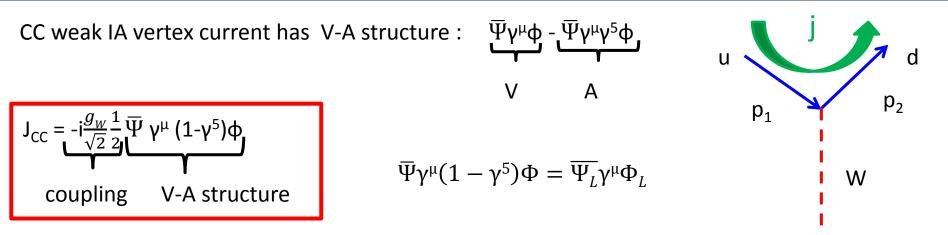
Particle Physics WS 2012/13 (21.12.2012)

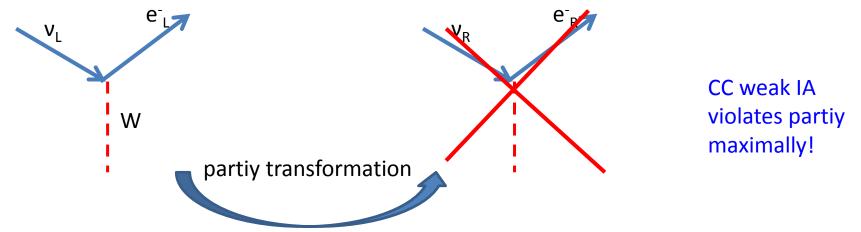
Stephanie Hansmann-Menzemer

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Reminder: Charge Current Weak IA

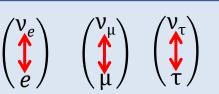


Only left handed chirality particles and right handed chirality anti-particles take part in charged current IA



V-A Coupling for Leptons & Quarks

for leptons

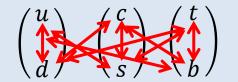


[Lepton flavour violation in SM introduced via neutrino mixing of order 10⁻⁵² (beyond experimental reach)]

transition only inside weak isospin doublets, with universal coupling constants $\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{8m_W}$

for quarks

$$\begin{pmatrix} u \\ \uparrow \\ d' \end{pmatrix} \begin{pmatrix} c \\ \uparrow \\ s' \end{pmatrix} \begin{pmatrix} t \\ \uparrow \\ b' \end{pmatrix}$$



d', s', b' : weak eigenstates ≠ d,s,b : mass/flavour eigenstates

 $\begin{pmatrix} d'\\ s'\\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub}\\ V_{cd} & V_{cs} & V_{cb}\\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d\\ s\\ b \end{pmatrix}$ Cabbibo-Kobayashi-Maswaka (CKM) Matrix

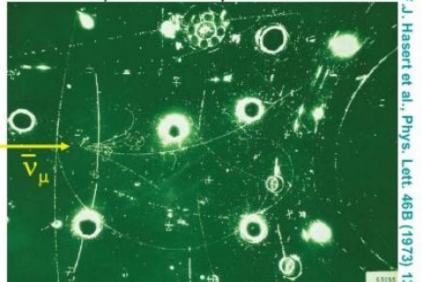
e.g. $B^+ \rightarrow J/\Psi K^+$ $B^+ \qquad \begin{bmatrix} u & V_{cs} & u \\ \overline{b} & V_{cs} & \overline{c} \end{bmatrix} K^+$ latrix V_{cb}^*

CC weak interaction is the only IA in the SM, which violates quark quantum numbers. It is the only one which allow for decay of heavy quarks!

Gargamelle: Discovery of Weak Neutral Current

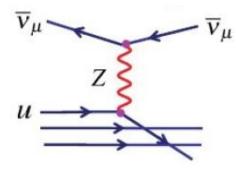
Weak neutral currents observed in Gargamelle bubble chamber in 1973:

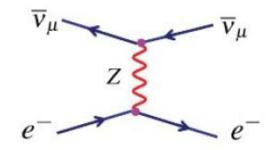
 $\overline{\nu}_{\mu} + N \rightarrow \overline{\nu}_{\mu} + \text{hadrons}$



$$\overline{\nu}_{\mu} + e^- \rightarrow \overline{\nu}_{\mu} + e^-$$

No muon in the final state, thus cannot be a CC interaction!





