Applications of Effective Theories in *B* Decays

(weak eff. Hamiltonian, factorization, HQET, SCET, and all that)

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particle **p**hysics **p**henomenology after the **H**iggs discovery

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quark **f**lavour physics and **e**ffective field **t**heories

Disclaimer:

The dynamics of strong and weak interactions in B-decays is very complex and has many faces . . .

. . . I will not be able to cover everything, . . .

. . . but I hope that some theoretical and phenomenological concepts become clearer.

. . . Some introductory remarks . . .

- Physical processes involve Different typical Energy/Length Scales:

  $\Rightarrow$ Short-distance Dynamics vs. Long-distance Dynamics

- e.g. for b-decays:

New physics	:	$\delta x \lesssim 1/\Lambda_{\rm NP}$
Electroweak interactions	:	$\delta x \sim 1/M_W$
Short-distance QCD(QED) corrections	:	$\delta x \sim 1/M_W \to 1/m_b$
Hadronic effects	:	$\delta x \sim 1/m_b$ (perturbative)
		$\delta x \gtrsim 1/\Lambda_{\rm had}$ (non-perturbative)

$\to$ Model-independent Parametrization of NP Effects

$\to$ Sequence of Effective Field Theories (EFT)

$\to$ Perturbative vs. Non-Perturbative Strong Interaction Effects

$\to$ QFT-Definition of Hadronic Input Parameters (Functions)

Central Notions / Theoretical Jargon

- **Factorization:**
  1. Separation of Scales in (RG-improved) Perturbation Theory
  2. Simplification of Exclusive Hadronic Matrix Elements

- **Operator-Product Expansion (OPE):**
  Short-distance expansion ($x \to 0$) of time-ordered operator products, corresponding to $|q^2| \to \infty$ in Fourier transform:

$$\int d^4x\, e^{iq\cdot x}\, T\left(\phi(x)\,\phi(0)\right) = \sum_i c_i(q^2)\, \mathcal{O}_i(0)$$

"Wilson Coefficients" $c_i(q^2)$

"Effective" Operators $\mathcal{O}_i(0)$

- **Effective (Quantum) Field Theories:**
  Effective Lagrangian / Hamiltonian:
  - Feynman rules reproduce the dynamics of low-energy modes.
  - High-energy (short-distance) information in coefficient (functions).

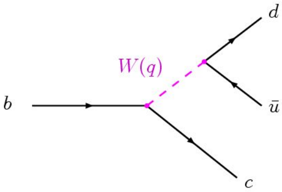
Outline

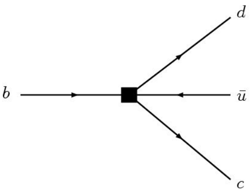
- Example: effectcive Hamiltonian for $b \to c d \bar{u}$ decays
  - separation of scales in loop diagrams
  - current-current operators (chirality, colour)
  - matching and running of Wilson coefficients
- Generalization to $b \to s(d)$ transitions
  - strong penguin operators
  - electroweak operators
- From $b \to s$ to $B \to K^* \ell^+ \ell^-$
  - naive factorization
  - small hadronic recoil (HQET)
  - large hadronic recoil (SCET/QCDF)

Example: $b \to c d \bar{u}$ decays

$b \to c d \bar{u}$ decay at Born level

Full theory (SM) $\to$ Fermi model





$g^2 \times$ current $\times$ W-propagator $\times$ current $\xrightarrow{|q| \ll M_W}$ effective coupling $\times$ local 4-quark operator

- Energy/Momentum transfer limited by mass of decaying b-quark.
- b-quark mass much smaller than W-boson mass.

$$\boxed{|q| \leq m_b \ll M_W}$$

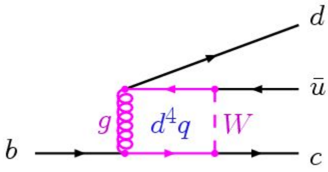
Effective Theory:

- Analogously to muon decay, transition described in terms of current-current interaction, with left-handed charged currents

$$J_\alpha^{(b\to c)} = V_{cb}\,[\bar{c}\,\gamma_\alpha(1-\gamma_5)\,b]\,, \qquad \bar{J}_\beta^{(d\to u)} = V_{ud}^*\,[\bar{d}\,\gamma_\beta(1-\gamma_5)\,u]$$

- Effective operators only contain light fields (!)
  ("light" quarks, leptons, gluons, photons).
- Effect of large scale M_W in effective Fermi coupling constant:

$$\boxed{\frac{g^2}{8M_W^2} \longrightarrow \frac{G_F}{\sqrt{2}} \simeq 1.16639\cdot 10^{-5}\,\mathrm{GeV}^{-2}}$$



- 4-momentum of the W-boson in the loop is an

  internal integration parameter d^4q,

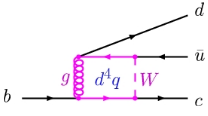
  each component taking values between $-\infty$ and $+\infty$.

$\Rightarrow$ We cannot simply expand in $|q|/M_W$!

$\Rightarrow$ Need a method to separate the cases $|q| \gtrsim M_W$ and $|q| \ll M_W$.

IR and UV regions in the Effective Theory

full theory $=$ IR region $\left(\begin{array}{c} |q| \ll M_W \\ M_W \to \infty \end{array} \right)$ $+$ UV region $\left(\begin{array}{c} |q| \gtrsim M_W \\ m_{b,c} \to 0 \end{array} \right)$



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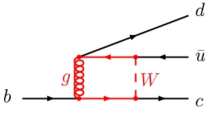

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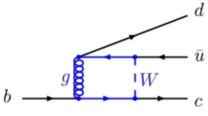

$$I(\alpha_s;\, \frac{m_b}{M_W}, \frac{m_c}{m_b})/G_F \simeq I_{IR}(\alpha_s;\, \frac{m_b}{\mu}, \frac{m_c}{m_b}) + I_{UV}(\alpha_s;\, \frac{\mu}{m_W})$$

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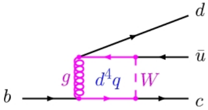
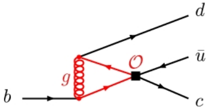


$\simeq$



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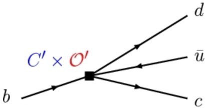

$$I(\alpha_s;\, \tfrac{m_b}{M_W},\, \tfrac{m_c}{m_b})/G_F \;\simeq\; \langle \mathcal{O} \rangle^{\rm loop}(\alpha_s;\, \tfrac{m_b}{\mu},\, \tfrac{m_c}{m_b}) \;+\; C'(\alpha_s;\, \tfrac{\mu}{m_W}) \times \langle \mathcal{O}' \rangle^{\rm tree}$$

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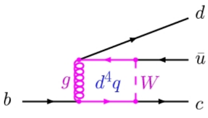

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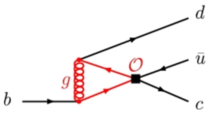

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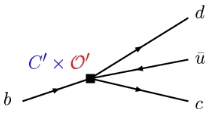

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$\updownarrow$

1-loop matrix element of operator $\mathcal{O}$ in Eff. Th.

