

# Quark Flavor Physics: Mixing & CPV

## Outline:

- Quark Flavor Physics
- Neutral Meson Mixing
- CP Violation in Interference between Mixing and Decay
- CP Violation in Mixing
- A words to direct CPV and  $\gamma$

# What is Flavor Physics?

Three generations carry the same charges under the Standard Model gauge group  $SU(3)_c \times SU(2)_L \times U(1)$ :

Leptons			Quarks		
$e$	$\mu$	$\tau$	$uuu$	$ccc$	$ttt$
$\nu_e$	$\nu_\mu$	$\nu_\tau$	$ddd$	$sss$	$bbb$

**Flavor is the feature** that distinguishes the generations.

# Quark Flavor within the Standard Model

Yukawa interaction couples fermions to Higgs. For the quarks:

$$\mathcal{L}_Y^{\text{quarks}} = -\frac{v}{\sqrt{2}} \left( \bar{d}_L Y_d d_R + \bar{u}_L Y_u u_R \right) + \text{h.c}$$

After electroweak  
symmetry breaking

$Y_d, Y_u$  are  $3 \times 3$  complex matrices in generation space

not diagonal  $\rightarrow$  flavor structure

Mass eigenstates of the quarks obtained by unitary transformations:

$$\tilde{u}_A = V_{A,u} u_A \quad \tilde{d}_A = V_{A,d} d_A \quad \text{and} \quad A = L, R \quad \text{where} \quad V_{A,q} V_{A,q}^\dagger = 1$$

$V_{A,q}$  are determined by requiring that the matrices  $M_{d,u}$  are diagonal:

$$M_d = \text{diag}(m_d, m_s, m_b) = \frac{v}{\sqrt{2}} V_{L,d} Y_d V_{R,d}^\dagger$$

# Quark masses

After this transformation quark masses appear as usual Dirac terms:

$$\mathcal{L}_Y^{\text{quarks}} = -\bar{\tilde{d}}_L M_d \tilde{d}_R - \bar{\tilde{u}}_L M_u \tilde{u}_R + \text{h.c.}$$

Up-type and down-type quarks cannot be diagonalized by the same matrix, i.e.  $V_{A,d} \neq V_{A,u}$ .  $\rightarrow$  net effect on flavor structure of charged current.

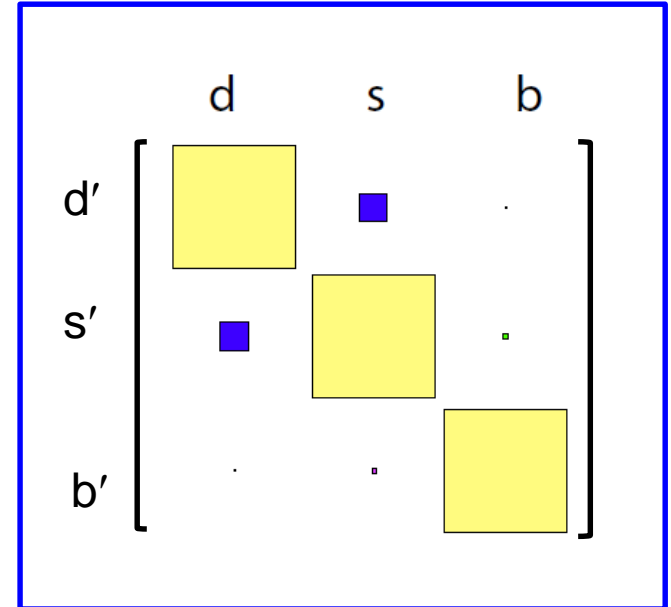
$$\mathcal{L}_{CC} = -\frac{g_2}{\sqrt{2}} \left( \bar{\tilde{u}}_L \gamma^\mu W_\mu^+ V_{CKM} \tilde{d}_L + \bar{\tilde{d}}_L \gamma^\mu W_\mu^- V_{CKM}^\dagger \tilde{u}_L \right)$$

$$\text{with } V_{CKM} = V_{L,u} V_{L,d}^\dagger \quad (\text{must be unitary})$$

# CKM Matrix

Complex and unitary 3×3 matrix:

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$



Complex 3×3 matrix: 18 parameters  
+ unitarity condition (9 parameters)  
+ removal of 5 unobservable quark phases  
→ 4 free parameter:  
**3 Euler angles and one phase  $\delta$**

# Unobservable Phases

Absolute phases of quarks are unobservable: possible redefinition

$$\begin{array}{lll}
 u_L \rightarrow e^{i\phi(u)} u_L & c_L \rightarrow e^{i\phi(c)} c_L & t_L \rightarrow e^{i\phi(t)} t_L \\
 d_L \rightarrow e^{i\phi(d)} d_L & s_L \rightarrow e^{i\phi(s)} s_L & b_L \rightarrow e^{i\phi(b)} b_L
 \end{array}$$