Neutral current IA appear with a significant rate: $R = \frac{\sigma_{NC}(\nu N \rightarrow \nu X)}{\sigma_{CC}(\nu N \rightarrow \mu X)} \sim 1/3$

Vector and Axial-Vector Couplings

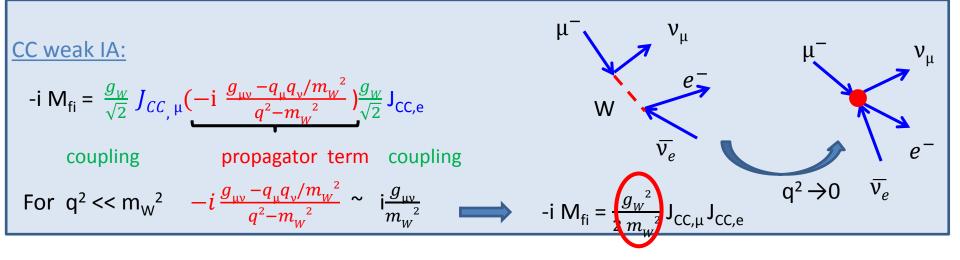
Vertex currents are linear combinations of vector and axial coupling. Relative contributions need to be determined from data. They are different for different particle species.

$$g_{L} = \frac{1}{2} (g_{V} + g_{A})$$
 $g_{R} = \frac{1}{2} (g_{V} - g_{A})$ $J_{NC} = \overline{u} \gamma^{\mu} (g_{R} \frac{1 + \gamma^{5}}{2} + g_{L} \frac{1 - \gamma^{5}}{2}) u$

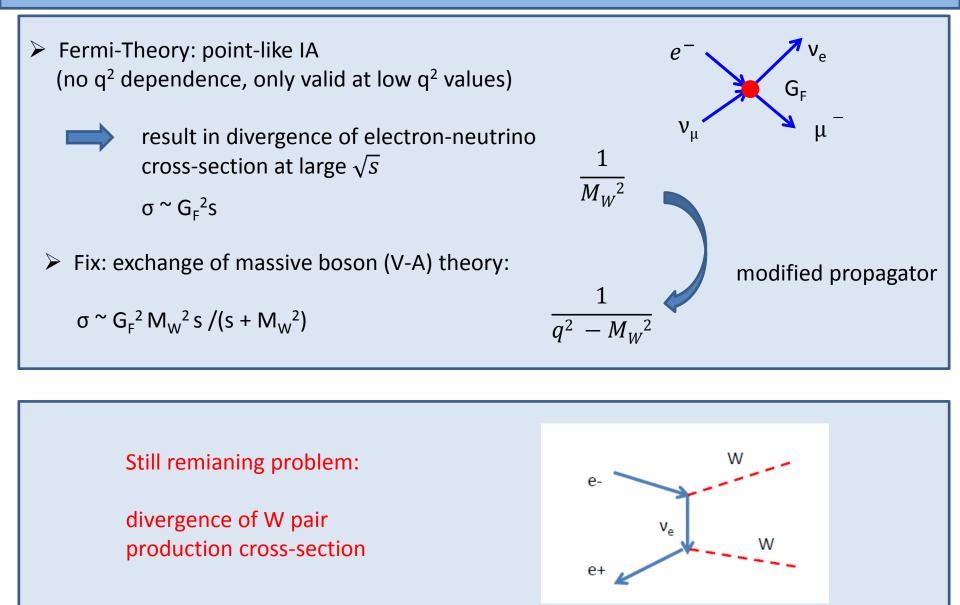
In case of neutrinos: $g_{L}^{\nu} = 1/2$ $g_{R}^{\nu} = 0$ $J_{NC}^{\nu} = \bar{u} \gamma^{\mu} \left(\frac{1}{2} \frac{1-\gamma^{5}}{2}\right) u$ pure left handed neutrino current (this itself is partive violation!)

