Signal Generation in Detectors

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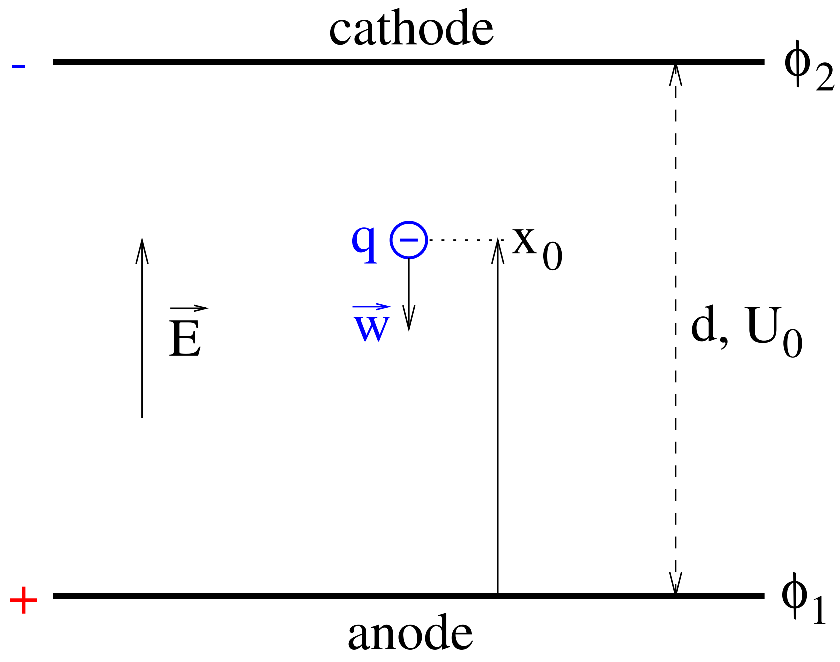
A short tutorial on how signals arise in detectors that collect (and maybe amplify) charges.

- System of electrodes + medium in which charges drift/multiply
- medium: gas, liquid or solid (semiconductors); 'insulator', i.e. no free charges, but charges liberated by primary event can drift in field.
- electrode potentials → desired field and drift geometry
- signals are due to induced currents/charges on the electrodes
- all electrodes carry signal, but not all signals are useful

Content:

1. The Simplest Case:
the Parallel Plate Condensor



Regard one free charge q (electron or ion)

Energy balance: work done against field = energy change of condensor
(both negative, energy is gained by charge, condensor charge decreases)

$$\boxed{q\vec{\nabla}\phi \cdot \mathrm{d}\vec{x} = \mathrm{d}q_i U_0}$$

$\dfrac{\partial}{\partial t}$:
$$i_{\mathrm{ind}} = \frac{q}{U_0}\vec{\nabla}\phi \cdot \vec{w}$$

charge : $\Delta q = \dfrac{q}{U_0}\displaystyle\int_{x_0}^{0}\vec{\nabla}\phi \cdot \mathrm{d}\vec{x} = -\dfrac{q}{U_0}\left(\phi(x_0) - \phi(0)\right) = -\dfrac{q}{U_0}\Delta\phi$

Remember: $\vec{E} = -\vec{\nabla}\phi, \quad U_0 = \phi_1 - \phi_2$

- current constant as long as q drifts, "recharging of capacity"
- final induced charge Δq depends on x_0

proportional to traversed potential $\Delta\phi$

Signals due to an electron/ion pair

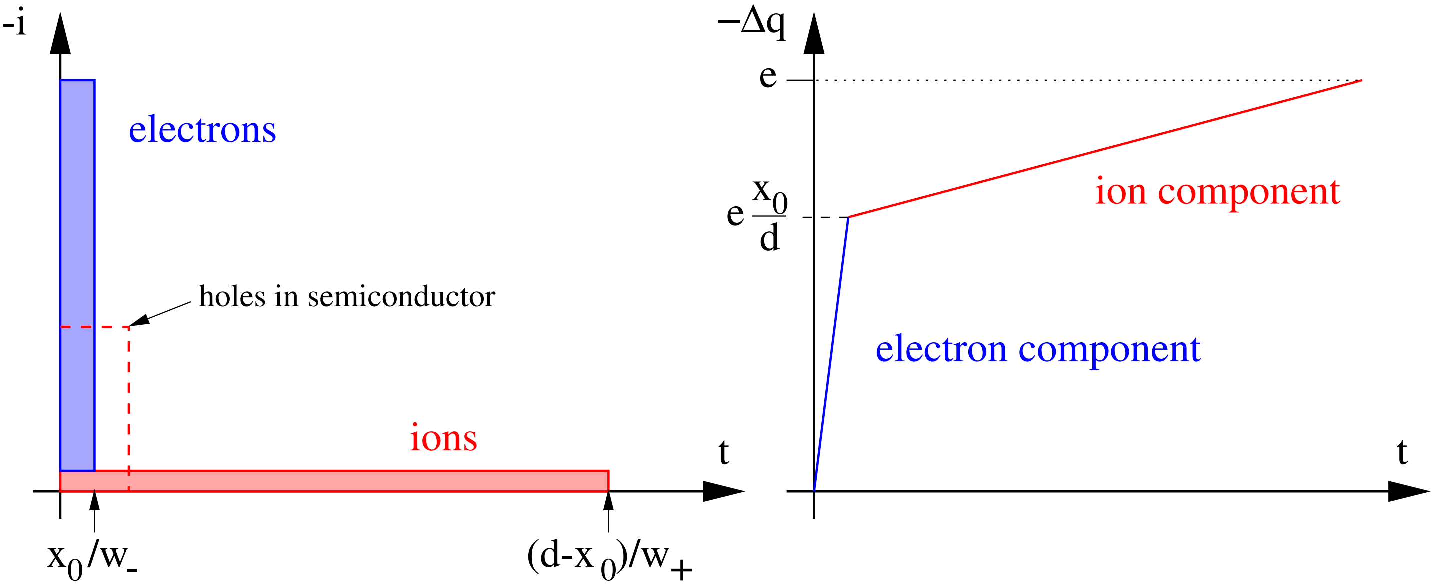
$\pm e$ start to drift from x_0

$$\begin{aligned} \Delta q_+ &= -\frac{e}{U_0}\left(\phi(x_0) - \phi(d)\right) \\ \Delta q_- &= +\frac{e}{U_0}\left(\phi(x_0) - \phi(0)\right) \end{aligned}$$

