Quark-Gluon Plasma Physics

7. Hanbury Brown–Twiss correlations

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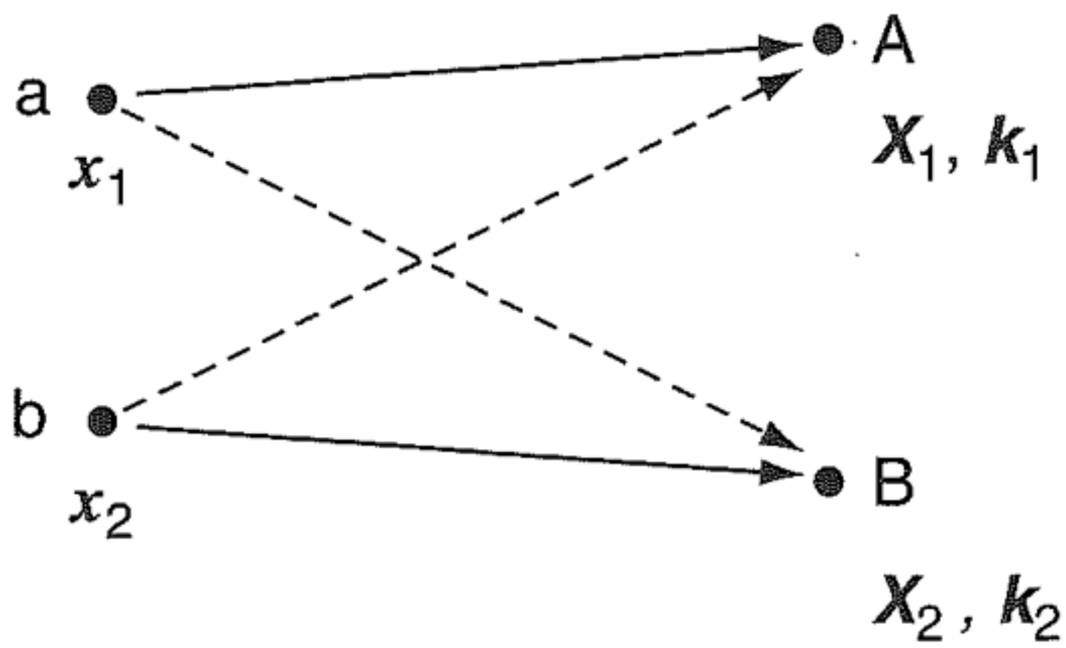
SS 2017

Momentum correlation of identical bosons emitted from two point sources



$|\vec{x}\rangle$: Single-particle state with exactly defined position (in position space)

Representation in momentum space:

$$\psi(\vec{k}) = \langle \vec{k} | \vec{x} \rangle = \frac{1}{\sqrt{V}} e^{-i\vec{k}\vec{x}}$$

Momentum undefined in this case:

$$P(\vec{k}) = |\psi(\vec{k})|^2 = \frac{1}{V} = \text{const.}$$

Two-particle wave function in momentum representation:

symmetrization of the two-particle wave function, bosons: "+"

$$\psi(\vec{k}_1, \vec{k}_2) = \langle \vec{k}_1, \vec{k}_2 | \vec{x}_1, \vec{x}_2 \rangle = \frac{1}{\sqrt{2}V} \left[e^{-i\vec{k}_1\vec{x}_1} e^{-i\vec{k}_2\vec{x}_2} + e^{-i\vec{k}_1\vec{x}_2} e^{-i\vec{k}_2\vec{x}_1} \right]$$

$$P(\vec{k}_1, \vec{k}_2) = |\psi(\vec{k}_1, \vec{k}_2)|^2 = \frac{1}{V^2}(1 + \cos(\Delta\vec{k} \cdot \Delta\vec{x}))$$

$\Delta\vec{k} \approx 0$ enhanced due to Bose-Einstein statistics

$$\Delta\vec{k} = \vec{k}_1 - \vec{k}_2, \;\; \Delta\vec{x} = \vec{x}_1 - \vec{x}_2$$

Spatially extended static particle source

Extended distribution $\rho(\vec{x})$ of incoherent particle sources:

Single particle: $$P(\vec{k}) = \int \mathrm{d}^3x \, \rho(\vec{x}) |\psi(\vec{k})|^2$$

Two particles: $$P(\vec{k}_1, \vec{k}_2) = \frac{1}{2} \int \mathrm{d}^3x_1 \mathrm{d}^3x_2 \, \rho(\vec{x}_1)\rho(\vec{x}_2)|\psi(\vec{k}_1, \vec{k}_2)|^2$$

Two-particle correlation function:

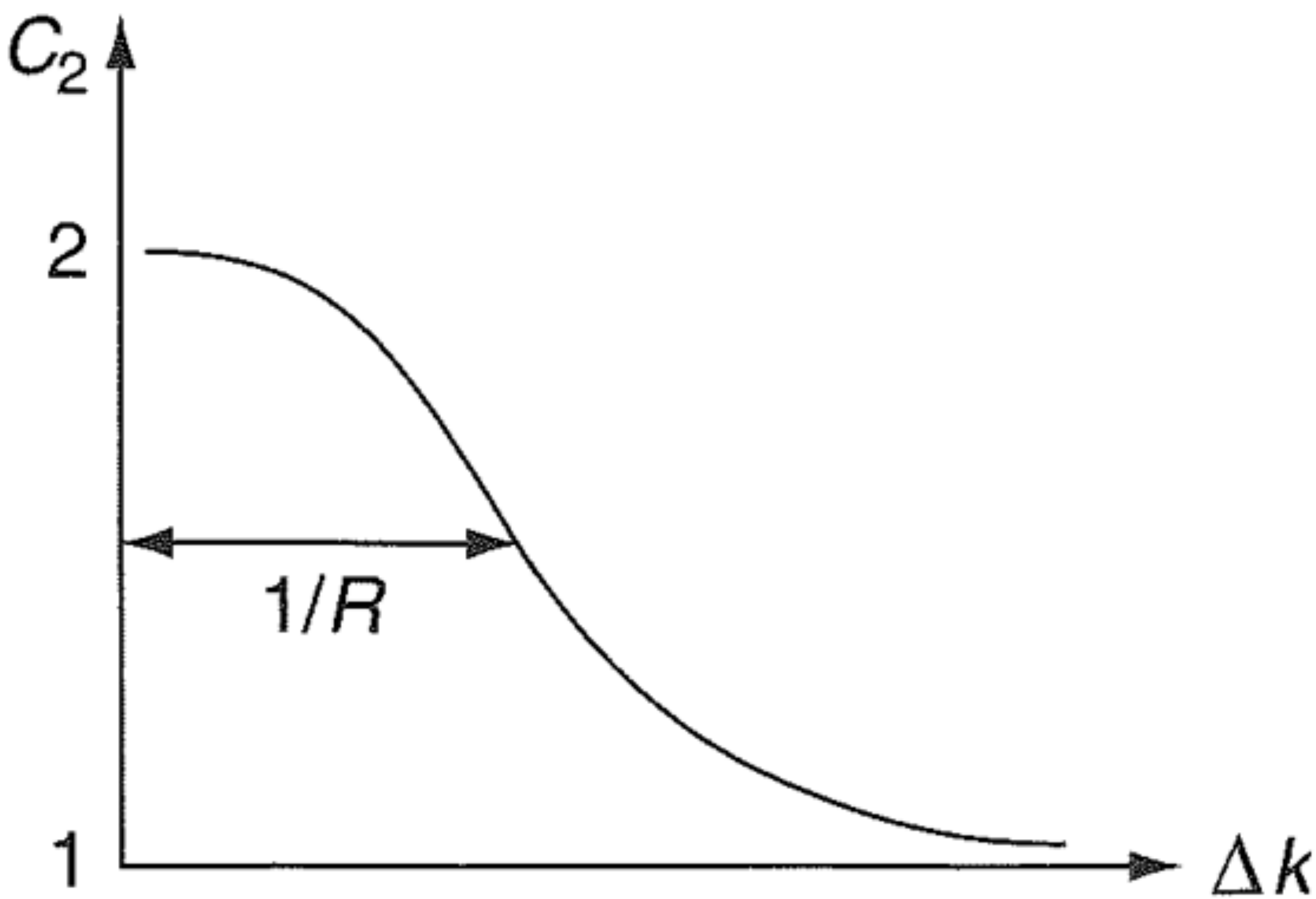
$$C_2 = \frac{\mathrm{d}^6 N/(\mathrm{d}^3 k_1 \mathrm{d}^3 k_2)}{(\mathrm{d}^3 N/\mathrm{d}^3 k_1)(\mathrm{d}^3 N/\mathrm{d}^3 k_2)} = \frac{2P(\vec{k}_1, \vec{k}_2)}{P(\vec{k}_1)P(\vec{k}_2)} = 1 + |\tilde{\rho}(\Delta \vec{k})|^2$$

