

# An Overview Of Neutron Decay

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## 1 Neutron Decay In The Context Of Nuclear Physics.

### 1.1 The Weak Interaction in Nuclei.

According to the Standard Model of particle physics the charged weak current is purely left-handed, i.e. it is an equal admixture of polar vector (V) and axial vector (A) currents of quarks and leptons with appropriate relative sign. In nuclear physics vector currents give rise to Fermi  $\beta$  -transitions with coupling constant  $G_V$  and spin-parity selection rule for allowed transitions:

$$\Delta I = 0, \text{ no parity change} \quad (1)$$

Axial currents give Gamow-Teller  $\beta$ -transitions with coupling constant  $G_A$  and spin-parity selection rule for allowed transitions:

$$\Delta I = 0, \pm 1, \text{ no } 0 \Rightarrow 0, \text{ no parity change} \quad (2)$$

The  $\beta$ -decay of free neutrons into protons

$$n \Rightarrow p + e^- + \bar{\nu}_e, \quad \frac{1}{2}^+ \Rightarrow \frac{1}{2}^+ \quad (3)$$

is allowed by both selection rules and is described as a mixed transition. One can therefore observe parity-violating effects in neutron decay associated with vector/axial vector interference.

### 1.2 Neutron Decay Parameters.

The principal kinematic parameters which govern neutron decay are:

$$\Sigma = (m_n + m_p)c^2 = 1877.83794 \text{ MeV}; \Delta = (m_n - m_p)c^2 = 1.29332 \text{ MeV} \quad (4)$$

$$\text{Kinetic energy of electrons : } 0 \leq T_e \leq 783 \text{ keV} \quad (5)$$

$$\text{Kinetic energy of protons : } 0 \leq T_p \leq 751 \text{ eV} \quad (6)$$

$$\text{Recoil parameter : } \delta = \Delta / \Sigma < 10^{-3} \quad (7)$$

Because the recoil parameter  $\delta$  is so small it follows that the momentum transfer dependence of all form factors may be neglected. This is also the reason why the neutron lifetime is so long. The current best value of the neutron lifetime is [1]:

$$\tau_n = 885.7 \pm 0.8 \text{ sec.} \quad (8)$$

This is greater by a factor of  $\sim 4.10^8$  than the lifetime of the muon which is the next longest lived elementary particle.

### 1.3 Measurement of the Neutron Lifetime.

Neutron lifetime experiments may be separated into two groups: the classical 'beam' methods and the more modern 'bottle' methods. In beam methods the number of decaying neutrons in a specified volume of neutron beam is recorded. These methods rely on the relationship:

$$\frac{dN(t)}{dt} = -\frac{N(t)}{\tau_n} \quad (9)$$

where  $N(t)$  is the number of neutrons in the source volume  $V$  at time  $t$ . To proceed further we require two additional relations:

$$\left\langle \frac{dN(t)}{dt} \right\rangle = n_d \frac{4\pi}{\Omega \varepsilon} \quad (10)$$

and

$$\langle N(t) \rangle = \rho_n V \quad (11)$$

where  $n_d$  is the number of neutron decays recorded per unit time in a detector of known solid angle  $\Omega$  and efficiency  $\varepsilon$ , and  $\rho_n$  is the neutron density. Assuming a  $4\pi$  collection solid angle, as in all recent variants of the technique, and unit efficiency  $\varepsilon$  for recording the number  $N_d$  of decays occurring per second in a known length  $L$  of beam, the value of  $\tau_n$  is given by

$$\tau_n = \frac{N_n L}{N_d \sigma_0 v_0 \eta} \quad (12)$$

Here  $N_n$  is the number of neutron-nucleus reactions detected per unit time in a neutron counter,  $\sigma_0$  is the cross section at some standard neutron velocity  $v_0$  (usually 2200m./sec.) and  $\eta$  is the surface density of neutron detector isotope. This result does not depend on the neutron velocity  $v$ , provided  $\sigma(v)$  scales as  $v^{-1}$ . Suitable reactions are:

$$^{10}\text{B}(n, \alpha)^7\text{Li} \quad (\sigma_0 = 3836 \pm 8b.), \quad (13)$$

$$^6\text{Li}(n, \alpha)^3\text{H} \quad (\sigma_0 = 941 \pm 3b.) \quad (14)$$

and

$$^3\text{He}(n, p)^3\text{H} \quad (\sigma_0 = 5327 \pm 10b.) \quad (15)$$

'Bottle' methods for the determination of  $\tau_n$  on the other hand rely on the integrated form of (9), i.e.

$$N(t) = N(0) e^{-t/\tau_n} \quad (16)$$

where  $N(t)$  is determined by recording the number of neutrons surviving to time  $t$  as a function of the number  $N(0)$  present in a fixed source volume at zero time. This is to be contrasted with the beam methods where it is the number of neutrons which fail to survive in a continually replenished source of neutrons which is recorded. Ever since the identification of a storable ultra-cold component of energy  $\leq 2.10^{-7}$  eV in the Maxwellian tail of the thermal flux from a reactor, the bottle methods have been favored since they do not rely

on the performance of a number of subsidiary experiments, e.g. determination of absolute cross-sections or the precise isotopic composition of neutron counters.

