Ultra-Relativistic Nuclear Collisions

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Outline

Introduction

Setting the Stage

An Example of an Experiment

Aspects of Relativistic Nuclear Collisions
  - Centrality
  - Particle Production
  - Density
  - Transverse Expansion Velocity
  - Equation of State
  - Spatial Extension

Summary
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Summary
A QGP-Reminder

- QGP ≈ state of deconfined quarks and gluons
- can be produced by heating and/or compressing hadronic matter → relativistic nuclear collisions

![Graph showing phase transitions including QGP, Hadron Gas, Color Superconductor, and CFL.][1]
A Comment on Theoretical Tools

QCD

- correct theory of strong interaction
- but: perturbation theory only applicable at high energies/short distances (running coupling)
- in QGP at non-asymptotic temperatures coupling relatively large

Thermal field theory

- QCD in thermal systems
- but: perturbative expansion (HTL) doesn’t converge very well
- application to heavy ion collisions questionable
A Comment on Theoretical Tools

QCD

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A Comment on Theory

AdS/CFT correspondence (Maldacena conjecture)

- relates strongly coupled conformal field theory to a weakly coupled type IIB string theory (supergravity)
- pro: many quantities become calculable
- con: QCD is not a conformal theory
- exciting but remains to be proven
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Summary
Natural Units

\[ \hbar = c = k_B = 1 \]

\[ [E] = [p] = [m] = [T] = [l^{-1}] = [t^{-1}] = \text{GeV} \]

usually: \[ [E] = [p] = [m] = [T] = \text{GeV} \]
\[ [l] = [t] = \text{fm} = 10^{-15} \text{ m} \]

extremely useful: \[ \hbar c = 0.2 \text{ GeVfm} = 1 \]
Coordinates and Useful Quantities

1-Particle Observables

- Longitudinal momentum: \( p_\parallel = |\vec{p}| \cos \vartheta \)
- Transverse momentum: \( p_\perp = |\vec{p}| \sin \vartheta \)
- Transverse mass: \( m_\perp = \sqrt{p_\perp^2 + m^2} \)
- Rapidity: \( y = \tanh^{-1}(\beta_\parallel) = \frac{1}{2} \ln \left(\frac{E+p_\parallel}{E-p_\parallel}\right) \)
- Pseudo-rapidity: \( \eta = -\ln \left(\tan \frac{\vartheta}{2}\right) = \frac{1}{2} \ln \left(\frac{p+p_\parallel}{p-p_\parallel}\right) \)

For \( E \gg m \): \( y \approx \eta \)
Coordinates and Useful Quantities

Global Observables

transverse energy: \[ E_\perp = \sum_i E_i \sin \theta_i \]

excitation energy: \[ E^* = E_{\text{cm}} - N_{\text{part}} m_N \]
\[ = (\gamma_{\text{beam}} N_{\text{part,beam}} + \gamma_{\text{target}} N_{\text{part,target}}) m_N - N_{\text{part}} m_N \]

kinetic energy of participating nucleons →
energy of the produced matter

isotropic source: \[ E_\perp = \frac{\pi}{4} E^* \]

zero-degree energy: \[ E_{\text{ZD}}: \text{energy deposited in small solid angle around beam axis} \rightarrow \text{sensitive to} \]
number of projectile spectator nucleons

ideally: \[ \frac{E_{\text{ZD}}}{E_{\text{beam}}} = \frac{N_{\text{spec}}}{A} \]
\[ \Rightarrow E_\perp \text{ and } E_{\text{ZD}} (E^*) \text{ complementary} \]
Coordinates and Useful Quantities

$E_r$ (GeV) vs. $E_{zd}$ (GeV)

$^{32}\text{S} + \text{nucleus}$

200 GeV/nucleon

Data

$^{\bullet}\text{Au}$
$^{\circ}\text{Ag}$
$^{\circ}\text{Cu}$
$^{\star}\text{S}$

[2]
Rapidity

\[ \beta = \frac{\beta_1 + \beta_2}{1 + \beta_1 \beta_2} \]
\[ y = \tanh^{-1} \beta \]
\[ = \tanh^{-1} \left( \frac{\beta_1 + \beta_2}{1 + \beta_1 \beta_2} \right) \]
\[ = \tanh^{-1} \beta_1 + \tanh^{-1} \beta_2 \]
\[ = y_1 + y_2 \]