C

Matrix Elements



Problems with Weak Interaction



Electro-weak Unification

Phenomenological approach to the Standard Model (SM):

- Prerequisites
- Weak isospin and weak hypercharge
- Couplings to gauge fields
- Feynman rules
- Generation of mass

Prerequisites

a) Fundamental fermions:

Leptons

$$\begin{pmatrix} \nu_{e} \\ e^{-} \end{pmatrix}^{L} \begin{pmatrix} \nu_{\mu} \\ \mu^{-} \end{pmatrix}^{L} \begin{pmatrix} \nu_{\tau} \\ \tau^{-} \end{pmatrix}^{L}$$
$$e_{R}^{-} \mu_{R}^{-} \tau_{R}^{-}$$
$$\begin{pmatrix} u \\ d' \end{pmatrix}^{L} \begin{pmatrix} c \\ s' \end{pmatrix}^{L} \begin{pmatrix} t \\ b' \end{pmatrix}^{L}$$

 $u_R, d_R c_R, s_R t_R, b_R$

Left-handed doublets

right-handed singletts

weak IA

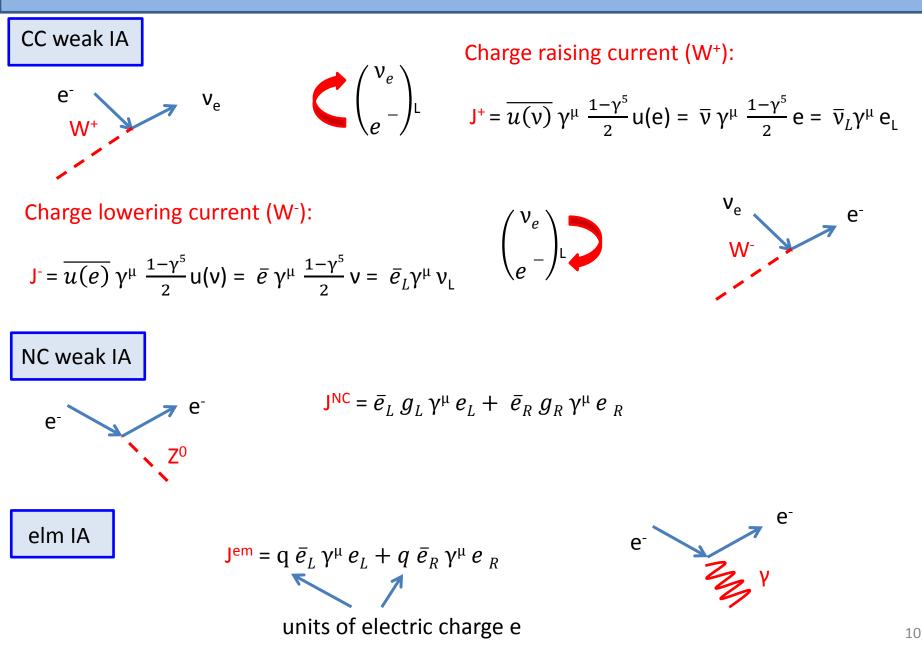
Quarks

b) Fundamental interactions:

- Charged current interaction: transition inside LH doublets
- Neutral current interaction: couples to LH and RH fermions

Electromagnetic interaction couples equally to LH and RH fermions

Fundamental Interactions



Weak Isospin and Weak Hypercharge

In analogy to the strong isospin one can describe the particles of the LH doublets as $T_3 = \pm \frac{1}{2}$ states of a particle with weak isospin T = $\frac{1}{2}$

$$\chi_{\rm L} = \begin{pmatrix} \nu \\ e \end{pmatrix}_{\rm L} \qquad \qquad T = \frac{1}{2}, T_3 = \pm \frac{1}{2}$$

As for (iso)spin one can use the raising and lowering operators defined by the Pauli matrices to express state transitions.

 $\tau^{\pm} = \frac{1}{2} (\tau^{1} \pm i \tau^{2}) \quad \tau^{i} = \sigma^{i} = Pauli-matrices$

The current can be written in the compact form: $J^{\pm} = \overline{\chi_L} \gamma^{\mu} \tau^{\pm} \chi_L$

From the SU(2) structure of isospin formalism, one expects that in addition to the currents J^{\pm} there exists a 3^{rd} neutral current J^{3} of the form:

$$J_{3} = \overline{\chi_{L}} \gamma^{\mu} T_{3} \tau_{3} \chi_{L} = \frac{1}{2} \overline{\nu_{L}} \gamma^{\mu} \nu_{L} - \frac{1}{2} \overline{e_{L}} \gamma^{\mu} e_{L}$$

Weak isospin triplett of LH fermion currents $J_i = \overline{\chi_L} \gamma^{\mu} \frac{\tau_i}{2} \chi_L$

Pauli-matrices

 $\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

 $\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ $\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

 $e_{L} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad v_{L} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

Electro-Weak Unification

The third current J³ is NOT equal to the weak neutral current J^{NC}:

J^{NC} contains LH and RH fermion contributions!

