High-accuracy astrometry and experimental foundations of General Relativity

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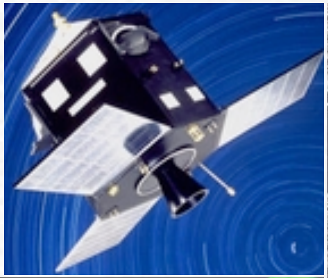
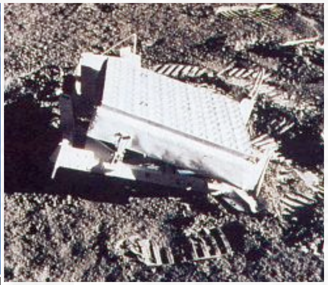
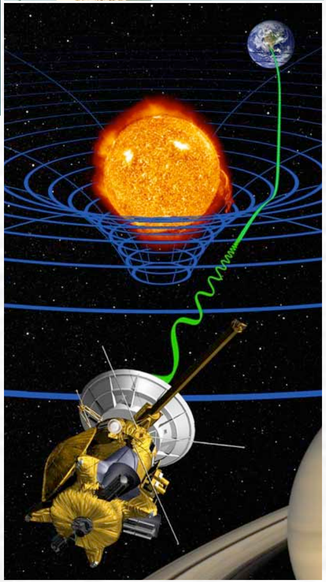
Why relativity?

Relativity in high-accuracy observations in the solar system

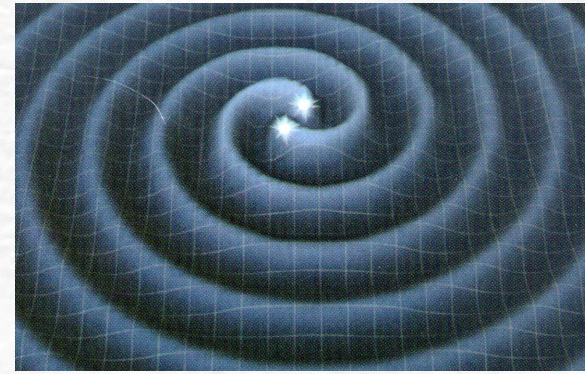
Several general-relativistic effects are seen
In the data with the following precisions:

- VLBI ± 0.0003
- HIPPARCOS ± 0.003
- Viking radar ranging ± 0.002
- Cassini radar ranging ± 0.000023
- Planetary radar ranging ± 0.0001
- Lunar laser ranging I ± 0.0005
- Lunar laser ranging II ± 0.007

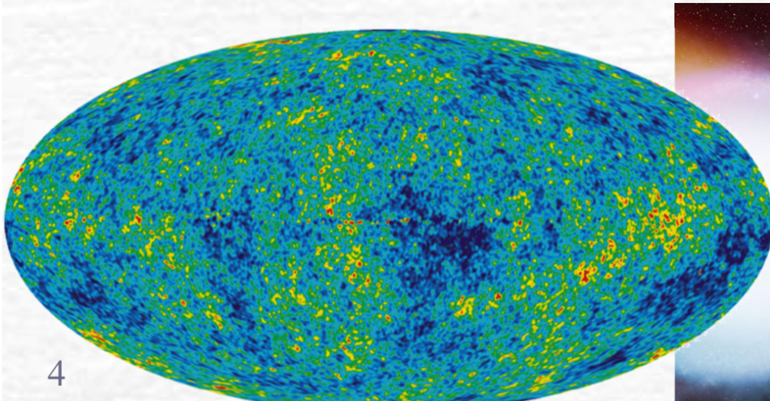



Relativity outside of the Solar system



- Gravitational microlensing
  a way to detect low-mass stars in the Galaxy
- Gravitational (macro-)lensing
- Pulsars in binaries (7 systems, 1 system of two pulsars!)

- Black holes of stellar masses (systems like Cyg X1)
- Black holes in the centers of galaxies and quasars
- Cosmology



Why general relativity?

- Newtonian models cannot describe observations:
  - many relativistic effects are many orders of magnitude larger than the observational accuracy

⇓

- The simplest theory which successfully describes all available observational data:

GENERAL RELATIVITY

Experimental foundations of Newtonian gravity

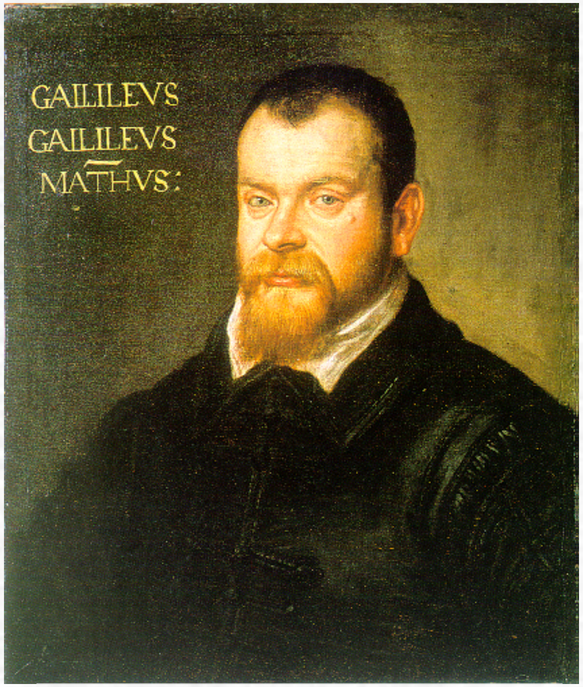
Newtonian gravity

Based on physical ideas of Galileo Galilei and empirical findings of Johannes Kepler, Isaac Newton has provided a clear mathematical model of gravity:

$$m_A \ddot{\mathbf{x}}_A = -\sum_{B \neq A} \frac{G m_A m_B}{\left|\mathbf{x}_A - \mathbf{x}_B\right|^3} \left(\mathbf{x}_A - \mathbf{x}_B\right)$$

Until 1859 the model explained all experimental facts within their observational accuracy

Johannes Kepler (1571-1630)



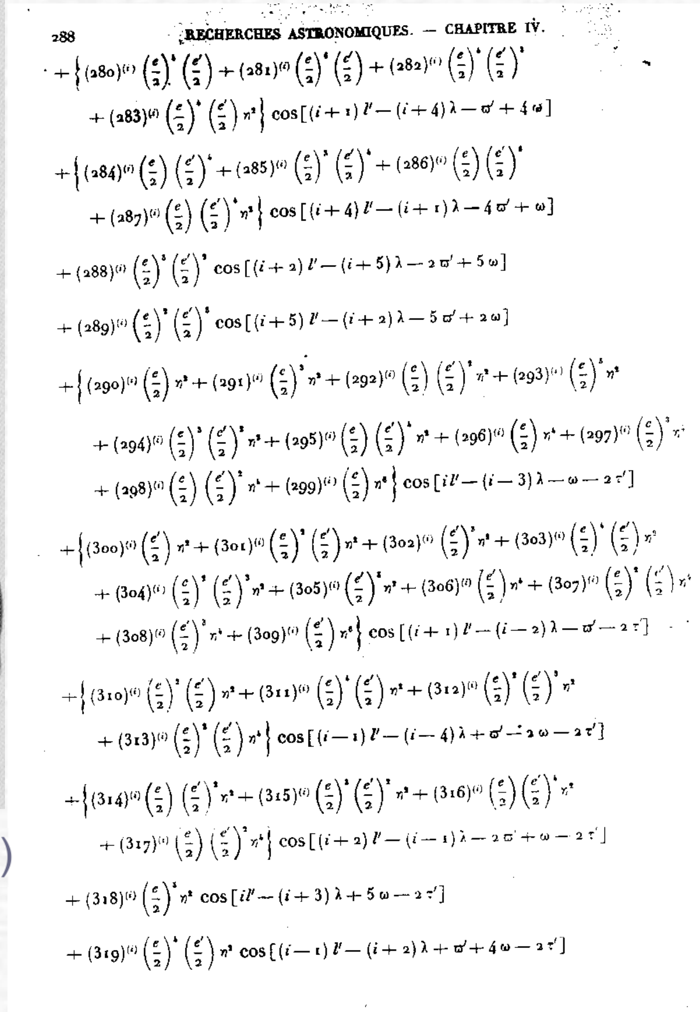
Galileo Galilei (1564-1642)

Isaac Newton (1643-1727)

Triumph of Newtonian gravity

Having performed analytical computations of incredible complexity
Urbain Leverrier 1846 has predicted the position of a new planet Neptune:

Urbain J.J. Leverrier (1811-1877)



288 RECHERCHES ASTRONOMIQUES. — CHAPITRE IV.

$$+\left\{(280)^{(i)}\left(\frac{e}{2}\right)^4\left(\frac{e'}{2}\right)+(281)^{(i)}\left(\frac{e}{2}\right)^6\left(\frac{e'}{2}\right)+(282)^{(i)}\left(\frac{e}{2}\right)^4\left(\frac{e'}{2}\right)^3\right.$$
$$\left.+(283)^{(i)}\left(\frac{e}{2}\right)^4\left(\frac{e'}{2}\right)\eta^2\right\}\cos[(i+1)l'-(i+4)\lambda-\varpi'+4\omega]$$

$$+\left\{(284)^{(i)}\left(\frac{e}{2}\right)\left(\frac{e'}{2}\right)^4+(285)^{(i)}\left(\frac{e}{2}\right)^3\left(\frac{e'}{2}\right)^4+(286)^{(i)}\left(\frac{e}{2}\right)\left(\frac{e'}{2}\right)^6\right.$$
$$\left.+(287)^{(i)}\left(\frac{e}{2}\right)\left(\frac{e'}{2}\right)^4\eta^2\right\}\cos[(i+4)l'-(i+1)\lambda-4\varpi'+\omega]$$

$$+(288)^{(i)}\left(\frac{e}{2}\right)^5\left(\frac{e'}{2}\right)^2\cos[(i+2)l'-(i+5)\lambda-2\varpi'+5\omega]$$

$$+(289)^{(i)}\left(\frac{e}{2}\right)^2\left(\frac{e'}{2}\right)^5\cos[(i+5)l'-(i+2)\lambda-5\varpi'+2\omega]$$

$$+\left\{(290)^{(i)}\left(\frac{e}{2}\right)\eta^2+(291)^{(i)}\left(\frac{e}{2}\right)^3\eta^2+(292)^{(i)}\left(\frac{e}{2}\right)\left(\frac{e'}{2}\right)^2\eta^2+(293)^{(i)}\left(\frac{e}{2}\right)^5\eta^2\right.$$
$$+(294)^{(i)}\left(\frac{e}{2}\right)^3\left(\frac{e'}{2}\right)^2\eta^2+(295)^{(i)}\left(\frac{e}{2}\right)\left(\frac{e'}{2}\right)^4\eta^2+(296)^{(i)}\left(\frac{e}{2}\right)\eta^4+(297)^{(i)}\left(\frac{e}{2}\right)^3\eta^4$$
$$\left.+(298)^{(i)}\left(\frac{e}{2}\right)\left(\frac{e'}{2}\right)^2\eta^4+(299)^{(i)}\left(\frac{e}{2}\right)\eta^6\right\}\cos[il'-(i-3)\lambda-\omega-2\tau']$$

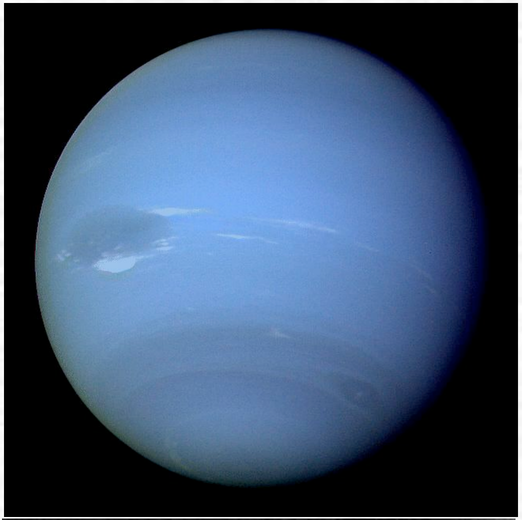
$$+\left\{(300)^{(i)}\left(\frac{e'}{2}\right)\eta^2+(301)^{(i)}\left(\frac{e}{2}\right)^2\left(\frac{e'}{2}\right)\eta^2+(302)^{(i)}\left(\frac{e'}{2}\right)^3\eta^2+(303)^{(i)}\left(\frac{e}{2}\right)^4\left(\frac{e'}{2}\right)\eta^2\right.$$
$$+(304)^{(i)}\left(\frac{e}{2}\right)^2\left(\frac{e'}{2}\right)^3\eta^2+(305)^{(i)}\left(\frac{e'}{2}\right)^5\eta^2+(306)^{(i)}\left(\frac{e'}{2}\right)\eta^4+(307)^{(i)}\left(\frac{e}{2}\right)^2\left(\frac{e'}{2}\right)\eta^4$$
$$\left.+(308)^{(i)}\left(\frac{e'}{2}\right)^3\eta^4+(309)^{(i)}\left(\frac{e'}{2}\right)\eta^6\right\}\cos[(i+1)l'-(i-2)\lambda-\varpi'-2\tau']$$

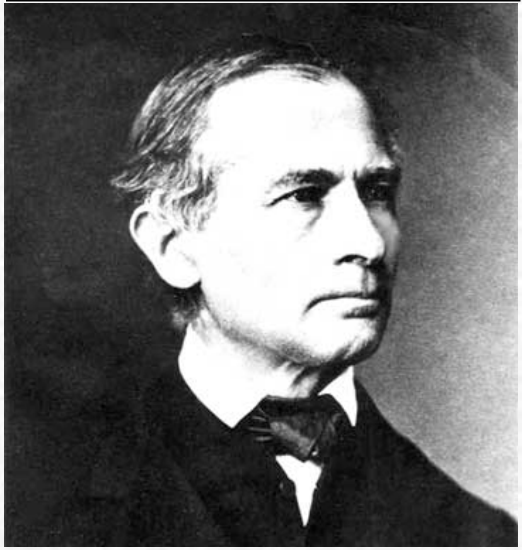
$$+\left\{(310)^{(i)}\left(\frac{e}{2}\right)^2\left(\frac{e'}{2}\right)\eta^2+(311)^{(i)}\left(\frac{e}{2}\right)^4\left(\frac{e'}{2}\right)\eta^2+(312)^{(i)}\left(\frac{e}{2}\right)^2\left(\frac{e'}{2}\right)^3\eta^2\right.$$
$$\left.+(313)^{(i)}\left(\frac{e}{2}\right)^2\left(\frac{e'}{2}\right)\eta^4\right\}\cos[(i-1)l'-(i-4)\lambda+\varpi'-2\omega-2\tau']$$

$$+\left\{(314)^{(i)}\left(\frac{e}{2}\right)\left(\frac{e'}{2}\right)^2\eta^2+(315)^{(i)}\left(\frac{e}{2}\right)^3\left(\frac{e'}{2}\right)^2\eta^2+(316)^{(i)}\left(\frac{e}{2}\right)\left(\frac{e'}{2}\right)^4\eta^2\right.$$
$$\left.+(317)^{(i)}\left(\frac{e}{2}\right)\left(\frac{e'}{2}\right)^2\eta^4\right\}\cos[(i+2)l'-(i-1)\lambda-2\varpi'+\omega-2\tau']$$

$$+(318)^{(i)}\left(\frac{e}{2}\right)^5\eta^2\cos[il'-(i+3)\lambda+5\omega-2\tau']$$

$$+(319)^{(i)}\left(\frac{e}{2}\right)^4\left(\frac{e'}{2}\right)\eta^2\cos[(i-1)l'-(i+2)\lambda+\varpi'+4\omega-2\tau']$$





Johann G. Galle (1812-1910)

Neptune was observed close to the predicted position
on 23.09.1846 by Johann Gottfried Galle in Potsdam

Assumptions of Newtonian gravity

The assumptions of Newtonian gravity can be read off the main equation:

$$m_A^{in}\ddot{\mathbf{x}}_A = -\sum_{B\neq A}\frac{G m_A^{gr} m_B^{gr}}{\left|\mathbf{x}_A - \mathbf{x}_B\right|^3}\left(\mathbf{x}_A - \mathbf{x}_B\right)$$

These are:

1) $m^{in} = m^{gr}$ Weak equivalence principle (WEP)

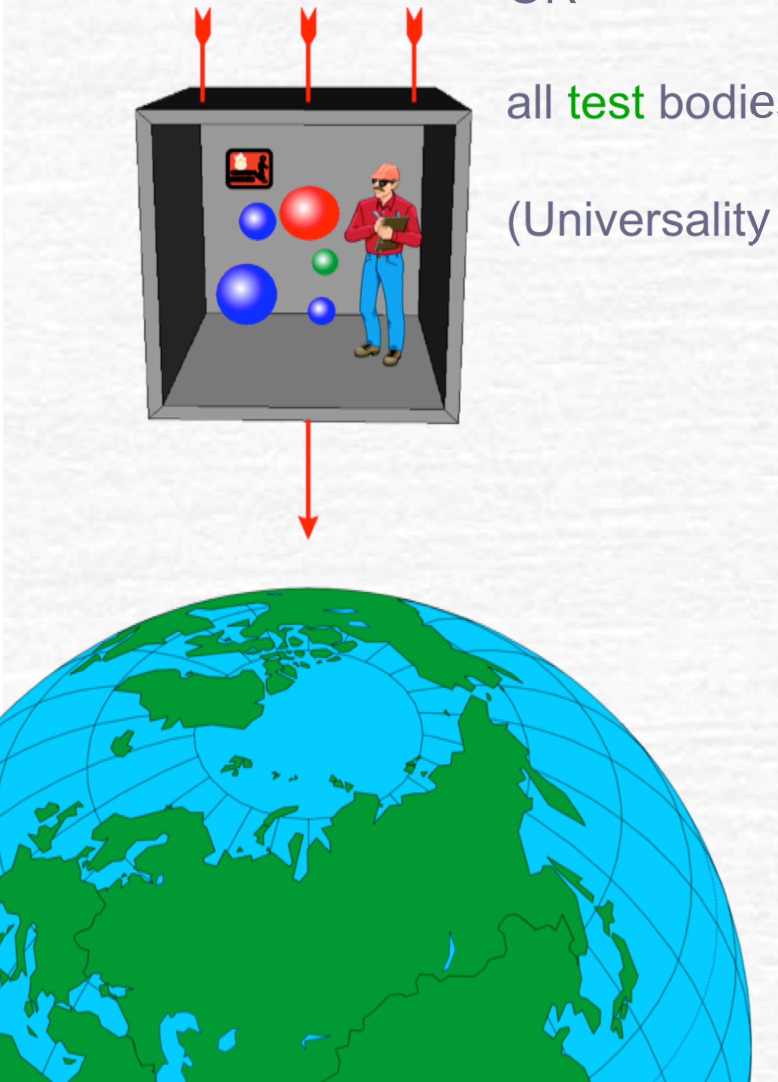
2) $G \neq G(t)$

3) $G \neq G(r)$

G is constant both in time and in space

Weak equivalence principle

$$m^{in} = m^{gr}$$



inertial mass is equal (or proportional) to gravity mass

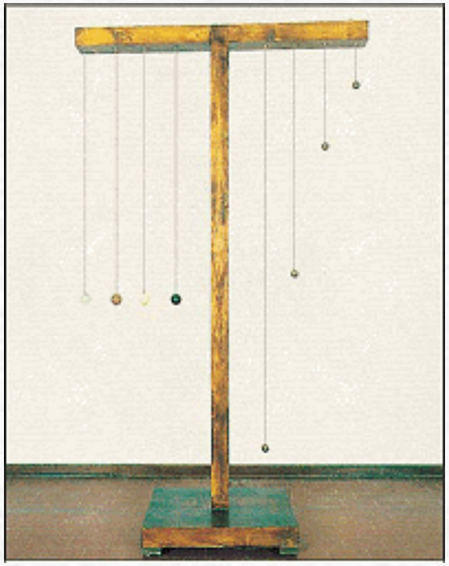
OR

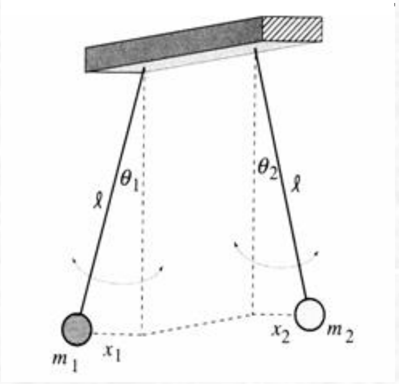
all test bodies fall with the same acceleration

(Universality of Free Fall: Einstein's elevator)

- The WEP was first tested by Galileo Galilei by "throwing things" from the Pisa tower: 0.02

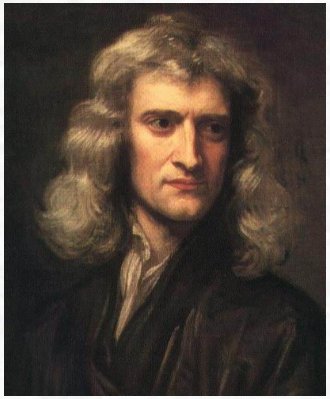
Weak equivalence principle: pendulum





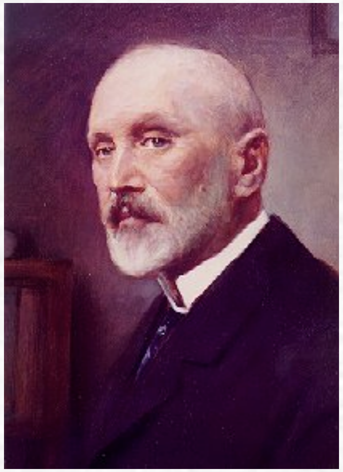
different materials – equal periods

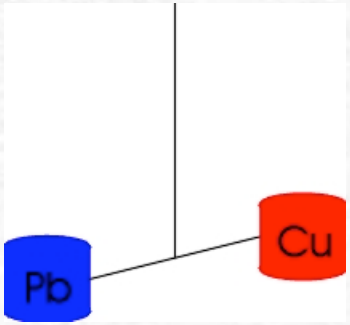
Galileo Galilei	(1590-1638)	0.02
Isaac Newton	(1680)	0.001
Friedrich Bessel	(1830)	0.000017

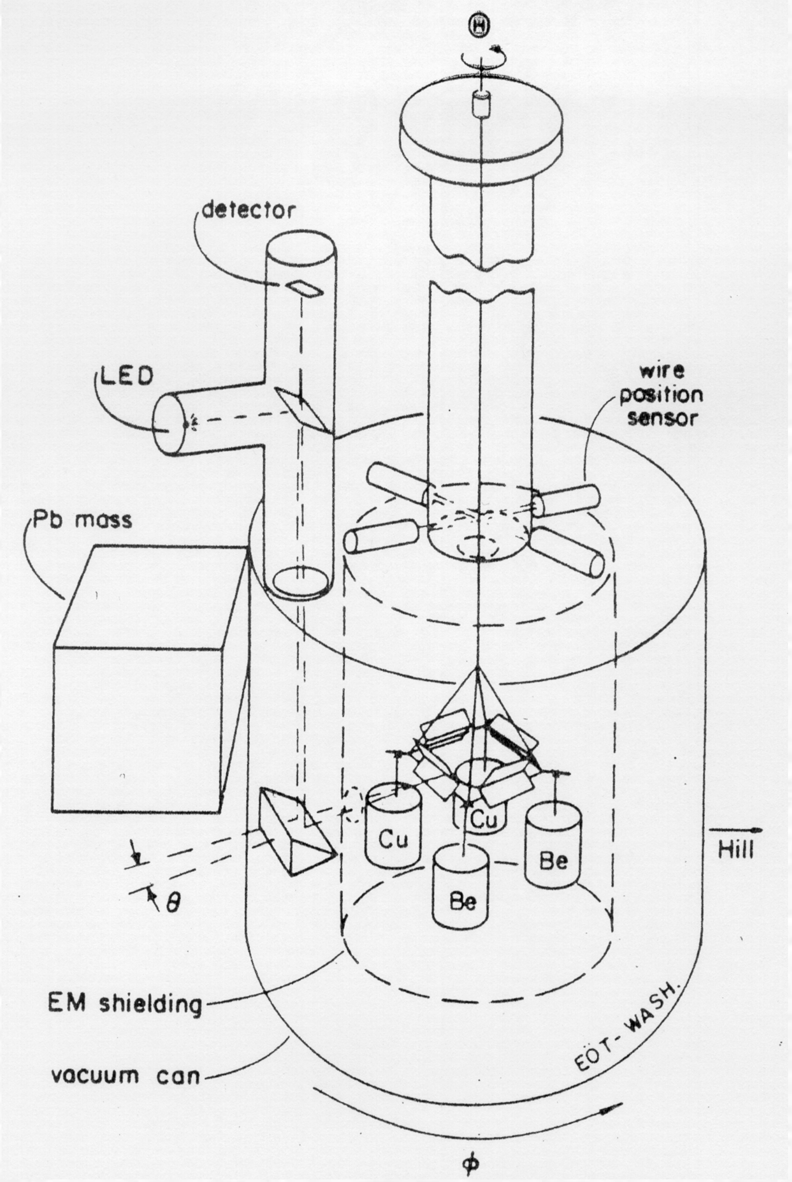


Friedrich Wilhelm Bessel
(1784-1846)

Weak equivalence principle: torsion pendulum







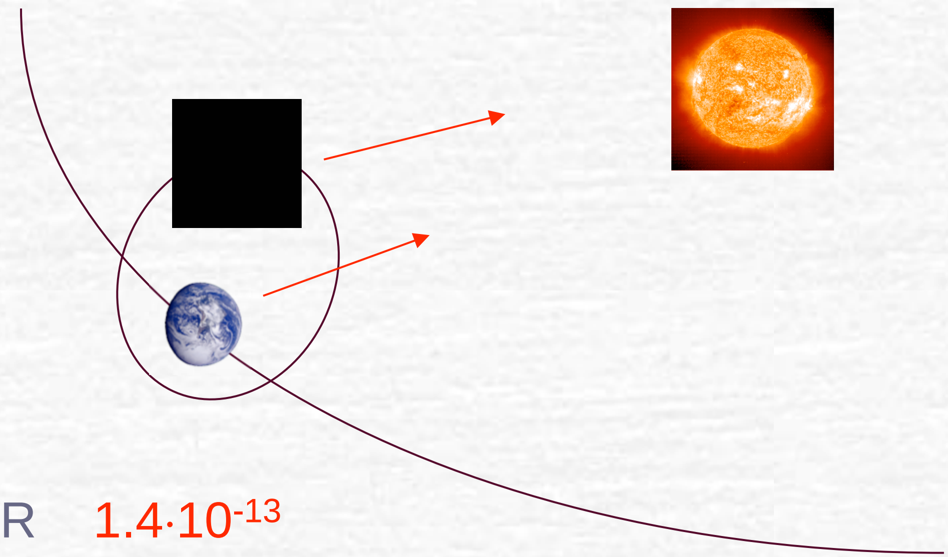
Loránd Eötvös (1848-1919)

detecting a torque on a hanging pendulum

Eötvös	(1909)	$5 \cdot 10^{-9}$
Braginsky-Panov	(1972)	10^{-12}
Adelberger	(2003)	$5 \cdot 10^{-13}$