$\uparrow$   
 Real numbers

Under phase transformation:

$$V \rightarrow \begin{pmatrix} e^{-i\phi(u)} & 0 & 0 \\ 0 & e^{-i\phi(c)} & 0 \\ 0 & 0 & e^{-i\phi(t)} \end{pmatrix} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} e^{i\phi(d)} & 0 & 0 \\ 0 & e^{i\phi(s)} & 0 \\ 0 & 0 & e^{i\phi(b)} \end{pmatrix}$$

$$V_{\alpha j} \rightarrow \exp[i(\phi(j) - \phi(\alpha))] V_{\alpha j}$$

$L^{phys}(f, G)$  invariant

$L(f, H)$  affected .... rephasing  $q_R$

# CP violation

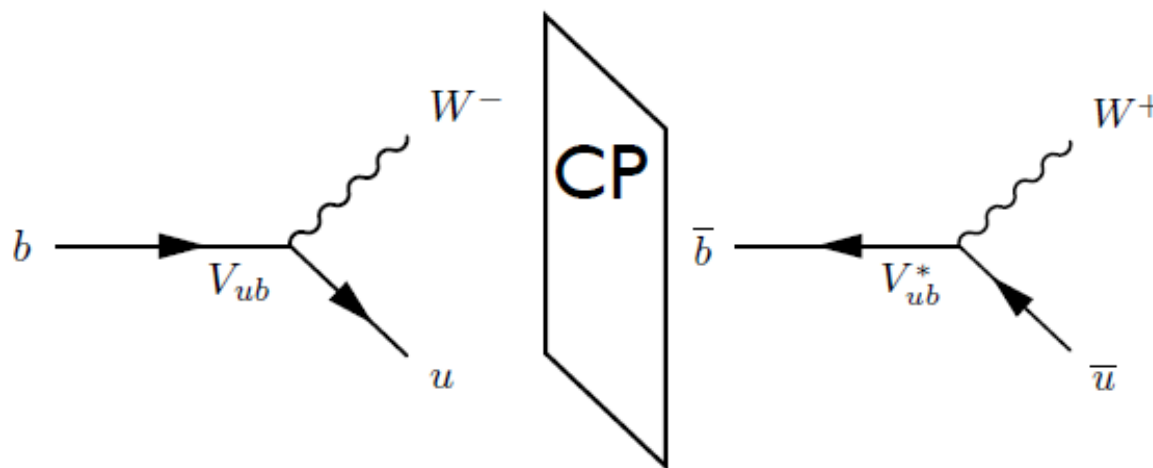
CP violation if  $V_{CKM}$  is complex:

$$\mathcal{L} = -\frac{g_2}{\sqrt{2}} \left[ V_{ub} \bar{u}_L \gamma^\mu b_L W_\mu^+ + V_{ub}^* \bar{b}_L \gamma^\mu u_L W_\mu^- \right]$$

// // // // //

$$\mathcal{L}_{CP} = -\frac{g_2}{\sqrt{2}} \left[ V_{ub} \bar{b}_L \gamma^\mu u_L W_\mu^- + V_{ub}^* \bar{u}_L \gamma^\mu b_L W_\mu^+ \right]$$

CP and T are anti-unitary operators  
 → complex conjugation

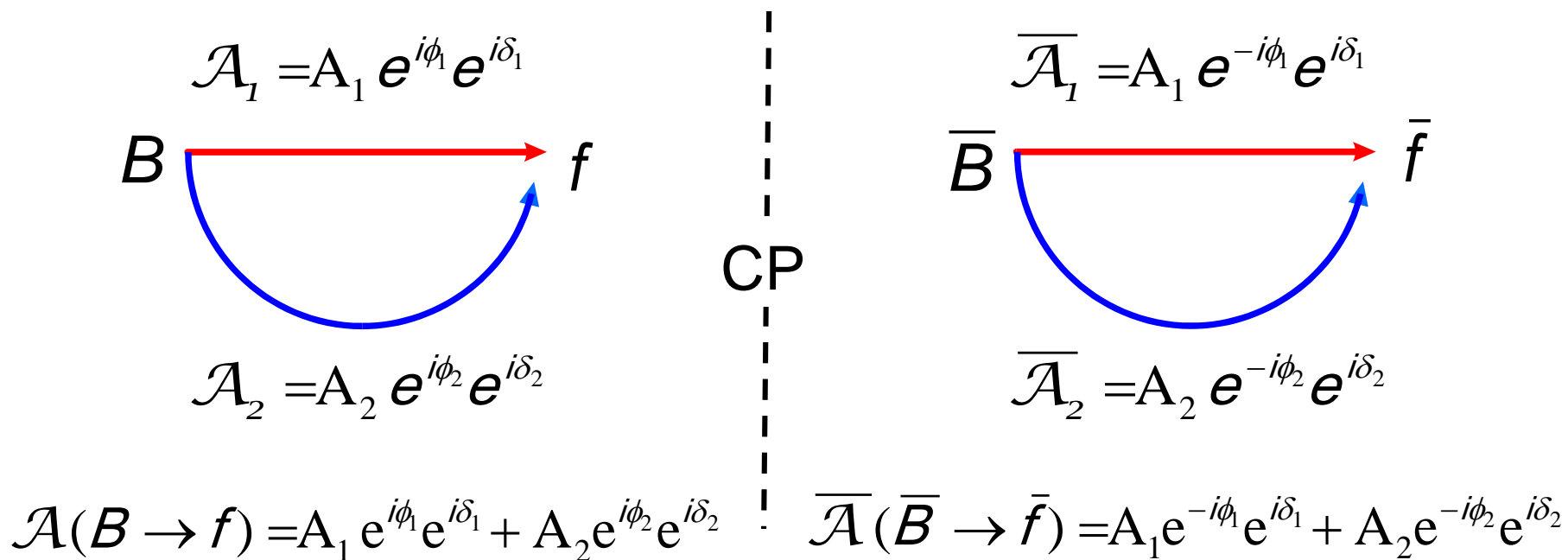


CP (T) violation possible if  $V_{ji} \neq V_{ji}^*$

# CP Violation in meson decays

CKM phase do not lead easily to measurable CPV asymmetries.