$\sum$: $\Delta q = e$

Practical problem: ion drift comparatively slow

$w_+ = 10^{-3} \ldots 10^{-2}\, w_-$ (except for semiconductors: typ. $w_+ \approx 0.5\, w_-$)



Induced current and charge for parallel plate case, ratio w_-/w_+ decreased for clarity

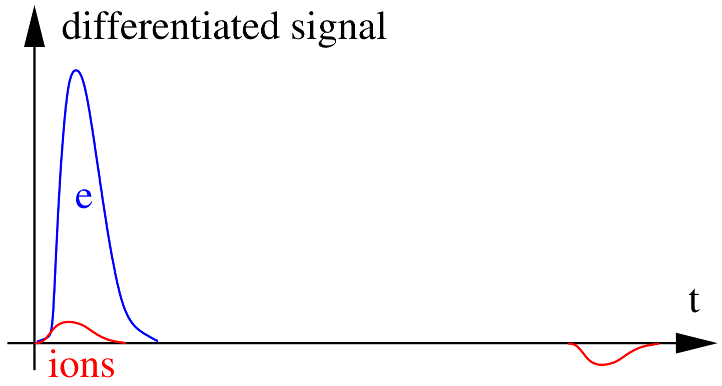
- signal during drift of charges
- induced current ends when charges arrive
- induced charge reaches constant value when charges arrive

Shaping of signals

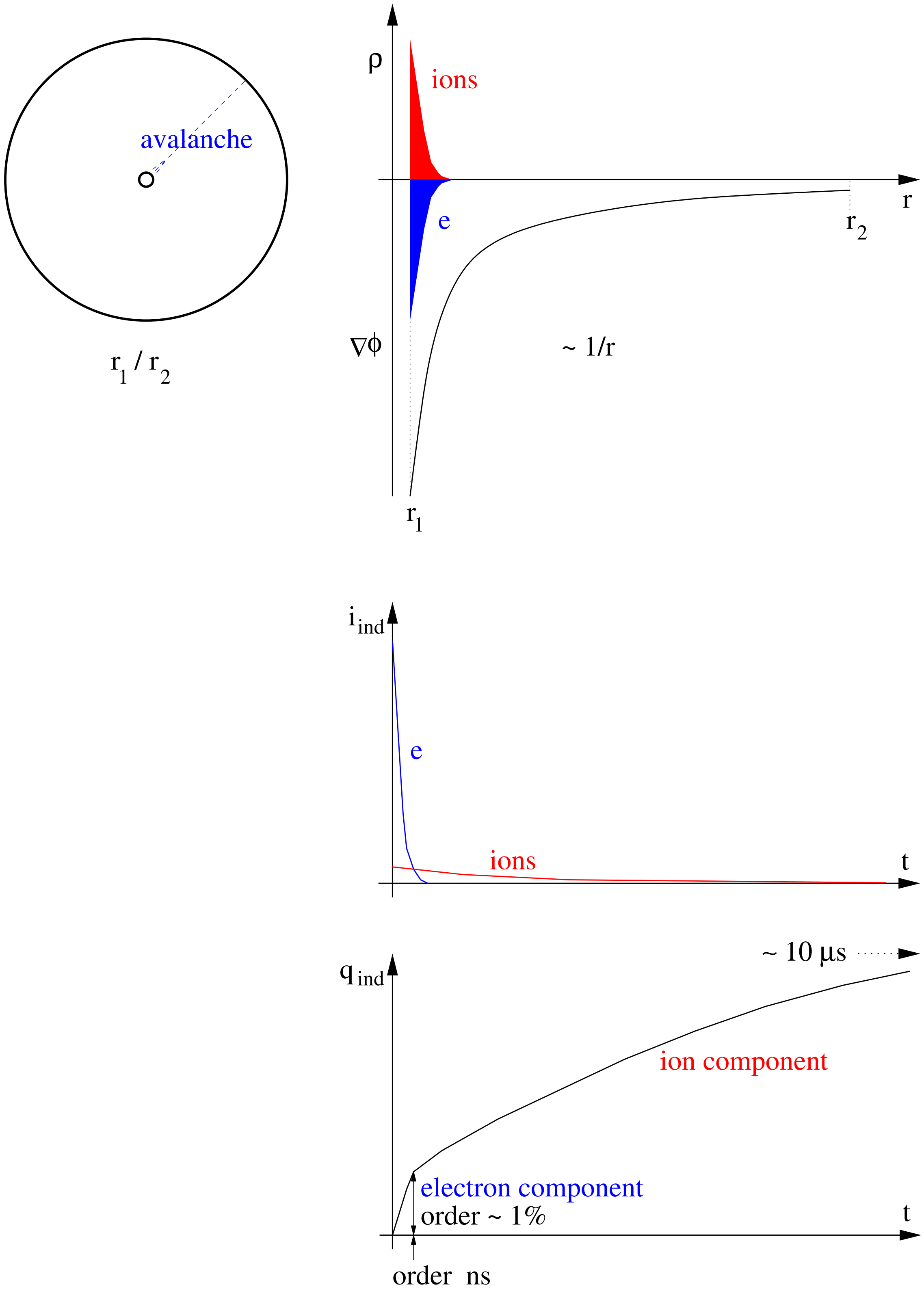
Differentiation of signal (for sake of speed)

$\Rightarrow$ slow ion component suppressed

However, this leads to undershoots at the time of ion arrival.



2. Wire Proportional Counter, qualitatively



In case of shaping the ion contribution may vary widely depending on integration time.

3. Many-Electrode Systems

- $\vec{w}$ depends on all potentials $\phi_{1...n}$
- induced currents depend on $\vec{w}$ and location of charge, but not directly on the ϕ_i
- energy balance argument not easily applicable

Formalism for general case

electrode #i, one drifting charge q at x_0

(for derivation see Appendix A)

$$\boxed{\begin{aligned} i^{\mathrm{i}}_{\mathrm{ind}} &= q\vec{\nabla}\Phi^{\mathrm{i}}_{\mathrm{ind}} \cdot \vec{w} \\ \Delta q^{\mathrm{i}}_{\mathrm{ind}} &= q\,\Phi^{\mathrm{i}}_{\mathrm{ind}}(x_0) \end{aligned}}$$

Induction Potential:

$$\boxed{\Phi^{\mathrm{i}}_{\mathrm{ind}}(x) = \frac{\phi(x)}{\phi_i}}$$

for case electrode i at potential ϕ_i, *all* other electrodes grounded

- dimensionless, $0 \leq \Phi^{\mathrm{i}}_{\mathrm{ind}}(x) \leq 1$
- $\Phi^{\mathrm{i}}_{\mathrm{ind}} = 1$ on electrode i, $\Phi^{\mathrm{i}}_{\mathrm{ind}} = 0$ on all other electrodes

Consequences:

- If one partner of an ion pair is collected by electrode i, the other on some other electrode then the full charge q is induced on electrode i.
- If both partners are collected on other electrodes then the final induced charge on electrode i is strictly 0, i.e. the induced current integrates to 0.
- Not true if signal is shaped!