Fourier transform of $\rho(\vec{x})$ with normalization $\tilde{\rho}(0) = 1$

Gaussian source:

$$\rho(\vec{x}) \propto \exp\left(-\frac{1}{2}\vec{x}^2/R^2\right) \quad \rightarrow \quad C_2 = 1 + \lambda \exp\left(-\Delta \vec{k}^2 \cdot R^2\right)$$

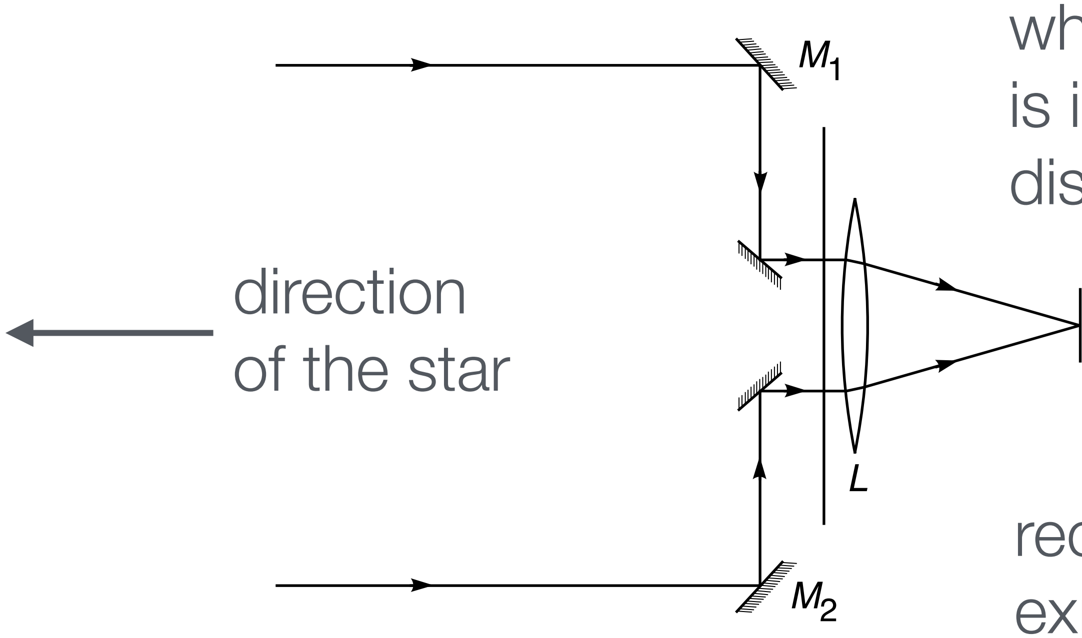
Width of the correlation function is a measures the source size



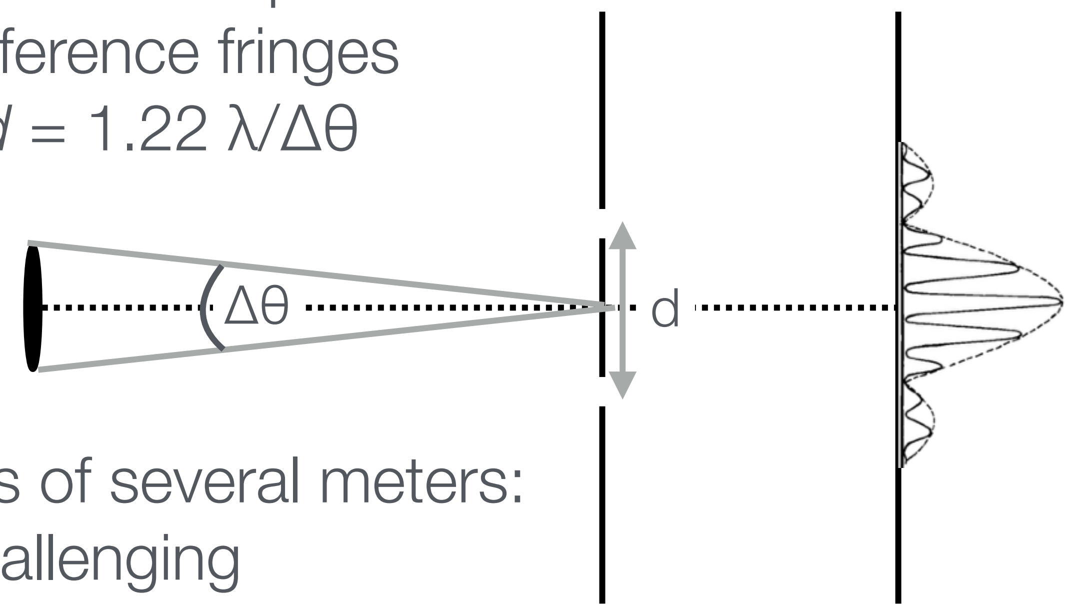
heavy ion collisions: typical dimensions 1–10 fm
→ interference at momentum differences of 20–200 MeV/c

Where the name comes from: Stellar intensity interferometry

Michelson stellar interferometry (measures spatial coherence of star light):

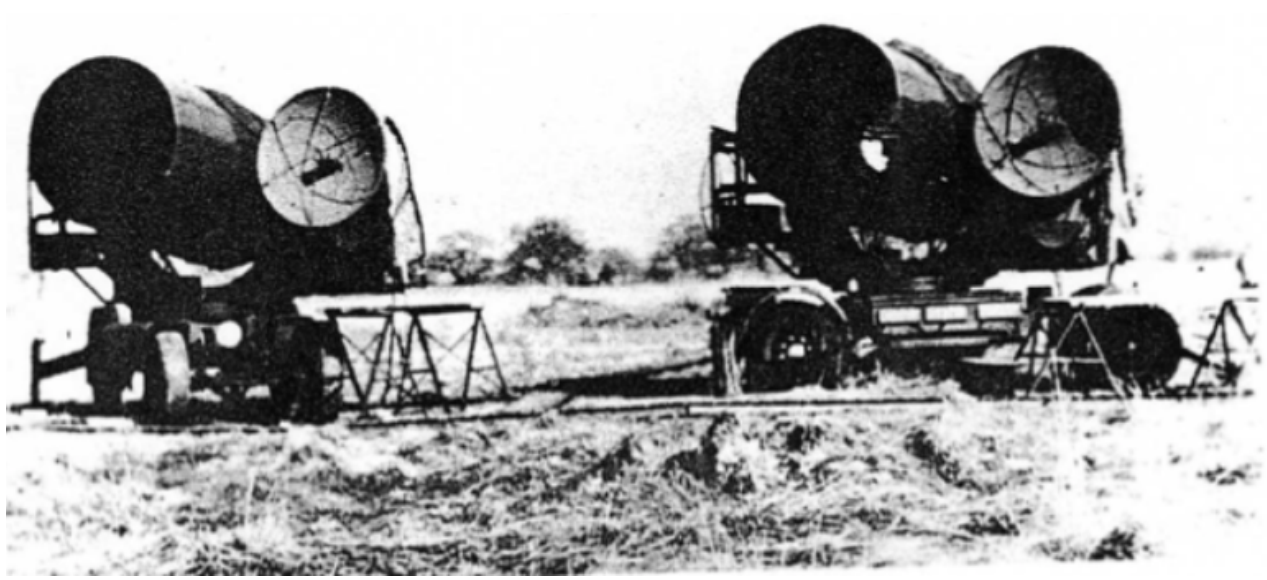


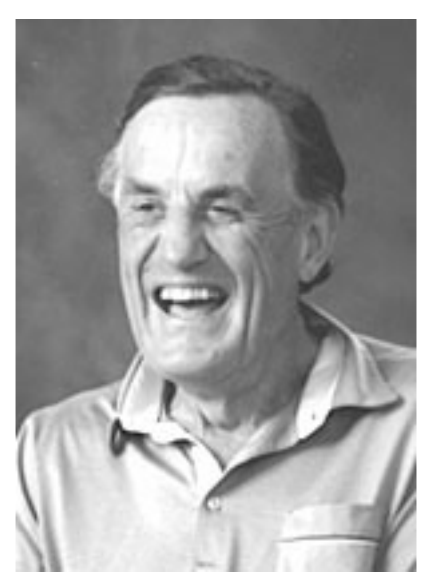
when the distance *d* of the pinholes is increased, interference fringes disappear when $d = 1.22\ \lambda/\Delta\theta$



requires distances of several meters: experimentally challenging

Robert Hanbury Brown conceived a method based on the correlations of intensity fluctuations (less sensitive to vibrations and atmospheric fluctuations):

