Limits of the RICH Technique

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Basic Formulae

$$\boxed{\text{basic Cherenkov relation:} \quad \cos\Theta = \frac{1}{n\beta}}$$

$$\text{threshold:} \quad \gamma_t = (1 - 1/n^2)^{-\frac{1}{2}}$$

$$\text{number of detected photons:} \quad N = N_0 L z^2 \sin^2\Theta$$

$$\text{asymptotic } (\beta \longrightarrow 1)\text{:} \quad N_{max} = \frac{N_0 L z^2}{\gamma_t^2}$$

$$\text{figure of merit:} \quad N_0 = \frac{\alpha}{\hbar c} \int Q\, T\, R\, \mathrm{d}E$$

$$\boxed{\text{convention:} \quad \eta = \beta\gamma \quad \Leftrightarrow \quad \eta^2 = \gamma^2 - 1}$$

$$\sin^2\Theta_{max} = \frac{1}{\gamma_t^2} = \frac{1}{\eta_t^2 + 1}$$

$$\tan^2\Theta_{max} = \frac{1}{\eta_t^2} = \frac{1}{\gamma_t^2 - 1}$$

$$\frac{\sin^2\Theta}{\sin^2\Theta_{max}} = \frac{N}{N_{max}} = 1 - \frac{\eta_t^2}{\eta^2}$$

The RICH observables

- space directions of N Cherenkov photons $\{\vec{u}_i\}$

  $\longrightarrow$ define a cone

  (spherical aberrations $\rightarrow$ sensitivity to *direction only* not exact but typically quite good)
- cone axis is mean flight direction $\vec{u}_p$ of particle in radiator
- Cherenkov angles $\Theta_i = \arccos \vec{u}_i \cdot \vec{u}_p$
- $\Theta_c = \langle \Theta_i \rangle$ is opening angle of cone
- track and $\vec{u}_p$ maybe also from other tracking elements

Pattern recognition:

identify sets $\{\vec{u}_i\}$ that originate from one particle

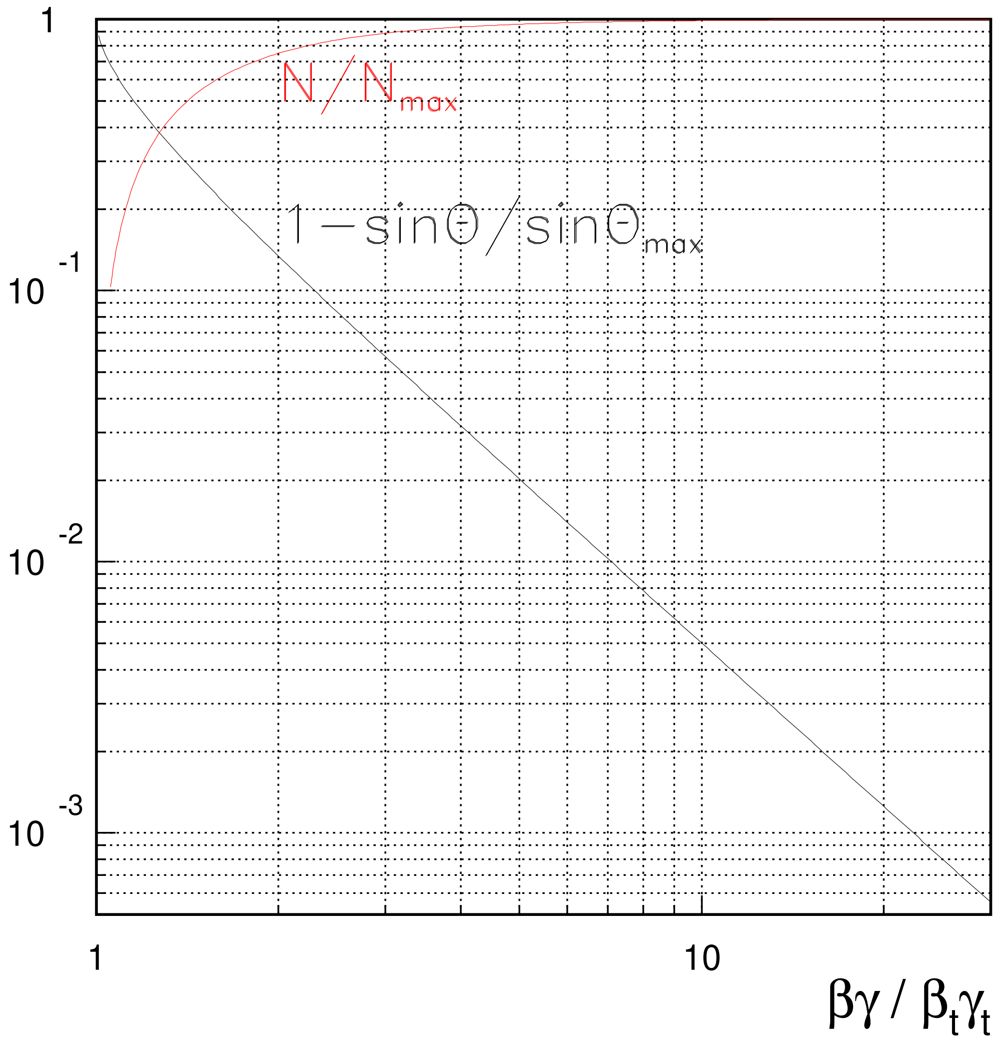
requires minimum number of detected photons per particle, depending on