Initial charge distributions and avalanches

The signal due to charge distributions has to be obtained by integration over the time-dependent distributions of electrons and ions:

$$i^{\mathrm{i}}_{\mathrm{ind}}(t) = \int \rho_-(x,t)\vec{\nabla}\Phi^{\mathrm{i}}_{\mathrm{ind}}(x)\cdot\vec{w}_-(x)\mathrm{d}^3x + \int \rho_+(x,t)\vec{\nabla}\Phi^{\mathrm{i}}_{\mathrm{ind}}(x)\cdot\vec{w}_+(x)\mathrm{d}^3x$$

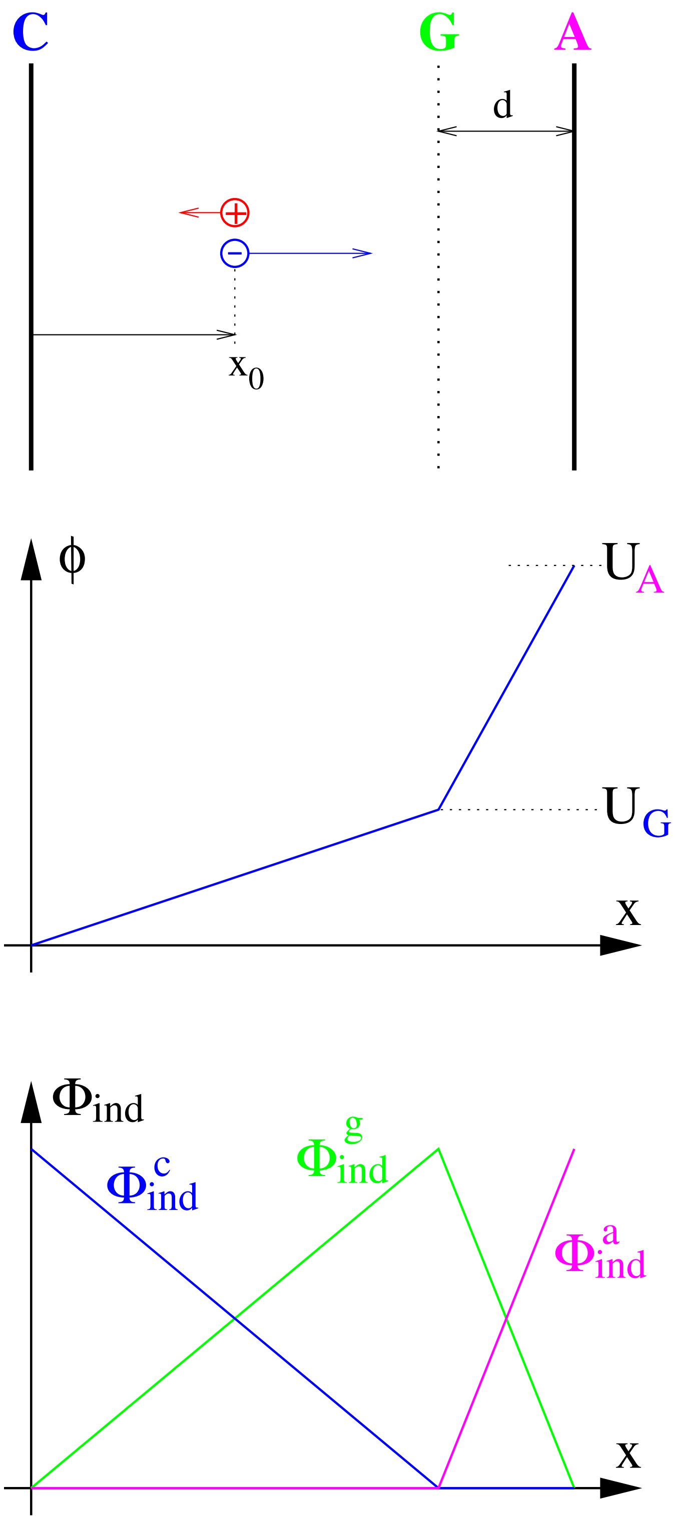
$$\Delta q^{\mathrm{i}}_{\mathrm{ind}}(t) = \int_{t_0}^{t} i^{\mathrm{i}}_{\mathrm{ind}}(t)\mathrm{d}t$$

Realistic shaped output signals are obtained by folding the charge or current signal with the corresponding response function of the electronics chain.