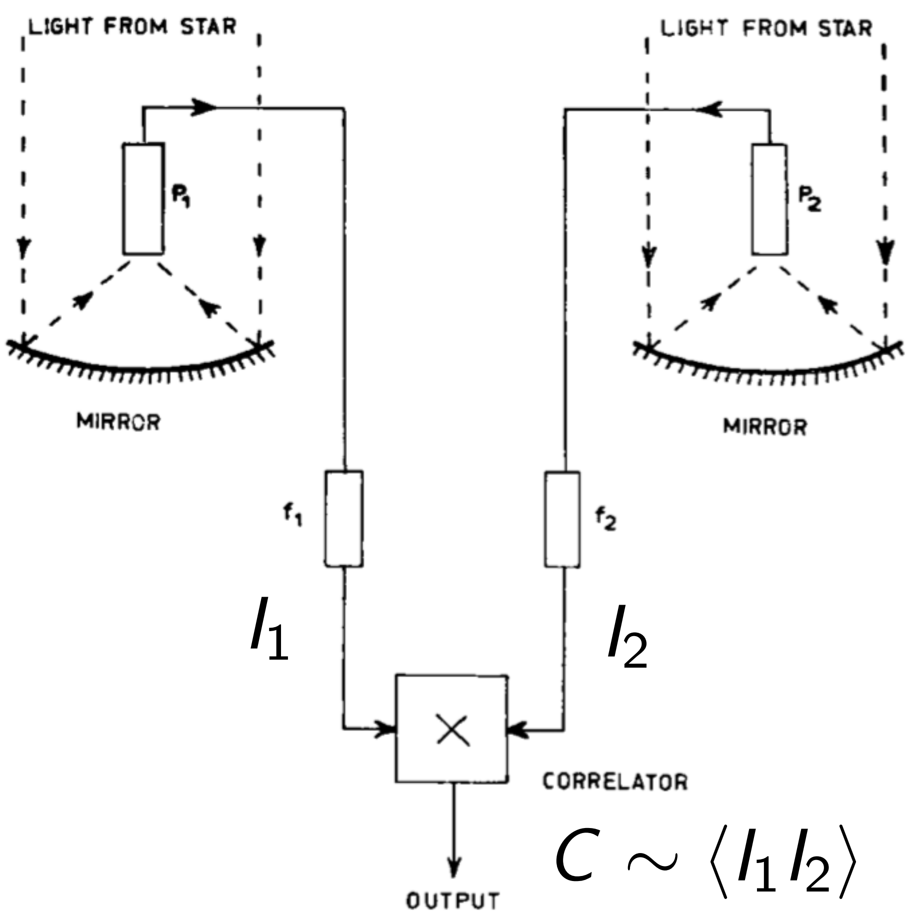


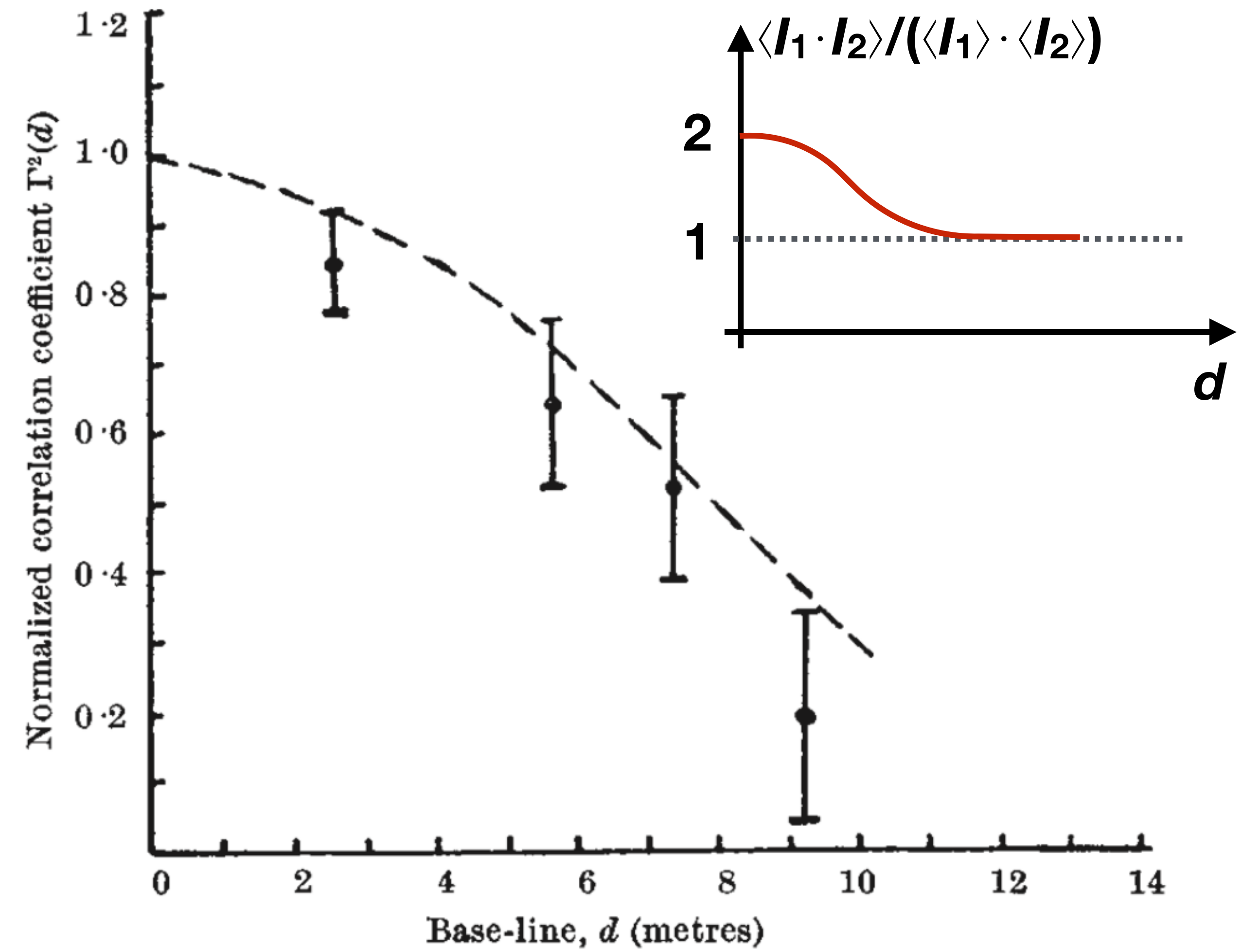


Robert Hanbury Brown
1916–2002
(link)

"As an engineer my education in physics had stopped far short of the quantum theory. Perhaps just as well ... ignorance is sometimes a bliss in science"