There are two principal neutron storage methods, magnetic confinement or storage in a closed vessel made from a material with suitable Fermi pseudo-potential. Magnetic confinement relies on the force

$$\mathbf{F} = -\nabla\{\mu_n \cdot \mathbf{B}(\mathbf{r})\} \quad (17)$$

which is exerted on the neutron magnetic moment  $\mu_n$  in an inhomogeneous magnetic field  $\mathbf{B}(\mathbf{r})$ . Since the sense of the force depends on the sign of the spin quantum number only one sign of the spin can be confined which means that, in principle, neutrons can always be lost from the source volume by spin-flipping which is a difficult loss mechanism to control. Alternatively in the case of storage in a material bottle the ideal relation (16) must be replaced by

$$N(t) = N(0)e^{-t(1/\tau_n + 1/\tau_w)} \quad (18)$$

where  $\tau_w(v)$  represents the lifetime for neutron loss through absorption or inelastic collisions of ultra-cold neutrons with the walls of the vessel. In general this is given by a relation of the form

$$\tau_w(v)^{-1} = \langle \mu(v) \rangle v / \lambda \quad (19)$$

where  $\langle \mu(v) \rangle$  is the loss rate per bounce averaged over all angles of incidence and the mean free path  $\lambda$  is a function of the geometry of the containing vessel. A number of techniques have been developed to estimate  $\tau_w(v)$  by using variable geometry and/or counting the number of up-scattered neutrons.

#### 1.4 Neutron Lifetime and the Big Bang.

The free neutron lifetime is also of significance in big bang cosmology, where it directly influences the relative abundance of primordial helium synthesized in the early universe. This is determined by the ratio of the neutron lifetime to the expansion time from that epoch at which neutrinos decouple from hadronic matter to the onset of nucleosynthesis [2].

The argument goes briefly as follows. At times  $t < 10^{-2}$  sec. and temperatures  $T > 10^{11}$  K the populations of neutrons and protons are kept in a state of thermal equilibrium, i.e.

$$X_n/X_p = e^{-(m_n - m_p)c^2/kT} \quad (20)$$

through the weak interactions

$$n + e^+ \rightleftharpoons p + \bar{\nu}_e; \quad p + e^- \rightleftharpoons n + \nu_e \quad (21)$$

At  $t \simeq 1$  sec. the freeze-out temperature  $T \simeq 10^{10}$  K is reached where the leptons decouple from the hadrons and neutrons begin to decay into protons according to (3). This process continues until a time  $t \simeq 180$  sec when the temperature has fallen to a value  $T \simeq 10^9$  K and deuterium formed by the capture of neutrons on protons remains stable in the thermal radiation field. This is followed by a sequence of strong interactions whose net effect is the conversion of all free neutrons into helium. Using the current value of the neutron lifetime, these considerations result in a relative helium abundance in the present day universe of about 25% in good agreement with observation.

### 1.5 Application to Solar Astrophysics.

The main source of solar energy derives from the proton-proton cycle of thermonuclear reactions, the end-point of which is the fusion of four protons into a helium nucleus with the release of positrons, photons and neutrinos. In the first step two protons interact weakly to form deuterium



An alternative reaction is the weak  $p - e - p$  process which occurs with a branching ration of approximately 0.25%



That the timescale is determined by the neutron lifetime stems from the fact that the governing reaction (22) is just inverse neutron decay with the spectator proton providing the energy, while the  $p - e - p$  interaction (21) is the corresponding electron capture process [3]. However since the two protons can interact weakly only in the  ${}^1S_0$  state because of the Pauli principle, and since the deuteron can exist only in the triplet state, it follows that the vector contribution to the underlying inverse neutron  $\beta$ -decay is forbidden and the weak capture of protons on protons proceeds at a rate proportional to  $|G_A|^2$ . To compute this rate it is therefore necessary to determine individual values for the weak coupling constants  $G_V$  and  $G_A$ .