- independent of M_W
- UV divergent $\to \mu$

$\updownarrow$

1-loop coefficient for new operator $\mathcal{O}'$ in EFT

- independent of $m_{b,c}$
- IR divergent $\to \mu$

Effective Operators for $b \to c\bar{d}u$

- short-distance QCD corrections preserve chirality;
- quark-gluon vertices induce second colour structure.

$$H_{\rm eff} = \frac{G_F}{\sqrt{2}}\, V_{cb} V_{ud}^* \sum_{i=1,2} C_i(\mu)\, \mathcal{O}_i + \text{h.c.} \qquad (b \to c d\bar{u})$$

- **Current-Current Operators:** $(b \to c d\bar{u},$ analogously for $b \to q q' \bar{q}''$ decays)

$$\begin{aligned} \mathcal{O}_1 &= (\bar{d}_L^a \gamma_\alpha u_L^b)\,(\bar{c}_L^b \gamma^\alpha b_L^a) \\ \mathcal{O}_2 &= (\bar{d}_L^a \gamma_\alpha u_L^a)\,(\bar{c}_L^b \gamma^\alpha b_L^b) \end{aligned}$$

- The Wilson Coefficients $C_i(\mu)$ contain all information about

  Short-Distance Physics $\equiv$ Dynamics above a Scale μ

Wilson Coefficients in Perturbation Theory

- 1-loop result:

$$C_i(\mu) = \left\{ \begin{array}{c} 0 \\ 1 \end{array} \right\} + \frac{\alpha_s(\mu)}{4\pi} \left(\ln \frac{\mu^2}{M_W^2} + \frac{11}{6} \right) \left\{ \begin{array}{c} 3 \\ -1 \end{array} \right\} + \mathcal{O}(\alpha_s^2)$$

Question : How do we choose the renormalization scale μ ?

Wilson Coefficients in Perturbation Theory

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Question : How do we choose the renormalization scale μ ?

Answer :

"Matching"

- For $\mu \sim M_W$ the logarithmic term is small, and $\frac{\alpha_s(M_W)}{\pi} \ll 1$

$\rightarrow$ $C_i(M_W)$ can be calculated in Fixed-order Perturbation Theory

- In this context, M_W is called the Matching Scale.

Anomalous Dimensions

- In order to compare with experiment / hadronic models, the matrix elements of EFT operators are needed at low-energy scale $\mu \sim m_b$
  - Only the combination
    $$\sum_i C_i(\mu)\, \langle \mathcal{O}_i \rangle(\mu)$$
    is μ-independent (in perturbation theory).

  $\Rightarrow$ Need Wilson coefficients at low scale !

- Scale dependence can be calculated in perturbation theory:
  - Loop diagrams in EFT are UV divergent
    $\Rightarrow$ anomalous dimensions (matrix):

$$\frac{\partial}{\partial \ln \mu} C_i(\mu) \equiv \gamma_{ji}(\mu)\, C_j(\mu) = \left(\frac{\alpha_s(\mu)}{4\pi}\, \gamma_{ji}^{(1)} + \ldots \right) C_j(\mu)$$

  - $\gamma = \gamma(\alpha_s)$ has a perturbative expansion.

RG Improvement ("running")

In our case:

$$\gamma^{(1)} = \begin{pmatrix} -2 & 6 \\ 6 & -2 \end{pmatrix} \qquad \begin{cases} \text{Eigenvectors: } C_\pm = \frac{1}{\sqrt{2}}(C_2 \pm C_1) \\ \text{Eigenvalues: } \gamma_\pm^{(1)} = +4,\ -8 \end{cases}$$

- Formal solution of differential equation: (separation of variables)

$$\ln \frac{C_\pm(\mu)}{C_\pm(M)} = \int_{\ln M}^{\ln \mu} d\ln\mu'\,\gamma_\pm(\mu') = \int_{\alpha_s(M)}^{\alpha_s(\mu)} \frac{d\alpha_s}{2\beta(\alpha_s)}\,\gamma_\pm(\alpha_s)$$

- Perturbative expansion of anomalous dimension and β-function:

$$\gamma = \frac{\alpha_s}{4\pi}\,\gamma^{(1)} + \ldots\,, \qquad 2\beta \equiv \frac{d\alpha_s}{d\ln\mu} = -\frac{2\beta_0}{4\pi}\,\alpha_s^2 + \ldots$$

$$C_\pm(\mu) \simeq C_\pm(M_W) \cdot \left(\frac{\alpha_s(\mu)}{\alpha_s(M_W)}\right)^{-\gamma_\pm^{(1)}/2\beta_0} \qquad \text{(LeadingLogApprox)}$$

Numerical values for $C_{1,2}$ in the SM [Buchalla/Buras/Lautenbacher 96]

operator:	$\mathcal{O}_1$	$\mathcal{O}_2$
$C_i(m_b)$:	-0.514 (LL) -0.303 (NLL)	1.026 (LL) 1.008 (NLL)

(modulo parametric uncertainties from $M_W, m_b, \alpha_s(M_Z)$ and QED corr.)

(potential) New Physics modifications:

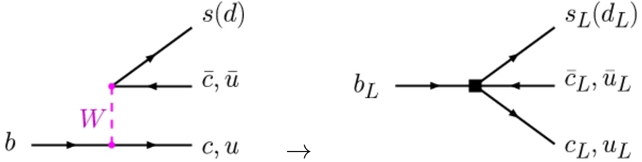
- new left-handed interactions (incl. new phases)

$$C_{1,2}(M_W) \to C_{1,2}(M_W) + \delta_{\rm NP}(M_W, M_{\rm NP})$$

- new chiral structures $\Rightarrow$ extend operator basis (LR,RR currents)

Next Example: $b \to s(d)$ transitions

$b \to s(d)\, q\bar{q}$ decays – Current-current operators



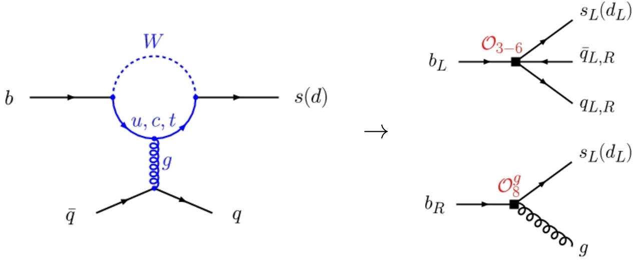
- Now, there are two possible flavour structures:

$$V_{ub} V^*_{us(d)}\; (\bar{u}_L \gamma_\mu b_L)(\bar{s}(d)_L \gamma^\mu u_L) \;\equiv\; \lambda_u\, \mathcal{O}_2^{(u)}\,,$$
$$V_{cb} V^*_{cs(d)}\; (\bar{c}_L \gamma_\mu b_L)(\bar{s}(d)_L \gamma^\mu c_L) \;\equiv\; \lambda_c\, \mathcal{O}_2^{(c)}\,,$$

- Again, α_s corrections induce independent colour structures $\mathcal{O}_1^{(u,c)}$.