To observe CP violation needs at least two amplitudes with different weak (sign flip under CP) and different strong (invariant under CP) amplitudes:



$$|\bar{\mathcal{A}}|^2 - |\mathcal{A}|^2 = 4 A_1 A_2 \sin(\phi_1 - \phi_2) \sin(\delta_1 - \delta_2)$$



# Wolfenstein Parametrization

Reflects the hierarchical structure of the CKM matrix

$\lambda, A, \rho, \eta$  with  $\lambda = 0.22$

$|V_{ub}| \times e^{-i\gamma}$

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4)$$

$|V_{td}| \times e^{-i\beta}$

$$V_{CKM} = \begin{pmatrix} 1 - \frac{\lambda^2}{2} - \frac{\lambda^4}{8} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda + A^2\lambda^5\left(\frac{1}{2} - \rho - i\eta\right) & 1 - \frac{\lambda^2}{2} - \frac{\lambda^4}{8}(1 + 4A^2) & A\lambda^2 \\ A\lambda^3(1 - \bar{\rho} - i\bar{\eta}) & -A\lambda^2 + A\lambda^4(1/2 - \rho - i\eta) & 1 - A^2\lambda^4/2 \end{pmatrix} + O(\lambda^6)$$

$|V_{ts}| \times e^{-i\beta_s}$

# Unitarity of CKM Matrix

$$V_{\text{CKM}}^\dagger V_{\text{CKM}} = 1$$

$$\begin{pmatrix} V_{ud}^* & V_{cd}^* & V_{td}^* \\ V_{ub}^* & V_{cb}^* & V_{tb}^* \\ V_{ub} & V_{cb} & V_{tb} \end{pmatrix} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{ud} & V_{us} & V_{ub} \\ V_{ud} & V_{us} & V_{ub} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Two definitions of  $\beta$  and  $\gamma$ . Which one is the correct definition, invariant against different phase conventions?

$$\Rightarrow V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0$$

Im  
▲

Unitarity triangle „db“

$(\bar{d} \bar{b})$

CKM Phases  $b \rightarrow u$

↓

$$\alpha \equiv \arg \left[ -\frac{V_{td} V_{tb}^*}{V_{ud} V_{ub}^*} \right] \quad \beta \equiv \arg \left[ -\frac{V_{cd} V_{cb}^*}{V_{td} V_{tb}^*} \right] \quad \gamma \equiv \arg \left[ -\frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*} \right]$$

Re →  $V_{cd} V_{cb}$

**CP Violation if Triangle has finite area !**

# More Triangles ...

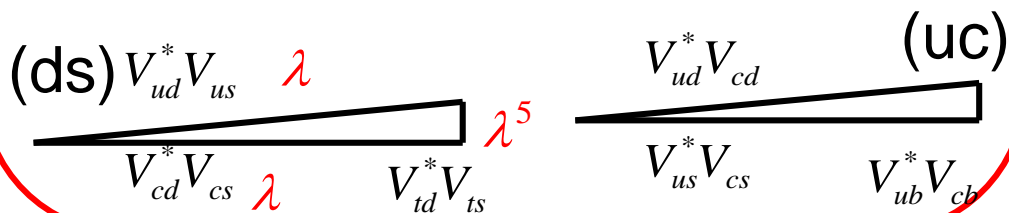
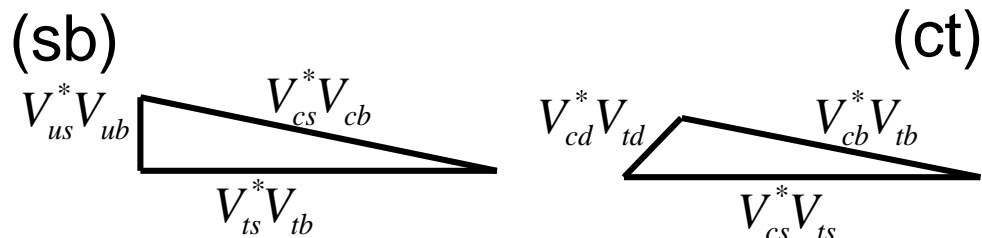
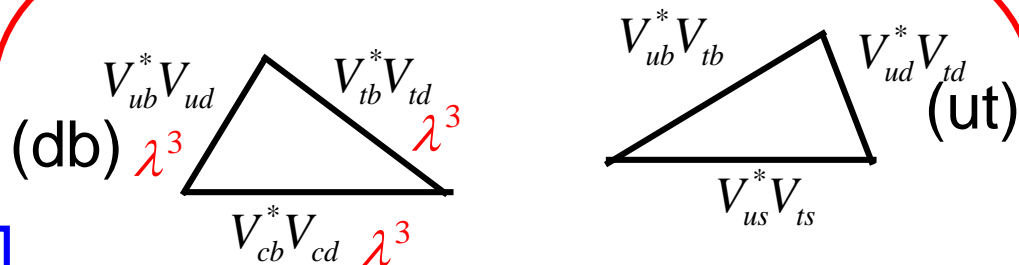
$$V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0 \text{ (db)}$$

