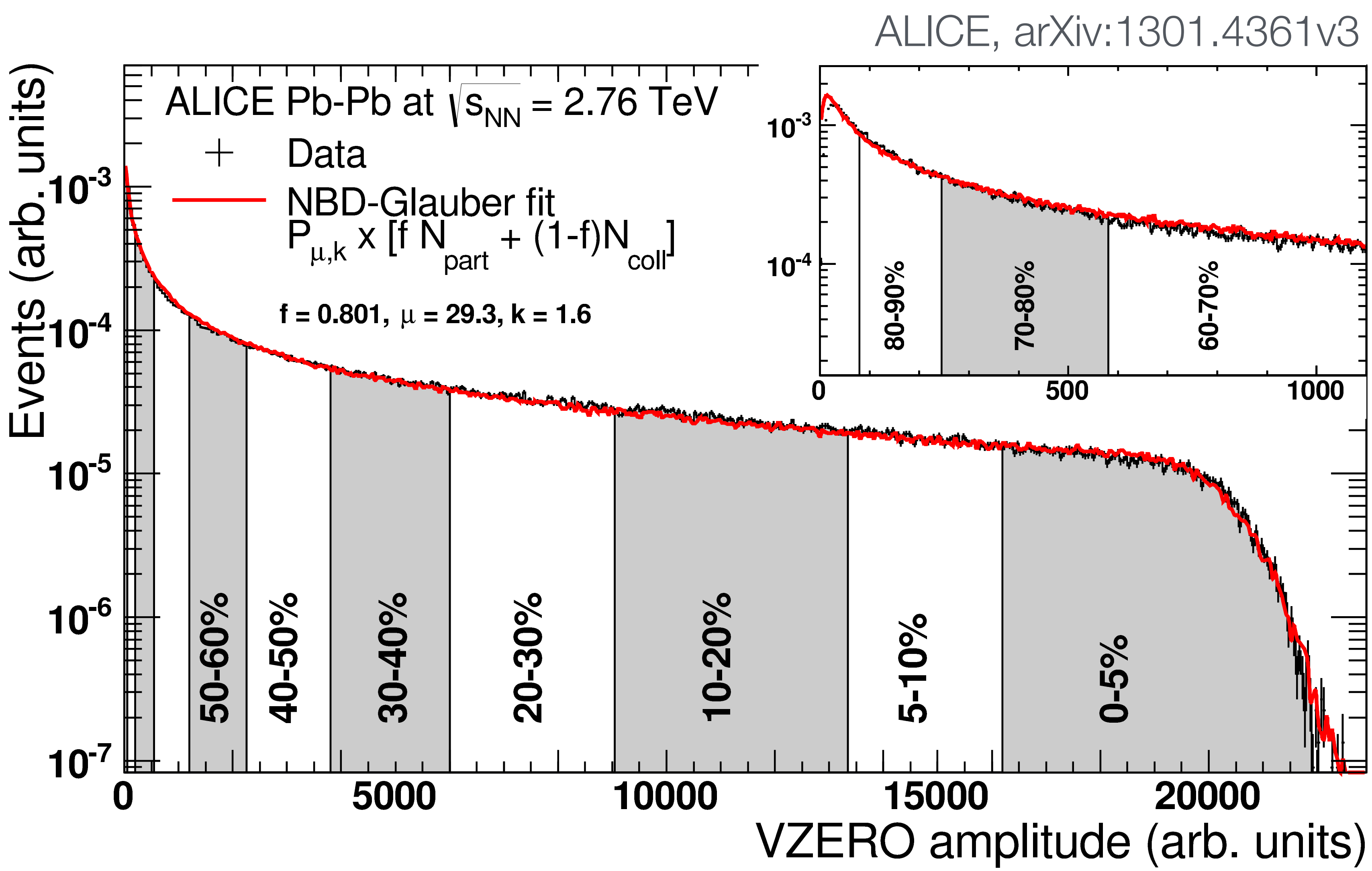
# Centrality selection: Forward and transverse energy

Example: Pb-Pb, fixed-target experiment (WA98, CERN SPS)




