Particle Physics WS 2012/13
(7.12.2012)

Stephanie Hansmann-Menzemer

Physikalisches Institut, INF 226, 3.101

Content of Today

- Feynman-Rules for Strong IA and Colour Factors
- Strong IA potential
- Experimental Test of QCD
  - Observation of Gluon
  - Measurement of spin of the gluon
  - Test of $SU(3)_C$ structure of strong IA
- Running of strong IA constant α_s

SU(3) Color

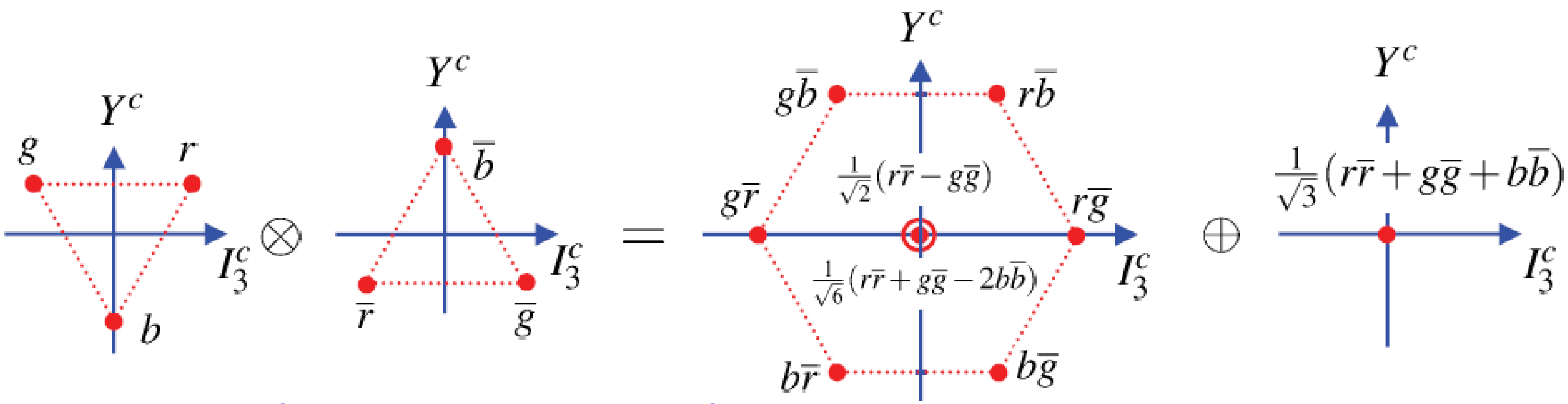
physics is invariant under rotation in color space

red, green and blue quarks are not distinguishable

This is an exact symmetry!

$$\begin{pmatrix} R' \\ G' \\ B' \end{pmatrix} = U \begin{pmatrix} R \\ G \\ B \end{pmatrix}$$

color | anti-color | octett state carry color (as well Y=0,I_3=0 state) | color neutral singulett state



It is believed (though not yet proven) that all free particles are colour neutral .

neutral = symmetric under rotation in colour space (Y_c = I_3=0 is not sufficient!)

Colour wave function of mesons: $\psi_C = \frac{1}{\sqrt{3}}(r\bar{r} + g\bar{g} + b\bar{b})$

Gluons consists of a combination of colour and anticolour, and have net color.

Gluons are represented by octett state.

Gell-Mann Matrices can be "associated" to Gluons!

Color SU(3): Quark states $R = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad G = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$R \leftrightarrow G \quad \lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$T_\pm = 1/2(\lambda_1 \pm i\lambda_2)$ $\quad r\bar{g}, \quad g\bar{r}$

$\frac{1}{\sqrt{2}}(r\bar{r} - g\bar{g})$

$R \leftrightarrow B \quad \lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}$

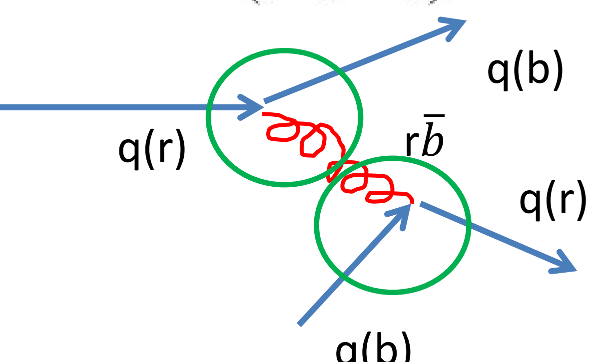
$V_\pm = 1/2(\lambda_4 \pm i\lambda_5)$ $\quad r\bar{b}, b\bar{r}$

$B \leftrightarrow G \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}$

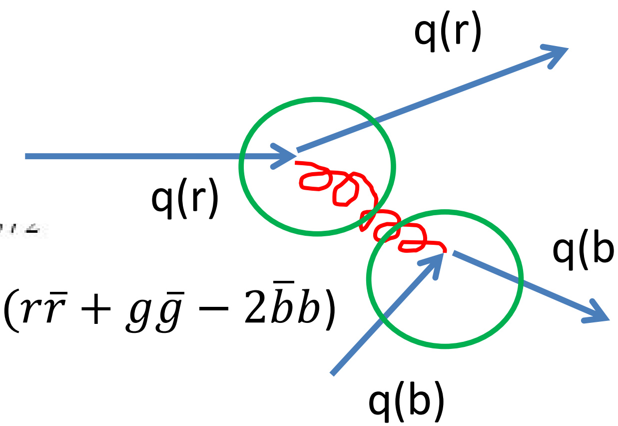
$U_\pm = 1/2(\lambda_6 \pm i\lambda_7)$ $\quad b\bar{g}, \bar{b}g$

$\lambda_8 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$

$\frac{1}{\sqrt{6}}(r\bar{r} + g\bar{g} - 2\bar{b}b)$



conservation of color
at each vertex



$\frac{1}{\sqrt{6}}(r\bar{r} + g\bar{g} - 2\bar{b}b)$

Symmetries define Interactions

Lagrangian must reflect the invariance of the symmetry transformation.

The Lagrangian of the free fermion ($L = i\bar{\psi}\,\gamma^{\mu}\,\partial_{\mu}\psi - m\bar{\psi}\psi$) is not invariant under any symmetry, thus need to add IA terms.

The Lagrangian defines the IA. To each Lagrangian, there correpsonds a set of Feynman rules.

QED: U(1)

$U = e^{i\alpha(x)}$

two transformation commute

Introduce 1 photon field A to get invariant Lagrangian

L = „$\bar{\psi}\psi$“ + „$e\bar{\psi}\psi A$"+"A^2"

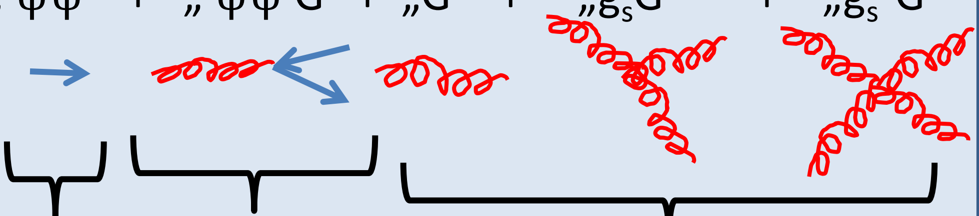
QCD: SU(3)

$U = e^{i\vec{\alpha}(x)\vec{\lambda}}$ λ_i; i=1,2,...,8

two transformation in genreral do not commute

introduce 8 gluon fields G_i to get invariant Lagrangian

L = „ $\bar{\psi}\psi$“ + „ $\bar{\psi}\psi$ G“ + „G^2“ + „$g_s G^3$“ + „$g_s^2 G^4$“



free fermion IA kinematic energy of gluon fields

$$L = i\bar{\psi}\,\gamma^{\mu}\,\partial_{\mu}\psi - m\bar{\psi}\psi \;-\; g_s(\bar{\psi}\gamma^{\mu}T_a\psi)G_{\mu}{}^{a} \;-\; \frac{1}{4}\,G_{\mu\nu}{}^{a}G_{a}{}^{\mu\nu}$$

The Quark-Gluon Interaction

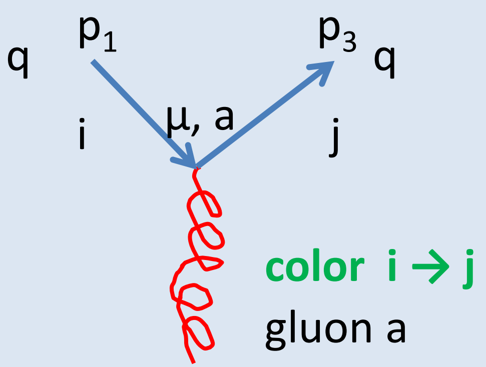
Particle wave functions $\psi(x) = u(x)\, e^{-ipx}$ → $c_j\, u(x)\, e^{-ipx}$

$$c_1 = \begin{pmatrix}1\\0\\0\end{pmatrix} \quad c_2 = \begin{pmatrix}0\\1\\0\end{pmatrix} \quad c_3 = \begin{pmatrix}0\\0\\1\end{pmatrix}$$

colour part of the fermion wave-function

Interaction term: $g_s(\bar{\psi}\gamma^{\mu}T_a\psi)G_{\mu}^{\ a}$

vertex factor part of propagator

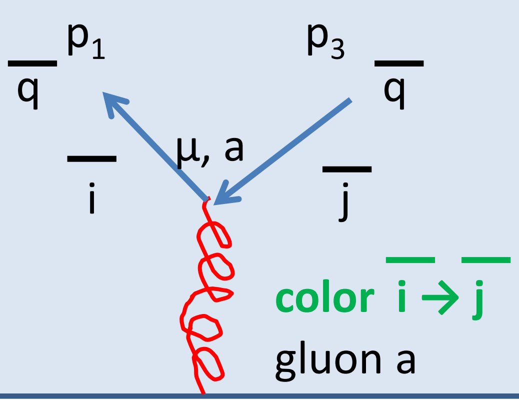


vertex factor: $\overline{u(p_3)}c_j^{\dagger}(-1/2\ i\ g_s\ \lambda^a\ \gamma^{\mu})\ c_i\ u(p_1)$

isolating the colour part: $c_j^{\dagger}\lambda^a c_i = \boldsymbol{\lambda_{ji}^{a}}$