$$V_{us} V_{ub}^* + V_{cs} V_{cb}^* + V_{ts} V_{tb}^* = 0 \text{ (sb)}$$

$$\begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

$$V_{cd} V_{td}^* + V_{cs} V_{ts}^* + V_{cb} V_{tb}^* = 0 \text{ (ct)}$$

$$V_{ud} V_{cd}^* + V_{us} V_{cs}^* + V_{ub} V_{cb}^* = 0 \text{ (uc)}$$



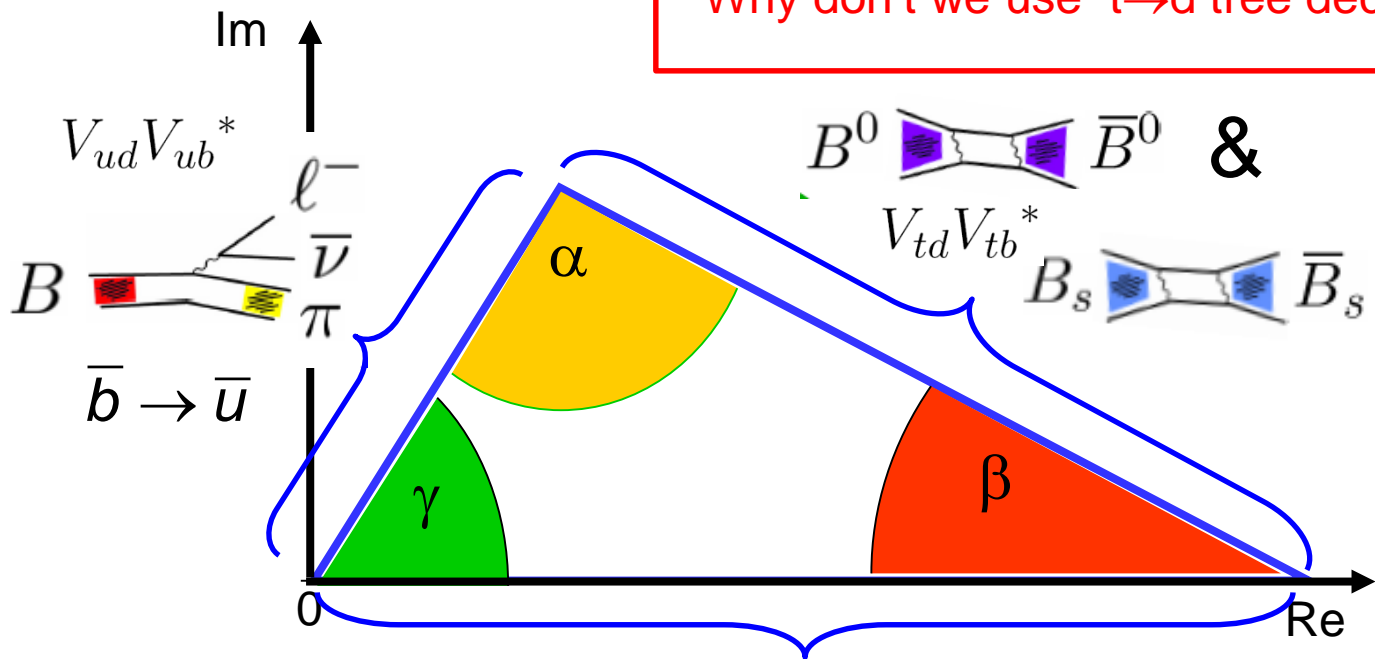
All 6 triangles have the same area:  $J_{CP}/2$

$J_{CP}$  is called Jarlskog invariant, it is a measure of CPV in Standard Model.

$$J_{CP} = \text{Im} \left( V_{ij} V_{kl} V_{il}^* V_{kj}^* \right) \approx 3 \cdot 10^{-5} \sim O(\lambda^6)$$

# Unitarity Triangle from B Decays

Why don't we use  $t \rightarrow d$  tree decays?