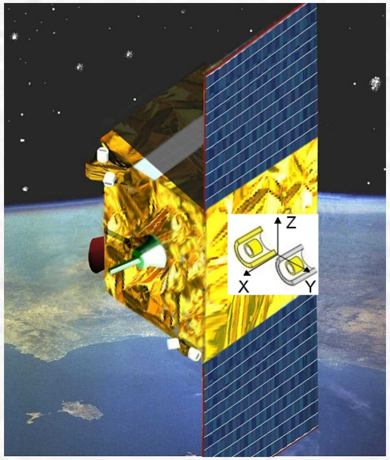
Weak equivalence principle: free fall

⇒

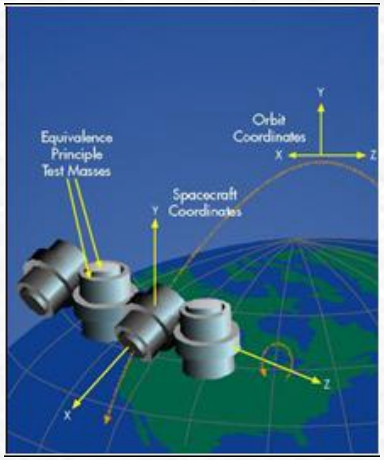


- freely falling Earth and Moon: LLR 1.4·10^{-13}

- freely falling test bodies on an orbit around the Earth: Microscope, GG, STEP



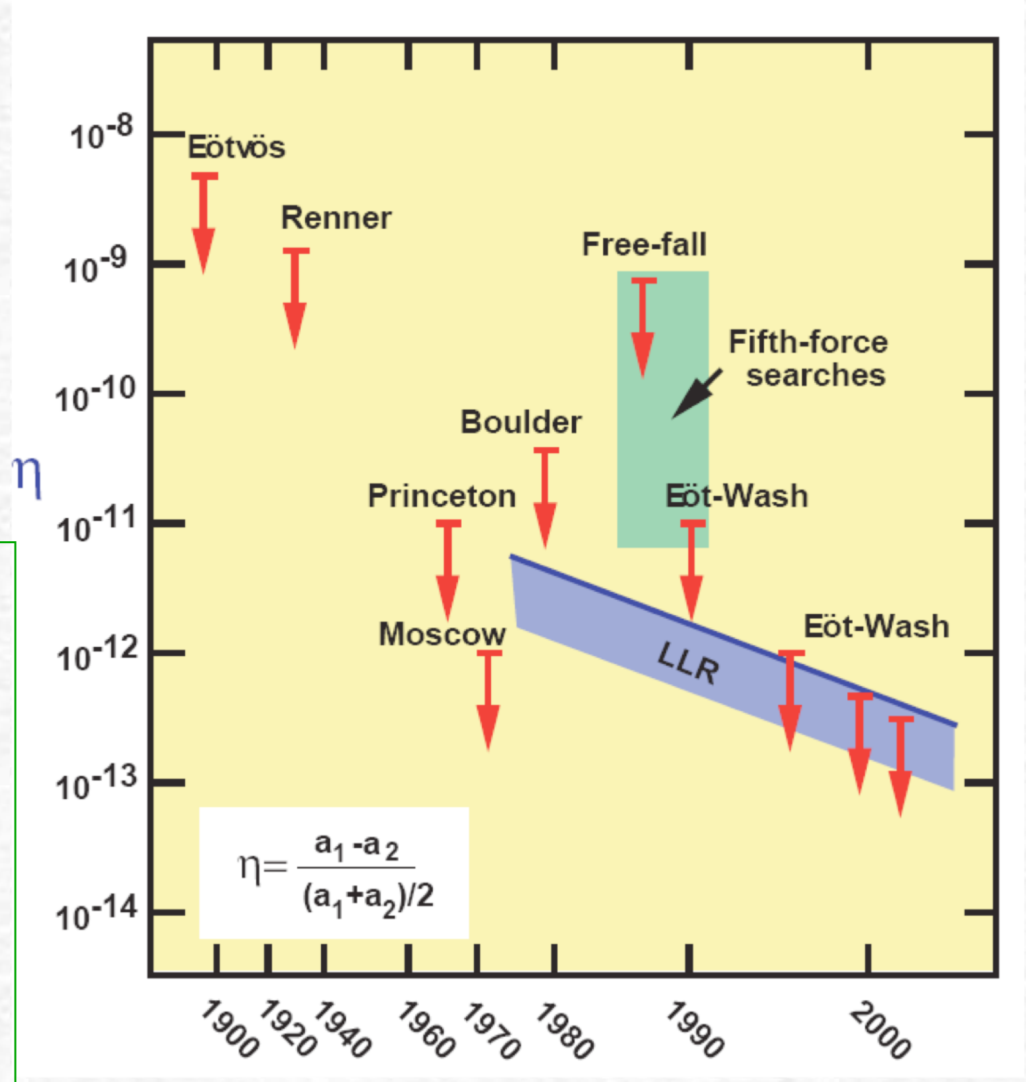
Microscope: 10^{-15}



STEP: 10^{-17}

Weak equivalence principle

Relative difference between accelerations of two different bodies



Will, 2005

Funded projects:

APOLLO (LLR): 10^{-14} @ 2015
MicroSCOPE: 10^{-15} @ 2012

Most ambitious unfunded idea:

STEP: 10^{-17}

Constancy of G in space

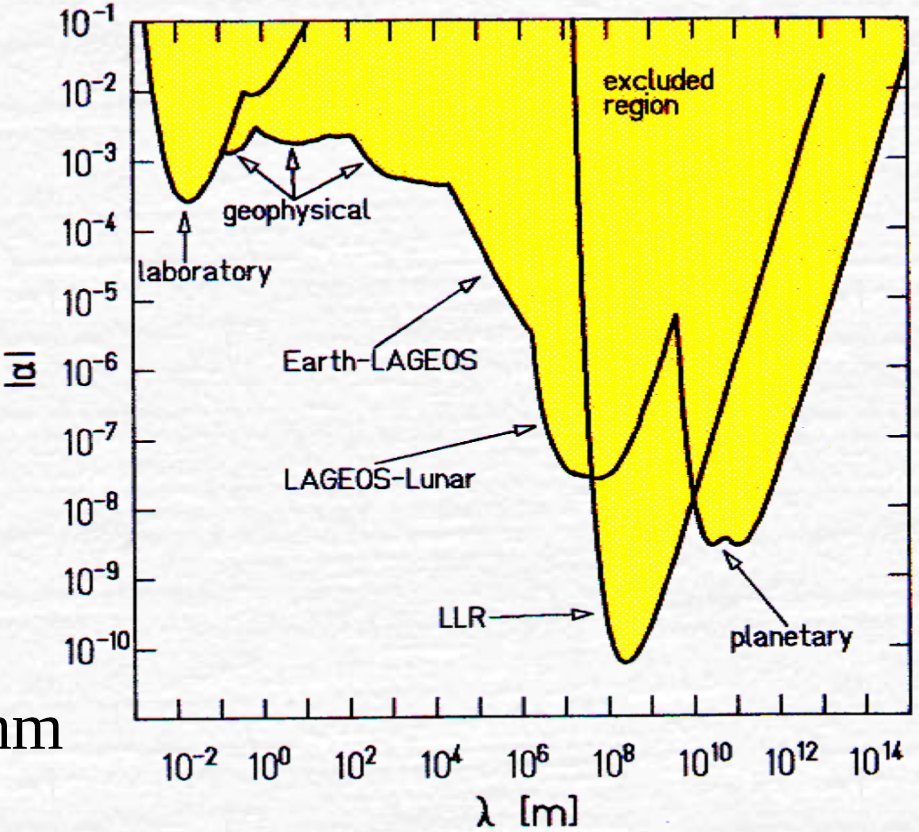
Various physical ideas related to the search of new kinds of interactions lead to a modified law of gravity with

$$G(r) = G\left(1 + \alpha\left(1 + \frac{r}{\lambda}\right)\exp\left(-\frac{r}{\lambda}\right)\right)$$

Fifth force (1986-1995): $\lambda \sim 100\ \mathrm{m}$

$$G(r) \approx G\, r^{-n}, \quad n = 1, 2, \ldots$$

Some ideas in the string theory: $\lambda < 1\ \mathrm{mm}$



No deviations were found between 10^{-5} m to 10^{13} m

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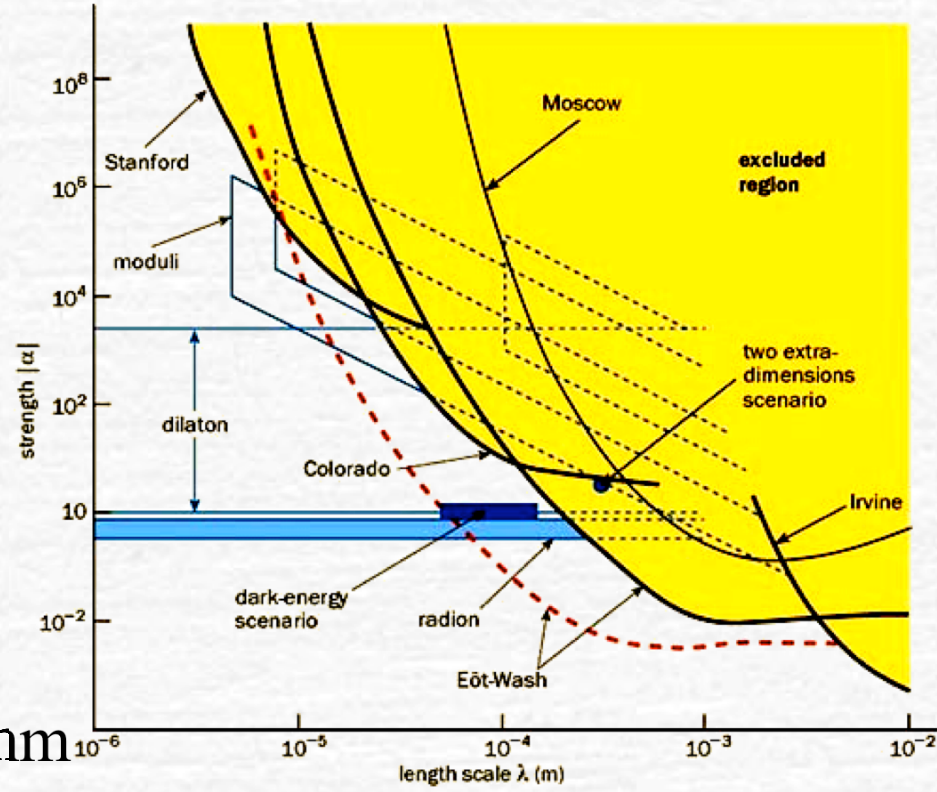

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No deviations were found between 10^{-5} m to 10^{13} m

Constancy of G in time

If G were time-dependent, the motion of planets would have a specific behaviour in time: linear drift of the periods of motion.

This can be tested in the solar system!

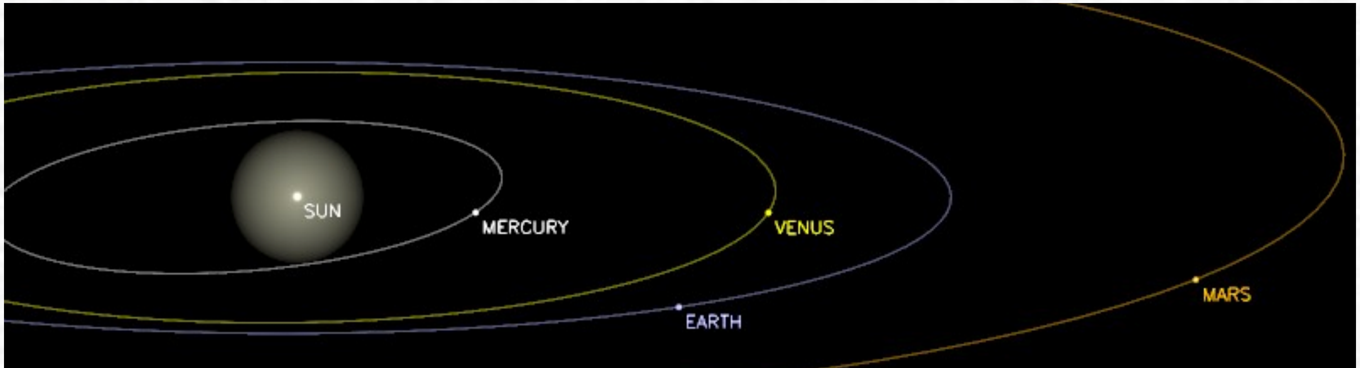
	$\dot{G}/G, \quad yr^{-1}$
Moon	$<7\cdot10^{-13}$
planets	$<5\cdot10^{-13}$
asteroids	$<10^{-10}$

Funded projects:

APOLLO (LLR): 10^{-14} @ 2015

Warning:
masses become time-dependent below 10^{-13} / yr !

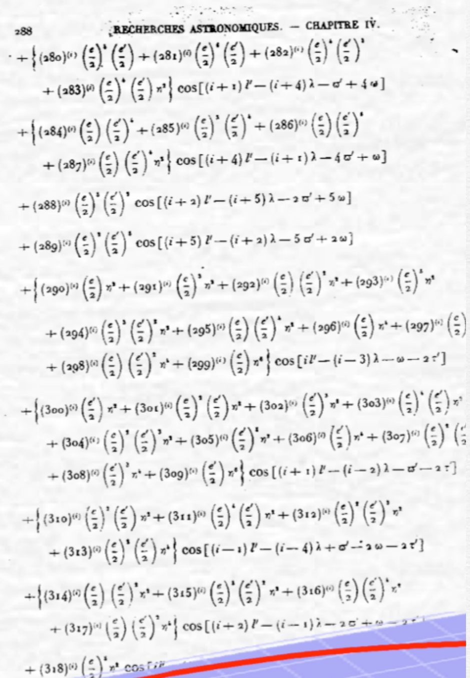
many independent groups confirm these results…



Newtonian gravity or General Relativity?

The first experimental fact contradicting Newtonian theory of gravity

The perihelion advance of Mercury discovered 1859 by Leverrier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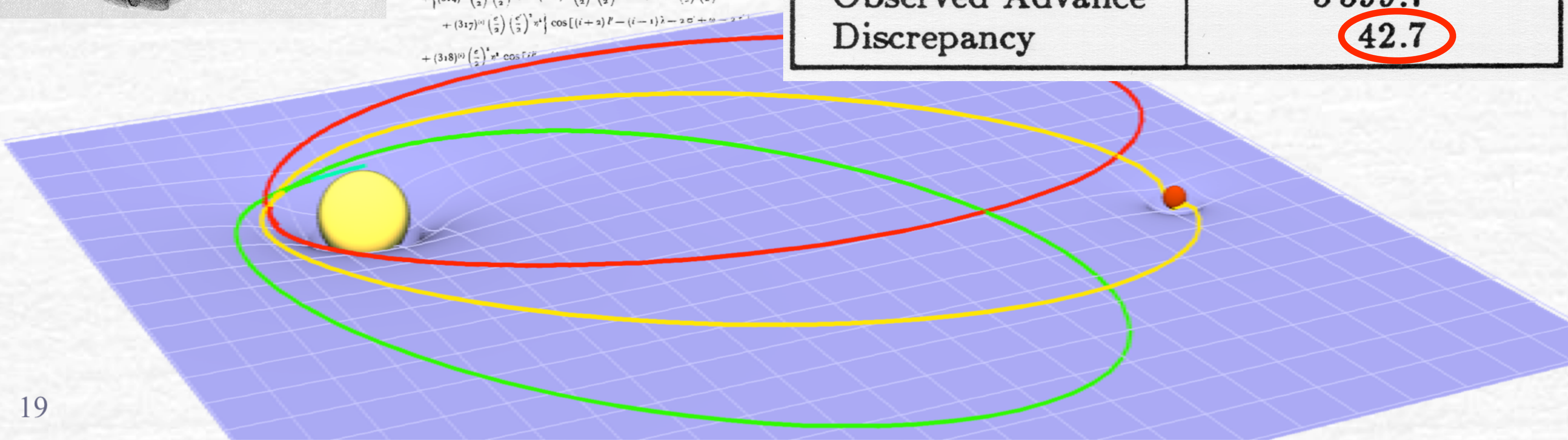
Cause of advance	Rate (″/century)
General precession (epoch 1900)	5 025.6
Venus	277.8
Earth	90.0
Mars	2.5
Jupiter	153.6
Saturn	7.3
Others	0.2
Sum	5 557.0
Observed Advance	5 599.7
Discrepancy	42.7



How to explain the perihelion advance?

Many ideas were proposed to explain
the anomalous perihelion advance of Mercury:

A) Additional bodies:

- additional planet between Mercury and Sun (Vulcan)
- rings of dust or minor bodies of very special forms and masses

B) Various modifications of the Newtonian attraction law

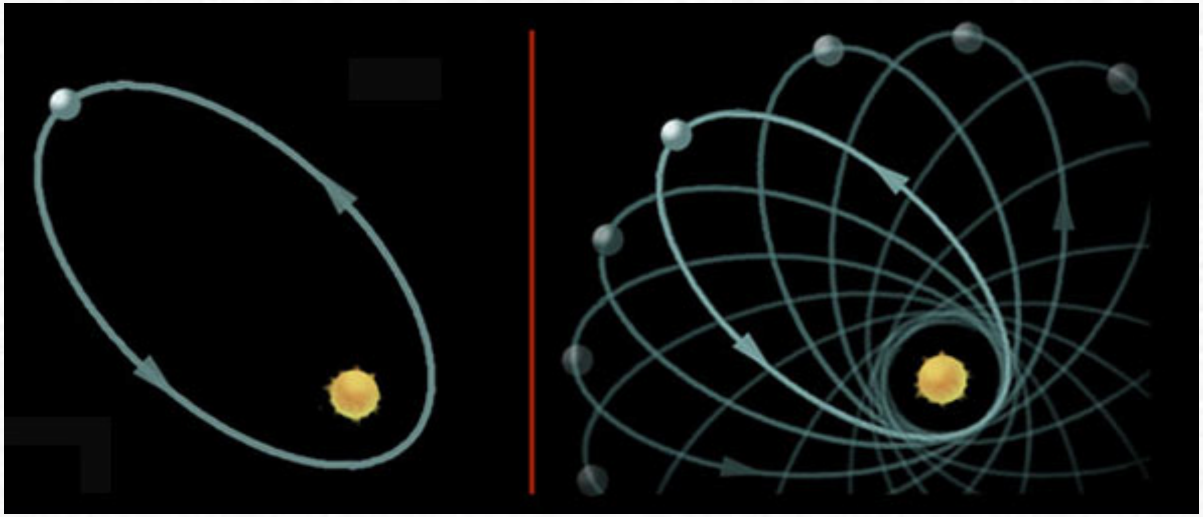
- $F \sim 1/r^{2+\varepsilon}$
- $F = F(r,v)$
- ...

All failed!

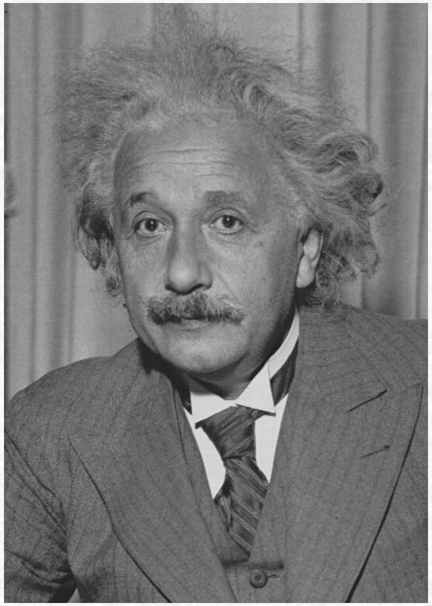
The problem was to find an explanation for the perihelion advance of Mercury, which does not destroy other predictions (e.g. motion of the Moon) of Newtonian gravity

… and the answer was

General Relativity Theory



Newtonian Gravity General Relativity

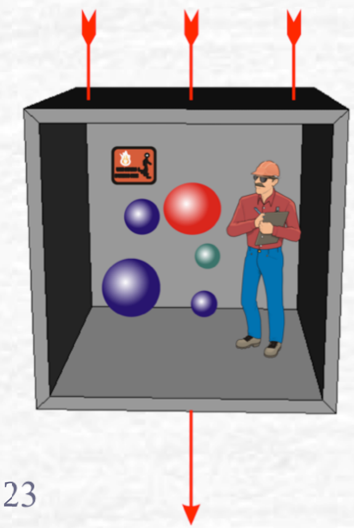


Experimental foundations of the Einstein Equivalence Principle

Einstein equivalence principle

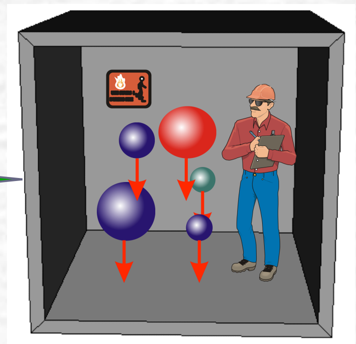
The Einstein Equivalence Principle (EEP) consists of 3 parts:

1. Weak Equivalence Principle (WEP) : no matter what bodies we observe
2. Local Lorentz Invariance (LLI): no matter how we move
   the outcome of any local non-gravitational experiment is independent of the velocity of the freely-falling reference frame in which it is performed.
3. Local Positional Invariance (LPI): no matter where and when
   the outcome of any local non-gravitational experiment is independent of where and when in the universe it is performed.



Free fall or at rest far away from all masses?

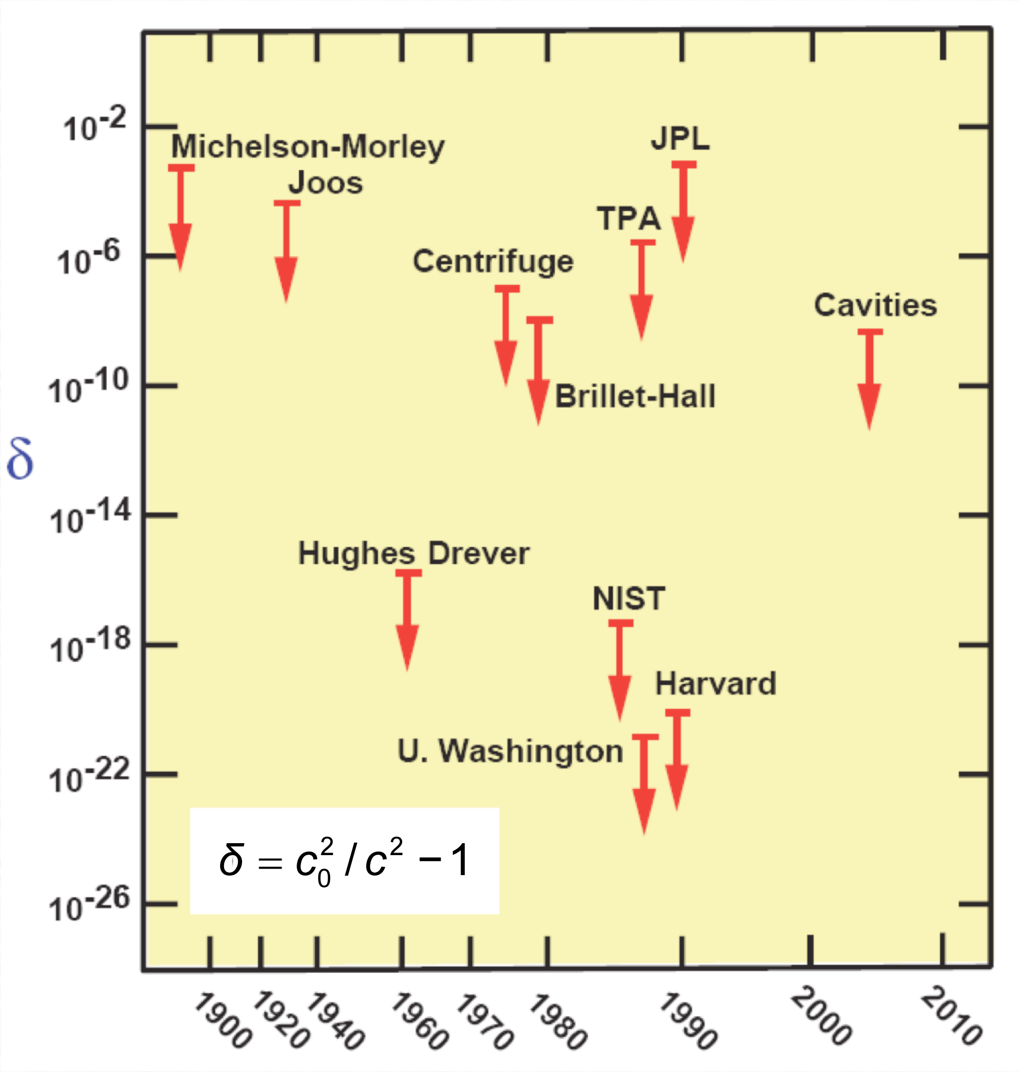
Accelerated (an elevator with thrusters) or
at rest in a homogeneous gravitational field?



No way to decide!