\( \Rightarrow \) The shape of rapidity distributions is invariant under Lorentz-transformations.
Rapidity

rapidity: relativistic analogue of (longitudinal) velocity

\[
\begin{align*}
\beta &= \frac{\beta_1 + \beta_2}{1 + \beta_1 \beta_2} \\
y &= \tanh^{-1} \beta \\
&= \tanh^{-1} \left( \frac{\beta_1 + \beta_2}{1 + \beta_1 \beta_2} \right) \\
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&= y_1 + y_2
\end{align*}
\]

⇒ The shape of rapidity distributions is invariant under Lorentz-transformations.
Accelerators and Beam Rapidity

AGS: \[ E_{\text{beam}} = 11 \text{ A GeV} \text{ Au+Au fixed target} \]
SPS: \[ E_{\text{beam}} = 158 \text{ A GeV} \text{ Pb+Pb fixed target} \]
RHIC: \[ \sqrt{s_{\text{NN}}} = 200 \text{ GeV} \text{ Au+Au collider} \]
LHC: \[ \sqrt{s_{\text{NN}}} = 5.5 \text{ TeV} \text{ Pb+Pb collider} \]
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Stages of a Nuclear Collision

(a) Lorentz-contracted nuclei
(b) nuclei overlap, scatterings occur
(c) nucleus remnants recede from interaction region leaving a dense and hot system behind
(d) system expands, cools and hadronises, hadrons scatter and resonances decay
Geometry

Centrality

\( b \): impact parameter

\[
\text{centrality} = \frac{\sigma}{\sigma_{\text{geo}}} \sim \frac{\int_0^b b' \, db'}{\int_0^{2R_A} b' \, db'} \propto b^2
\]

[3]
Geometry

Glauber-models

- characterise collision by
  - number of participating nucleons \( N_{\text{part}}(b) \)
  - number of binary nucleon-nucleon collisions \( N_{\text{bin}}(b) \)

- rule of thumb:
  - soft (low momenta) particle production scales with \( N_{\text{part}} \)
  - hard (high momentum transfer) processes scale with \( N_{\text{bin}} \)
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Challenges

General Complications

- QGP not directly observable
- have to infer QGP properties from hadronic final state
- complicated space-time evolution
- complex multi-particle dynamics

Experimental Challenges

- high multiplicity (RHIC: up to ~4000 charged particles)
- many measurements have huge background
- this background contains structures and correlations
- it fluctuates
An Example for an Experiment: STAR

**STAR Detector**

Silicon Vertex Tracker: position and momentum
Time Projection Chamber: momentum and position
Time Of Flight: velocity
E-M Calorimeter: energy

⇒ combining information from different subdetectors allows for particle identification
A Central Au+Au Event in the STAR Detector
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Summary

Centrality

\[ y \]

\[ x \]

\[ b \]

\[ R_A \]

\[ \eta/TdE \]

\[ E_{2D}/E = A_{PS}/32 \]

Counts

1

10

10

2

10

3

10

4

(GeV)

0 200 400 600 800 1000 1200

[2]

[5]

[6]
Centrality

- $Q_{BBC}$: charge in Beam Beam Counter (detector at $3 < |\eta| < 4$ measuring number of charged particles)

- complication: incomplete measurement of spectators

Collisions with increasing centrality have

- increasing activity away from beam rapidity (transverse energy, number of produced particles, total charge etc.).

- decreasing activity near beam rapidity, i.e. decreasing number of spectator nucleons.