However there is a 2nd NC containing LH and RH fermion contributions: J^{em}

→ Treat both neutral currents, J^{em} and J^{NC}, simultaneously:

As both currents contain RH contributions it should be possible to contruct a linear Combinations which couples only to LH fermions:

Two linear combinations of J^{em} and J^{NC}:

 $J_3 = sin^2 \Theta_W J^{em} + J^{NC}$ Choose Θ_W such that RH fermions components in J³ vanish

 $1/2 J^{Y} = \cos^2 \Theta_{W} J^{em} - J^{NC}$

 $J_3 + 1/2J^{Y} = J^{em}$

 \rightarrow J^Y = 2J^{em} - 2J₃

J₃ completes the weak isospin current triplett Jⁱ J^Y is called hypercharge current, couples via hypercharge

Hypercharge

$$J^{\gamma} = 2 J^{em} - 2 J^{3} = 2Q \overline{\Psi} \gamma^{\mu} \Psi - 2 T_{3} \overline{\Psi} \gamma^{\mu} \Psi = 2(Q-T_{3}) \overline{\Psi} \gamma^{\mu} \Psi$$

$$\gamma^{\mu} \Psi$$

Hypercharge operator: $Y = 2[Q-T_3]$

(Gell-Mann Nishijima Formula)

Electro-weak quantum numbers

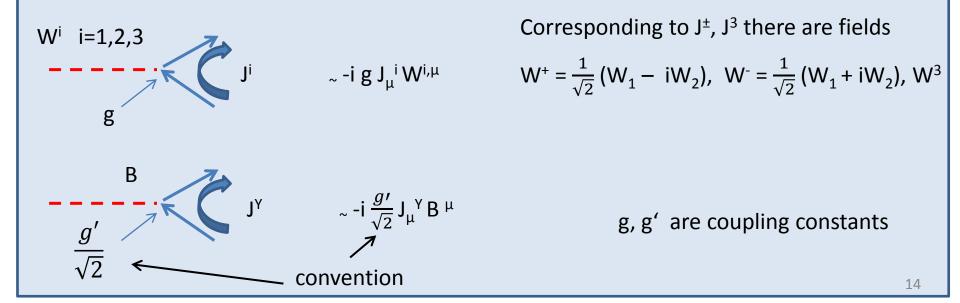
Leptons	Т	T ₃	Q	Y
v _e e _L		+1/2 -1/2	0 -1	-1 -1
e _R	0	0	-1	-2

Quarks	Т	T ₃	Q	Y
u _L	1/2	+1/2	2/3	1/3
d' _L	1/2	-1/2	-1/3	1/3
u _R	0	0	-1	-2
d _R	0	0	-1/3	-2/3

Current Coupling to the Gauge Fields/Bosons

Reminder QED: Coupling to photon is described by IA with the the photon (gauge) field

In electro-weak theory the coupling between boson and fermions is defined in analogy to the coupling of the photon to the fermions currents in QED. There are in total 4 boson fields:



Gauge Bosons

While the charged boson fields W[±] correspond to the observed W bosons, the neutral fields B and W³ only correspond to linear combinations of the observed photon and Z boson:

$$A_{\mu} = B_{\mu} \cos \Theta_{W} + W^{3}_{\mu} \sin \Theta_{W}$$

$$Z_{\mu} = -B_{\mu} \sin \Theta_{W} + W^{3}_{\mu} \cos \Theta_{W}$$

$$B_{\mu} = A_{\mu} \cos \Theta_{W} - Z_{\mu} \sin \Theta_{W}$$

$$W^{3}_{\mu} = A_{\mu} \sin \Theta_{W} + Z_{\mu} \cos \Theta_{W}$$

the mixing of is introduced

the mixing of the neutral fields which is introduced here ad hoc is generated through the Symmetry Breaking of the Higgs-mechanism.

The weak mixing angle Θ_W (Weinberg angle) is defined by the coupling constants to A^{μ} and Z^{μ} .

