Center for Electronic Correlations and Magnetism
University of Augsburg

Superfluid Helium-3:

Universal Concepts for Condensed Matter and the Big Bang

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Helium

Two stable Helium isotopes: ^4He, ^3He

^{4}He: air, oil wells, …

Janssen/Lockyer/Secci (1868)

Ramsay (1895)
Cleveit (UO_2)

$\frac{^4\text{He}}{\text{air}} \approx 5\times10^{-6}$	$\left.\frac{^3\text{He}}{^4\text{He}}\right\|_{\text{air}} \approx 1\times10^{-6}$

^{3}He: $^6_3Li + ^1_0n \rightarrow ^3_1H + \alpha$

$\downarrow$

^3_2He

Research on macroscopic samples of ^{3}He only since 1947

^{4}He: Coolant, Welding, Balloons
^{3}He: - Contrast agent in medicine
- Neutron detectors
- ^{3}He-^{4}He dilution refrigerators (quantum computers!)

Helium

Atoms: spherical, hard core diameter ~ 2.5 Å

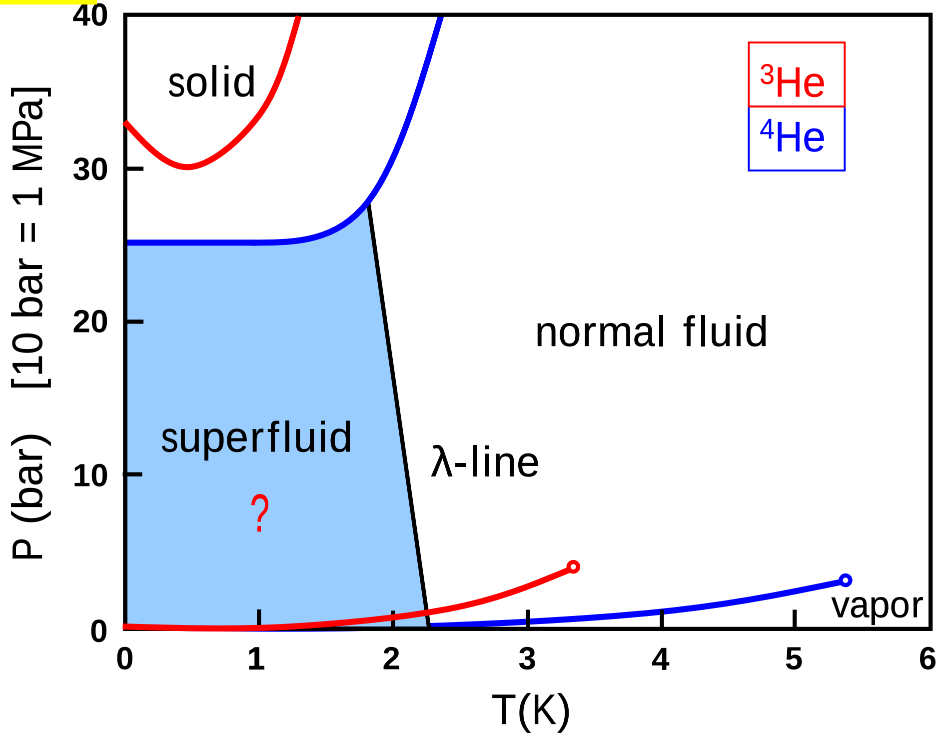
Interaction:
- hard sphere repulsion
- van der Waals dipole/multipole attraction

Boiling point: 4.2 K, ^{4}He Kamerlingh Onnes (1908)

3.2 K, ^{3}He Sydoriak *et al.* (1949)

Dense, simple liquids { isotropic; short-range interactions; extremely pure }

Helium

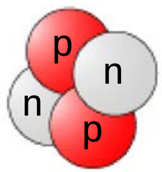
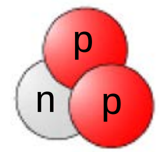
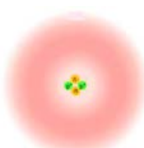


T→0, P $\lesssim$ 30 bar: Helium remains liquid

$$\lambda \propto \frac{\hbar}{\sqrt{k_B T}} \xrightarrow{T \to 0}$$

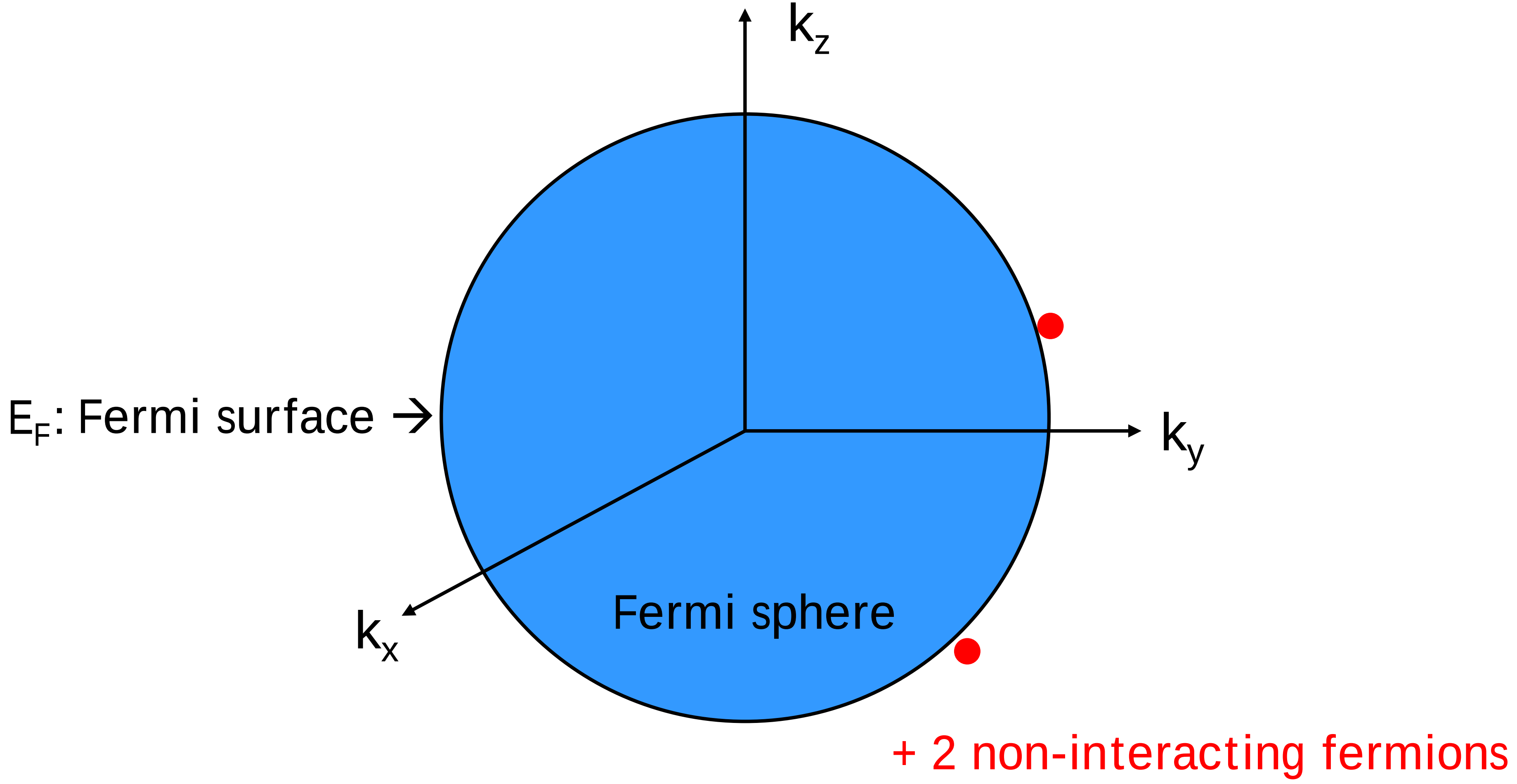
quantum phenomena on a macroscopic scale

Helium

	^4He	^3He
Electron shell:	$2\ e^-$, $S = 0$	
Nucleus:	 $S = 0$	 $S = \frac{1}{2}\hbar$
Atom(!) is a	 Boson	 Fermion → Quantum liquids
Phase transition	$T_\lambda = 2.2$ K Bose-Einstein condensation → superfluid with frictionless flow	T_c = ???