→ use a combination of the two to experimentally determine centrality
Centrality

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⇒ use a combination of the two to experimentally determine centrality
**Stopping**

nuclear stopping power: amount of kinetic energy lost by projectiles

⇒ stopping means that protons get shifted to midrapidity

⇒ mean rapidity shift of projectiles: $\Delta y \simeq 2$
Stopping

\[ \frac{\text{d}N}{\text{d}y} \text{net-protons} \]

\[ y_{\text{CM}} \]

\[ \text{Ratio} \]

\[ N_{\pi/\pi^+} \]

\[ N_{K/K^+} \]

\[ N_{\bar{p}/p} \]

\[ 0 \]

\[ 1 \]

\[ 2 \]

\[ 3 \]

\[ 4 \]

\[ \Rightarrow \text{increasing beam energy we go from stopping to transparency} \]

\[ \Rightarrow \text{valence quark part of wave function gets more and more Lorentz-contracted while sea cannot become smaller than } \sim 1 \text{ fm (uncertainty principle)} \rightarrow \text{collisions at high energy dominated by sea-sea interactions} \]
Increasing beam energy we go from stopping to transparency

- Valence quark part of wave function gets more and more Lorentz-contracted while sea cannot become smaller than $\sim 1 \text{ fm}$ (uncertainty principle) $\Rightarrow$ collisions at high energy dominated by sea-sea interactions
Increasing beam energy we go from stopping to transparency.

Valence quark part of wave function gets more and more Lorentz-contracted while sea cannot become smaller than \( \sim 1 \text{ fm} \) (uncertainty principle) \( \rightarrow \) collisions at high energy dominated by sea-sea interactions.
Total Energy

\[ E = m_\perp \cosh y \Rightarrow E_{\text{tot}} = \sum_{\text{species}} \int dy \frac{dN}{dy} \langle m_\perp \rangle \cosh y \]

<table>
<thead>
<tr>
<th>particle</th>
<th>energy [GeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p )</td>
<td>3108</td>
</tr>
<tr>
<td>( \bar{p} )</td>
<td>428</td>
</tr>
<tr>
<td>( K^+ )</td>
<td>1628</td>
</tr>
<tr>
<td>( K^- )</td>
<td>1093</td>
</tr>
<tr>
<td>( \pi^+ )</td>
<td>5888</td>
</tr>
<tr>
<td>( \pi^- )</td>
<td>6117</td>
</tr>
<tr>
<td>( \pi^0 )</td>
<td>6004</td>
</tr>
<tr>
<td>( n )</td>
<td>3729</td>
</tr>
<tr>
<td>( \bar{n} )</td>
<td>513</td>
</tr>
<tr>
<td>( K^0 )</td>
<td>1628</td>
</tr>
<tr>
<td>( \bar{K}^0 )</td>
<td>1093</td>
</tr>
<tr>
<td>( \Lambda )</td>
<td>1879</td>
</tr>
<tr>
<td>( \bar{\Lambda} )</td>
<td>342</td>
</tr>
</tbody>
</table>

Total: 33.4 TeV

\( E_{\text{beam}} \cdot N_{\text{part}} = 35 \text{ TeV} \)

Produced: 24.8 TeV

\( \Rightarrow 74 \% \) of beam energy goes into particle production
Rapidity Distribution

- isotropic particle source at rest: \( \frac{dN_1}{d \cos \vartheta} = \frac{N_{1,\text{tot}}}{2} \)

\[ y = \frac{1}{2} \ln \left( \frac{E+p \cos \vartheta}{E-p \cos \vartheta} \right) \]
\[ \Rightarrow \cos \vartheta = \frac{E}{p} \tanh y \]
\[ \Rightarrow \frac{dN_1}{dy} = \frac{dN_1}{d \cos \vartheta} \frac{d \cos \vartheta}{dy} \]
\[ = \frac{N_{1,\text{tot}}}{2} \frac{E}{p} \text{sech}^2 y \]

- moving isotropic source: \( \frac{dN_1}{dy} = \frac{N_{1,\text{tot}}}{2} \frac{E}{p} \text{sech}^2 (y + y_s) \)

- picture of nuclear collision: particles need proper time \( \tau_{\text{de}} \) to form → time-dilated in lab frame \( \gamma \tau_{\text{de}} \) → superposition if independent moving sources

\[ \frac{dN}{dy} = \int dy' \frac{dN_1}{dy'} (y + y') \propto \tanh(y + y_{\text{max}}) - \tanh(y - y_{\text{max}}) \]

\[ -y_{\text{max}} \]
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  $= \frac{N_{1,\text{tot}}}{2} \frac{E}{p} \text{sech}^2 y$