### 1.6 Determination of the Weak Coupling Constants.

The neutron lifetime  $\tau_n = t_n / \ln(2)$ , where the half-life  $t_n$  is commonly employed in nuclear physics, is given by the formula

$$ft_n = \frac{2\pi^3 \ln(2) \hbar^3}{m_e^5 c^4} \cdot [|G_V|^2 + 3|G_A|^2]^{-1} = \frac{K}{|G_V|^2} \cdot [1 + 3|\lambda|^2]^{-1} \quad (24)$$

where  $K = (8120.271 \pm 0.012) \cdot 10^{-10} \text{ GeV}^{-4} \text{ sec.}$ , and

$$\lambda = G_A / G_V \quad (25)$$

The factor  $f$  is the integral of the Fermi Coulomb-corrected phase space function  $F(E_e)$  which, including the outer radiative corrections  $\delta_R > 0$ , has the value [4]

$$f(1 + \delta_R) = 1.71489 \pm 0.00002 \quad (26)$$

If isospin invariance of the strong interactions and conservation of the weak vector current are assumed, then  $|G_V|$  may be determined from the  $ft$ -values of the sequence of pure Fermi superallowed  $0^+ \Rightarrow 0^+$  nuclear positron emitters through the formula

$$\overline{ft(1 - \delta_C)(1 + \delta_R)(0^+ \Rightarrow 0^+)} = K / |G_V|^2 \quad (27)$$

where each nuclear decay has been individually corrected, incorporating factors  $(1 - \delta_C) < 1$  for isospin symmetry-breaking and  $(1 + \delta_R) > 1$  for the nucleus-dependent radiative correction. It follows that the values of  $|G_V|$  and  $|G_A|$  can be determined from a combination of equations (24) to (27). To determine the relative sign of  $G_V$  and  $G_A$  it is necessary to observe some phenomenon which relies on Fermi/Gamow-Teller interference and this requires the availability of polarized neutrons. Such phenomena allow the direct determination  $\lambda$  and thus  $G_V$  and  $G_A$  can each be determined in both sign and magnitude from neutron decay alone, in which case uncertainties associated with nuclear structure effects do not arise.

## 2 Neutron Decay In The Context Of Particle Physics

### 2.1 The Cabibbo-Kobayashi-Maskawa Matrix

The neutron and proton form the components of an isospin doublet and are the lightest constituents of the lowest SU(3) flavor octet, each of whose sub-multiplets is characterized by its isospin (I) and its hypercharge (Y). A quantum number alternative to hypercharge is the strangeness  $S=Y-B$  where the baryon number B has the value unity. Flavor SU(3) symmetry is based on neglect of the difference in mass between the u- and d-quarks on the one hand, and the s-quark on the other, and is severely broken. Because of the near equality of the u- and d-quark masses the isospin SU(2) symmetry is much more closely realized, a result which is derived from a dynamic global gauge symmetry of the QCD Lagrangian which is expressed in the conservation of the weak vector current.

In increasing order of mass the octet contains an isodoublet  $\{n,p; I=1/2, Y=1\}$ , an isosinglet  $\{\Lambda^0; I=0, Y=0\}$ , an isotriplet  $\{\Sigma^-, \Sigma^0, \Sigma^+; I=1, Y=0\}$  and a heavy isodoublet, the so-called cascade particles  $\{\Xi^-, \Xi^0, I=1/2, Y=-1\}$ . The  $\Sigma^0$  decays electromagnetically into the  $\Lambda^0$  which has the same value of Y and differs only in the value of I which is not conserved by the electromagnetic interaction. Semi-leptonic weak decays within the octet are characterized according to whether they are hypercharge conserving (e.g.  $n \Rightarrow p, \Sigma^- \Rightarrow \Lambda^0$  and  $\Sigma^+ \Rightarrow \Lambda^0$ ), or hypercharge violating (e.g.  $\Sigma^- \Rightarrow n, \Sigma^+ \Rightarrow n$  and  $\Xi^- \Rightarrow \Lambda^0$ ).

It was Cabibbo's original insight to note and appreciate the significance of the fact that the vector coupling constants corresponding to hypercharge conserving weak decays  $G_V (\Delta Y = 0)$ , and hypercharge non-conserving weak decays  $G_V (\Delta Y = 1)$  satisfied the empirical relations

$$G_V (\Delta Y = 0) = G_F \cdot \cos(\theta_c); \quad G_V (\Delta Y = 1) = G_F \cdot \sin(\theta_c) \quad (28)$$

where the Fermi coupling constant  $G_F$  is determined from the lifetime of the muon and the Cabibbo angle  $\theta_c \simeq 0.23$ . The result (28) is interpreted to mean that the charged vector bosons  $W^\pm$  which mediate the weak interaction couple to the mixtures of quark mass eigenstates

$$d' = d \cdot \cos(\theta_c) + s \cdot \sin(\theta_c); \quad s' = -d \cdot \sin(\theta_c) + s \cdot \cos(\theta_c) \quad (29)$$

rather than to the mass eigenstates of the down ( $d$ ) and strange ( $s$ ) quarks themselves.