Local Lorentz Invariance

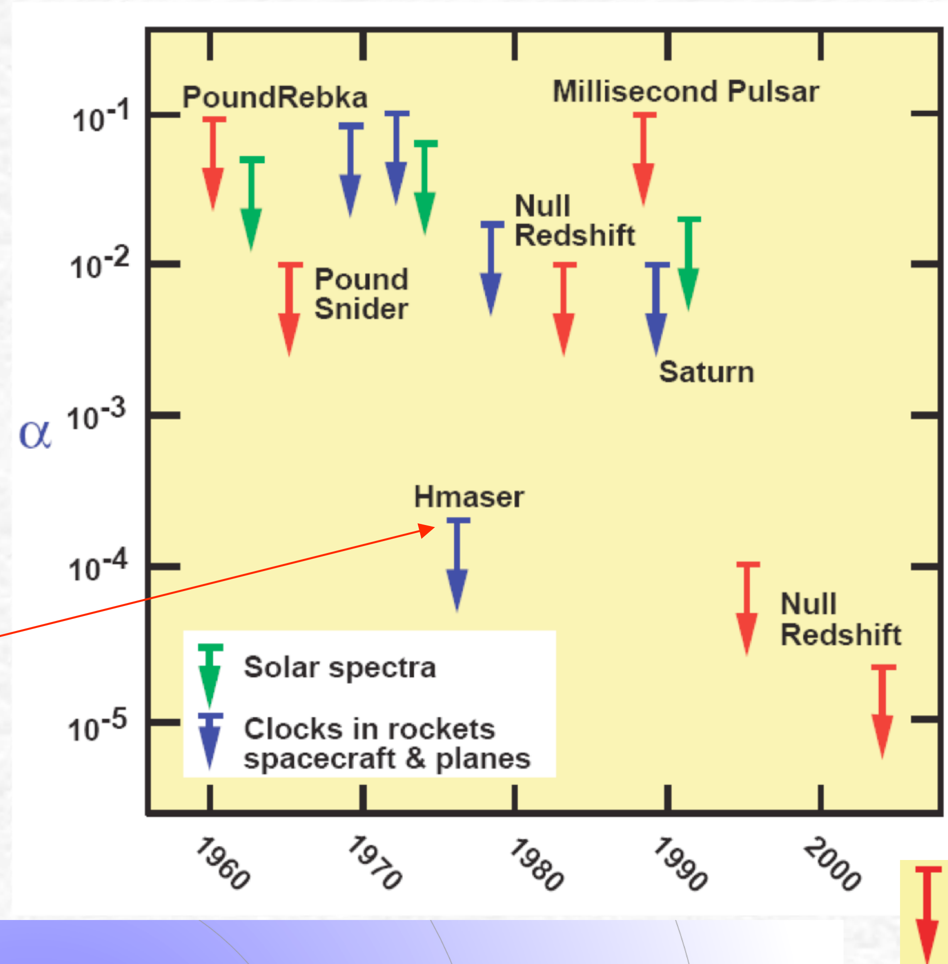
The degree of the violation of Lorentz Invariance in electromagnetism



Will, 2005

Local Positional Invariance

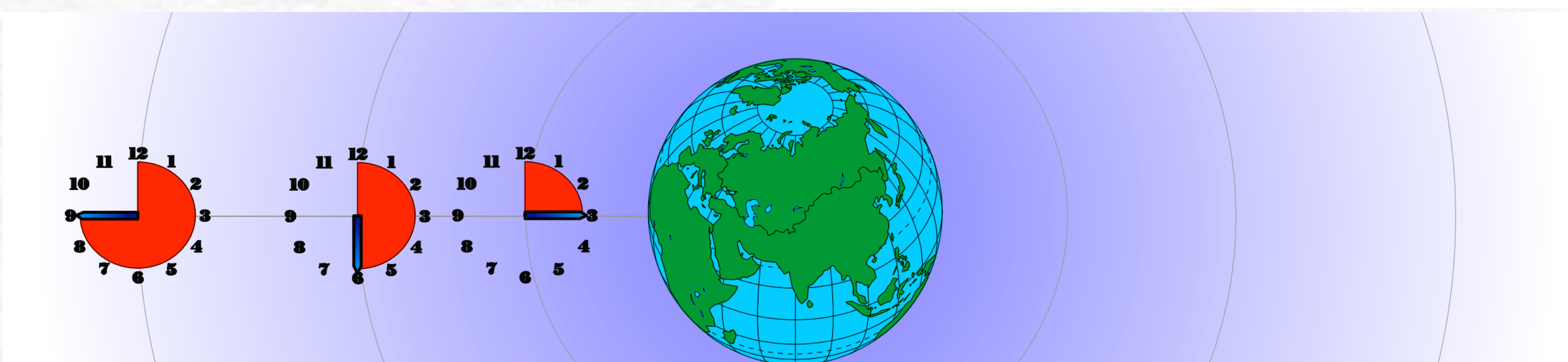
One aspect of the LPI can be tested by measuring the gravitational red shift of clocks



degree of the violation of the gravitational red shift

$$\frac{\Delta \nu}{\nu} = (1+\alpha)\frac{\Delta U}{c^2}$$

GP-A



Gravity Probe A (1976)

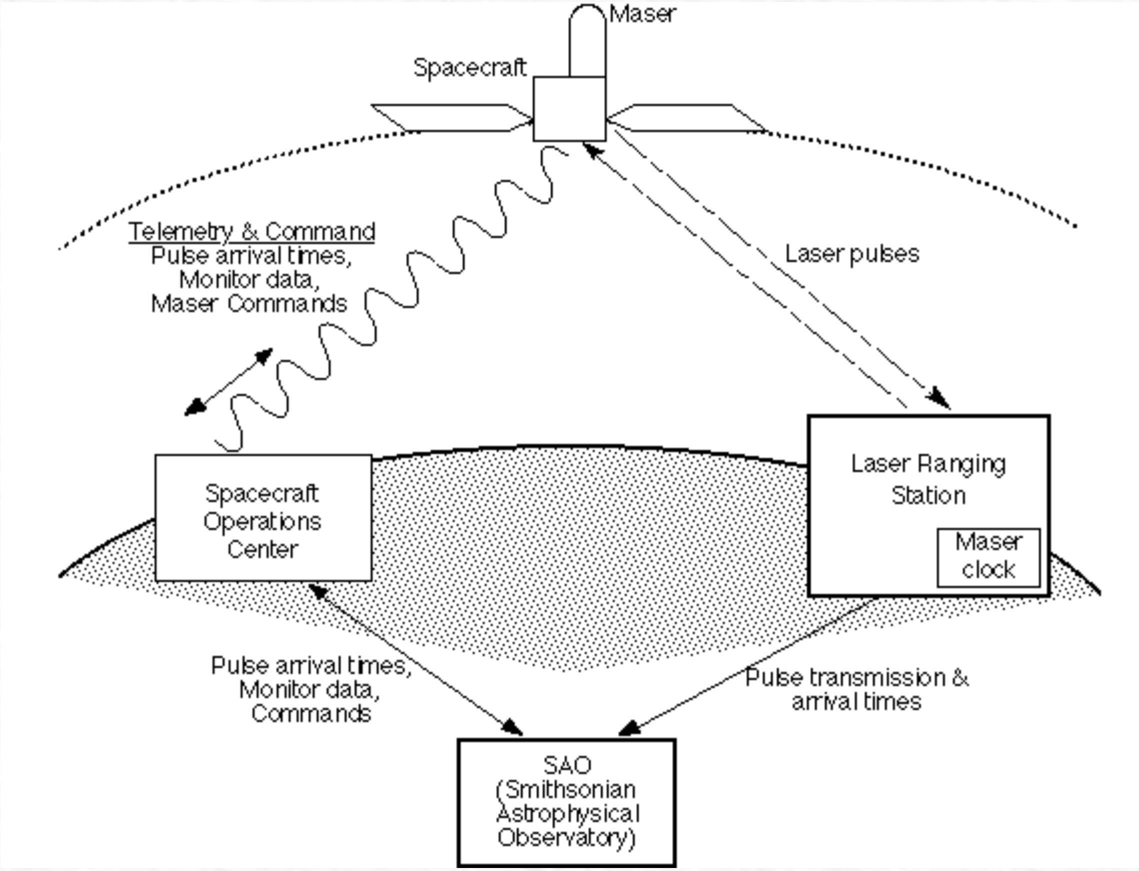
Preparations for GPS

H-maser clock
sent to the height of
10000 km by Scout rocket

$$\frac{\Delta \nu}{\nu} = \left(1+\alpha\right)\frac{\Delta U}{c^2}$$

Result

$$\left|\alpha\right| < 2 \cdot 10^{-4}$$



Most ambitious funded(!) idea:

ACES: 10^{-6} in 2015

This formula for α=0 is now used,
e.g. for GPS, at the engineering level!

General Relativity or other metric theories?

Metric theories of gravity

- If the Einstein Equivalence Principle is valid,
  gravitation must be a phenomenon of curved space-time
  described by a metric theory of gravity.

- A theory of gravity is called metric theory of gravity if:

  - space-time is endowed with a symmetric metric
  - the trajectories of freely falling test bodies are geodesics of that metric
  - in local freely falling reference frames, the non-gravitational laws of physics are those written in the language of special relativity

- General Relativity is the simplest metric theory of gravity

- There are very many others

Parametrized post-Newtonian (PPN) formalism

- K. Nordtvedt, C. Will (1970-)

- covers a class of possible metric theories of gravity
  in the weak-field slow-motion (post-Newtonian) approximation:

  many metric theories of gravity were investigated and a generic form of the post-Newtonian metric tensor of a system of N bodies was derived.

- the metric tensor contains 10 numerical ad hoc parameters.

- Two most important parameters are γ and β $(\gamma = \beta = 1$ in GRT$)$

- All predictions of the theories can be expressed using these parameters

General Relativity predicts the perihelion advance

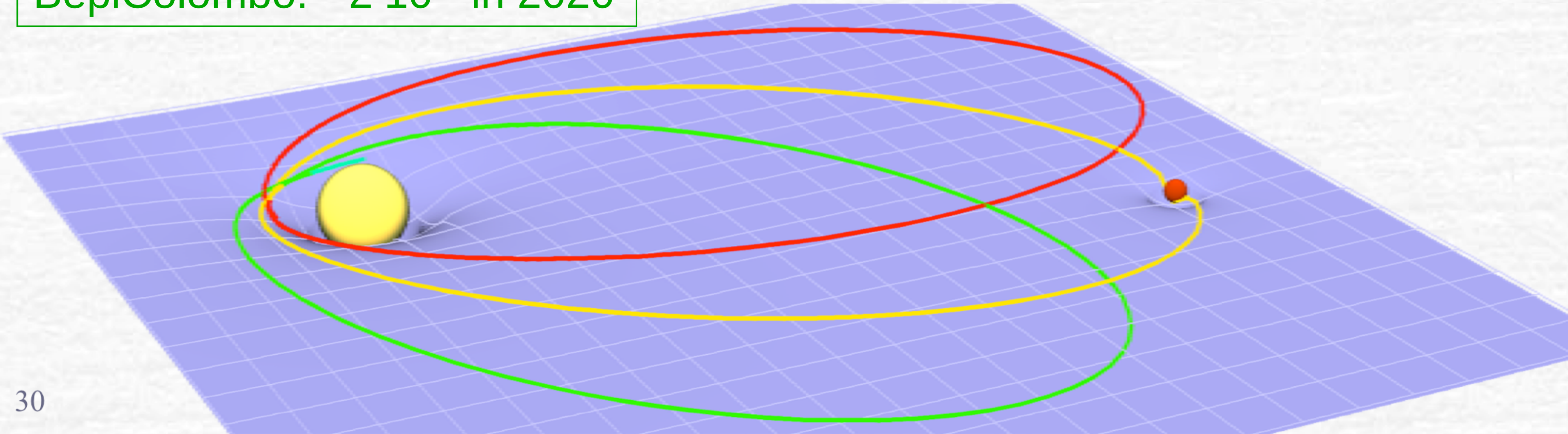
Einstein's General Theory of Relativity has naturally explained the perihelion advance of Mercury.

$$\Delta\omega = 2\pi \frac{(2\gamma + 2 - \beta) G M_\odot}{c^2 a (1 - e^2)}, \quad \text{rad per revolution}$$

Modern precision of the perihelion shift: ~10^{-3}

Funded projects:

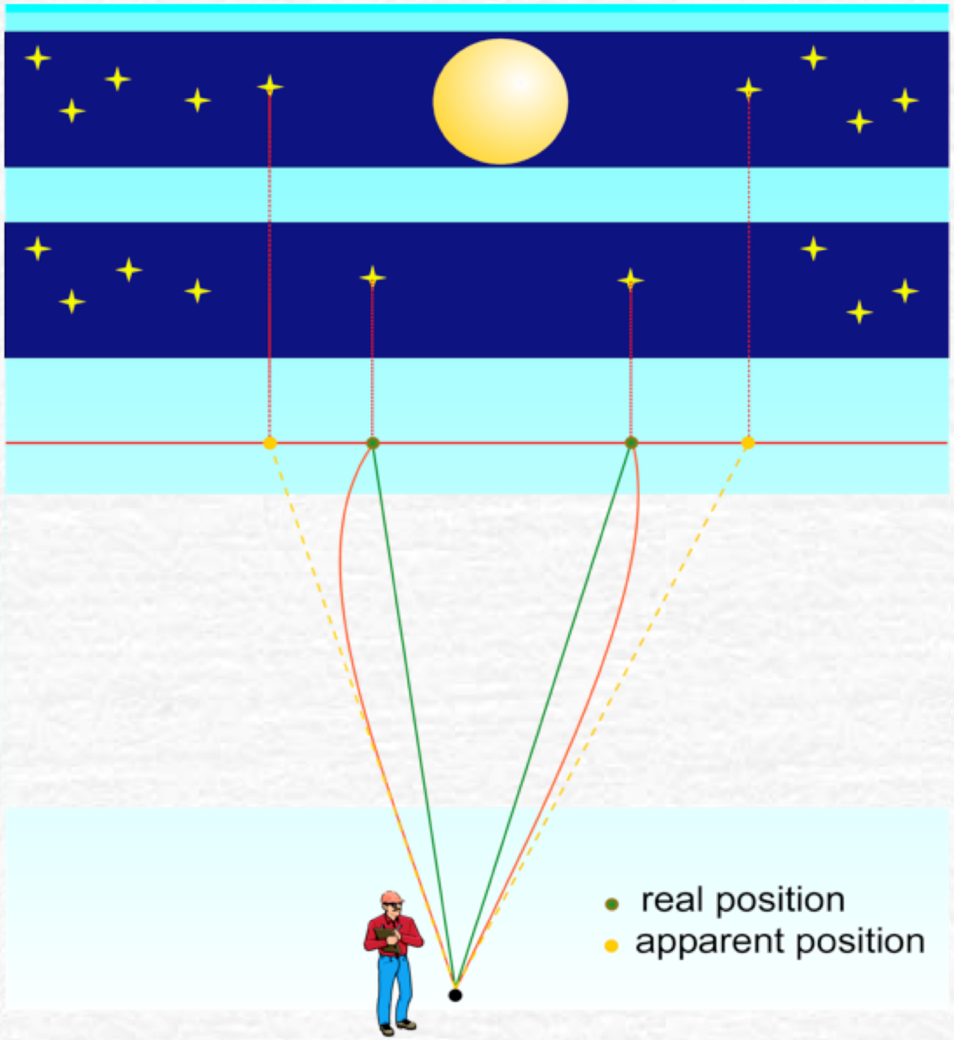
BepiColombo: $2 \cdot 10^{-6}$ in 2020



Second test of General Relativity: light deflection

Light deflection from the Sun: 1.75”

$$\Delta\varphi = \frac{2(1+\gamma)GM_{\odot}}{c^2 d}$$



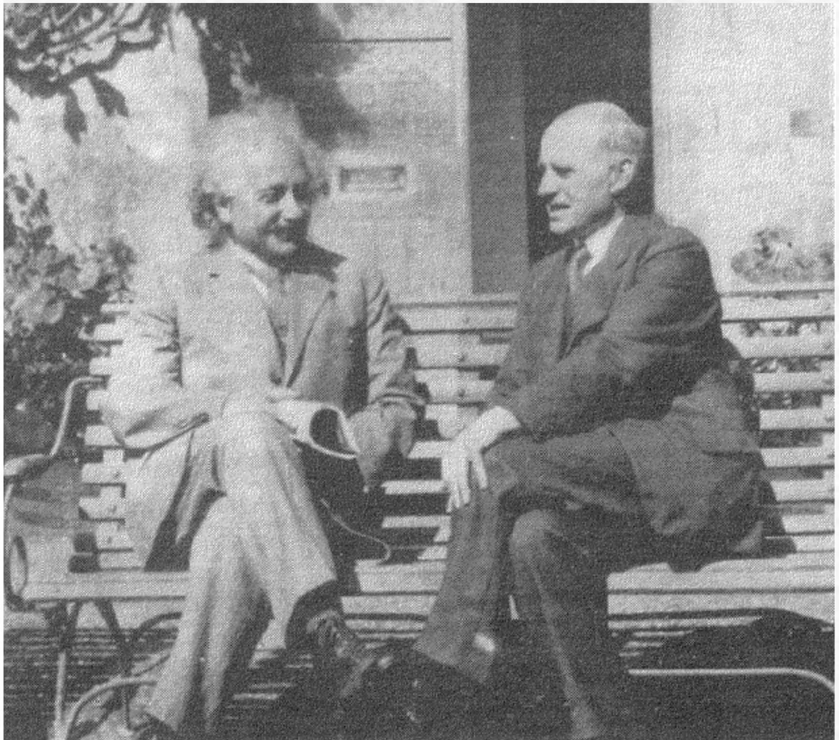
Second test of General Relativity: light deflection

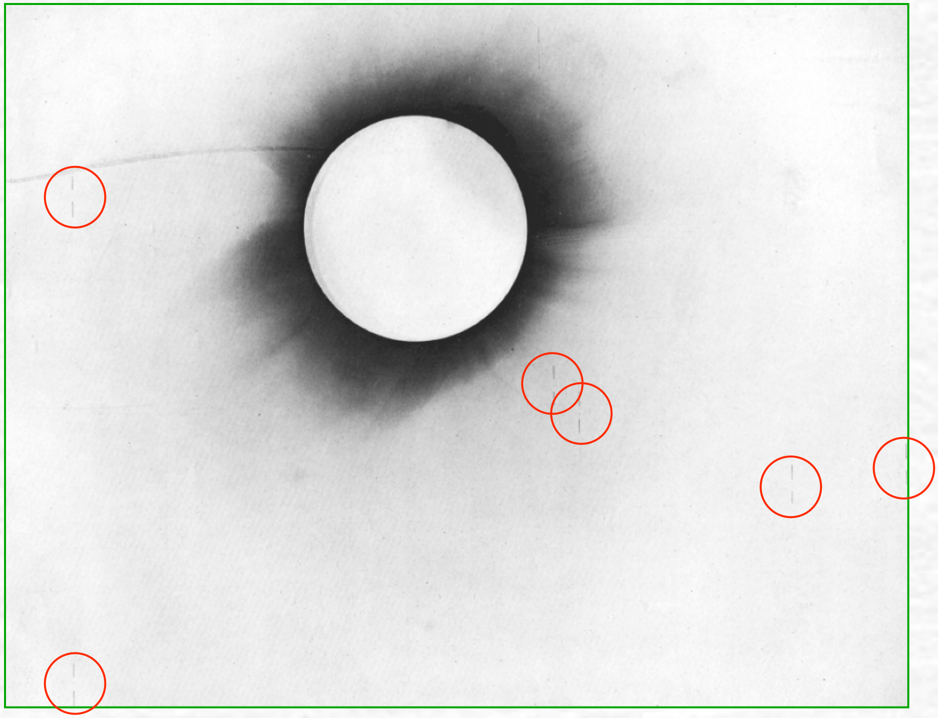
Eddington's expedition measures the deflection during the total solar eclipse
29 May 1919: Sobral (Brazil), Principe (island close to Africa)

Conceivable outcomes:

- No deflection = 0
- Newton = 0.87″
- Einstein = 1.75″

Einstein and Eddington, Cambridge, 1930





one of the Eddington's photographs of the 1919 eclipse, presented in:

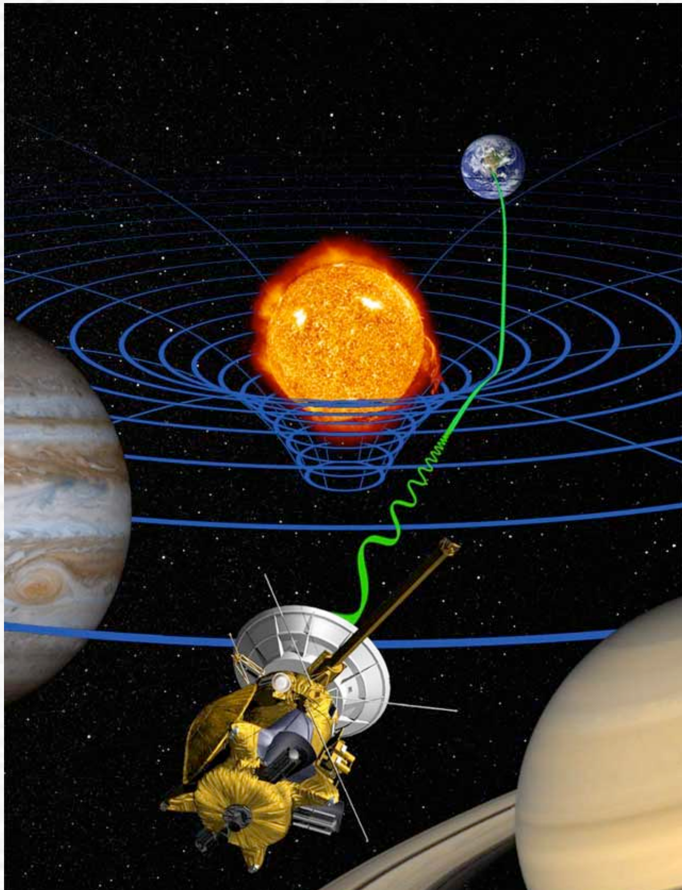
Dyson, F.W., Eddington, A.S., & Davidson, C.R. 1920
Mem. R. Astron. Soc., **220**, 291-333:

$1.98'' \pm 0.12''$
$1.61'' \pm 0.30''$

Third test of General Relativity: Shapiro delay

Light needs a bit longer to go from the emitter to the receiver
than the distance between them divided by c

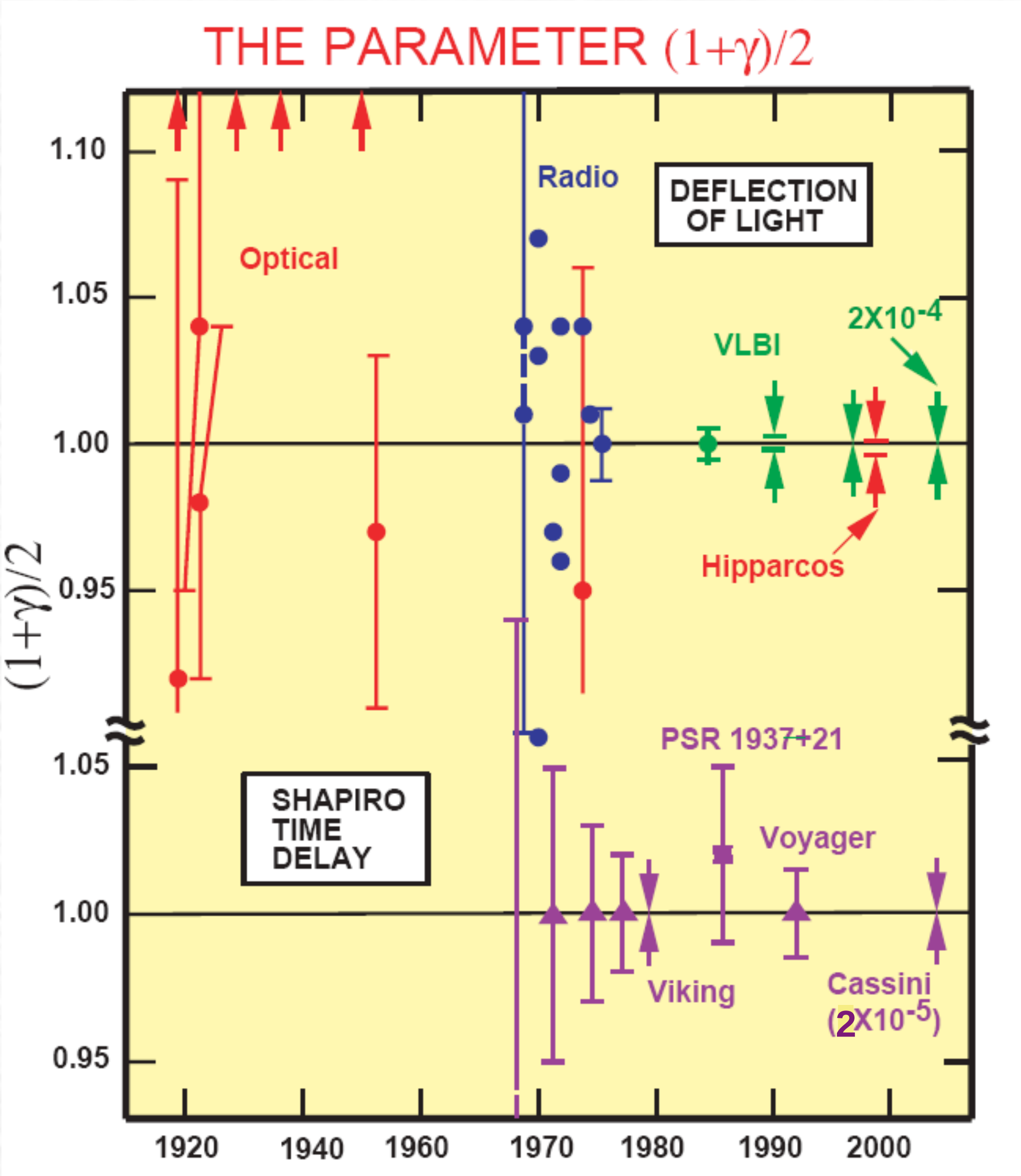
$$t = \frac{1}{c}\left|\mathbf{x}_{\mathrm{emitter}} - \mathbf{x}_{\mathrm{receiver}}\right| + \frac{(1+\gamma)GM_{\odot}}{c^3} F(\mathbf{x}_{\odot}, \mathbf{x}_{\mathrm{emitter}}, \mathbf{x}_{\mathrm{receiver}})$$



Discovered by Irwin Shapiro in 1964
as a theoretical prediction of General Relativity

First measured by the Shapiro's team at the end
of the 1960s with an accuracy of 10%

Light propagation: modern tests



Funded projects:

Gaia & Bepi Colombo:

$>10^{-6}$ in 2020

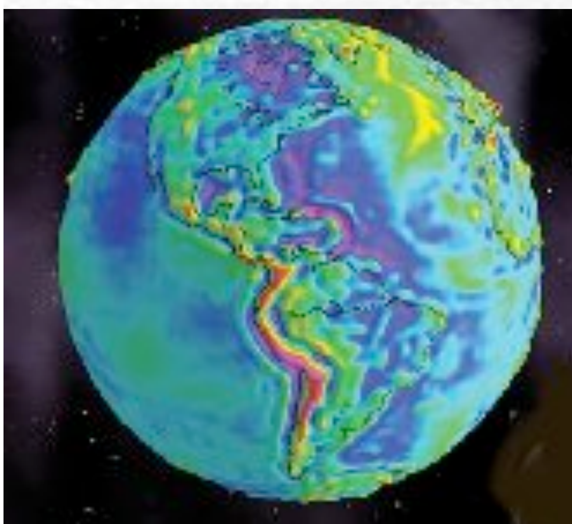
Most ambitious unfunded idea:

