Hydrodynamical Model
and
Shear Viscosity from Black Holes
(η/s from AdS/CFT)

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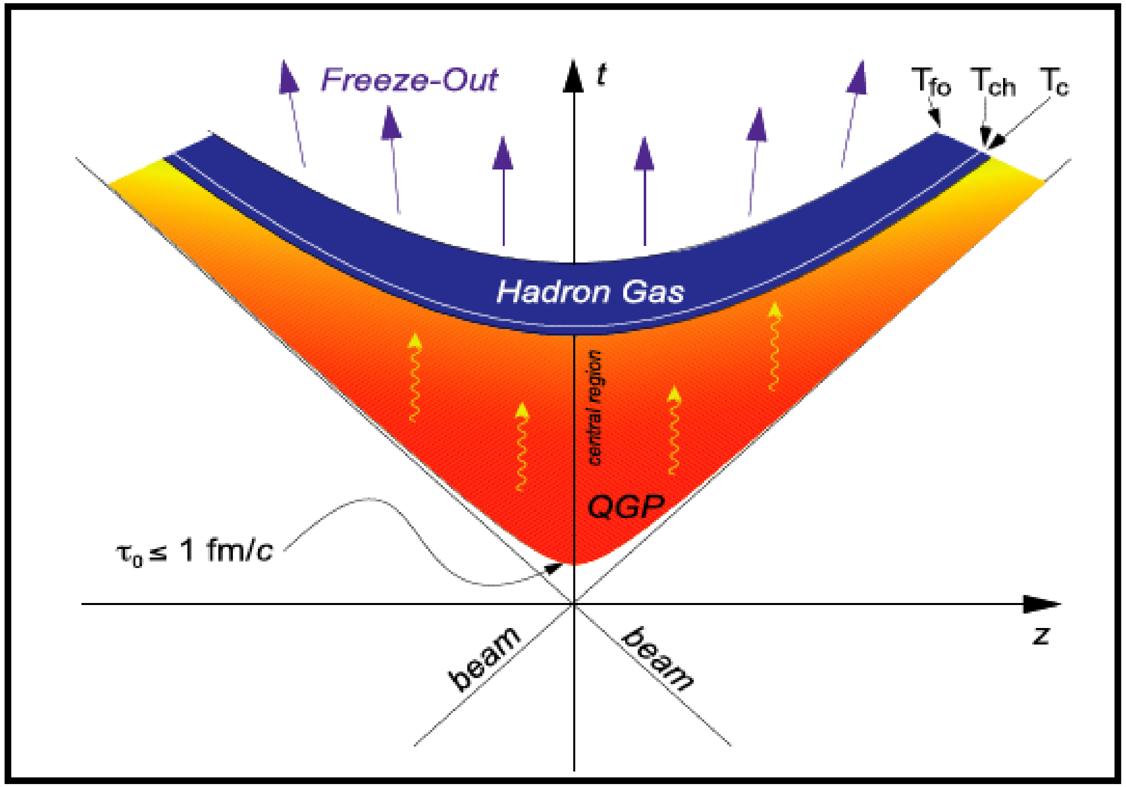
Space-time evolution



Plot: courtesy of R. Stock.

- **QGP life time**
  10 fm/c ≈ $3 \bullet 10^{-23}$ s

- **thermalization time**
  0.2 fm/c ≈ $7 \bullet 10^{-25}$ s
  → hydrodynamical expansion until freeze-out
  simplest model: only longitudinal expansion, 1d
  → Bjorken model

- **collision time**
  $2R/\gamma = 0.005$ fm/c
  ≈ $2 \bullet 10^{-26}$ s

Hydrodynamical model description

Some basic concepts

Relativistic Hydrodynamics (I)

The energy-momentum tensor $T^{\mu\nu}$ is the four-momentum component in the μ direction per three-dimensional surface area perpendicular to the ν direction.

$$\Delta\mathbf{p} = (\Delta E, \Delta p_x, \Delta p_y, \Delta p_z)$$
$$\Delta\mathbf{x} = (\Delta t, \Delta x, \Delta y, \Delta z)$$

$$\mu = \nu = 0 : \quad T_R^{00} = \frac{\Delta E}{\Delta x \Delta y \Delta z} = \frac{\Delta E}{\Delta V} = \varepsilon$$

$$\mu = \nu = 1 : \quad T_R^{11} = \frac{\Delta p_x}{\Delta t \Delta y \Delta z}$$

force in *x* direction acting on a surface Δ*y* Δ*z* perpendicular to the force → pressure

$$T^{\mu\nu} = \begin{pmatrix} \text{energy density} & \text{energy flux density} \\ \text{momentum density} & \text{momentum flux density} \end{pmatrix} \equiv \begin{pmatrix} \varepsilon & \vec{j}_\varepsilon \\ \vec{g} & \Pi \end{pmatrix}$$

Relativistic Hydrodynamics (II)

Isotropy in the fluid rest implies that

the energy flux T^{0j} and the momentum density T^{j0} vanish

and that $\Pi^{ij} = P\,\delta_{ij}$

$$T_R^{\mu\nu} = \begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & P & 0 & 0 \\ 0 & 0 & P & 0 \\ 0 & 0 & 0 & P \end{pmatrix}$$

Off-diagonal elements ≠ 0 in case of viscous hydrodynamics, not considered here

→ ideal (perfect) fluid.

See also Ollitrault, arXiv:0708.2433.