$b \to s(d)\, q\bar{q}$ decays – strong penguin operators

- New feature: **Penguin Diagrams** $\to$ additional operator structures



smaller Wilson coefficients
(suppressed by α_s / loop factor)

- Strong penguin operators: $\mathcal{O}_{3-6}$
- Chromomagnetic operator: $\mathcal{O}_8^g$

Question : CKM factor of Penguin Pperators? (for $m_{u,c} \ll m_t$)
Answer :

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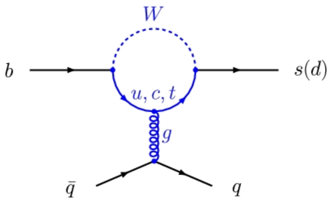

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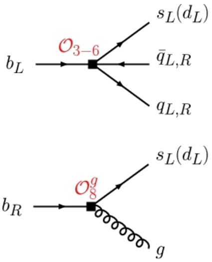

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Question : CKM factor of Penguin Pperators? (for $m_{u,c} \ll m_t$)

Answer : $-\lambda_t = (\lambda_u + \lambda_c) = -V_{tb} V_{ts(d)}^*$

Eff. Hamiltonian for $b \to s(d)q\bar{q}$ decays (QCD only)

$$H_{\rm eff} = \frac{G_F}{\sqrt{2}} \sum_{i=1,2} C_i(\mu) \left(\lambda_u \, \mathcal{O}_i^{(u)} + \lambda_c \, \mathcal{O}_i^{(c)}\right) - \frac{G_F}{\sqrt{2}} \lambda_t \sum_{i=3}^{6} C_i(\mu) \, \mathcal{O}_i - \frac{G_F}{\sqrt{2}} \lambda_t \, C_8^g(\mu) \, \mathcal{O}_8^g$$

$$\mathcal{O}_3 = (\bar{s}_L^a \gamma_\mu b_L^a) \sum_{q \neq t} (\bar{q}_L^b \gamma^\mu q_L^b) \,, \qquad \mathcal{O}_4 = (\bar{s}_L^a \gamma_\mu b_L^b) \sum_{q \neq t} (\bar{q}_L^b \gamma^\mu q_L^a) \,,$$

$$\mathcal{O}_5 = (\bar{s}_L^a \gamma_\mu b_L^a) \sum_{q \neq t} (\bar{q}_R^b \gamma^\mu q_R^b) \,, \qquad \mathcal{O}_6 = (\bar{s}_L^a \gamma_\mu b_L^b) \sum_{q \neq t} (\bar{q}_R^b \gamma^\mu q_R^a) \,,$$

$$\mathcal{O}_8^g = \frac{g_s}{8\pi^2} \, m_b \, (\bar{s}_L \, \sigma^{\mu\nu} \, T^A \, b_R) \, G_{\mu\nu}^A \,.$$

- gluon couples to left- and right-handed currents.
- chromomagnetic operator requires one chirality flip !

(m_s is set to zero)

Matching and running for strong penguin operators

- Matching coefficients depend on top mass,

$$C_i = C_i(\mu, x_t)\,, \qquad x_t = m_t^2/M_W^2$$

- Matching of chromomagnetic operator is scheme-dependent. Usually, one considers scheme-independent linear combination:

$$C_8^{g,\,\mathrm{eff}} = C_8^g + \sum_{i=1}^{6} z_i\, C_i$$

- Again, operators mix under RG running (→ anomalous-dimension matrix)

Electroweak Corrections

- Penguin and box diagrams with additional γ/Z exchange:

  $\rightarrow$ Electroweak Penguin Operators $\mathcal{O}_{7-10}$

$$\mathcal{O}_7 = \frac{2}{3}\,(\bar{s}_L^a \gamma_\mu b_L^a) \sum_{q\neq t} e_q\,(\bar{q}_L^b \gamma^\mu q_L^b)\,, \quad \mathcal{O}_8 = \frac{2}{3}\,(\bar{s}_L^a \gamma_\mu b_L^b) \sum_{q\neq t} e_q\,(\bar{q}_L^b \gamma^\mu q_L^a)\,,$$

$$\mathcal{O}_9 = \frac{2}{3}\,(\bar{s}_L^a \gamma_\mu b_L^a) \sum_{q\neq t} e_q\,(\bar{q}_R^b \gamma^\mu q_R^b)\,, \quad \mathcal{O}_{10} = \frac{2}{3}\,(\bar{s}_L^a \gamma_\mu b_L^b) \sum_{q\neq t} e_q\,(\bar{q}_R^b \gamma^\mu q_R^a)\,.$$

  depend on electromagnetic charge of final state quarks !

  $\rightarrow$ Electromagnetic operators $\mathcal{O}_7^\gamma$
  main contribution to $b \to s(d)\gamma$ and $b \to s(d)\ell^+\ell^-$ decays.

  $\rightarrow$ Semileptonic operators $\mathcal{O}_{9V}$, $\mathcal{O}_{10A}$
  another main contribution to $b \to s\ell^+\ell^-$ decays.

  [see below]

  $\rightarrow$ electroweak corrections to matching coefficients

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main contribution to $b \to s(d)\gamma$ and $b \to s(d)\ell^+\ell^-$ decays.

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$$
\begin{aligned}
\mathcal{O}_{9V} &= (\bar{s}_L \, \gamma_\mu \, b_L)\,(\bar{\ell}\,\gamma^\mu \, \ell)\,, \\
\mathcal{O}_{10A} &= (\bar{s}_L \, \gamma_\mu \, b_L)\,(\bar{\ell}\,\gamma^\mu \gamma_5 \, \ell)
\end{aligned}
$$

another main contribution to $b \to s\ell^+\ell^-$ decays.

