Leslie's Cube and the Demonstration of Kirchhoff's Radiation Law

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Kirchhoff's law of thermal radiation [1] states that the emissivity of a body in thermal equilibrium, i.e. the energy radiated relative to a black body, equals its absorbance, i.e. the fraction of incident power absorbed by the body. Leslie's cube, first introduced by John Leslie in the early 19th century [2], provides an elegant way to experimentally demonstrate this fundamental law. We have set up a simple demonstration experiment allowing one to show Kirchhoff's law of thermal radiation in the classroom using Leslie's cube in combination with two absorbing surfaces of equal area. The demonstration can serve as a motivation experiment to enter the general discussion of black-body radiation, its properties and importance for various phenomena in nature [3].

An alternative formulation of Kirchhoff's law of thermal radiation, is that the ratio of the emitted power per unit surface area E to the absorbance A of a body in thermal equilibrium only depends on the temperature (and not on, e.g., surface properties). Thus, the ratio E/A is given by a universal function solely depending on the temperature. By definition, the absorbance of a black body is equal to unity, and therefore this universal function describes the well-known emission properties of a black body. Kirchhoff's law thus forms the conceptual basis for Planck's general law on the spectral power density of black-body radiation, one of the corner stones of modern thermodynamics and the starting point of quantum mechanics [4].

The basic idea is to prepare a rotatable metallic cube (or, as in our case, a cuboid) at a given temperature with its surfaces consisting of different textures. In this way, one can directly compare the emitted power of the different surfaces at a fixed temperature under equivalent conditions. The radiated energy is detected as the heat absorbed by a metal plate, the surface texture of which may also be varied. In our realization shown in Fig. 1, an aluminum cuboid is heated evenly by four equal heating resistors placed inside at the four quadratic surfaces of the cuboid (side length 4 cm, wall thickness 4 mm). One outer surface is polished to mirror quality, the others being roughened by abrasive blasting. The rough surface opposing the reflecting one is colored with black varnish. The third surface is sprayed with white varnish, while its opposing surface is left in its rough state without any further manipulation.

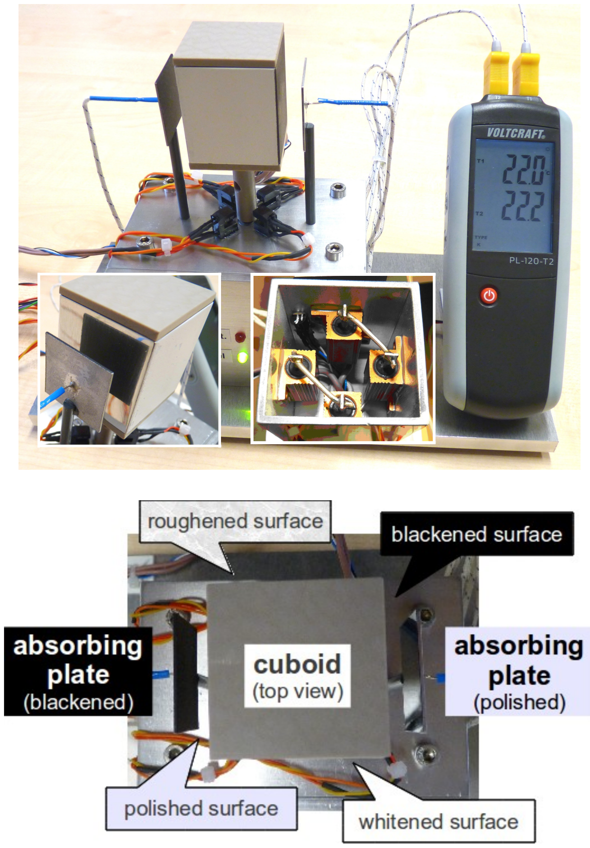

Fig. 1: *Leslie's cube for the demonstrating Kirchhoff's law of thermal radiation. (upper picture) The cuboid is heated by four resitive heaters inside. The temperature of the plates is measured by thermistors and monitored by a digital voltmeter. Insets: Details of the setup. (left) reflecting cuboid surface opposing the blackened plate, (right) view inside the cuboid showing the heating resistors. (lower picture) Top view of the cuboid. The four side surfaces of the cuboid are treated in different ways (polished, roughened, blackened roughened, withened roughened). The cuboid can be rotated by a computer controlled motor around the central axis (perpendiculat to the drawing plane) to expose two absorbing plates at the opposite sides to different surface textures. The absorbing plates have a polished and rough blackened surface, respectively.*