- number of particles producing rings
- externally determined particle direction $\vec{u}_p$?
- background hits

  (particles close to threshold, radiator scintillations, diffuse reflections, ...)
- desired efficiency

$$\boxed{\text{typically } \langle N \rangle \gtrsim 10}$$

Cherenkov Angle and Hits vs Momentum



- momentum range scales with mass ($p = \beta\gamma m = \eta m$)

- upper η limit determined by $\sigma_{\Theta_c} = \sigma_\Theta/\sqrt{N}$:

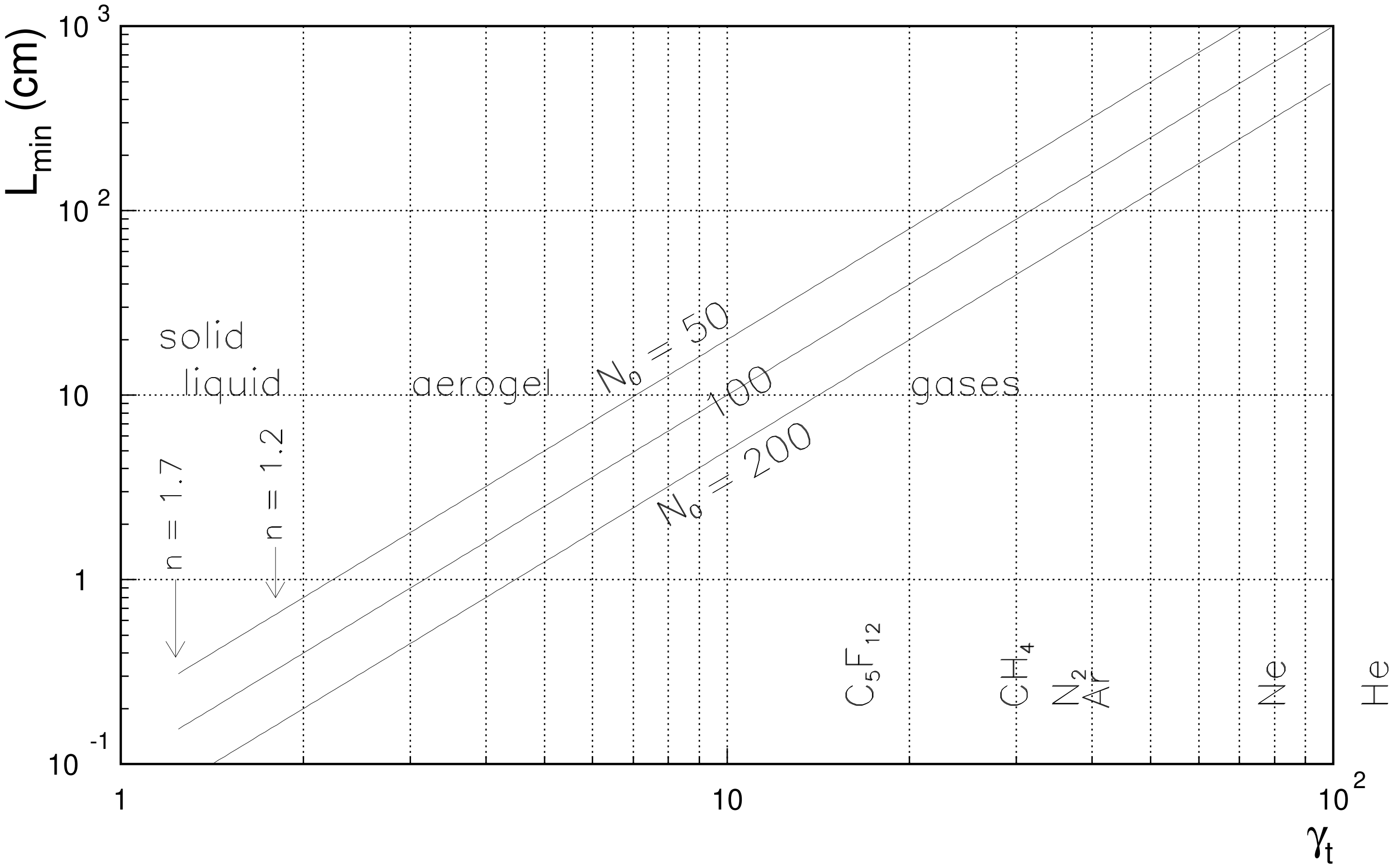
$$\eta_{max} \approx \eta\sqrt{\frac{\sigma_\eta}{\eta} \Big/ \frac{\sigma_{\Theta_c}}{\Theta_{max}}}$$