Interacting Fermions (Fermi liquid): **Ground state**

Landau (1956/57)

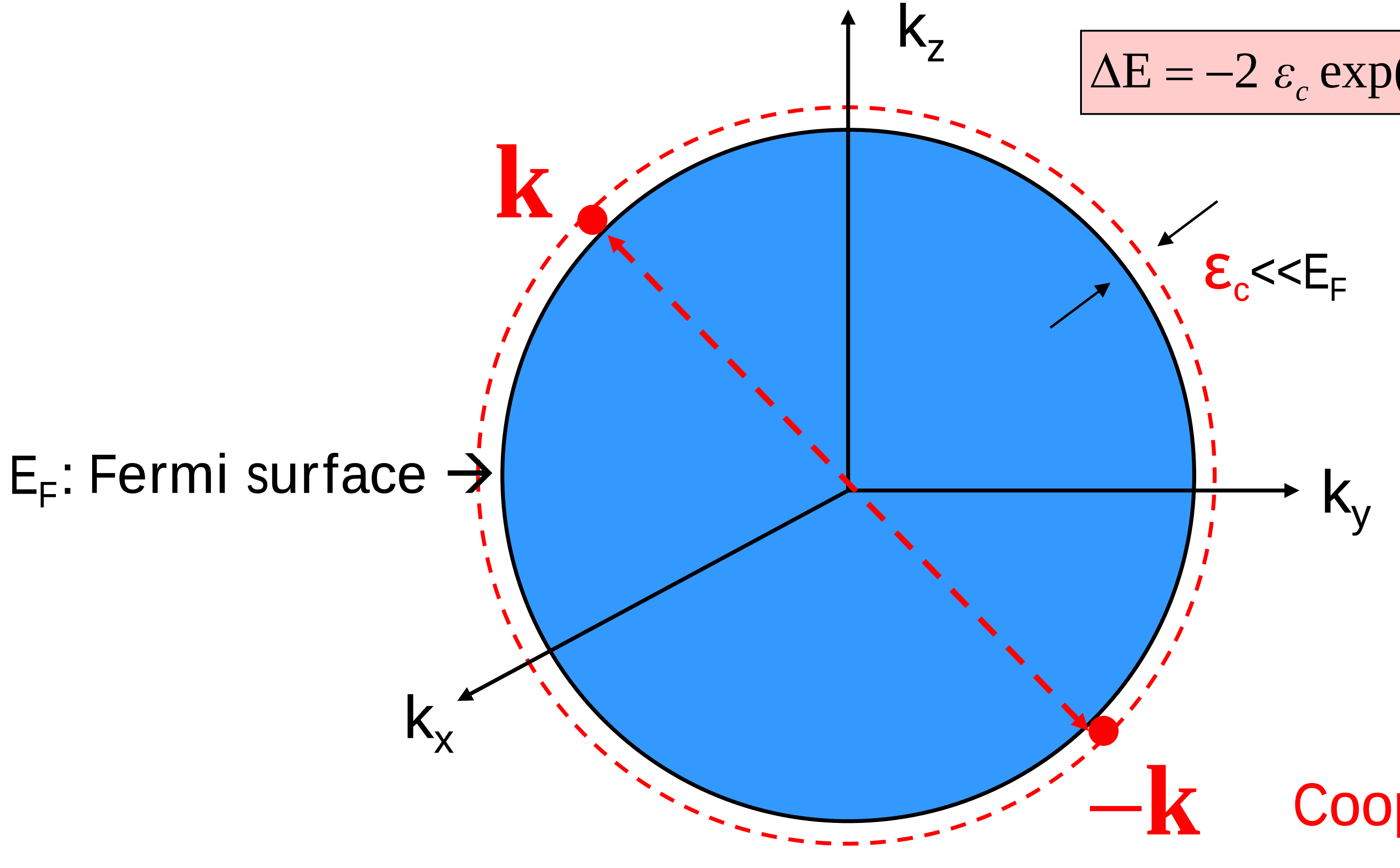


Interacting Fermions (Fermi liquid): ~~**Ground state**~~

Arbitrarily weak attraction $\Rightarrow$ instability

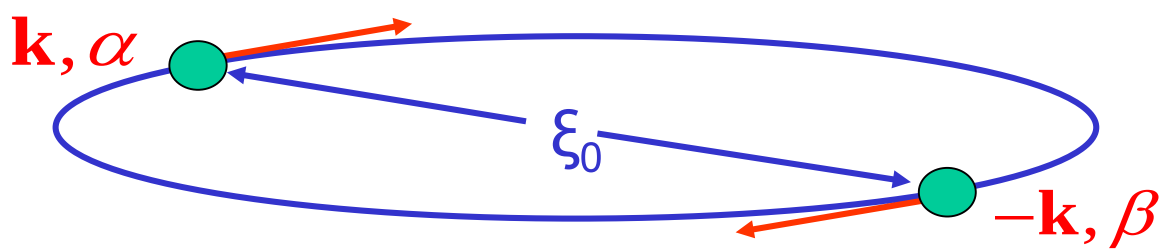
Cooper (1956)

$$\Delta E = -2\,\varepsilon_c \exp(-2/N(0)|V_L|)$$



Universal property of Fermi systems

Cooper pair $(\mathbf{k},\alpha;-\mathbf{k},\beta)$



Antisymmetry

$$\Psi_{L=0,2,4,...} = \psi(\mathbf{r})\left|\uparrow\downarrow - \downarrow\uparrow\right\rangle$$

S=0 (singlet)

$$\Psi_{L=1,3,5,...} = \psi_+(\mathbf{r})\left|\uparrow\uparrow\right\rangle + \psi_0(\mathbf{r})\left|\uparrow\downarrow + \downarrow\uparrow\right\rangle + \psi_-(\mathbf{r})\left|\downarrow\downarrow\right\rangle$$

S=1 (triplet)

L = 0 (“s-wave”): isotropic pair wave function

L > 0 (“p,d,f,... -wave”): anisotropic pair wave function

^3He: Strongly repulsive interaction → L > 0 expected

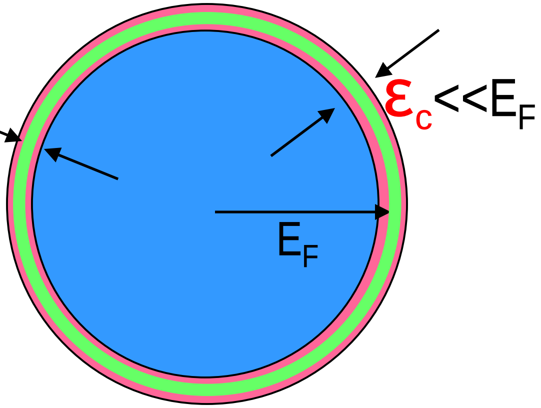
BCS theory

Bardeen, Cooper, Schrieffer (1957)

Generalization to macroscopically many Cooper pairs

→ Pair condensate

Energy gap Δ(T)
here: L=0 (s-wave)



transition temperature $$T_c = 1.13\, \varepsilon_c \exp(-1/N(0)|V_L|)$$

ε_c, V_L: Magnitude ? Origin ? → T_c ?

Thanksgiving 1971: Transition in ^{3}He at T_c = 0.0026 K

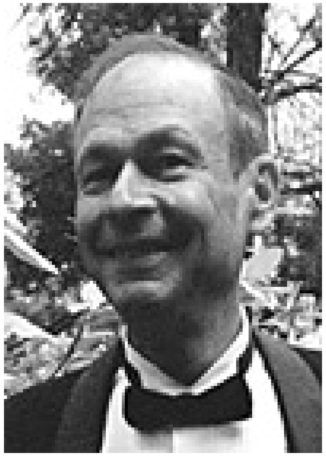
Osheroff, Richardson, Lee (1972)
Osheroff, Gully, Richardson, Lee (1972)

The Nobel Prize in Physics 1996

"for their discovery of superfluidity in helium-3"

David M. Lee
Cornell (USA)

Douglas D. Osheroff
Stanford (USA)

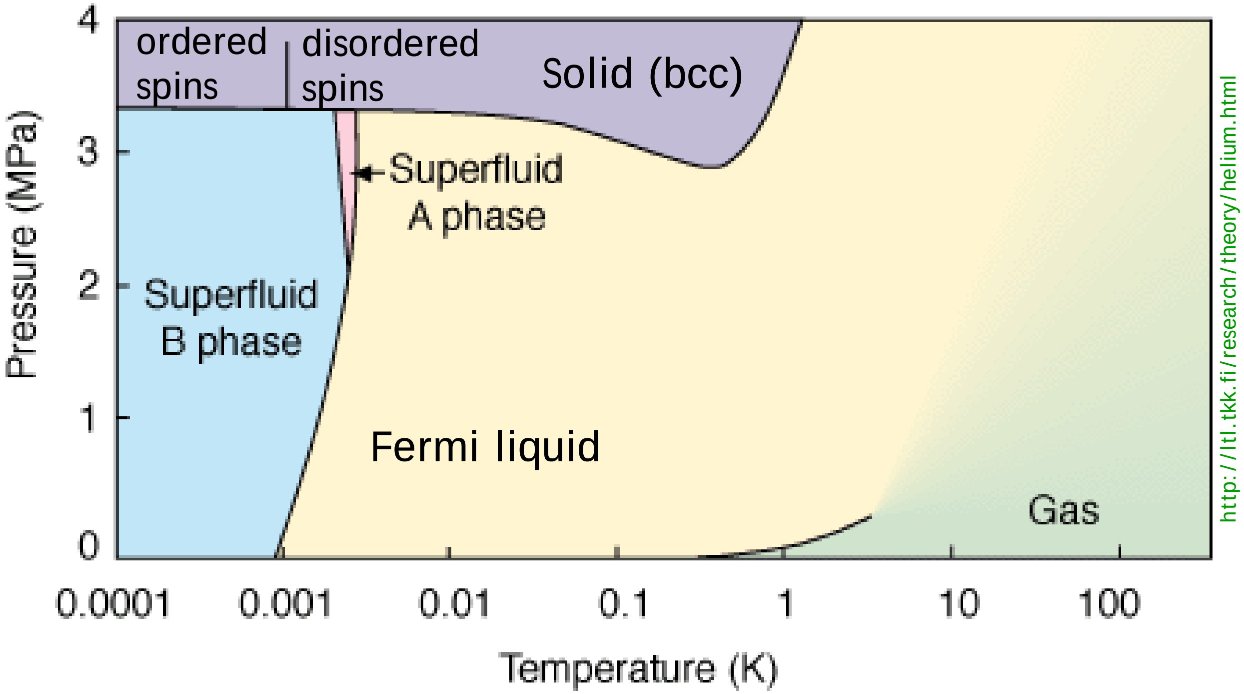


Robert C. Richardson
Cornell (USA)

Phase diagram of Helium-3

P-T phase diagram

Dense, simple liquid
- isotropic
- short-range interactions
- extremely pure
- nuclear spin S=1/2



