Two Particle Interferometry – Determination of 'Size' of Fireball
The method: Pion HBT interferometry

When phase space volume smaller than $\Delta p_x \Delta x \approx \hbar$ is considered, chaotic system of identical non-interacting particles exhibits quantum fluctuations following Bose-Einstein or Fermi statistics

First application in astrophysics (Hanbury Brown and Twiss) $\rightarrow$ size of stars

$$C_2 \propto \frac{P_2(\vec{p}_1, \vec{p}_2)}{P_1(\vec{p}_1)P_2(\vec{p}_2)} = 1 \pm \chi(\vec{p}_1 - \vec{p}_2)$$

$$\Delta r = \frac{\hbar c}{q} = \frac{197 \text{MeV}}{q} \text{fm}$$

in heavy ion physics typical dimensions 1-10 fm $\rightarrow$ momentum differences of 20-200 MeV/c

more complications, but also more information for non-static source: duration of emission, space-momentum correlations due to expansion, strong & EM interaction, decays of resonances ... measure $C_2$ as function of $\Delta p_x, \Delta p_y, \Delta p_z$ for all $y, p_t, m$
The two particle correlation function is related via Fourier transformation to source emission function

e.g. a Gaussian source profile in x direction leads to a two boson correlation function \( C(p_x^1, p_x^2) = 1 + \exp(-q_x^2/2\sigma_x^2) \)

typically there is also correlation due to Coulomb repulsion, data need to be corrected before fitting correlation function to extract geometry of source
The vector momentum difference as analyzed in terms of its 3 components:

**Bertsch-Pratt Parametrization**  \( p_1 - p_2 = q = (q_{\text{long}}, q_{\text{side}}, q_{\text{out}}) \rightarrow R_{\text{long}}, R_{\text{side}}, R_{\text{out}} \)

\[
C(\vec{q}, \vec{P}) = \frac{\left\{ n\vec{p}_1 n\vec{p}_2 \right\}}{\left\{ n\vec{p}_1 \right\} \left\{ n\vec{p}_2 \right\}} = 1 + \lambda e^{-\left( q_{\text{out}}^2 R_{\text{out}}^2 + q_{\text{side}}^2 R_{\text{side}}^2 + q_{\text{long}}^2 R_{\text{long}}^2 + 2 q_{\text{out}} q_{\text{long}} R_{\text{ol}}^2 \right)}
\]

**longitudinal:**

\[ q_{\text{long}} = \vec{p}_{z1} - \vec{p}_{z2} \]

**in LCMS:**

\[ p_{z,1} + p_{z,2} = 0 \]

**transverse:**

\[ \vec{k}_t = \frac{1}{2} \left( \vec{p}_{t1} + \vec{p}_{t2} \right) \]

\[ q_t = \vec{p}_{t1} - \vec{p}_{t2} \]

\[ q_{\text{side}} \perp \vec{k}_t, \quad q_{\text{out}} \parallel \vec{k}_t \]

**\( R_{\text{long}} \):** longitudinal expansion and lifetime \( \tau_f \)

**\( R_{\text{side}} \):** transverse size, radial expansion

**\( R_{\text{out}} \):** as \( R_{\text{side}} \) but also duration of emission \( \Delta \tau \)
3D information

\[ k = \frac{1}{2} (\mathbf{p}_2 + \mathbf{p}_1) \quad q = \mathbf{p}_2 - \mathbf{p}_1 \]

3 “radii” by using 3-D vector \( \mathbf{q} \)
HBT in expanding systems

space-momentum correlations:
pions with similar momenta (small q) are
preferentially emitted from close-by
regions
→ length of homogeneity: (smaller than geometrical dimension of system)

\[ \Delta v_{\text{coll}} \approx \sqrt{\frac{T_f}{m_t}}, \quad m_t = \sqrt{m_\pi^2 + k_t^2} \]

\( \ell_{\text{thermal}} \) vs. geometrical length scale

\[ R_{\text{HBT}} = \min\left( R_{\text{geo}}, R_{\text{therm}} \right) \]

→ HBT radius parameters depend on \( m_t \)

→ differential HBT-analysis as function of \( m_t \)
contains information about source expansion
$R_{long} - \text{longitudinal expansion and lifetime}$

$R_{long} = \tau_f \cdot \sqrt{\frac{T_f}{m_t}}$  \hspace{1cm} (Y. Sinyukov)

- duration of expansion
- thermal velocity

**Hubble-Expansion**

$H_{LB} = 1.4 \cdot 10^{42} \frac{km}{s \cdot Mpc}$

$\tau = \frac{1}{H_{LB}} = 6 - 8 \text{ fm/c} \approx 2 \cdot 10^{-23} \text{ s}$

For $T_f = 160 - 120$ MeV

![Graph showing $R_{long}$ as a function of $1/\sqrt{m_t}$ for different energy bins.](image_url)
in hydrodynamic model:

\[ R_{side} = \frac{R_{geo}}{\sqrt{1 + \eta_f^2 m_t / T_f}} \]

\( \eta_f^2 \): rapidity of transverse expansion

(U. Heinz, B. Tomasik, U. Wiedemann)

average transverse expansion velocity

\( < v_t > = 0.5-0.6c \) \ for \( T_f = 160 - 120 \) MeV
Generally:

$$R_{out} \approx R_{side}$$

At 158 AGeV:
Short but finite duration of emission

$$\Delta \tau \approx 2 \text{ fm/c}$$

from

$$\Delta \tau^2 \frac{1}{\beta_t^2} \cdot (R_{out}^2 - R_{side}^2)$$
HBT radius parameters: AGS – SPS – RHIC

\[ R_{\text{long}} : \text{increase SPS } \rightarrow \rightarrow \text{RHIC?} \]

\[ \text{…but by and large a weak } E_{\text{beam}} \text{ dependence} \]

\[ \rightarrow \text{is there a universal freeze-out criterion?} \]
Fit of particle spectra and 2 particle correlations using so-called blast wave model - Retiere/Lisa nucl-th/0312024
how can one understand very short duration of pion emission?

one possibility: as system hadronizes simultaneously chemical freeze-out (hadron abundancies are fixed) and thermal freeze-out (spectra and momentum correlations are fixed)

Broniowski, Baran, Florkowski, nucl-th/0212053

assuming $T_f = 160$ MeV and linear transverse velocity profile with radius and $\langle \beta_t \rangle = 0.47 \quad \beta_{\text{max}} = 0.64$

$\sqrt{s} = 130$ GeV