- lower limit from pattern recognition: $\dfrac{N}{N_{max}} = 1 - \dfrac{\eta_t^2}{\eta^2}$

  e.g. $\eta = \sqrt{2}\eta_t \;\Leftrightarrow\; N = \frac{1}{2}N_{max}$ and $\sin\Theta = \sqrt{1/2}\,\sin\Theta_{max}$

Minimum Radiator Length

vs threshold and N_0 for asymptotic $N_{max} = 10$



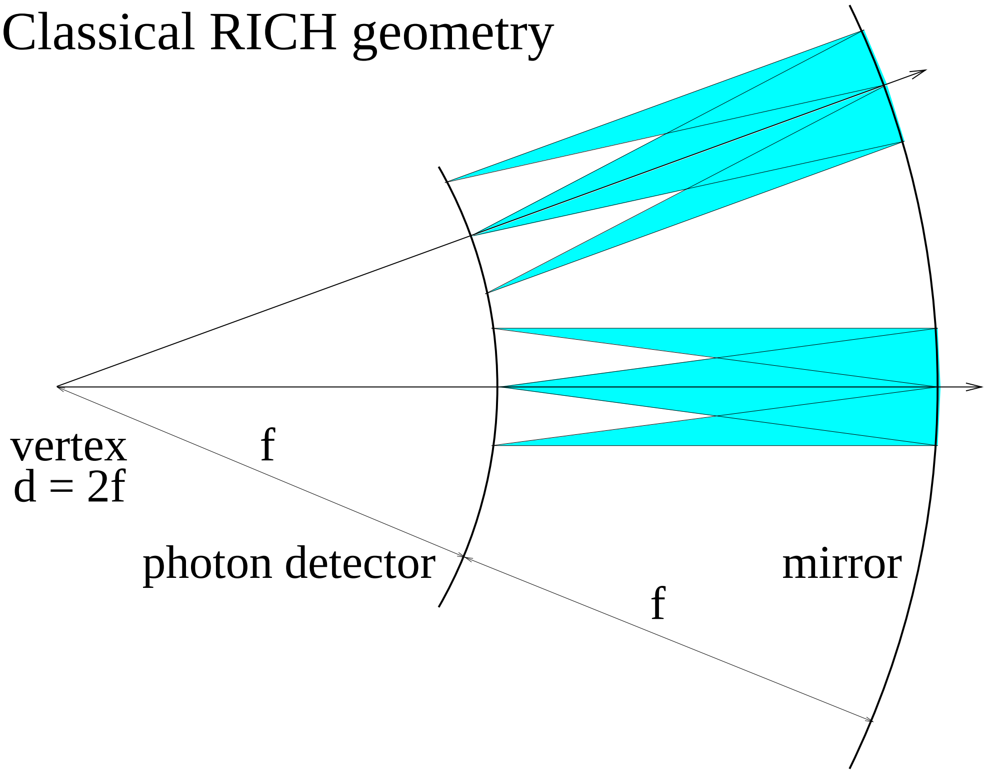
- solids: $L_{min} \lesssim 1$ cm
  need tricks to avoid internal reflection problem
  (tilted radiator, sawtooth surface, DIRC)
- liquids: $L_{min} \lesssim 1$ cm
  H_2O
  C_5F_{12}, C_6F_{14}, $n = 1.21$, UV-transparent
- aerogel $n = 1.02 \ldots 1.07$, diffuse scattering?
- gases: $L_{min} \gtrsim 1$ m