Phase diagram of Helium-3

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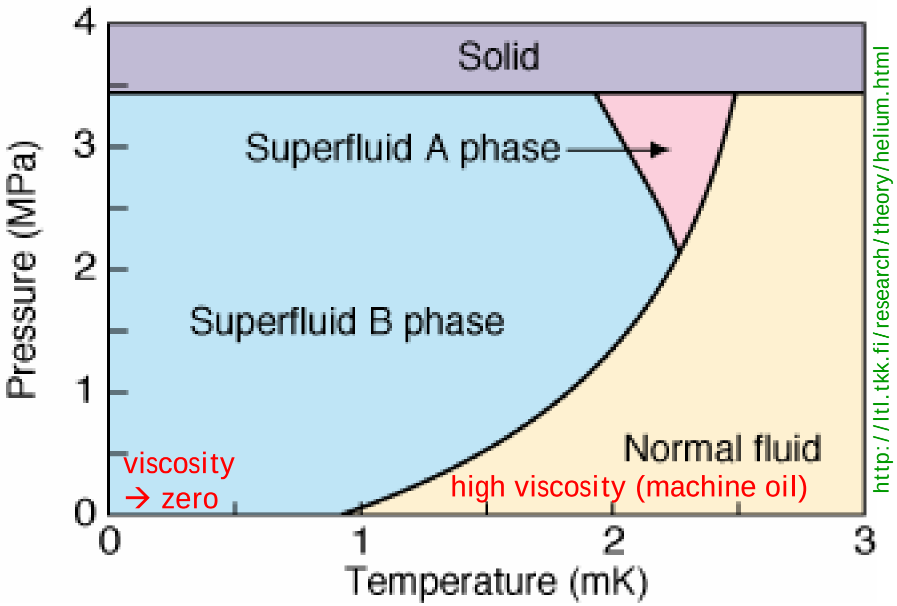
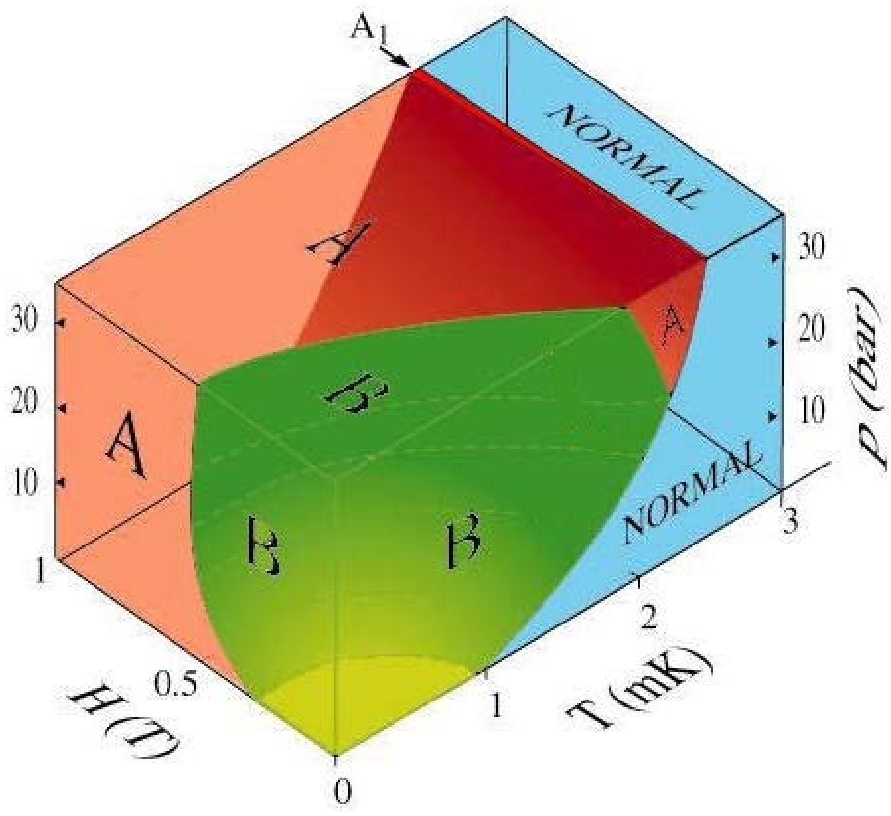
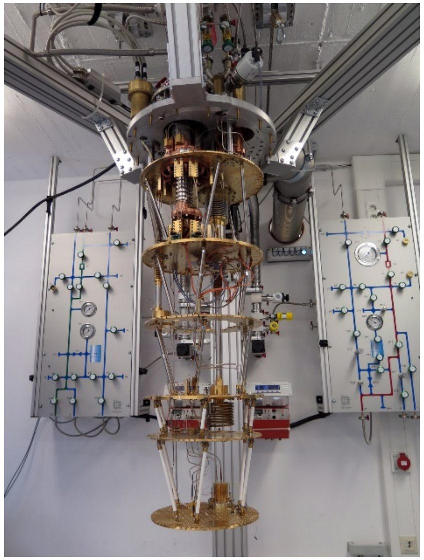


Phase diagram of Helium-3

P-T-H phase diagram

http://ltl.tkk.fi/images/archive/ab.jpg

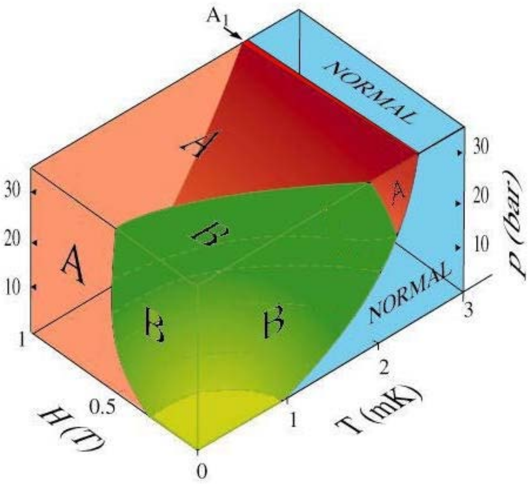




Millikelvin Cryostat WMI Garching

"Very (ultra) low temperatures": $T \ll T_{boiling}$ ~ 3 K
and $\ll T_{backgr.\ rad.}$ ~ 3 K

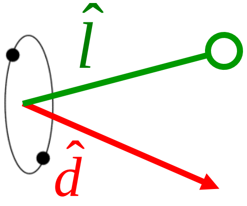
Superfluid phases of ^{3}He



Experiment: Osheroff, Richardson, Lee, Wheatley, …
Theory: Leggett, Wölfle, Mermin, …

L=1, S=1 ("p-wave, spin-triplet")
in all 3 phases

→ anisotropy directions
in every ^{3}He Cooper pair



orbital part

spin part

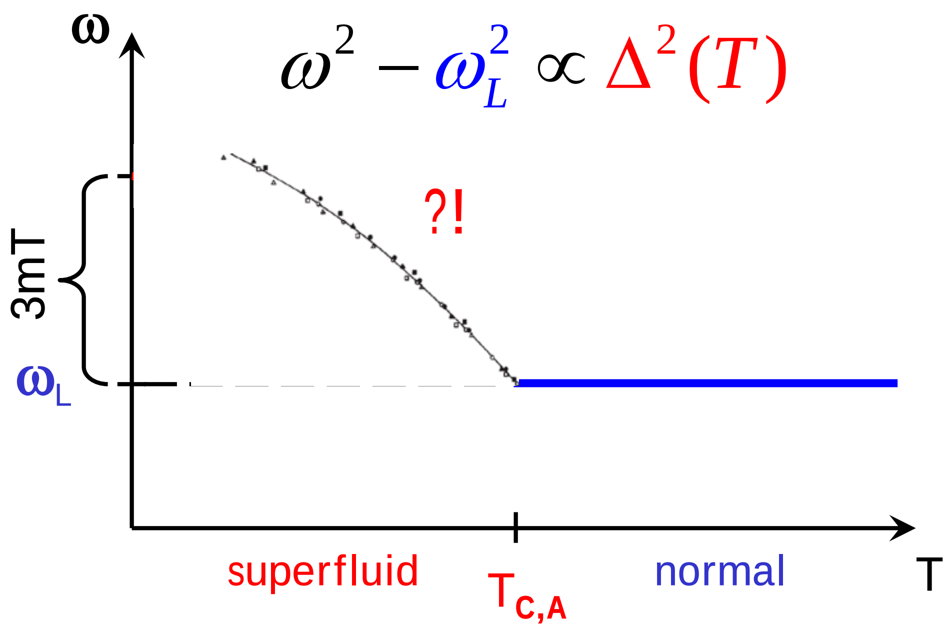
Attraction due to spin fluctuations → S=1

Anderson, Brinkman (1973)

... and a mystery!

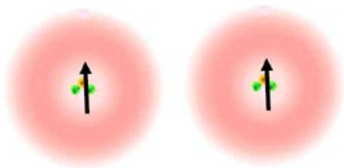
NMR experiment on nuclear spins $I=\frac{1}{2}\hbar$

Osheroff *et al.* (1972)



← Larmor frequency: $\omega_L = \gamma H$

Shift of ω_L ⟺ spin-nonconserving interactions

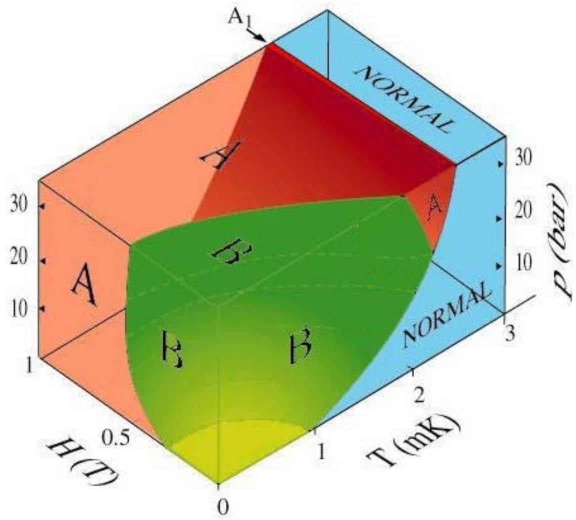


→ nuclear dipole interaction $g_D \sim 10^{-7} K \ll T_C$

Origin of frequency shift ?!