⟹ vertex factor: $\overline{u(p_3)}(-1/2\ i\ g_s\ \lambda_{ji}^{a}\ \gamma^{\mu})\ u(p_1)$

for quarks



vertex factor: $\overline{v(p_1)}c_i^{\dagger}(-1/2\ i\ g_s\ \lambda^a\ \gamma^{\mu})\ c_j\ v(p_3)$

isolating the colour part: $c_i^{\dagger}\lambda^a c_j = \boldsymbol{\lambda_{ij}^{a}}$

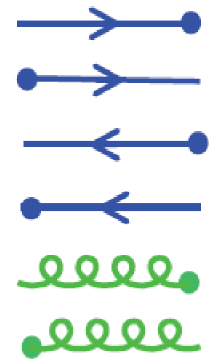
⟹ vertex factor: $\overline{v(p_1)}(-1/2\ i\ g_s\ \lambda_{ij}^{a}\ \gamma^{\mu})\ v(p_3)$

for anti-quarks

Feynman Rules for QCD

External Lines

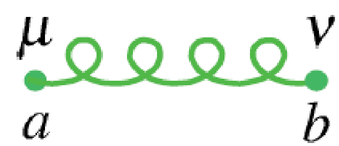
spin 1/2	incoming quark	$u(p)$	
	outgoing quark	$\overline{u}(p)$	
	incoming anti-quark	$\overline{v}(p)$	
	outgoing anti-quark	$v(p)$	
spin 1	incoming gluon	$\varepsilon^{\mu}(p)$	
	outgoing gluon	$\varepsilon^{\mu}(p)^{*}$	



Internal Lines (propagators)

spin 1 gluon

$$\frac{-ig_{\mu\nu}}{q^2}\delta^{ab}$$

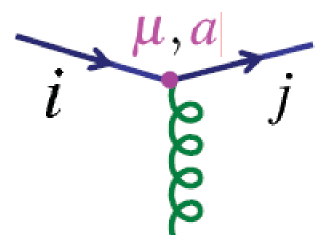


a, b = 1,2,…,8 are gluon colour indices

Vertex Factors

spin 1/2 quark

$$-ig_s\tfrac{1}{2}\lambda^{a}_{ji}\gamma^{\mu}$$



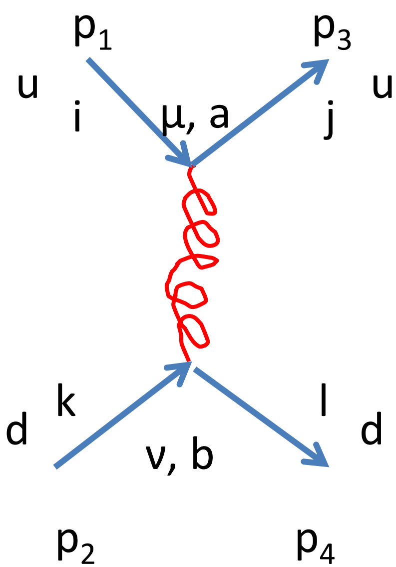
i, j = 1,2,3 are quark colours,

λ^a a = 1,2,..8 are the Gell-Mann SU(3) matrices

+ 3 gluon and 4 gluon interaction vertices

Matrix Element $-iM$ = product of all factors

Matrix Element for Quark-Quark Scattering



coulor indices: i,j,k,l

In this example the colour flow ik → jl

a,b are the gluon index. δ_{ab} ensures that a=b, same gluon is „emitted“ at a and „absorbed“ at b

$$iM = [\overline{u(p_3)}\left(\frac{1}{2}g_s\,\lambda_{ji}{}^{a}\gamma^{\mu}\right)u(p_1)]\,\frac{-ig^{\mu\nu}\delta^{ab}}{q^2}\,[\overline{u(p_4)}\left(\frac{1}{2}g_s\,\lambda_{lk}{}^{b}\gamma^{\nu}\right)u(p_2)]\,]$$

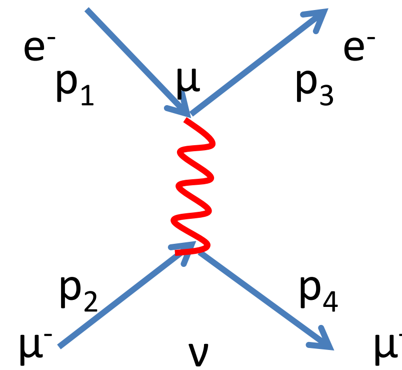
where summing over a,b and μ, ν is implied.

$$M = -\frac{g_s{}^2}{4}\lambda_{ji}{}^{a}\lambda_{lk}{}^{a}\,\frac{1}{q^2}\,g^{\mu\nu}[\overline{u(p_3)}\gamma^{\mu}u(p_1)]\,[\overline{u(p_4)}\,\gamma^{\nu}u(p_2)]\,]$$

QCD vs. QED

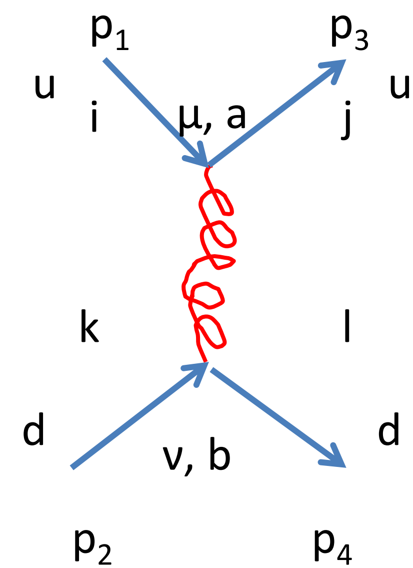
QED matrix element:

$$M = -\, e^2 \frac{1}{q^2}\, g^{\mu\nu} [\overline{u(p_3)}\gamma^{\mu} u(p_1)]\; [\overline{u(p_4)}\,\gamma^{\nu} u(p_2)]\;]$$



$e^2 \rightarrow g_s^{\,2}$ $\alpha^2 = \frac{e^2}{4\pi} \rightarrow \alpha_s^{\,2} = \frac{g_s^{\,2}}{4\pi}$

+ add. color factor $C(ik \rightarrow jl) = \frac{1}{4} \sum \lambda_{ji}^{a} \lambda_{lk}^{a}$



QCD matrix element:

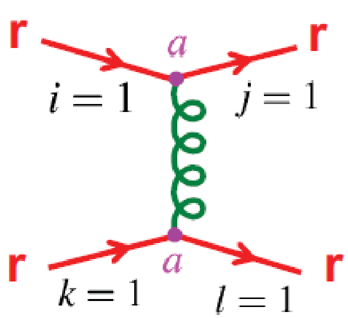
$$M = -\, \frac{g_s^{\,2}}{4} \lambda_{ji}^{a} \lambda_{lk}^{a} \frac{1}{q^2}\, g^{\mu\nu} [\overline{u(p_3)}\gamma^{\mu} u(p_1)]\; [\overline{u(p_4)}\,\gamma^{\nu} u(p_2)]$$

Evaluation of QCD Colour Factors

- QCD colour factors reflect the gluon states that are involved

$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ $\lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$\lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$ $\lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}$	$\lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ $\lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}$	$\lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ $\lambda^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$
Gluons: $r\bar{g}, g\bar{r}$	$r\bar{b}, b\bar{r}$	$g\bar{b}, b\bar{g}$	$\frac{1}{\sqrt{2}}(r\bar{r} - g\bar{g})$ $\frac{1}{\sqrt{6}}(r\bar{r} + g\bar{g} - 2b\bar{b})$

➊ Configurations involving a single colour



- Only matrices with non-zero entries in **11** position are involved

$$C(rr \to rr) = \frac{1}{4}\sum_{a=1}^{8} \lambda^a_{11}\lambda^a_{11} = \frac{1}{4}(\lambda^3_{11}\lambda^3_{11} + \lambda^8_{11}\lambda^8_{11})$$

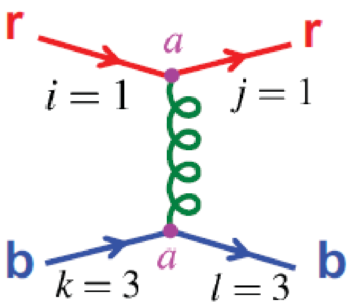
$$= \frac{1}{4}\left(1 + \frac{1}{3}\right) = \frac{1}{3}$$