In the Standard Model of Particle Physics these ideas are extended to three quark generations where the couplings effective for the weak semi-leptonic decays of quarks are described by the Cabibbo-Kobayashi-Maskawa (CKM) matrix [5], which rotates the quark mass eigenstates ( $d, s, b$ ) to the weak eigenstates ( $d', s', b'$ ):

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \quad (30)$$

where

$$V_{ud} \simeq V_{cs} \simeq \cos(\theta_c); \quad V_{us} \simeq -V_{cd} \simeq \sin(\theta_c) \quad (31)$$

Since, assuming that no more than three quark generations exist, the CKM matrix must be unitary, its nine elements can be expressed in terms of only four real parameters, three of which can be chosen as real angles and the fourth as a phase. If this phase is not an integral multiple of  $\pi$ , then CP symmetry is violated. For this to be possible the number of quark

generations must be at least three. In the present context the unitarity of the CKM matrix requires that

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 \quad (32)$$

and the role of neutron  $\beta$ -decay centers on the determination of the largest matrix element  $V_{ud}$ .

## 2.2 Neutron Decay in the Standard Model.

In the Standard Model the weak interaction responsible for neutron decay is given as the contraction of a leptonic current  $J_\mu^l(x)$  and a hadronic current  $J_\mu^h(x)$ , where, in the convention that the operator  $(1-\gamma_5)/2$  projects out the left-handed field components,

$$J_\mu^l(x) = \bar{e}\gamma_\mu(1-\gamma_5)\nu_e; \quad J_\mu^h(x) = \bar{d}\cdot V_{ud}\gamma_\mu(1-\gamma_5)u \quad (33)$$

Since the leptons have no strong interactions the matrix element of the weak leptonic current is relatively simple, i.e.

$$\langle e^-\bar{\nu}_e|J_\mu^l(0)|0\rangle = \langle \bar{u}_e|\gamma_\mu(1-\gamma_5)|u_{\nu_e}\rangle \quad (34)$$

where  $u_e$  and  $u_{\nu_e}$  are Dirac spinors describing electron and neutrino respectively. However since the quarks are strongly interacting particles confined in nucleons the hadronic matrix elements are in principle limited only by the requirements of Lorentz invariance and maximal parity violation. Thus we find for the matrix element of the vector current

$$\langle p|J_\mu^{h,V}(0)|n\rangle = \langle \bar{v}_p|g_V(q)\gamma_\mu - i\frac{\hbar}{2m_p c}g_{WM}(q)\sigma_{\mu\nu}q_\nu + \frac{\hbar}{2m_p c}g_S(q)q_\mu|v_n\rangle \quad (35)$$

where  $v_n$  and  $v_p$  are neutron and proton spinors respectively,  $q_\mu$  is the 4-momentum transfer and  $g_i(i = V, WM, S)$  represent form factors corresponding to the bare vector, induced weak magnetism and induced scalar interactions respectively. As noted in section 1.2, for neutron decay all form factors may be evaluated at  $q = 0$ . Conservation of the vector current then requires that

$$g_V(0) = 1, \quad g_{WM}(0) = \kappa_p - \kappa_n = 3.70, \quad g_S(0) = 0. \quad (36)$$

where  $\kappa_p = 1.79$ , and  $\kappa_n = -1.91$ , are the *anomalous* magnetic moments of neutron and proton respectively, expressed in units of the nuclear magneton. Since weak magnetism is a term of recoil order it makes only a very small correction to the vector matrix element in neutron decay and is totally absent in pure Fermi decays. An alternative test of the conserved weak vector current theorem in action outside the regime of baryon decays is the pure Fermi  $0^- \Rightarrow 0^-$   $\beta$ -decay  $\pi^+ \Rightarrow \pi^0 + e^+ + \nu_e$ . The induced scalar interaction is also ruled out on the separate grounds that, having the wrong transformation properties under the G-parity transformation, it is second class and therefore does not contribute to  $\beta$ -decays within an isospin multiplet [6].