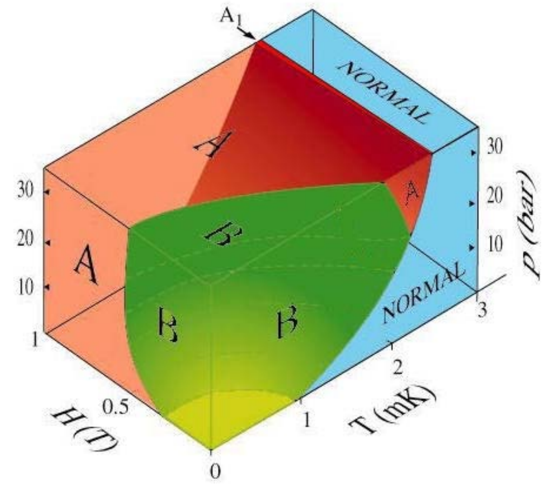
Leggett (1973)

The superfluid phases of ^3He



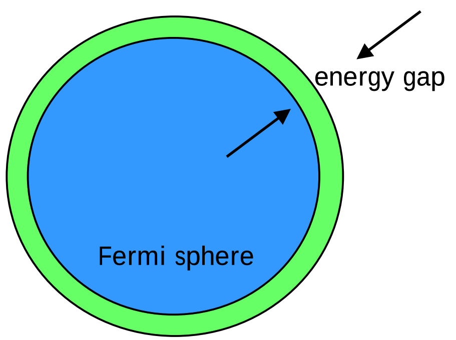
B-phase

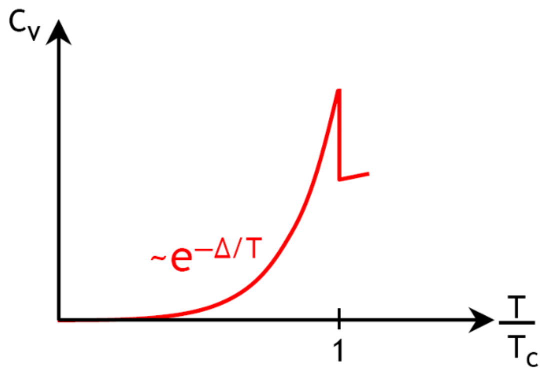
All spin states $|\uparrow\uparrow\rangle$, $|\uparrow\downarrow + \downarrow\uparrow\rangle$, $|\downarrow\downarrow\rangle$ occur equally



$$\Delta(\mathbf{k}) = \Delta_0$$

Balian, Werthamer (1963)
Vdovin (1963)





"(pseudo-) isotropic state" $\leftrightarrow$ s-wave superconductor

Weak-coupling theory: stable for all $T<T_c$

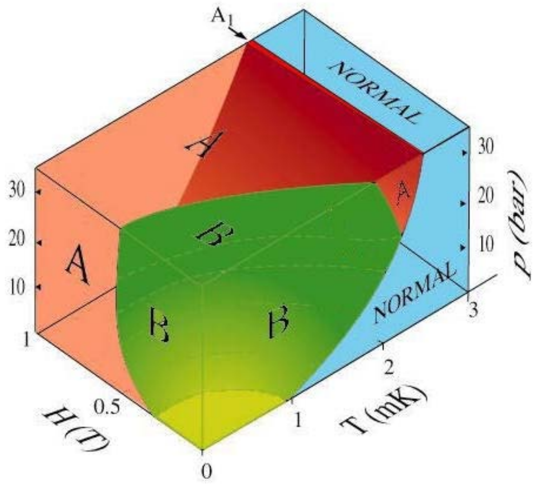
A-phase

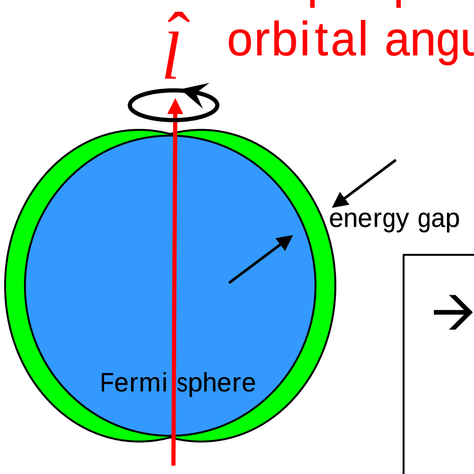
Spin states $|\uparrow\uparrow\rangle$, $|\downarrow\downarrow\rangle$ occur equally

→ strong gap anisotropy

$$\Delta(\hat{k}) = \Delta_0 \sin(\hat{k}, \hat{l})$$

Anderson, Morel (1961)





Energy gap with point nodes

"axial (chiral) state"

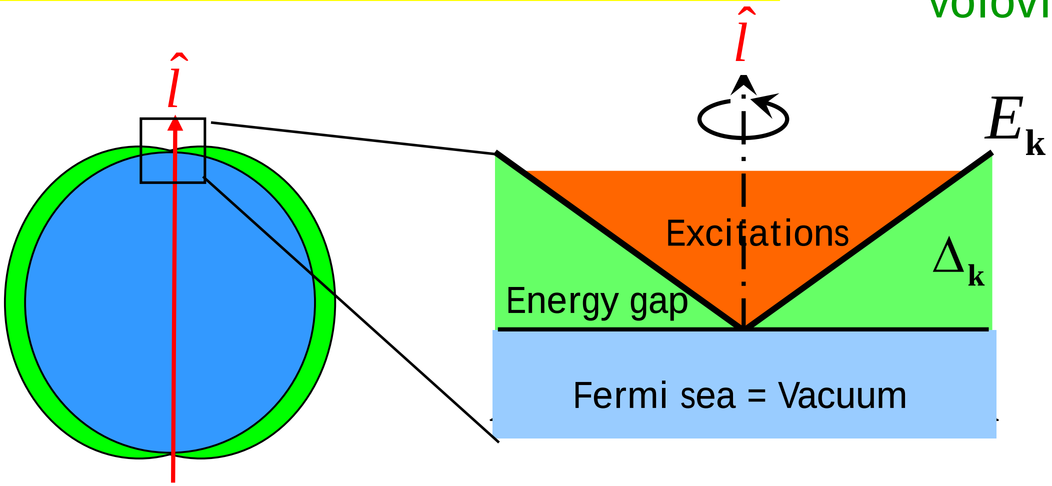
→ helped to understand

- "heavy fermion" superconductors ($CeCu_2Si_2$, UPt_3, ...)
- high-T_c (cuprate) superconductors

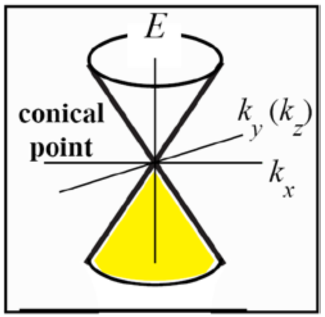
Strong-coupling effect

^{3}He-A: Spectrum near nodes

The Universe in a Helium Droplet, Volovik (2003)



$\Leftrightarrow$



Fermi point: spectral flow of fermionic charge

$$E_{\mathbf{k}}^2 = \mathrm{v}_{\mathrm{F}}^2 (k - k_F)^2 + \Delta_0^2 \sin^2(\hat{k}, \hat{l}) = \mathrm{g}^{ij} p_i p_j$$

Lorentz invariance

$$\mathrm{g}^{ij} = \mathrm{v}_{\mathrm{F}}^2 l_i l_j + \left(\frac{\Delta}{k_F}\right)^2 (\delta_{ij} - l_i l_j)$$

$$e = \begin{cases} +1 & \hat{k} \parallel +\hat{l} \\ -1 & \hat{k} \parallel -\hat{l} \end{cases}$$

chirality “up”

chirality “down”

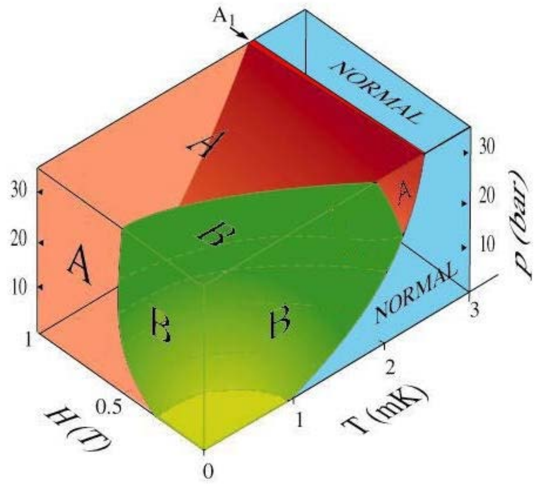
→ Chiral (Adler) anomaly in ^{3}He-A observed

Bevan *et al.* (1997)