Sides from CP conserving observables

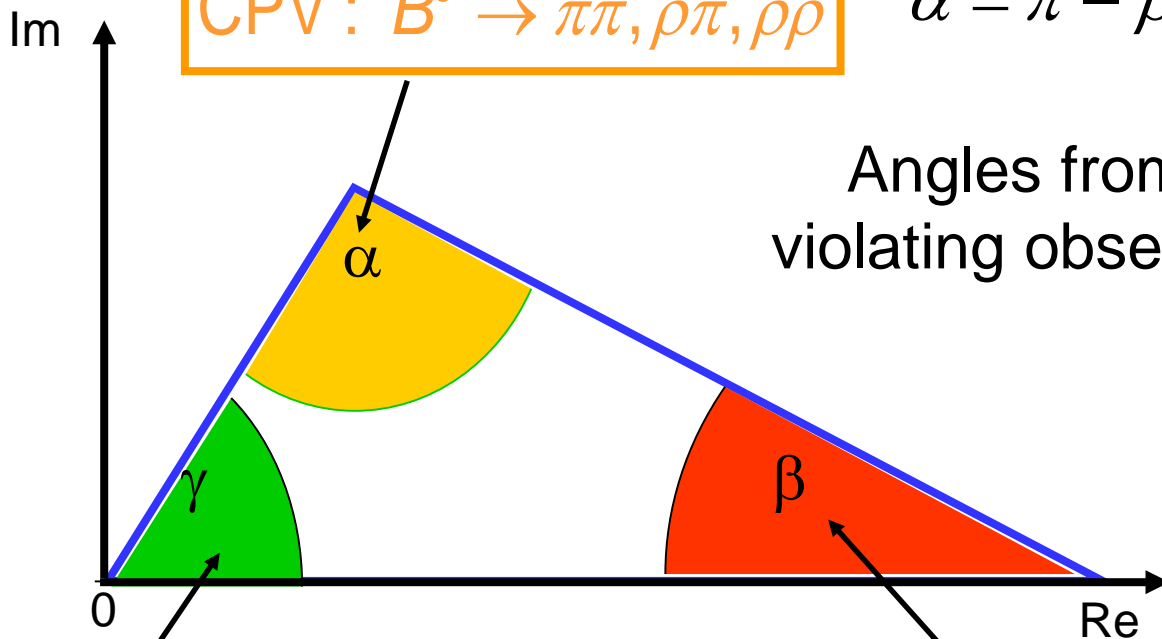
# Unitarity Triangle from B Decays

(time-dependent)

CPV :  $B^0 \rightarrow \pi\pi, \rho\pi, \rho\rho$

$$\alpha = \pi - \beta - \gamma$$

Angles from CP violating observables



CPV :  $B^0 \rightarrow DK^{(*)}, DK_S^0, K\pi, D^*\pi$   
 $B_S^0 \rightarrow D_S K, KK$

(time integrated)

CPV :  $B^0 \rightarrow J/\psi K_S^0$

(time-dependent)

# Neutral Meson Mixing

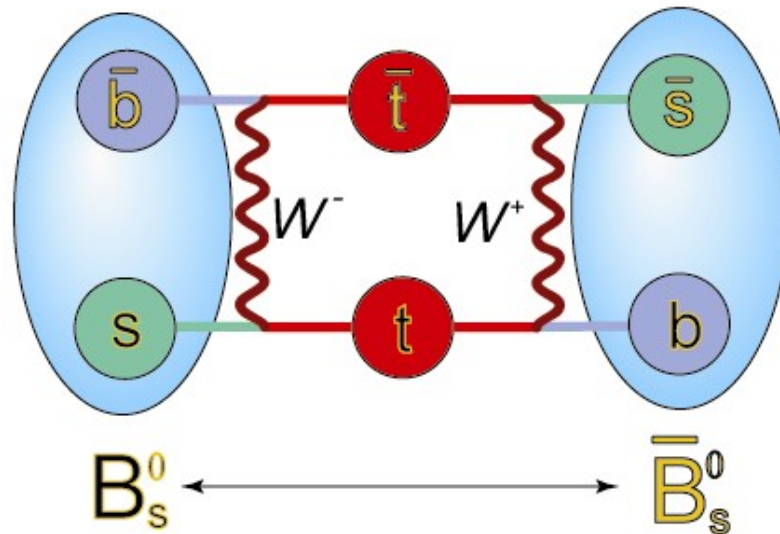
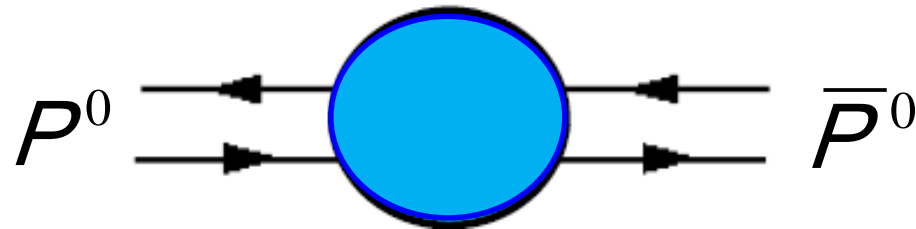


Figure from <http://www.gridpp.ac.uk/news/?p=205>

# Mixing Phenomenology



$$i \frac{d}{dt} \begin{pmatrix} |P^0(t)\rangle \\ |\bar{P}^0(t)\rangle \end{pmatrix} = \left( \mathbf{M} - \frac{i}{2} \mathbf{\Gamma} \right) \begin{pmatrix} |P^0(t)\rangle \\ |\bar{P}^0(t)\rangle \end{pmatrix}$$

No mass eigenstates

Non-hermitian  $\rightarrow P^0$  decays

CPT

$$m_{11} = m_{22} = m$$

$$\Gamma_{11} = \Gamma_{22} = \Gamma$$

$$\begin{pmatrix} M - \frac{i}{2} \Gamma & M_{12} - \frac{i}{2} \Gamma_{12} \\ M_{12}^* - \frac{i}{2} \Gamma_{12}^* & M - \frac{i}{2} \Gamma \end{pmatrix}$$