Relativistic Hydrodynamics (III)

Energy-momentum tensor (in case of local thermalization)
after Lorentz transformationto the lab frame:

$$T^{\mu\nu} = (\varepsilon + P)\, u^\mu u^\nu - P\, g^{\mu\nu}$$

metric tensor diag(1,-1,-1,-1)

Energy density
and pressure in
the co-moving system

4-velocity: $u^\mu = \mathrm{d}x^\mu / \mathrm{d}\tau$

$= \gamma(1, \vec{v})$

Energy and momentum conservation:

$$\partial_\mu T^{\mu\nu} = 0, \quad \nu = 0, \ldots, 3$$

$$\partial_\mu = (\frac{\partial}{\partial t}, \vec{\nabla})$$

in components:

$$\left\{ \begin{array}{l} \frac{\partial}{\partial t}\varepsilon + \vec{\nabla}\vec{j}_\varepsilon = 0 \;\; \text{(energy conservation)} \\ \frac{\partial}{\partial t} g_i + \nabla_j \Pi_{ij} = 0 \;\; \text{(momentum conservation)} \end{array} \right.$$

Conserved quantities, e.g., baryon number:

$$j^\mu_B(x) = n_B(x)\, u^\mu(x), \qquad \partial_\mu j^\mu_B(x) = 0 \quad \Leftrightarrow \quad \frac{\partial}{\partial t} N_\mathrm{B} + \vec{\nabla}(N_\mathrm{B}\vec{v}) = 0$$

continuity equation

$$N_\mathrm{B} = \gamma n_\mathrm{B}$$

Ingredients of Hydro - models

- Equation of motion and baryon number conservation:

$$\partial_\mu T^{\mu\nu} = 0, \quad \partial_\mu j^\mu_{\mathrm{B}}(x) = 0$$

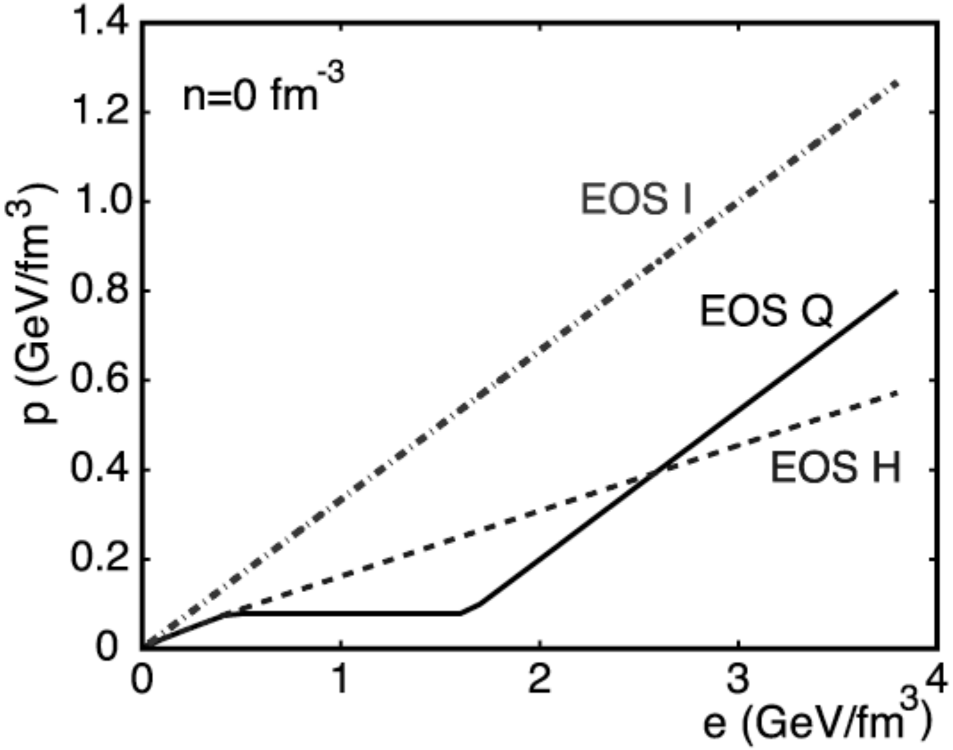
- 5 equations for 6 unknowns:

$$(u_x, u_y, u_z, \varepsilon, P, n_{\mathrm{B}})$$

- Equation of state: $P(\varepsilon, n_{\mathrm{B}})$
- (needed to close the system)
- Initial conditions,

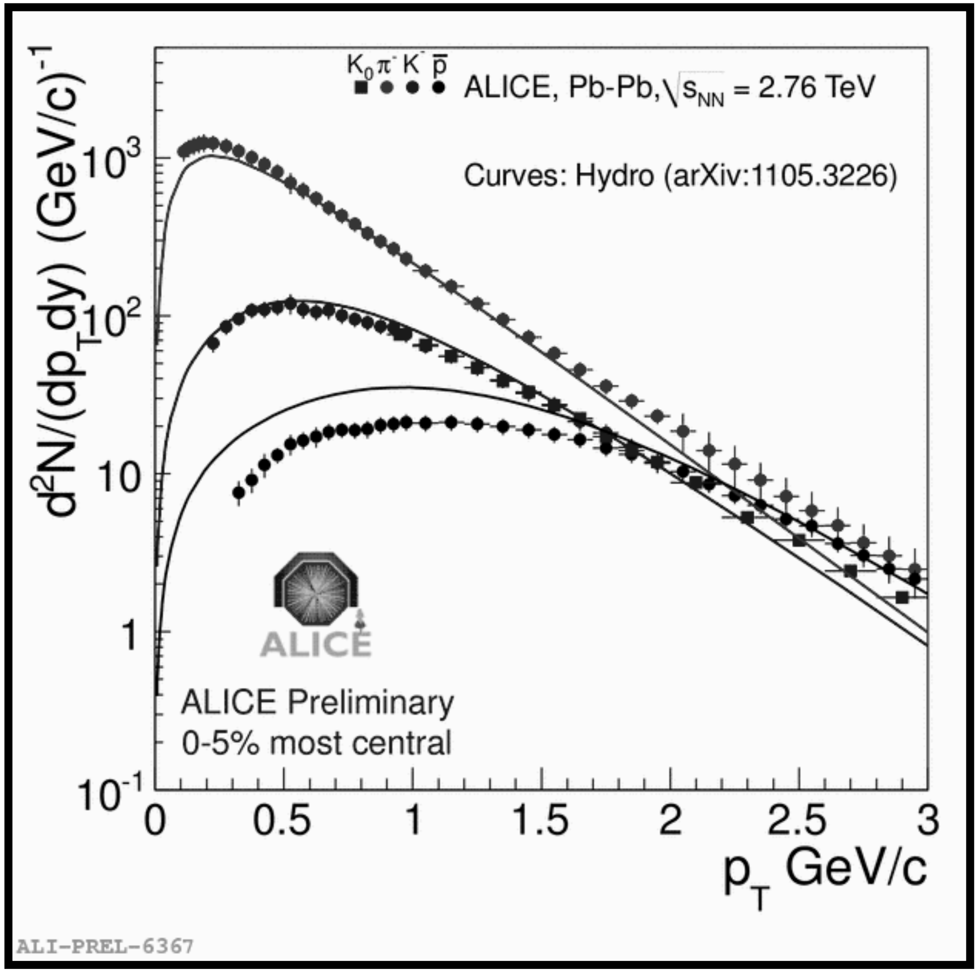
  e.g., from Glauber calculation
- Freeze-out condition, fluid → hadrons