A_1-phase

in finite magnetic field

Only spin state $|\uparrow\uparrow\rangle$

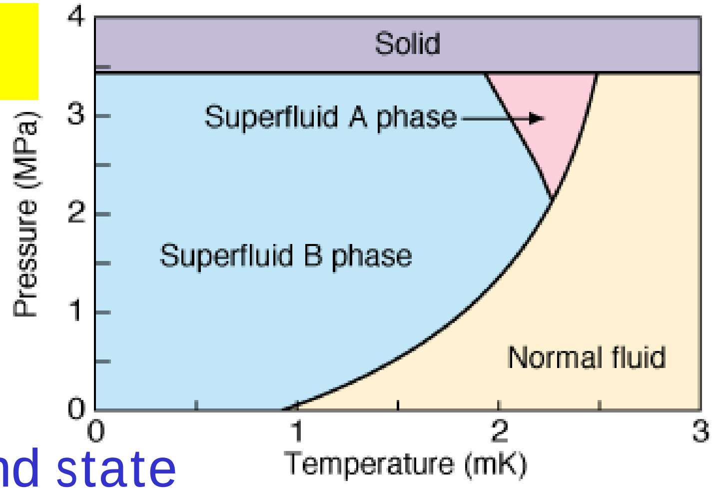


Long-range ordered magnetic **liquid**

Broken Symmetries
&
Long-Range Order

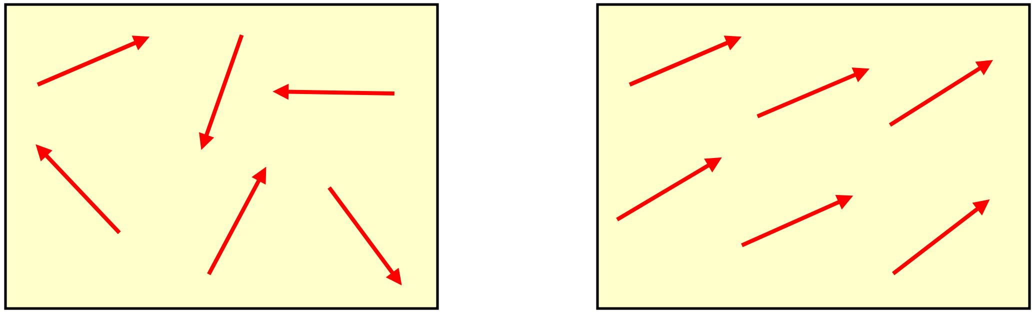
Broken Symmetries & Long-Range Order

Normal ^{3}He $\leftrightarrow$ ^{3}He-A, ^{3}He-B:
2nd order phase transition



$T<T_c$: higher order, lower symmetry of ground state

I. Ferromagnet



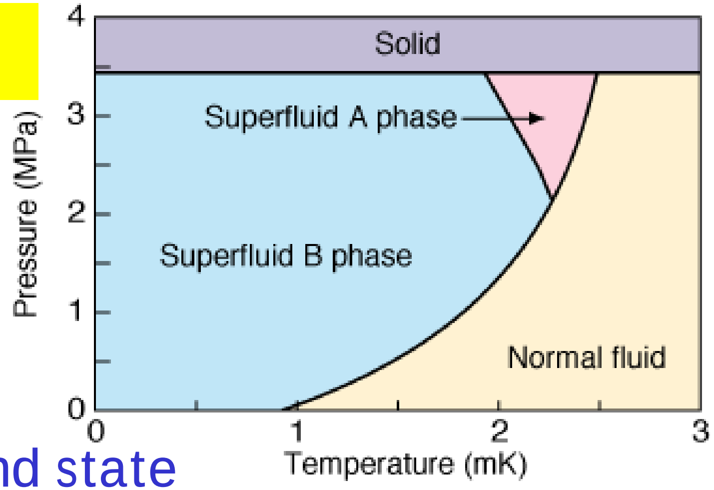
$T>T_c$ | $T<T_c$

Average magnetization: $\langle \mathbf{M} \rangle = 0$ | $\langle \mathbf{M} \rangle \neq 0$ Order parameter

Symmetry group: SO(3) | $U(1) \subset SO(3)$

$T<T_C$: SO(3) rotation symmetry in spin space spontaneously broken

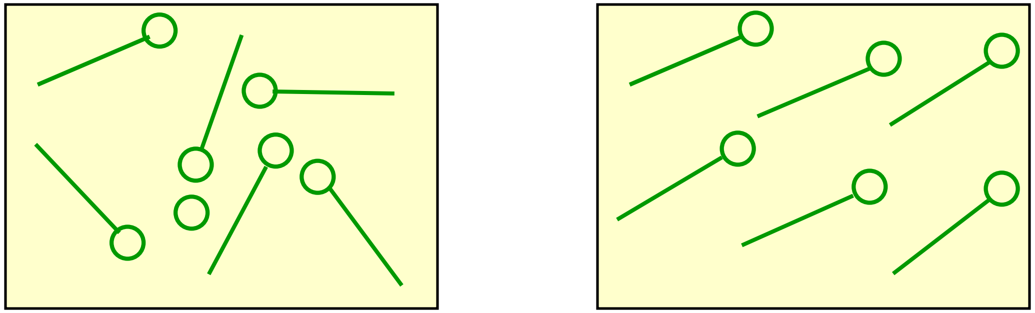
Broken Symmetries & Long-Range Order



Normal ^{3}He ↔ ^{3}He-A, ^{3}He-B:
2nd order phase transition

$T<T_c$: higher order, lower symmetry of ground state

II. Liquid crystal

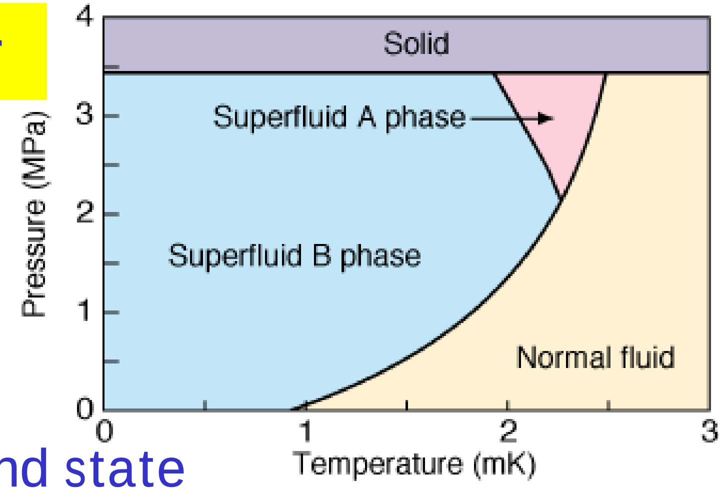


$T>T_c$ $T<T_c$

Symmetry group: SO(3) U(1) ⊂ SO(3)

$T<T_C$: SO(3) rotation symmetry in real space spontaneously broken

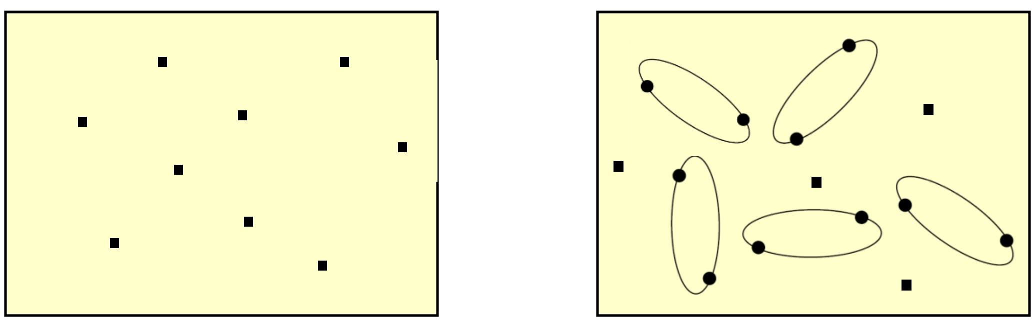
Broken Symmetries & Long-Range Order



Normal $^3He \leftrightarrow$ ^3He-A, ^3He-B:
2nd order phase transition

$T<T_c$: higher order, lower symmetry of ground state

III. Conventional superconductor



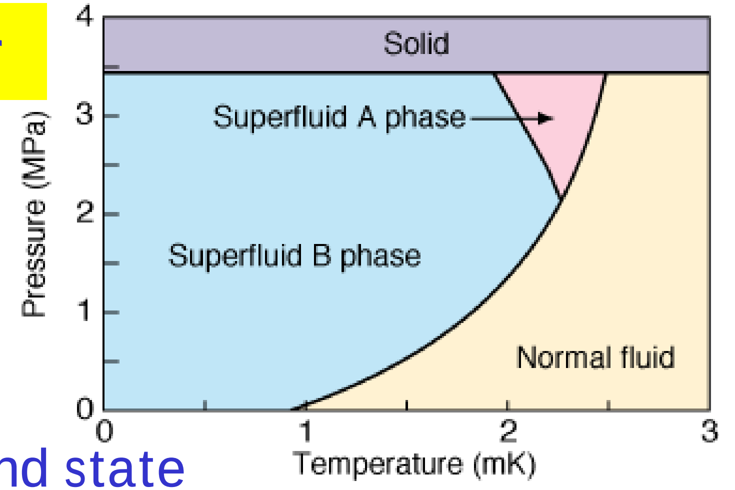
$T>T_c$ $T<T_c$

Pair amplitude $\left\langle c^\dagger_{\mathbf{k}\uparrow} c^\dagger_{-\mathbf{k}\downarrow} \right\rangle =$ 0 $\quad \Delta e^{i\phi}$ complex order parameter

Gauge transf. $c^\dagger_{\mathbf{k}\sigma} \to c^\dagger_{\mathbf{k}\sigma} e^{i\varphi}$: gauge invariant $\quad$ not gauge invariant