Angular diameter of Sirius from HBT Correlations





Nature, Nov. 10, 1956, Vol. 178

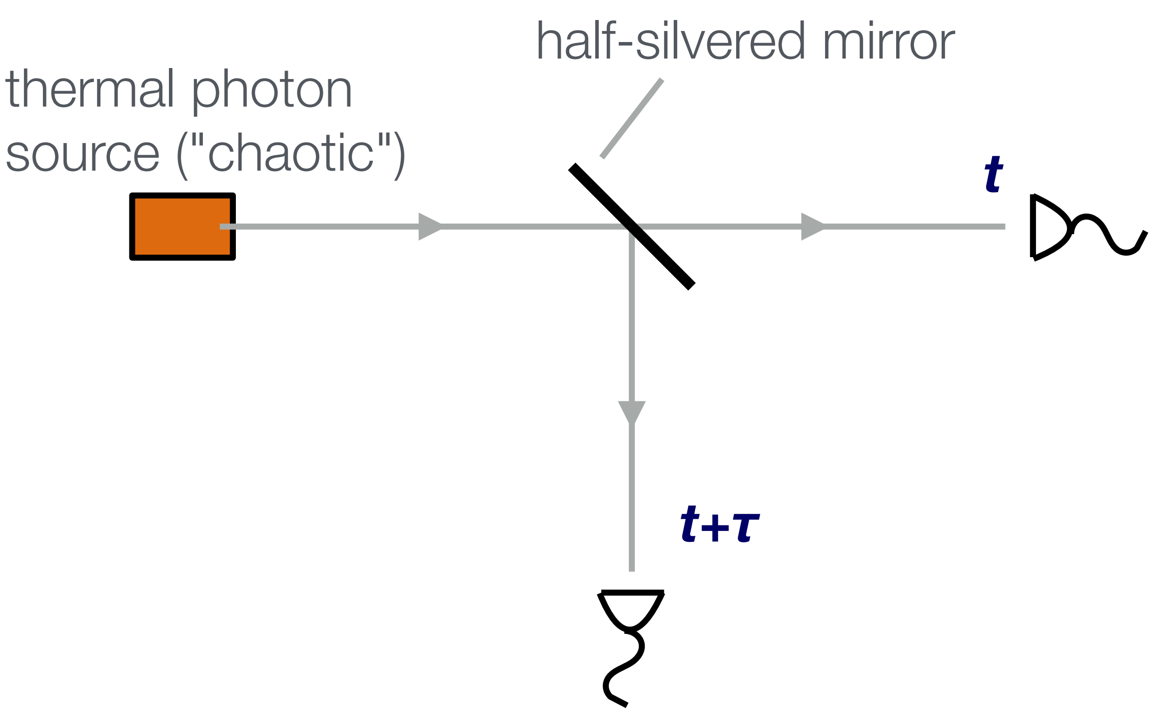
A TEST OF A NEW TYPE OF STELLAR INTERFEROMETER ON SIRIUS

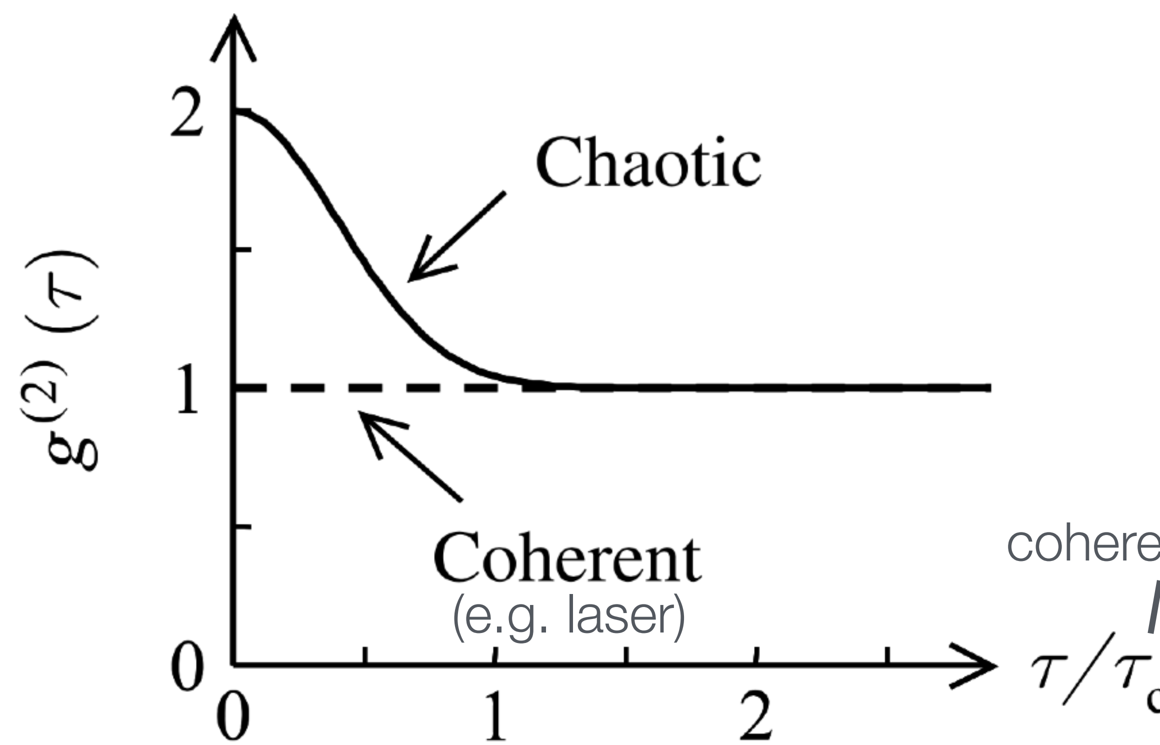
By R. HANBURY BROWN
Jodrell Bank Experimental Station, University of Manchester
AND
Dr. R. Q. TWISS
Services Electronics Research Laboratory, Baldock

Angular diameter of Sirius from intensity interferometry: $3.1 \cdot 10^{-8}$ rad

Hanbury Brown and Twiss tested their technique in the laboratory

$$g^{(2)}(\tau) = \frac{\langle I(t) I(t+\tau) \rangle}{\langle I(t) \rangle \langle I(t+\tau) \rangle}$$





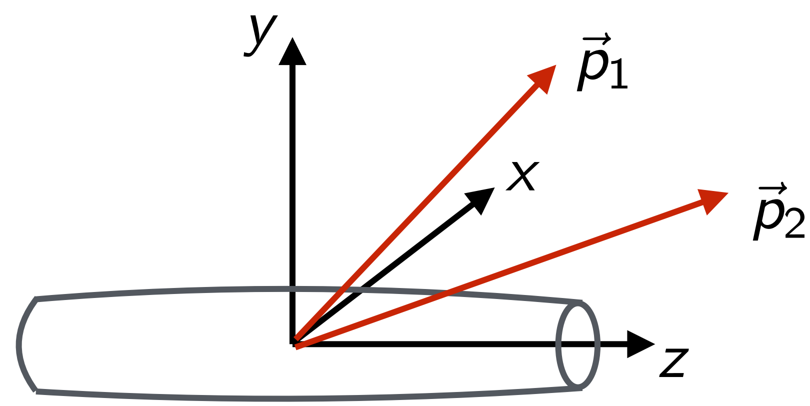
P. Dirac (1958): "Each photon interferes only with itself; interference between different photons never occurs"
→ No! Applies to conventional interference experiments, but not to HBT

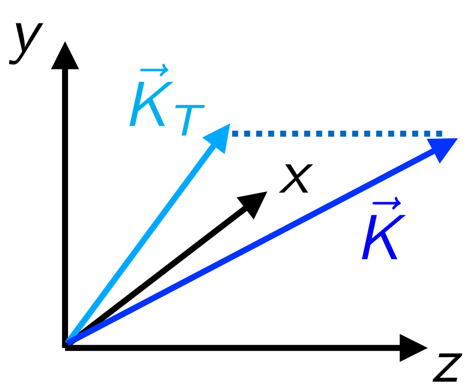
Hanbury Brown-Twiss experiment: milestone for the field of "quantum optics"

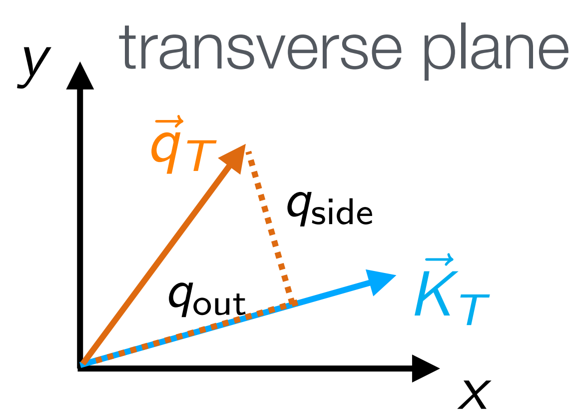
Back to heavy ions: Bertsch-Pratt variables: q_{out}, q_{side}, q_{long}

$C_2(\vec{q}, \vec{K})$ $\qquad \vec{q} = \vec{p}_1 - \vec{p}_2 \qquad \vec{K} = \dfrac{\vec{p}_1 + \vec{p}_2}{2}$