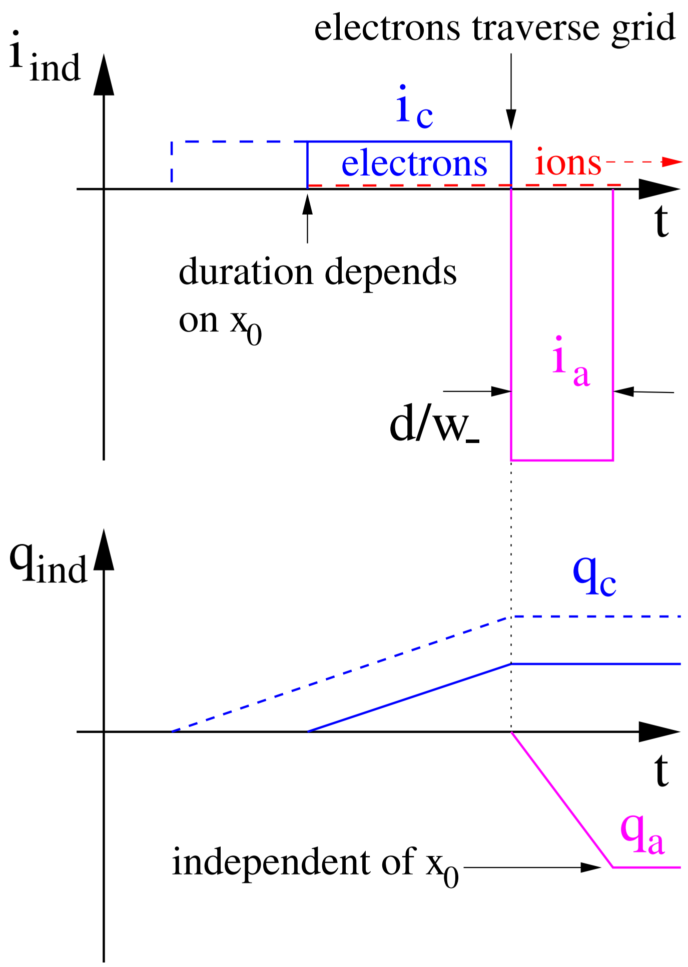
3.1 Example: Frisch Grid

Used in ionization chambers to make anode signal amplitude independent of location of primary ionization.

