Kirchhoff-Insitut für Physik
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KIP CIP Pool (1.401)

Exercises for Statistical Methods in Particle Physics

http://www.physi.uni-heidelberg.de/~nberger/teaching/ws13/statistics/statistics.php

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Exercise 10: Confidence limits and the CL_s method

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Hand-in solutions by 24:00, 8. January 2014

Please send your solutions to obrandt@kip.uni-heidelberg.de by 8.1.2014 (note that exceptionally this is not the Sunday before the exercises session), 24:00, punctually. Make sure that you use *SMIPP:Exercise10* as subject line. If plots are requested, please include print statements to produce pdf files in your code, and provide the plots separately. Please add comments to your source code explaining the steps. Test macros and programs before sending them off...

"Is this a new discovery or just a statistical fluctuation?" Statistics offers various approaches to quantify this in a scientific manner by providing confidence level (CL) benchmarks. However, due care has to be exercised when interpreting these statistical benchmarks, in particular one should know exactly what the obtained numbers mean and, sometimes even more important, what they do *not* mean.

1 Confidence limits for Poisson processes

In this Problem, we want to derive "by hand" the CLs for a basic counting experiment which can be modelled by a Poisson process with mean λ.

1.1 Confidence limits for a given expectation value

Write a program that lists, for fixed values of $\lambda = 1, 2, ...12$, the number of observed events n such that at most 10% of the probability are (i) above n, (ii) below n, (iii) above n and below n' (central confidence interval).

1.2 Confidence limits given a measured value

In a similar spirit, write a program that lists, for fixed number of *observed* events $n = 0, 1, 2, ...12$, the (i) upper, (ii) lower, and (iii) central CLs on λ at 90% CL. Describe the conceptual difference between this case and that discussed in 9.1.1.
Hint: In the above problem one will need to numerically find the values of λ which solve the equations such as to obtain the desired CL, which can be done by a scanning or bisection approach.
(ex_10_1.C/.py)

2 Confidence limits for Poisson processes in the context of elementary particle physics

In the context of particle physics, considering a simple counting experiment where there are contributions from signal (S) and background (B) processes, the number of signal (n_S) and background events (n_B) can be treated as independent Poisson variables with means λ_S and λ_B, respectively. For simplicity, we consider a pure Poisson process, i.e. λ_B is assumed to be known with infinite precision. The total number of events $n = n_S + n_B$ is therefore a Poisson varialbe with mean $\lambda = \lambda_S + \lambda_B$, and the probability to observe n events is then given by:

$$p(n; \lambda_S; \lambda_B) = \frac{(\lambda_S + \lambda_B)^n}{n!} \exp(-[\lambda_S + \lambda_B])$$

Typically, one is interested in the question whether the observed number of events in data is due to a statistical fluctuation of the background, assuming no signal is present. This can be quantified by the probability to observe n_{obs} or more events from background alone:

$$p(n \geq n_{\text{obs}}) = \sum_{n=n_{\text{obs}}}^{\infty} p(n; \lambda_S = 0; \lambda_B) = 1 - \sum_{n=0}^{n_{\text{obs}}-1} \frac{\lambda_B^n}{n!} \exp(-\lambda_B)\,.$$

As an example, assume a counting experiment in which 5 events are observed, while $\lambda_B = 1.8$ background events are expected.

2.1 Establishing the presence of signal

Is this a significant (=3σ) excess to establish the presence of signal? In other words, calculate the probability of observing $n_{\text{obs}} = 5$ or more events assuming the presence of background only with the expectation value is $\lambda_B = 1.8$ using Poisson statistics.

2.2 Upper limit on the number of signal events

Determine an upper limit $\lambda_S^{\max}$ for the number of signal events at a 95% CL. Such a limit is defined by the expected number of signal events $\lambda_S^{\max}$ where the probability of measuring n_{obs} or fewer events reaches 5% assuming a Poisson statistic with mean $\lambda_B + \lambda_S^{\max}$. To (numerically) find the answer, perform an interval search starting from the probabilities to observe $n_B + n_S^{\min}$ and $n_B + n_S^{\max}$ or less events. Stop the search when the uncertainty, i.e. the difference of the limits of the interval, is less than 10^{-5}.
(`ex_10_2.C/.py`)

3 Confidence limit determination with MC approaches

Verify the limit determined in above Problem 9.2.2 with toy MC experiments. In each toy experiment generate a random number according to a Poisson distribution with a mean value of $\lambda_B + \lambda_S^{\max}$. Then count the number of experiments in which this random number is less or equal n_{obs}. By construction, the fraction of these events should be 5%.
(`ex_10_3.C/.py`)

Please turn

4 The CL_s method

Read the comprehensive description of the CL_s method in its today's form by A.L. Read [1], based on the preparatory work of G. Zech. It was first widely used in searches for the Higgs boson by the LEP experiments, and is still the baseline tool at the LHC. Try to answer the following questions:

- What is *coverage*?
- What is *flip-flopping*?
- In this context, what is *conservative*?
- What happens if the background estimate is too large?
- What if it is too small?
- Would you consider yourself a Bayesian or a Frequentist? Why?

(`ex_10_4.txt`)

5 The implementation of the CL_s method in ROOT

The CL_s method is implemented in the root class `TLimit`, which is a C++ copy of the first-time fortran implementation by T. Junk [2]. Assume you perform a search as a simple counting experiment assuming purely statistical uncertainties, where you expect 100 background events. Using `TLimit`, answer the following questions:

- If you expect 10 signal events and observe a total of 90, 100, 110 events, what are your expected and observed confidence levels for the signal plus background hypothesis using the CL_s method?
- If you observe 100 events with the same background expectation, what signal size can you exclude at the 95% confidence level?
- How much would you have to reduce the background to be able to exclude a 10 event signal at the 95% confidence level?

(`ex_10_5.C/.py`)

Literatur

[1] A.L. Read, *Modified frequentist analysis of search reslts*, `http://cdsweb.cern.ch/record/451614/files/p81.ps.gz?version=1`, (2000).

[2] T. Junk, *Confidence Level Computation for Combining Searches with Small Statistics*, `http://root.cern.ch/root/doc/TomJunk.pdf`, (1999).