Similarly find

$$C(rr \to rr) = C(gg \to gg) = C(bb \to bb) = \frac{1}{3}$$

Evaluation of QCD Colour Factors

❷ Other configurations where quarks don't change colour e.g. $rb \to rb$



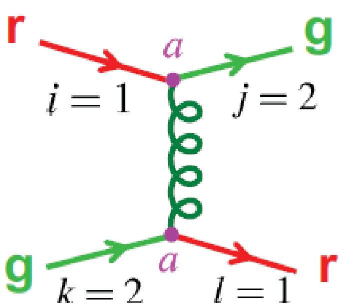
- Only matrices with non-zero entries in 11 and 33 position are involved

$$C(rb \to rb) = \frac{1}{4}\sum_{a=1}^{8}\lambda_{11}^{a}\lambda_{33}^{a} = \frac{1}{4}(\lambda_{11}^{8}\lambda_{33}^{8})$$

$$= \frac{1}{4}\left(\frac{1}{\sqrt{3}}\cdot\frac{-2}{\sqrt{3}}\right) = -\frac{1}{6}$$

Similarly $C(rb \to rb) = C(rg \to rg) = C(gr \to gr) = C(gb \to gb) = C(br \to br) = C(bg \to bg) = -\frac{1}{6}$

❸ Configurations where quarks swap colours e.g. $rg \to gr$



- Only matrices with non-zero entries in 12 and 21 position are involved

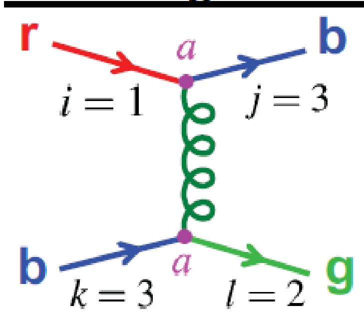
Gluons $r\bar{g}, g\bar{r}$

$$C(rg \to gr) = \frac{1}{4}\sum_{a=1}^{8}\lambda_{21}^{a}\lambda_{12}^{a} = \frac{1}{4}(\lambda_{21}^{1}\lambda_{12}^{1} + \lambda_{21}^{2}\lambda_{12}^{2})$$

$$= \frac{1}{4}(i(-i)+1) = \frac{1}{2}$$

$$C(rb \to br) = C(rg \to gr) = C(gr \to rg) = C(gb \to bg) = C(br \to rb) = C(bg \to gb) = \frac{1}{2}$$

❹ Configurations involving 3 colours e.g. $rb \to bg$



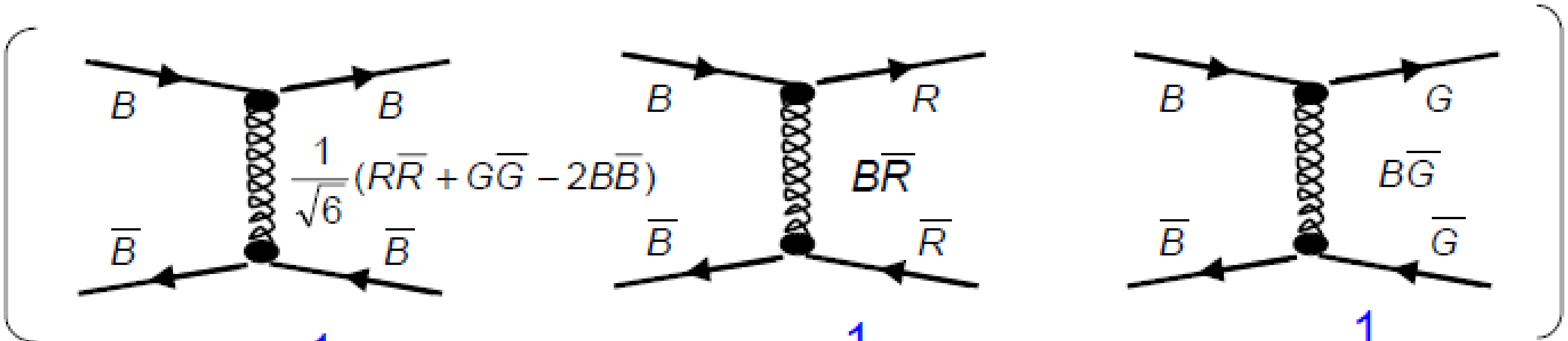
- Only matrices with non-zero entries in the 13 and 32 position
- But none of the λ matrices have non-zero entries in the 13 and 32 positions. Hence the colour factor is zero

★ colour is conserved

Colour Factor for Mesons

Colour-factor for q$\bar{q}$ color singulett state: $\psi = \frac{1}{\sqrt{3}}\,(r\bar{r} + b\bar{b} + g\bar{g}\,)$

3 ×



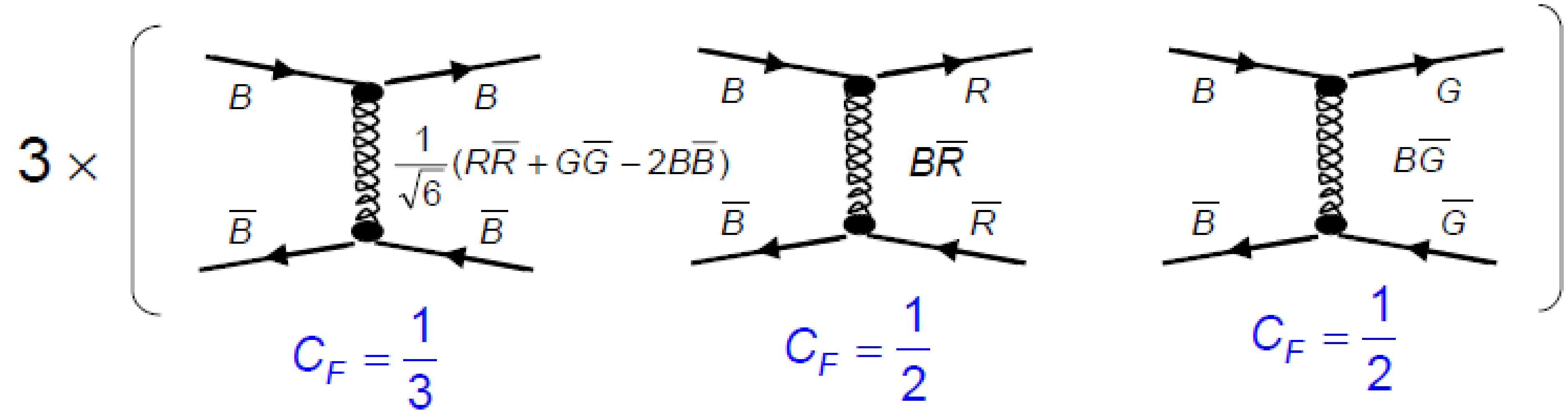
$C_F = \frac{1}{3}$ $\quad$ $C_F = \frac{1}{2}$ $\quad$ $C_F = \frac{1}{2}$

C(b$\bar{b} \rightarrow b\bar{b}) = \frac{1}{4}\sum_{a=1}^{8} \lambda_{33}{}^{a}\lambda_{33}{}^{a}$ $\quad$ C(b$\bar{b} \rightarrow r\bar{r}) = \frac{1}{4}\sum_{a=1}^{8} \lambda_{13}{}^{a}\lambda_{31}{}^{a}$ $\quad$ C(b$\bar{b} \rightarrow g\bar{g}) = \frac{1}{4}\sum_{a=1}^{8} \lambda_{23}{}^{a}\lambda_{32}{}^{a}$

$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$\lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$	$\lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$	$\lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$
$\lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$\lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}$	$\lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}$	$\lambda^8 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$

Colour Factor for Mesons

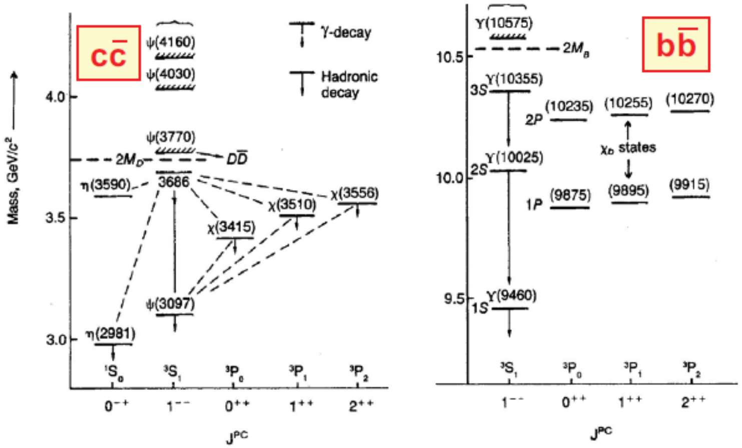
Colour-factor for $q\bar{q}$ color singulett state: $\psi = \frac{1}{\sqrt{3}}\,(r\bar{r} + b\bar{b} + g\bar{g}\,)$



Colour factor of meson: $C_F = 3 * \frac{1}{\sqrt{3}} * \frac{1}{\sqrt{3}} * \left(\frac{1}{3} + \frac{1}{2} + \frac{1}{2}\right) = +\frac{4}{3}$

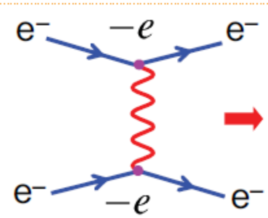
Computing the colour factor for colour octett states, e.g. $\psi = \frac{1}{\sqrt{2}}\,(r\bar{r} - g\bar{g})$ results in negative colour factors (see homeworks) ⟹ sign of colour factors determine if the potential is attractive or repulsive (see next slides).