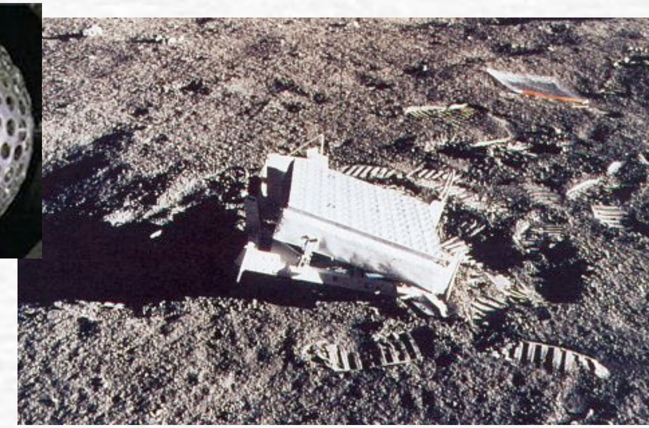
LATOR: $10^{-8} - 10^{-9}$

Will, 2005

Relativistic precession: experimental status

- LLR: geodetic precession <1% (Newhall et al., 1996; …)
- SLR: Lense-Thirring precession 10% (Ciufolini, Pavlis, 2004; Ries, 2008)
- VLBI & Earth rotation: geodetic precession 30% (Krasinsky, 2006)



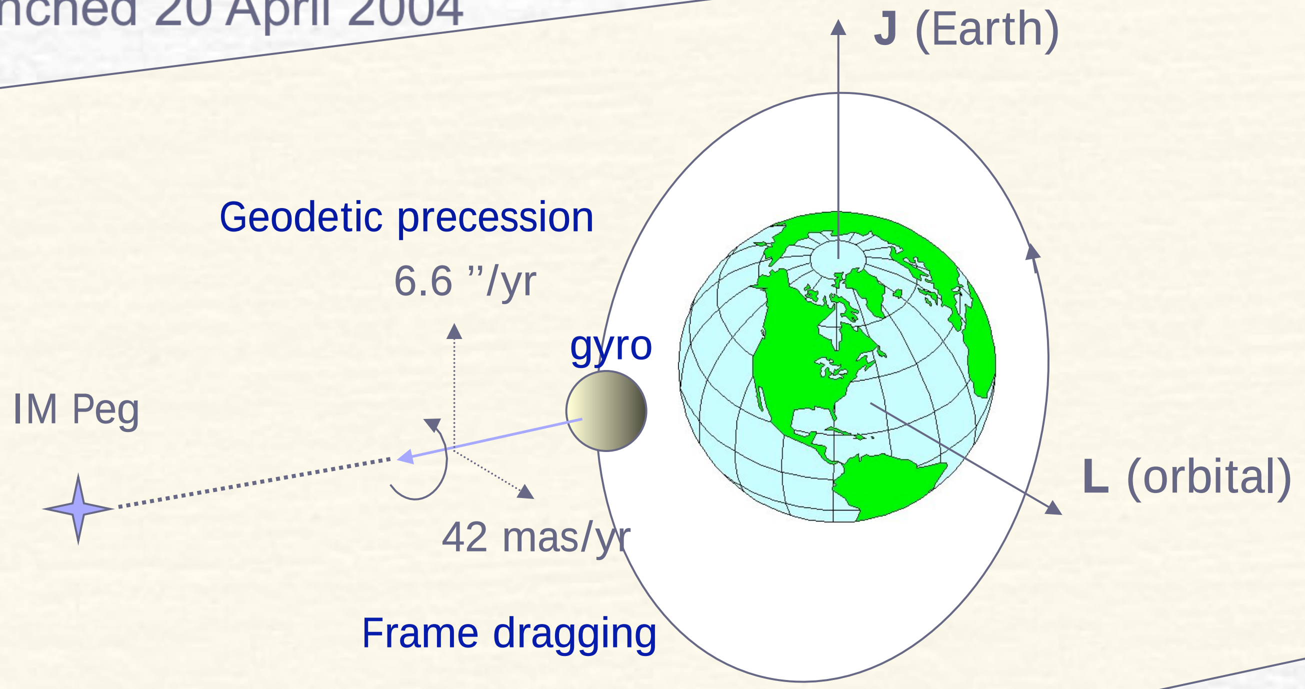



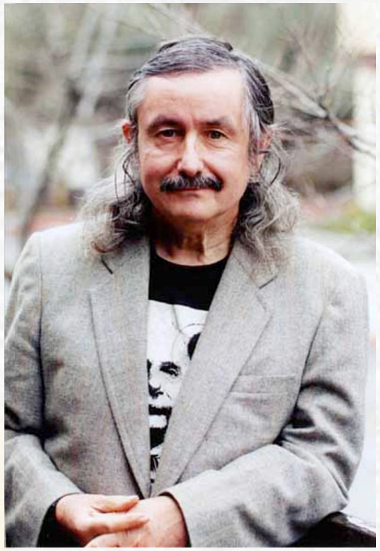
Relativistic precession: experimental status

- Gravity Probe B

the longest lasting experiment in modern history (1959-2010?)

launched 20 April 2004

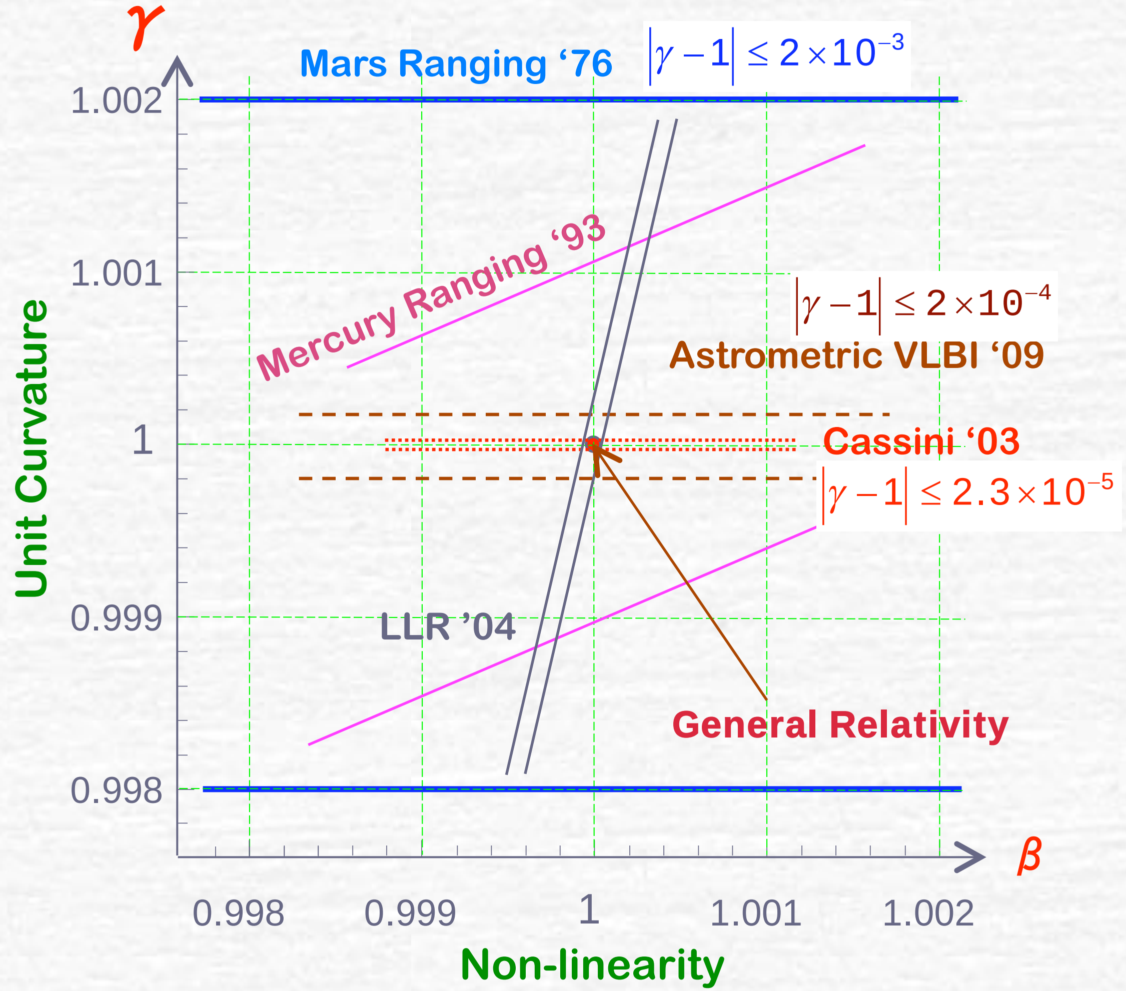




Francis Everitt

14% confirmation of the frame dragging
0.08% confirmation of geodetic precession ???

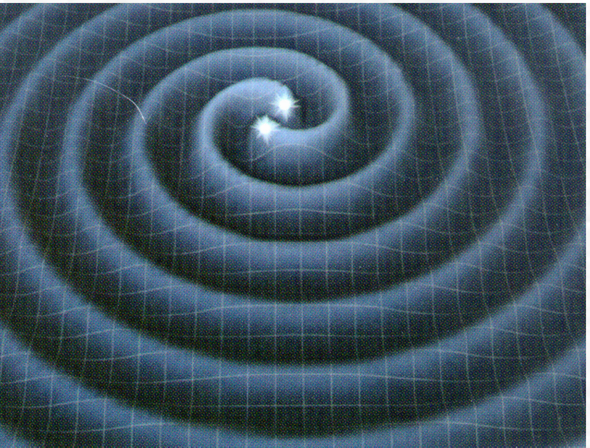
Testing in β-γ plane of the PPN formalism

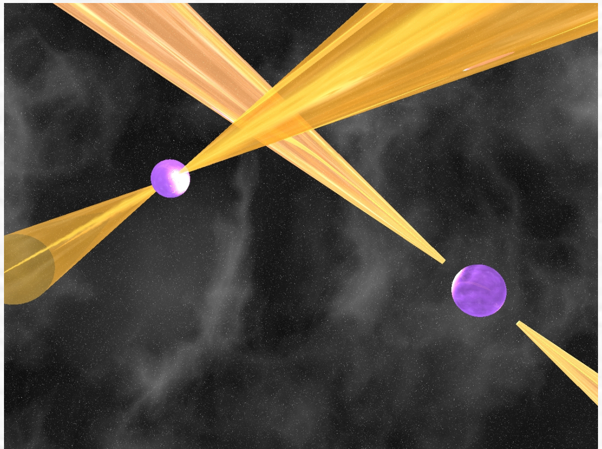


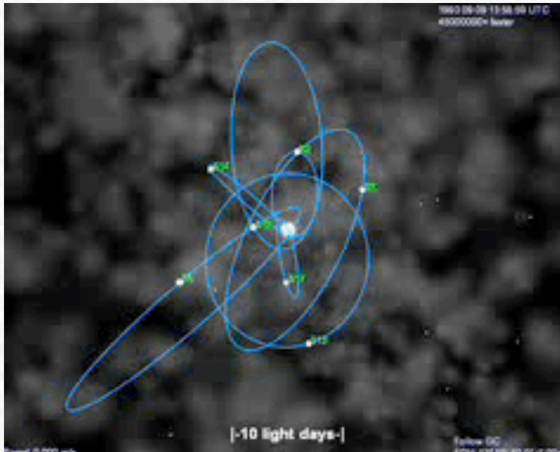
Strong field tests of General Relativity

Strong field tests

- Binary pulsar B1913+16: indirect evidence for gravity waves: 0.2%
- Double pulsar PSR J0737-3039A/B: more precise 0.06%
- Existence of black holes:
    - stellar mass (Cyg X1)
    - supermassive black hole in the centers of galaxies
        - IR measurements of the stellar orbits around the center of Milky Way





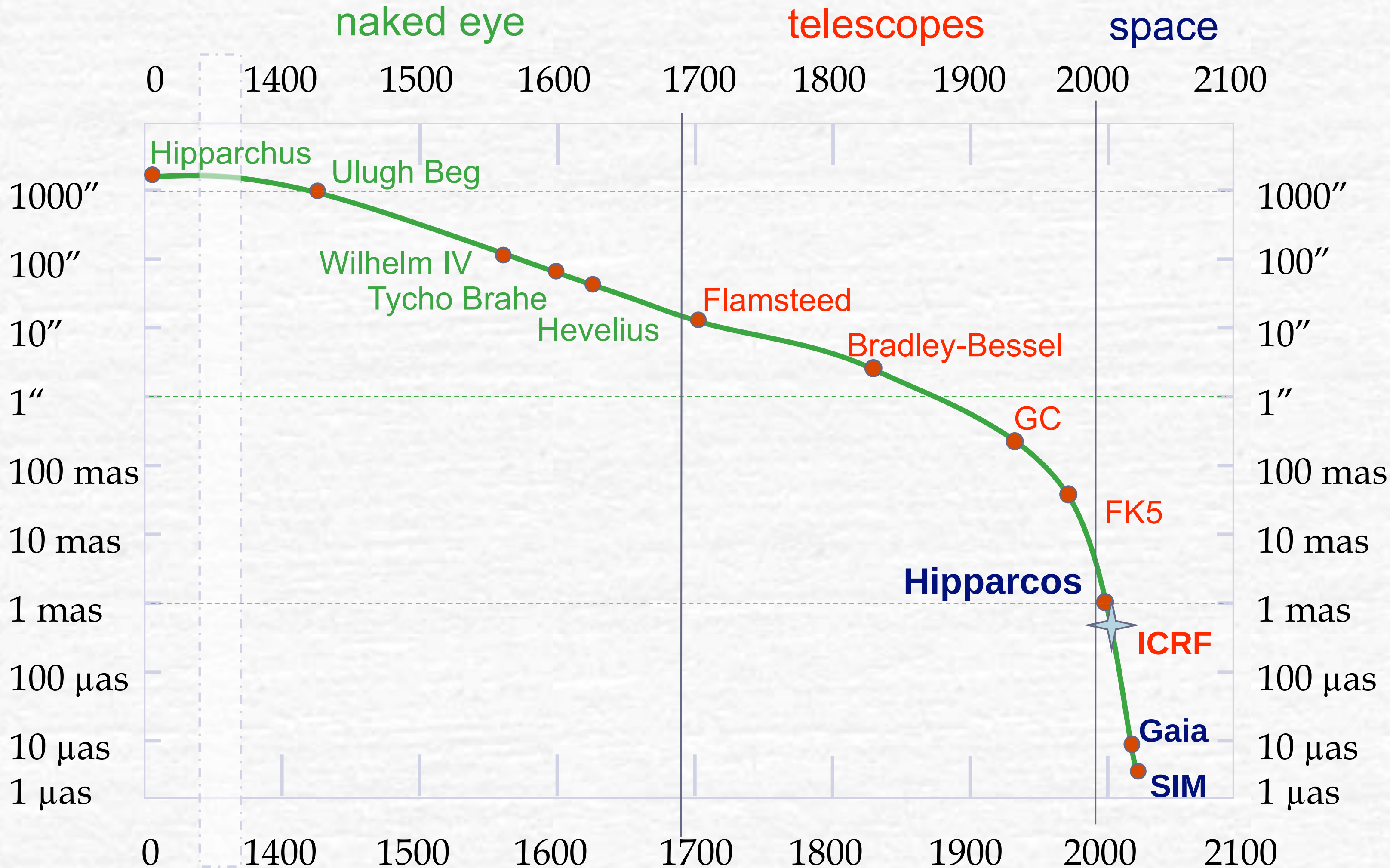


Why to test further?