  (Cooper-Frye formalism)



EOS I: ultra-relativistic gas $P = \varepsilon/3$
EOS H: resonance gas, $P \approx 0.15\ \varepsilon$
EOS Q: phase transition,
QGP ↔ resonance gas

LHC: Identified particle spectra

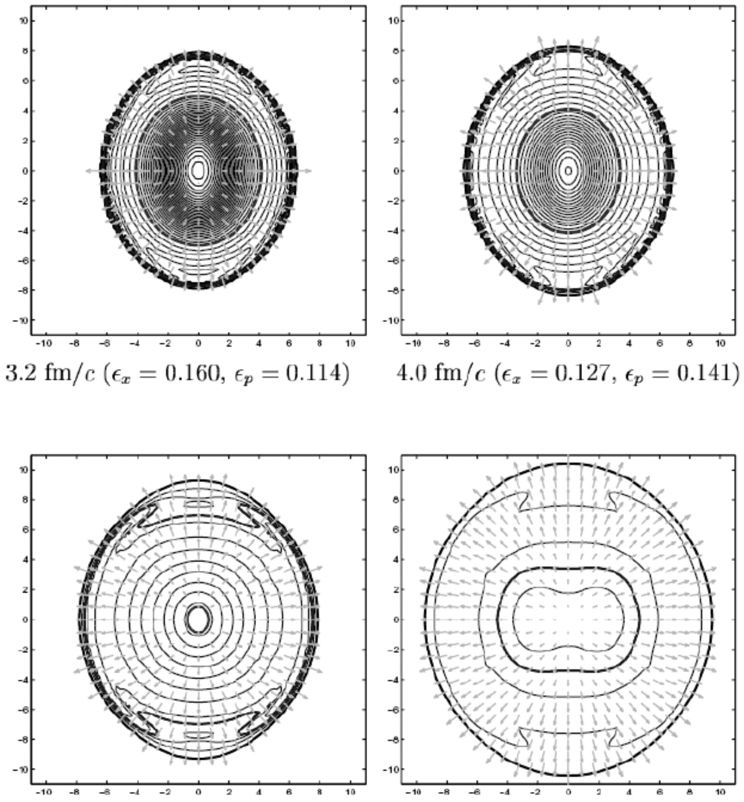


Initial conditions fixed by pion abundance

Protons overestimated

Annihilation of protons and anti-protons in the hadron phase ?

Elliptic flow in Hydro - models



Au+Au at $b = 7\,\mathrm{fm}$

3.2 fm/c ($\epsilon_x = 0.160$, $\epsilon_p = 0.114$)

4.0 fm/c ($\epsilon_x = 0.127$, $\epsilon_p = 0.141$)

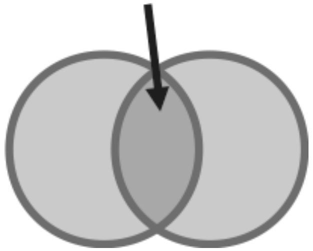
5.6 fm/c ($\epsilon_x = 0.067$, $\epsilon_p = 0.147$)

8.0 fm/c ($\epsilon_x = 0.003$, $\epsilon_p = 0.123$)

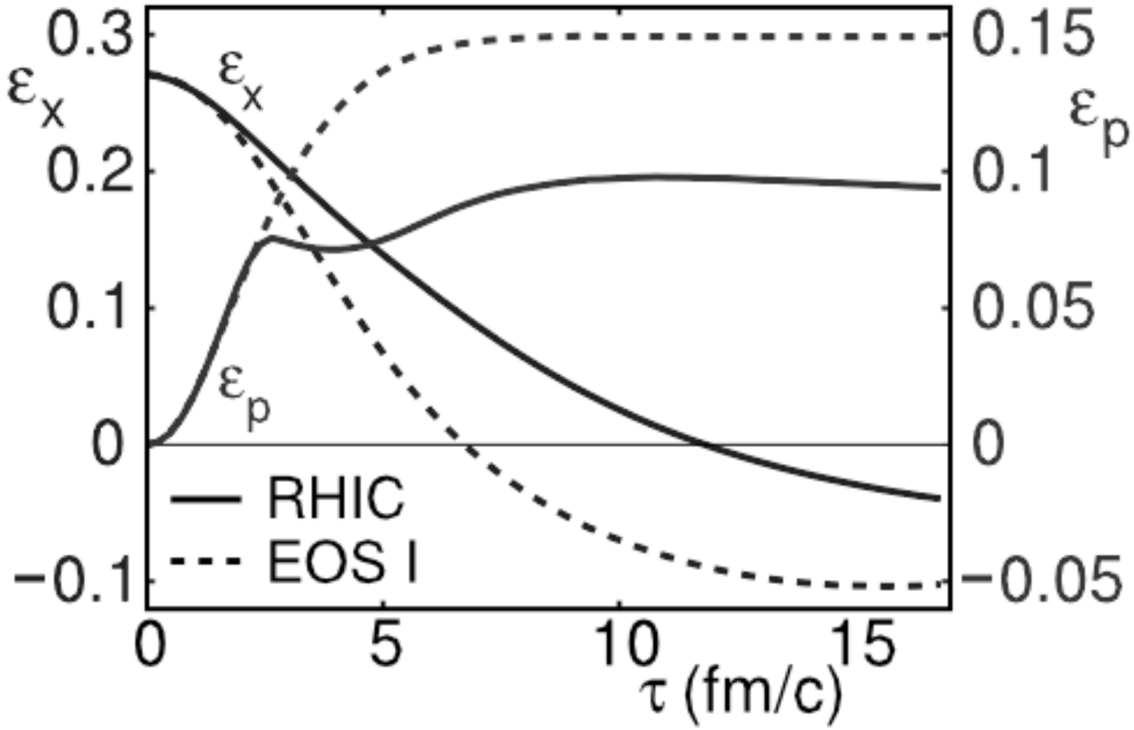
Elliptic flow is “self-quenching”:The cause of elliptic flow, the initial spacial anisotropy, decreases as the momentum anisotropy increases