Gauge Bosons

The fermion coupling to the neutral fields are given by

$$B_{\mu} = A_{\mu} \cos \Theta_{W} - Z_{\mu} \sin \Theta_{W}$$
$$W^{3}_{\mu} = A_{\mu} \sin \Theta_{W} + Z_{\mu} \cos \Theta_{W}$$

 $(J^{Y} = 2J^{em} - 2J_{3})$

$$-i g J_{\mu}^{3} W^{3,\mu} - i \frac{g'}{2} J^{\mu}_{\mu} B^{\mu} = -i [g \sin \Theta_{W} J_{\mu}^{3} + \frac{g'}{2} \cos \Theta_{W} J^{\mu}_{\mu}] A^{\mu}$$
$$+ i [g \cos \Theta_{W} J_{\mu}^{3} - \frac{g'}{2} \sin \Theta_{W} J^{\mu}_{\mu}] Z^{\mu}$$

Fermion coupling to the photon

$$-ie J_{\mu}^{em} A^{\mu} = -ie [J_{\mu}^{3} + \frac{1}{2} J^{\gamma}] A^{\mu}$$

Comparison of the coefficients give:

 $e = g \sin \Theta_W$ $e = g' \cos \Theta_W$ $\implies \tan \Theta_W = g/g'$

Fermion coupling to the Z boson

+ i
$$[g \cos \Theta_W J_{\mu}^3 - \frac{g'}{2} \sin \Theta_W J_{\mu}^y] Z^{\mu} = i \frac{g}{\cos \Theta_W} [\cos \Theta_W^2 J_{\mu}^3 - \frac{1}{2} \sin \Theta_W^2 J_{\mu}^y] Z^{\mu}$$

 $= i \frac{g}{\cos \Theta_W} [\cos \Theta_W^2 J_{\mu}^3 - \sin \Theta_W^2 (J^{em} - J^3)] Z^{\mu}$
 $= i \frac{g}{\cos \Theta_W} [J_{\mu}^3 - \sin \Theta_W^2 J^{em}] Z^{\mu}$

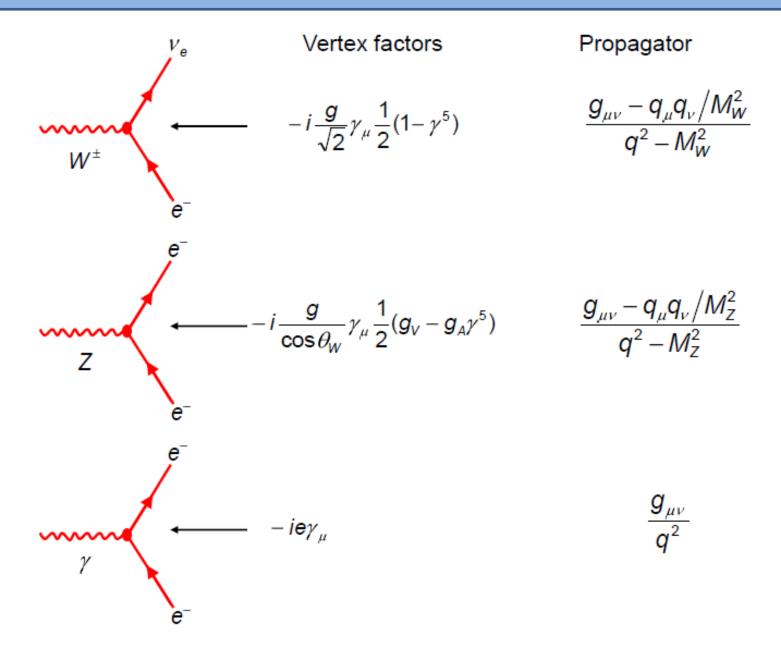
Fermion Coupling to Z Boson

$$\begin{split} \overline{\Psi}\gamma^{\mu} \ \frac{1}{2}(g_{V} - g_{A}\gamma^{5})\Psi &= \mathsf{J}^{\mathsf{NC}} \\ &= \mathsf{J}^{3}\text{-}\sin^{2}\Theta_{\mathsf{W}}\mathsf{J}^{\mathsf{em}} \\ &= \mathsf{T}_{3}\overline{\Psi}\gamma^{\mu} \ \frac{1-\gamma^{5}}{2}\tau_{3}\Psi - \mathsf{q}\sin^{2}\Theta_{\mathsf{W}}\overline{\Psi}\gamma^{\mu}\Psi \end{split}$$