Important Radiator Properties

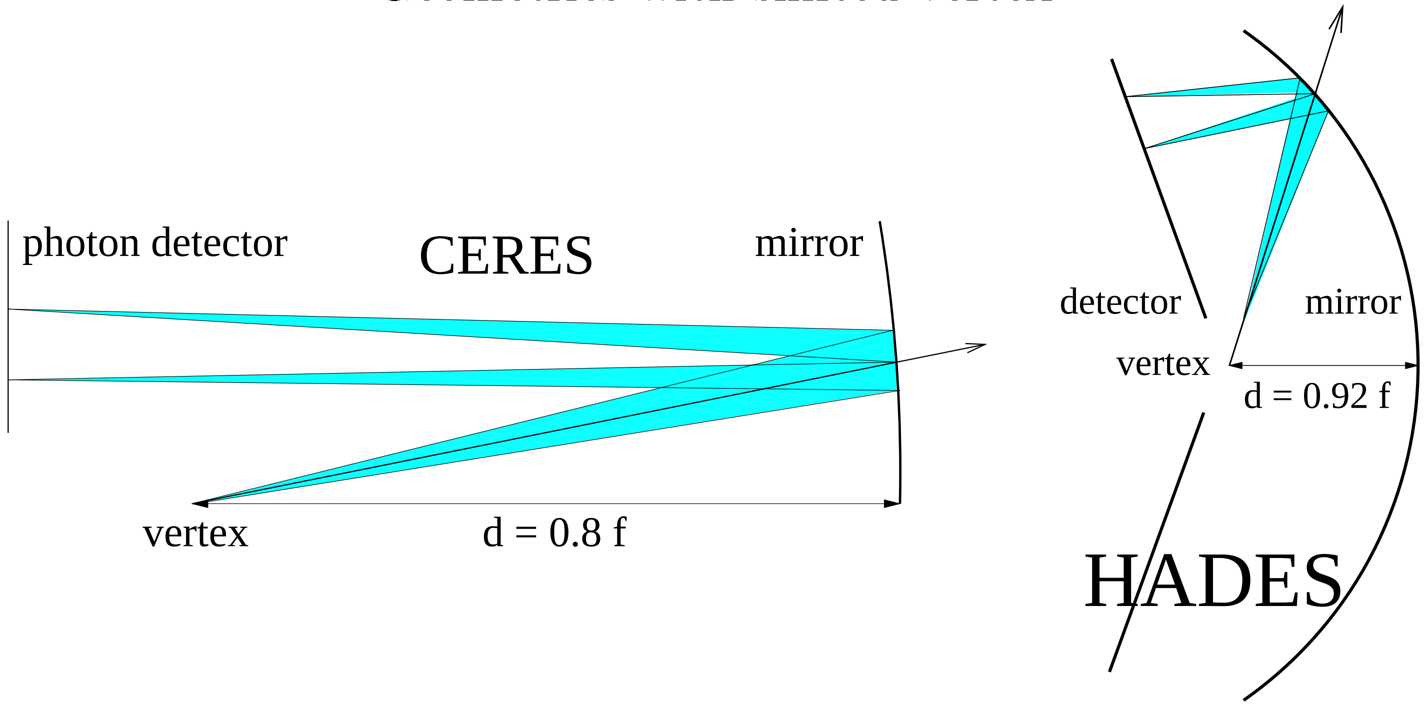
- transparency
- purification
- scintillations?
- chromatic dispersion
- multiple scattering
- safety issues