Symmetry group U(1) $\quad$ –

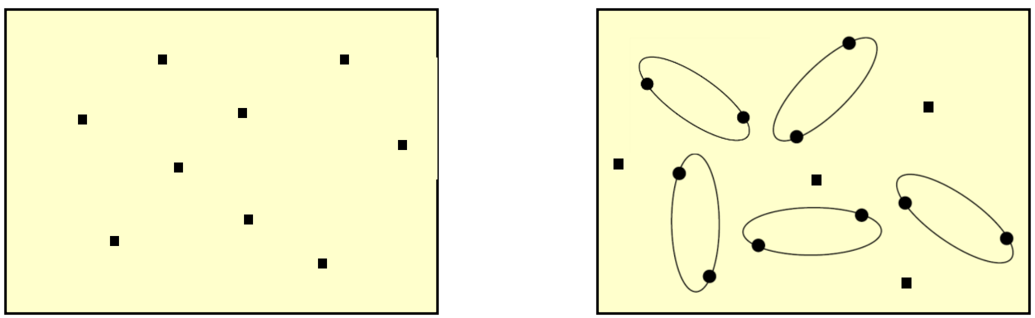
Broken Symmetries & Long-Range Order



Normal ^{3}He ↔ ^{3}He-A, ^{3}He-B:
2nd order phase transition

$T<T_c$: higher order, lower symmetry of ground state

III. Conventional superconductor



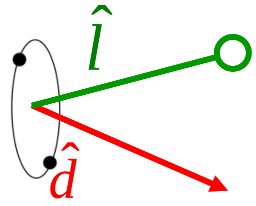
$T>T_c$ $T<T_c$

$T<T_C$: U(1) “gauge symmetry“ spontaneously broken

Broken symmetries in superfluid ^{3}He

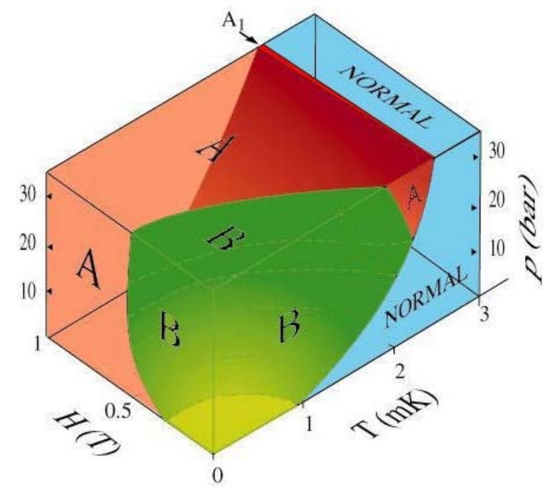
L=1, S=1 in all 3 phases

Cooper pair:



orbital part

spin part



Quantum coherence in	phase (complex order parameter)	Superfluid,
	anisotropy direction in real space	liquid crystal
	anisotropy direction in spin space	magnet

Characterized by $2\times(2L+1)\times(2S+1)$ = 18 real numbers

3x3 order parameter matrix $A_{i\mu}$

$SO(3)_S \times SO(3)_L \times U(1)_\varphi$ symmetry spontaneously broken — Leggett (1975)

$\simeq$ $SU(2)_L \times SU(2)_R \times U(1)_Y$ for electroweak interactions — Pati, Salam (1974)

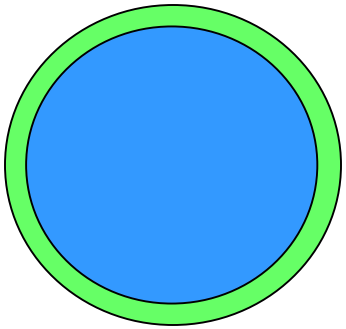
Broken symmetries in superfluid ^{3}He

Mineev (1980)
Bruder, Vollhardt(1986)

3He-B $SO(3)_S \times SO(3)_L \times U(1)_\varphi$ symmetry broken

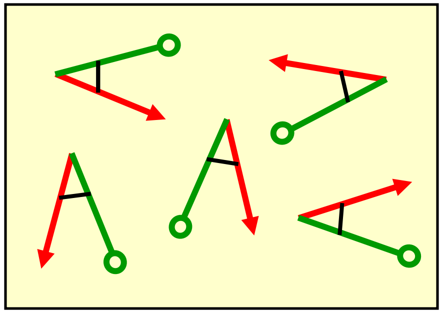
$SO(3)_{S+L}$ —

„Unconventional" pairing



Spontaneously broken spin-orbit symmetry Leggett (1972)

Cooper pairs



Fixed relative orientation

Broken symmetries in superfluid ^{3}He

Mineev (1980)
Bruder, Vollhardt(1986)

3He-B $SO(3)_S \times SO(3)_L \times U(1)_\varphi$ symmetry broken

$SO(3)_{S+L}$ —

„Unconventional" pairing

Relation to particle physics

Isodoublet $\begin{pmatrix} u \\ d \end{pmatrix}_L , \begin{pmatrix} u \\ d \end{pmatrix}_R$ chiral invariance

Global symmetry $SU(2)_L \times SU(2)_R$

$q\bar{q}$ condensation ("Cooper pair")

$SU(2)_{L+R}$

Goldstone excitations (bosons) 3 pions

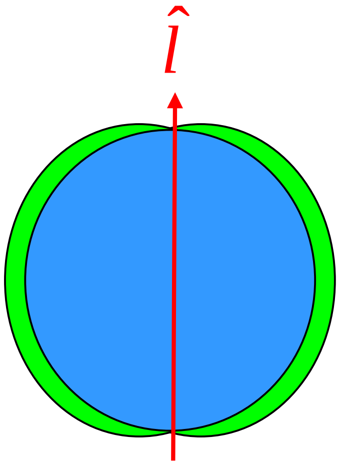
Broken symmetries in superfluid ^{3}He

Mineev (1980)
Bruder, Vollhardt(1986)

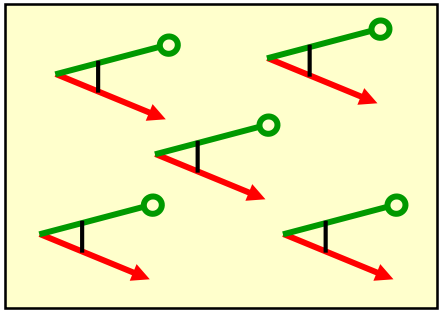
3He-A $SO(3)_S \times SO(3)_L \times U(1)_\varphi$ symmetry broken

$U(1)_{S_z} \times U(1)_{L_z - \varphi}$

„Unconventional" pairing

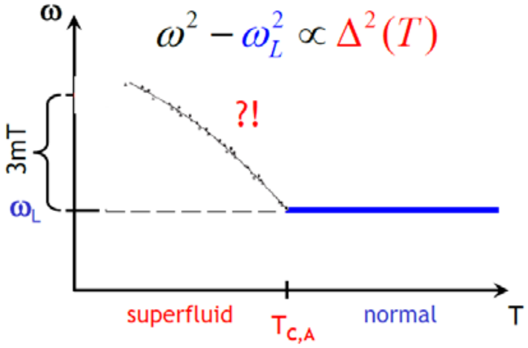


Cooper pairs



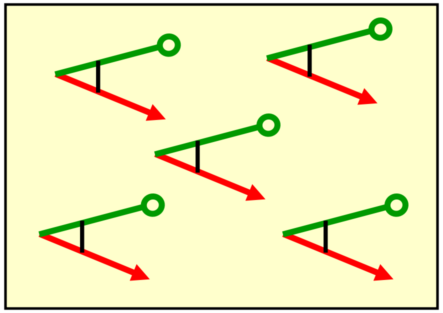
Fixed absolute orientation

... solves the NMR mystery



Superfluid ^{3}He - a quantum amplifier

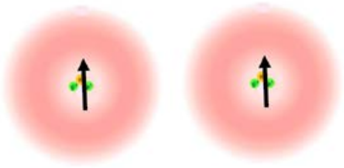
Cooper pairs in ^{3}He-A



Fixed absolute orientation

What fixes the relative orientation of $\hat{d}, \hat{l}$?

→ Interaction of nuclear dipoles ("spin-orbit coupling") :

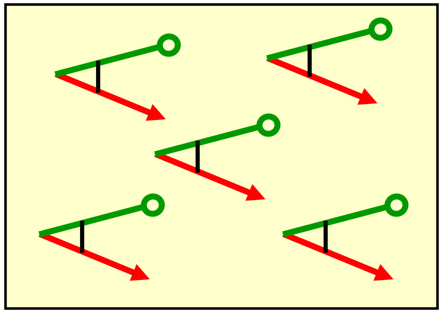


Dipole-dipole coupling of ^{3}He nuclei: $g_D \sim 10^{-7} K \ll T_C$

Unimportant ?!

Superfluid ^{3}He - a quantum amplifier

Cooper pairs in ^{3}He-A



Fixed absolute orientation

But:
- Long-range order in $\hat{d}, \hat{l}$
- $g_D \sim 10^{-7} K$: tiny, but lifts degeneracy of relative orientation

$\Downarrow$ quantum coherence

$\hat{d}, \hat{l}$ locked in all Cooper pairs at a fixed angle

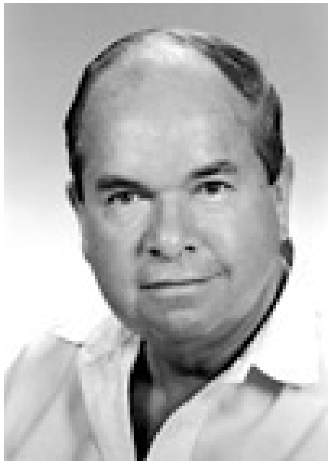
$\Downarrow$

NMR frequency increases: $\omega^2 = (\gamma H)^2 + g_D \Delta^2(T)$ Leggett (1973)

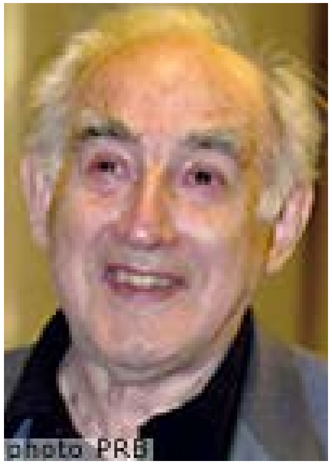
→ Nuclear dipole interaction is macroscopically measurable

The Nobel Prize in Physics 2003

"for pioneering contributions to the theory of superconductors and superfluids"