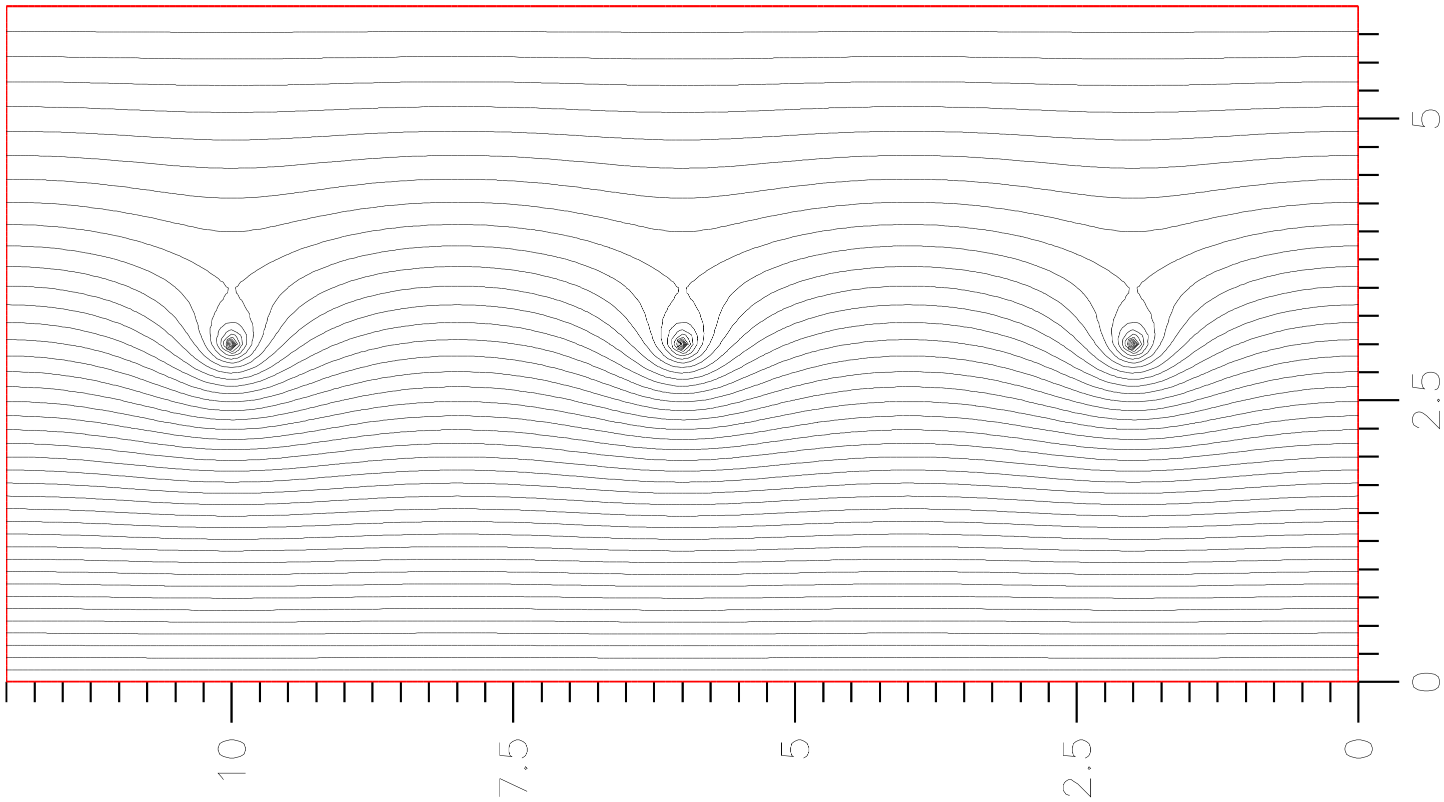
Geometry and potentials



Induced signals



MWPC's, cathode readout

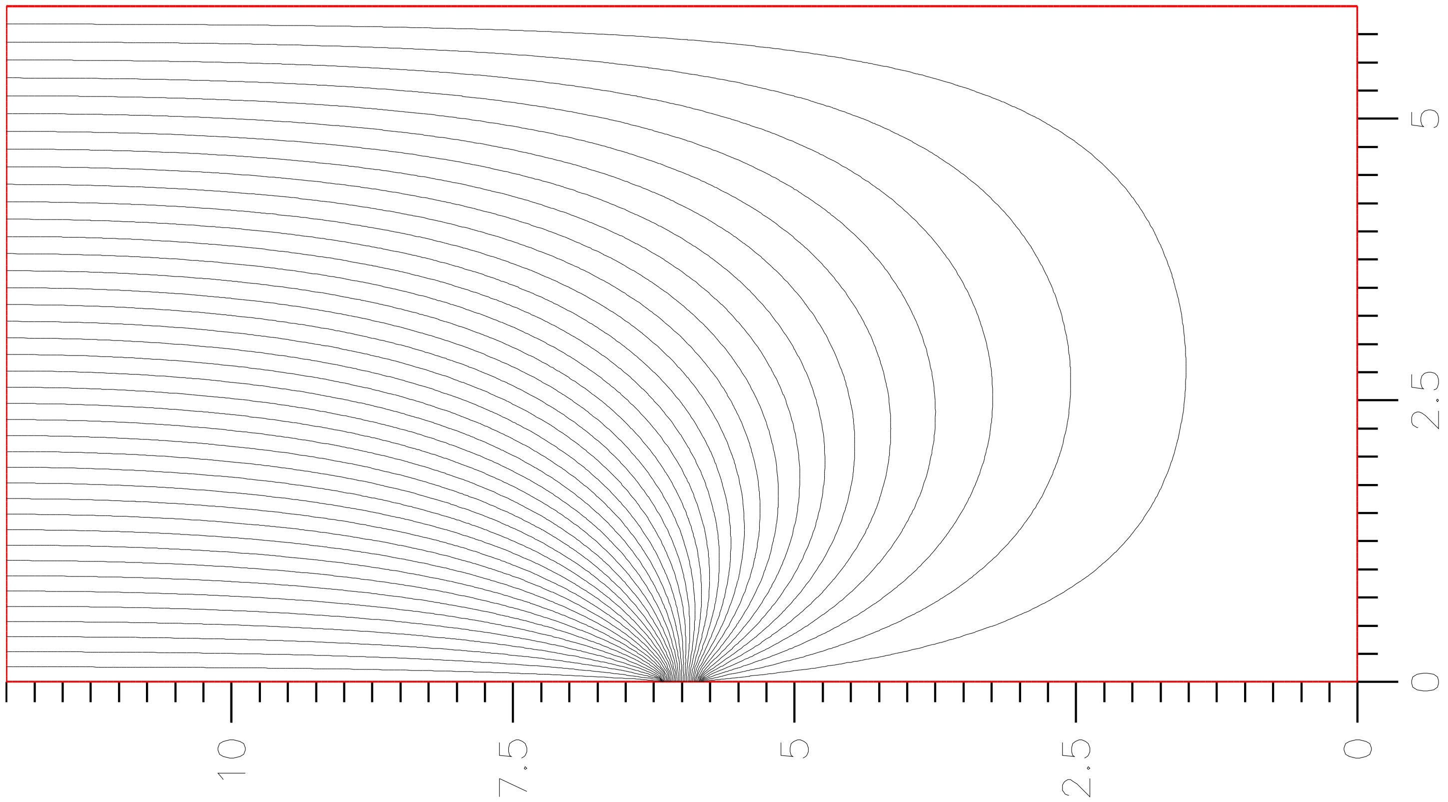


Induction potential of the lower cathode (equipotentials with equal potential spacing). There are saddle points above the anodes.

Note that the potential near the anode wires is nearly cylinder-symmetric, i.e. the induced signals from an avalanche close to the wire with short shaping time would not depend on the location of the avalanche around the wire. For avalanches on the side opposite to the signal cathode, the induced current will change sign when the positive ions cross the saddle point. The integrated induced charge will return to zero if the integration time is long enough to accommodate the ion collection.

The induction potential looks the same if every other wire is a grounded potential wire! The drift field would be different, however.

Anode pad readout



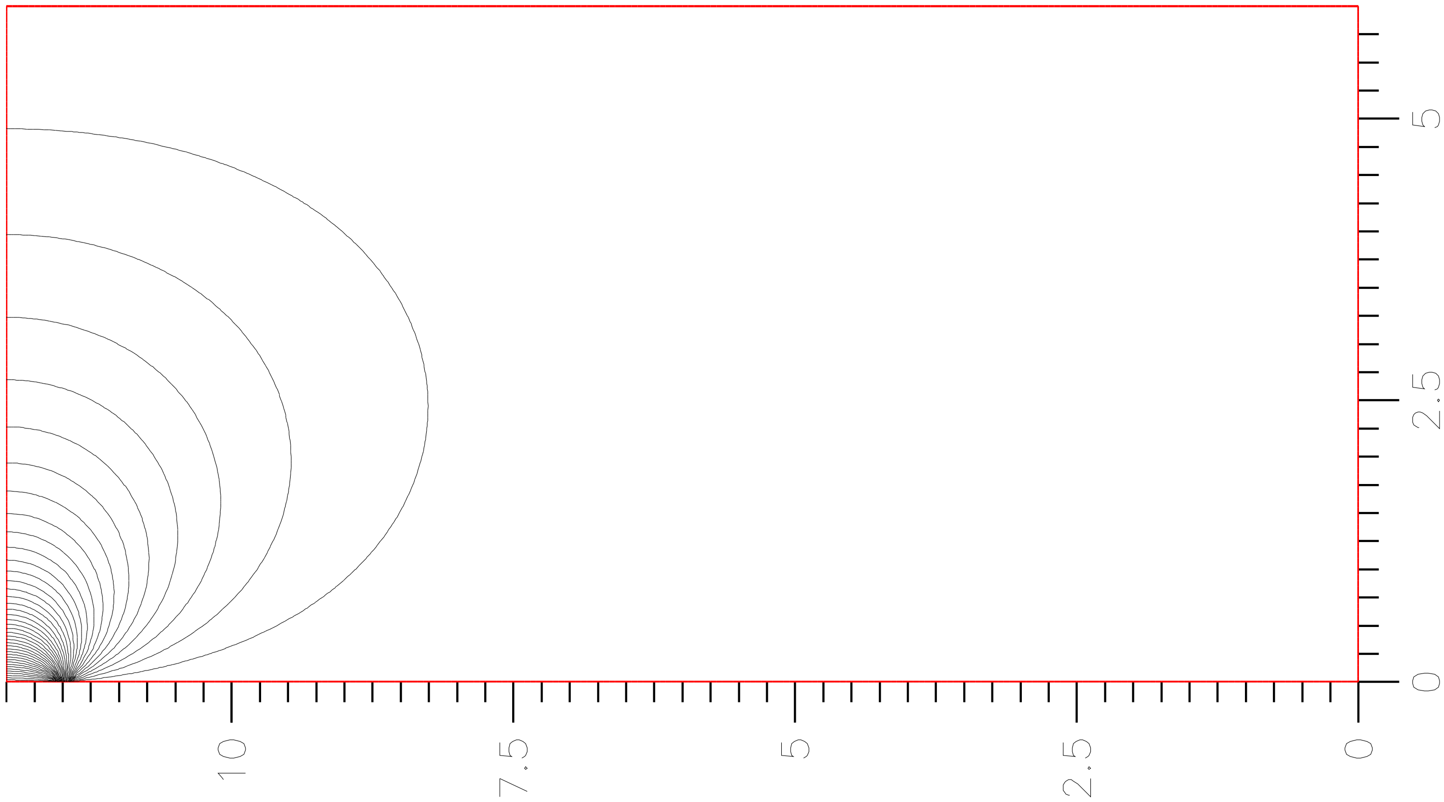
Induction potential of an anode pad (lower left half) comparable to the gap size. The spacing between the pads is 0.4 mm, the gap is 6 mm. The opposite electrode is an idealized grid, it could be a Frisch grid, anode of a PPAC, or a GEM-like structure. Drift would be always nearly perpendicular to the pad surface (vertical in the figure).