RICH Optics Schemes

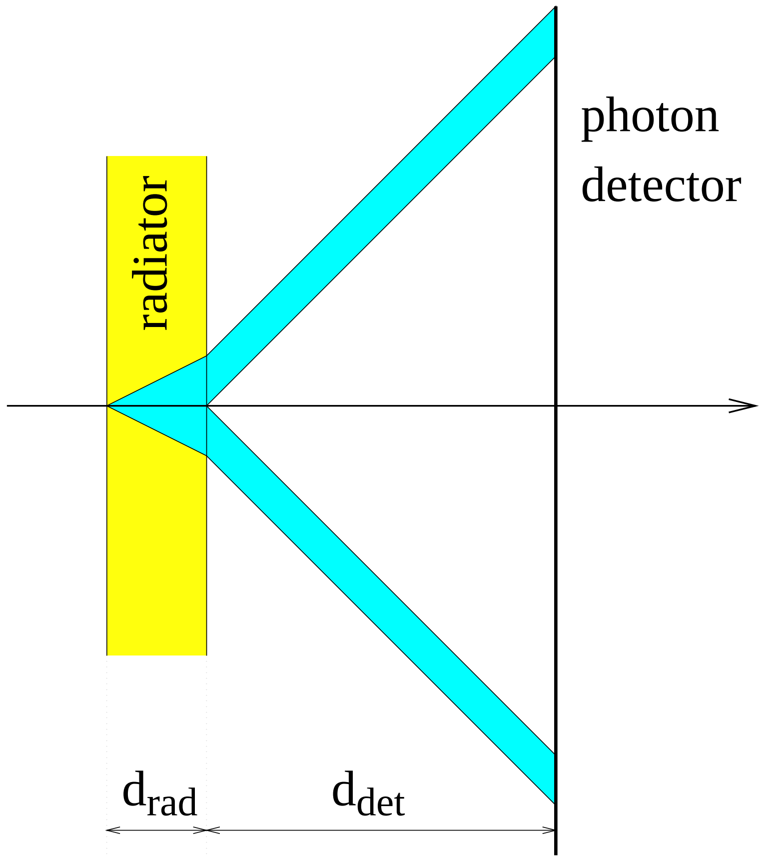
Classical RICH geometry



Geometries with shifted vertex



Proximity Focusing Scheme



- resolution σ_{Θ_c} determined by d_{rad}/d_{det}
- minimize d_{rad} to pattern recognition limit given by minimum allowed $\langle N \rangle$

  typically $d_{rad} \lesssim 1$ cm
- quite short designs possible

- for solids: internal reflection problem for $n \gtrsim 1.35$
- solutions: tilted radiator or facetted surface

  $\rightarrow$ complicated patterns instead of rings

Photon Detectors

- gas detectors with photo-sensitive gas (TMAE, TEA, ...)
- gas detectors with solid photocathode (CsI, ... ?)
  - for high rates and use in trigger:
    fast RICH design: integration times of ∼10 ns possible
  - windowless systems possible for special radiator gases
- photomultipliers
  - arrays
  - multianode, position-sensitive
- HPD hybrid photo diodes
  - position-sensitive readout

RICH Mirrors

- required optical quality moderate: cheap glas techniques possible with $\sigma_\Theta \lesssim 0.2$ mrad
- thin mirrors on carbon fiber laminate (1 mm = 0.4% X_0)
- reflector: Aluminum with MgF_2 protective layer.

  UV-reflectivity up to about 10 eV
- UV-reflectivity requires very small surface roughness (few nm!)

Proven figures of merit N_0

for some combinations detector/window

photon detector	window	mirror	N_0	ref
TMAE / gas	quartz	yes	80	several
TMAE / gas	CaF_2	yes	135	CERES
reflective CsI / gas	CaF_2	yes	120	HADES
reflective CsI / gas	quartz	–	41	HMPID
PM (visible)	glas	yes	47	HERMES
PM (visible)	glas + PTP	yes	55	SPHINX
PM (170 nm)	quartz	yes	~100	SELEX

UV-cutoff of some radiator and window media

material	C_6F_{14}	quartz	C_2H_6	CH_4	CaF_2	MgF_2	LiF
eV	7.3	7.7	7.8	9	10	10.8	11.7

Contributions to Cherenkov Angle Resolution

- optics: mirror quality, spherical aberration, flat detectors
- chromatic aberrations: Δn over bandwidth
- readout

  position resolution of gas detectors $\lesssim 300\ \mu$m and better
- multiple scattering
- track bending due to magnet field

$$\sigma_{\Theta}^2 = \sigma_{opt}^2 + \sigma_{msc}^2 + \sigma_{mag}^2 + \sigma_{read}^2$$

$$\sigma_{\Theta_c} = \frac{\sigma_{\Theta}}{\sqrt{N}}$$

Ring Center Resolution = Tracking Resolution of the RICH:

$$\sigma_{\Theta_p} = \sqrt{\frac{2}{N-2}}\,\sigma_{\Theta}$$

Multiple scattering

Mean particle direction in radiator deviates from initial one:

$$\sigma_{\Theta_p} = \frac{\Theta_0}{\sqrt{2}}$$

(Θ_0: rms deflection after traversal, projection into plane)

σ_{Θ_p} only relevant for connection with external track.