Anisotropy in momentum space

Anisotropy in coordinate space



Anisotropy in momentum space



Ulrich Heinz, Peter Kolb, arXiv:nucl-th/0305084

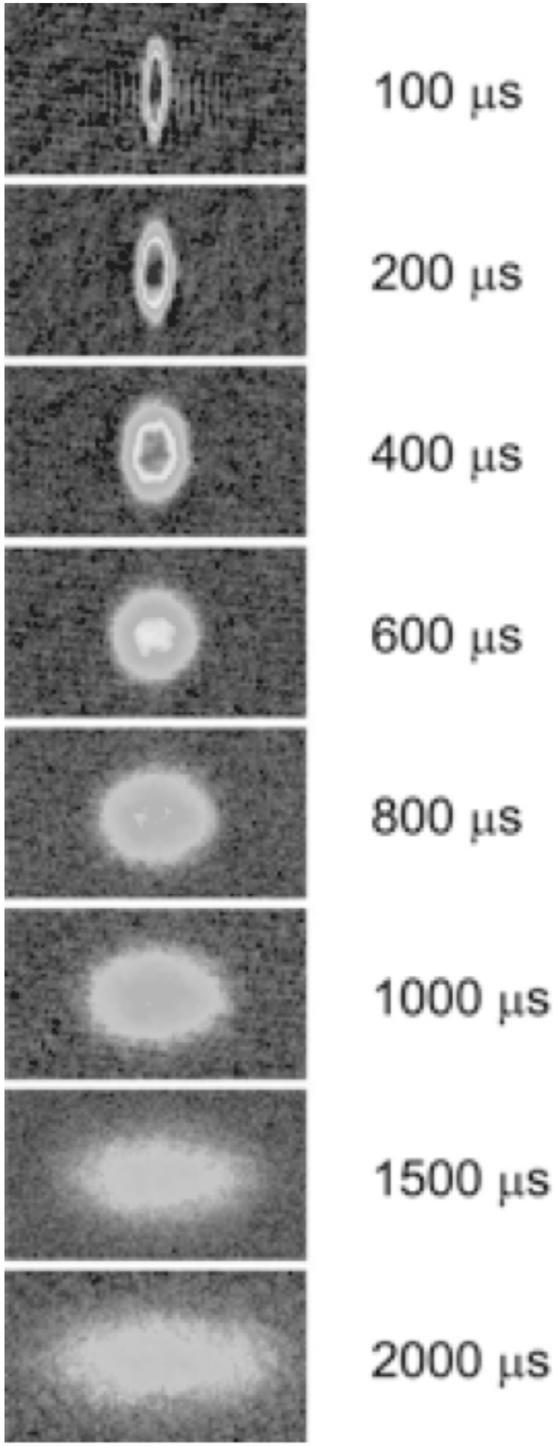
In hydrodynamic models the momentum anisotropy develops in the early (QGP)phase of the collision. Thermalization times of less than 1 fm/c are needed to describe the data.

Cold atomic gases

200 000 Li-6 atoms in an highly anisotropic trap (aspect ratio 29:1)

Very strong interactions between atoms (Feshbach resonance)

Once the atoms are released the one observed a flow pattern similar to elliptic flow in heavy-ion collisions



Lesson III

- **First results** from **ALICE** at LHC show large **increase** in **energy density** (**factor 2-3** compared to RHIC)
- **longer life-time** of qgp
- **larger collective flow** effects
- **anisotropic flow** comparable to **ultra-low viscosity**
- triangular flow sensitive to initial energy density fluctuations and viscosity/entropy ratio
- Hydrodynamical model provides framework to characterize QGP, i.e. equation of state, viscosity/entropy ratio

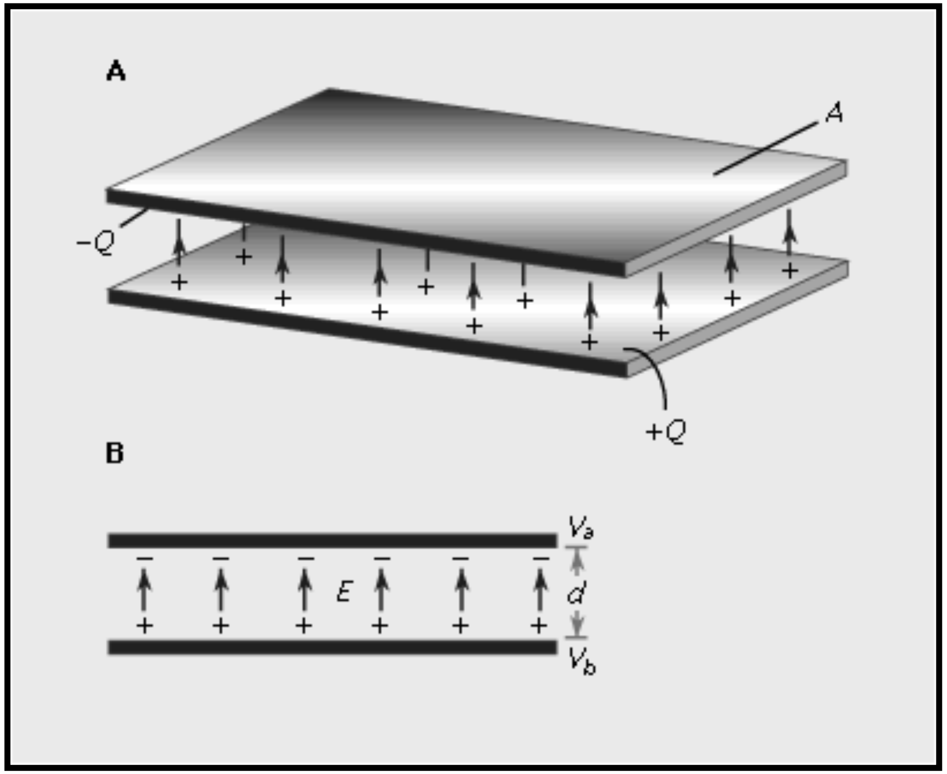
Shear Viscosity from Black Holes
(η/s from Ads/CFT)

What is this all about ?