Our demonstration experiment has been laid out is such a way that it can be used as an independent exhibit, e.g. for an interactive showcase, which can be steered from an external control panel. All electric components (resistive heating circuit, rotation motor of the cuboid) are controlled by a microprocessor which allows press-buttom operation of the experiment. Heating of the cube to approximately 340 K is provided the heating resistors (0.5 Ω, 12 W) by a standard electric power circuit. Rotation of the cuboid by 90 degrees is controlled using photoelelectric sensors at the buttom of the holding shaft which provide the feedback to the driving circuit of the rotation motor. The absorbing plates and the cube are mounted with posts made of PEEK (Polyether Ether Ketone) exhibiting low heat conductance while withstanding temperatures up to 600 K. Further details on the setup are available from the authors upon request.

The heat emitted by the surfaces is absorbed by two aluminum plates of equal area mounted in front of two opposite surfaces of the cube, as shown in Fig. 1. One of the absorbing plates is polished to the same surface quality as the mirror surface of the cube, the other plate has a rough surface with black varnish. The temperature of the plates is measured by thermocouples and displayed by a digital voltmeter.The cube can be rotated around its central axis thus exposing the absorbing plates to different heat emitting surfaces. In thermal equilibrium, the temperature T of the plate is proportional to the product of the total emitted heat power per unit area E of the cube's surface opposing the plate, and the absorbance A of the plate:

$$T \propto E \cdot A. \qquad (1)$$

Here, we assumed that the temperature of the cube is well above room temperature, which ensures that the temperature of the plate is solely determined by the radiation equilibrium due to negligible effects of convection and heat conduction. By turning the cube, which is kept at a fixed temperature by running a constant constant current through the heating resistors, one can now qualitatively compare emissivities and absorbances of different surfaces. By exposing a plate to the different surfaces of the cube and measuring temperature of the plate, one readily finds that the blackened surface emits the largest amount of heat radiation, while the polished one emits the lowest amount of heat. Therefore, one observes the important fact that, at a given cube temperature, $E_b > E_r$ where the indices b and r indicate the black-painted and reflecting mirror surface, respectively. Similarly, when being subsequently exposed to the same surface of the cube at constant temperature, one confirms the common expectation that the blackened plate absorbs more heat than the polished one, therefore $A_b > A_r$.

In the classroom, Kirchhoff's law of thermal radiation is best demonstrated by performing two differential temperature measurements under identical surrounding conditions: First, the blackened absorption plate is exposed to the blackened cube surface, which, by construction of the cuboid, will expose the mirror plate to the mirror cube surface. One finds a higher temperature of the blackened plate as compared to the reflecting plate. The temperatures differ typically by 4 to 5 degrees, from which one can directly conclude that the product of emitted power times the absorbance of a blackened surface is larger than that of a reflecting surface.

For a second measurement, one simply rotates the cuboid by 180 degrees while keeping all other conditions identical. Now the blackened surfaces opposes the mirror plate, while the blackened plate is exposed to the mirror surface of the cuboid After equilibration, one observes $T_b \approx T_r$ (roughly 2 degrees above room temperature at a cube temperature of about 340 K) from which one directly deduces as a major result, using Eq. (1), the radiation law of Kirchhoff:

$$\frac{E_b}{A_b} = \frac{E_r}{A_r}. \qquad (2)$$

By changing the temperature of the cube one can verify that the measured temperatures of the plates indeed remain equal thus showing the general validity of Eq. (2). Using Eq. (2) one can then easily derive from the first measurement follows that both $E_b > E_r$ and $A_b > A_r$.

From these two simple measurements one therefore directly deduces Kirchhoff's law: different surface textures result in different emittances and absorbances, but the ratio of both remains a constant. This experiment thus provides an excellent way to further discuss the realisation and emission properties of a black body, defined by $A_{bb} = 1$, and to introduce its importance in many natural phenomena, e.g. the sun's spectrum or the green house effect.

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References

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