Direction of particle *within radiator* deviates from mean direction
$\longrightarrow$ smearing of Cherenkov ring:

$$\boxed{\sigma_{\Theta_i} = \frac{\Theta_0}{\sqrt{6}}}$$

Cherenkov angle resolution $\sigma_{\Theta_c} = \sigma_{\langle\Theta_i\rangle} = \sigma_{\Theta_i}/\sqrt{N}$ independent of radiator length!

For *multiple scattering dominated ring quality* σ_{Θ_i}:
$\longrightarrow$ measure momentum p via σ_{Θ_i}
(e.g. AQUA-RICH: particle identification without external momentum measurement)

Magnetic field deflection

$$\Delta\Theta = \frac{0.3(\mathrm{GeV/c\,m^{-1}\,T^{-1}})B_\perp L}{p}$$

$$\sigma_{\Theta_i} = \frac{0.3(\mathrm{GeV/c\,m^{-1}\,T^{-1}})B_\perp L}{\sqrt{12}\,p}$$

same p-scaling as multiple scattering, but

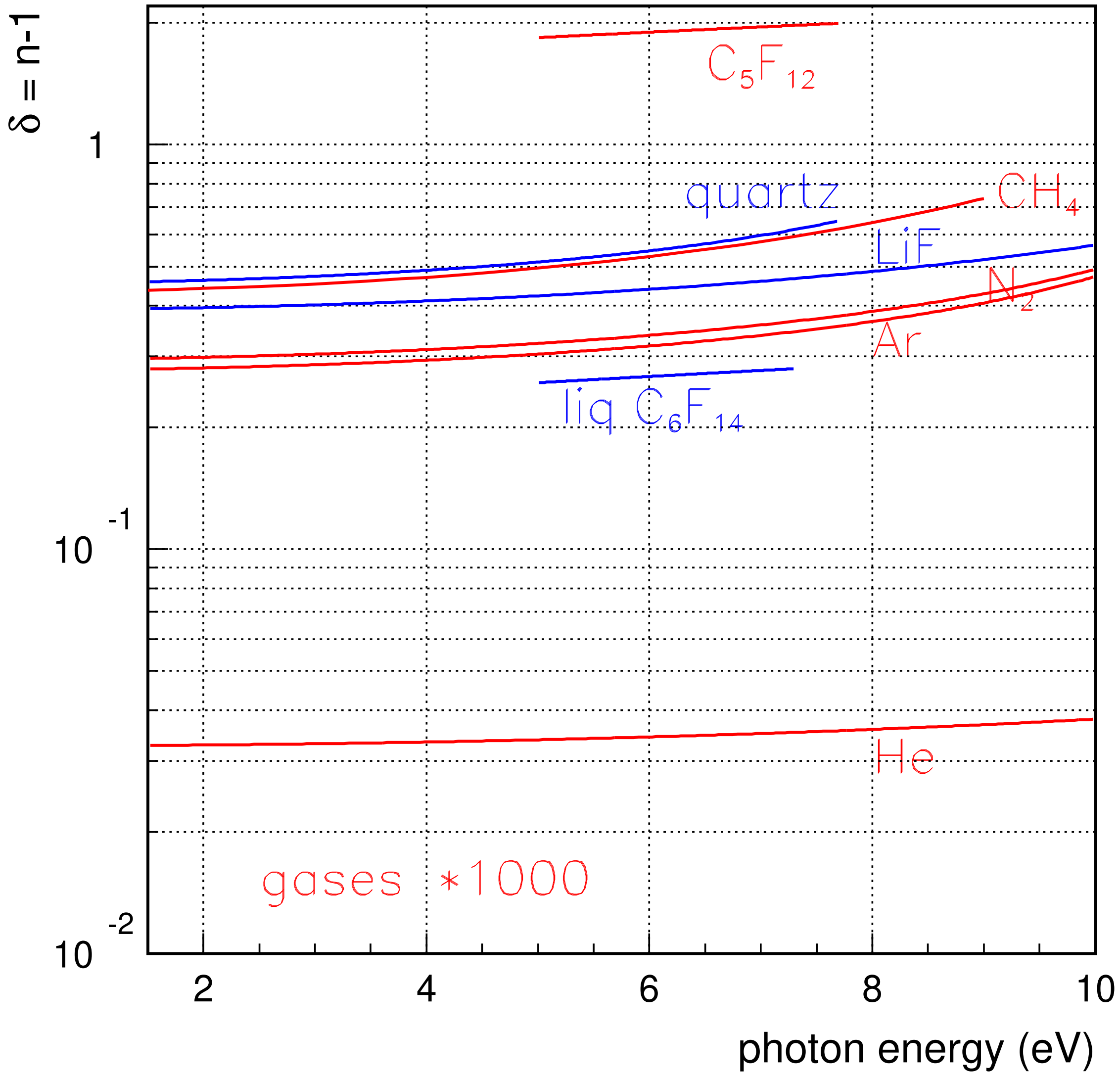
$$\Theta_{msc} \propto \sqrt{L}$$

$$\Theta_{mag} \propto L$$

Examples

material	L	$\Theta_0/\sqrt{6}$ @ 1 GeV/c	$B_\perp$ for same σ_Θ
LiF	1 cm	1.5 mrad	1.6 T
N_2	100 cm	0.9 mrad	10 mT

Radiator Chromatic Dispersion



- the prize for large N_0
- resulting error: $$\sigma_{\Theta_i} = \frac{\sigma_n}{\beta n^2 \sin\Theta}$$

for $\beta \approx 1$ and $n \approx 1$: $$\frac{\sigma_{\Theta_i}}{\Theta_i} = \frac{1}{2} \cdot \frac{\sigma_n}{n-1}$$

σ_n: rms spread of n over bandwidth weighted with QTR

Angular Resolution and PID

Separation of m_1 and m_2 at momentum p