Both $E_T$ and $E_{ZDC}$ can be used to define centrality classes

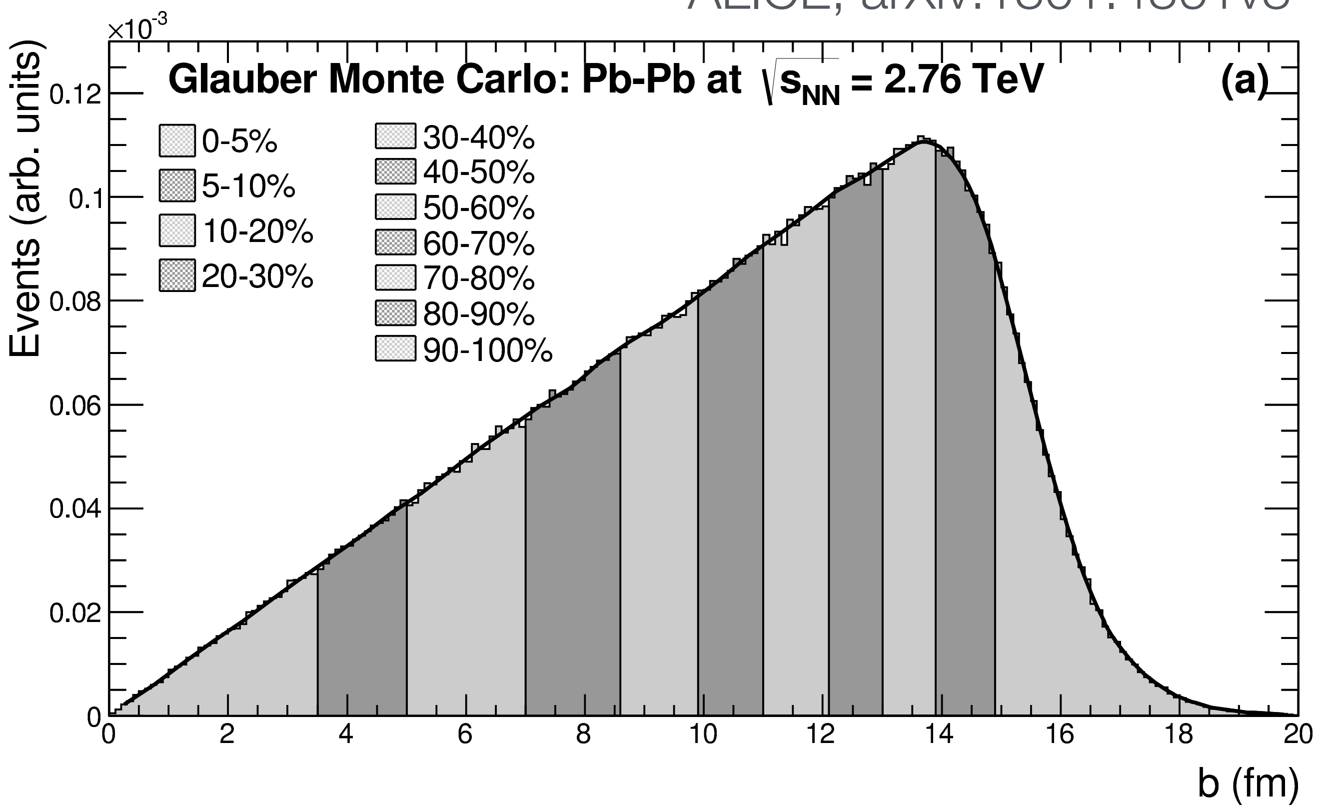
# Centrality Selection: Charged-Particle Multiplicity




- Measure charged particle multiplicity
  - ALICE: VZERO detectors ($2.8 < \eta < 5.1$ and $-3.7 < \eta < -1.7$)
  - Assumption: $\langle N_{ch} \rangle(b)$ increases monotonically with decreasing $b$
- Define centrality class by selecting a percentile of the measured multiplicity distribution (e.g. 0-5%)
  - Need Glauber fit to define "100%" (background at low multiplicities)

# How $\langle N_{part} \rangle$, $\langle N_{coll} \rangle$, and $\langle b \rangle$ are Assigned to an Experimental Centrality Class?

ALICE, arXiv:1301.4361v3




- Glauber Monte Carlo
  - Find impact parameter interval $[b_1, b_2]$ which corresponds to the same percentile
  - Average $N_{part}(b)$, $N_{coll}(b)$, etc over this interval
- Example: Pb-Pb at $\sqrt{s_{NN}}$ = 2.76 TeV
  - $\sigma_{NN}$(inel) = (64 ± 5) mb

| Centrality | $b_{min}$ (fm) | $b_{max}$ (fm) | $\langle N_{part} \rangle$ | RMS | (*sys.*) | $\langle N_{coll} \rangle$ | RMS | (*sys.*) | $\langle T_{AA} \rangle$ 1/mbarn | RMS 1/mbarn | (*sys.*) 1/mbarn |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0–5% | 0.00 | 3.50 | 382.7 | 17 | 3.0 | 1685 | 140 | 190 | 26.32 | 2.2 | 0.85 |
| 5–10% | 3.50 | 4.94 | 329.4 | 18 | 4.3 | 1316 | 110 | 140 | 20.56 | 1.7 | 0.67 |
| 10–20% | 4.94 | 6.98 | 260.1 | 27 | 3.8 | 921.2 | 140 | 96 | 14.39 | 2.2 | 0.45 |
| 20–40% | 6.98 | 9.88 | 157.2 | 35 | 3.1 | 438.4 | 150 | 42 | 6.850 | 2.3 | 0.23 |
| 40–60% | 9.88 | 12.09 | 68.56 | 22 | 2.0 | 127.7 | 59 | 11 | 1.996 | 0.92 | 0.097 |
| 60–80% | 12.09 | 13.97 | 22.52 | 12 | 0.77 | 26.71 | 18 | 2.0 | 0.4174 | 0.29 | 0.026 |
| 80–100% | 13.97 | 20.00 | 5.604 | 4.2 | 0.14 | 4.441 | 4.4 | 0.21 | 0.06939 | 0.068 | 0.0055 |