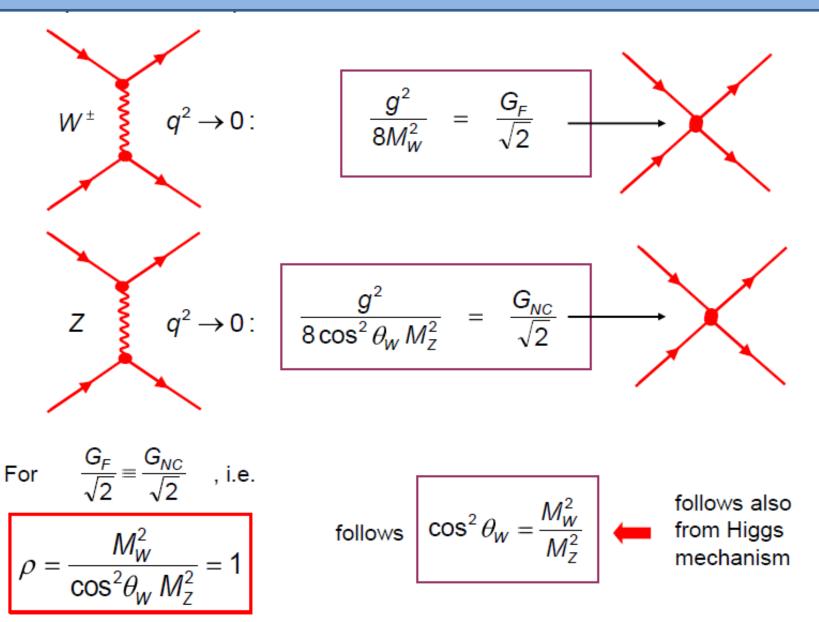
 \rightarrow g_V = T₃ - 2q sin² Θ_W g_A = T₃

	g _v	g _A
ν	1/2	+1/2
charged lepton 1-	-1/2+2sin ² Θ_{W}	-1/2
u-type-quark	+1/2-4/3sin ² 0 _W	+1/2
d-type-quark	-1/2+2/3sin ² 0 _w	-1/2

Feynman Rules



Comparison of the $q^2 \rightarrow 0$ limit with 4-Fermion Ansatz



Massive gauge bosons?

So far: Electroweak unification

However: Z, W are assigned ad hoc, for the moment they are still mass less Need additional trick to assign masses to all particles (quarks, exchange bosons, ...)



Symmetry breaking via the Higgs-Mechanism

A scalar field with a non-vanishing vacuum expectation value v, couples to the Boson and fermion fields and generates the marticles masses through these couplings. (more details in future lectures)

For the boson masses one finds:
$$M_W = \frac{1}{2} v g$$

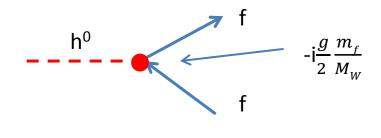
 $M_Z = \frac{1}{2} v \sqrt{g^2 + g'^2}$
 $M_Z = \frac{g}{\sqrt{g^2 + g'^2}} = \cos \Theta_W$
 $g \sin \Theta_W = g' \cos \Theta_W$

For the Higgs mass itself one finds: $M_{H} = 2v^{2}\lambda$

Parameter λ describes the potential and can as v not be predicted by theory.

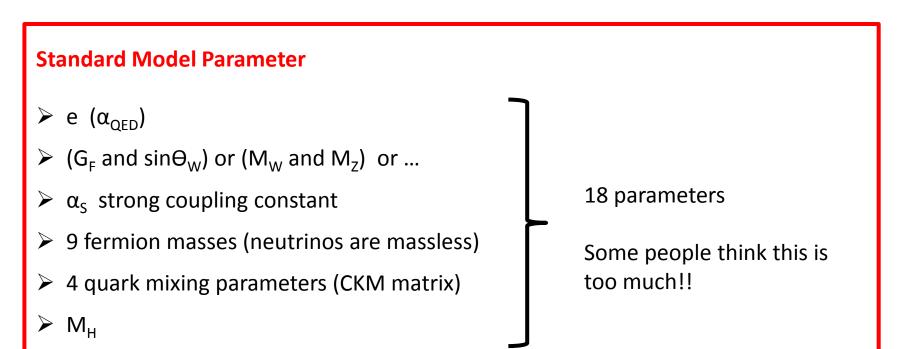
Parameters of the SM

Higgs coupling to fermions and fermion masses



with fermion masses $mf = \frac{G_f v}{\sqrt{2}}$

G_f is an unknown fermion dependent (Yukawa) Coupling constant.