The corresponding axial matrix element is

$$\langle p|J_\mu^{h,A}(0)|n\rangle = \left\langle \bar{v}_p|g_A(q)\gamma_\mu\gamma_5 - i\frac{\hbar}{2m_p c}g_T(q)\sigma_{\mu\nu}q_\nu\gamma_5 + \frac{\hbar}{2m_p c}g_P(q)q_\mu\gamma_5|v_n\rangle \quad (37)$$

The axial current is not conserved which means that the form factor  $g_A(0)$  is nucleon structure dependent and has to be determined experimentally. Since the induced tensor form

factor  $g_T(0)$  is also ruled out as second class, and the operator  $q_\mu \gamma_5$  does not contribute to allowed decay between nuclear states of the same parity, it follows that the axial matrix element depends only on the single constant  $g_A(0)$  which, given that  $g_V(0) = 1$ , becomes identical with the empirical constant  $\lambda$  introduced in (23).

### 3 The Correlation Coefficients In Neutron Decay.

#### 3.1 Polarized Neutron Decay.

In a pure Fermi transition nuclear polarization is not possible and in a pure Gamow-Teller transition only the  $M=\pm 1$  lepton magnetic substates contribute to the correlation between the nuclear spin and the lepton momenta. This is the origin of the parity violation phenomenon first observed in the decay of  $^{60}\text{Co}$ . However, in a mixed transition such as in neutron decay, interference can arise between the singlet and triplet magnetic substates with  $M=0$ . As a consequence, depending on the sign of  $\lambda$ , either the electron or the antineutrino asymmetry will be enhanced as compared with pure Gamow-Teller decay, the other being reduced in proportion.

Experimental study of the angular and polarization coefficients which characterizes the decay of unpolarized and polarized neutrons offers an alternative route to the determination of  $\lambda$ . These involve carrying out measurements of the neutron spin polarization  $\sigma_n$ , and perhaps the electron polarization  $\sigma_e$ , together with some combination of the energies  $E_e, E_{\bar{\nu}}, E_p$  and momenta  $\mathbf{p}_e, \mathbf{p}_{\bar{\nu}}, \mathbf{p}_p$  of the three particles in the final state. The transition rate for a polarized neutron can then be written [7]:

$$dW(\sigma, \mathbf{p}_e, \mathbf{p}_{\bar{\nu}}) \propto F(E_e) d\Omega_e d\Omega_{\bar{\nu}} \left\{ 1 + a \frac{\mathbf{p}_e \cdot \mathbf{p}_{\bar{\nu}}}{E_e E_{\bar{\nu}}} + \frac{bm_e}{E_e} + \langle \sigma_n \rangle \left( A \frac{\mathbf{p}_e}{E_e} + B \frac{\mathbf{p}_{\bar{\nu}}}{E_{\bar{\nu}}} + D \frac{\mathbf{p}_e \times \mathbf{p}_{\bar{\nu}}}{E_e E_{\bar{\nu}}} + R \frac{\sigma_e \times \mathbf{p}_e}{E_e} + \dots \right) \right\} \quad (38)$$

where the neutron polarization  $\sigma_n$  has been averaged over all wavelengths and positions within the neutron beam and some less significant correlations have been omitted. The three correlation coefficients  $a, A$  and  $B$ , which have finite values within the Standard Model, are given in lowest order by the relations:

$$a = \frac{1 - |\lambda|^2}{1 + 3|\lambda|^2}, \quad A = -2 \frac{|\lambda|^2 + \text{Re}(\lambda)}{1 + 3|\lambda|^2}, \quad B = 2 \frac{|\lambda|^2 - \text{Re}(\lambda)}{1 + 3|\lambda|^2} \quad (39)$$

where the possibility has been left open that the coupling constant ratio  $\lambda$  might be complex signalling a break-down of time reversal invariance in the weak interaction. Each of these coefficients has to be corrected by inclusion of radiative corrections plus additional terms of recoil order including weak magnetism. However certain linear combinations of these coefficients exist which are independent of radiative corrections to lowest order in the fine structure constant  $\alpha$  omitting cross terms of order  $\alpha q$  or  $\alpha(E_e/m_p) \ln(m_p/E_e)$ . These relations are [8]:

$$f_1 = 1 + A - B - a = 0; \quad f_2 = aB - A^2 - -A = 0 \quad (40)$$

The possibility of a breakdown in T-invariance is tested in a measurement of the T-odd, P-even triple correlation coefficient D which is given by the expression

$$D = \frac{2\text{Im}(\lambda)}{1 + 3|\lambda|^2} \quad (41)$$

In order to establish a violation of T-invariance it is necessary to identify some feature of the decay which changes sign under reversal of the time but not under inversion of the coordinate system. The term  $\sigma_n \cdot (\mathbf{p}_e \times \mathbf{p}_{\bar{\nu}})$  possesses the desired property.