$\mathbf{M}$  and  $\mathbf{\Gamma}$  hermitian:

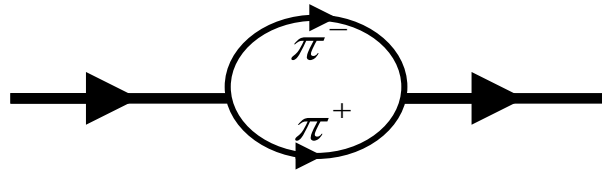
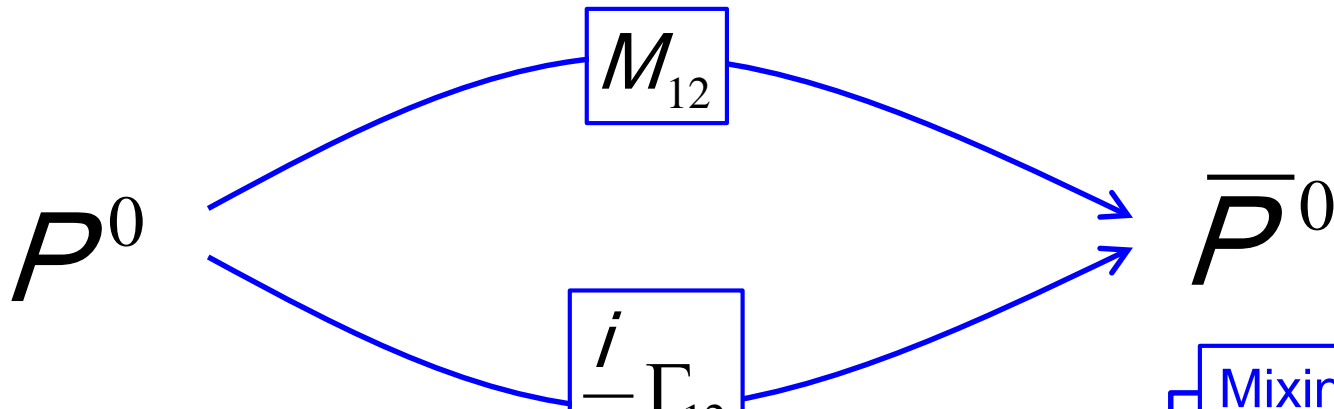
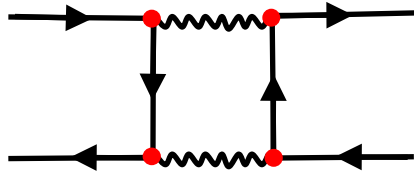
$$m_{21} = m_{12}^*$$

$$\Gamma_{21} = \Gamma_{12}^*$$

Off – diagonal elements describe the mixing.

# Mixing Phenomenology

„short distant, virtual states“



„long distant, on-shell states“

for  $K^0$  very important, for  $B^0$  small

Mixing parameters

$$|M_{12}| \quad |\Gamma_{12}|$$

$$\phi_{M/\Gamma} = \arg\left(-\frac{M_{12}}{\Gamma_{12}}\right)$$



# Mass eigenstates

Mass eigenstates are obtained by diagonalizing the matrix:

$$\begin{aligned} |P_a\rangle &= p|P^0\rangle + q|\overline{P^0}\rangle & \text{with } m_a, \Gamma_a & \implies |P_a(t)\rangle = e^{-im_a t} \cdot e^{-\frac{1}{2}\Gamma_a t} |P_a(0)\rangle \\ |P_b\rangle &= p|P^0\rangle - q|\overline{P^0}\rangle & \text{with } m_b, \Gamma_b & \implies |P_b(t)\rangle = e^{-im_b t} \cdot e^{-\frac{1}{2}\Gamma_b t} |P_b(0)\rangle \end{aligned}$$

complex coefficients  $|p|^2 + |q|^2 = 1$

$P_a$  and  $P_b$  are not necessary orthogonal:  $\langle P_b | P_a \rangle = |p|^2 - |q|^2 \neq 0$

The mass (physical) states are usually labeled by the properties which distinguish them the best:  $K_S, K_L$ ;  $B_H, B_L$ ;  $D_1, D_2$ ;

For  $p = q = 1/\sqrt{2}$ :  $P_a = P_1$  (CP +) ,  $P_b = P_2$  (CP -)

# Mixing Parameters

$$\Delta m = m_b^{\text{H}} - m_a^{\text{L}} \quad \Delta\Gamma = \Gamma_b^{\text{H}} - \Gamma_a^{\text{L}} \quad \Delta\Gamma = \Gamma_a^{\text{L}} - \Gamma_b^{\text{H}}$$

$$m = \frac{1}{2}(m_b + m_a) \quad \Gamma = \frac{1}{2}(\Gamma_a + \Gamma_b)$$

$$\frac{q}{p} = \pm \sqrt{\frac{H_{21}}{H_{12}}} = \pm \sqrt{\frac{2M_{12}^* - i\Gamma_{12}^*}{2M_{12} - i\Gamma_{12}}}$$