intentionally left blank

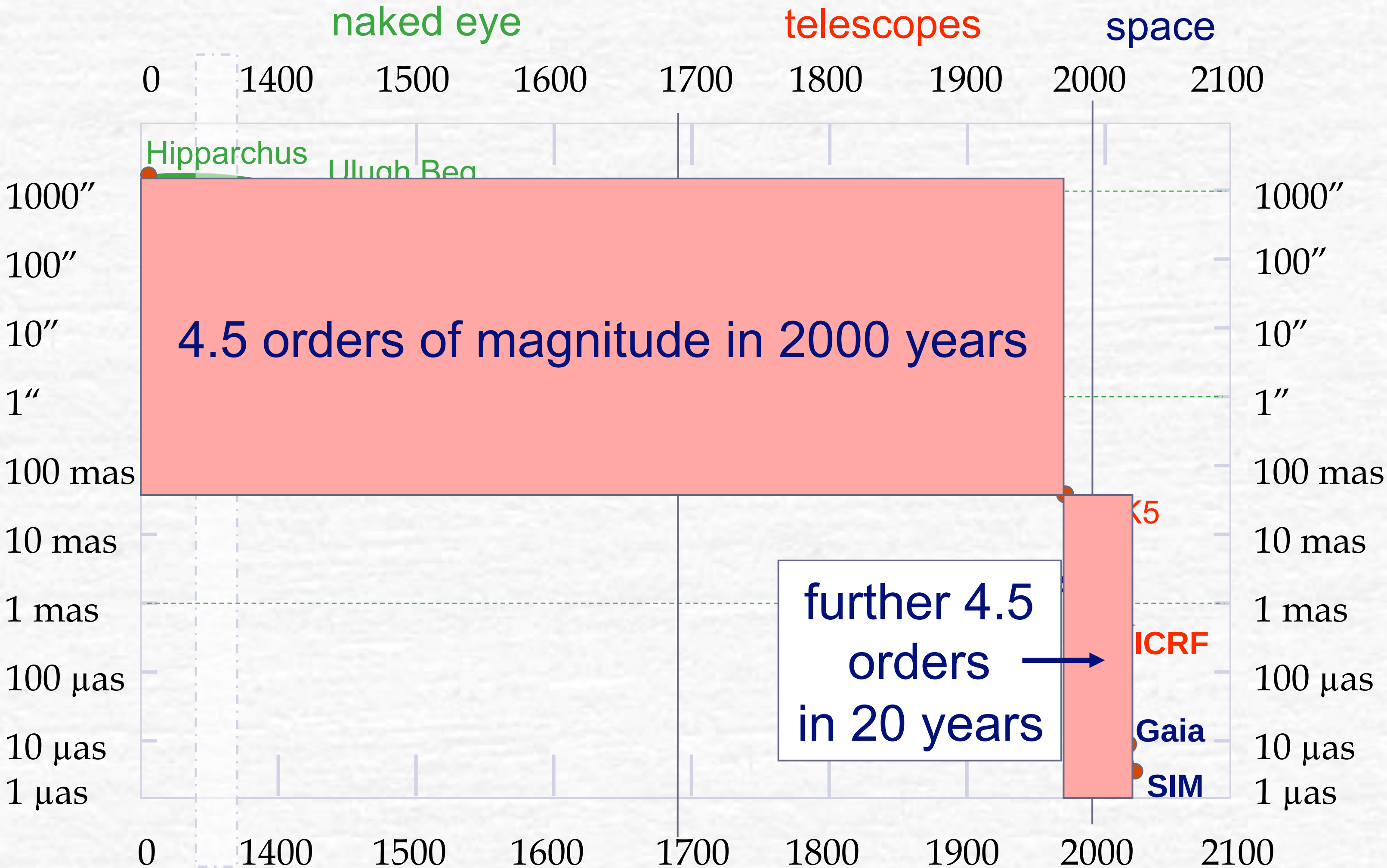
High-accuracy astrometry

Accuracy of astrometric observations

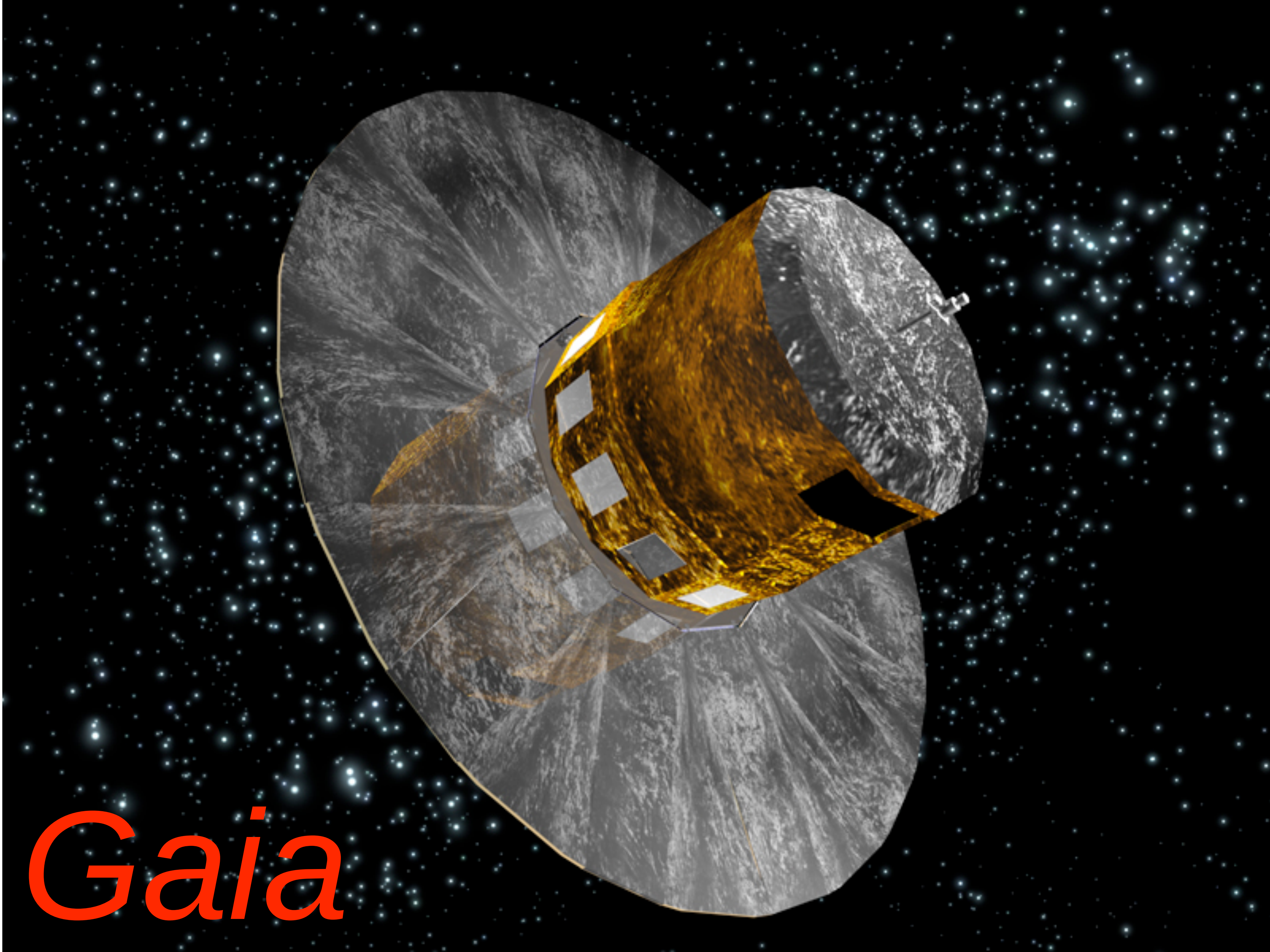


1 µas is the thickness of a sheet of paper seen from the other side of the Earth

Accuracy of astrometric observations

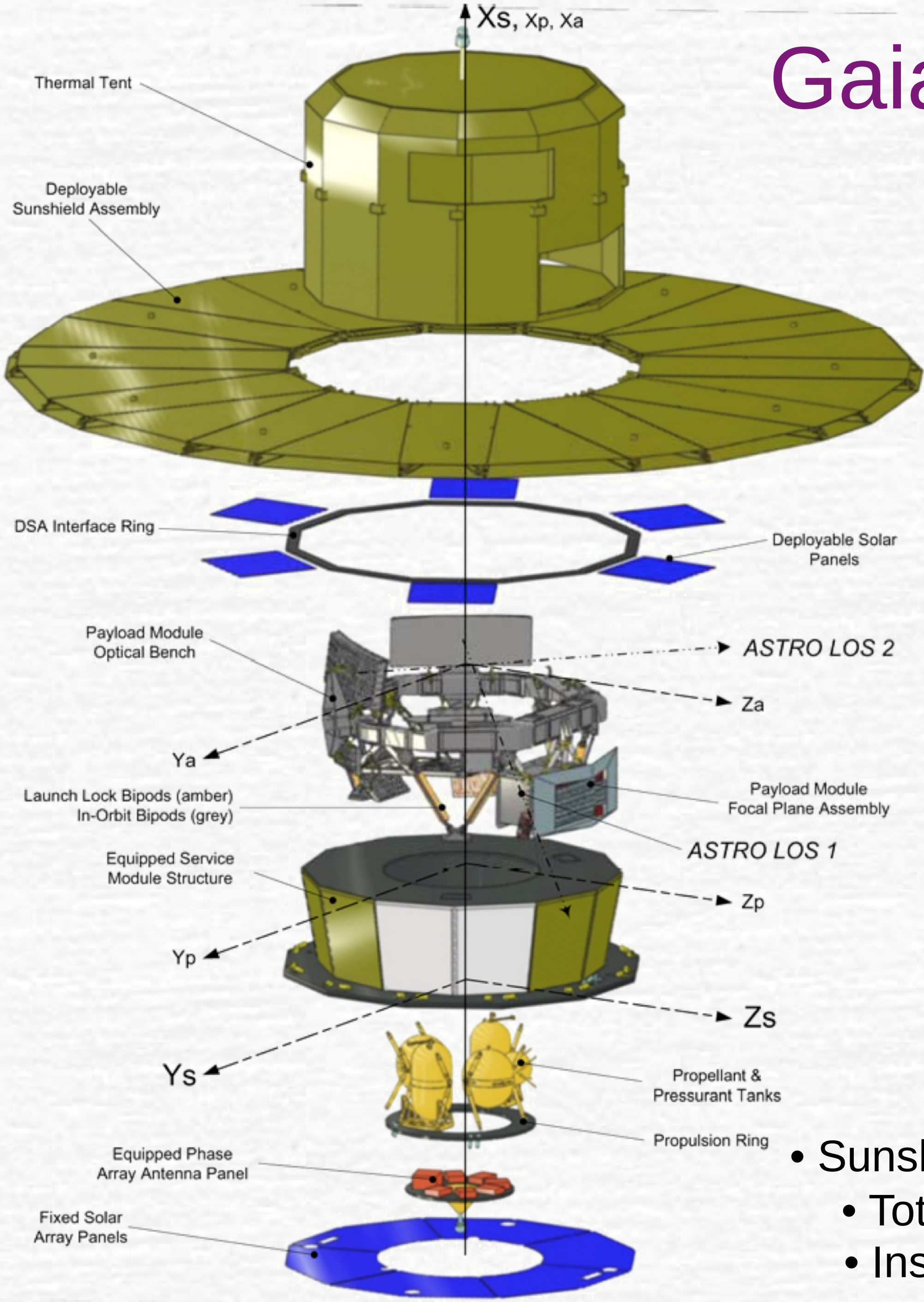


1 µas is the thickness of a sheet of paper seen from the other side of the Earth


Gaia

Gaia: satellite

ESA mission

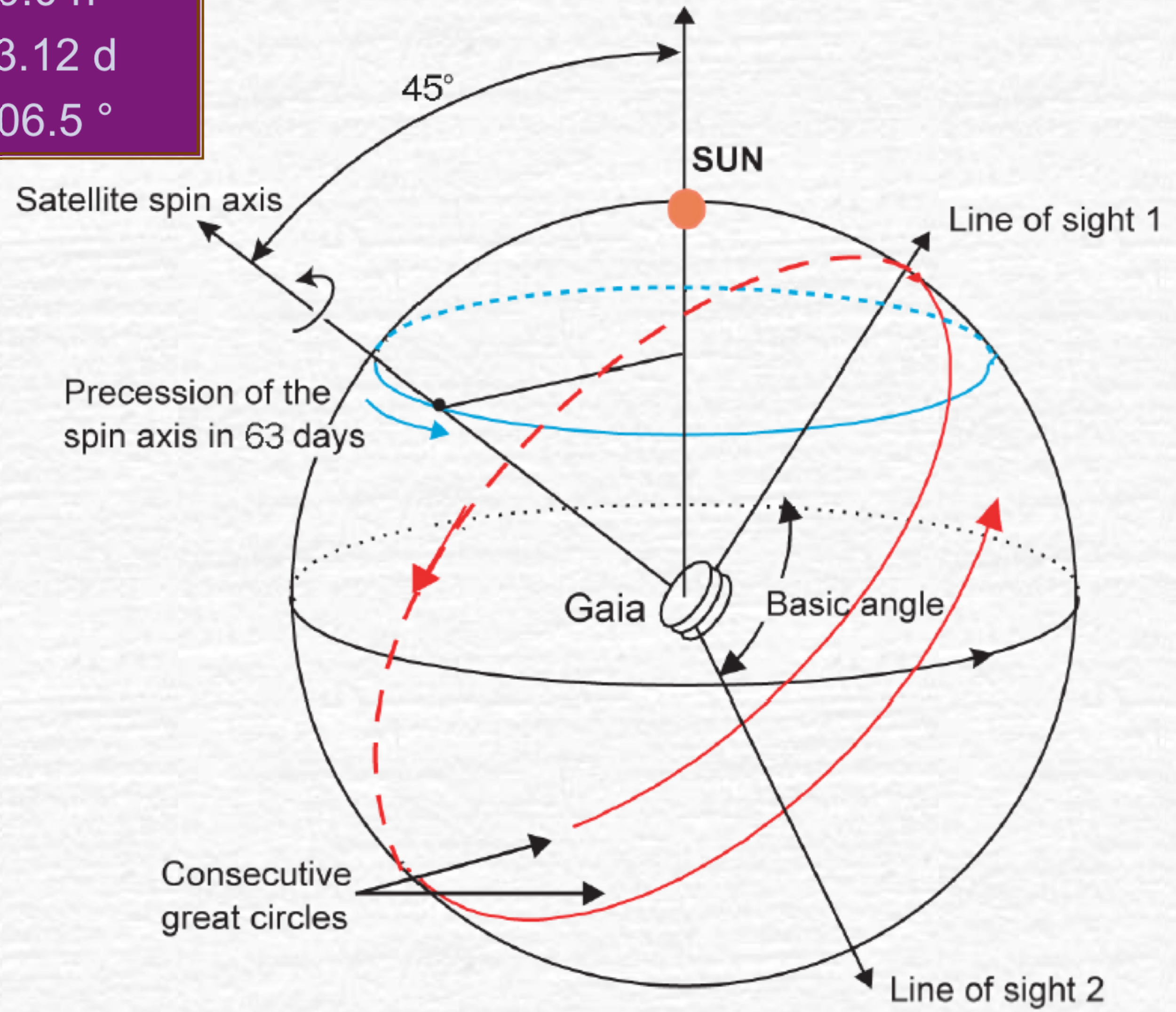


- Sunshield diameter: 11 m
- Total mass: 1700 kg
- Instruments: 800 kg

Figures courtesy EADS-Astrium

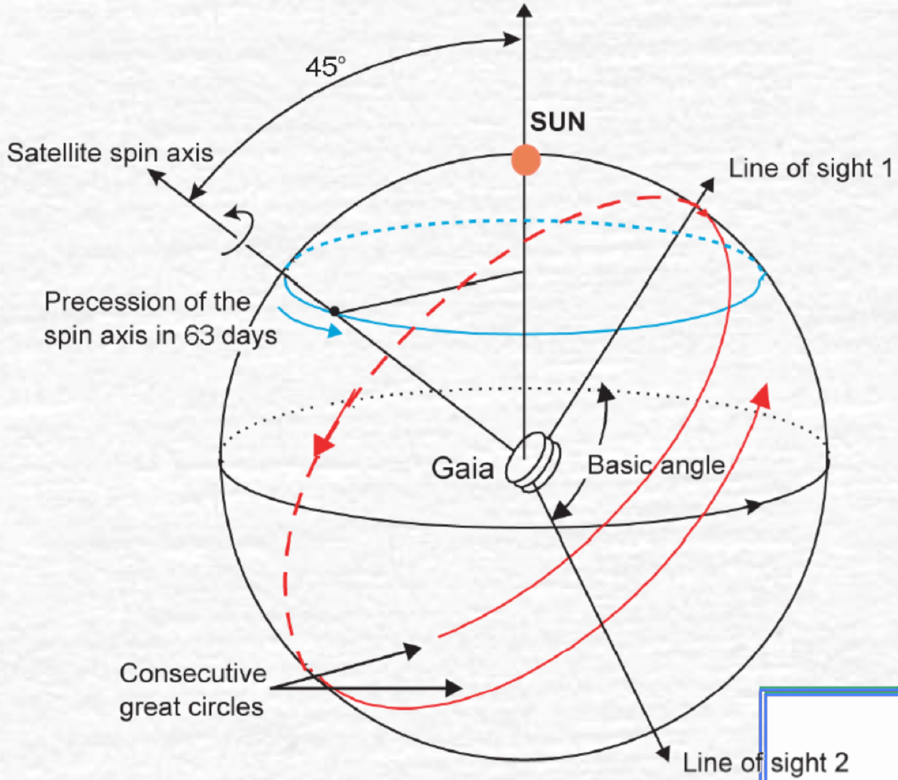
Gaia: scanning satellite

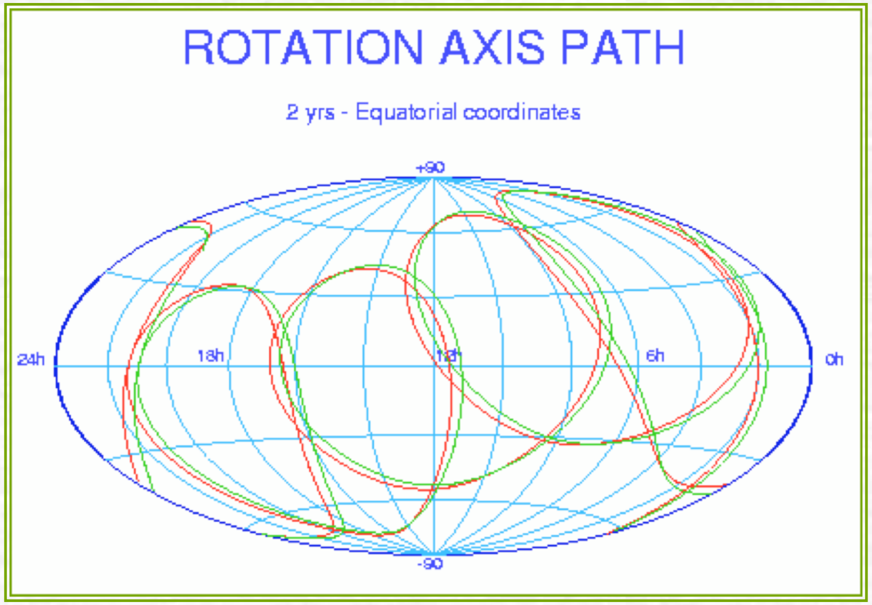
spin/Sun angle	45°
scan rate	60 "/s
period	6.0 h
precession	63.12 d
basic angle	106.5 °



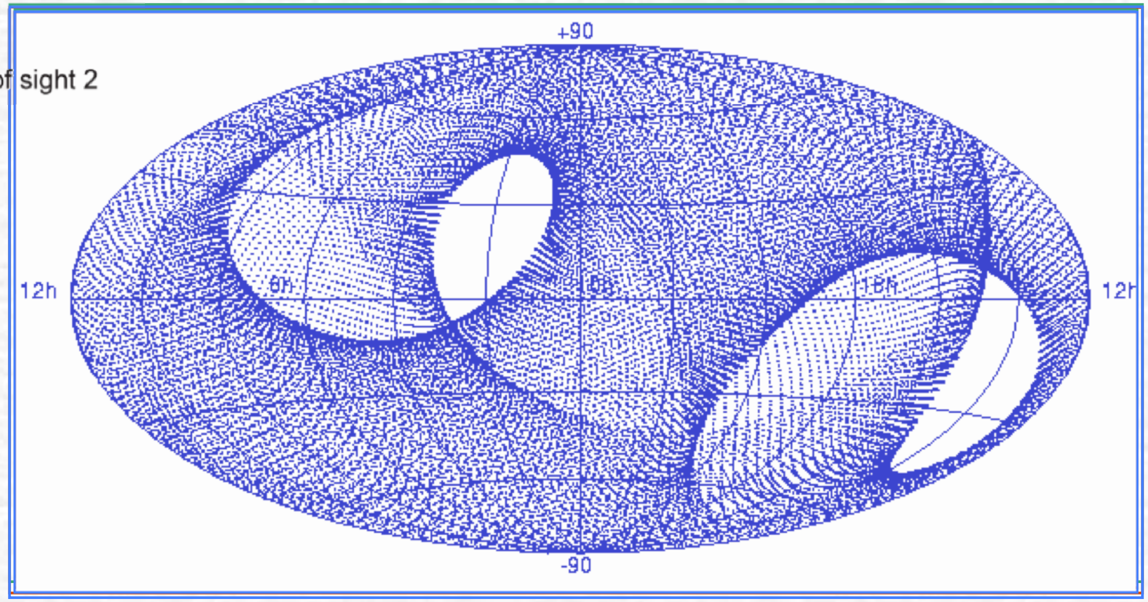
 Figures courtesy Karen O’Flaherty

Gaia: scanning satellite





100 days



Figures courtesy Francois Mignard

Gaia: observation distribution

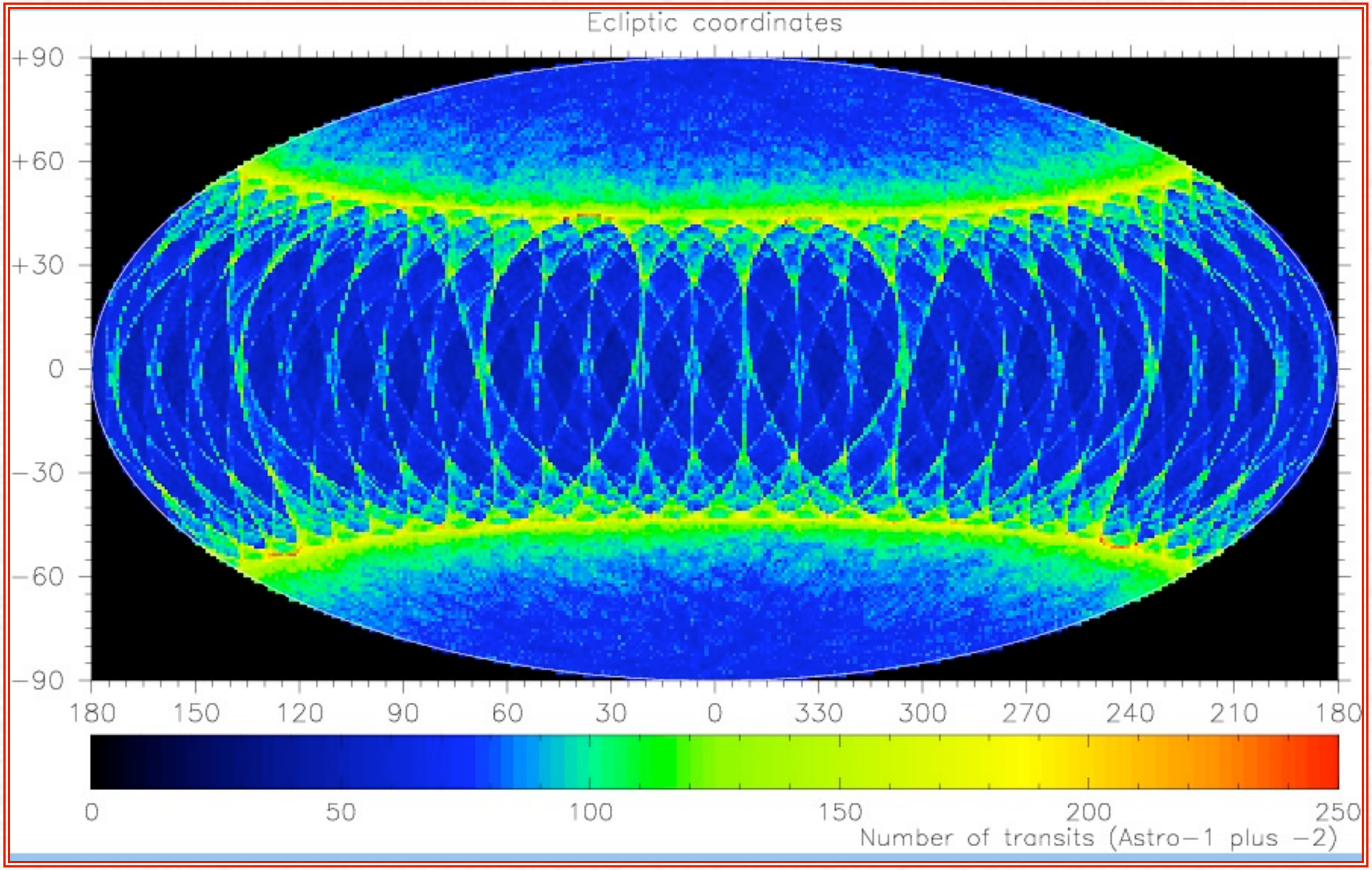
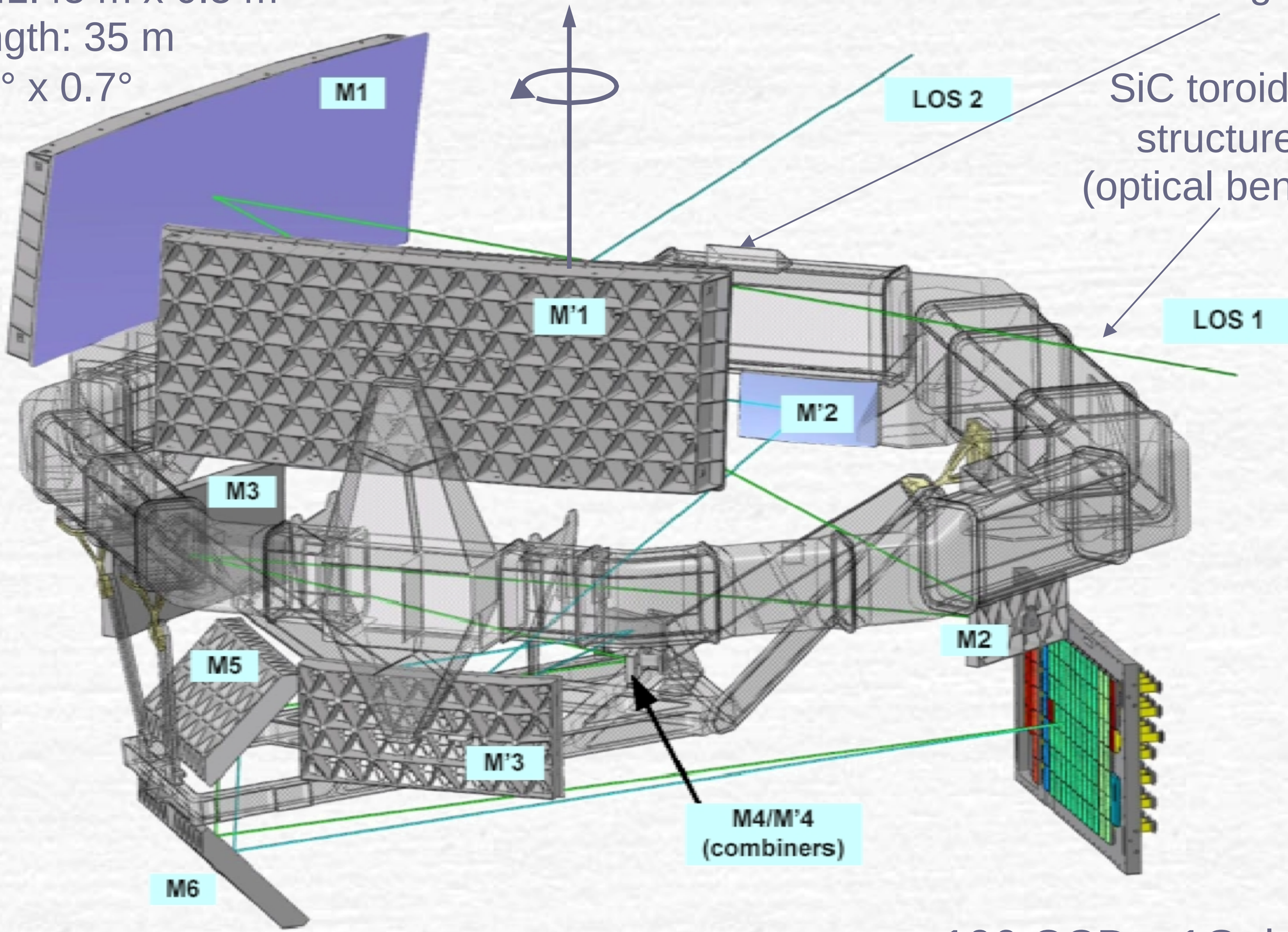


 Figure courtesy Jos de Bruijne, 2003

Gaia: telescope

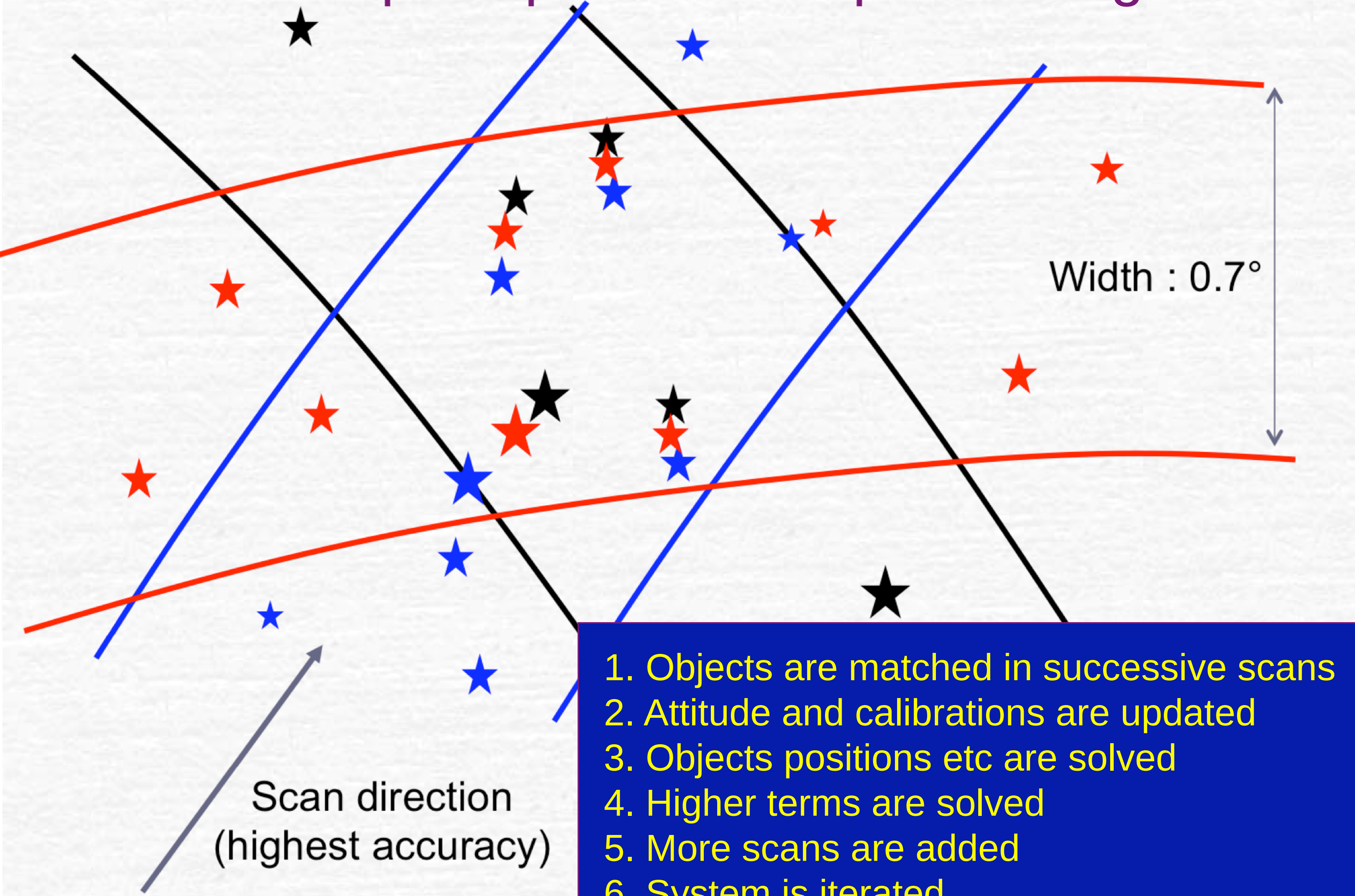
2 SiC primary mirrors at 106.5°
Aperture:1.45 m x 0.5 m
Focal length: 35 m
FOV: 1.6° x 0.7°



106 CCDs, 1Gpixel, TDI

Figure courtesy EADS-Astrium

Gaia: principles of data processing



1. Objects are matched in successive scans
2. Attitude and calibrations are updated
3. Objects positions etc are solved
4. Higher terms are solved
5. More scans are added
6. System is iterated

The challenge of data processing

- Parameters
  - At least 5 parameters for each star: $5 \cdot 10^9$
  - 4 parameters of orientation each 15 seconds: 10^8
  - 2000 calibration parameters per day: $4 \cdot 10^6$
  - global parameters (e.g., PPN γ): 10^2

- Observations

  about 1000 raw images for each star: 10^{12}

- Data volume: 1 PB (iterative data processing)
- Computational efforts: $\sim 10^{21}$ flops
- Direct least squares solution is impossible…

Expected astrometric accuracy

G2V star

V	6–13	14	15	16	17	18	19	20
π	8	13	21	34	54	89	154	300
μ	5	7	11	18	29	47	80	158
α, δ	6	10	16	25	40	66	113	223