[see below]

→ electroweak corrections to matching coefficients

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[see below]

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Summary: Effective Theory for b-quark decays

"Full theory" $\leftrightarrow$ all modes propagate Parameters: $M_{W,Z}, M_H, m_t, m_q, g, g', \alpha_s \ldots$	$\uparrow \mu > M_W$
$C_i(M_W) = C_i\big|_{\rm tree} + \delta_i^{(1)} \frac{\alpha_s(M_W)}{4\pi} + \ldots$	matching: $\mu \sim M_W$
"Eff. theory" $\leftrightarrow$ low-energy modes propagate. High-energy modes are "integrated out". Parameters: $m_b, m_c, G_F, \alpha_s, C_i(\mu) \ldots$	$\downarrow \mu < M_W$
$\frac{\partial}{\partial \ln \mu} C_i(\mu) = \gamma_{ji}(\mu)\, C_j(\mu)$	anomalous dimensions
Expectation values of operators $\langle O_i \rangle$ at $\mu = m_b$. All dependence on M_W absorbed into $C_i(m_b)$	resummation of logs

From $b \to s$ to $B \to K^* \ell^+ \ell^-$

0[th] approximation:

Hadronic amplitudes expressed in terms of seven Form Factors for Tensor, Vector, and Axialvector $b \to s$ currents,

$$T_{1,2,3}(q^2)\,, \quad A_{0,1,2}(q^2)\,, \quad V(q^2)$$

multiplied by Wilson Coefficients $C_{7,9,10}(\mu)$ and kinematic factors.

- form factors include non-perturbative bound-state effects for $B \to K^*$ transitions
- to be taken from light-cone sum rules (small q^2) or lattice QCD (large q^2)

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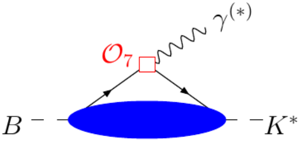

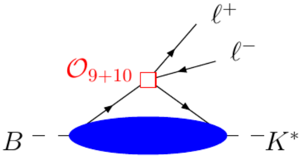

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The case of small hadronic recoil energy → HQET

The heavy *b*-quark:

- Heavy quark approximately behaves as **Static Source of Colour**

$$p_b^\mu \quad = \quad m_b v^\mu + k^\mu \,, \qquad \text{with } |k^\mu| \ll m_b$$

k^μ: **soft** (residual) momentum. $v^\mu = (1, \vec{0})$: *B*-meson **velocity**.

- The *b*-quark propagator is approximated as

$$\frac{i}{\not p_b - m_b + i\epsilon} = \frac{i\,(\not p_b + m_b)}{p_b^2 - m_b^2 + i\epsilon} \simeq \frac{i m_b\,(\not v + 1)}{2 m_b\, v \cdot k + i\epsilon} \quad = \quad \frac{i}{v \cdot k + i\epsilon}\,\frac{1 + \not v}{2}$$

This corresponds to a kinetic term for an **effective *b*-quark field** h_v

$$\mathcal{L}_{\rm kin} \quad = \quad \bar h_v\,(i\, v \cdot \partial)\, h_v \,, \qquad \text{with} \quad (\not v - 1)\, h_v = 0$$

→ **Heavy Quark Effective Theory** (HQET)

The case of small hadronic recoil energy $\to$ HQET

- "full theory" (QCD with weak-decay currents) matched onto HQET , only contains the "good components" of the b-quark Dirac spinor,

$$b(x) \quad \to \quad e^{-im_b v\cdot x}\, h_v(x)\,, \quad \text{with } \not{v}\, h_v = h_v \qquad (\mu \leq m_b)$$

- QCD part:

$$\bar{b}(x)\,(i\not{D} - m_b)\,b(x) \qquad \longrightarrow \qquad \bar{h}_v(x)\,(iv\cdot D)\,h_v(x) + \ldots$$

- decay currents, e.g.

$$\bar{q}(x)\,\Gamma\, b(x) \qquad \longrightarrow \qquad \sum_i c_i^\Gamma(\mu)\, e^{-im_b v\cdot x}\,\bar{q}(x)\,\Gamma_i\, h_v(x) + \ldots$$

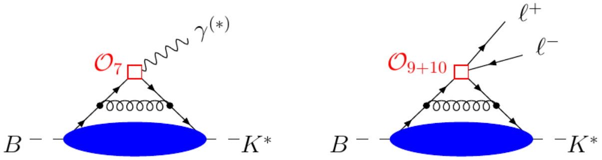
Consequences:

$\Rightarrow$ relative orientation of heavy-quark spin irrelevant in the limit $m_b \to \infty$

$\Rightarrow$ reduction of independent $B \to K^*$ form factors from $7 \to 4$

$\Rightarrow$ additional gluon-radiation costs factors of $1/m_b$ (higher-dim. operators in HQET)

NLO vertex corrections to matching coefficients:



Remember:

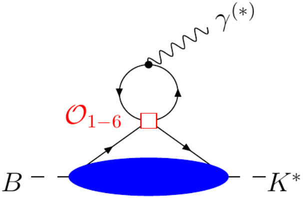
- QCD@m_b: hard gluons (with $|p| \sim m_b$) and soft gluons ($|p| \ll m_b$)
- HQET@m_b: only soft gluons

- radiative corrections to form-factor symmetry relations (from hard gluons)
- calculable in perturbation theory, since $\alpha_s(m_b) \ll 1$

Complications:

Also Hadronic Operators contribute:

- LO:



- Effect can be absorbed into effective Wilson coefficients,

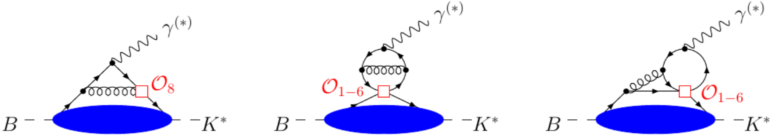
$$C_7^{\text{eff}} = C_7 + \sum_{i=1}^{6} y_i \, C_i \,, \qquad C_9^{\text{eff}}(q^2) = C_9 + \sum_{i=1}^{6} f_i(q^2) \, C_i$$

- removes scheme-dependence in the definition of Wilson coefficients (see above)

Complications:

Also Hadronic Operators contribute:

- NLO:



...

- $\mathcal{O}(\alpha_s(m_b))$ contributions require evaluation of 2-loop diagrams
- Higher-order terms have sizeable numerical impact!

Further Complications:

Quark loops can form hadronic vector resonances if $q^2 \simeq m_V^2$

- Particularly relevant for charm loop $\leftrightarrow$ charmonium resonances

$$\text{e.g.} \qquad B \to J/\psi(\to \ell^+\ell^-)\, K^* \quad \ldots$$

- Requires some modelling of “quark-hadron duality” assumption
- Resonant contributions interfere with (large) tree-level contribution from C_9
  $\Rightarrow$ sensitive to real part of charm loop (not a Breit-Wigner)
  (in contrast to R-ratio in $e^+e^- \to$ hadrons $\sim$ imaginary part of quark loops)

- HQET analysis only valid for q^2 above narrow $\bar{c}c$ states (J/ψ, ψ')
- theory predictions must be averaged over sufficiently large region of q^2

The case of large hadronic recoil energy → SCET

Fast (massless) light quarks in K^* with large energy $E_{K^*} \sim \mathcal{O}(m_b/2)$:

- Quarks move approximately **collinear** to their parent mesons.

$$p_{\rm coll}^\mu = p_+ \frac{n_-^\mu}{2} + p_\perp^\mu + p_- \frac{\bar{n}_+^\mu}{2}, \qquad \text{with } p_- \ll |p_\perp| \ll p_+$$

$p_\perp^\mu$: **small transverse momentum.** $\qquad n_\pm^\mu = (1, 0_\perp, \pm 1)$: **light-like**.