Form of QCD Potential: Small Distances



Comparision of postironium (e^+e^-) spectroscopy and quarkonium spectroscopy motivate that QED and QCD potential the same at small distances! Masses of c, b quarks (1.5/5 GeV) large compared to electron mass, thus test potential at smaller distances.

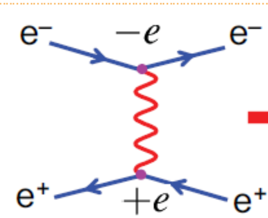
QED



$$V(r) = +\frac{\alpha}{r}$$

Repulsive Potential

Static

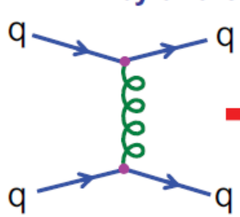


$$V(r) = -\frac{\alpha}{r}$$

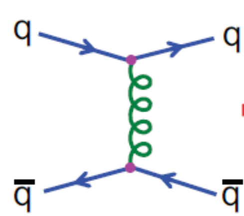
Attractive Potential

QCD

★ by analogy with QED expect potentials of form



$$V(r) = +C\frac{\alpha_S}{r}$$



$$V(r) = -C\frac{\alpha_S}{r}$$

★ Whether it is a attractive or repulsive potential depends on sign of colour factor

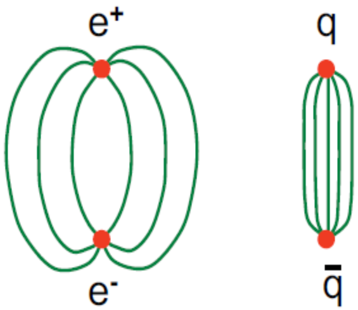
This is by no means a proof,
just an illustration!

Form of QCD Potential: Long Distances

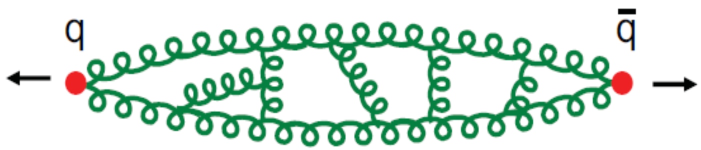
★ Gluon self-interactions are believed to give rise to colour confinement

★ Qualitative picture:

- Compare QED with QCD
- In QCD "gluon self-interactions squeeze lines of force into a flux tube"



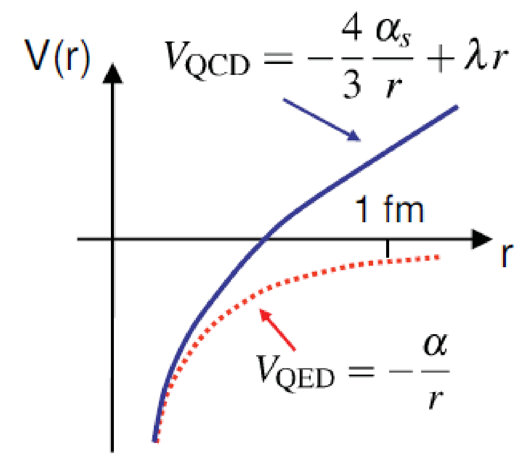
★ What happens when try to separate two coloured objects e.g. $q\bar{q}$



- Form a flux tube of interacting gluons of approximately constant energy density $\sim 1\,\mathrm{GeV/fm}$

➡ $V(r) \sim \lambda r$

- Require infinite energy to separate coloured objects to infinity
- Coloured quarks and gluons are always confined within colourless states
- In this way QCD provides a plausible explanation of confinement – but not yet proven (although there has been recent progress with Lattice QCD)



e.g. potential of a meson

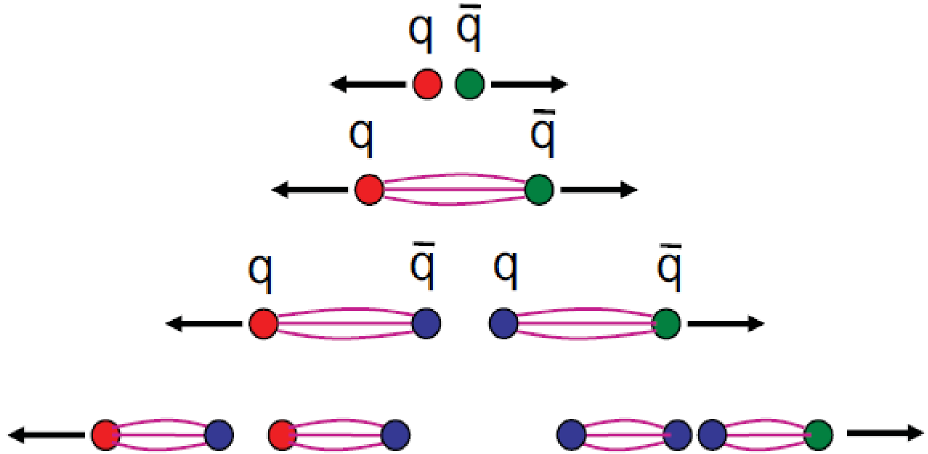
Hadronisation and Jets

★ Consider a quark and anti-quark produced in electron positron annihilation

i) Initially Quarks separate at high velocity

ii) Colour flux tube forms between quarks

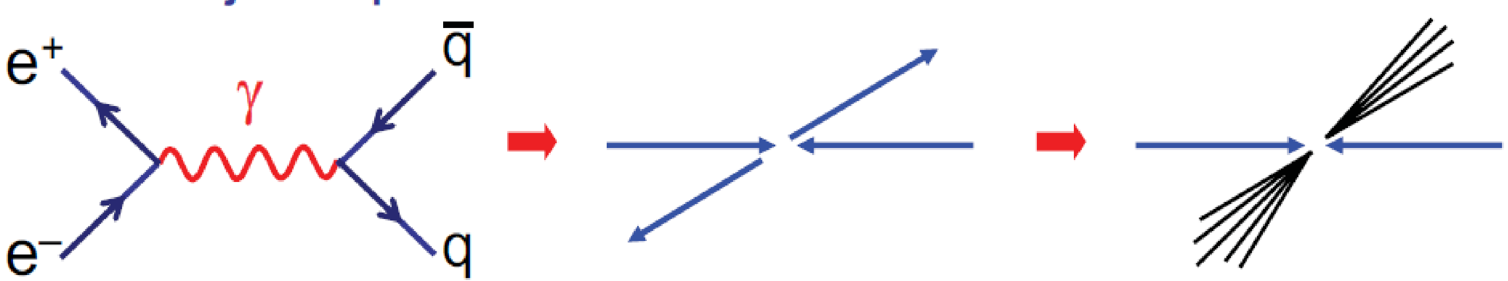
iii) Energy stored in the flux tube sufficient to produce $q\bar{q}$ pairs



iv) Process continues until quarks pair up into jets of colourless hadrons

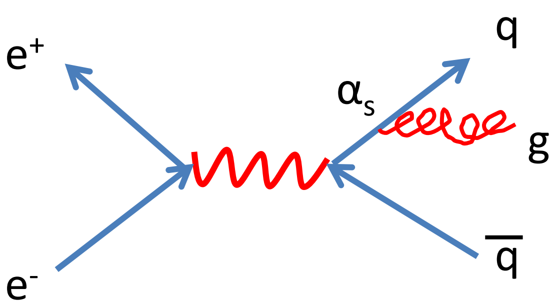
★ This process is called **hadronisation**. It is not (yet) calculable.