### 3.2 Non-Standard Model Contributions to the Correlation Coefficients

The leading coefficients  $a, A$  and  $B$  are each sensitive to right-handed contributions to the weak interaction irrespective of any possible contribution from scalar or tensor couplings. For example in left-right symmetric models the coefficient  $A$  takes the form[9]

$$A = -2 \frac{|\lambda|^2(1 + y^2) + \text{Re}(\lambda)(1 - xy) + T_1}{1 + x^2 + 3|\lambda|^2(1 + y^2) + T_2} \quad (42)$$

where  $T_1$  and  $T_2$  are small terms of recoil order,  $x \simeq \delta - \zeta$ ,  $y \simeq \delta + \zeta$ ,  $\delta$  is the square of the ratio of the mass of the light W-boson which couples to left-handed currents to the mass of the postulated heavy W-boson coupling to right handed currents and  $\zeta$  is the mixing angle. In these models  $D$  is linear in  $\zeta$  and is particularly sensitive to a T-violating coupling of a left-handed lepton to a right-handed quark.

When the possibility is allowed for contributions from scalar and tensor couplings then both the Fierz interference coefficient  $b$  and the T-violating coefficient  $R$  receive finite contributions. Specifically

$$b = b_F + b_{GT} \propto \text{Re}(G_V G_S^* + G'_V G_S'^*) - 3\text{Re}(G_A G_T^* + G'_A G_T'^*) \quad (43)$$

and

$$\begin{aligned} R = R_F + R_{GT} \propto & -\text{Im}(G'_A G_S^* + G_A G_S^*) + \\ & + \text{Im}(3\text{Re}(G_A G_T^* + G'_A G_T'^*) + G'_V G_T^* + G_V G_T'^*) \end{aligned} \quad (44)$$

where in this case it is necessary quite generally to distinguish between coupling constants which are P-conserving (e.g.  $G_V$ ) and P-non-conserving (e.g.  $G'_V$ ). The Fermi coefficients  $b_F$  and  $R_F$  are particularly sensitive to the scalar coupling of a right-handed lepton to any quark while the Gamow-Teller coefficients are sensitive to the tensor coupling of a right-handed lepton to a left-handed quark [10].

## 4 Measurement Of The Correlation Coefficients.

### 4.1 The Electron-Antineutrino Angular Correlation Coefficient $a$ .

Since the electron spectrum in allowed  $\beta$ -decay is determined by the Fermi phase space factor  $F(E_e)$  alone, it is insensitive to the details of the weak interaction. Thus, up to the discovery of parity violation, the correlation coefficient  $a$  was the only parameter available to provide such information. Also since the operator  $\mathbf{p}_e \cdot \mathbf{p}_{\bar{\nu}}$  commutes with the total angular momentum of the leptons, and therefore does not mix singlet and triplet operators, it follows that the correlation coefficient



$$a = \frac{1 - |\lambda|^2}{1 + 3|\lambda|^2}; \quad \frac{\delta|\lambda|}{|\lambda|} \simeq 0.27 \frac{\delta a}{a} \simeq 1\% \quad (45)$$

contains no Fermi/Gamow-Teller interference terms apart from small terms of recoil order.

It is, of course, impracticable to measure the correlation between the electron and antineutrino momenta directly, since efficient detectors of antineutrinos do not exist. In practice therefore only two indirect methods have been employed. These are (a) measuring the momentum spectrum of electrons emitted into a given range of angles referred to the proton momentum and (b) measuring the proton spectrum [11]. The experimenter is therefore presented with a choice between electron spectroscopy and proton spectroscopy and both methods have been explored...

It turns out that, up to the present, the measurement of the proton spectrum has proved the more fruitful and two studies of this nature have been completed. These have used (a) proton magnetic spectroscopy and (b) a Penning trap with adiabatic focusing. Both experiments have required the addition of post acceleration of the protons to energies of order 20-30 keV and have each reached precisions on  $a$  at the level of 5%. Because this correlation measures the anomaly in  $|\lambda|$  rather than  $|\lambda|$  itself the resultant error in  $|\lambda|$  is reduced to  $\simeq 1.4\%$ .

Angular correlation measurements have the great advantage that it is not necessary that the neutrons be polarized and this route to the determination of  $|\lambda|$  has yet to achieve its true potential.