Extensive simulations: Jos de Bruijne, Lennart Lindegren, 2009

1. Redder stars better, blue stars worse

2. In some regions (with ecliptic longitude about 45°) a factor 1.2-2.8 better

Schedule

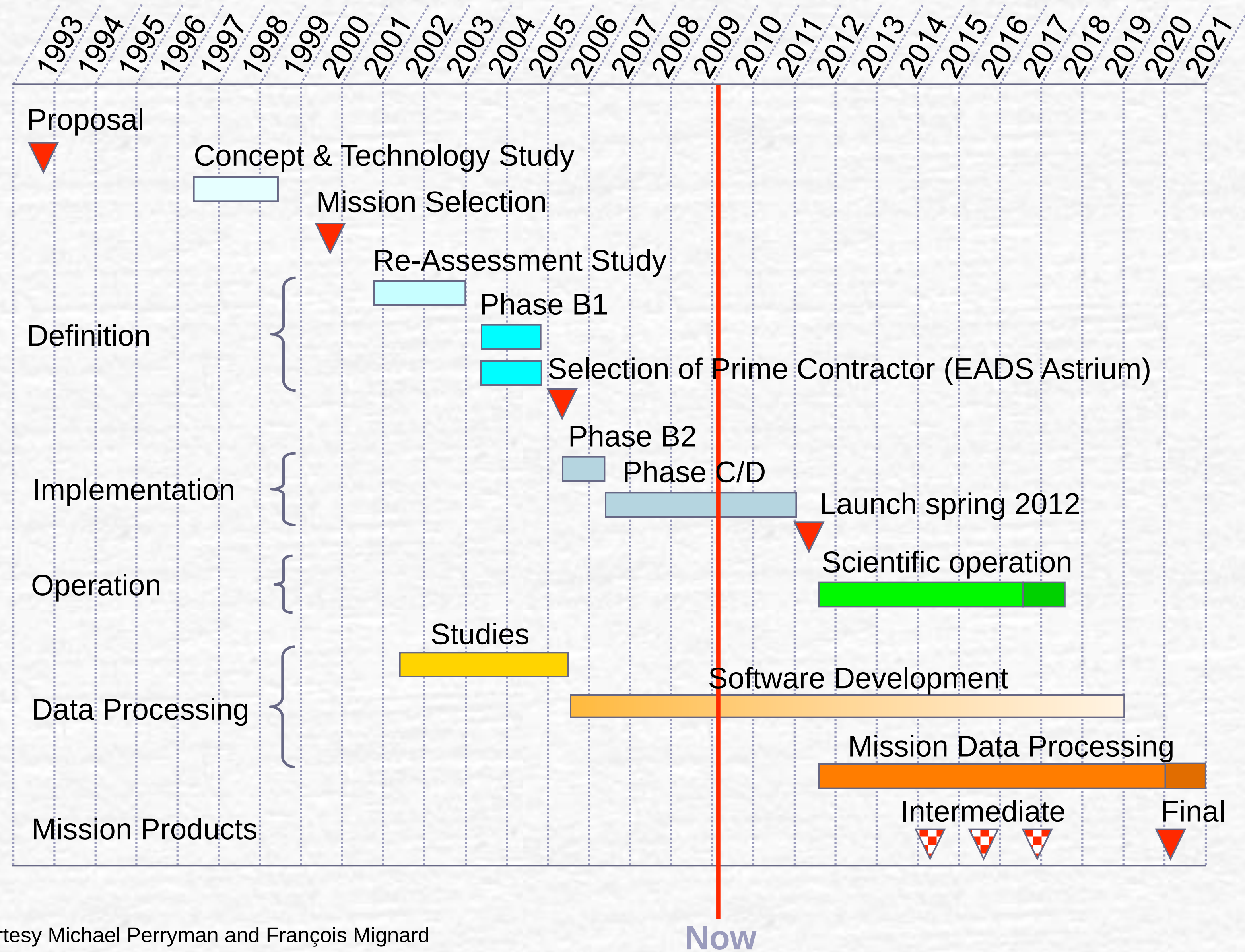
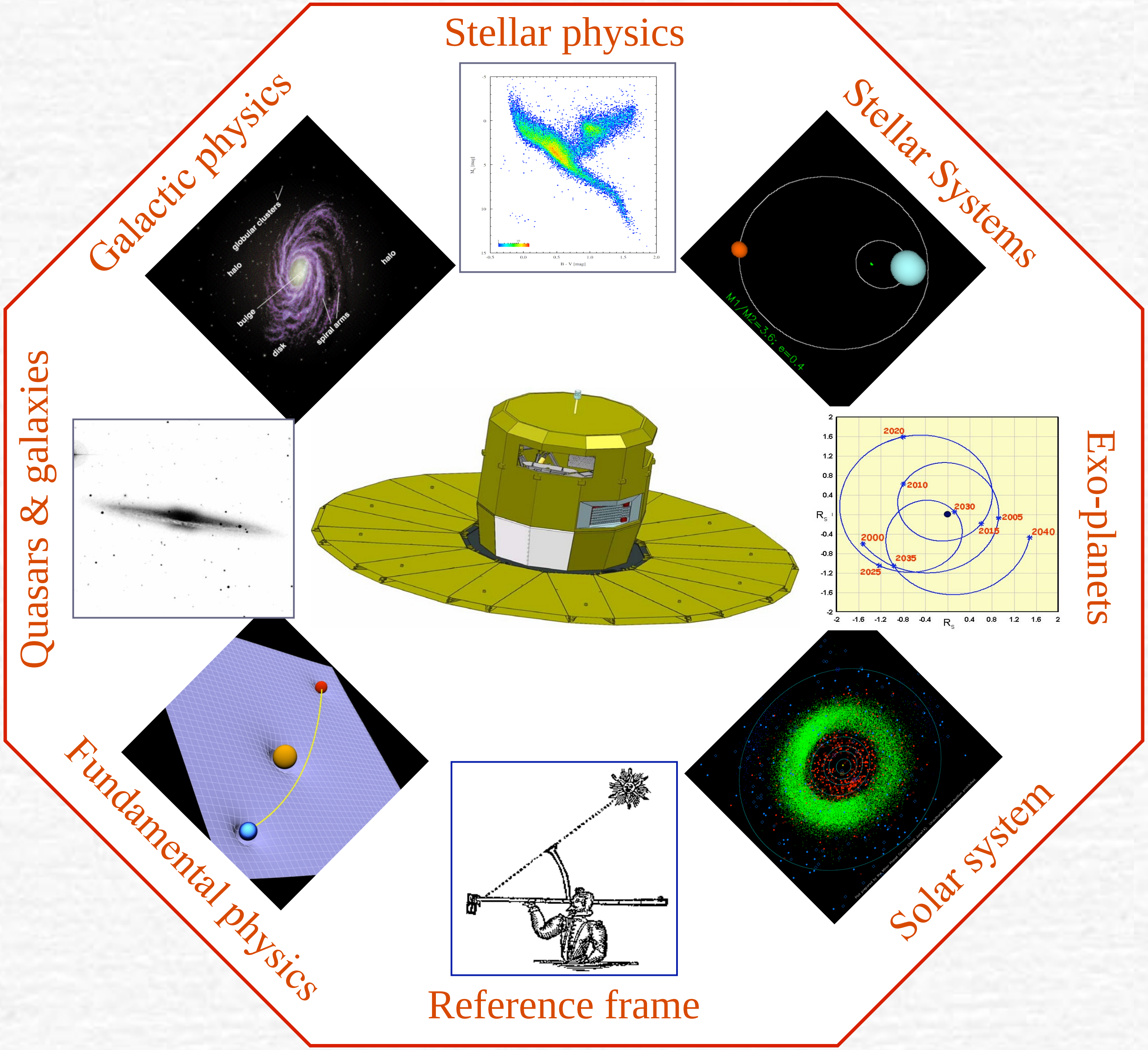


Figure courtesy Michael Perryman and François Mignard

Gaia: goals in brief

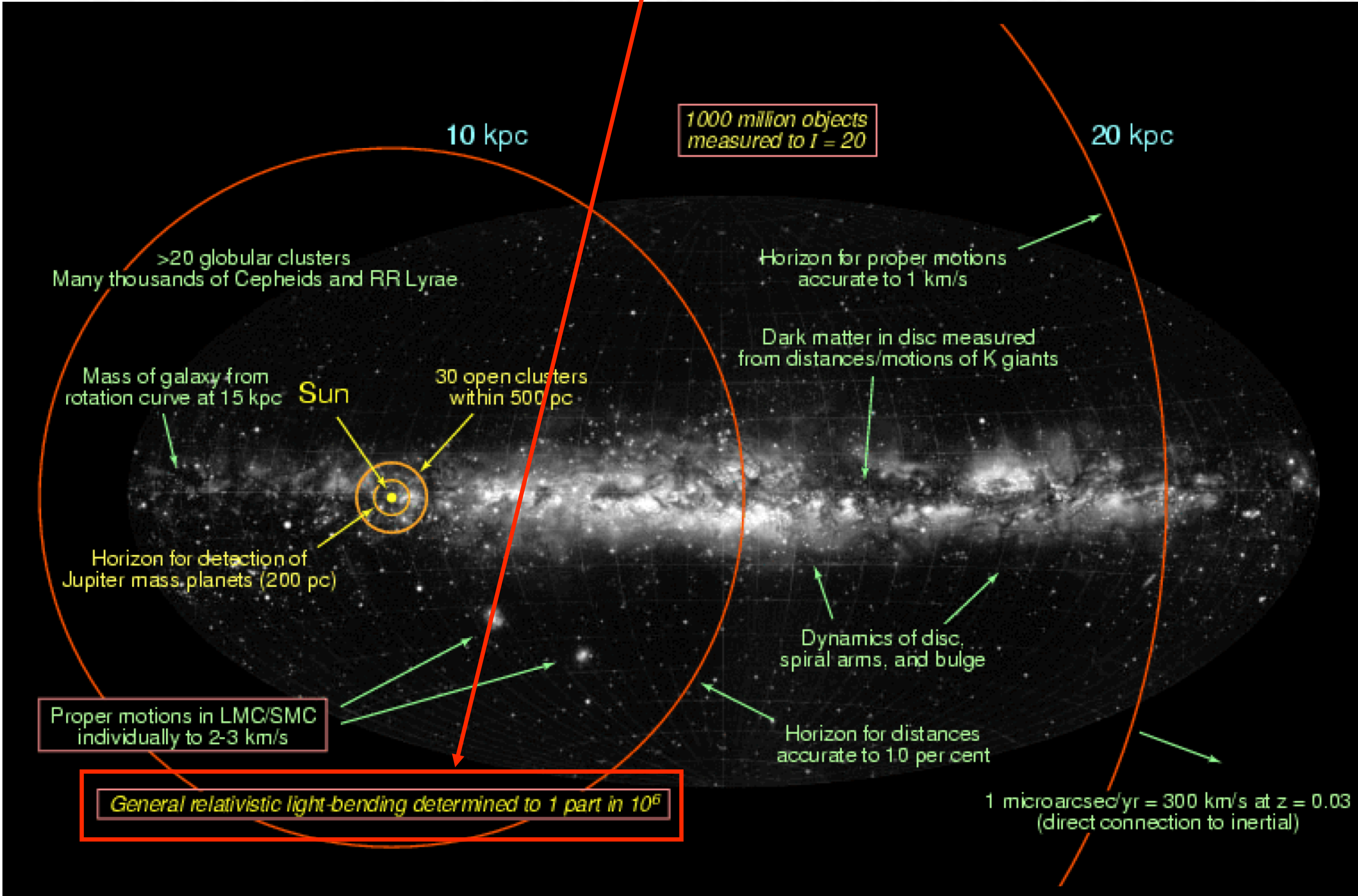


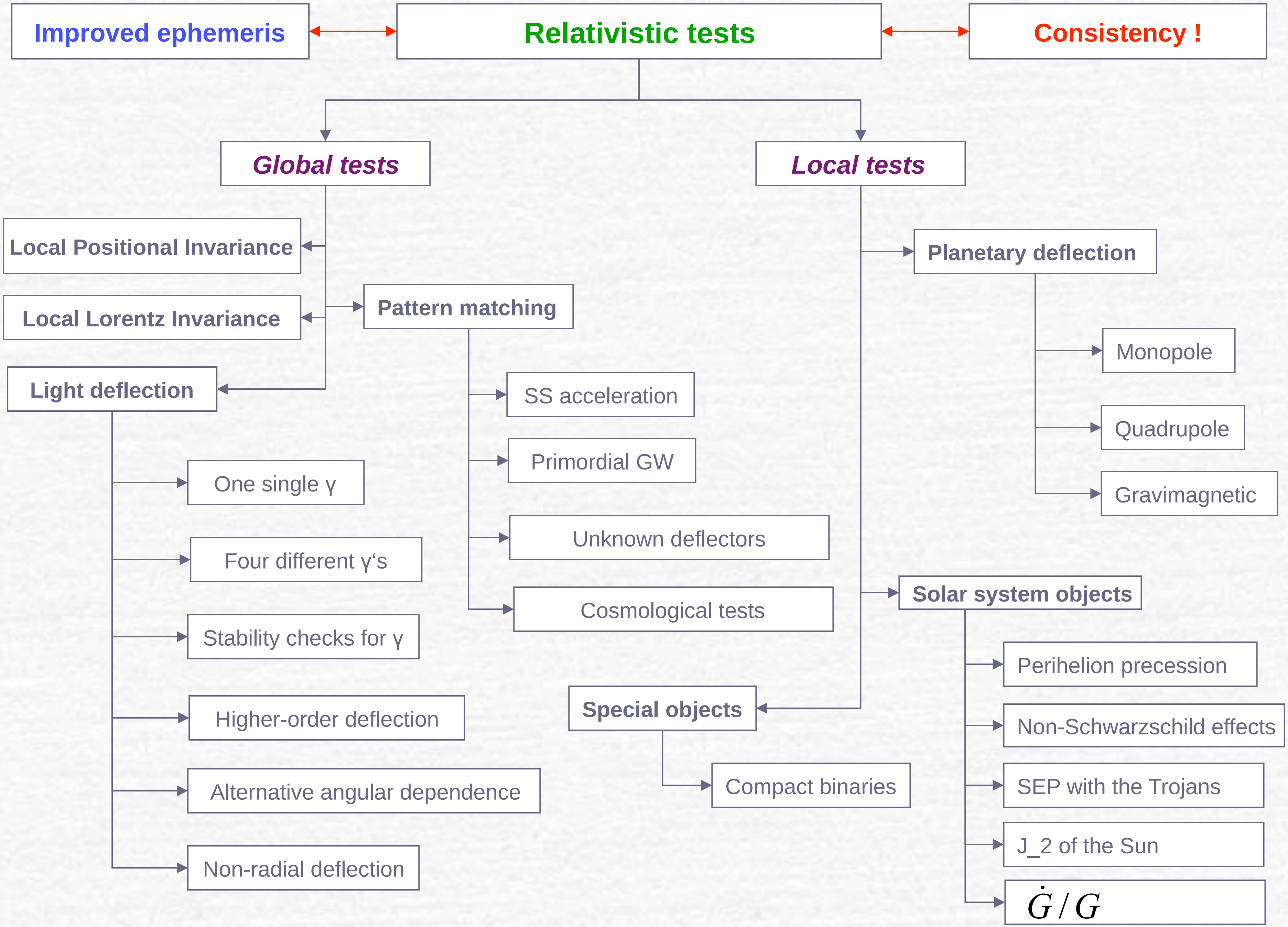
Gaia: goals in brief

- Mapping of the Milky Way: galactic kinematics and dynamics
- Stellar physics (classification, L, log g, Teff, [Fe/H])
- Distance scale (geometric, HR diagrams, cepheids, RR Lyr)
- Age of the Universe (globular clusters, distance and luminosity)
- Dark matter (potential tracers)
- Reference frame (quasars)
- Extra-solar planets (astrometry, photometric transits)
- Solar system objects (survey, taxonomy, masses)
- Fundamental physics (relativity experiments)

Testing Relativity with Gaia

Relativity as a driving force for Gaia





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$\dot{G}/G$
Special objects
Compact binaries

Necessary condition: consistency of the whole data processing chain

- Any kind of inconsistency is very dangerous for the quality and reliability of the estimates

- Consistent relativistic model for 1μas astrometry

Gaia Relativity Model

- General-relativistic modelling of all relevant processes

- Consistent use of the IAU reference systems for all parts of the data modelling and processing

  - motion of solar system
  - motion of Gaia
  - light propagation
    - aberration
    - light deflection: monopole (pN and ppN),
quadrupole
gravitomagnetic (translational)
  - Description of observed objects:
    - orbit
    - parallax, proper motion, radial velocity

- Gaia catalog is a model of the solar system/Galaxy/Universe in the BCRS coordinates

- The model restores the coordinate picture/model from observables…

Necessary condition: consistency of the whole data processing chain

- Any kind of inconsistency is very dangerous for the quality and reliability of the estimates
- Consistent relativistic model for 1μas astrometry
- The whole data processing and all the auxiliary information should be assured to be compatible with the PPN formalism (or at least GR)
    - planetary ephemeris: coordinates, scaling, constants
    - Gaia orbit: coordinates, scaling, constants
    - astronomical constants
    - ???
- Monitoring of the consistency during the whole project

Example:
Optical aberrations by a rotating instrument

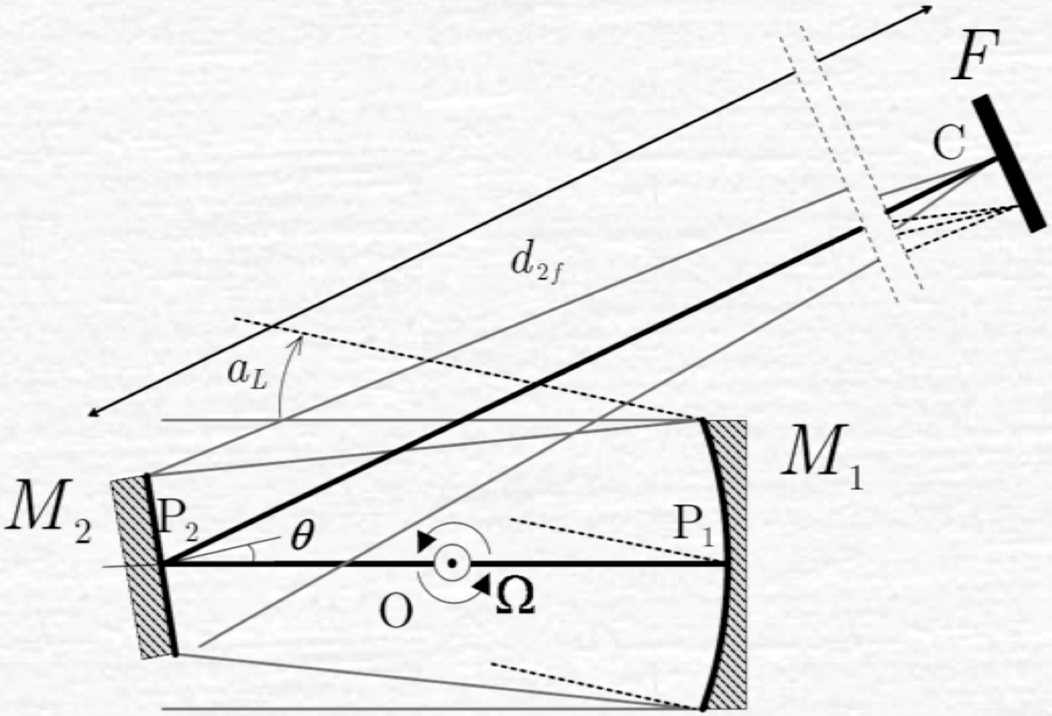
- Two special-relativistic effects modifying PSF of a rotating instrument:
  - Finite light velocity leads to propagation delays within telescope; these delays depend on the position in the field of view
  - Special-relativistic change of the reflection law (Einstein, 1905)

- Reassessment for Gaia was necessary

(Anglada, Klioner, Soffel, Torra, 2007, Astronomy & Astrophysics, 462, 371)

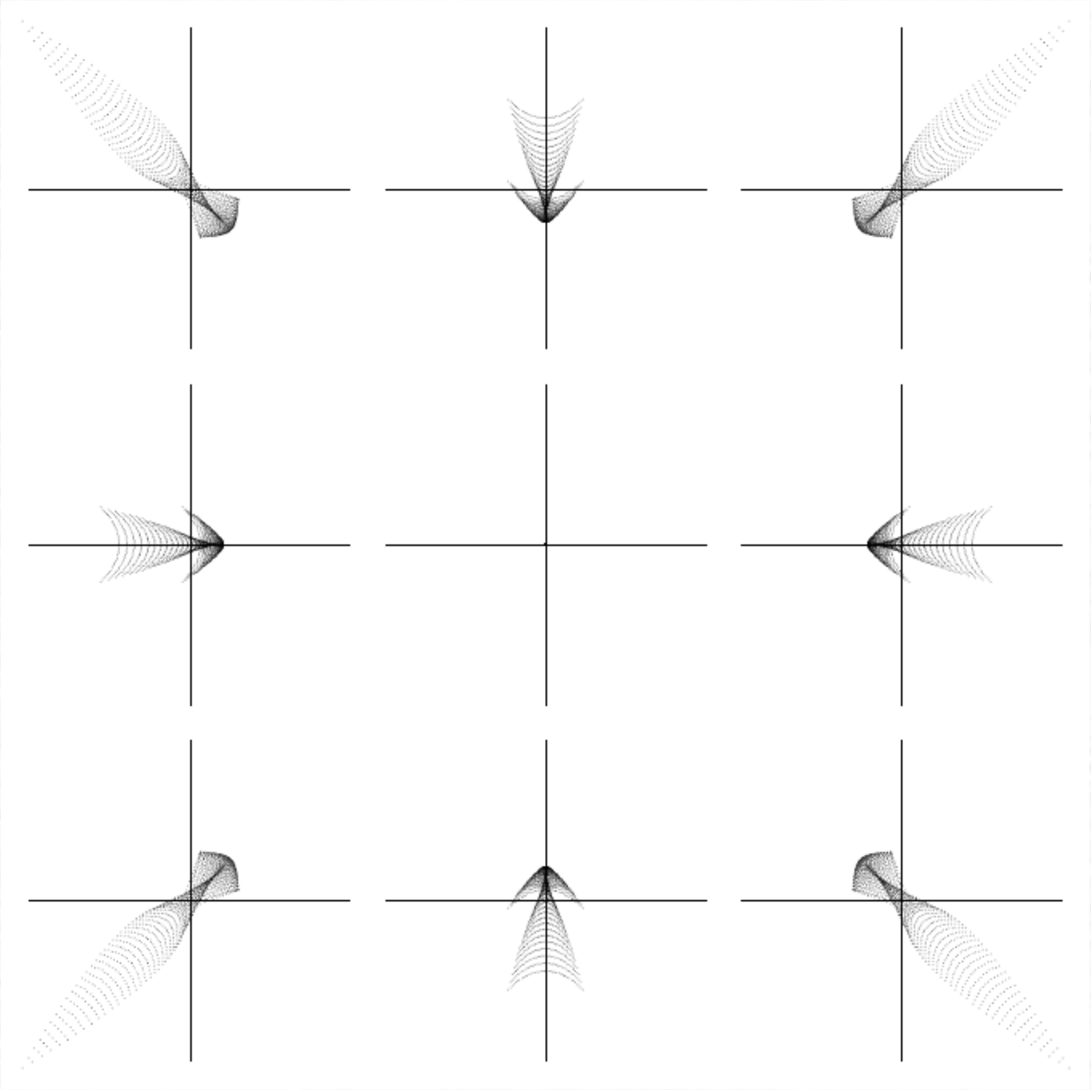
Optical aberrations by a rotating instrument

- Model instrument



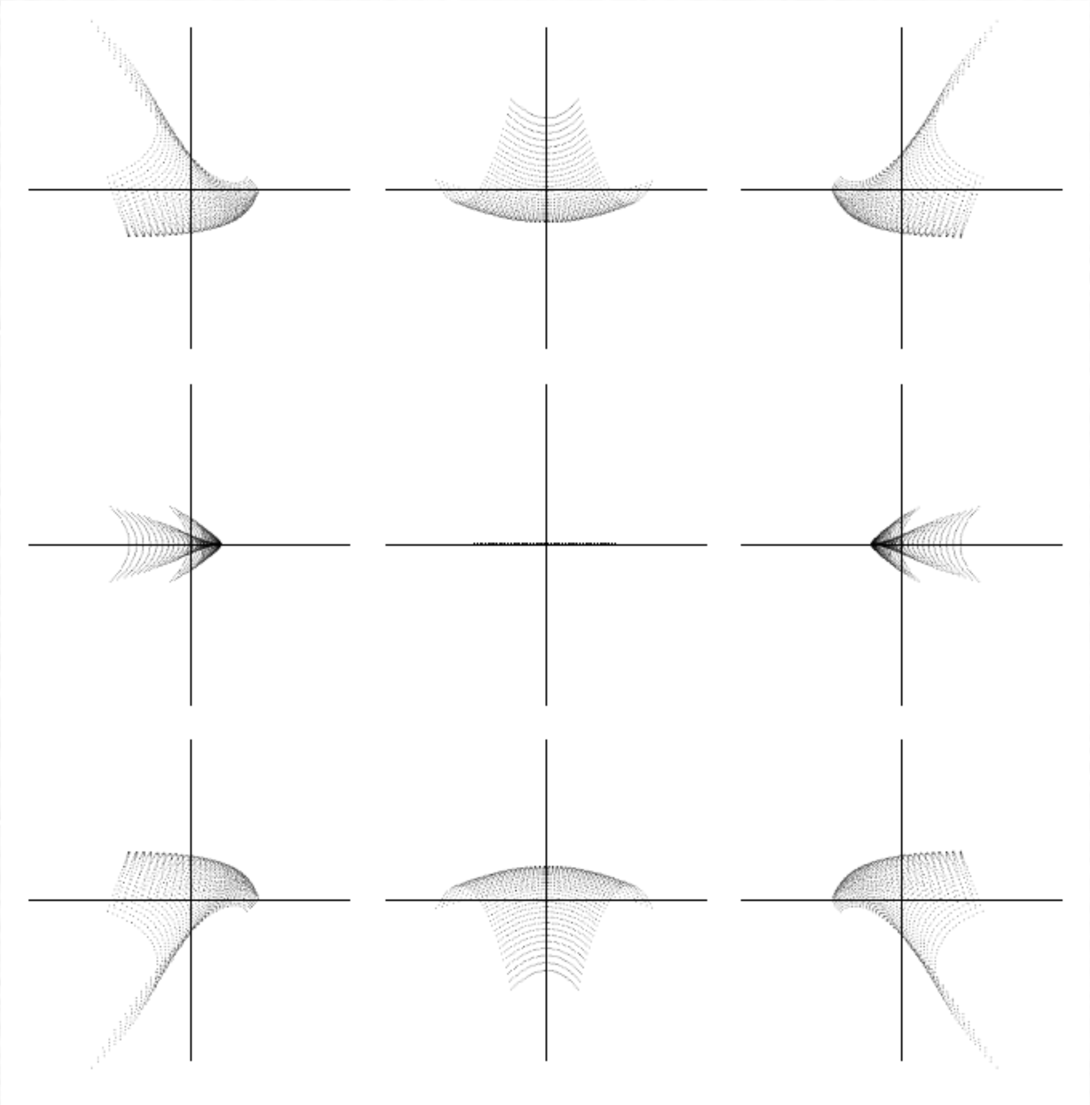
Optical aberrations by a rotating instrument

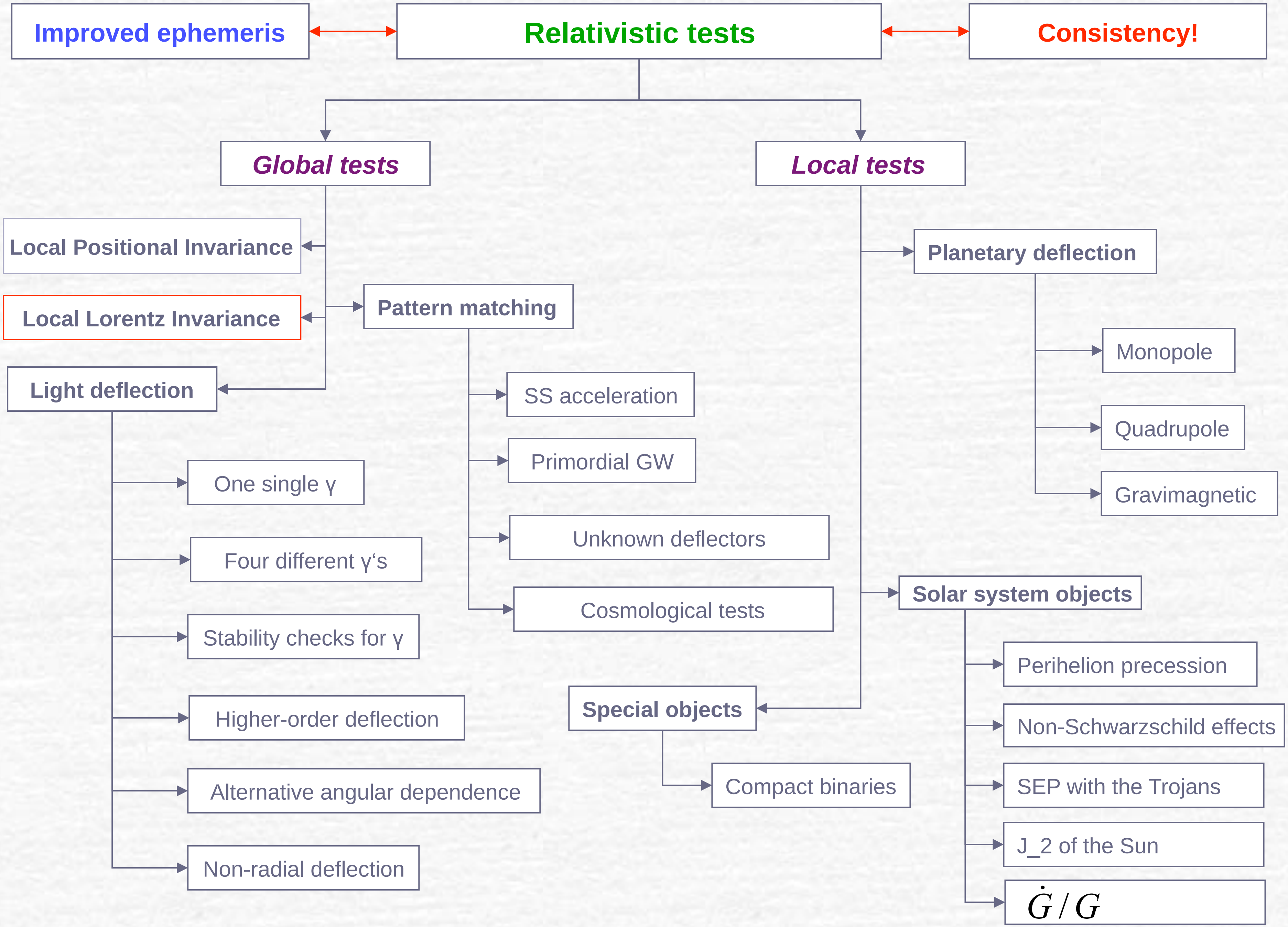
- Aberration patterns by the instrument at rest