projection onto the transverse plane: $\vec{q}_T, \ \vec{K}_T$





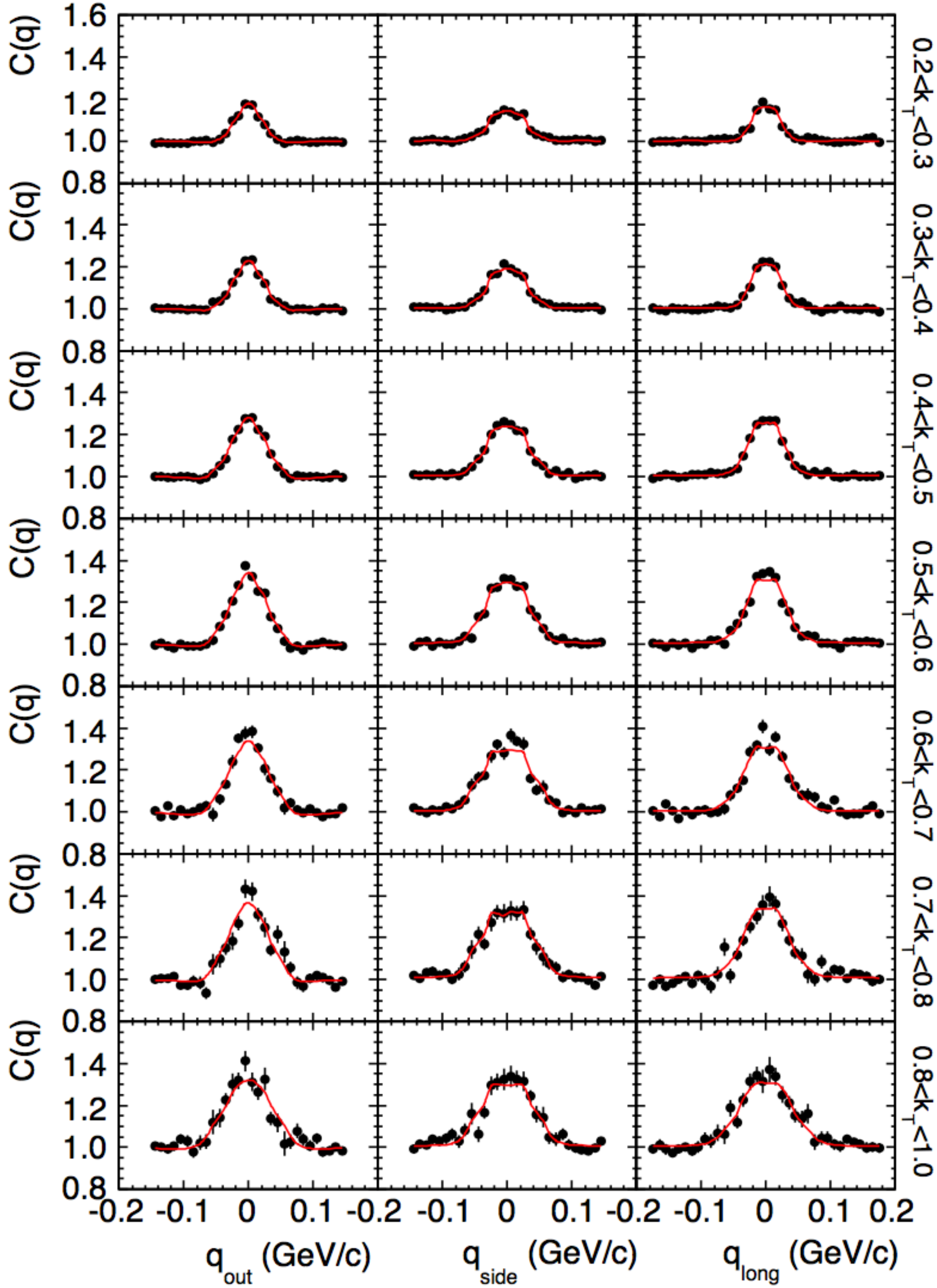


$$\vec{e}_{\mathrm{out}} = \frac{\vec{K}}{|\vec{K}|}$$

$$\vec{e}_{\mathrm{side}} = \vec{e}_{\mathrm{out}} \times \vec{e}_z$$

$q_{\mathrm{side}} = \vec{q} \cdot e_{\mathrm{side}} \qquad q_{\mathrm{out}} = \vec{q} \cdot \vec{e}_{\mathrm{out}} \qquad q_{\mathrm{long}} = \vec{q} \cdot \vec{e}_z \quad \rightsquigarrow \quad C_2(q_{\mathrm{out}}, q_{\mathrm{side}}, q_{\mathrm{long}})$

G. Bertsch, Phys. Rev. C37 (1988) 1896



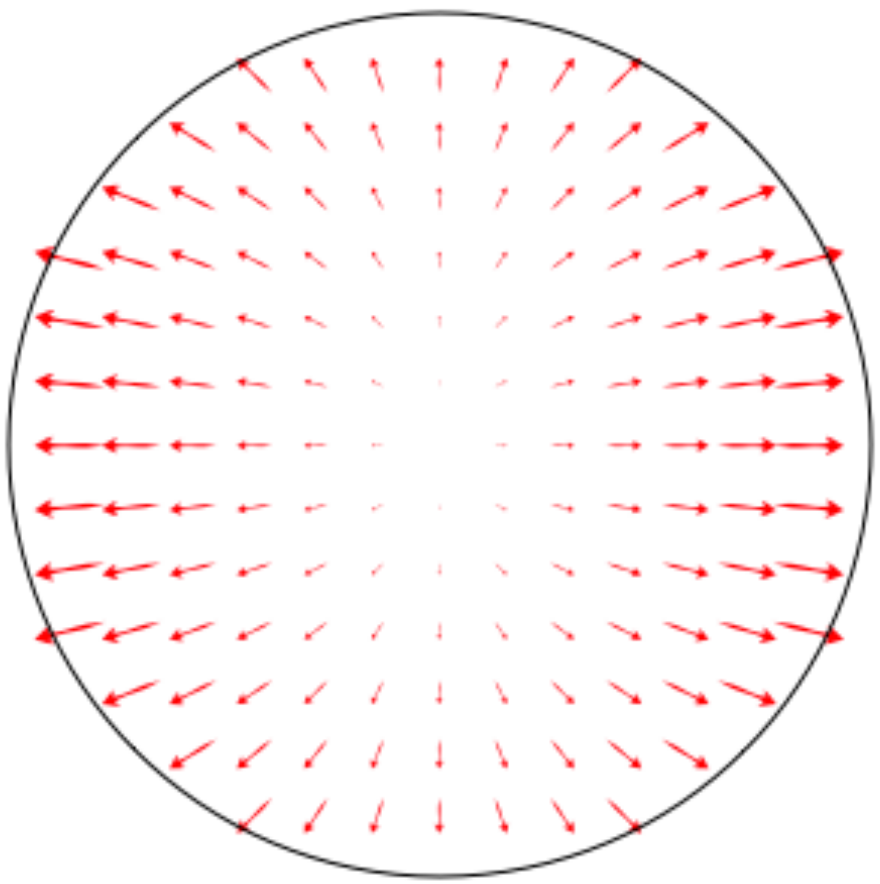
Two-Pion Bose-Einstein correlations

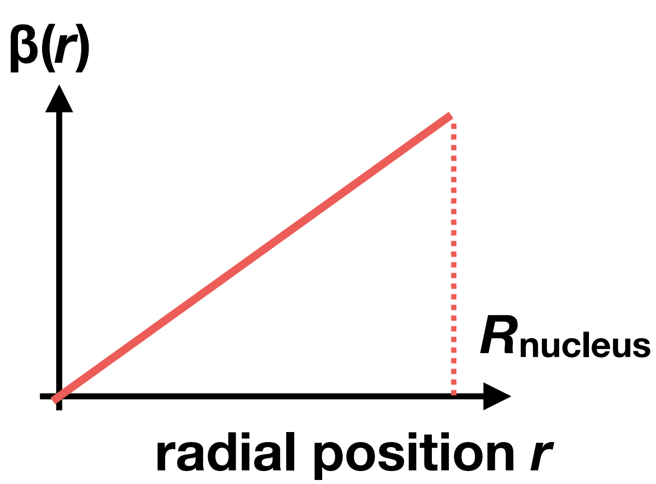
$\pi^-\pi^-$ correlation function in central Pb-Pb collisions at $\sqrt{s_{NN}}$ = 2.76 TeV

projection on q_a (a = out, side, long) axis was done for -30 MeV $< q_b, q_c <$ 30 MeV

characteristic width: 30–40 MeV/c

Effect of collective expansion: Apparent reduction of the source size





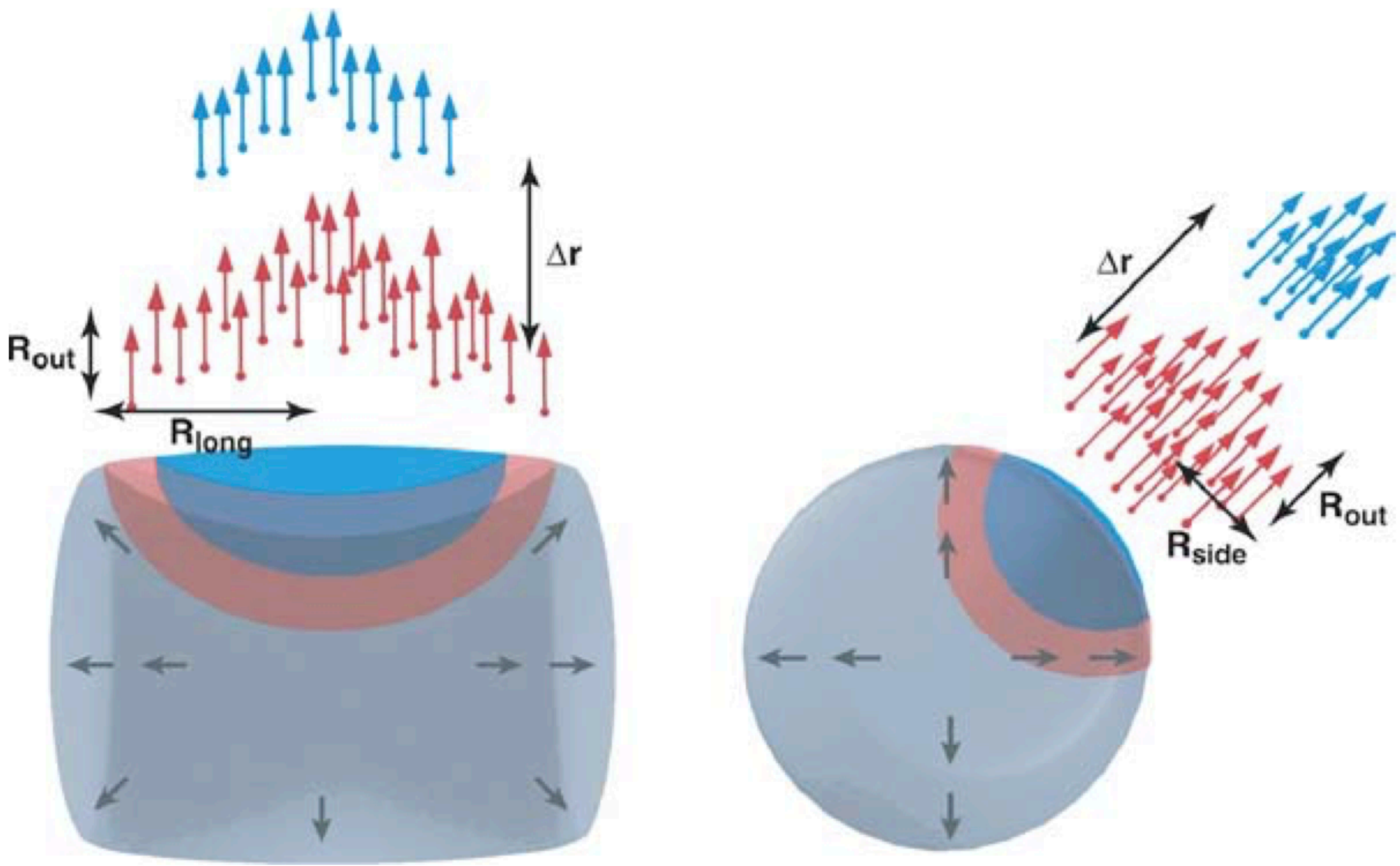
Only particles emitted from nearby space points have similar momenta, i.e., small momentum differences