For electrons collected well inside the pad, a constant induced current is seen with a duration equal to the total drift through the gap.

Near the edge of the pad, the current would start lower and rise till the electron reaches the pad.

Beyond the edge of the pad, the induced current would change sign when the electron reaches the turn-over of the induction potential lines, with zero integrated charge when the electron has reached the neighbor pad.

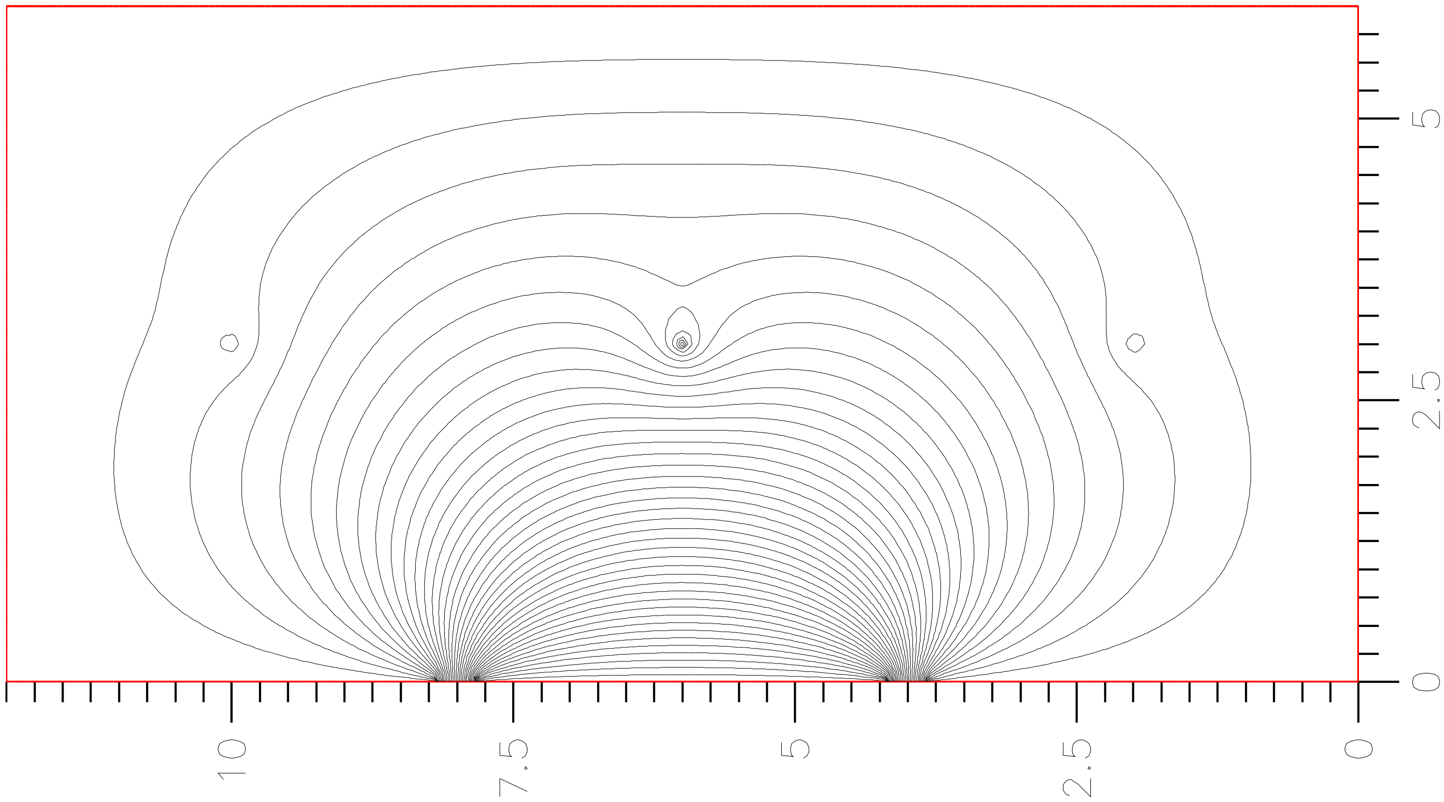
Anode pad readout, narrow pads



Here the pads have a pitch of 1 mm, 0.2 mm inter-pad spacing, electrode gap 6 mm. The left edge is a symmetry line, i.e. the center of the pad.

The main difference to the large pad is that the induced current is never constant during drift, but rises quite steeply towards the end of drift.