- The sign of  $q/p$  determines whether  $m_a$  or  $m_b$  is heavier: the usual choice is  $\Delta m > 0$ :  $q/p > 0 \Rightarrow$  “+” sign.
- Attention: this conventions is not fixing the sign of  $\Delta\Gamma$ .  
The experiment has to tell whether CP even/odd lives longer.

for B mesons:

$$\Delta m = 2|M_{12}|$$

$\Gamma_{12} \ll M_{12}$ :

$$\Delta\Gamma = 2|\Gamma_{12}|\cos\varphi_{M/\Gamma}$$

$\Delta\Gamma$  small

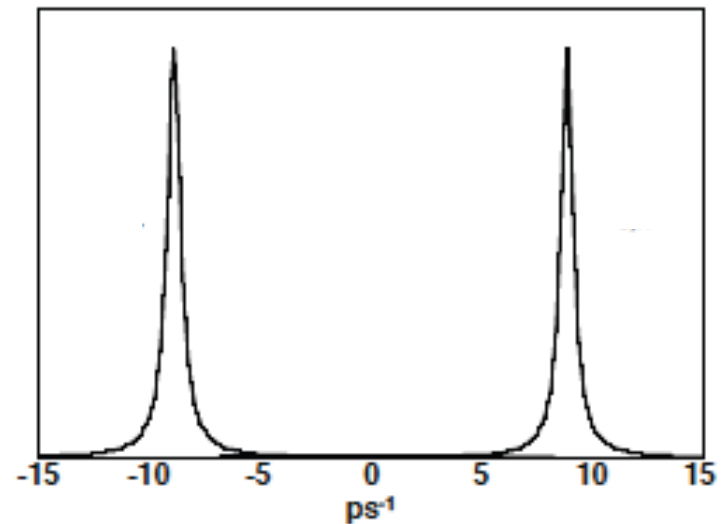
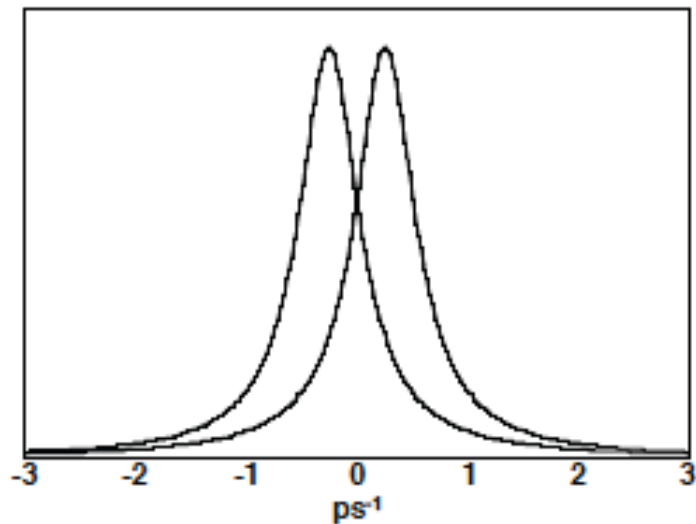
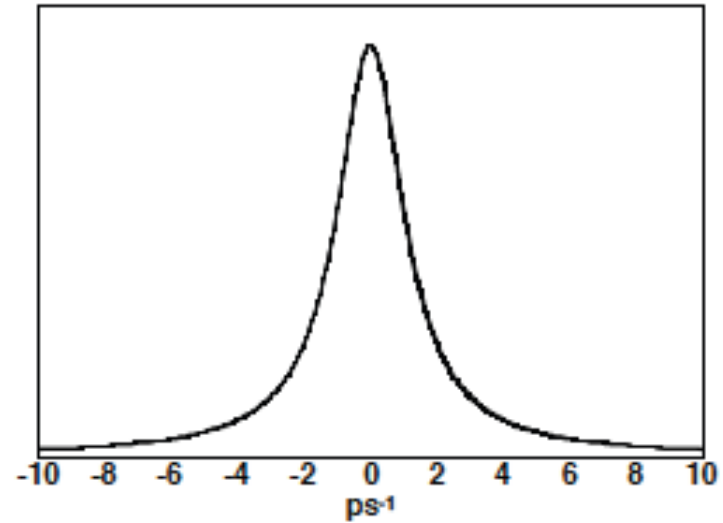
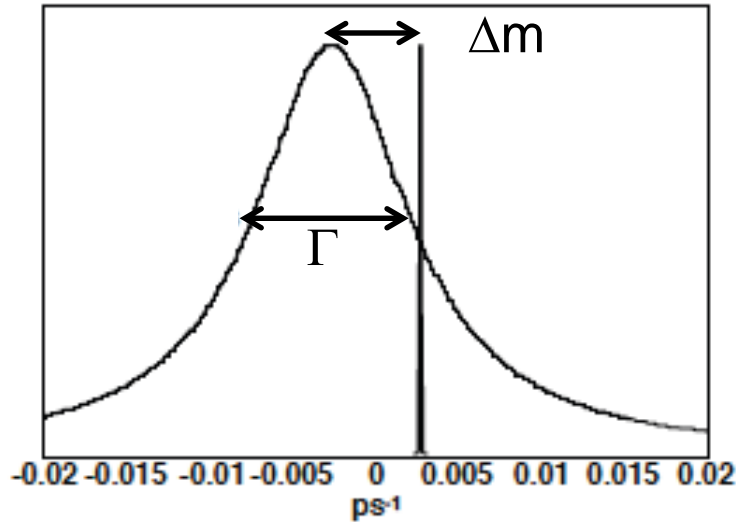
$$\Delta m = 2|M_{12}|$$