Space-momentum correlations from collective radial expansion lead to an apparent reduction of the source size

static source:
no dependence on pair momentum $\vec{K}$

expanding source:
HBT radii depend on $\vec{K}$

Gaussian HBT radii R_{out}, R_{side}, R_{long}



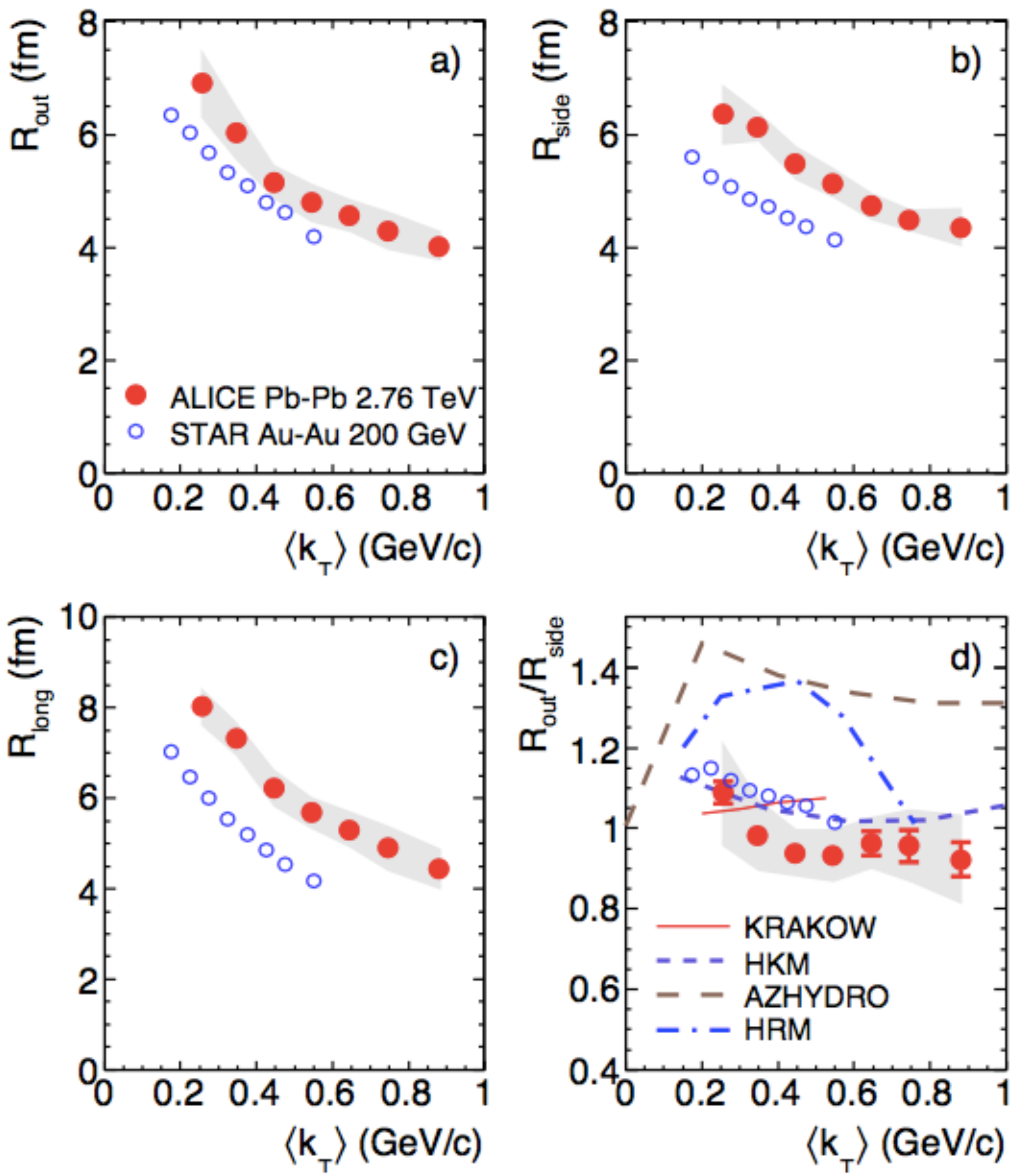
correction for Coulomb interaction

$$C(\vec{q}) = N[(1-\lambda) + \lambda K(q_{inv})(1 + G(\vec{q}))]$$

$$G(\vec{q}) = \exp[-(R_{out}^2 q_{out}^2 + R_{side}^2 q_{side}^2 + R_{long}^2 q_{long}^2)]$$

k_T dependence of HBT radii: signature of radial flow

0-5% most central Pb-Pb



ALICE, arXiv:1012.4035

Parameterization inspired by blast wave model

$$R_{\rm side} \approx R_{\rm out} \approx \frac{R_{\rm geom}}{\sqrt{1 + m_T \beta_{\rm surf}^2 / T}}$$

PHENIX, arXiv:nucl-ex/0201008v3

HBT radii larger at the LHC: Effect of stronger radial flow?

In some models prolonged lifetime of the source leads to $R_{\rm out}/R_{\rm side} > 1$

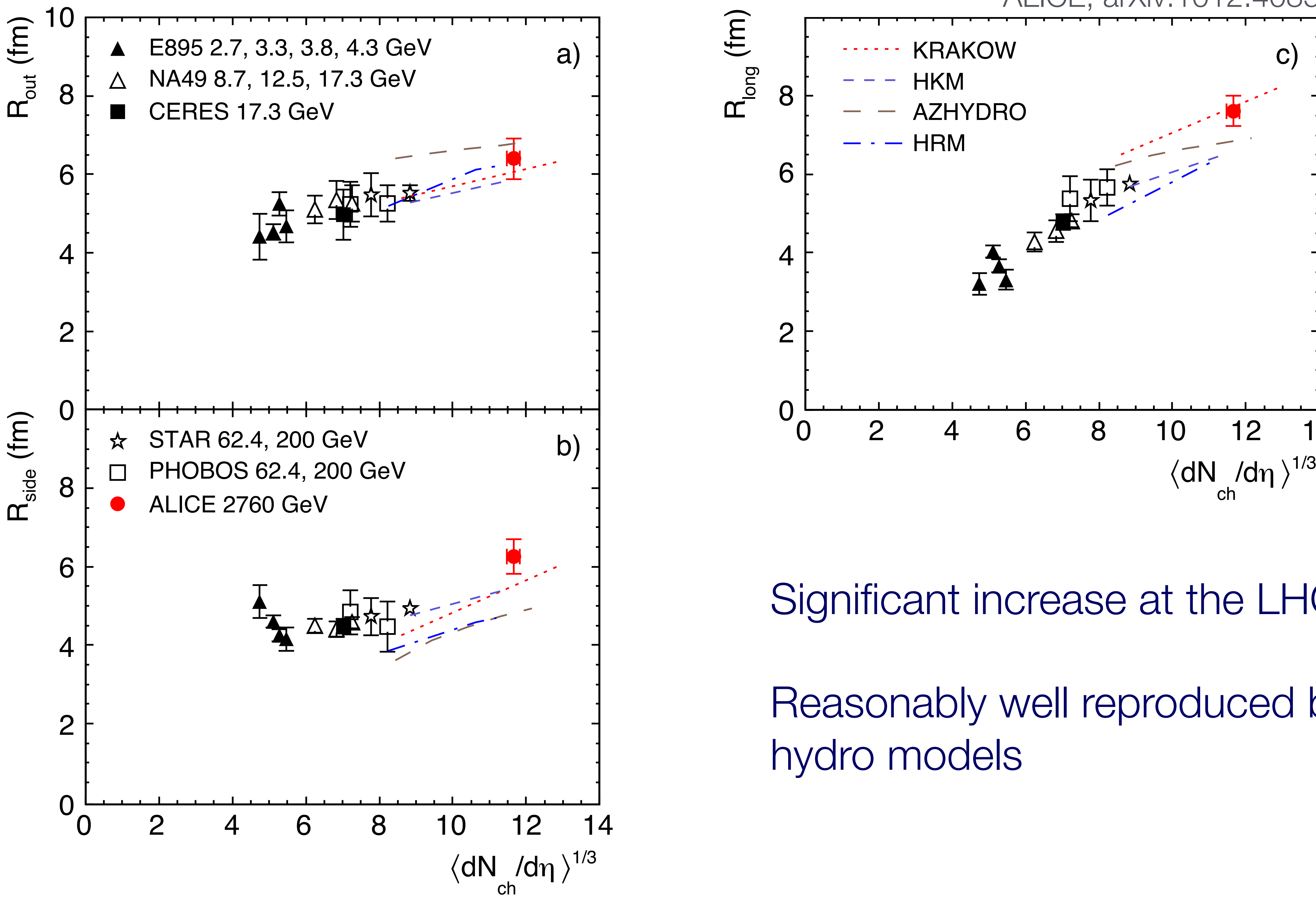
Not seen in data

HBT radii larger than 1d rms radii of the nuclei:

