3. Gas Detectors

3.1. General introduction

principle
- ionizing particle creates primary and secondary charges via energy loss by ionization (Bethe-Bloch, chapter 2)
  \( N_0 \) electrons and ions
- charges drift in electric field
- generally gas amplification in the vicinity of an anode wire
- signal generation

different operational modes depending on strength of electric field

after F. Sauli, 1977 lecture notes
Charge carriers in layer of thickness $L$ for a mean energy $W$ to produce electron-ion pair

- mean number

about 2-6 times the primary number (see chapter 2)

important for spatial resolution: secondary ionization by $\delta$ – electrons happens on length scale $10 \, \mu m$

e.g. $T_e = 1 \, \text{keV}$ in isobutane $R = 20 \, \mu m$

- ionization statistics

$\lambda = 1/n_e \cdot \sigma_I$ mean distance between ionization events with cross section $\sigma_I$

mean number of ionization events $\langle n \rangle = L/\lambda$

Poisson distribution about mean $\langle n \rangle$

and specifically probability for no ionization

$P(0) = \exp -\langle n \rangle = \exp (- L/\lambda)$

efficiency of gas detectors allows determination of $\lambda$ and hence $\sigma_I$

$\lambda$ (cm)

typical values:

<table>
<thead>
<tr>
<th>Gas</th>
<th>$\sigma_I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>He</td>
<td>$10^{-22}$ cm$^2$ or 100 b</td>
</tr>
<tr>
<td>air</td>
<td>0.053</td>
</tr>
<tr>
<td>Xe</td>
<td>0.023</td>
</tr>
</tbody>
</table>
3.2. Charge Transport

3.2.1 Ion mobility

drift along field lines in external E-field plus superimposed random thermal motion

ion transfers in collisions with gas atoms typically half of its energy $\rightarrow$ kinetic energy of ion is approximately thermal energy

$$\left\langle T_{\text{ion}}(E) \right\rangle \approx \left\langle T_{\text{ion}}(\text{thermal}) \right\rangle = \frac{3}{2} kT$$

drift velocity in direction of $\vec{E}$: (thermal velocity has random orientation relative to $\vec{E}$)
assume at $t = 0$, $u_e = 0$ and typical collision time $\tau$

directly prior to collision

$$\vec{u}_e = \vec{a} \cdot \tau = \frac{eE}{m} \cdot \tau$$

mean drift velocity of ion

$$\left\langle v_0 \right\rangle = \frac{1}{2} u_e = \frac{eE}{2m} \cdot \tau = \mu_+ \left\langle E \right\rangle$$

where $\tau \propto \lambda \propto 1 / \sigma_+ \equiv$ constant since $\left\langle T \right\rangle$ essentially thermal

$\mu_+ \equiv$ ion mobility
electrons drift towards anode of a gas detector in a given field with a constant velocity, measurement of drift time allows to determine point of ionization

$$\Delta t = \frac{L}{v_D}$$

equation of motion of electron in superimposed $\vec{E}$ and $\vec{B}$ – fields (Langevin):

$$m \frac{d\vec{v}}{dt} = e\vec{E} + e(\vec{v} \times \vec{B}) + \vec{Q}(t)$$

with instantaneous velocity $\vec{v}$ and a stochastic, time dependent term $Q(t)$ due to collisions with gas atoms

assume:
- collision time $\tau$
- $\vec{E}$ and $\vec{B}$ constant between collisions
- consider $\Delta t \gg \tau$ (averaging)
Drift velocity \( v_0 \) is given by

\[
\langle m \frac{d\vec{v}}{dt} \rangle = e (\vec{E} + \langle \vec{v} \rangle \times \vec{B}) - \frac{m}{c} \vec{v}_D
\]

\[ \vec{v}_D = \vec{u}_- \vec{E} \]

\[ \mu_+ = \frac{e\vec{v}}{m} = \mu \]

\[ \omega = \frac{e\vec{B}}{m} \]

Compared to ions \( \mu_+ \ll \mu_- \) since \( M \gg m \)

- **2 types of gases:**

  a) **Hot gases:** atoms with few low-lying levels, electron loses little energy in a collision with atom \( \rightarrow T_e \gg kT \)

    Acceleration in E-field and friction lead to constant \( v_D \) for a given \( \vec{E} \)

    "Free fall with friction"

    \[ \lambda(T_e) = \lambda(|\vec{E}|) \]

    \[ \mu \propto \frac{1}{\delta(|\vec{E}|)} \]

    **But**

    \[ \lambda(T_e) = \lambda(|\vec{E}|) \]