Summary: Historical Background

1934: Fermi-Theorie of point like IA

 $\mathsf{M}_{\mathsf{fi}} = \mathsf{G}_{\mathsf{F}} \, \mathsf{g}_{\mu\nu} \, [\overline{\Psi}\gamma^{\mu}\Psi] [\overline{\Psi}\gamma^{\nu}\Psi$

Later extended to describe partiy:

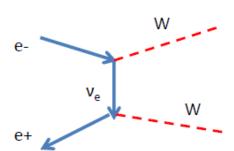
 $\mathsf{M}_{\mathsf{fi}} = \frac{G_F}{\sqrt{2}} \mathsf{g}_{\mu\nu} \left[\overline{\Psi} \gamma^{\mu} (1 - \gamma^5) \Psi \right] \left[\overline{\Psi} \gamma^{\nu} (1 - \gamma^5) \Psi \right]$

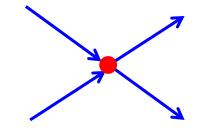
Then extended to include propagator term and massive exchange boson

$$\mathsf{M}_{\mathsf{fi}} = \frac{g_{W}}{\sqrt{2}} \,\overline{\Psi} \frac{1}{2} \gamma^{\mu} \left(1 - \gamma^{5}\right) \Psi \frac{g_{\mu\nu} - q_{\mu}q_{\nu}/mW^{2}}{q^{2} - mW^{2}} \frac{g_{W}}{\sqrt{2}} \,\overline{\Psi} \frac{1}{2} \gamma^{\mu} \left(1 - \gamma^{5}\right) \Psi$$

Problem: W production cross section violates unitary (~ s)

1967: Glashow, Salem and Weinberg Electroweak unification W





Summary: Electroweak Unification

In analogy to strong IA, introduce weak isospin

Left handed particles from isospin doublets $T = \frac{1}{2}$, $T_3 = \pm \frac{1}{2}$

$$\begin{pmatrix} \nu_e \\ e^{-} \end{pmatrix}^{L} \begin{pmatrix} \nu_{\mu} \\ \mu^{-} \end{pmatrix}^{L} \begin{pmatrix} \nu_{\tau} \\ \tau^{-} \end{pmatrix}^{L} \begin{pmatrix} u \\ d' \end{pmatrix}^{L} \begin{pmatrix} c \\ s' \end{pmatrix}^{L} \begin{pmatrix} t \\ b' \end{pmatrix}^{L}$$

Right handed particles form isospin singletts T = 0

$$e_R^{-}, \mu_R^{-}, \tau_R^{-}, u_R^{-}, d_R^{-}, c_R^{-}, s_R^{-}, t_R^{-}, b_R^{-}$$

 W_1 , W_2 , W_3 are generators of SU(2)_{iso}

Ladder operators $W^{\pm} = W_1 - / + i W_2$

W₃ eigenvalue: third component of weak isospin

Hypercharge $Y = 2(Q-T_3)$

Electroweak theory described by $SU(2)_{iso} \times U_{Y}(1)$

4 mass less gauge fields W_1 , W_2 , W_3 and B

Couple to left handed particles only

v_e e⁻

couples to left and right handed particles

Summary: Electroweak Unification

 $SU_{iso}(2) \times U_{\gamma}(1)$ contains subgroup $U_{em}(1)$

A is not identical to the both neutral currents B and W_3 ;

 $B \neq A$ and $W_3 \neq A$ elm. IA does not couple to left handed neutrinos!

Solution: A is linear combination of W₃ and B

$A_{\mu} = B_{\mu} \cos \Theta_{W} + W_{\mu}^{3} \sin \Theta_{W}$ $Z_{\mu} = -B_{\mu} \sin \Theta_{W} + W_{\mu}^{3} \cos \Theta_{W}$	— massless photon — massive Z boson
$B_{\mu} = A_{\mu} \cos \Theta_{W} - Z_{\mu} \sin \Theta_{W}$ $W^{3}_{\mu} = A_{\mu} \sin \Theta_{W} + Z_{\mu} \cos \Theta_{W}$	

Weinberg angle Θ_{W} defined by couplings of A and Z.