- Collinear quark propagator is approximated as

$$\frac{i\,\not{p}_{\rm coll}}{p_{\rm coll}^2 + i\epsilon} \simeq \frac{i\,p_+ \not{n}_-}{p_+ p_- + p_\perp^2 + i\epsilon} = \frac{i}{p_- + p_\perp^2/p_+ + i\epsilon}\,\frac{\not{n}_-}{2}$$

This corresponds to a kinetic term for an **effective collinear field** ξ_c

$$\mathcal{L}_{\rm kin} = \bar{\xi}_c \left(i\,n_- \cdot D + i\not{D}_\perp \frac{1}{in_+ D}\, i\not{D}_\perp \right) \frac{\not{n}_+}{2}\, \xi_c\,, \qquad \text{with} \quad \not{n}_-\, \xi_c = 0$$

→ **Soft Collinear Effective Theory** (SCET)

The case of large hadronic recoil energy → SCET

Soft-collinear interactions:

- Invariant mass of a gluon coupled to soft-collinear quark current:

$$(k_{\rm soft} - p_{\rm coll})^2 \quad \simeq \quad -p_+ \, (n_- \cdot k) \sim \mathcal{O}(E \, \Lambda_{\rm had})$$

→ **hard-collinear modes** (relevant for spectator interactions)

Subtlety: Soft-collinear vertices have to be **multipole-expanded** according to the different sizes for the typical wave-lengths involved.

Heavy-to-light currents:

- A generic heavy-to-light current (with arbitrary Dirac matrix Γ) matches onto:

$$\bar q(0) \, \Gamma \, Q(0) \quad \longrightarrow \quad \bar\xi_c(0) \, \Gamma \, h_v(0) + \ldots$$

→ **Soft Collinear Effective Theory** (SCET)

The case of large hadronic recoil energy $\to$ SCET

$\Rightarrow$ distinguish different kind of modes for light quarks and gluons:

name	energy	$\lvert\vec{p}_z\rvert$	$\lvert p^2\rvert, \lvert\vec{p}_\perp^2\rvert$
“hard”:	$\mathcal{O}(m_b)$	$\mathcal{O}(m_b)$	$\mathcal{O}(m_b^2)$
“hard-collinear’:	$\mathcal{O}(m_b)$	$\mathcal{O}(m_b)$	$\mathcal{O}(m_b \Lambda_{\rm had})$
”collinear“:	$\mathcal{O}(m_b)$	$\mathcal{O}(m_b)$	$\mathcal{O}(\Lambda_{\rm had}^2)$
”soft“:	$\mathcal{O}(\Lambda_{\rm had})$	$\mathcal{O}(\Lambda_{\rm had})$	$\mathcal{O}(\Lambda_{\rm had}^2)$

The case of large hadronic recoil energy $\to$ SCET

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"soft":	$\mathcal{O}(\Lambda_{\rm had})$	$\mathcal{O}(\Lambda_{\rm had})$	$\mathcal{O}(\Lambda_{\rm had}^2)$

two-step matching:

- remove hard modes $\longrightarrow$ SCET-1 $\times$ HQET
- remove hard-collinear modes $\longrightarrow$ SCET-2 $\times$ HQET

The case of large hadronic recoil energy → SCET

⇒ distinguish different kind of modes for light quarks and gluons:

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two-step matching:

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Form-factor relations in SCET

- energetic quarks → two components of the Dirac spinor are subleading:

$$\text{collinear fields:} \qquad q(x) \quad \to \quad \frac{\not{n}_- \not{n}_+}{4}\, \xi_c(x)\,,$$

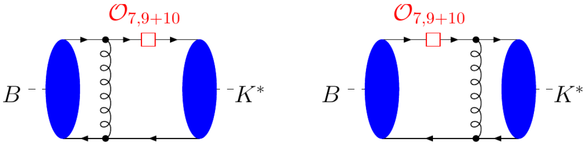
⇒ further reduction of independent form factors in $B \to K^*$

$$7 \text{ (QCD)} \longrightarrow 4 \text{ (HQET)} \longrightarrow 2 \text{ (SCET} \times \text{HQET)}$$

- leading-order predictions depend on less hadronic unknowns ✓
- Feynman rules for perturbative corrections in SCET more complicated !
- power corrections $\sim 1/E$ might be more important, since $E \lesssim m_b/2$?

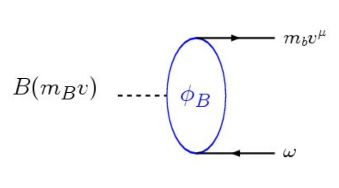
Radiative corrections to form-factor relations in SCET

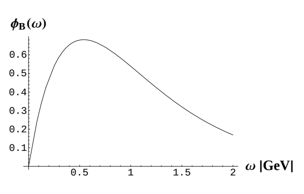
- vertex diagrams from integrating out hard modes
  (similar as for HQET – see above)
- new feature:
  spectator diagrams from integrating out hard-collinear modes



- contribute at leading power of $1/m_b$ expansion
- perturbatively calculable as long as $\alpha_s \ll 1$ for $\mu^2 \sim m_b \Lambda_{\rm had}$
- new hadronic input functions that describe the momentum distribution of spectator quarks $\to$ light-cone distribution amplitudes for B-meson and K^*-mesons.

New ingredient: LCDA for the B-meson





where $\omega = n_- \cdot k_{\bar{q}}$

- Phenomenologically relevant:

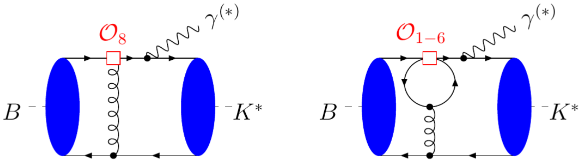
$$\langle \omega^{-1} \rangle_B = \int_0^\infty \frac{d\omega}{\omega} \phi_B(\omega) \approx 2 \text{ GeV}^{-1} \qquad \text{(at } \mu = \sqrt{m_b \Lambda} \simeq 1.5 \text{ GeV)}$$

(from QCD sum rules [Braun/Ivanov/Korchemsky])

(from HQET parameters [Lee/Neubert])

Further complications:

- spectator scattering with hadronic operators,
  when virtual photon is radiated from any of the internal quark lines !

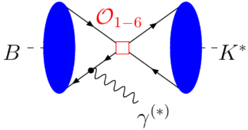


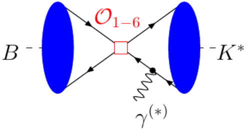
etc.

- hadronic input functions are the same as above (i.e. LCDAs)
- all internal dynamics is perturbative, as long as $m_b \ll \Lambda_{\rm had}$
- but power corrections $\sim \Lambda_{\rm had}/m_b$ may spoil the picture . . .