Optical aberrations by a rotating instrument

- Aberration patterns by the rotating instrument

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Local Lorentz Invariance

- Motivated by ideas about quantum gravity, a tremendous amount of effort over the past decade has gone into testing Lorentz invariance in various regimes.
- Details: **David Mattingly, Living Rev. Relativity, 8, (2005), 5**
- Simplest approach: Robertson, 1948; Mansouri, Sexl, 1977:

preferred frame1: (T, X^i)

light velocity is constant: $c^2 dT^2 = dX^2 + dY^2 + dZ^2$

frame 2: (t, x^i)

light velocity is no longer constant…

e.g. frame 1 could be the frame where the Cosmic Microwave Background looks isotropic: $v_\odot \approx 370 \text{ km/s}$

$$\alpha = 11.2^{\text{h}}, \delta = \text{-}6.4^\circ$$

Local Lorentz Invariance

- Transformation between these two frames:

$$dT = \frac{1}{a}(dt + \frac{v}{c^2}dx)$$
$$dX = \frac{1}{b}dx + \frac{v}{a}(dt + \frac{v}{c^2}dx)$$
$$dY = \frac{1}{d}dy$$
$$dZ = \frac{1}{d}dz$$

$$a = 1 + \alpha\frac{v^2}{c^2} + \mathcal{O}(c^{-4})$$
$$b = 1 + \beta\frac{v^2}{c^2} + \mathcal{O}(c^{-4})$$
$$d = 1 + \delta\frac{v^2}{c^2} + \mathcal{O}(c^{-4})$$

Special Relativity: $\alpha = -1/2,\ \beta = 1/2,\ \delta = 0$

- Light velocity in frame 2:

$$dt = \frac{dl}{c}\left[1 - (\beta - \alpha - 1)\frac{v^2}{c^2} - \left(\frac{1}{2} - \beta + \delta\right)\sin^2\theta\,\frac{v^2}{c^2}\right] + \mathcal{O}(c^{-4})$$

Local Lorentz Invariance

- Three classic experiments:

$$P_{MM} = 1/2 - \beta + \delta$$

Michelson-Morley: orientation dependence

$$P_{KT} = \beta - \alpha - 1$$

Kennedy-Thorndike: velocity dependence

$$P_{IS} = \left|\alpha + 1/2\right|$$

Ives-Stillwell: contraction, dilation

$$P_{MM} = 9.4\,(\pm 8.1) \times 10^{-11}$$

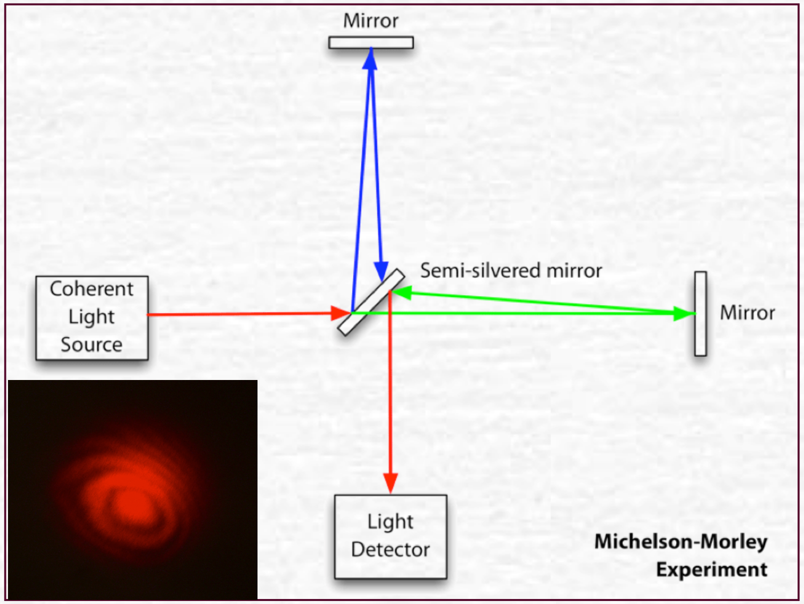
Stanwix et al, PRD 74 (2006) 081101

$$P_{KT} = -3.1\,(\pm 6.9) \cdot 10^{-7}$$

Wolf et al, PRL 90 (2003) 060402

$$P_{IS} < 2.2 \cdot 10^{-7}$$

Saathoff et al, PRL 91 (2003) 190403



LLI and aberration

- Special-relativistic aberration is given by

$$\boldsymbol{s}' = \left(\boldsymbol{s} - \left[\frac{\gamma}{c} - (\gamma - 1)\frac{\boldsymbol{v}\cdot\boldsymbol{s}}{v^2}\right]\boldsymbol{v}\right)\frac{1}{\gamma(1 - \boldsymbol{v}\cdot\boldsymbol{s}/c)},$$

$$\gamma = \left(1 - v^2/c^2\right)^{-1/2},$$

$$\boldsymbol{v} = \dot{x}_o\left(1 + \frac{2}{c^2}U(t, \boldsymbol{x}_o)\right)$$

standard Lorentz transformations

- Expanding in powers of $\boldsymbol{k} = \boldsymbol{v}/c$

$$\boldsymbol{s}'' = \boldsymbol{s}$$

$$+(\boldsymbol{s}\cdot\boldsymbol{k})\,\boldsymbol{s} - \boldsymbol{k}$$

$$-\frac{1}{2}(\boldsymbol{s}\cdot\boldsymbol{k})\,\boldsymbol{k} - \frac{1}{2}k^2\,\boldsymbol{s} + (\boldsymbol{s}\cdot\boldsymbol{k})^2\,\boldsymbol{s}$$

LLI and aberration

- Using the Mansouri-Sexl generalization of the Lorentz transformation

(Klioner, Zschocke, Soffel, Butkevich, 2008)

$$s'' = s$$

$$+(\boldsymbol{s}\cdot\boldsymbol{k})\,\boldsymbol{s} - \boldsymbol{k}$$

$$-\frac{1}{2}(\boldsymbol{s}\cdot\boldsymbol{k})\,\boldsymbol{k} - \frac{1}{2}k^2\,\boldsymbol{s} + (\boldsymbol{s}\cdot\boldsymbol{k})^2\,\boldsymbol{s}$$

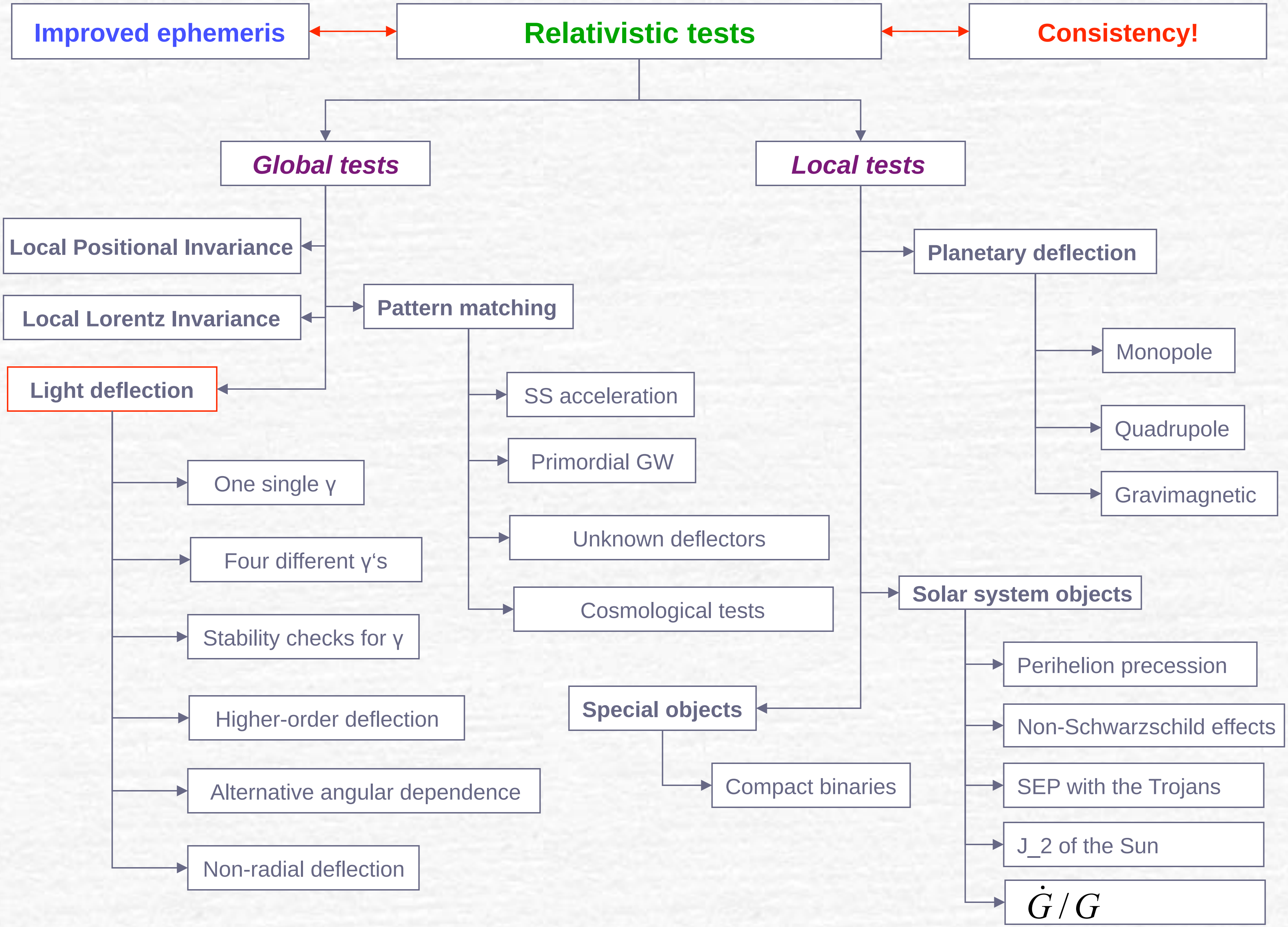
$$-\eta\,(\boldsymbol{s}\cdot\boldsymbol{K})\,\boldsymbol{k} - \eta\,(\boldsymbol{s}\cdot\boldsymbol{k})\,(\boldsymbol{k}+\boldsymbol{K}) + \eta\,(\boldsymbol{s}\cdot\boldsymbol{k})^2\,\boldsymbol{s} + 2\eta\,(\boldsymbol{s}\cdot\boldsymbol{k})\,(\boldsymbol{s}\cdot\boldsymbol{K})\,\boldsymbol{s}$$

The same parameter as in the Michelson-Morley experiment

$$\eta \equiv P_{MM} = 1/2 - \beta + \delta$$

$$\boldsymbol{K} = \boldsymbol{V}/c$$

$\boldsymbol{V}$ is the velocity of the solar system (BCRS) relative to the preferred frame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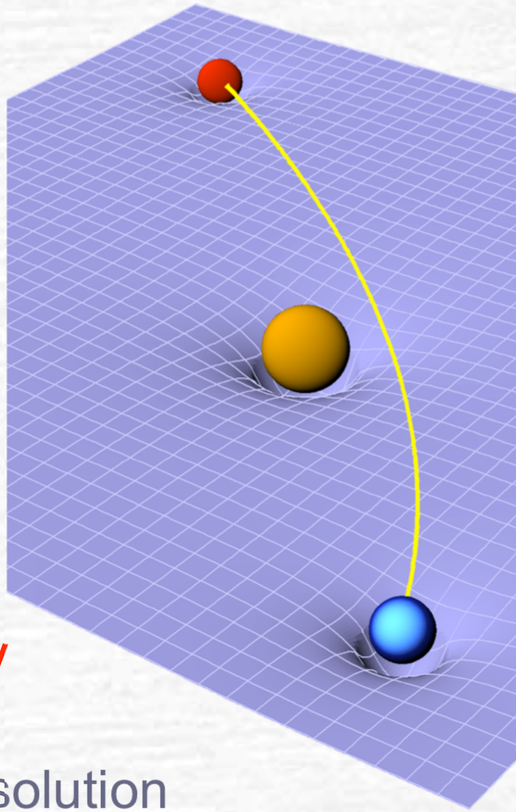
PPN γ from light deflection

- Most precise test possible with Gaia

$$\sigma_\gamma > 10^{-6}$$

- Properties of the Gaia measurements

  - optical,
  - deflection (not Shapiro),
  - wide range of angular distances,
  - full-scale simulations of the experiments



- Problems with some of the „current best estimates" of γ

  1. special fits of the post-fit residuals of a standard solution (e.g., missed correlations leads to wrong estimates of the uncertainty);
  2. no special simulations with faked data to check what kind of effects we are really sensitive to

Challenge: systematic errors

- Light deflection
  (both the general-relativistic one and any sort of alternatives)
  can be mimicked by some systematic changes in calibration parameters:

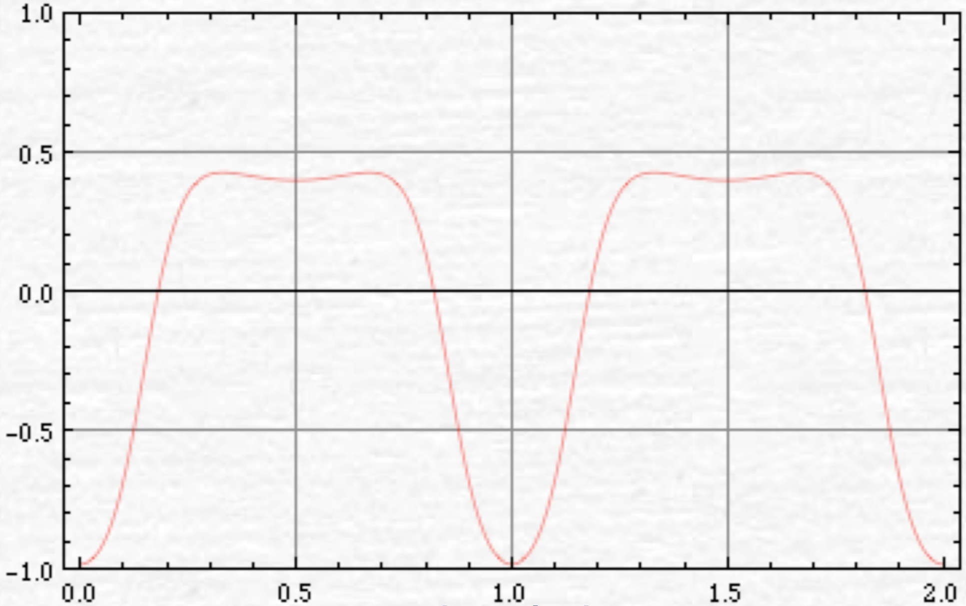
  basic angle variation, errors in velocity of the satellite, etc.

- Special care must be taken!!!

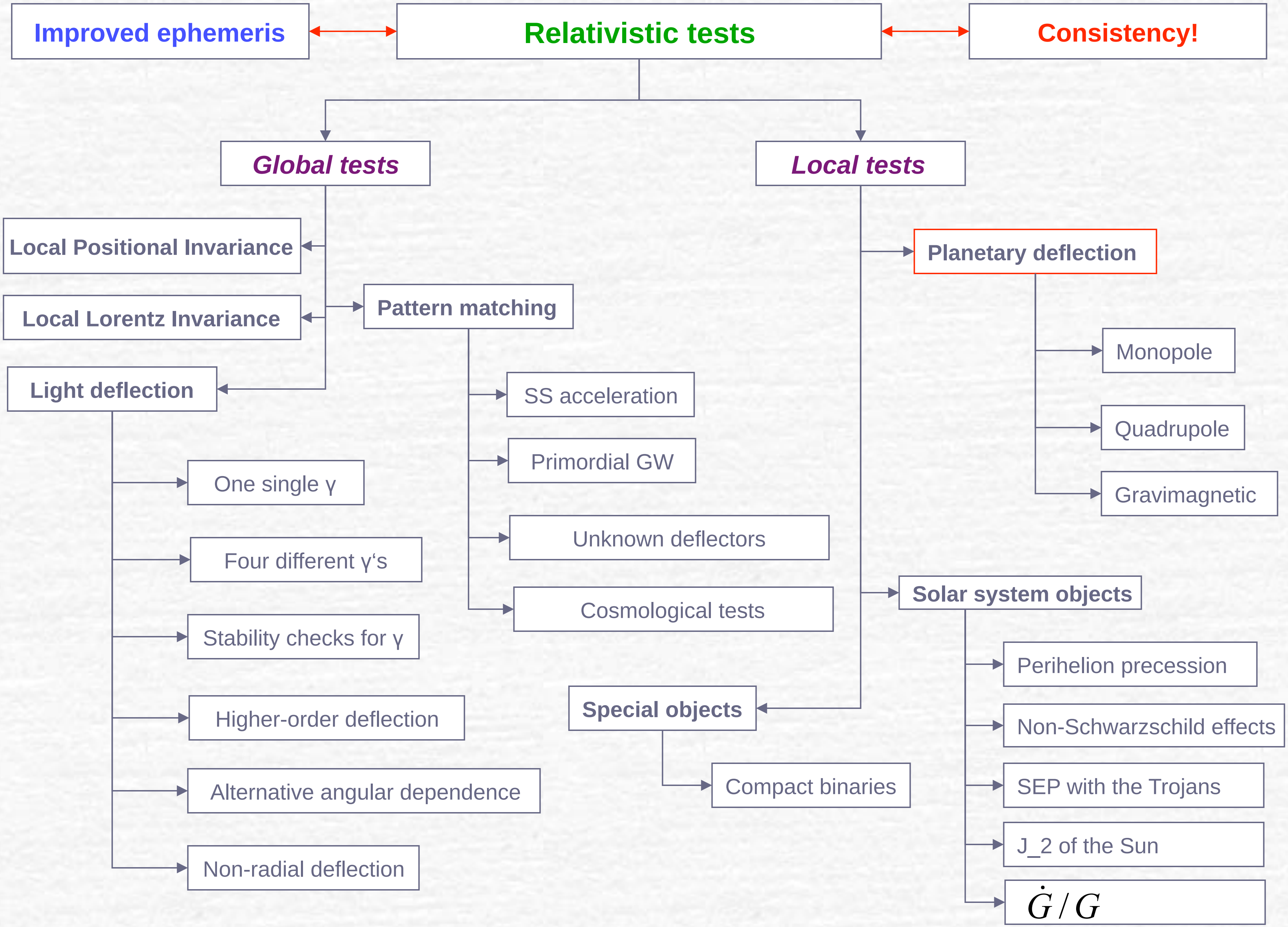
- Very serious efforts are made to control such subtle issues

- Example of an enemy:

  a signal in the basic angle
  that mimics a change of γ



rotational phase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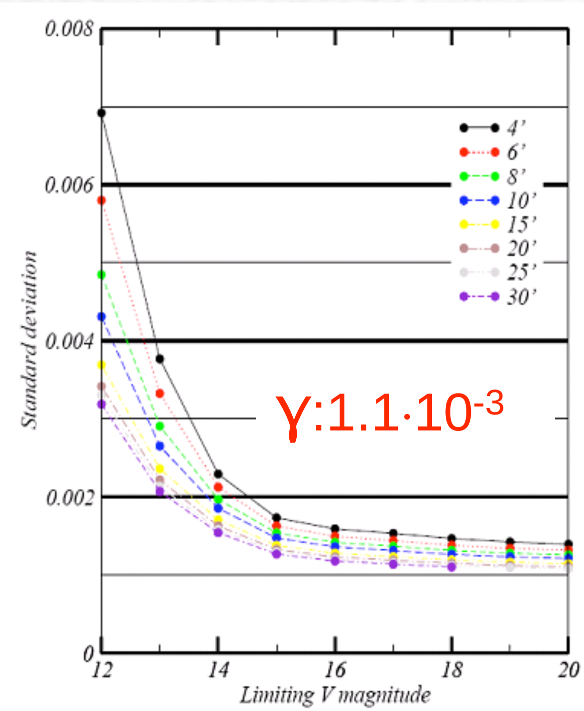
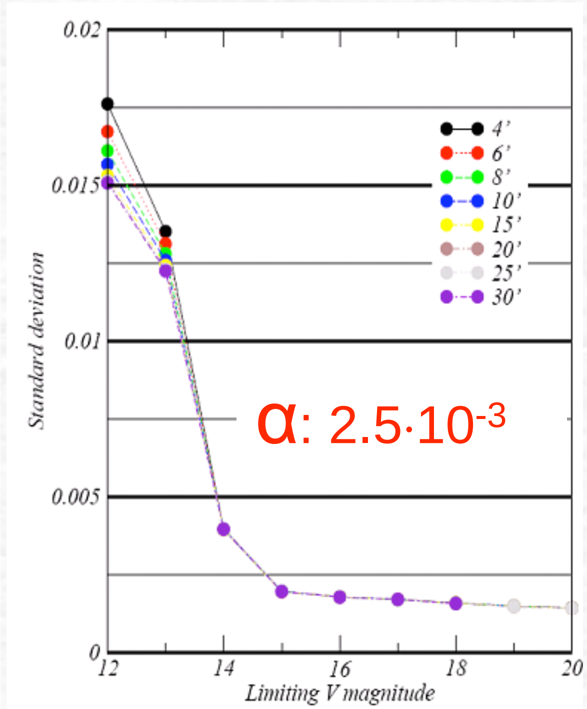
Light deflection from the planets

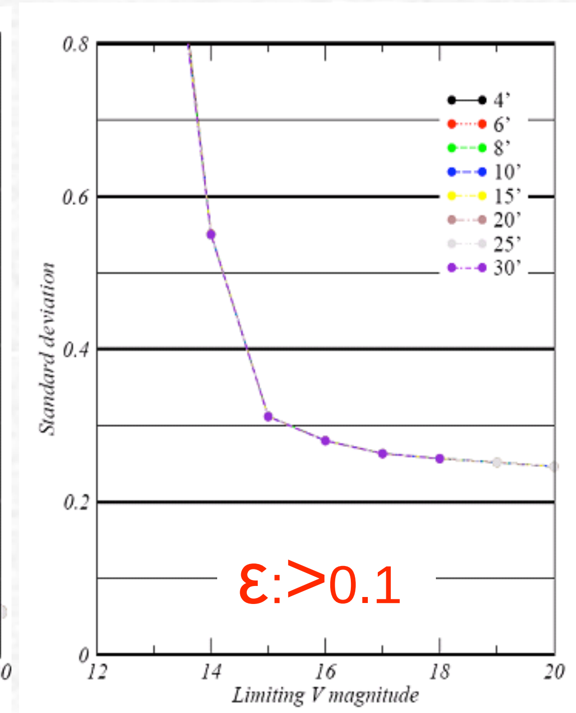
Jupiter: monopole



gradient-gravitomagnetic



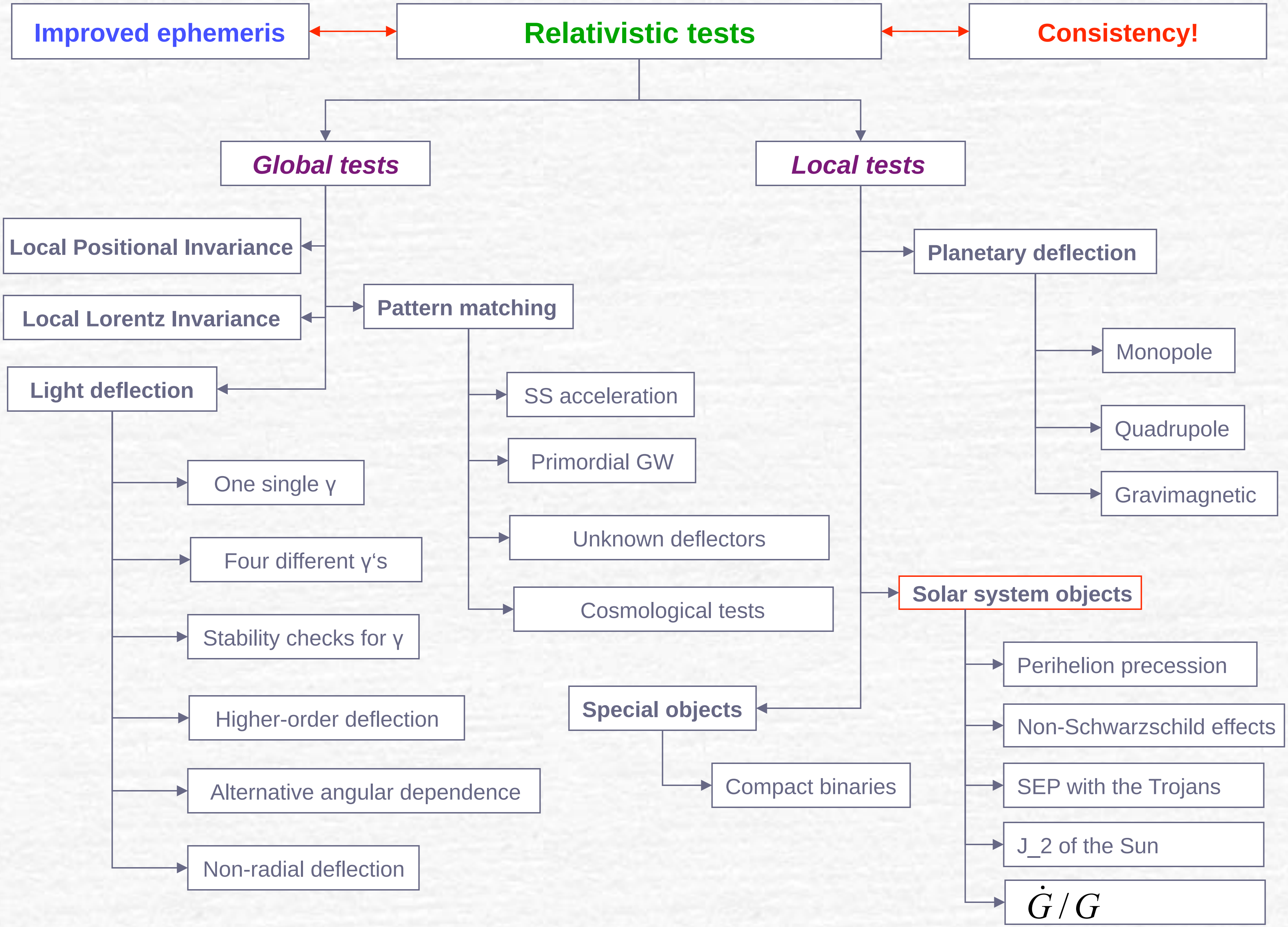
quadrupole



Anglada-Escudé, Klioner, Torra, 2006
Crosta, Mignard, 2006

For other planets the results are worse: 0.1-0.007 for the monopole

Problem: rings, dust, gas, etc. in the vicinity of the giant planets



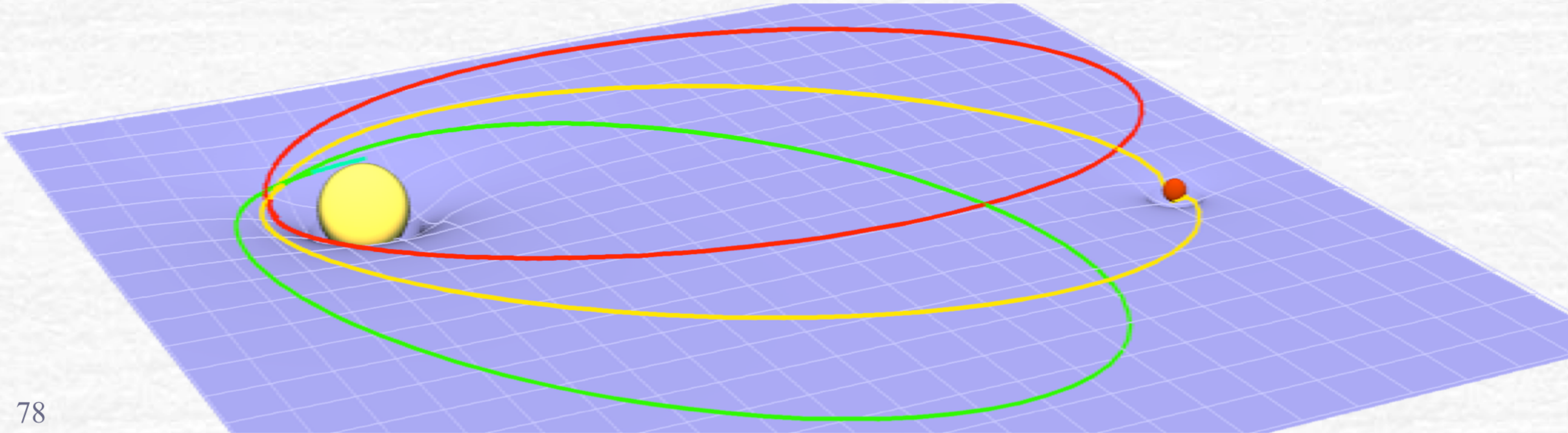
Improved ephemeris
Relativistic tests
Consistency!
Global tests
Local tests
Local Positional Invariance
Local Lorentz Invariance
Light deflection
One single γ
Four different γ's
Stability checks for γ
Higher-order deflection
Alternative angular dependence
Non-radial deflection
Pattern matching
SS acceleration
Primordial GW
Unknown deflectors
Cosmological tests
Planetary deflection
Monopole
Quadrupole
Gravimagnetic
Solar system objects
Perihelion precession
Non-Schwarzschild effects
SEP with the Trojans
J_2 of the Sun
$\dot{G}/G$
Special objects
Compact binaries