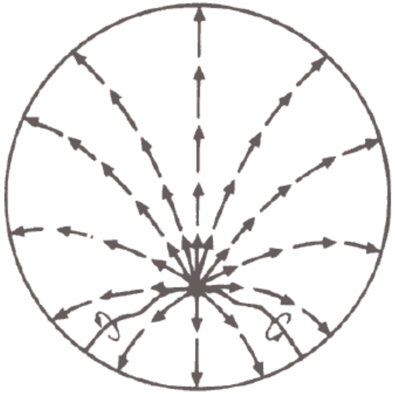
Alexei A. Abrikosov
USA and Russia



Vitaly L. Ginzburg
Russia

Anthony J. Leggett
UK and USA

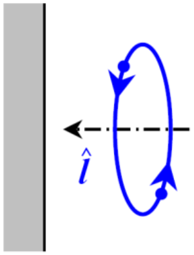
Order-parameter textures and topological defects

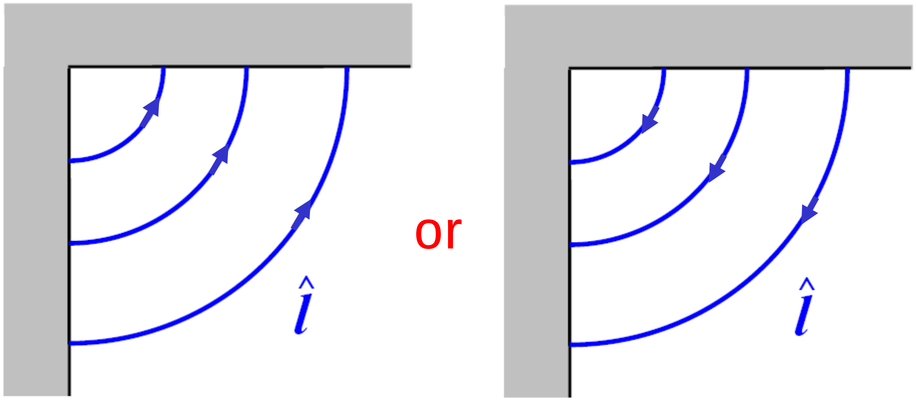


Order-parameter textures in ^{3}He-A

Orientation of the macroscopic anisotropy directions $\hat{d}, \hat{l}$:

1) Walls → $\hat{l}$





Chirality exp. confirmed:

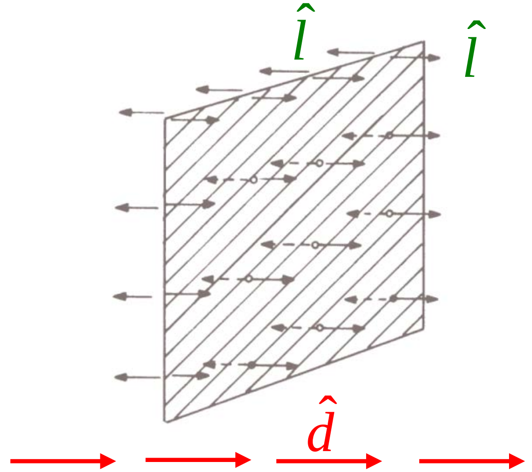
Walmsley, Golov (2012)

2) Magnetic field → $\hat{d}$

→ *Textures* in $\hat{d}, \hat{l}$ ↔ liquid crystals

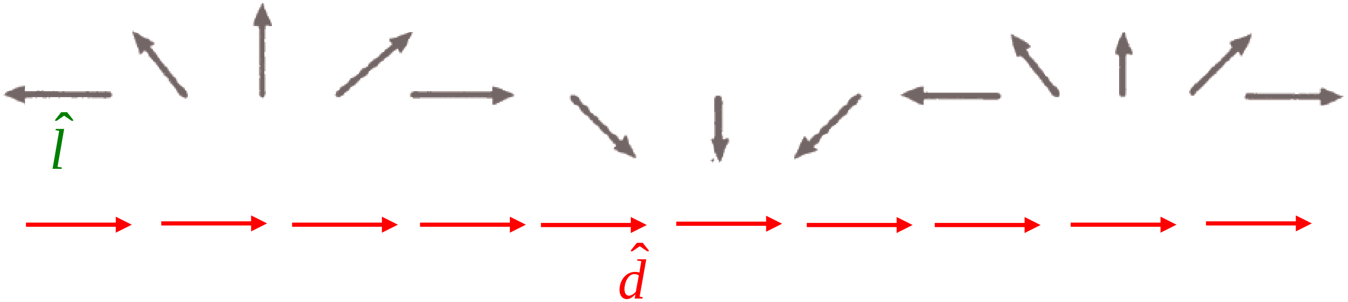
Order-parameter textures and topological defects in ^{3}He-A

D=2: domain walls (solitons) in $\hat{d}$ or $\hat{l}$ because of Z_2 symmetry



Single domain wall

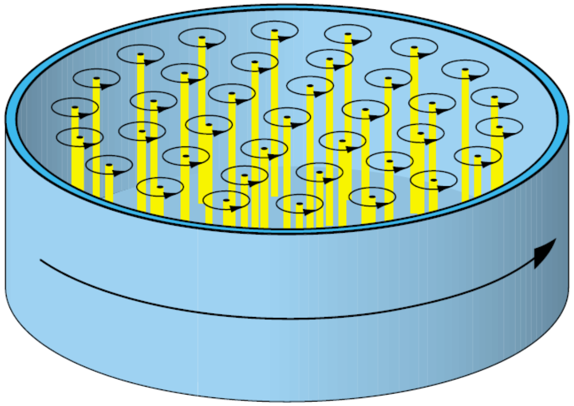
Cannot be removed by local operation
→ topological defect



Domain wall lattice

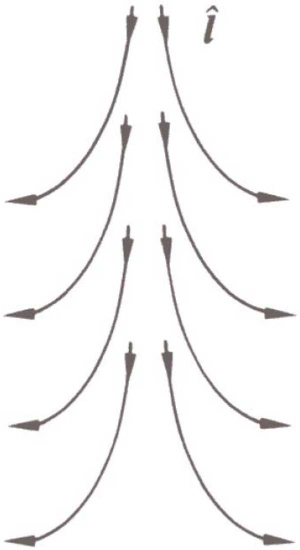
Order-parameter textures and topological defects in ^{3}He-A

D=1: Vortices



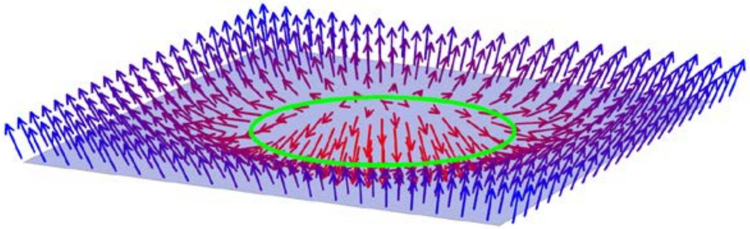
Vortex formation by rotation

http://ltl.tkk.fi/research/theory/vortex.html



e.g., Mermin-Ho vortex
(non-singular)

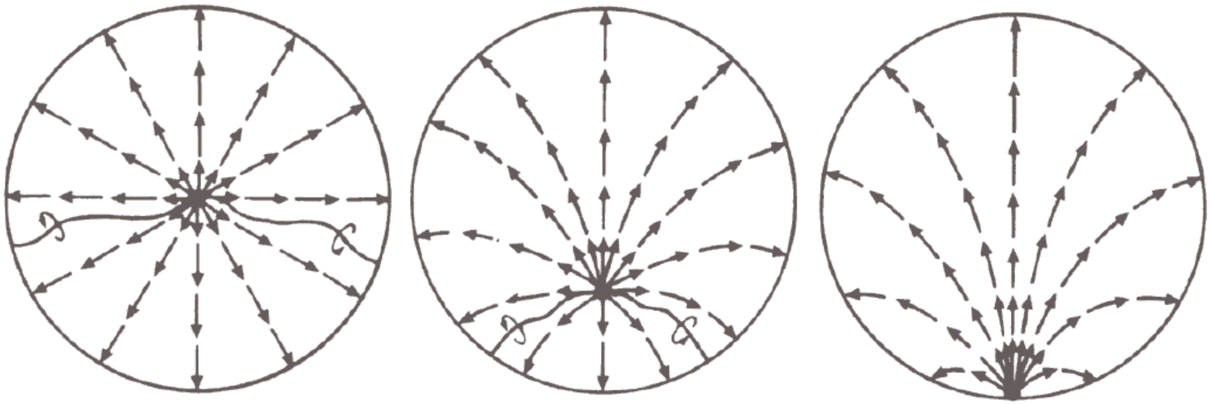
Thin film of ^{3}He-A



Skyrmion vortex Volovik (2003), Sauls (2013)

Order-parameter textures and topological defects in ^{3}He-A

D=0: Monopoles



"Boojum" in $\hat{l}$-texture of ^{3}He-A

Defect formation by
- rotation
- geometric constraints
- rapid crossing through symmetry breaking phase transition

Big bang simulation

in the low-temperature lab

Universality in symmetry-breaking phase transitions

High symmetry, short-range order — $T>T_c$

	Spins:	Helium:	Universe:
	para-magnetic	normal liquid	Unified forces and fields

$T=T_c$ — Phase transition

Broken symmetry, long-range order

	Spins	Helium	Universe
	ferromagnetic	superfluid	elementary particles, fundamental interactions
Defects:	domain walls	vortices, etc.	cosmic strings, etc. Kibble (1976)

$T<T_c$

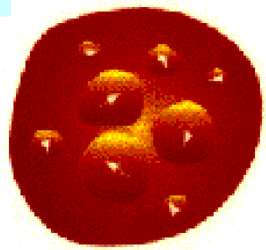
nucleation of galaxies?