★ The main consequence is that at collider experiments **quarks and gluons** observed as jets of particles



Discovery of Gluon

discovery of 3-jet events by Tasso collaboration in 1977 at PETRA ($\sqrt{s} \sim 20\ GeV$)



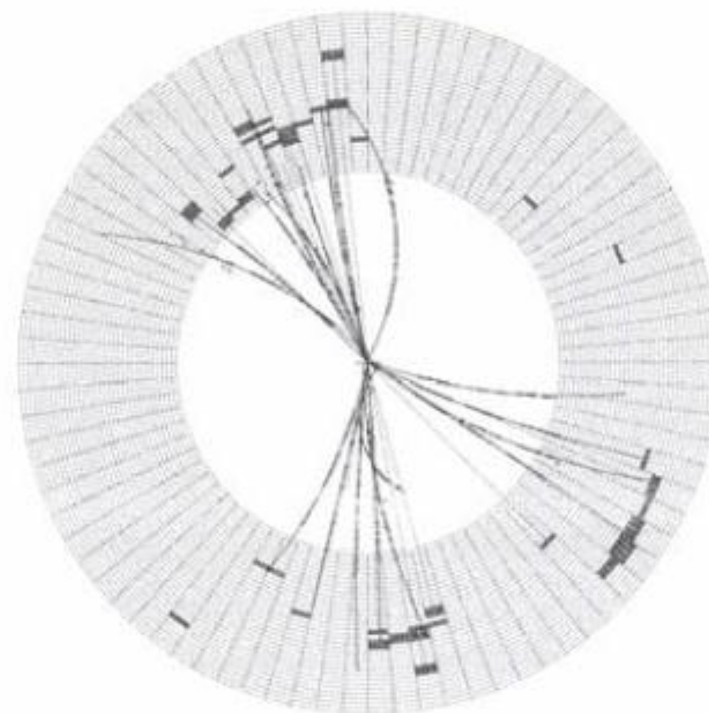


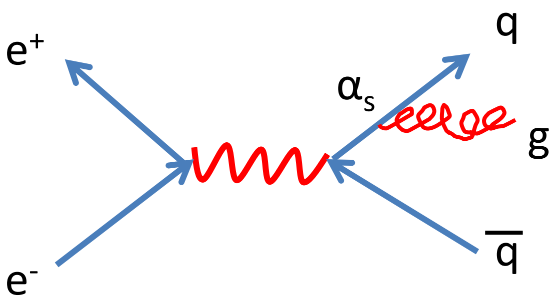
Fig. 11.12 A three-jet event observed by the JADE detector at PETRA

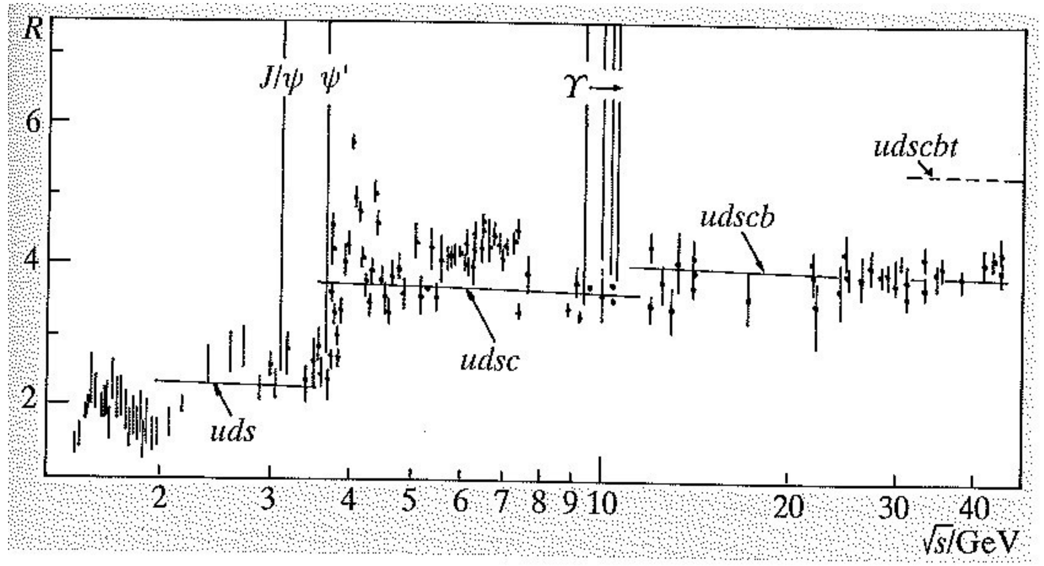
Interpreted as quark anti-quark pair which emits an additional hard gluon.

$$\frac{\#\ of\ three\ jet\ events}{\#\ of\ two\ jet\ events} \sim 0.15$$

⇒ α_s is large!

Reminder: Evidence for Color





q	Z_i^2	$R[\sqrt{s} \leq 2m(q)]$
u	4/9	4/3
d	1/9	5/3
s	1/9	2
c	4/9	10/3
b	1/9	11/3
t	4/9	5

N_c=3 „more or less" confirmed by data!

$$R = \frac{\sigma(e^+e^- \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

$$= N_c \sum_i^u Z_i^2 \ \left(1 + \frac{\alpha_s}{\pi} + 1.411\ \frac{\alpha_s 2}{\pi^2} + \ \ldots\right)$$

original R-factor computation ignored higher QCD corrections (due to large size of α_s) not negligible!

Spin of the Gluon

Ellis-Karlinger angle

Ordering of 3 jets: $E_1 > E_2 > E_3$

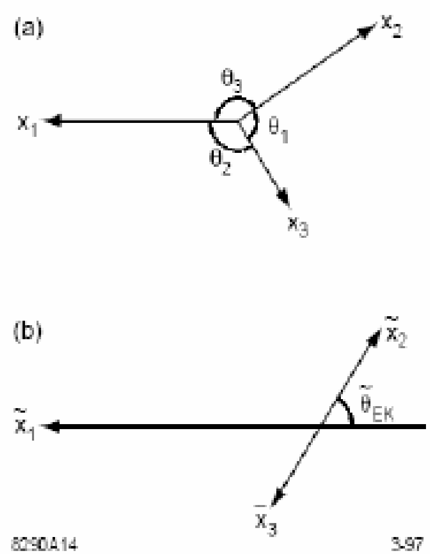


Figure 8: (a) Representation of the momentum vectors in a three-jet event, and (b) definition of the Ellis-Karliner angle.

Measure direction of jet-1 in the rest frame of jet-2 and jet-3: θ_{EK}

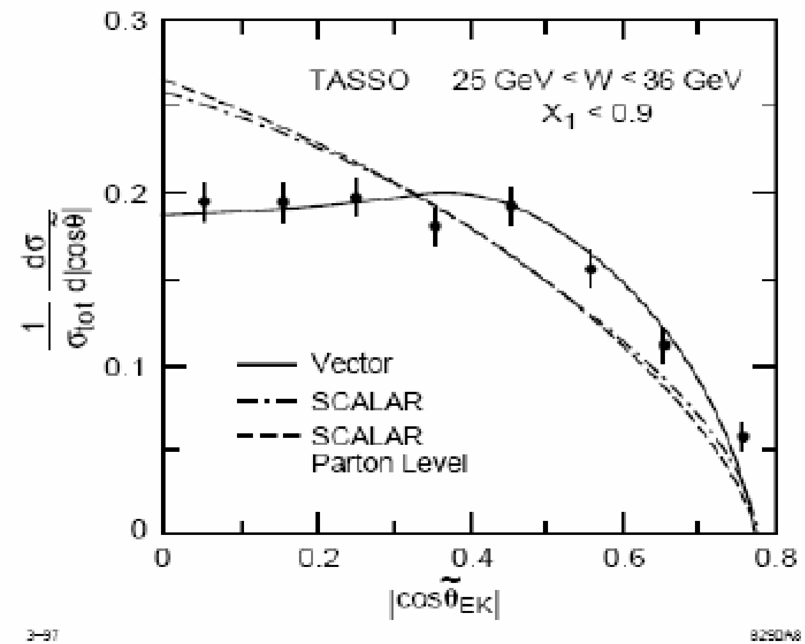


Figure 9: The Ellis-Karliner angle distribution of three-jet events recorded by TASSO at $Q \sim 30$ GeV [18]; the data favour spin-1 (vector) gluons.

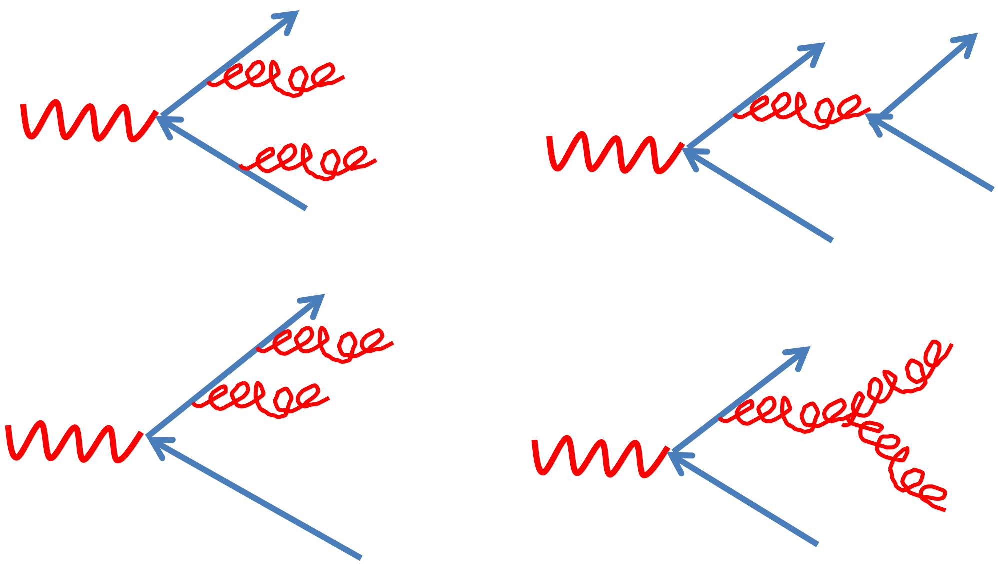
Gluon spin J=1

Multi-Jet Events and Gluon Self Coupling

Gluon self-coupling is a direct consequence of non-abelian SU(3) gauge symmetry!

⟺

Test of gluon-self coupling (strenght) is a test of SU(3).