Further complications:

- also, some annihilation topologies are leading power
- no α_s suppression (but small Wilson coefficient and/or CKM factors)





etc.

- when a time-like photon is radiated from an internal quark line, it very much behaves like a vector meson (same quantum numbers)

Using dispersion relations / analyticity

Idea: **[Bobeth et al. 17]** (here: for γ-radiation of charm-loop only)

- make use of theoretical predictions for unphysical kinematics, with *space-like* momenta,

$$-m_b^2 \ll -q^2 \ll 0$$

- perform clever change of variables, encorporating the open-charm threshold

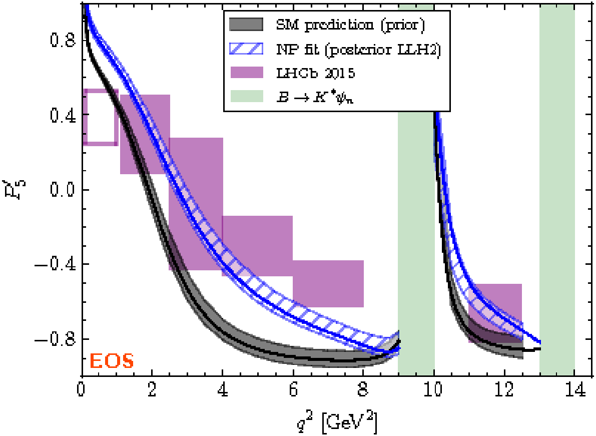
$$z(q^2) \equiv \frac{\sqrt{4M_D^2 - q^2} - \sqrt{4M_D^2 - t_0}}{\sqrt{4M_D^2 - q^2} + \sqrt{4M_D^2 - t_0}}$$

where the parameter t_0 can be chosen to minimize $|z|$ in a chosen q^2-interval.
(for instance $|z| < 0.52$ for $-7\ \text{GeV}^2 \leq q^2 \leq M^2_{\psi(2S)}$)

- truncated Taylor expansion in the new variable z
- include information from resonant decays $B \to J/\psi(\psi')[\to \ell^+\ell^-]K^*$

$\Rightarrow$ theory predictions can be extended to values $m_\rho^2 \ll q^2 < M^2_{\psi(2s)}$

E.g. the famous angular observable P_5':



[Bobeth/Chrzaszcz/van Dyk/Virto]

Summary

Weak b-quark decays described by Effective Hamiltonian:

- **Current-current** and **Penguin** and **Box** operators.
- **Wilson Coefficients** encode short-distance dynamics in SM or NP.
- QCD effects between m_W and m_b via **Renormalization-Group**.

Exclusive Amplitudes for semi-leptonic FCNC decays:

- **Hadronic Matrix Elements** of $\mathcal{O}_i$ contain QCD dynamics below m_b.
- **“Naive” Factorization** in terms of form factors.
- **Factorization Theorems**: soft and collinear modes in **HQET** / **SCET**.

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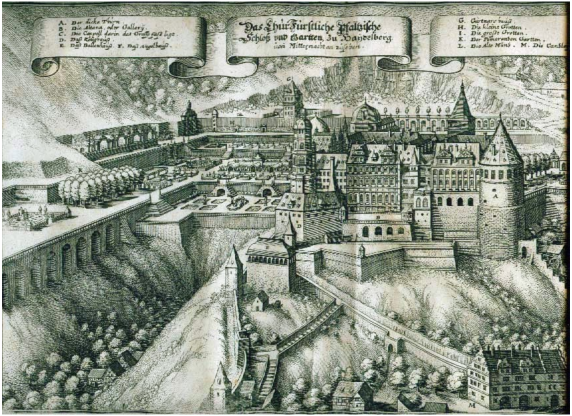
Summary

” When looking for New Physics in Beauty, . . .
. . . do not forget about the Beautiful Complexity of Old Physics ! ”



Backup Slides

From flavour anomalies to SMEFT

- Low-energy perspective: Explain current flavour anomalies by significant change of the Wilson coefficients C_9 and/or C_{10} at the electroweak scale.
- BSM perspective: $b \to s\ell^+\ell^-$ operators receive additional contributions from (virtual) exchange of new heavy particles, (e.g. of leptoquarks or Z'-bosons.

Model-independent approach → SMEFT

- use EFT framework, if new particles are much heavier than 200 GeV.
- construct effective operator basis from requiring manifest symmetry with respect to the SM gauge group, $SU(3)_C \times SU(2)_L \times U(1)_Y$.
- *all* SM particles (quarks, leptons, gauge bosons, Higgs doublet) may appear explicitly as fields in the effective operators.
- effect of new particles encoded in (a priori) unknown Wilson coefficients.

E.g. analogue of the operator O_9 written in SM-invariant manner as

$$-\frac{C_S^{ij\alpha\beta}}{\bar{\Lambda}_{\rm NP}^2}\,(\bar{Q}_L^i\gamma_\mu Q_L^j)(\bar{L}^\alpha\gamma^\mu L^\beta) \quad \text{or} \quad -\frac{C_T^{ij\alpha\beta}}{\bar{\Lambda}_{\rm NP}^2}\,(\bar{Q}_L^i\gamma_\mu\sigma^a Q_L^j)(\bar{L}^\alpha\gamma^\mu\sigma^a L^\beta)$$

where Q_L and L are $SU(2)_L$ quark and lepton doublets.

From $b \to cd\bar{u}$ to $\bar{B}^0 \to D^+\pi^-$

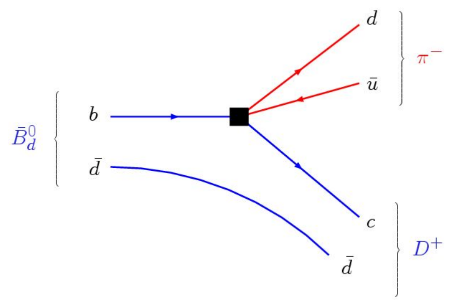
- In experiment, we cannot see the quark transition directly.
- Rather, we observe exclusive hadronic transitions, described by hadronic matrix elements, like e.g.

$$\langle D^+\pi^-|\mathcal{H}_{\rm eff}^{b\to cd\bar{u}}|\bar{B}_d^0\rangle = V_{cb}V_{ud}^* \frac{G_F}{\sqrt{2}} \sum_{i=1,2} C_i(\mu)\, r_i(\mu)$$

$$r_i(\mu) = \langle D^+\pi^-|\mathcal{O}_i|\bar{B}_d^0\rangle \Big|_\mu$$

- The hadronic matrix elements r_i contain QCD (and also QED) dynamics below the scale $\mu \sim m_b$.