    **And**

    Not constant

Typical drift velocity: \( v_D = 3-5 \text{ cm/\mu s} \) for 90 % Ar / 10 % CH\(_4\) (methane)
b) **cold gases**: many low-lying degrees of freedom
→ electrons lose kinetic energy they gain in between collisions
(similar to ions)
\[ T_e \approx kT \]
\[ \mu \approx \text{constant} \]
\[ v_D \propto |E| \]

examples: Ar/CO\textsubscript{2} or Ne/CO\textsubscript{2}
in latter \[ \mu \approx 7 \times 10^{-3} \text{ cm}^2/\mu\text{s V} \text{ at 10\% CO}_2 \] or \[ v_D = 2 \text{ cm/\mu s at 300 V/cm} \]
\[ 3.5 \times 10^{-3} \text{ cm}^2/\mu\text{s V} \text{ 20\%} \]
3.2.3. Electron loss

with some probability a free electron is lost during drift

a) recombination \( \text{ion}^+ + e^- \)
decrease in number of negative/positive charge carriers

\[
\frac{d\mathbf{v}}{dt} = \rho_v \cdot n^+ n^- \\
\text{coefficient of recombination} \approx 10^{-7} \text{ cm}^3/\text{s}
\]

generally not important
b) electron attachment
   on electro-negative molecules, probability could be large
   \[ e^- + M \rightarrow M^- \] for \[ T_e \approx 1 \text{ eV} \]
   otherwise dissociative attachment
   \[ e^- + XY \rightarrow X + Y^- \]
   for gases like \( O_2, Cl_2, \) freon, \( SF_6 \) probability per collision order of \( 10^{-4} \)
   capture coefficient \( p_c \) is strongly energy dependent (e.g. in \( O_2 \) there is a minimum at \( 1 \text{ eV} \)
   “Ramsauer effect”)
   per second electron undergoes order of \( 10^{11} \) collisions → for drift time of \( 10^{-6} \) s fraction lost
   \( X \) loss depends on partial oxygen pressure
   \[ X_{loss} = 10^{-4} \cdot 10^{11}/s \cdot 10^{-6}s \cdot PO_2/PA_{tm} \]

   \( \rightarrow \) less than 1% lost for \( PO_2/PA_{tm} \leq 10^{-3} \)

   remark: in presence of certain quencher gases such as \( CO_2 \) the effect of \( O_2 \) is enhanced
   by multistep catalytic reaction
   - 10 ppm \( O_2 \) can lead to 10 % loss within 10 \( \mu s \) \( \rightarrow \) need to keep oxygen out of gas
3.2.4. Diffusion

original ionization trail diffuses (spreads apart) with drift time ↔ effect on space point and momentum resolution, ultimate limit

a) only thermal motion ($|\vec{E}| = |\vec{B}| = 0$)
mean thermal velocity

$$\langle v \rangle = \frac{\lambda}{c}$$

$$\langle T \rangle = \frac{1}{2} m \langle v \rangle^2$$

time between collisions
mean free path

for a pointlike source at time $t = 0$, collisions between electrons and gas atoms (molecules) → smearing

spread of charge cloud at time of first collision

$$R^2 = 2 \lambda^2$$

and after $n = t / \tau$ collisions

$$\sigma^2(t) = 2\lambda^2 \frac{t}{\tau}$$

define diffusion coefficient

$$D = \frac{\sigma^2(t)}{2t}$$

for $|\vec{E}| = |\vec{B}| = 0$

$$D = D_0 = \frac{\lambda_0^2}{\tau} = \frac{2\langle T \rangle}{m} \frac{c}{\tau}$$
diffusion is \textit{isotropic}
longitudinal diffusion coefficient
transverse diffusion coefficient

→ after time $t$ charge cloud has width
\[
\sigma(t) = \sqrt{(D \cdot 2t)}
\]
respectively, in each dimension
\[
\sigma_x(t) = \sigma_y(t) = \sigma_z(t) = \sqrt{(1/3 \cdot D \cdot 2t)}
\]

charge distribution

diffusion equation:

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{j} = 0 \]

\[ \mathbf{j} = -D \nabla \rho \]

\[ \frac{\partial \rho}{\partial t} - D \Delta \rho \]

charge density defined by

\[ N(x) = c \cdot \exp\left( -\frac{x^2}{2\sigma_x^2} \right) \]

\[ \rho(r, t) = c \cdot \exp\left(-\frac{r^2}{4Dt}\right) \]

solved by
hot gases: \( \langle T \rangle \gg \frac{3}{2} \frac{k}{\mu} T \)  

D large

cold gases: \( \langle T \rangle \approx \frac{3}{2} \frac{k}{\mu} T \)  