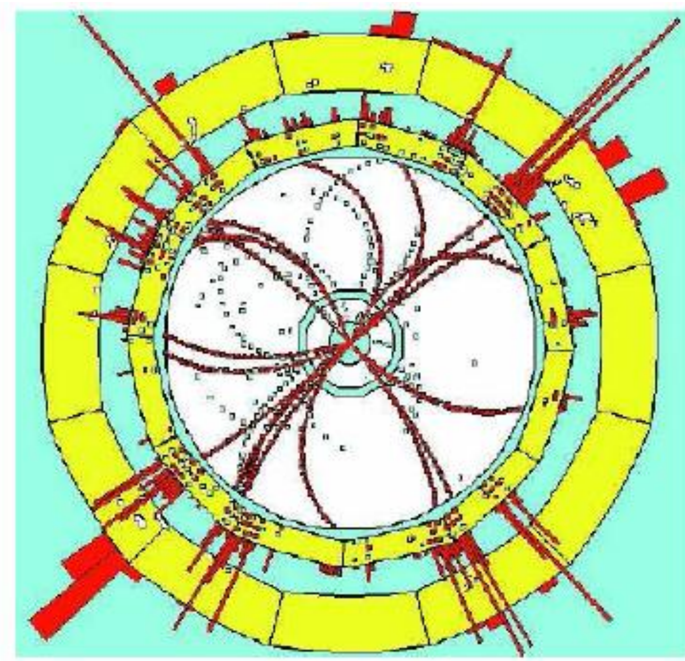


Figure 1: Hadronic event of the type $e^+e^- \rightarrow 4$ jets recorded with the ALEPH detector at LEP-I

Rate of four jet events depend on the colour factors of the involved vertices:
$q \rightarrow gq$, $g \rightarrow q\bar{q}$, $g \rightarrow gg$

Multi-Jet Algorithms

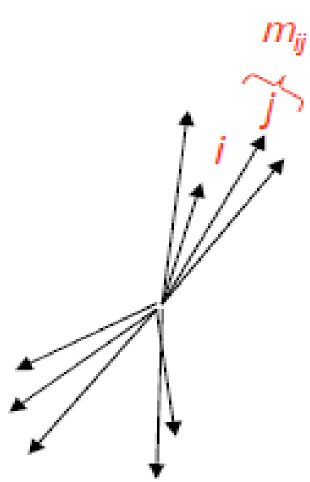
Jet Algorithm

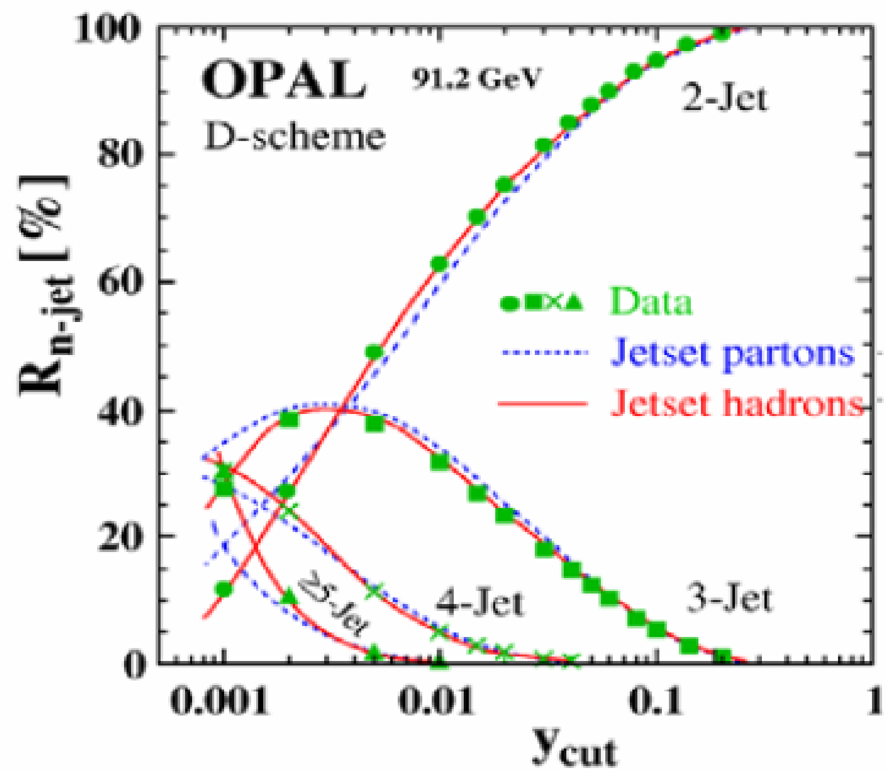
Hadronic particles are i and j grouped to a pseudo particle k as long as the invariant mass is smaller than the **jet resolution parameter**:

$$\frac{m_{ij}^2}{s} < y_{cut}$$

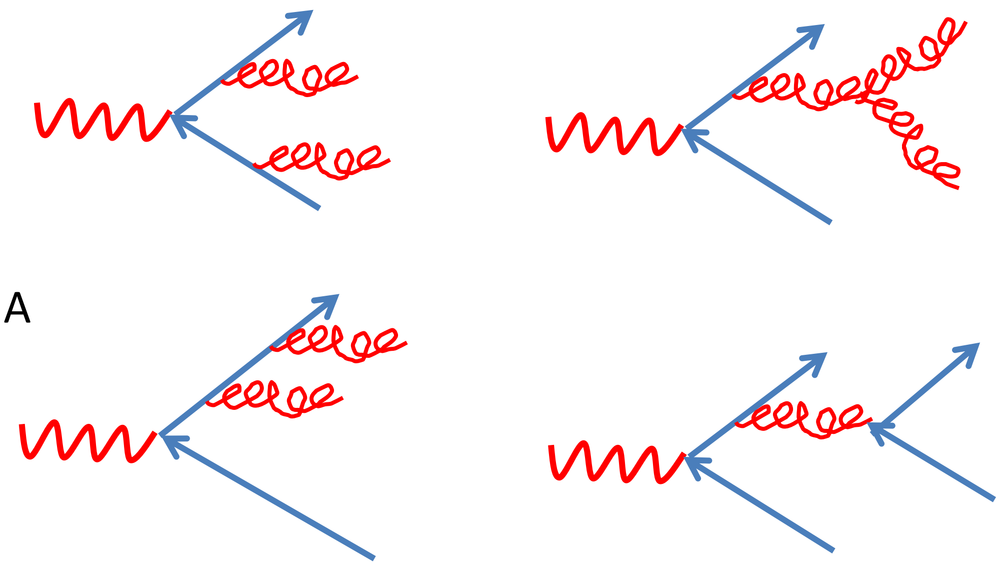
m_{ij} is the invariant mass of i and j.

Remaining pseudo particles are **jets.**





4 Jet Events

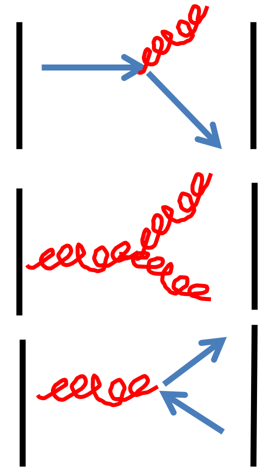


A

B+C

D

(General) colour factors:



$|\ldots|^2 \sim C_F = 4/3$

$|\ldots|^2 \sim N_c = 3$

$|\ldots|^2 \sim T_f = 1/2$

Value of colour factors are a direct consequence of SU(3)

$$\frac{1}{\sigma} d\sigma^4 = \left(\frac{\alpha_s C_F}{\pi}\right)^2 \left[F_A + \left(1 - \frac{1}{2}\frac{N_C}{C_F}\right) F_B + \frac{N_C}{C_F} F_C + \frac{T_F}{C_F} N_f F_D\right]$$