#### 4.2 The Electron-Neutron Spin Asymmetry Coefficient $A$ .

The correlation coefficient

$$A = -2 \frac{|\lambda|^2 + \text{Re}(\lambda)}{1 + 3|\lambda|^2}; \quad \frac{\delta\lambda}{\lambda} \simeq 0.24 \frac{\delta A}{A} \simeq 0.23\% \quad (46)$$

has been subjected to an enormous amount of experimental study going back to the 1950's. It has provided the most precise value for the parameter  $\lambda$  both in magnitude and sign, and therefore for the CKM matrix element  $V_{ud}$  based on neutron decay alone [12]. This information has been largely derived from studies over the past  $\simeq 15$  years at the ILL, Grenoble using the electron spectrometer PERKEO in its various forms. The current world average value for  $\lambda$  is [1]:

$$\lambda = -1.2670 \pm 0.0030 \quad (47)$$

Like the  $a$ -coefficient, the  $A$ -coefficient has the great advantage that it measures the anomaly in  $\lambda$ . However it relies critically on  $\simeq 1$  MeV electron spectroscopy, and, although this is in general easier to perform than  $\simeq 1$  keV proton spectroscopy, it has not proved possible to extend the electron spectrum down to the lowest energies. However the measurement of  $A$  suffers from the great disadvantage that the neutrons must be polarized and the neutron polarization must be measured to an accuracy  $\geq 99\%$  and this is not easy. Fortunately discrepancies between the values of the polarization derived using polarizer/analyser combinations based on supermirrors and  ${}^3\text{He}$  filters appear to have been satisfactorily resolved.

#### 4.3 The Antineutrino-Neutron Spin Asymmetry Coefficient $B$

The correlation

$$B = 2 \frac{|\lambda|^2 - \text{Re}(\lambda)}{1 + 3|\lambda|^2}; \quad \frac{\delta\lambda}{\lambda} \simeq 2.0 \frac{\delta B}{B} \quad (48)$$

is quite insensitive to the value of  $\lambda$ . Its measurement has the disadvantages that it requires both that the neutrons be polarized and that proton spectroscopy be performed. For both these reasons it has tended to be neglected as a topic for study. However, for the same reason that it is insensitive to the precise value of  $\lambda$ , it is very sensitive to contributions from right-handed bosons and recent measurements have succeeded in setting a limit  $m_{BR} > 284.3 \text{ GeV}/c^2$  for the mass of the heavy W-boson which is postulated to couple to right-handed currents [13].

A recent encouraging development has been the simultaneous measurement of  $A$  and  $B$  whose ratio is therefore independent of neutron polarization [14].

#### 4.4 The Triple Correlation Coefficient $D$ .

This coefficient

$$D = \frac{2\text{Im}(\lambda)}{1 + 3|\lambda|^2} \quad (49)$$

is measured by counting coincidences between electrons and protons detected in counters set at appropriately selected angles for a given sign of the neutron spin. The spin is then reversed and the relevant counting rate asymmetry is recorded.

The  $D$ -coefficient is of second order in the  $T$ -violating phase in the CKM matrix and is expected to be vanishingly small. Currently it is known to vanish at a level of about 0.1% from neutron decay, and to marginally better precision from the decay of  $^{19}\text{Ne}$ . However, since the  $T$ -symmetry is non-unitary and is generated by a non-linear operator, a violation can be mimicked by final state electromagnetic interactions which in this instance appear at a level of about 0.001%.

## 5 Additional Experimental Possibilities.

### 5.1 The Proton-Neutron Spin Asymmetry Coefficient $\alpha$ .

The individual coefficients  $A$  and  $B$  each have terms in  $|\lambda|^2$  deriving from the axial vector interaction, in addition to terms in  $\text{Re}(\lambda)$  generated through polar vector/axial vector interference. Suppose, instead, one were to measure the correlation  $\alpha \sigma_n \cdot \mathbf{p}_p$  by detecting the complete range of proton energies but without recording electron coincidences. Then, since this is a parity-violating term and no lepton is detected, it satisfies the conditions of Weinberg's interference theorem [15], and is therefore proportional to  $\text{Re}(\lambda)$  with no term in  $|\lambda|^2$ . The corresponding expression for the coefficient  $\alpha$  is given by [16]:

$$\alpha = C \frac{4\lambda}{1 + 3|\lambda|^2}, \quad C = 0.27484, \quad \frac{\delta\lambda}{\lambda} \simeq 1.5 \frac{\delta\alpha}{\alpha} \quad (50)$$

where the kinematic constant  $C$  comes from the double integral over electron and proton energies, and includes Coulomb, recoil order and radiative corrections. Since in lowest order the correlation  $\alpha$  is proportional to  $(A+B)$  it is also relatively insensitive to the value of  $\lambda$ .

The principle of an experiment is quite straightforward. Recoil protons from the decay of longitudinally polarized neutrons are collected in a magnetic field of order 5T, where the

maximum radius of the cyclotron orbit is  $\prec 1mm$ . If  $N^+(N^-)$  denote the numbers of protons with momenta parallel (anti-parallel) to the neutron spin, then  $N^\pm = N_0\{1 \pm \alpha\langle\sigma_n\rangle/2\}$  and the appropriate counting rate asymmetry can be computed.