Rapid thermal quench through 2nd order phase transition   Bäuerle *et al.* (1996)

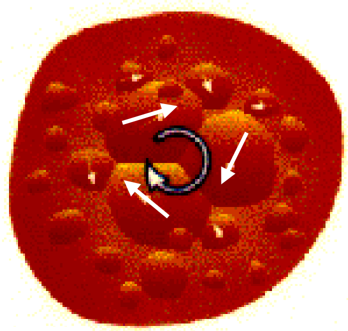
1. Local temperature $T \gg T_c$

→ Expansion + rapid cooling through T_c

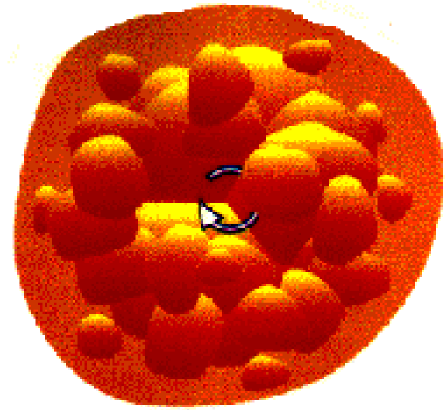
2. Nucleation of independently ordered regions



Clustering of ordered regions
→ Defects form

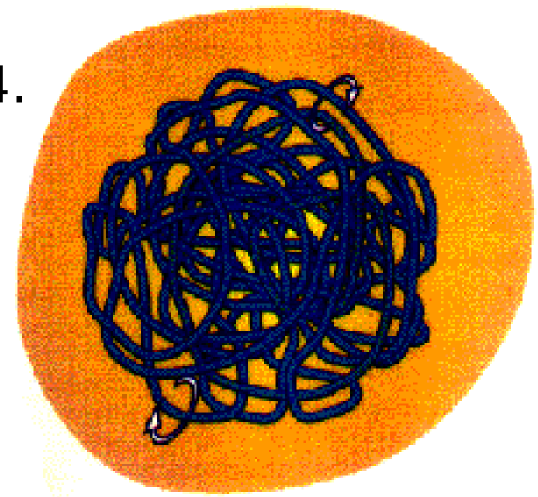


3.



Defects overlap

4.



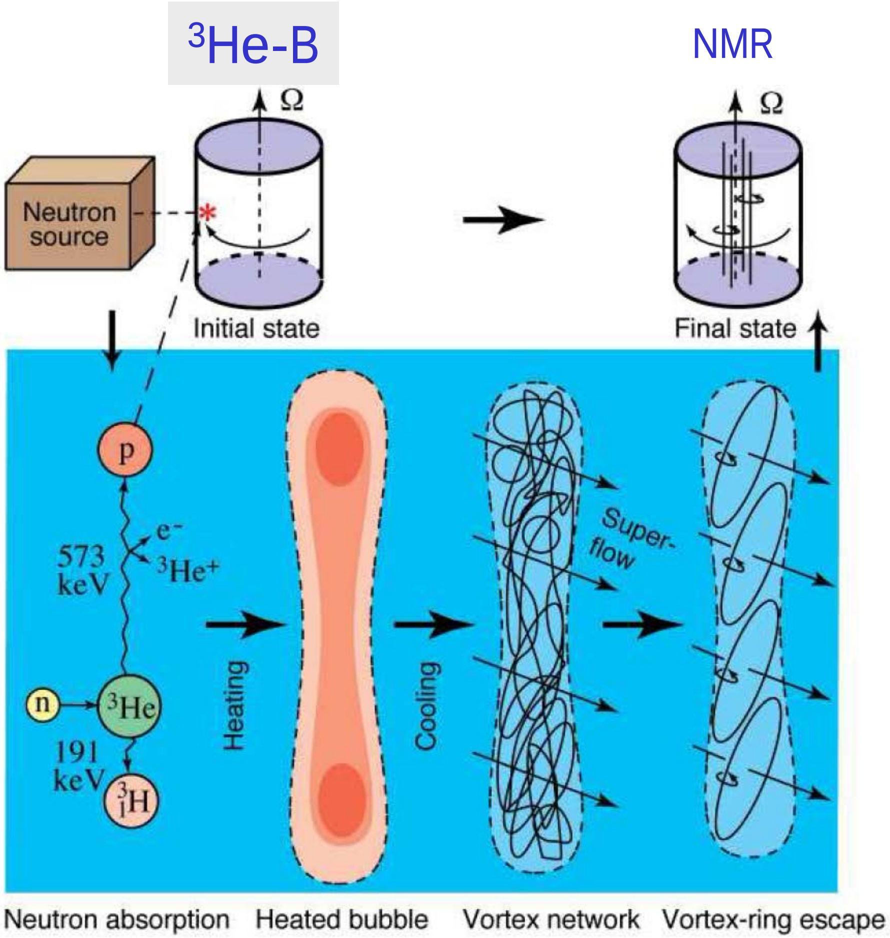
Vortex tangle

Estimate of density of defects: Zurek (1985)

"Kibble-Zurek mechanism" of defect formation: How to test?

Big-bang simulation in the low-temperature laboratory

Grenoble: Bäuerle *et al.* (1996), Helsinki: Ruutu *et al.* (1996)



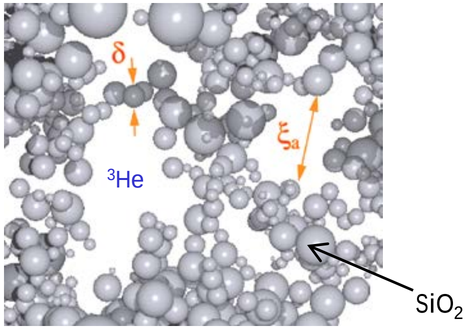
Measured vortex-tangle density:

Quantitative support for Kibble-Zurek mechanism

Current research on superfluid ^3He

1. Influence of **disorder** on superfluidity

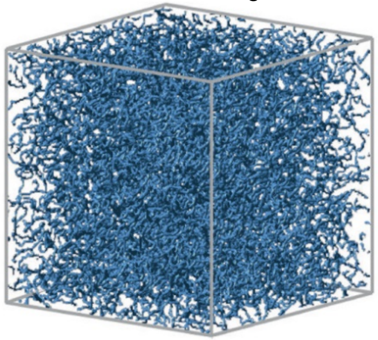
^3He in 98% open silica aerogel



Halperin *et al.* (2008)

2. **Quantum Turbulence**
   No friction – only vortex tangle
   Origin of decay of pure superfluid turbulence?
   Test systems: ^4He-II, ^3He-B

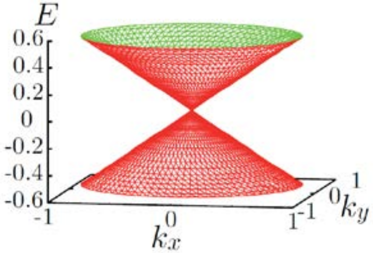
Vortex tangle



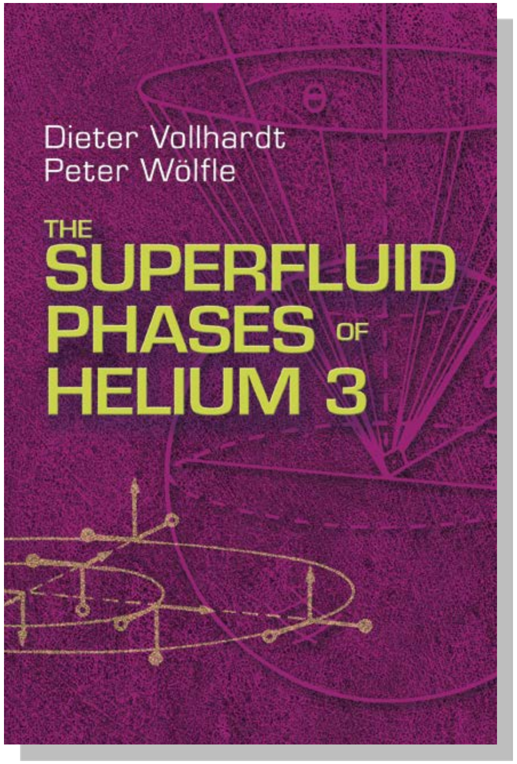
Tsubota (2008)

3. **Majorana fermions** (e.g., zero-energy Andreev bound states at surfaces in ^3He-B)

Majorana cone for ^3He-B in a thick slab



Tsutsumi, Ichioka, Machida (2011)



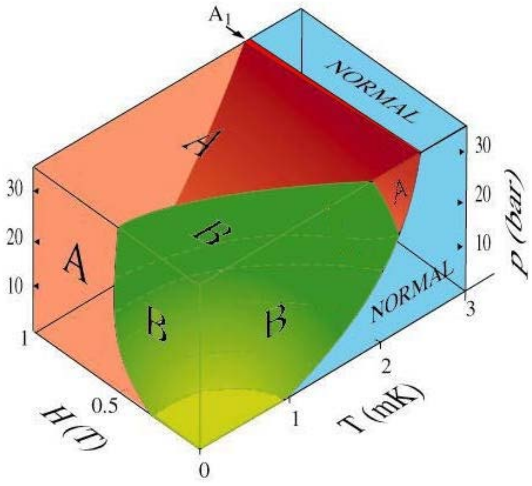
The Superfluid Phases of Helium 3

D. Vollhardt and P. Wölfle

(Taylor & Francis, 1990), 656 pages

Reprinted by Dover Publications (2013)

Conclusion



Superfluid Helium-3:

- Anisotropic superfluid (p-wave, spin-triplet pairing)
  - Cooper pairs with internal structure
  - 3 different bulk phases with many novel properties
- Large symmetry-group broken
  - Close connections with high energy physics
  - Zoo of topological defects