Group	N_C	C_F	T_F
U(1)	0	1	1
SU(N)	N	(N2-1)/2N	1/2
SU(3)	3	4/3	1/2

F_A, F_B, F_C, F_D depend on kinematics and not on symmetry group

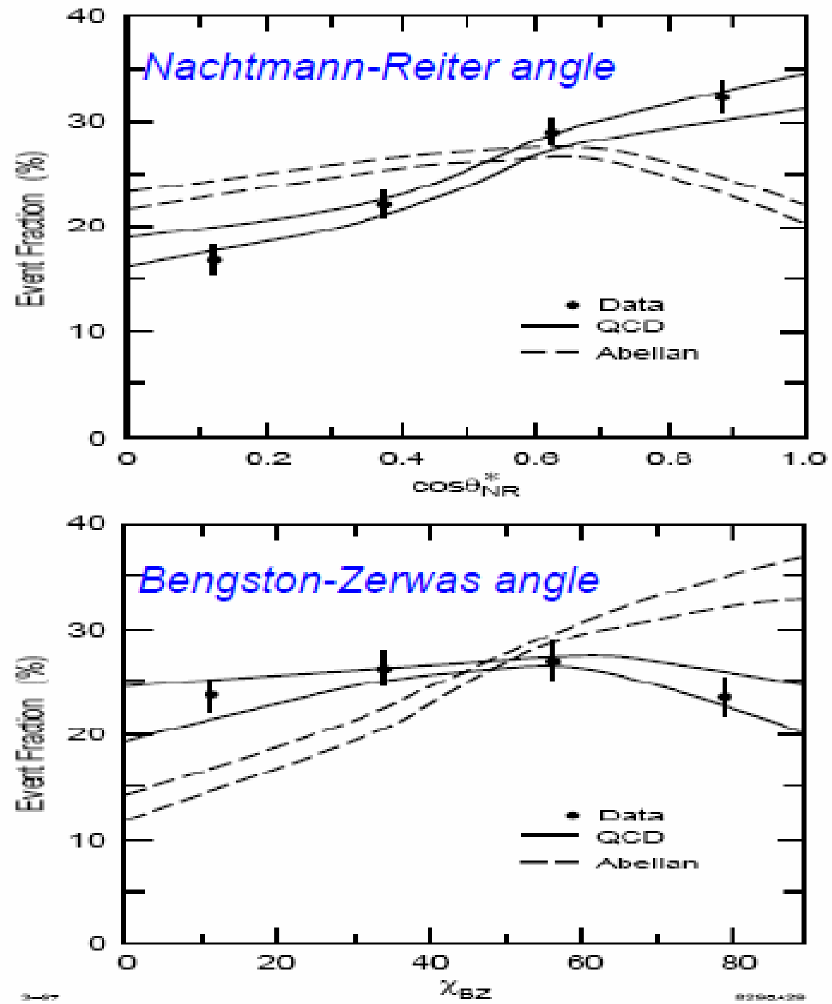
Angular Correlation of jets in 4-jet events

Exploiting the angular distribution of 4-jets:

- Bengston-Zerwas angle
  $\cos \chi_{BZ} \sim (\overrightarrow{p_1} \; x \; \overrightarrow{p_2})(\overrightarrow{p_3} \; x \; \overrightarrow{p_4})$

- Nachtmann-Reiter angle
  $\cos \Theta_{NR} \sim (\overrightarrow{p_1} \; - \; \overrightarrow{p_2})(\overrightarrow{p_3} \; - \; \overrightarrow{p_4})$

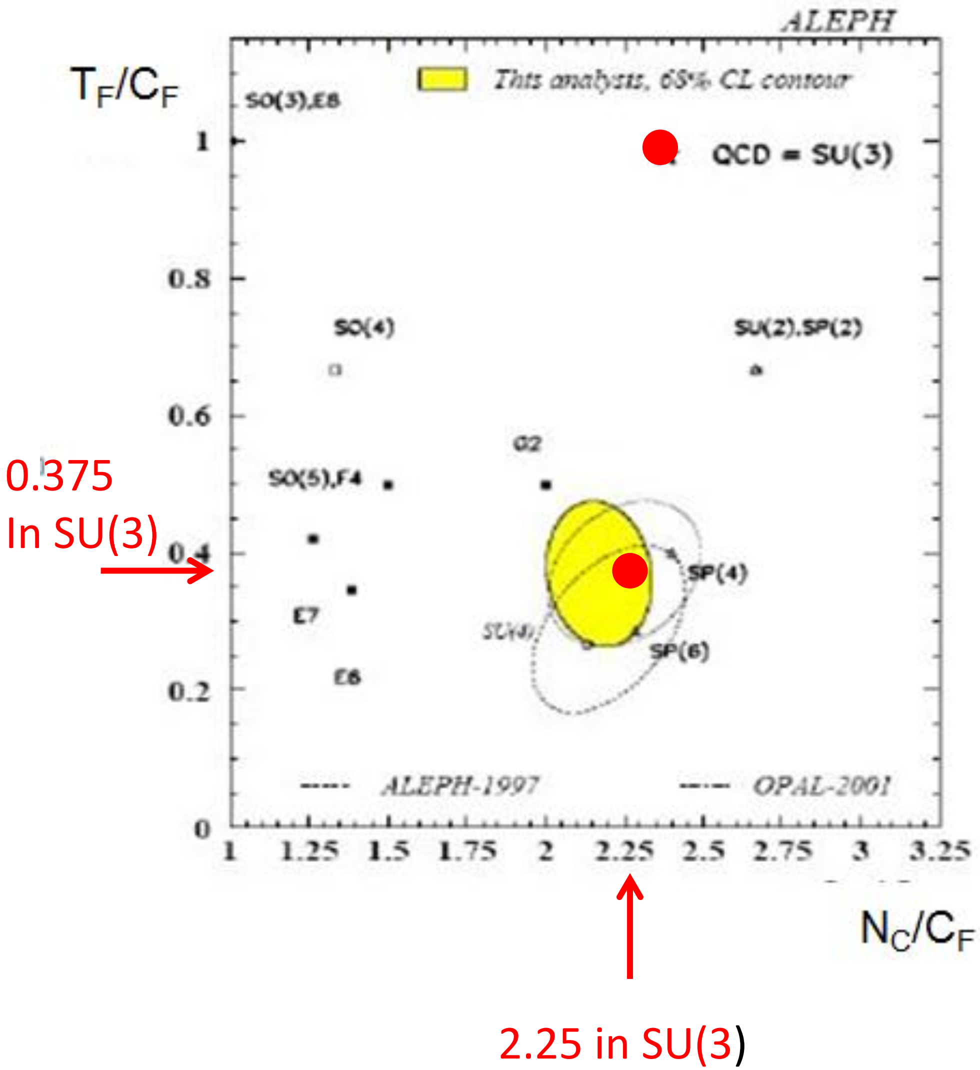
Allow to measure the ratios T_F/C_F and N_C/C_F
SU(3) predicts: $T_F/C_F = 0.375$ and $N_C/C_F = 2.25$



If $N_C/C_F \neq 0 \rightarrow$ contribution from gluon self-coupling in the 4-jet events

^{2} $\sim N_C$

Test of SU(3) Symmetry

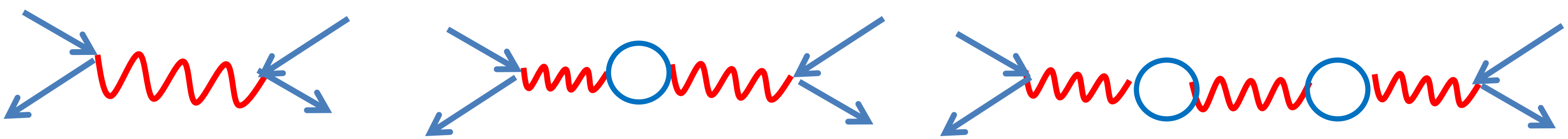


one example of a measurements at the ALEPH experiment (at LEP)

Many more similar analysis exist and confirm precisely SU(3) structure of QCD!

Strong coupling constant α_s

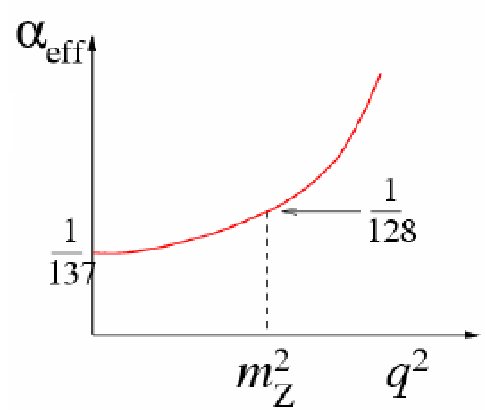
QED: Running coupling constants
due to higher order propgator corrections



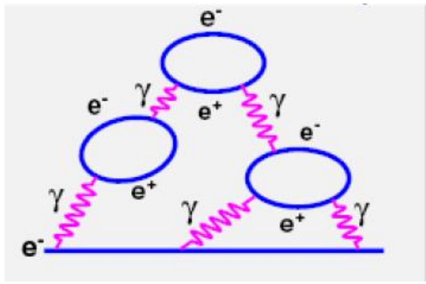
$$\alpha_{eff}(q^2) = \frac{\alpha_0}{1 - \frac{\alpha}{3\pi}\sum_f Q_f^2 \log\frac{q^2}{m_f^2}}$$

$\alpha_0 = 1/137$

note „-“ sign



screeining of bare charge



QCD:

What is the q^2 dependence of α_s?