Relativistic effects with asteroids

I. Schwarzschild effects due to the Sun: perihelion precession

Historically the first test of general relativity

Object		$\Delta\omega$ (″/cty)	$e\Delta\omega$ (″/cty)	$a(AU)$	e	i (°)
Mercury		42.98	8.84	0.39	0.21	7.00
Venus		8.62	0.06	0.72	0.01	3.39
Earth		3.84	0.06	1.00	0.02	0.00
Mars		1.35	0.12	1.52	0.09	1.85



Perihelion precession (12.09.05: 253113)

Object	number	$\Delta\omega$ (″/cty)	$e\Delta\omega$ (″/cty)	$a(AU)$	e	i (°)
Mercury		42.98	8.84	0.39	0.21	7.00
2004 XY60		32.14	25.63	0.64	0.80	23.79
2000 BD19		26.83	24.02	0.88	0.90	25.68
1995 CR		19.95	17.33	0.91	0.87	4.03
1999 KW4	66391	22.06	15.19	0.64	0.69	38.89
2004 UL		15.06	13.96	1.27	0.93	23.66
2001 TD45		17.12	13.30	0.80	0.78	25.42
1999 MN		18.48	12.30	0.67	0.67	2.02
2000 NL10		14.45	11.80	0.91	0.82	32.51
1998 SO		16.39	11.45	0.73	0.70	30.35
1999 FK21	85953	16.19	11.38	0.74	0.70	12.60
2004 QX2		11.05	9.97	1.29	0.90	19.08
2002 AJ129		10.70	9.79	1.37	0.91	15.55
2000WO107		12.39	9.67	0.91	0.78	7.78
2005 EP1		12.50	9.60	0.89	0.77	16.19
Phaethon	3200	10.13	9.01	1.27	0.88	22.17

Relativistic effects with asteroids

Schwarzschild effects due to the Sun: perihelion precession

For Gaia:

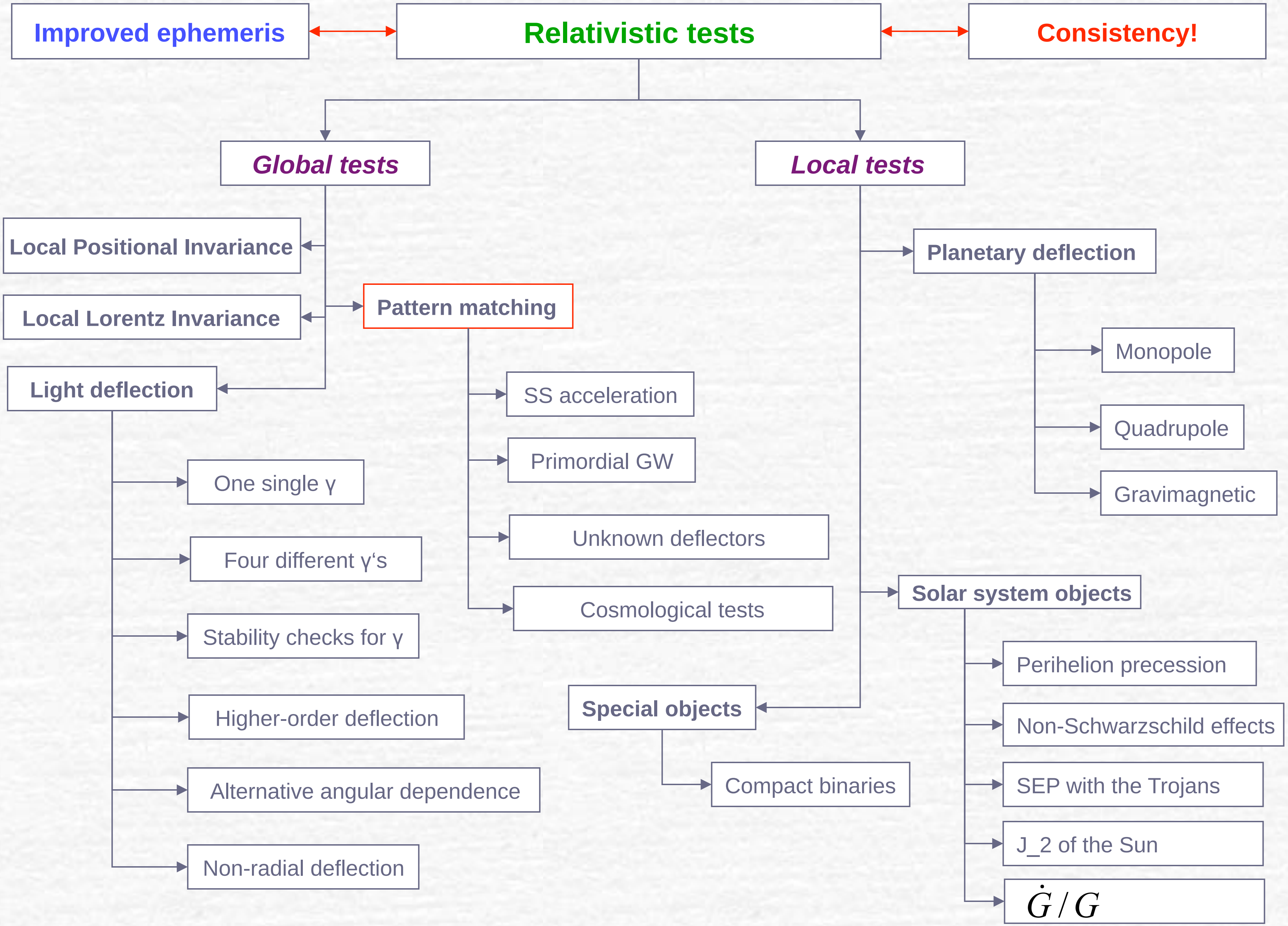
Hestroffer, Berthier, Mouret, Mignard, 2004-

Preliminary results with limited number of sources and with perihelion only:

$$\sigma_{\beta} < 10^{-3}$$

$$\sigma_{J_2} < 10^{-7}$$

$$\sigma_{\dot{G}/G} < 5 \times 10^{-13} \text{ yr}^{-1}$$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Reference frame

- accurate positions, proper motions and parallaxes of a dense net of objects

  > 1500 deg^2

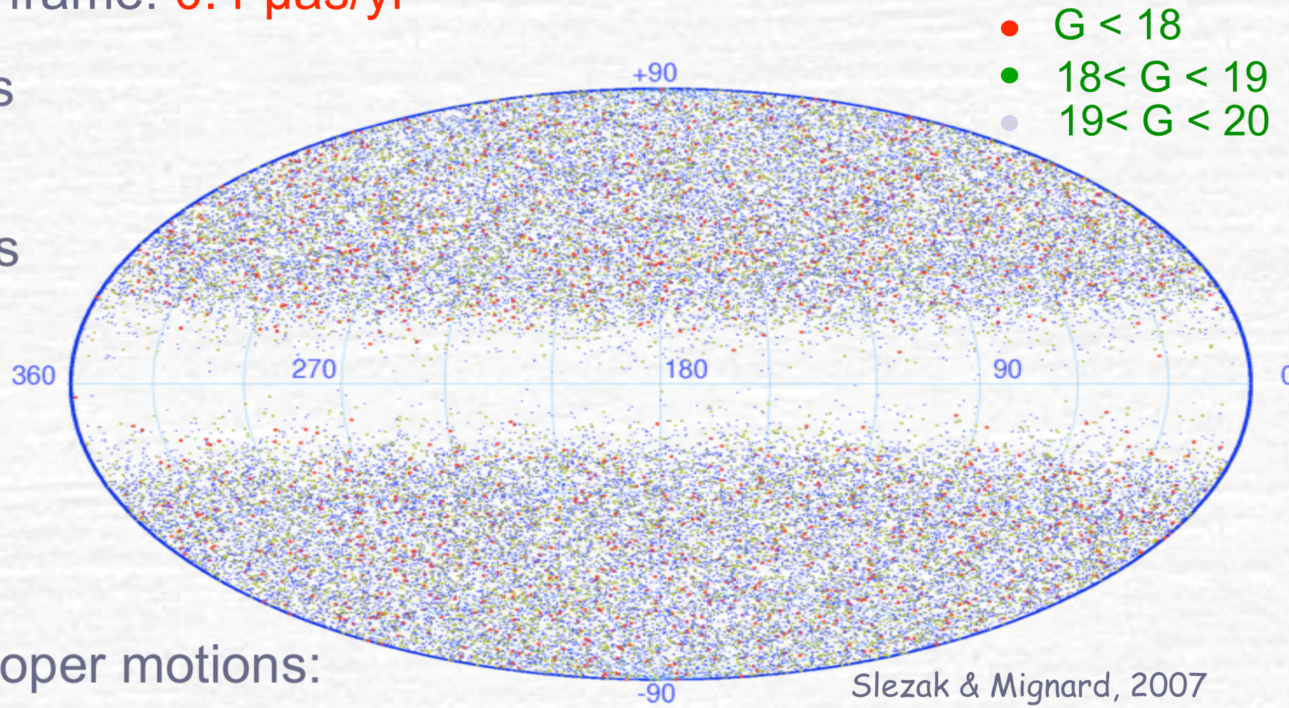
- direct link to the extragalactic objects (500000 quasars are expected)

  recognized photometrically, sample cleaned up astrometrically

  accuracy of the frame: 0.4 µas/yr

- 20000 primary sources with G<18

- long-lived frame: errors < 1mas for 40 years at G=18



Slezak & Mignard, 2007

- by-products:

  pattern matching in proper motions:

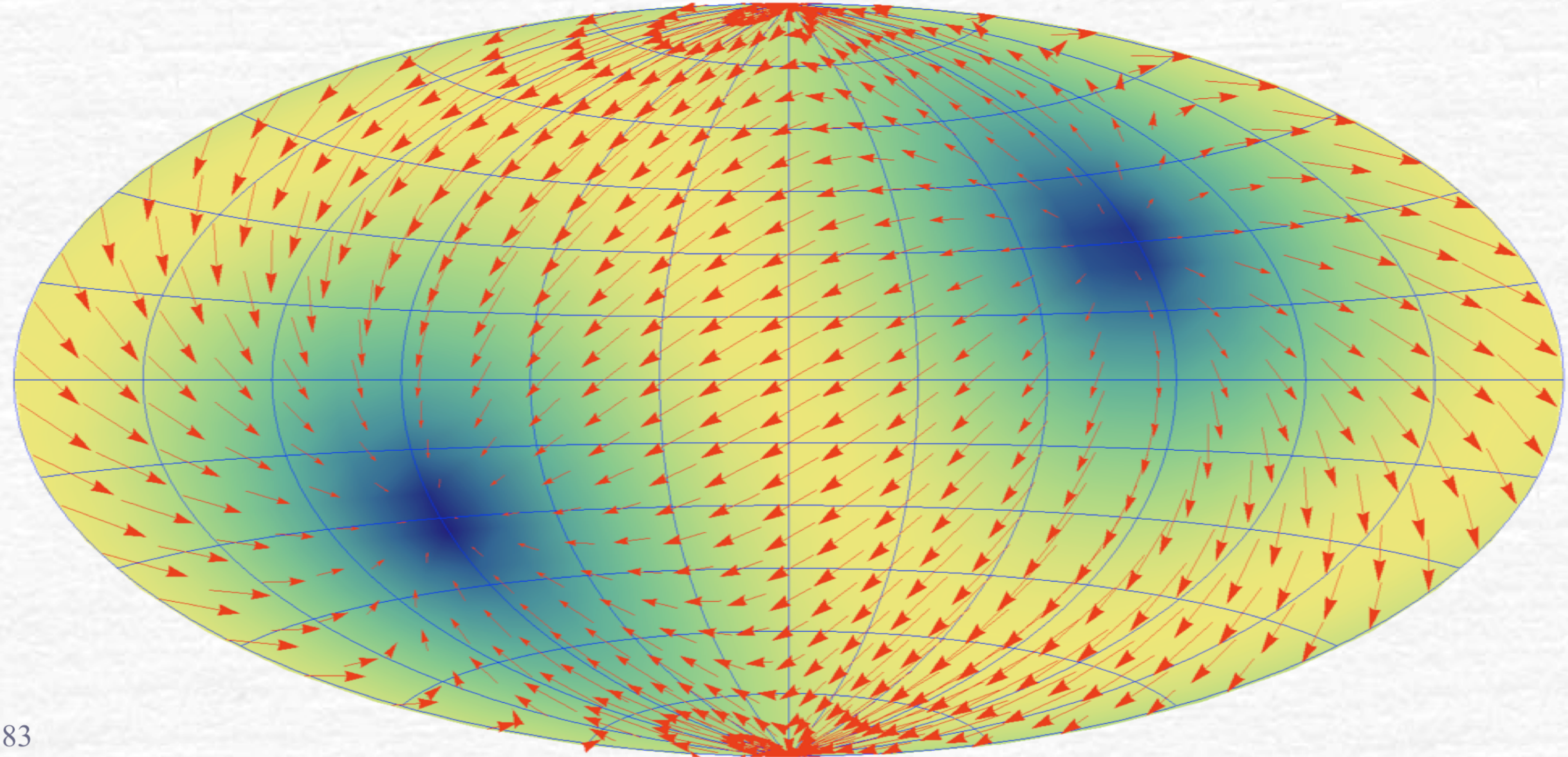
  individual transverse motion 20 µas/yr , systematic - < 1 µas/yr

Pattern matching in positions/proper motions

Example: a pattern of proper motion from the acceleration of Solar system towards the center of the Galaxy

$$\mu_\alpha \cos\delta = -a_x \sin\alpha + a_y \cos\alpha,$$

$$\mu_\delta = -a_x \cos\alpha \sin\delta - a_y \sin\alpha \sin\delta + a_z \cos\delta$$



Pattern matching in proper motions

I. Acceleration of the Solar system relative to remote sources leads to a time dependency of secular aberration: ~5 μas/yr

- constraint for the galactic potential model
- important for the binary pulsar test of relativity (at 1% level)

Mathematics:

expansion of the proper motion field into vector spherical harmonics

$$\vec{\mu} = \sum_{n=0}^{\infty} \sum_{m=0}^{n} a_{nm}^{E} \vec{\mathbf{Y}}_{nm}^{E} + a_{nm}^{M} \vec{\mathbf{Y}}_{nm}^{M}$$

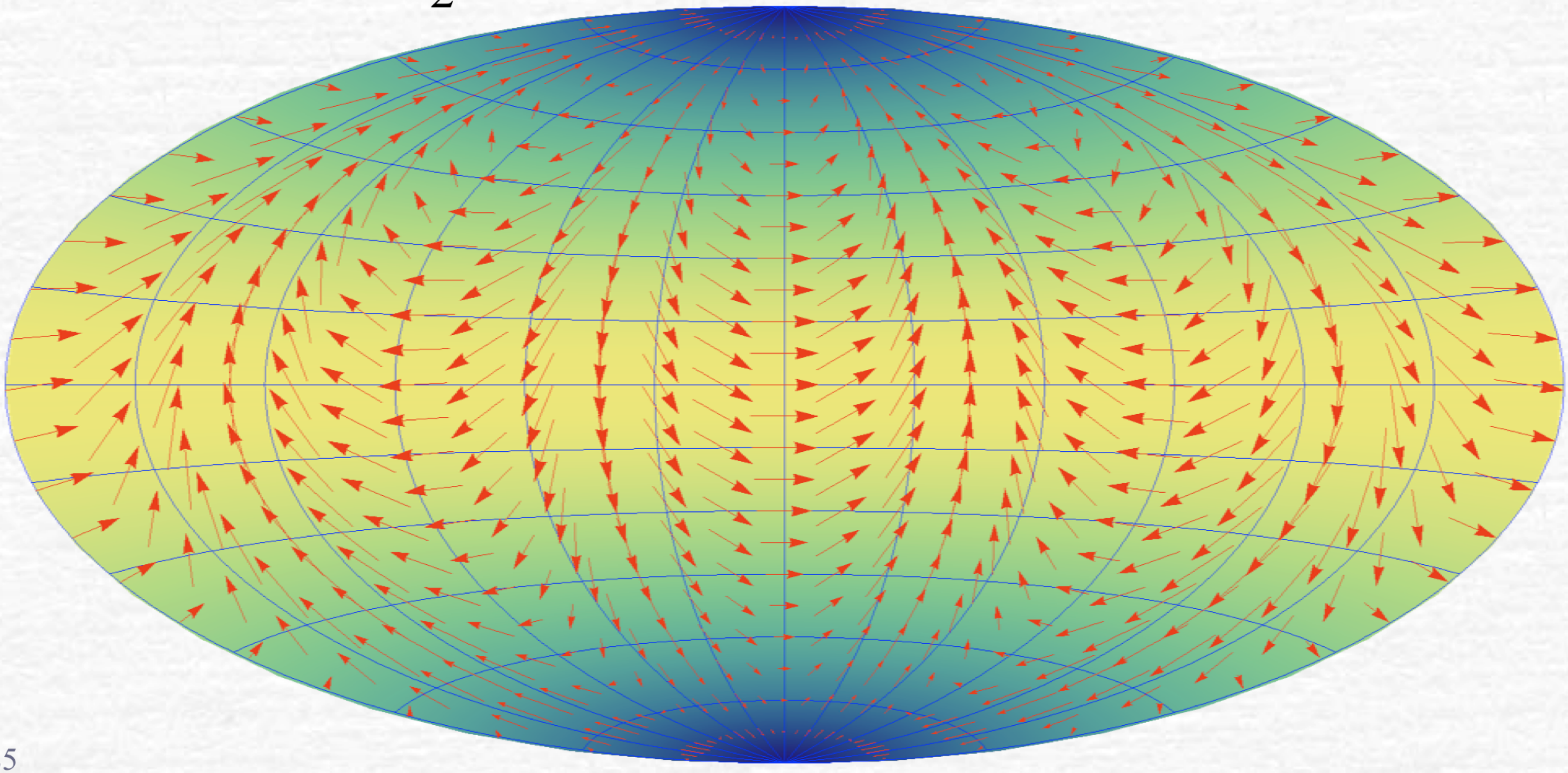
the coefficients for n=1 give - rotations
- the solar system acceleration

Gaia will measure the acceleration with at least 10% accuracy
Accuracy limit of Gaia is $\delta a \simeq 2 \times 10^{-11}\ \mathrm{m/s^2}$

Pattern matching in positions/proper motions

Example: a GW of strain *h* and frequency *ω* propagating in the direction δ=90°:

$$\vec{\mu} = \frac{1}{2}\omega h \sin\omega T \cos\delta\left(\cos 2\alpha\, \vec{\boldsymbol{e}}_{\delta} + \sin 2\alpha\, \vec{\boldsymbol{e}}_{\alpha}\right)$$



Pattern matching in positions/proper motions

II. Constraint on very low frequency gravitational waves:

- constraint of stochastic GW flux with $\nu < 10^{-8}$ Hz
  (similar study done for VLBI: Gwinn et al., ApJ, 1997)

  Harmonic coefficients for n>1 give the GW-flux constraints…

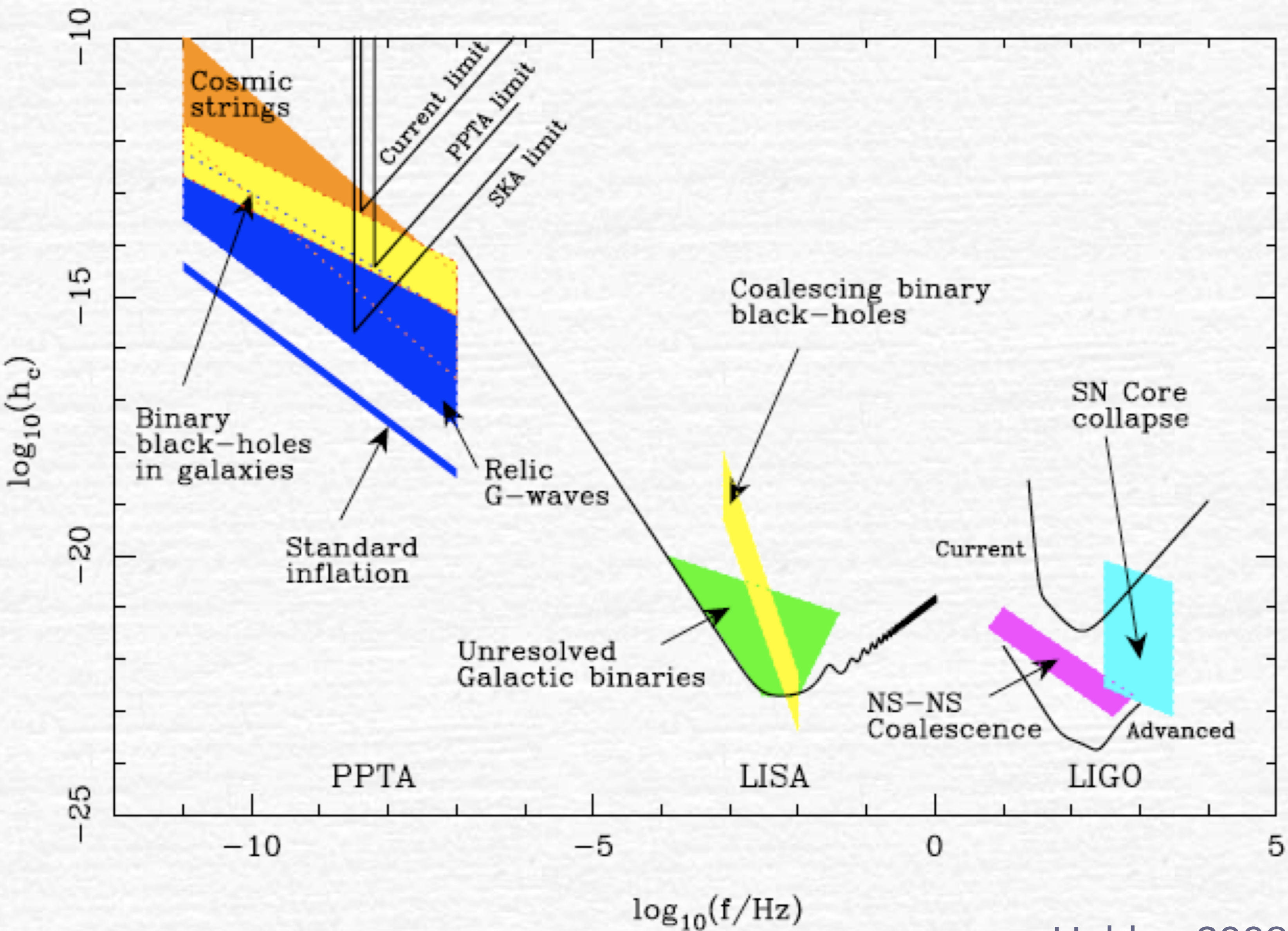
  $$h \simeq 10^{-12} \quad \text{for } \nu < 10^{-8}\,\text{Hz}$$

- attempts to fit a pattern of apparent motions induced by an individual GW with much higher frequencies

  up to 10^{-2} Hz
  using all the stars one can hope to get an improvement of up to 5 orders of magnitude $h \simeq 10^{-17}$???

  Sensitivity analysis is ongoing… Systematic errors…
  Do not take these estimates seriously!

Gravitational Wave Spectrum



Hobbs, 2008

One sentence from each part

- General Relativity has many applications in astronomy
- General Relativity is a well-established physical theory with many applications (even at the engineering level)
- Astrometry is making a stunning progress nowadays reaching the level of 1 microarcsecond
- Gaia will provide a variety of new relativistic tests