D small

\[
\Theta = \frac{2}{3} \frac{\langle T \rangle}{m} \gamma \quad \text{and} \quad \mu = \frac{e}{m} \gamma
\]

\[\varepsilon_k = \frac{2}{3} \langle T \rangle = \frac{e}{\mu}\]

can define a characteristic energy

for hot gas the same characteristic energy is reached at much lower T

Fig. 2.9. Characteristic energy of electrons in Ar and CO\textsubscript{2} as a function of the reduced \( \varepsilon \). The electric field under normal conditions is also indicated. The parameters refer to temperatures at which the measurements were made [SCH 76]
b) diffusion in B-field
\[ \vec{B} = B\vec{e}_z \]
along B no Lorentz force

\[ D_L(\vec{B}) = D_{0L} = \frac{1}{3} D_0 \]

in transverse direction Lorentz force helps charge cloud together, i.e. it counteracts diffusion

\[ D_T(B) = \frac{D_0 T}{1 + \omega t \tau^2} \]

for \( \vec{B} \) large \( \rightarrow \omega t \gg 1 \) \( D_T(B) \ll D_{0T} \)

e.g. Ar/CH\(_4\) \( B = 1.5 \) T \( D_T(1.5 \) T) \( \cong 1/50 \) D

transverse \( \sigma^2 \) as function of L
c) diffusion in electric field: ordered drift along field superimposed to statistical diffusion
mobility \( \mu \) is function of \( \langle T \rangle \)

\[ \vec{v}_0 = \mu(\langle T \rangle) \cdot \vec{E} \]

\[ D_L \neq \frac{1}{\varepsilon} D_T \]

\[ \sigma_x = \sqrt{2D_L} = \sqrt{\frac{2eE}{cE}} \]

longitudinal diff. in E-field

\[ \delta^2(t) = 2D_L t = 2D_L \frac{v_0}{\mu} = \frac{3kT}{e\varepsilon} L_D \]

\[ \sigma^2(t) / L_D = \frac{3kT}{e\varepsilon} \]

in hot gases: for large \( E \), \( D_L < D_T \)
and values are large
in cold gases: \( D_L \approx D_T \) small

\[ \text{Graph showing variation of } \sigma_x \text{ with } E \text{ for different gases.} \]
3.2.5 Exact solution

of drift and diffusion by solving a “transport equation”

electron density distribution \( f(t, \mathbf{r}, \mathbf{v}) \)

Boltzmann – equation:

\[
\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f + \frac{\partial}{\partial \mathbf{v}} \hat{g} = \mathcal{Q}(t)
\]

flow term

external forces

collision term (stochastic)

\[
\hat{g} = (\frac{\mathbf{E}}{m} + \mathbf{v} \times \mathbf{\Omega}) f
\]

numerical solution with codes such as Magboltz & Garfield
Garfield + Magboltz calculation

Figure 7: Drift velocity (top left), Lorentz angle (top right), longitudinal and transverse diffusion constants (middle) and longitudinal and transverse diffusion constants normalized to the square root of the number of charge carriers (bottom) for different mixtures of noble gas and CO₂.
3.3. Gas amplification

in case anode is a (thin) wire, E-field in vicinity of wire very large \( E \propto \frac{1}{r} \)
and electron gains large kinetic energy
in order to obtain large $E$ and hence large $\Delta T$ \rightarrow very thin wire \ (r_i \approx 10 - 50 \ \mu m) \n
within a few wire radii \rightarrow \Delta T$ large enough for secondary ionization
strong increase of $E \rightarrow$ avalanche formation for $r \rightarrow r_i$
First Townsend coefficient $\alpha$

number of electrons

$N(x) = N_0 \exp(\alpha x)$

gas gain

mean free path

since $l_0 = \frac{1}{\alpha} = \frac{1}{N \sigma(t)}$

and

$T = T(G)$

and

$\alpha = \alpha(x)$

gas gain

typically

up to $10^6$ possible in proportional mode

limit: discharge (spark) at $\alpha x \approx 20$ or $G = 10^8$ “Raether - limit”
gas gain and ionization by collisions
1. Townsend coefficient

Fig. 4.5. Energy dependence of the cross section for ionization by collision [104, 139, 140].
excitation of gas generates UV – photons which in turn can lead to photoeffect in gas and on cathode wire contributing thus to avalanche

$$\gamma = \frac{\# \text{ photo effect events}}{\# \text{ avalanche electrons}}$$

gas gain including photoeffect

$$G_{\gamma} = G + G(\gamma G) + G(\gamma G)^2 + \ldots = \frac{G}{1 - \gamma G}$$

gas gain including photoeffect

limit: $\gamma G \to 1$ continous discharge independent of primary ionization

to prevent this, add to gas so-called quench-gas which absorbs UV photons strongly leading to excitation and radiationless transitions

examples: $\text{CH}_4$, $\text{C}_4\text{H}_{10}$, $\text{CO}_2$
gas gain by photo effect and second Townsend coefficient