$\sqrt{\langle r_{\rm Pb}^2 \rangle}/\sqrt{3} \approx 3.2\,\mathrm{fm}$ $\sqrt{\langle r_{\rm Au}^2 \rangle}/\sqrt{3} \approx 3\,\mathrm{fm}$

Energy dependence of R_{out}, R_{side}, and R_{long}

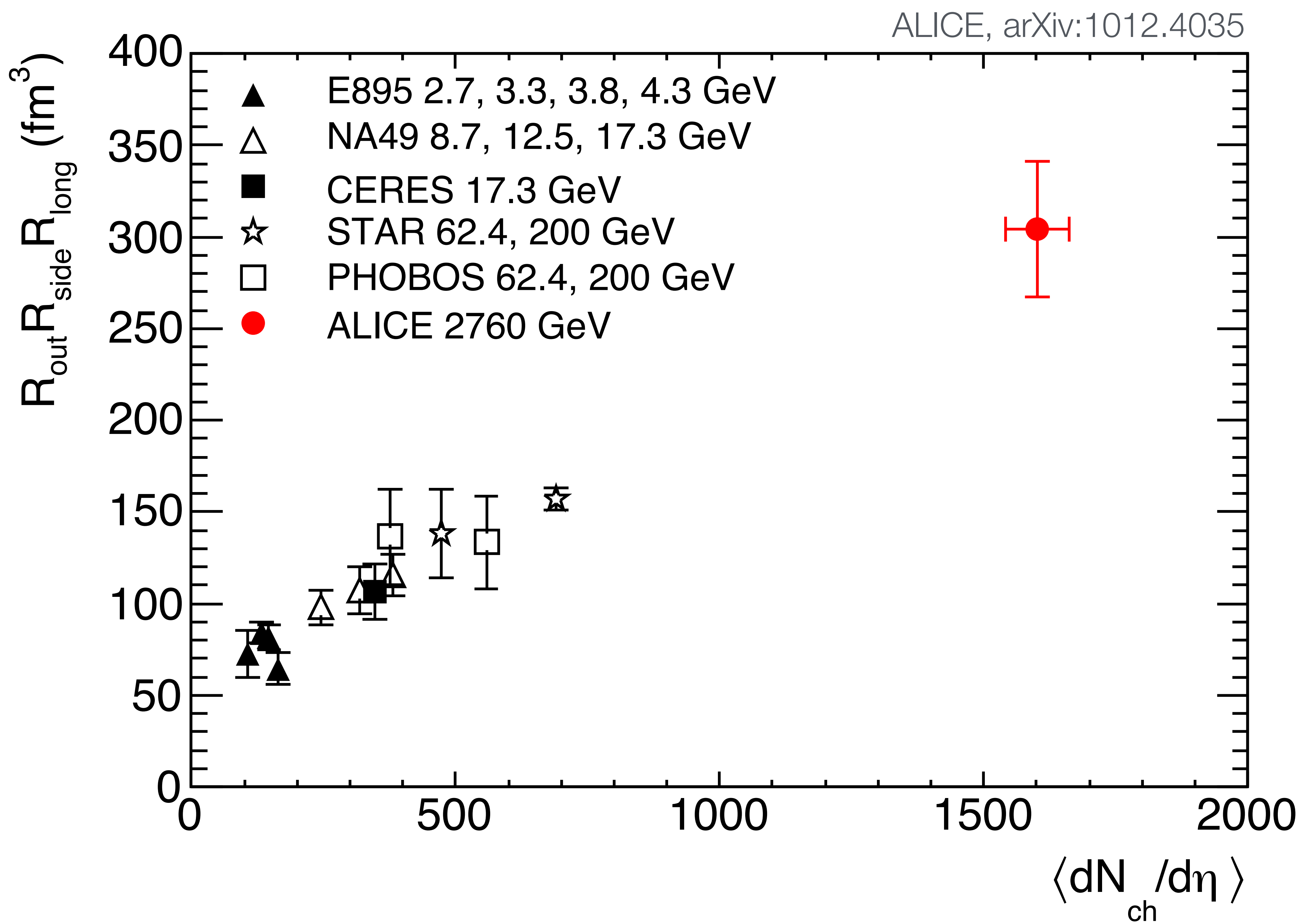
ALICE, arXiv:1012.4035



Significant increase at the LHC

Reasonably well reproduced by hydro models

Energy dependence of $R_{out} \times R_{side} \times R_{long}$



Freeze-out volume appears to scale linearly with $dN_{ch}/d\eta$

Factor 2 increase from RHIC to LHC: same factor as for $dN_{ch}/d\eta$

R_{long}: longitudinal expansion of the fireball

Bjorken expansion: $v_z = z/t$

R_{long} determined by the distance one can move before the collective velocity overwhelms the thermal velocity:

$$R_{\text{long}} \approx \frac{v_{\text{therm}}}{\mathrm{d}v_z/\mathrm{d}z}$$

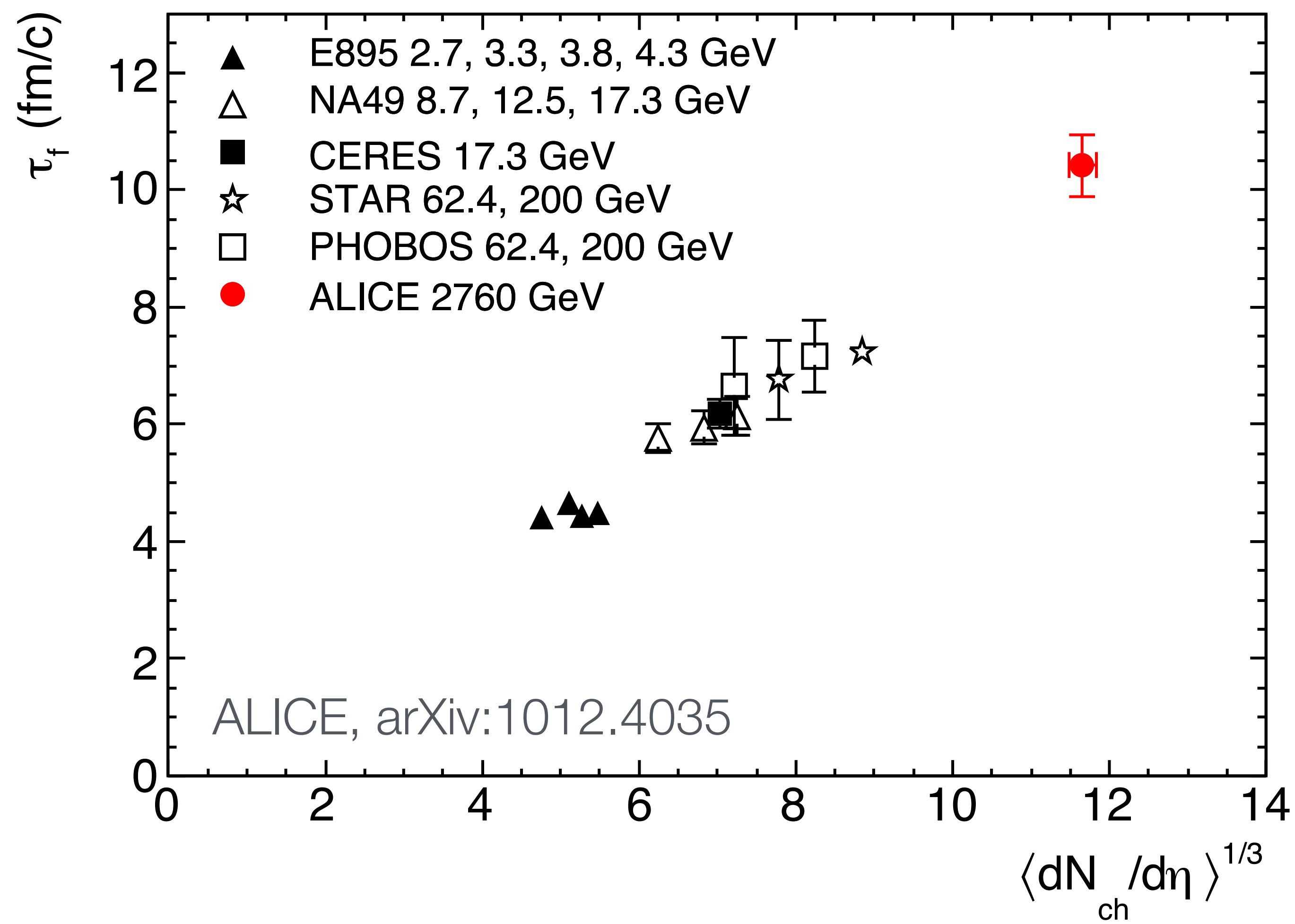
Thermal velocity (non relativistic):

