Exercise 6: Signal and Background

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Please send your solutions to `nberger@physi.uni-heidelberg.de` until 26. 11. 2012, 12:00. Put your answers (including plots, no (zip-) archives) in an email (subject line *SMIPP:Exercise06*). Test your programs before sending them off...

1. **Signal and background** Go back to exercise 5.2 and again simulate equal amounts of kaons (Landau distribution with mean 2 and "sigma" 0.4) and pions (1, 0.3) in a five layer energy loss particle identification detector. Form a truncated mean by throwing away the largest measurement each. Now make plots of the efficiency, purity, signal to background ratio, signal to $\sqrt{background}$ ratio for both pions and kaons as a function of the cut value. How does this change if only 10% of the particles are kaons?
(Attach .C or .py files and suitable plots)

2. **Unbinned Likelihood - Searching for a Resonance** Assume we are studying an invariant mass distribution in some decay channel (e.g. the $\pi^+\pi^-$ mass in in $J/\psi \to \gamma\pi^+\pi^-$ or the $t\bar{t}$ mass in $pp \to t\bar{t}X$ or whatever your favourite physics example is. Assume this mass distribution is dominated by a single resonance, which we can describe by a Breit-Wigner distribution (`TRandom3::BreitWigner()`) of mass 2 (GeV/c^2 for the pions, TeV/c^2 for the tops) and width 0.1 (same units). Generate 100 events (masses m_i) according to that distribution. Now construct the log likelihood function

$$\log L = \log\left(\prod_{i=1}^{N} BW(m_i, M, \Gamma)\right) = \sum_{i=1}^{N} \log\left(BW(m_i, M, \Gamma)\right), \qquad (1)$$

where you can use `TMath::BreitWigner()` for the Breit-Wigner function. Assume that you know the width and scan the assumed resonance mass M and plot $\log L$ as a function of the mass. The maximum of that curve is your measured value, 1 σ errors are where your likelihood changes by 0.5 on either side. What is your result?
(Attach .C or .py files and suitable plots)

3. **Unbinned Likelihood - Searching for a Resonance II** Take the example from above, but now add in 2000 equidistributed background events in the mass range from 1 to 3 (units again up to you). Throw away Breit-Wigner events with masses outside that range. Now construct a background-only likelihood function (flat) and a signal + background likelihood function, where you leave the relative normalisation free. Assuming you know mass and width of the resonance, now perform a scan as above in that normalisation parameter. Then compare the $\log L$ for

the background only hypothesis and the (optimal) signal + background hypothesis.
(Attach .C or .py files and suitable plots)

4. **Unbinned Likelihood - Searching for a Resonance III** Again take the example from above and perform a toy MC test of your findings. Perform the experiment 1000 times, generating only background; how often do you find a likelihood ratio ($\log L$ difference) larger than in 3? Do the same whilst generating the resonance and the background, how often do you find a likelihood ratio smaller than in 3? If you had not generated it yourself, how sure would you be that there is a resonance?
(Attach .C or .py files and suitable plots)