General Considerations

- Strong coupling → quantum effects large
- Use AdS/CFT correspondence
- Holographic duality: relate string theory of higher dimension to 4-d gauge theory on the boundary
- Limit of strong coupling: string theory → classical gravity (GR)

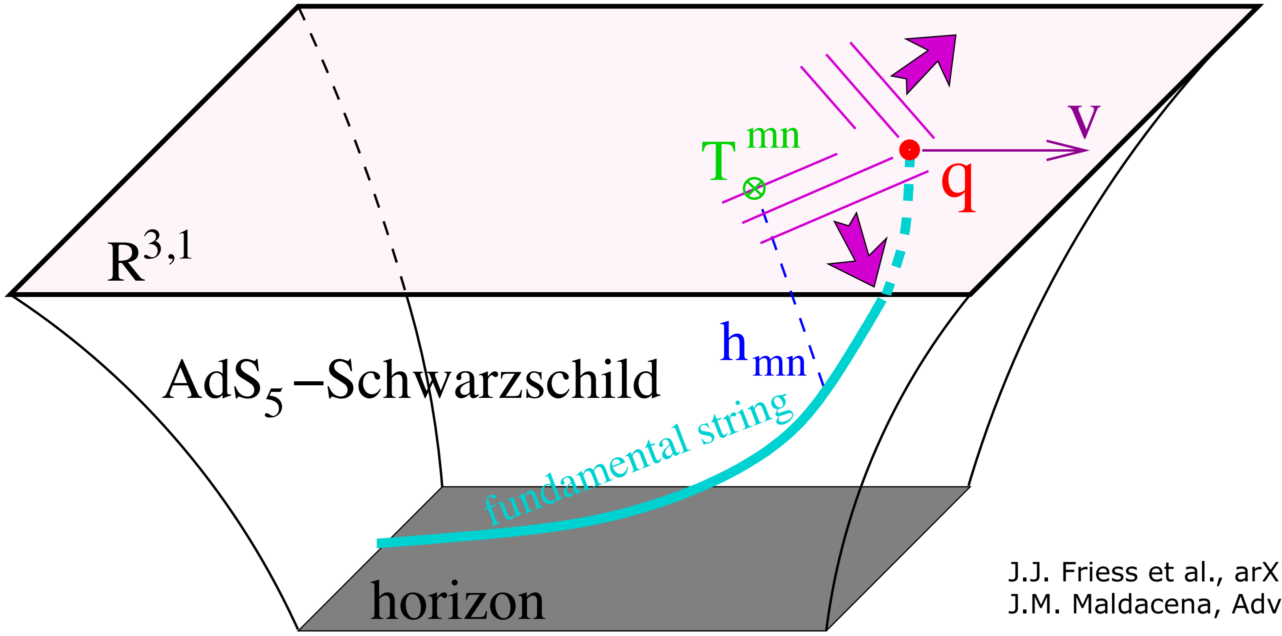
Parallel Plate Capacitor



Source: http://www.britannica.com

- Bulk: 3-d space between plates
- Fluctuations of the field in the bulk induce fluctuations of electric charges on the surface (boundary)
- Correlations of surface charges correlated to bulk field

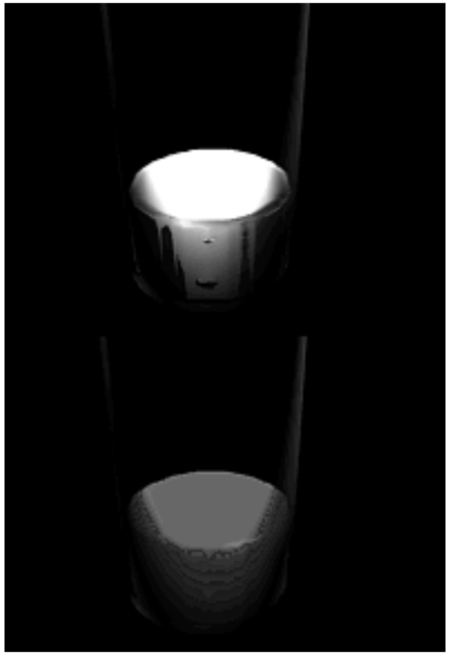
AdS/CFT correspondence



J.J. Friess et al., arXiv:0607022 [hep-th] (2006);
J.M. Maldacena, Adv.Theor.Math.Phys. 2 (1998) 231-252

- Maldacena conjecture: string theory and conformal QFT mathematically equivalent
- String theory: 10 dimensions
- E.g. Anti-de-Sitter Space (AdS) in 5dim + 5dim background
- Conformal field theory lives on 4dim boundary of 5dim AdS
- String theory becomes classical GR at boundary