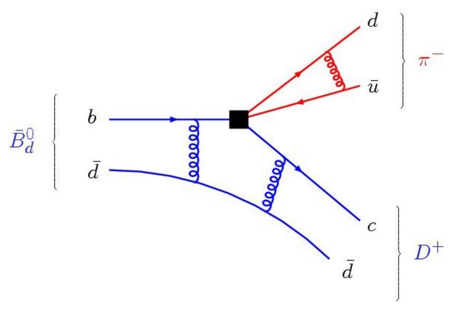
"Naive" Factorization of hadronic matrix elements



$$r_i = \underbrace{\langle D^+ | J_i^{(b\to c)} | \bar{B}_d^0 \rangle}\ \underbrace{\langle \pi^- | J_i^{(d\to u)} | 0 \rangle}$$

- Quantum fluctuations above $\mu \sim m_b$ already in Wilson coefficients

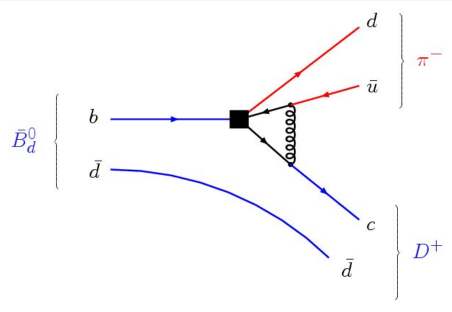
"Naive" Factorization of hadronic matrix elements



$$r_i = \underbrace{\langle D^+ | J_i^{(b\to c)} | \bar{B}_d^0 \rangle}_{\text{form factor}} \; \underbrace{\langle \pi^- | J_i^{(d\to u)} | 0 \rangle}_{\text{decay constant}}$$

- Part of (low-energy) gluon effects encoded in simple/universal had. quantities

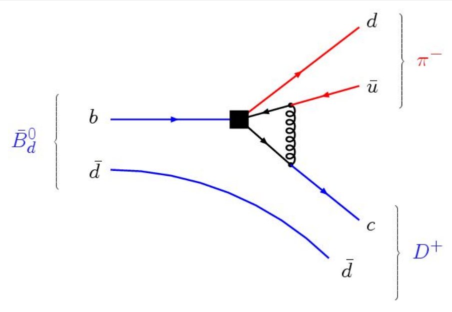
"Naive" Factorization of hadronic matrix elements



$$r_i = \underbrace{\langle D^+ | J_i^{(b\to c)} | \bar{B}_d^0 \rangle}\ \underbrace{\langle \pi^- | J_i^{(d\to u)} | 0 \rangle}$$

Question : Why is naive factorization not exact ?

"Naive" Factorization of hadronic matrix elements



$$r_i(\mu) = \underbrace{\langle D^+ | J_i^{(b\to c)} | \bar{B}_d^0 \rangle}\ \underbrace{\langle \pi^- | J_i^{(d\to u)} | 0 \rangle} + \text{corrections}(\mu)$$

Answer : Gluon cross-talk between π^- and $B \to D$

QCD factorization

[Beneke/Buchalla/Neubert/Sachrajda 2000]

- light quarks in π^- have large energy (in B rest frame)
- gluons from the $B \to D$ transition see "small colour-dipole"

$\Rightarrow$ corrections to naive factorization dominated by gluon exchange at short distances $\delta x \sim 1/m_b$

New feature: Light-cone distribution amplitudes $\phi_\pi(u)$

- Short-distance corrections to naive factorization given as convolution

$$r_i(\mu) \simeq \sum_j F_j^{(B\to D)} \int_0^1 du \left(1 + \frac{\alpha_s C_F}{4\pi} t_{ij}(u,\mu) + \ldots\right) f_\pi \, \phi_\pi(u,\mu)$$

- $\phi_\pi(u)$: distribution of momentum fraction u of a quark in the pion.
- $t_{ij}(u,\mu)$: perturbative coefficient function (depends on u)
- $F_j^{(B\to D)}$: form factors known from $B \to D\ell\nu$

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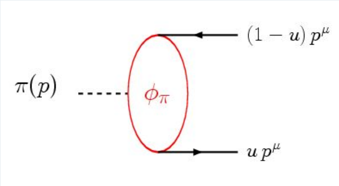
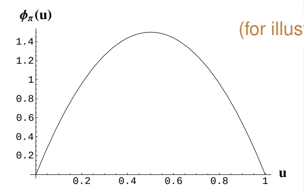
Light-cone distribution amplitude for the pion





(for illustration only)

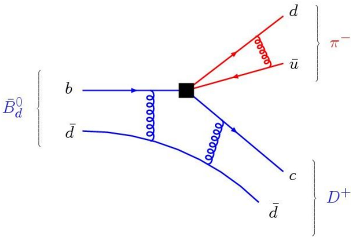
- Exclusive analogue of parton distribution function:
  - PDF: probability density (all Fock states)
  - LCDA: probability amplitude (one Fock state, here: $q\bar{q}$)
- Phenomenologically relevant

$$\langle u^{-1}\rangle_\pi = \int_0^1 \frac{du}{u}\,\phi_\pi(u) \simeq 3.3 \pm 0.3$$

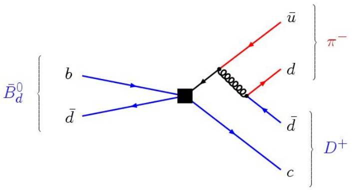
[from sum rules, lattice, exp.]

Complication: Annihilation in $\bar{B}_d \to D^+\pi^-$

Second topology for hadronic matrix element possible:



"Tree" (class-I)

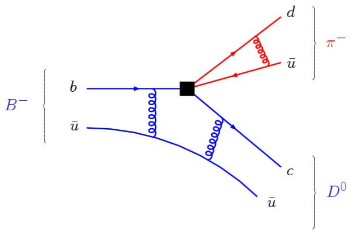


"Annihilation" (class-III)

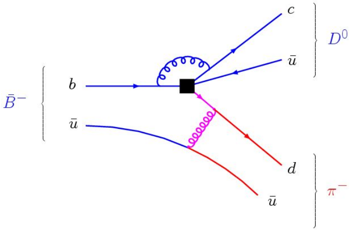
- annihilation is formally power-suppressed by $\Lambda_{\rm had}/m_b$
- more difficult to estimate (colour-dipole argument does not apply!)

Still more complicated: $B^- \to D^0 \pi^-$

Second topology with spectator quark going into light meson:



"Tree" (class-I)

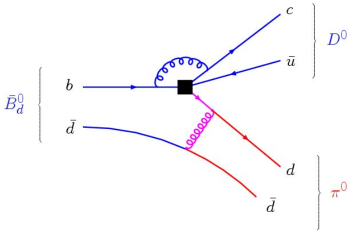


"Tree" (class-II)

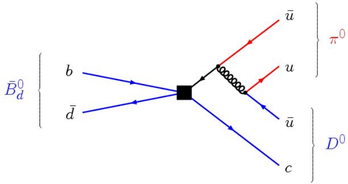
- class-II amplitude does not factorize into simpler objects
  (again, colour-transparency argument does not apply)
- again, it is power-suppressed compared to class-I topology

Non-factorizable: $\bar{B}^0 \to D^0\pi^0$

In this channel, class-I topology is absent:



"Tree" (class-II)



"Annihilation" (class-III)

- The whole decay amplitude is power-suppressed!
- Naive factorization is not even a first-order approximation!