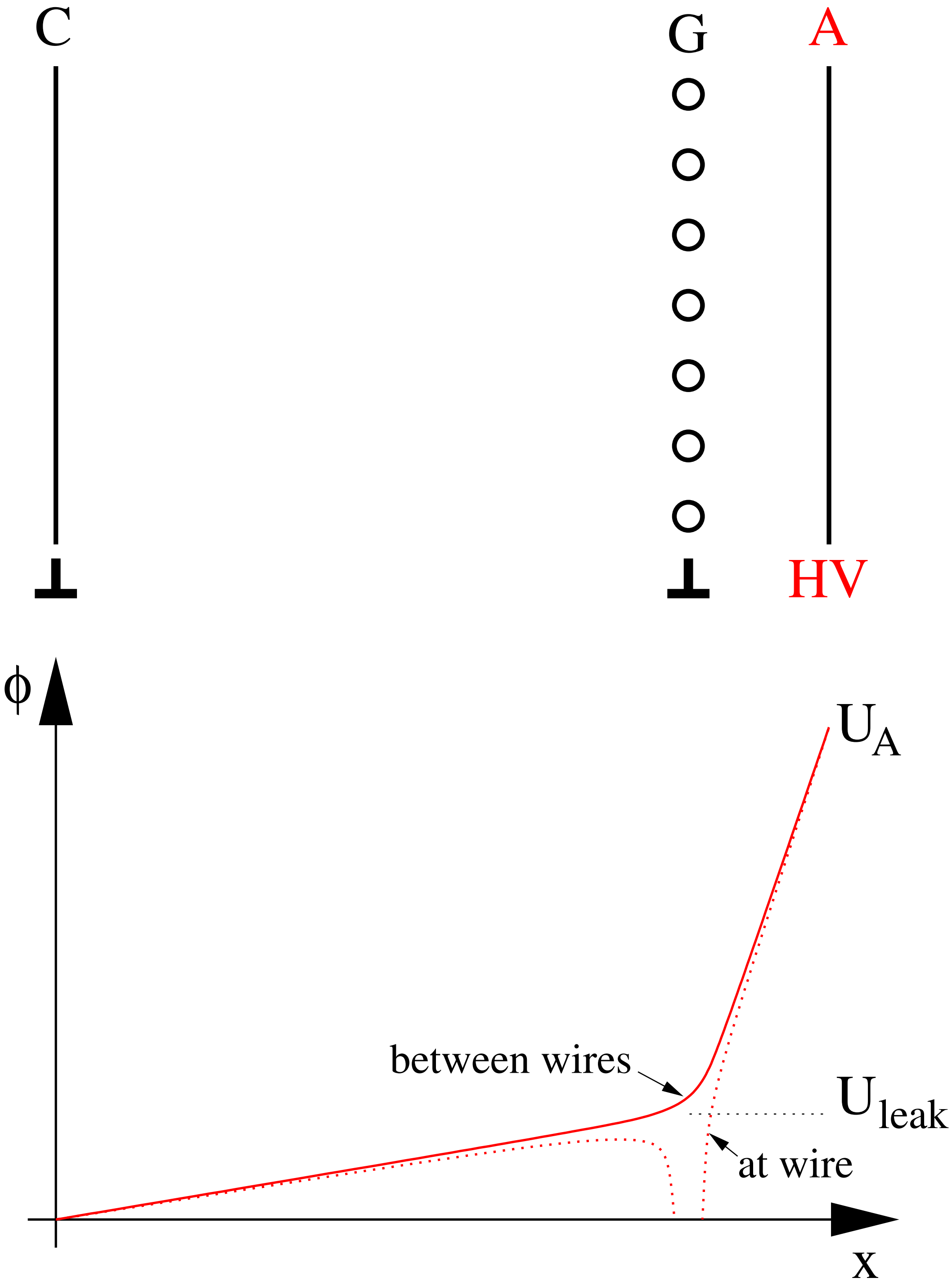
MWPC with pad readout



Induction potential for MWPC with pads with the same spacing as the wires. The more realistic case would be a pad division orthogonal to the wire spacing, but that is a 3D problem not treated here. Qualitatively, the results would be similar, however.

- Note again the symmetry of the induction field near the wires.
- The induction field is much weaker around the next wire, causing the fall-off of the signal as the avalanche position moves away from the pad center.

Appendix A:
Imperfect Screening of Real Grids



Potential at a real grid

A fraction σ of the potential U_0 applied to electrode A is visible through the grounded grid ('Durchgriff', screening factor $1/\sigma$), conformal mapping gives the result:

$$\begin{aligned} U_{\text{leak}} &= \sigma U_0 \\ \sigma &= \frac{l}{d+l} \qquad \text{with} \quad l = \frac{a}{2\pi}\left(\frac{(piR)^2}{a^2} + \ln\frac{a}{2\pi R}\right) \\ \sigma &\approx \frac{a}{2\pi d}\ln\frac{a}{2\pi R} \qquad \text{for } R \ll a \end{aligned}$$

This formulation can be rewritten in a way that makes it applicable to the case that electrode A is also a grid or mesh (making the field inhomogenous near electrode A):

$$\begin{aligned} U_{\text{leak}} &= \sigma d \frac{U_0}{d} = d_0 E \\ d_0 &= \frac{a}{2\pi}\ln\frac{a}{2\pi R} \end{aligned}$$

where E is the homogeneous electric field on the righthand side of the shielding grid.

Example: $\left.\begin{aligned} a &= 1 \text{ mm} \\ R &= 10\ \mu\text{m} \\ E &= 1 \text{ kV/cm} \end{aligned}\right\} \Rightarrow \begin{aligned} d_0 &= 0.44 \text{ mm} \\ U_{\text{leak}} &= 44 \text{ V} \end{aligned}$

Appendix B: Signal Induction in Multi-Electrode Systems

For an electron moving in a two-conductor system, i. e., a condenser, one can apply a simple energy conservation argument to derive the magnitude and variation of the induced charge. Moving a point charge q by $d\vec{x}$ corresponds to a change in potential energy $dE_{\mathrm{pot}} = q\vec{\nabla}\Phi \cdot d\vec{x}$, where $\Phi(\vec{x})$ is the potential. This change must correspond to a change in the energy stored in the condenser field $dE = d(\frac{1}{2}Q^2/C) = dQ \cdot U_0$ with $U_0 = \Phi_1 - \Phi_2$. The change of the condenser charge then corresponds to

$$dQ = q\frac{1}{U_0}\vec{\nabla}\Phi(x) \cdot d\vec{x} \tag{1}$$

(dQ can either be balanced by an external power supply maintaining the potential difference U_0, or it leads to a voltage change $dU = dQ/C$ in the absence of an external supply.) We see that dQ is independent of the actual potential U_0 applied. If the point charge is transported from one electrode to the other we get for the integrated induced charge

$$Q = \int dQ = \frac{q}{U_0}\int_0^U \vec{\nabla}\Phi(\vec{x})d\vec{x} = q \tag{2}$$

i. e., the induced charge is equal to the amount of charge moving from one electrode to the other. If, on the other hand, a close pair of *opposite and equal* charges starts to move from the point $\vec{x}_0$ to the opposite electrodes, we get

$$Q = \int dQ = \frac{q_\oplus}{U_0}\int_{\Phi(\vec{x}_0)}^{\Phi_2} \vec{\nabla}\Phi(\vec{x})d\vec{x} + \frac{q_\ominus}{U_0}\int_{\Phi(\vec{x}_0)}^{\Phi_1} \vec{\nabla}\Phi(\vec{x})d\vec{x} = \pm q \tag{3}$$

i. e., if the pair has reached the opposite electrodes the total induced charge equals the charge of one of the carriers $q = |q_\oplus| = |q_\ominus|$. A change of the condenser charge by q corresponds to equal but opposite changes of the charges on the two electrodes, the induced charge beeing opposite to the charge collected.