Viscosity



Source: wikipedia

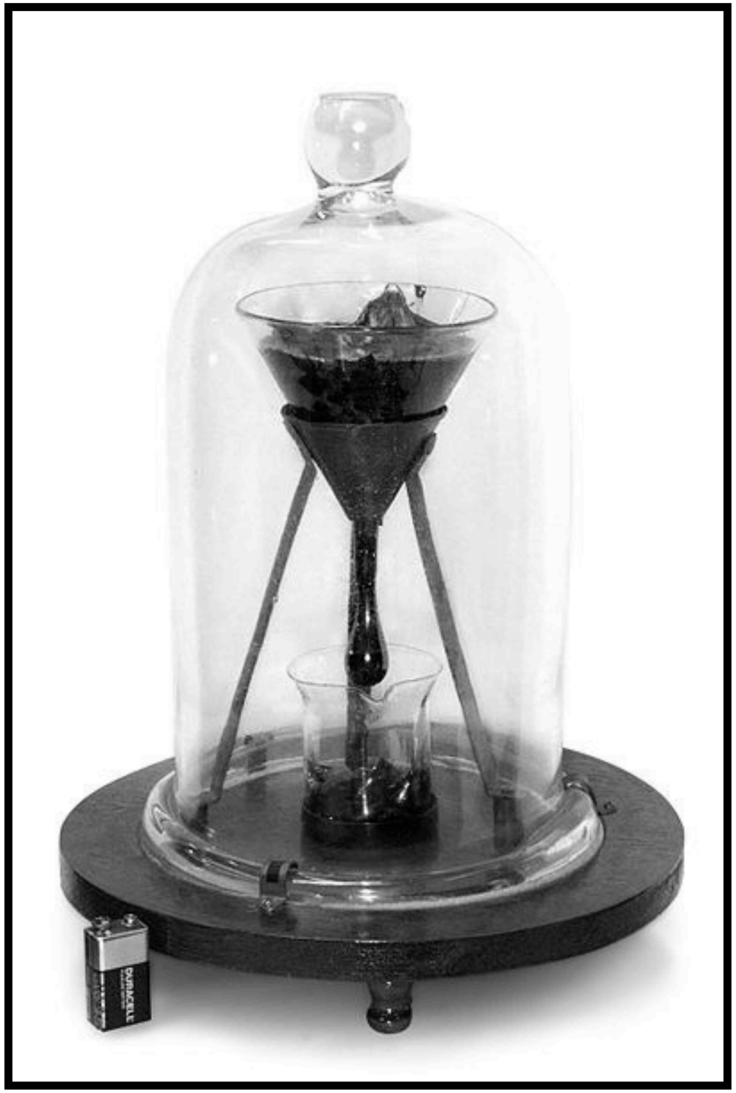
- Viscosity is a measure of a fluid resisting to flow
- ‘Fluid with smaller viscosity makes bigger splash’
- Due to friction between neighbouring particle of a fluid moving at different velocity
- Temperature dependent !
- Shear viscosity, bulk viscosity, …
- Symbol: η
- Unit: Pa s

Viscosity: some numbers

	T(K)	**μ(Pa s)**
Air	291.15	18.27×10^{-6}
Water	293	1×10^{-3}
Honey	293	2 – 10
Peanut butter	293	250
Pitch	293	2.3×10^{8}
QGP	2 000 000 000 000	gargantuan

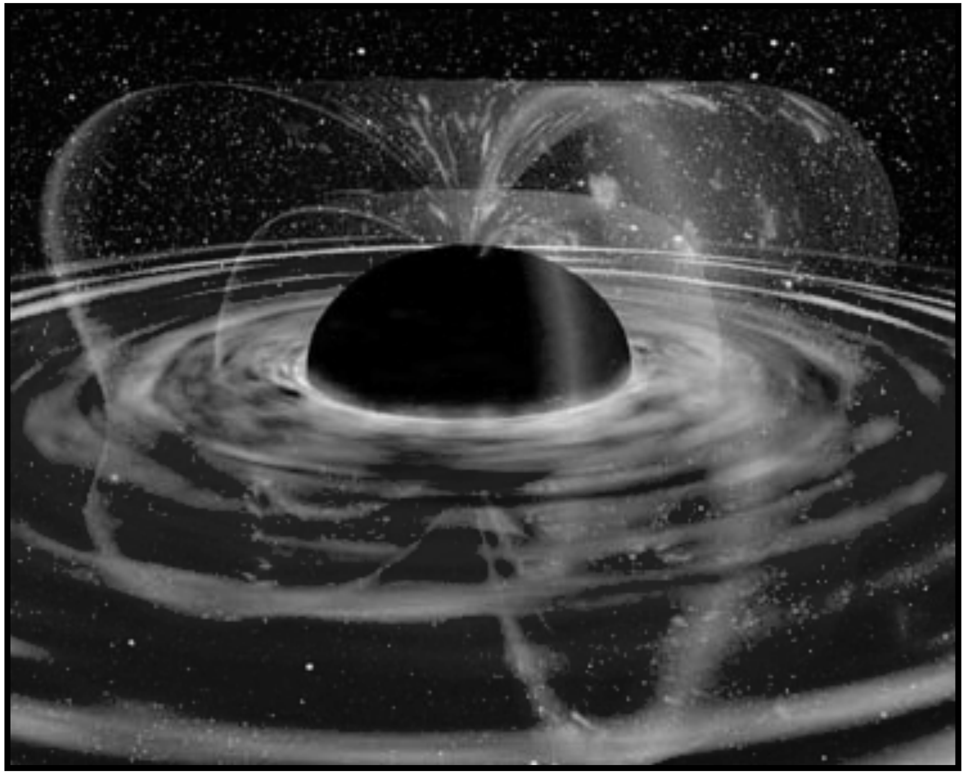
Source: wikipedia

Pitch drop experiment
University of Queensland



- Running since 83 years
- After 3 years of consolidation
- 8 drops fell so far
- No-one ever saw a drop falling
- 9th drop is about to fall

Black Hole



Source: http://media.photobucket.com

- Black hole, mass M
- Temp. $T = \frac{\hbar c^3}{8\pi G M k_B}$
- Entropy

  S = A/4·($k_B c^3$/Ghbar)

  A: area of horizon of boundary
- Physics of the interior region projected onto boundary: hologram

Holographic Principle

- Conjectured by ‘t Hooft
- Quantum gravity in (d+1) dimensions ⇔

  equivalent theory living on d-dimensional boundary

  ⇒ holographic dual

AdS/CFT Correspondence

- Fields that propagate in the bulk have well defined values at asymptotic infinity (boundary)
- Asymptotic values behave like field and coupling at the boundary
- Anti-de Sitter spacetime: negative curvature
- Holographic duals are sometimes gauge theories
- E.g. $\mathrm{AdS}_5 \Leftrightarrow$ N=4 Super Yang-Mills