Isospin analysis for $B \to D\pi$

- Employ isospin symmetry between (u, d) of strong interactions.
- Final-state with π ($I = 1$) and D ($I = 1/2$) described by only two isospin amplitudes:

$$\begin{aligned} \mathcal{A}(\bar{B}_d \to D^+\pi^-) &= \sqrt{\frac{1}{3}}\,\mathcal{A}_{3/2} + \sqrt{\frac{2}{3}}\,\mathcal{A}_{1/2}\,, \\ \sqrt{2}\,\mathcal{A}(\bar{B}_d \to D^0\pi^0) &= \sqrt{\frac{4}{3}}\,\mathcal{A}_{3/2} - \sqrt{\frac{2}{3}}\,\mathcal{A}_{1/2}\,, \\ \mathcal{A}(B^- \to D^0\pi^-) &= \sqrt{3}\,\mathcal{A}_{3/2}\,, \end{aligned}$$

- QCDF: $\mathcal{A}_{1/2}/\mathcal{A}_{3/2} = \sqrt{2} +$ corrections, relative strong phase $\Delta\theta$ small

Isospin amplitudes from experimental data [Fleischer et al., arXiv:1012.2784]

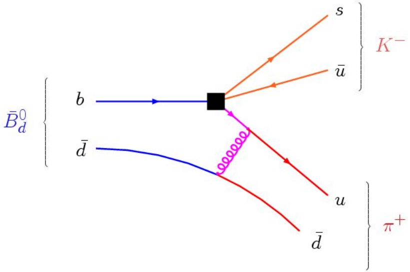
$$\left|\frac{\mathcal{A}_{1/2}}{\sqrt{2}\,\mathcal{A}_{3/2}}\right| = 0.676 \pm 0.038\,, \qquad \cos\Delta\theta = 0.930^{+0.024}_{-0.022}$$

(similar for $B \to D^*\pi$)

$\to$ Corrections to QCDF sizeable — Strong phases remain small

$B \to \pi\pi$ and $B \to \pi K$

Naive factorization:



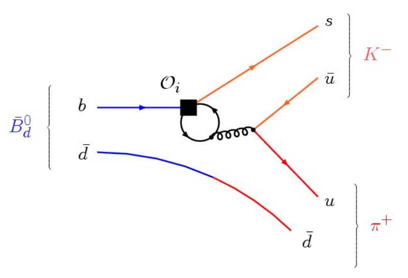
- Both final-state mesons are light and energetic.
- Colour-transparency argument applies for class-I and class-II topologies.
- $B \to \pi(K)$ form factors at large recoil fairly well known (QCD sum rules)

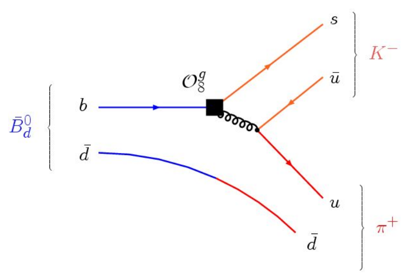
QCDF for $B \to \pi\pi$ and $B \to \pi K$ decays (BBNS 1999)

Factorization formula has to be extended:

- Vertex corrections are treated as in $B \to D\pi$
  - Include penguin (and electroweak) operators from $H_{\rm eff}$.
  - Take into account new (long-distance) penguin diagrams! (→ Fig.)
- Additional perturbative interactions involving spectator in B-meson (→ Fig.)
  - Sensitive to the distribution of the spectator momentum ω
    $\longrightarrow$ light-cone distribution amplitude $\phi_B(\omega)$

Additional diagrams for QCDF corrections in $B \to \pi K$ (example)

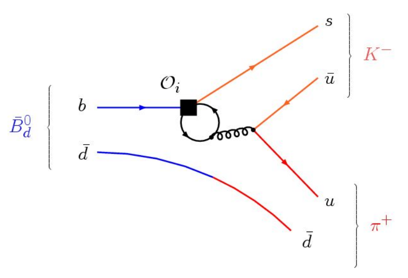


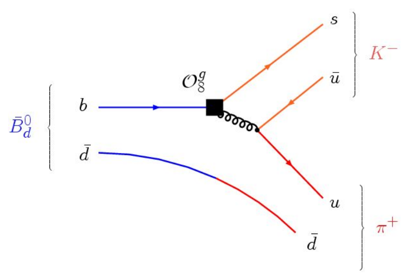


$\longrightarrow$ additional contributions to the hard coefficient functions $t_{ij}(u,\mu)$

$$r_i(\mu)\Big|_{\rm hard} \simeq \sum_j F_j^{(B\to\pi)}(m_K^2) \int_0^1 du \left(1 + \frac{\alpha_s}{4\pi}\, t_{ij}(u,\mu) + \ldots\right) f_K\, \phi_K(u,\mu)$$

Additional diagrams for QCDF corrections in $B \to \pi K$ (example)





$\longrightarrow$ additional contributions to the hard coefficient functions $t_{ij}(u, \mu)$

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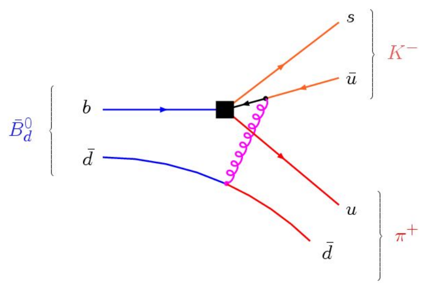
Spectator corrections in QCDF



$\longrightarrow$ additive correction to naive factorization

$$\Delta r_i(\mu)\Big|_{\text{spect.}} = \int du\, dv\, d\omega \left(\frac{\alpha_s}{4\pi} h_i(u, v, \omega, \mu) + \ldots \right) \times f_K\, \phi_K(u,\mu)\, f_\pi\, \phi_\pi(v,\mu)\, f_B\, \phi_B(\omega,\mu)$$

Distribution amplitudes for all three mesons involved!

Complications for QCDF in $B \to \pi\pi, \pi K$ etc.

- Annihilation topologies are numerically important. BBNS use conservative model estimates.
- Some power-corrections are numerically enhanced by "chiral factor"

$$\frac{\mu_\pi}{f_\pi} = \frac{m_\pi^2}{2 f_\pi \, m_q}$$

- Many decay topologies interfere with each other.
- Many hadronic parameters to vary.

$\to$ Depending on specific mode, hadronic uncertainties sometimes quite large.