To measure  $N^\pm$ , set the orientation of the neutron spin parallel to the magnetic field and reflect the protons from a  $\simeq 1kV$  electrostatic potential barrier so that protons of both senses of momentum enter the detector which is maintained at about  $-30kV$ . Thus the counting rate is given by

$$C_1 = N^+ + N^- + b \quad (51)$$

where  $b$  is the background. When the reflecting potential barrier is removed the new counting rate is

$$C_2 = N^+ + \beta N^- + b \quad (52)$$

where  $\beta \ll 1$  represents that fraction of protons initially moving away from the detector which is reflected back into the detector by magnetic mirror action. The procedure is now repeated with the neutron spin direction reversed giving corresponding counting rates  $C'_1, C'_2$  and background  $b'$ . The counting rate asymmetry is then given by

$$\frac{(C'_1 - C'_2) - (C_1 - C_2)}{(C'_1 - C'_2) + (C_1 - C_2)} = \alpha\langle\sigma_n\rangle \quad (53)$$

The experiment only works on the assumption that the proton counter background in the energy range  $\leq 30keV$  is weak in comparison to the signal strength which is certainly not true in the case that the neutrons are polarized using a supermirror.

## 5.2 Two-Body Decay of the Neutron and Right-Handed Currents

When a neutron undergoes  $\beta$ -decay there is a small branching ratio  $\simeq 4.10^{-6}$  that the final state should contain an antineutrino and a hydrogen atom i.e.

$$n \Rightarrow H + \bar{\nu}_e, \quad (54)$$

where the hydrogen atom is created in an S-state. Since this is a two-body decay, momentum conservation ensures that antineutrino and hydrogen atom each carry off unique energies with

$$\mathbf{p}_{\bar{\nu}} + \mathbf{p}_H = 0, \quad E_{\bar{\nu}} = 783keV, \quad T_H = 352eV \quad (55)$$

Although the higher S-levels decay spontaneously, hydrogen atoms created in the metastable 2S state can exist in one of four decoupled hyperfine levels  $|M_e, M_p\rangle$  with populations  $W_i (i = 1 - 4)$ , where  $\mathbf{n}_H = \mathbf{p}_H/p_H$  and

$$\left| \frac{1}{2}, \frac{1}{2} \right\rangle; \quad W_1 = 2(1 + |\lambda|^2)\{1 + \sigma_n \cdot \mathbf{n}_H\} \simeq 0.57\% \quad \text{when } \sigma_n \cdot \mathbf{n}_H = 0 \quad (56)$$

$$\left| -\frac{1}{2}, \frac{1}{2} \right\rangle; \quad W_2 = 8|\lambda|^2\{1 - \sigma_n \cdot \mathbf{n}_H\} \simeq 55.13\% \quad \text{when } \sigma_n \cdot \mathbf{n}_H = 0 \quad (57)$$

$$\left| \frac{1}{2}, -\frac{1}{2} \right\rangle; \quad W_3 = 2(1 - |\lambda|^2)\{1 - \sigma_n \cdot \mathbf{n}_H\} \simeq 44.28\% \quad \text{when } \sigma_n \cdot \mathbf{n}_H = 0 \quad (58)$$

$$\left| -\frac{1}{2}, -\frac{1}{2} \right\rangle; \quad W_4 = 2(1 + |\lambda|^2)\{1 + \sigma_n \cdot \mathbf{n}_H\} \equiv 0 \quad (59)$$

The population  $W_4$  vanishes identically in the case that the weak interaction is purely left-handed, and this is a result which depends on conservation of angular momentum only. Thus exploiting the neutron polarization to suppress the populations  $W_2$  and  $W_3$ , observation of a finite population  $W_4 \neq 0$  would provide an unambiguous signature for the existence of right-handed currents [17].

### 5.3 Radiative Neutron Decay.

Radiative decay of the free neutron

$$n \Rightarrow p + e^- + \bar{\nu}_e + \gamma, \quad (60)$$

also described as inner bremsstrahlung, has a branching ratio at the level of 0.1%. The matrix element for the process consists of two terms; a term describing electron photon emission and a term describing proton photon emission. Both terms contain infra-red divergences which cancel. However because  $|\lambda| \neq 1$ , contrary to the situation in the case of the muon which has no strong interactions, the total radiative correction depends on the ultra-violet cut-off parameter  $\Lambda$ . Thus the simplest experiments designed to detect the inner bremsstrahlung provide a measure of the outer radiative correction only.

Experiments designed to measure the branching ratio for radiative neutron decay by detecting triple coincidences between electron, proton and gamma are currently under way at the ILL Grenoble [18].

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