Strong coupling constant α_s

„screening“ „anti-screening“

Effective strong coupling $\alpha_s(Q^2)$

$$\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1+\alpha_s(\mu^2)\frac{1}{12\pi}(33-2n_f)\log\frac{Q^2}{\mu^2}}$$

$$\beta_0 = \frac{1}{12\pi}(33-2n_f) > 0$$

n_f = # quark flavors (5)
μ^2 = renormalization scale
conventionally $\mu^2 = M_Z^2$

sign of β_0 and thus Q^2 dependence of α_s, depends on number of quark flavours

Effective strong coupling $\alpha_s(Q^2)$

$$\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1+\alpha_s(\mu^2)\,\beta_0\log\frac{Q^2}{\mu^2}}$$

For $Q^2 \to \infty$, $\alpha_s \to 0$
at large Q^2 quarks are
asymtotically free → Quark Parton Model
(Gross&Wilczek (1973), Politzer (1974)

at small values of Q^2, pertubative theory doesn‘t work anymore ($\alpha_s >> 1$), this happens around Λ_{QCD} ~200 MeV (value has to come from experiment, see later)

$$\alpha_s(Q^2) = \frac{1}{\beta_0\log\frac{Q^2}{\Lambda_{QCD}^2}}$$

Strong coupling constant α_s

Effective strong coupling $\alpha_s(Q^2)$

$$\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1+\alpha_s(\mu^2)\,\beta_0 \log\frac{Q^2}{\mu^2}}$$

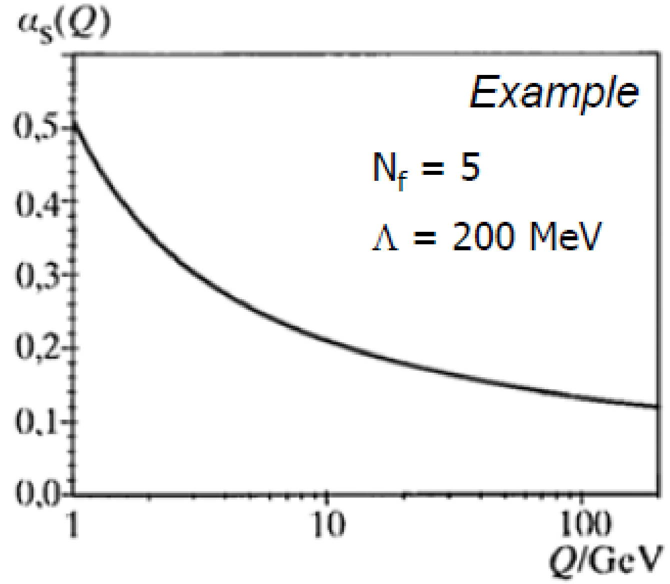
at small values of Q^2, pertubative theory doesn‘t work anymore ($\alpha_s \sim 1$)
This is the reason why hadronic computations are extremly hard to perform

If $\alpha_s(\mu^2)$ is around 1 and Q^2/μ^2 rather large: formular simplifies:

$$\alpha_s(Q^2) = \frac{1}{\beta_0 \log\frac{Q^2}{\Lambda_{QCD}^2}}$$

get Λ_{QCD} value from data!

$\Lambda_{QCD} \sim 200$ MeV [~ 1 fm]



Nobel Prize in 2004

The Nobel Prize in Physics 2004

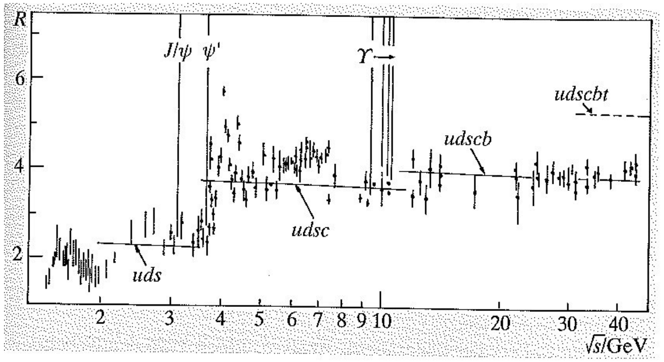
David J. Gross	**H. David Politzer**	**Frank Wilczek**

„for the discovery of asymptotic freedom in the theory of the strong interaction“

Nobel prize was awarded after a lot of experimental results from HERA ($e^{\pm}p$ collider) confirmed this hypothesis

Measurement of α_s

α_s measurements are done at fixed scale Q^2: $\alpha_s(Q^2)$

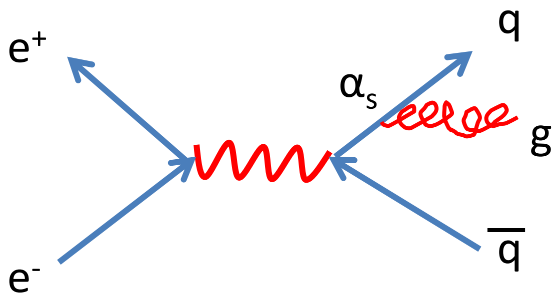


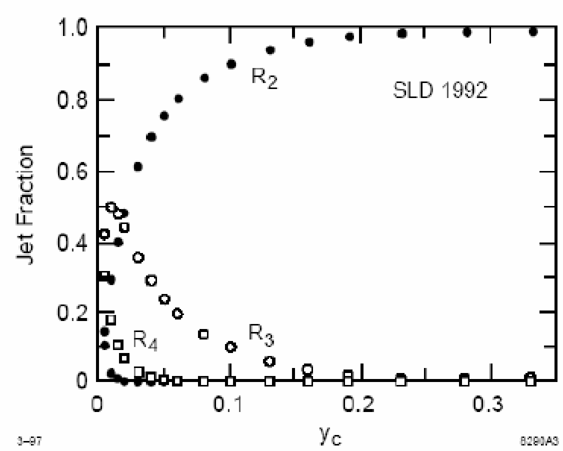
I) α_s from hadronic cross section in e+e- collisions

$$R_{had} = \frac{\sigma(ee \rightarrow hadrons)}{\sigma(ee \rightarrow \mu\mu)} = 3 \sum Q_q^{\,2} \left(1 + \frac{\alpha_s}{\pi} + 1.411 \frac{\alpha_s^{\,2}}{\pi^2} + \ \ldots \right)$$

unfortunately not very precise ...

II) α_s from hadronic event shape variables





n-jet rate as function of y_{cut}

3-jet rate $R_3 = \frac{\sigma_{3\_jet}}{\sigma_{had}}$

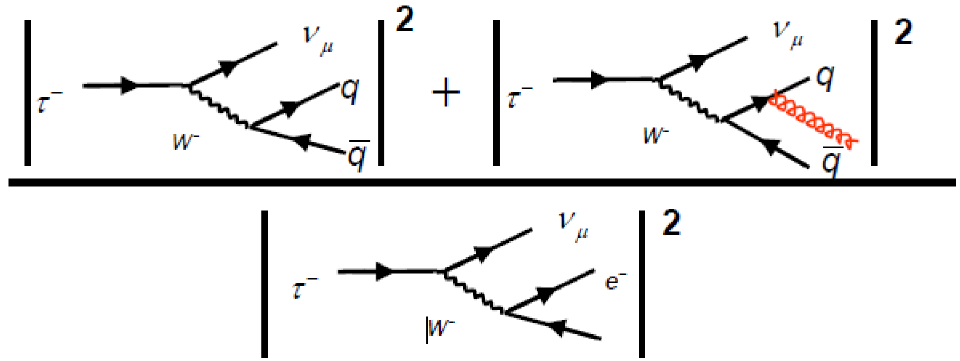
measure R_3 as function of jet paramter y
(similarly other event shape variables can be used)

Measurement of α_s

III) α_s from hadronic τ decays

$$R^{\tau}_{had} = \frac{\Gamma(\tau \to \nu_\tau + hadrons)}{\Gamma(\tau \to \nu_\tau + e + \bar{\nu}_e)} \sim f(\alpha_s)$$

R^{τ}_{had} =

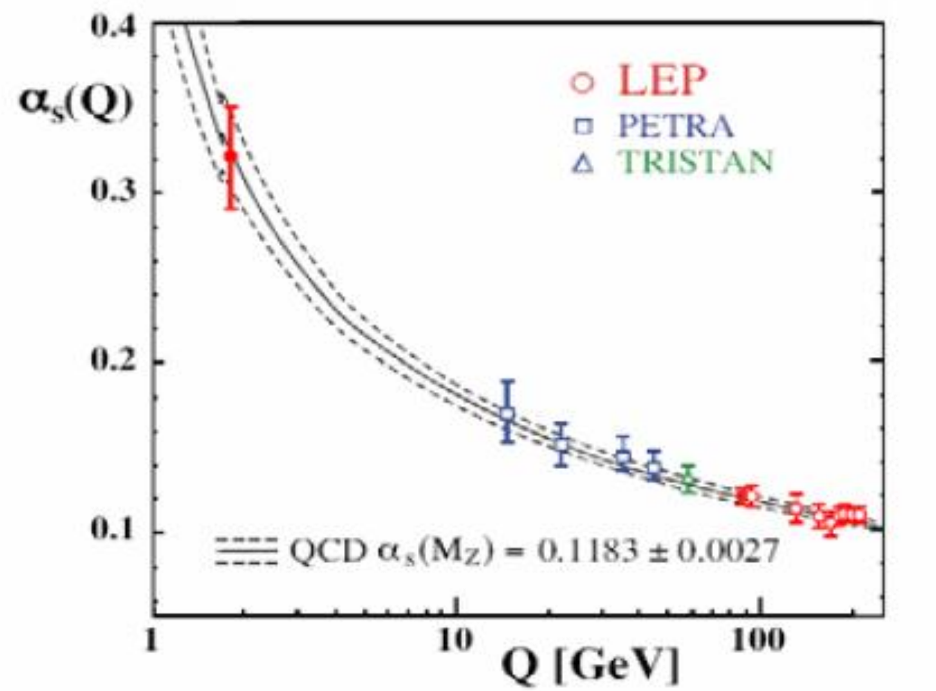


$$R^{\tau}_{had} = \quad R^{\tau,0}_{had} \left(1 + \frac{\alpha_s(m_\tau^{\,2})}{\pi} + \ldots\right)$$

IV) α_s from DIS (deep inelastic scattering)

Running of α_s and Asymptotic Freedom

$$\alpha_s(Q^2) = \frac{1}{\beta_0 \log(Q^2/\Lambda^2_{QCD})}$$



τ mass

~ Z^0 mass