$$\Delta \sin^2 \Theta_c = \frac{m_2^2 - m_1^2}{n^2 p^2} \quad \longrightarrow \quad n_\sigma = \frac{\beta^2 \Delta m^2}{2p^2} \Big/ \frac{\sigma_\beta}{\beta}$$

Upper p limit for separation by n_σ standard deviations, expressed in $\sigma_{\Theta_c} = \cot \Theta_c \frac{\sigma_\beta}{\beta}$:

$$\boxed{p_{max} = \left(\frac{\beta^2 \Delta m^2 \eta_t}{2 n_\sigma \, \sigma_{\Theta_c}} \right)^{\frac{1}{2}}}$$

Lower limit from decreasing N/N_{max}:

$$p_{min} \approx 1.4 \, \eta_t \, m$$

Example: $\eta_t = 30$, $\sigma_{\Theta_c} = 0.5$ mrad, 3σ separation
$\Rightarrow p_{max} = 33$ GeV/c for Kπ and 80 GeV/c for pK
$p_{min} = 6, 20, 40$ GeV/c for π, K, p

Examples for chromatic aberrations

radiator	LiF	C_6F_{14}	C_5F_{12}	N_2	He
L	1 cm	1 cm	0.5 m	2 m	10 m
γ_t	1.35	1.62	16	41	120
bandwidth (eV)	6 - 7.7	6 - 7.3	5.5 - 7.7	5.5 - 7.7	5.5 - 10
$\sigma_n = \Delta n/\sqrt{18}$	0.009	0.0028	$30 \cdot 10^{-6}$	$12 \cdot 10^{-6}$	$5 \cdot 10^{-6}$
σ_{Θ_i} (mrad)	5.4	2.8	0.45	0.40	0.13
σ_{Θ_c} (mrad)	1.8	0.9	0.14	0.13	0.042
chromatic/msc	5	5	10	30	60
3σ separ. $K\pi$					
p_{max} (GeV/c)	4	7	50	100	330
p_{min} (GeV/c)	0.6	0.88	11	28	83

Summary

What are the true limitations of the RICH technique?

- radiator dimensions: N_0
  - quantum efficiency of photon conversion
  - bandwidth (threshold, window, mirror)
- resolution limit: chromatic dispersion of radiator