$$v_{\text{therm}} = \sqrt{T/m_{\text{T}}}$$

This gives:

$$R_{\text{long}} \approx \tau_f \sqrt{T/m_T}$$

duration of emission



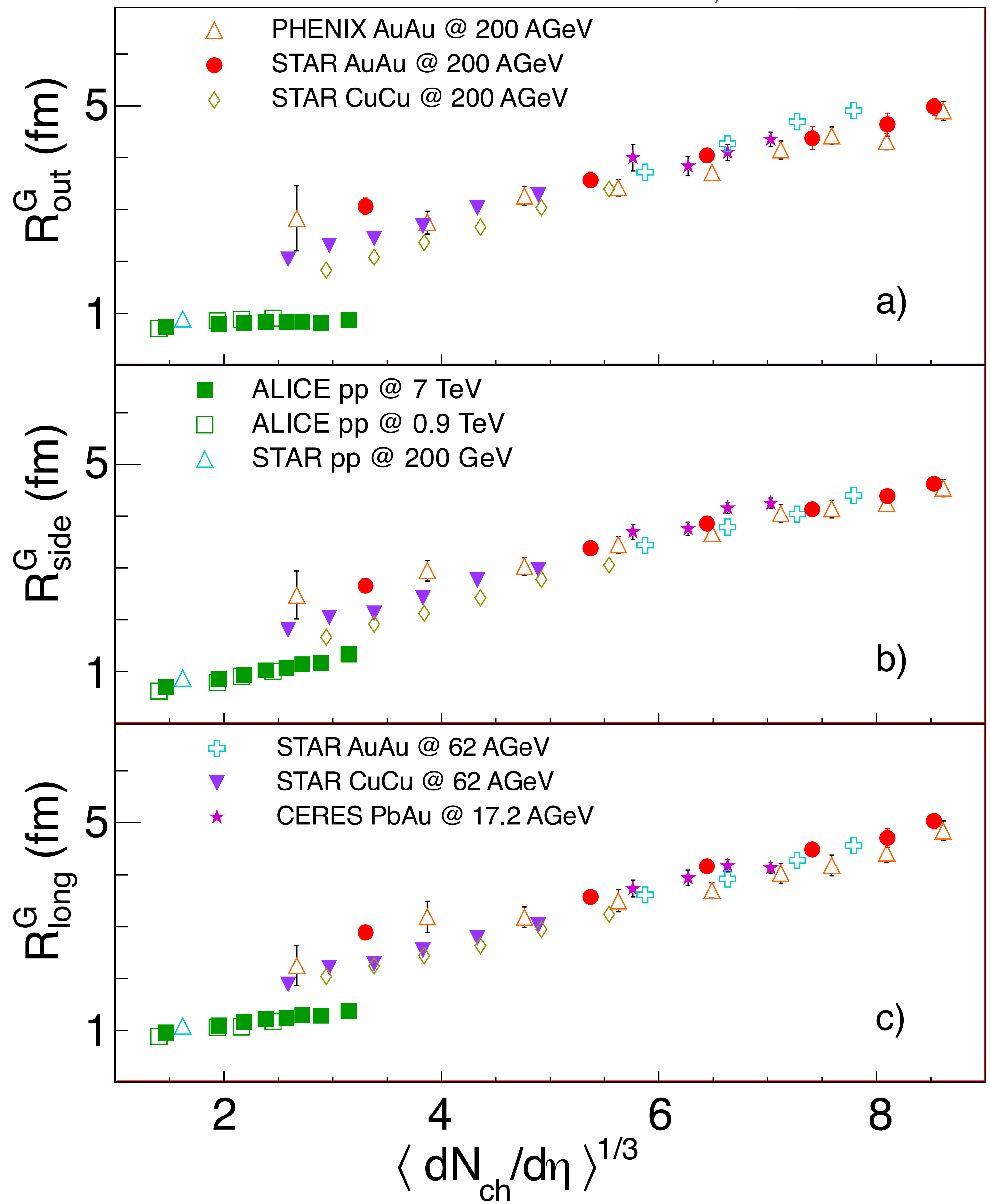
ALICE used:

$$R_{\text{long}}(k_T) = \tau_f \sqrt{\frac{T}{m_T} \frac{K_2(m_T/T)}{K_1(m_T/T)}}$$

Duration of particle emission at the LHC: about 10 fm/c

Small systems (I): HBT radii in pp

ALICE, arXiv:1101.3665

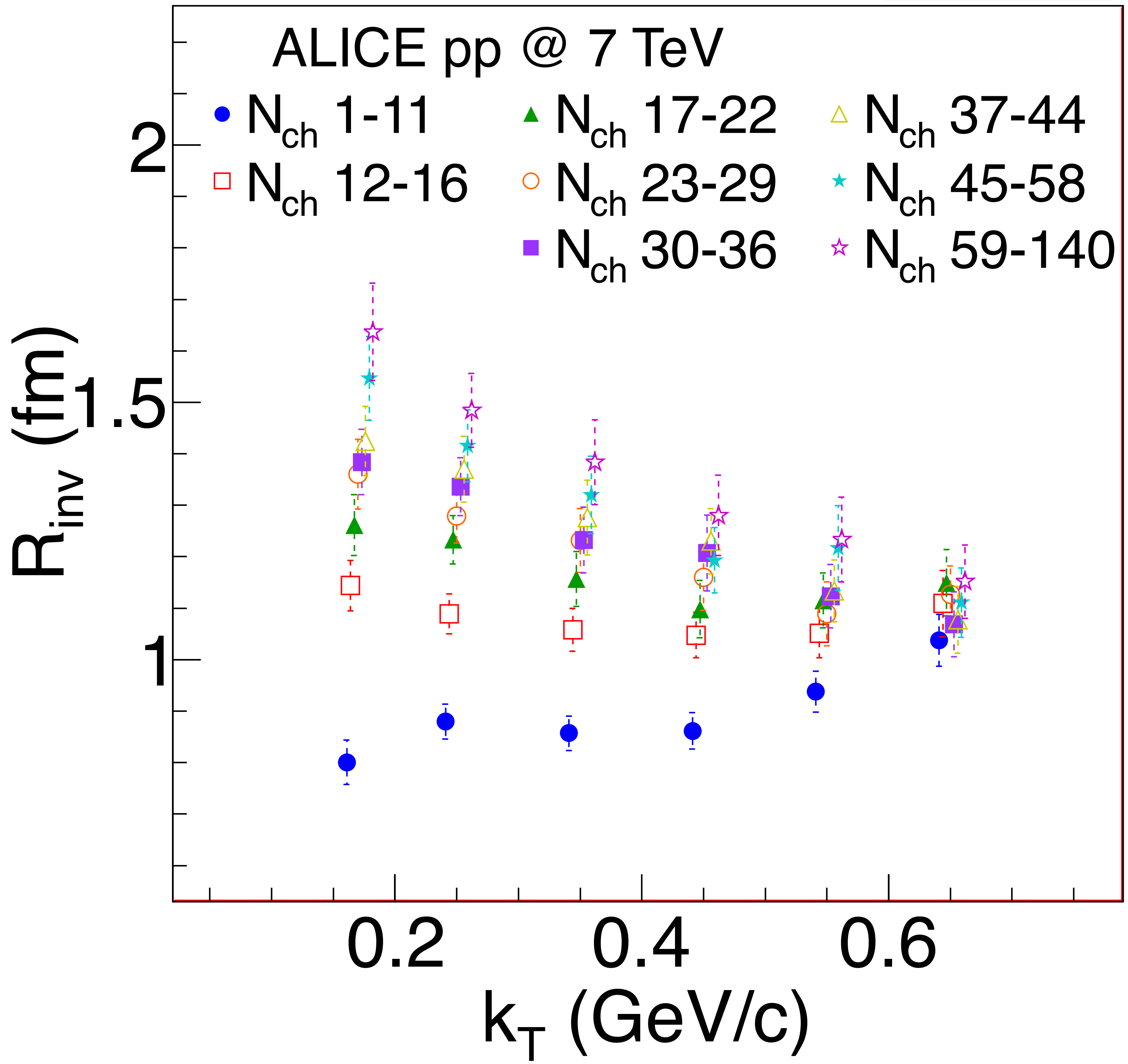


HBT radii in pp smaller than in AA at same $dN_{ch}/d\eta$

→ initial geometry matters

Small systems (II): Evidence for radial flow

ALICE, arXiv:1101.3665



Summary

- HBT correlations: effect of Bose-Einstein statistics
- Name comes from stellar intensity interferometry
- Information about the freeze-out volume and lifetime of the source
- HBT radii depend on pair momentum k_T: evidence for radial flow
- Freeze-out volume appears to scale linearly with $dN_{ch}/d\eta$:
  Approximately constant particle-density at freeze-out for different $\sqrt{s_{NN}}$