AdS$_5$×S$_5$ Geometry

- AdS$_5$: 5 dimensional Anti-de-Sitter space
- Infinitesimal line element

$$ds^2 = \frac{r^2}{L^2}(-dt^2 + dx^2) + \frac{L^2}{r^2}dr^2 + L^2 d\Omega_5^2$$

S$_5$: 5 dimensional sphere, neglect

- r: radial coordinate
- R = const.: 3+1 dim. flat Minkowski space
- R → ∞: boundary
- L: curvature radius

AdS$_5$×S$_5$ Geometry, cont’ed

$$ds^2 = \frac{r^2}{L^2}(-dt^2 + dx^2) + \frac{L^2}{r^2}dr^2$$

- Require L >> l_s, (classical approx.)
- ‘t Hooft coupling: $\lambda = g^2_{YM} N_c$
- $(L/l_s)^4 = \lambda$
- Classical approx. works at strong coupling

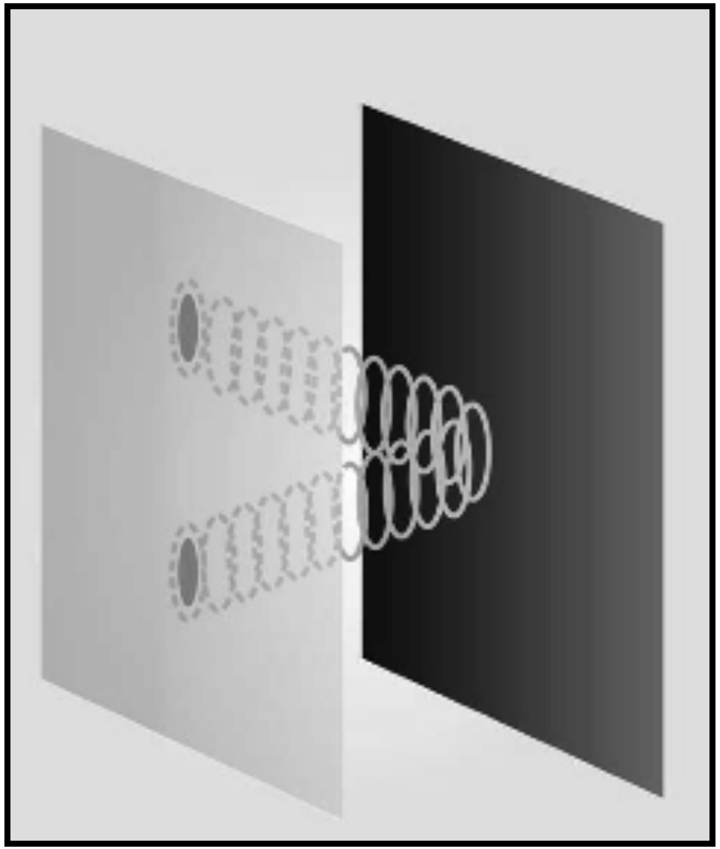
AdS$_5$×S$_5$ Geometry, cont’ed

- Rewrite for AdS$_5$ black hole

$$ds^2 = \frac{(\pi T L)^2}{u}\left(-(1-u^2)dt^2 + dx^2\right) + \frac{L^2}{4u^2(1-u^2)}du^2$$

- u = (r_0/r), r_0: Schwarzschild (horizon) radius
- Horizon at u = 1
- Boundary limit: u = ε, then $\varepsilon \rightarrow 0$

Ask the AdS/CFT Dictionary…



Source: Physics Today, p29, May 2010

- η from $T^{\mu\nu}$ (Kubo's formula)
- $T^{\mu\nu}$ corresponds to graviton $h^{\mu\nu}$
- Graviton is disturbance in $g_{\mu\nu}$
- Graviton at boundary propagates in the bulk and is scattered back
- Cross section $\propto$ surface A
- Entropy s $\propto$ surface A
- η/s does not depend on A

KSS bound on η/s

$$\frac{\eta}{s} \cong \frac{1}{4\pi} \cdot \frac{\hbar}{k_B} \left\{ 1 + \frac{15\zeta(3)}{\lambda^{3/2}} + \ldots \right\}$$

Classical approximation

Conjectured lower bound

from string theory

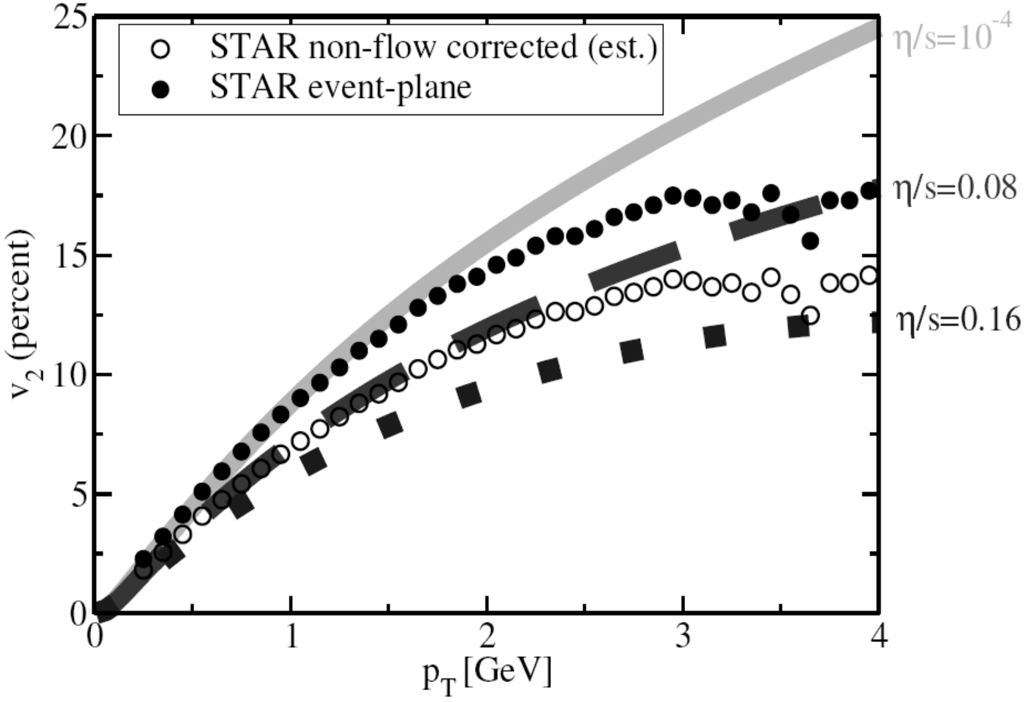
$$\frac{\eta}{s} = \frac{1}{4\pi} \cdot \frac{\hbar}{k_B} \left(1 - \frac{1}{2N_C} \right)$$