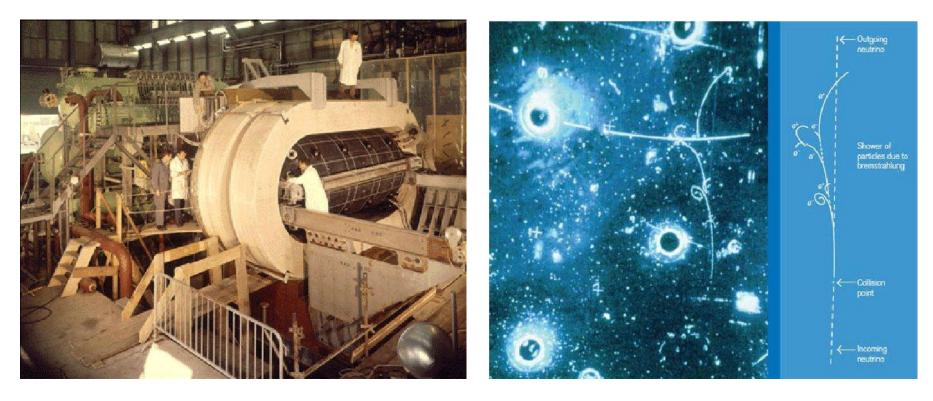
From experiment $\sin \Theta_W^2 = 0.23143 \pm 0.00015$ $\cos \Theta_W = \frac{M_W}{M_Z}$

Problem: no mass term in Lagrangian (otherwise spoils gauge invariance, thus theory is unphysical)

Require dedicated mechanism to create masses \rightarrow HIGGS mechanism (1964)

Next event: Discovery of Neutral Currents

NC was first theoretical introduced in GSW theory in 1967 and then discovered in 1973



 $\overline{
u}_{\mu} + e^-
ightarrow \overline{
u}_{\mu} + e^-$

Despite this experimental "proof" GSW model not widely accepted before 1977 t'Hooft and Veltmann demonstrated reonormalization of this theory.

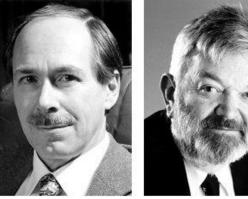
Nobel Prizes

Nobel Prize for Glashow, Salem and Weinberg (1979)



Sheldon Glashow, Abdus Salam, and Steven Weinberg sharing the Nobel Prize, 1979

Nobel Prize for t'Hooft and Veltmann (1999)

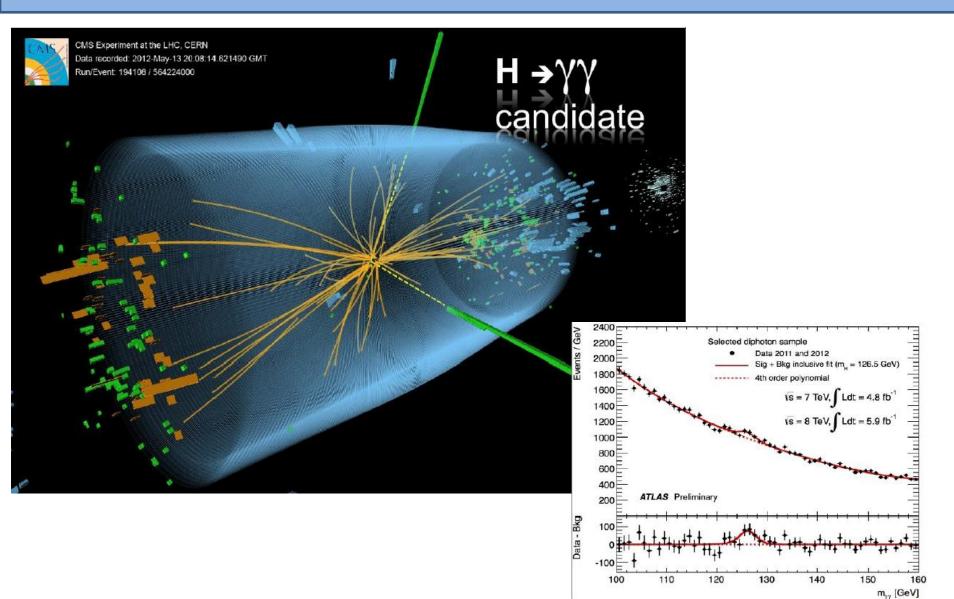


Gerardus 't Hooft

Martinus J.G. Veltman

The Nobel Prize in Physics 1999 was awarded jointly to Gerardus 't Hooft and Martinus J.G. Veltman "for elucidating the quantum structure of electroweak interactions in physics"

Discovery of the Higgs Boson 2012 at the LHC!





All the best wishes for X-mass and the New Year!