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  \]
- picture of nuclear collision: particles need proper time \(\tau_{de}\) to form \(\rightarrow\) time-dilated in lab frame \(\gamma \tau_{de}\) \(\rightarrow\) superposition if independent moving sources

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\( \Rightarrow \cos \vartheta = \frac{E}{p} \tanh \gamma \)

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Space-Time Picture

- particle formation time: \( t = \gamma \tau_{\text{de}} \) from moment of projectile overlap at \( t = 0 \) and \( z = 0 \)
- particles at rest (\( \gamma = 1 \)) are formed at midrapidity and at \( z = 0 \)
- moving particles are formed at higher rapidity and travel a distance \( \beta/\gamma \tau_{\text{de}} \) before formation

\( \Rightarrow \) Particles with high rapidity are produced at high \( z \)

\( \Rightarrow \) The rapidity is related to the point of particle emission (in coordinate space).

\[
y = \frac{1}{2} \ln \left( \frac{E + p_\parallel}{E - p_\parallel} \right) = \frac{1}{2} \ln \left( \frac{\gamma m + \gamma m v}{\gamma m - \gamma m v} \right) = \frac{1}{2} \ln \left( \frac{t + z}{t - z} \right) = Y
\]

\( Y \): space-time rapidity

NB: We have ignored the transverse expansion.
Density

Bjorken’s density estimate:  
\[ \epsilon_0 = \frac{1}{\pi R^2 \tau_0} \frac{dE_\perp}{d\eta} \bigg|_{\eta \to 0} \]

\[ \tau = \sqrt{t^2 - z^2} : \text{proper time} \]
\[ \tau_0 : \text{equilibration time} \quad (\tau_0 \approx 0.2 \ldots 1 \text{ fm}) \]
\[ \epsilon_0 = \epsilon(\tau_0) : \text{early energy density} \]

\[ \frac{dE_\perp}{d\eta} \bigg|_{\eta=0} \approx \frac{dE_\perp}{dy} \bigg|_{y=0} = \pi R^2 \epsilon(\tau) \frac{dz}{dy} \bigg|_{y=0} = \pi R^2 \epsilon(\tau) \tau \]

\[ \epsilon \tau = \epsilon_0 \tau_0 \text{ from entropy conservation} \]

- for \( \tau_0 = 1 \text{ fm} \)
  - AGS: \( \epsilon_0 = 1.4 \text{ GeV fm}^{-3} \)
  - SPS: \( \epsilon_0 = 3 \text{ GeV fm}^{-3} \)
  - RHIC: \( \epsilon_0 = 5 \text{ GeV fm}^{-3} \)
- estimated density needed to form QGP \( 1 \text{ GeV fm}^{-3} \)
Transverse Expansion Velocity

thermal source: \( \frac{dN}{m_\perp dm_\perp} \propto \exp \left( \frac{m_\perp - m_0}{T_{\text{kin}}} \right) \)
Transverse Expansion Velocity

- $T_{kin}$: kinetic freeze-out temperature (‘temperature at last interaction’)
- spectrum of exactly exponential
- inverse slope $T_{kin}$ depends on particle mass

$\Rightarrow$ characteristic of transverse flow: $T_{kin}^{\text{eff}} \approx T_{kin} + m_0 \beta_r^2 / 2$

$\Rightarrow$ need a hydrodynamic calculation
Transverse Expansion Velocity

\[ \frac{dN}{m_\perp dm_\perp} \propto \int_0^R rdr \frac{m_\perp}{l_0} \left( \frac{p_\perp \sinh \rho}{T_{\text{kin}}} \right) K_1 \left( \frac{p_\perp \cosh \rho}{T_{\text{kin}}} \right) \]

with \( \rho = \tanh^{-1} \beta_r \) and \( \beta_r(r) = \beta_s \left( \frac{r}{R} \right)^n \); \( 0 \leq r \leq R \)

\( n \approx 1 \) — analogous to Hubble expansion

\[ \frac{\alpha_{\text{trig}}}{\alpha_{\text{geom}}} \]

\[ \phi(\text{ss}) \]

\[ \Omega(\text{sss}) \]

\[ \text{Chemical freeze-out temperature} \]

\[ T_{\text{ch}} \]

\[ \text{Collective velocity} \quad \langle \beta_T \rangle \quad (c) \]