Potentially lower bound from SU(2)

Some remarks

- Relativistic fluid, but bound does not depend on speed of light
- N=4 Super Yang-Mills is **not** QCD
- $N_c = 3$, not large
- No confinement
- Quarks are massless
- However, details might not matter too much, system driven by temperature and degrees of freedom

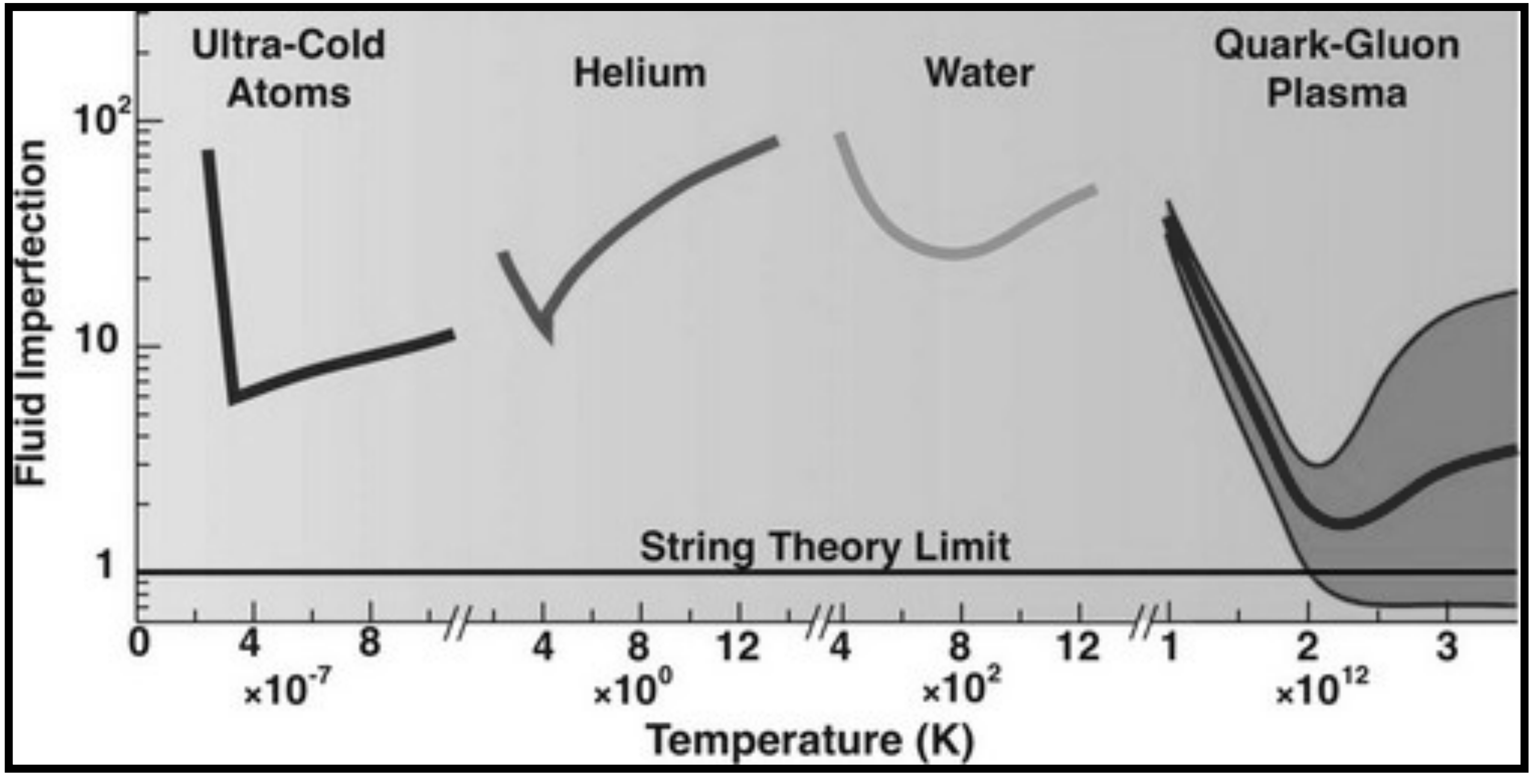
Non-ideal Hydro-dynamics



M.Luzum and R. Romatschke, PRC 78 034915 (2008); P. Romatschke, arXiv:0902.3663.

- Spectra and flow reproduced by ideal hydrodynamics calcs.
- Shear viscosity to entropy density ratio close to AdS/CFT bound
- viscosity leads to decrease in v_2, ultra-low viscosity sufficient to describe data
- Hydro-limit exceeded at LHC ?

$\eta\backslash s$ from experiments



- η huge, but also entropy s huge
- QGP close to conjectured bound → ‘prefect liquid’
- Cold atom gases (T = 10^{-7} K) have very similar properties, e,g, collective flow, etc.

References

- T. Schaefer and D. Teaney,
  Rep. Prog. Phys. 72 (2009), 126001, p22-26.
- P.K. Kovtun, D.T. Son, and A.O. Starinets,
  Phys. Rev. Lett. 94 (2005) 111601.
- J.M. Maldacena, TASI 2003 Lectures on AdS/CFT
  correspondence, arXiv:hep-th/0309246.
- C.V. Johnson and P. Steinberg,
  Physics Today (May 2010) 29.

Lesson IV

- shear viscosity / entropy density (η/s) ultralow in QGP

- however, shear viscosty AND entropy density large in QGP, only ratio becomes small !

- close to the conjectured bound from AdS/CFT correspondence

- common features of many-body systems (QGP vs ultra-cold atomic gases) over 20 orders of magnitude in temperature