Fig. 4.6. Energy dependence of the cross section for photoionization
3.4. Ionization chamber

no gas gain, charges move in electric field and induce signal in electrodes

2 electrodes form parallel plate capacitor
consider motion of a free charge $q$: electric field does work, capacitor is charged (lowering in energy of capacitor)

$$q \dot{\vec{\nabla}} \varphi \cdot d\vec{x} = dq_i \cdot U_0$$

leads to induced current

$$I_{\text{ind}} = \frac{q}{U_0} \dot{\vec{\nabla}} \varphi \cdot \vec{v}_b$$

with $\vec{c} = -\dot{\vec{\nabla}} \varphi$ and $U_0 = \varphi_1 - \varphi_2$
- current is constant while charge is drifting
- total induced signal (charge) independent of $x_0$
- signal induced by electrons
- signal induced by ions

\[
\Delta q_- = \frac{N_e}{U_0} \left( \phi(x_0) - \phi_e \right)
\]
\[
\Delta q_+ = -\frac{N_e}{U_0} \left( \phi(x_0) - \phi_e \right)
\]

\[
\Delta q = N_e
\]

**Practical problem:** ion drift comparatively slow

\[w_+ = 10^{-3} \ldots 10^{-2} w_-\] (except for semiconductors: typ. $w_+ \approx 0.5 w_-$)

Induced current and charge for parallel plate case, ratio $w_-/w_+$ decreased for clarity

signal generated during drift of charges
- induced current ends when charges reach electrodes
- induced charge becomes constant (total number $N_e$)
- signal shaping by differentiation (speed of read-out) $\rightarrow$ suppresses slow ion component

\[ \text{change in potential } dU = \frac{dQ}{C} \]

usually electronic signal shaping needed
signal shaping by RC – filter

\[ \Delta Q^{+} \] is charge induced in anode by motion of ions and electrons for total number of ionization events in gas \( N_e \)

\[ \Delta U = \Delta U^{-} + \Delta U^{+} = \frac{\Delta Q^{-}}{C} + \frac{\Delta Q^{+}}{C} \]

**without filter**

\[ \Delta Q = N_e \frac{\phi(x_0)-\phi_i}{u_0} = N_e \frac{x_0}{\partial t} \]

\[ d - x_0 = v^{+} \Delta t^{+} \]

**with filter**

\[ \Delta U = \frac{N_e}{C} \]

\[ v^{+}RC \left(1 - \exp\left(-\frac{\Delta t^{+}}{RC}\right)\right) \]

**Damping of ion component**

Fast rise and decrease of signal but now pulse height depends on \( x_0 \)
trick: introduce additional grid “Frisch–grid”

while electrons drift towards Frisch-grid, no induced signal on anode, only on FG

as soon as electrons pass Frisch-grid, signal induced on anode

choose $U_G$ such that E-field is unchanged

electrons pass grid

difficulty: small signals

example 1 MeV particle stops in gas

$$N_e = \frac{10^6 eV}{35 eV} \approx 3 \times 10^4$$

$$C = 100 \mu F$$

$$\frac{1}{10^{-10} F}$$

$$\Delta U_{max} = \frac{3 \times 10^4 \times 1.6 \times 10^{-19} C}{10^{-10} F} = 4.6 \times 10^{-5} V$$

need sensitive, low-noise preamplifier
application: e.g. cylindrical ionization chamber for radiation dosimetry

\[ \Delta Q^- = \frac{N_e}{U_0} \int E(r) dr = \frac{N_e}{\ln(r_a/r_i)} \ln \left( \frac{r_i}{l_0} \right) \]

\[ \Delta U^- = \frac{\Delta Q^-}{C} \]

\[ \Delta U^+/\Delta U^- = \frac{\ln(r_a/l_0)}{\ln(r_i/l_0)} \]

in cylindrical geometry, ion signal dominates by typically factor 100
- cylindrical capacitor filled with air
- initially charged until potential $U_0$
- ionization continuously discharges capacitor
- reduction of potential $\Delta U$ is measure for integrated absorbed dose
  (view via electrometer, e.g.)

**other applications:** measure energy deposit of charged particle, should be highly ionizing (low energy) or even stop (then measure total kin. energy) nuclear physics experiments with energies of 10 to 100 MeV combination of $\Delta E$ and $E$ measurements -> particle id (nuclei)