[11]
Equation of State

- mean free path $\ll$ system size $\rightarrow$ hydrodynamical description
- pressure gradient steeper in x-direction
- collective flow develops preferentially in x-direction
- particle distribution shows azimuthal anisotropy
- anisotropy directly sensitive to equation of state
- mostly sensitive to early times, when eccentricity is largest
Equation of State

\[ E \frac{d^3 N}{d^3 \mathbf{p}} = \frac{d^2 N}{2\pi p_\perp dp_\perp dy} \left( 1 + \sum_{n=1}^{\infty} 2v_n \cos(n\phi) \right) \]

elliptic flow: \( v_2 = \langle \cos(2\phi) \rangle \)

- hydrodynamic calculations with QGP EOS do good job at top SPS energies and RHIC
- suggests thermalisation and QGP formation
Spatial Extension

- Hanbury Brown - Twiss interferometry: interferometry of identical particles (originally used to determine size of stars)
- measure momenta $\vec{k}_1$ and $\vec{k}_2$ of two pions emitted from $\vec{x}_1$ and $\vec{x}_2$, respectively, at different positions
- indistinguishable particles $\rightarrow$ interference
- transition probability:

$$|\psi_{12}|^2 = \frac{1}{2V^2} \left| e^{-i\vec{k}_1 \cdot \vec{x}_1} e^{-i\vec{k}_2 \cdot \vec{x}_2} + e^{-i\vec{k}_1 \cdot \vec{x}_2} e^{-i\vec{k}_2 \cdot \vec{x}_1} \right|^2$$

$$= \frac{1}{V^2} \left( 1 + \cos(\Delta \vec{k} \cdot \Delta \vec{x}) \right)$$

![Spatial Extension Diagram]
Spatial Extension

- probability of observing $\vec{k}_1$ and $\vec{k}_2$ in emission from continuous source

$$P(\vec{k}_1, \vec{k}_2) = \frac{1}{2} \int d^3x_1 d^3x_2 \rho(\vec{x}_1) \rho(\vec{x}_2) |\Psi_{12}|^2$$

- probability of observing a single particle with $\vec{k}_i$

$$P(\vec{k}_i) = \int d^3x_i \rho(\vec{x}_i) ||\langle \vec{k}_i | \vec{x}_i \rangle||^2$$

- correlation function

$$C_2 \equiv \frac{d^6N}{d^3k_1 d^3k_2} \left( \frac{d^3N}{d^3k_1} \frac{d^3N}{d^3k_2} \right)^{-1} = \frac{2P(\vec{k}_1, \vec{k}_2)}{P(\vec{k}_1)P(\vec{k}_2)} = 1 + |\tilde{\rho}(\Delta \vec{k})|^2$$

where $\tilde{\rho}$ is the Fourier transform of the density distribution
Spatial Extension

⇒ can infer size of source from correlation function
  ▶ in practice: fit a 3d Gaussian to data
  ▶ life is more complicated and more interesting with an expanding source
    ▶ radii depend on transverse momentum of pair
    ▶ $R_{\text{out}}/R_{\text{side}} > 1$ for long duration of hadron emission
Spatial Extension

- transverse rms radii larger than nuclei [13]
- radii decrease with pair transverse momentum
- extended, expanding source ($\beta_r$ consistent with $m_\perp$ spectra)
- $R_{out}/R_{side} \sim 1$
Outline

Introduction

Setting the Stage

An Example of an Experiment

Aspects of Relativistic Nuclear Collisions
  Centrality
  Particle Production
  Density
  Transverse Expansion Velocity
  Equation of State
  Spatial Extension

Summary
Summary

What we have learned about the properties of the hot and dense matter produced in relativistic nuclear collisions:

- low beam energies: stopping; high beam energies: transparency (proton & antiproton rapidity distributions)
- longitudinal expansion (rapidity distribution of charged particles)
- transverse expansion ($m_\perp$-spectra, HBT radii, elliptic flow)
- at top SPS energies and RHIC: QGP formation (density, elliptic flow)
- early thermalisation (elliptic flow)
References

[1] from a sketch by K. Rajagopal


[8] W. Busza and R. Ledoux,
References II