However, in gas counters the mobilities of the two carrier types, ions and electrons, differ by about three orders of magnitude, and usually the integration of induced currents $i(t) = dQ/dt$ is only performed for the faster electrons. If the movement of the ion can be neglected within the collection time of the electron we get for the induced charge due to electron movement

$$Q = \frac{q_\ominus}{U_0}\int_{\Phi_1}^{\Phi(\vec{x}_0)} \vec{\nabla}\Phi(\vec{x})d\vec{x} = q\frac{(\Phi(x_0) - \Phi_1)}{U_0} \tag{4}$$

which depends on the starting position $\vec{x}_0$, but not on the static potentials applied to the electrodes.

The extension of the energy conservation argument to a multi-conductor system (> 2 electrodes) fails, since the resulting shape of the field lines depends on the relative voltages on the different conductors. When the path of the point charge is prescribed, however, the induced charge on each conductor does not depend on their static voltages. This is due to the linear superposition character of the electric field. The amount of induced charge, which can be visualized as due to the number of field lines starting at the point source and ending on the conductor in consideration giving rise to surface image charges, is not affected by arbitary changes of the static potentials. It is thus clear that the problem can be split in two parts: (i) we choose a conductor geometry and potentials on these conductors in order to achieve a desired shape of the field lines along which electrons will drift, (ii) the induced charge on a selected conductor is obtained by integrating the image charges due to the moving point charge. In the following we will derive a general, but nevertheless practical formulation of the induced charge problem.

The potential $\Phi(x)$ of a system of charges in a volume V with boundary conditions on a surface S is given by Green's second law [see e.g. Jackson, Classical Electrodynamics]

$$\Phi(\vec{x}) = \int_V \rho(\vec{x})G(\vec{x},\vec{x}')d^3x' + \frac{1}{4\pi}\oint_S \left\{ G(\vec{x},\vec{x}')\frac{\partial\Phi}{\partial n'} - \Phi(\vec{x}')\frac{\partial G(\vec{x},\vec{x}')}{\partial n'} \right\} da' \tag{5}$$

where the Green's function $G(\vec{x},\vec{x}')$ is only a function of the *geometry* of the problem. For a system of separate conductors $S_1,\ldots,S_n$ set to potentials $\Phi_1,\ldots,\Phi_n$, i.e., Dirichlet boundary conditions, we can rewrite eq. 5 due to the vanishing of $G(\vec{x},\vec{x}')$ on the surfaces:

$$\Phi(\vec{x}) = \int_V \rho(\vec{x})G(\vec{x},\vec{x}')d^3x' - \frac{1}{4\pi}\sum_{i=1}^{n}\Phi_i \oint_{S_i} \frac{\partial G(\vec{x},\vec{x}')}{\partial n'}da' \tag{6}$$

For a point charge $\rho(\vec{x}) = q\delta(\vec{x}-\vec{x}_0)$ eq. 6 further simplifies:

$$\Phi(\vec{x}) = q\cdot G(\vec{x},\vec{x}_0) - \frac{1}{4\pi}\sum_{i=1}^{n}\Phi_i \oint_{S_i} \frac{\partial G(\vec{x},\vec{x}')}{\partial n'}da' \tag{7}$$

The surface charge on conductor S_i as a function of the position of the point charge $\vec{x}_0$ is given by the integral of the normal component of the electric field $E_n = \partial\Phi/\partial n$:

$$Q^{(i)}_{\text{ind}}(\vec{x}_0) = -\frac{1}{4\pi}\oint_{S_i} \frac{\partial\Phi(\vec{x})}{\partial n}da \tag{8}$$

If the general expression for $\Phi(\vec{x})$ of eq. 7 is inserted, we obtain:

$$Q^{(i)}_{\text{ind}}(\vec{x}_0) = -\frac{q}{4\pi}\oint_{S_i} \frac{\partial G(\vec{x},\vec{x}_0)}{\partial n}ds + \frac{1}{(4\pi)^2}\sum_{j=1}^{n}\Phi_i \oint_{S_j} \frac{\partial^2 G(\vec{x},\vec{x}')}{\partial n\partial n'}dada' \tag{9}$$

The first term is proportional to the magnitude of the point charge and depends on its position $\vec{x}_0$, whereas the second term depends only on the geometry by the normal derivative of the Green's function and on the static potentials, but not on the charges $\rho(\vec{x})$. On the other hand, if all static potentials are zero, only the first term survives which represents the image charge *due to the point charge.* Since we are only interested in this part of the induced charge, we write:

$$Q^{(i)}_{\text{ind}}(\vec{x}_0) = -\frac{q}{4\pi}\oint_{S_i} \frac{\partial G(\vec{x},\vec{x}_0)}{\partial n}ds \tag{10}$$

If we now compare the left side of eq. 10 with eqs. 6, 7, we see that the induced charge $Q^{(i)}_{\text{ind}}(\vec{x}_0)$ due to a point charge is proportional to the potential for the case of no charges $\rho(\vec{x})$ and all potentials Φ_j except Φ_i set to zero, i. e.

$$Q^{(i)}_{\text{ind}}(\vec{x}_0) = q\frac{\Phi(\vec{x}_0)}{\Phi_i} = q\Phi^{(i)}_{\text{ind}}(\vec{x}_0) \tag{11}$$

The dimensionless normalized potential $\Phi^{(i)}_{\rm ind} = \Phi(\vec{x})/\Phi_i$ of electrode S_i, obtained by grounding all other electrodes, is called the *induction potential* of electrode S_i, it varies between unity on S_i and zero on all other electrodes. For the simple case of 2 electrodes, eq. 11 is trivially equivalent to eq. 1. For a charge moving with velocity $\vec{v}$ at the space point $\vec{x}$, differentiation along the path yields

$$i^{(i)}_{\rm ind} = \frac{dQ^{(i)}_{\rm ind}}{dt} = q\vec{\nabla}\Phi^{(i)}_{\rm ind}(\vec{x}) \cdot \vec{v} \tag{12}$$

In summary, in a multiconductor system the induced signal on a selected electrode is governed by its induction potential, which is the normalized potential for the case of all other electrodes set to zero potential.

P. Glässel, D. v. Harrach, H.J. Specht, unpublished