$$|q/p| \approx 1$$

# Neutral Mesons $K^0, D^0, B^0, B_s^0$ ?

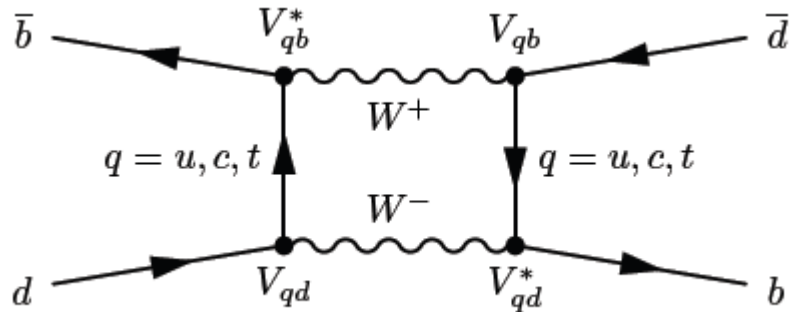
Slide from  
M. Gersabeck

Labeling of physical states: heavy/light, short/long, CP-even/CP-odd



# Theoretical predictions $\Delta m_d$

Question: What happens if quarks have the same mass?



$$\begin{aligned}
 t - \bar{t} &: \quad \propto m_t^2 |V_{tb} V_{td}^*|^2 \quad \propto m_t^2 \lambda^6 \\
 c - \bar{c} &: \quad \propto m_c^2 |V_{cb} V_{cd}^*|^2 \quad \propto m_c^2 \lambda^6 \\
 c - \bar{t}, \bar{c} - t &: \quad \propto m_c m_t V_{tb} V_{td}^* V_{cb} V_{cd}^* \quad \propto m_c m_t \lambda^6
 \end{aligned}$$

u quark can be replaced using unitarity  $V_{CKM}$

$$M_{12} \approx \frac{G_F^2}{12\pi^2} (V_{td}^* V_{tb})^2 M_W^2 S_0(x_t) B_B f_B^2 M_B \eta_B$$

$$\Delta m \approx 2 |M_{12}|$$

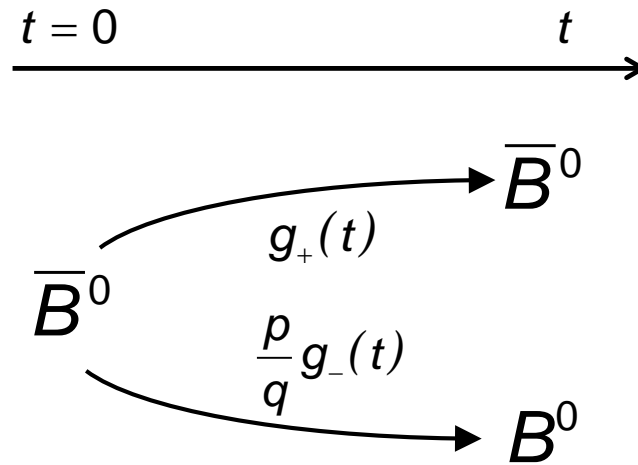
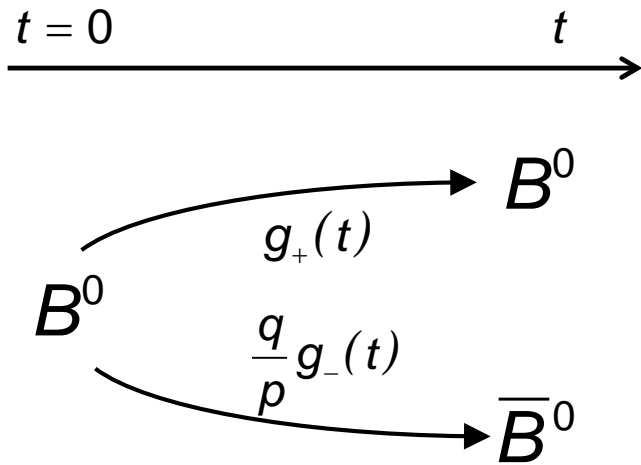
$$B_S: \quad (V_{ts}^* V_{tb})^2 \sim \lambda^4 \text{ about } \times 25 \text{ larger}$$

$S_0(m_t^2/m_W^2)$  = Loop-function (Inami-Lim) = result of box diagramm.

$B_B f_B^2$  = non-perturbative hadronic effects

$\eta_B$  = perturbative QCD corrections

# Time evolution of $B^0$ ( $P^0$ )



$$|B^0\rangle = \frac{1}{2p}(|B_L\rangle + |B_H\rangle)$$

$$|\bar{B}^0\rangle = \frac{1}{2q}(|B_L\rangle - |B_H\rangle)$$

CP Violation  
in mixing:

$$\left| \frac{q}{p} \right| \neq 1$$

# Mixing phenomenology

Mixed/ unmixed probability:  $\Delta\Gamma \approx 0$

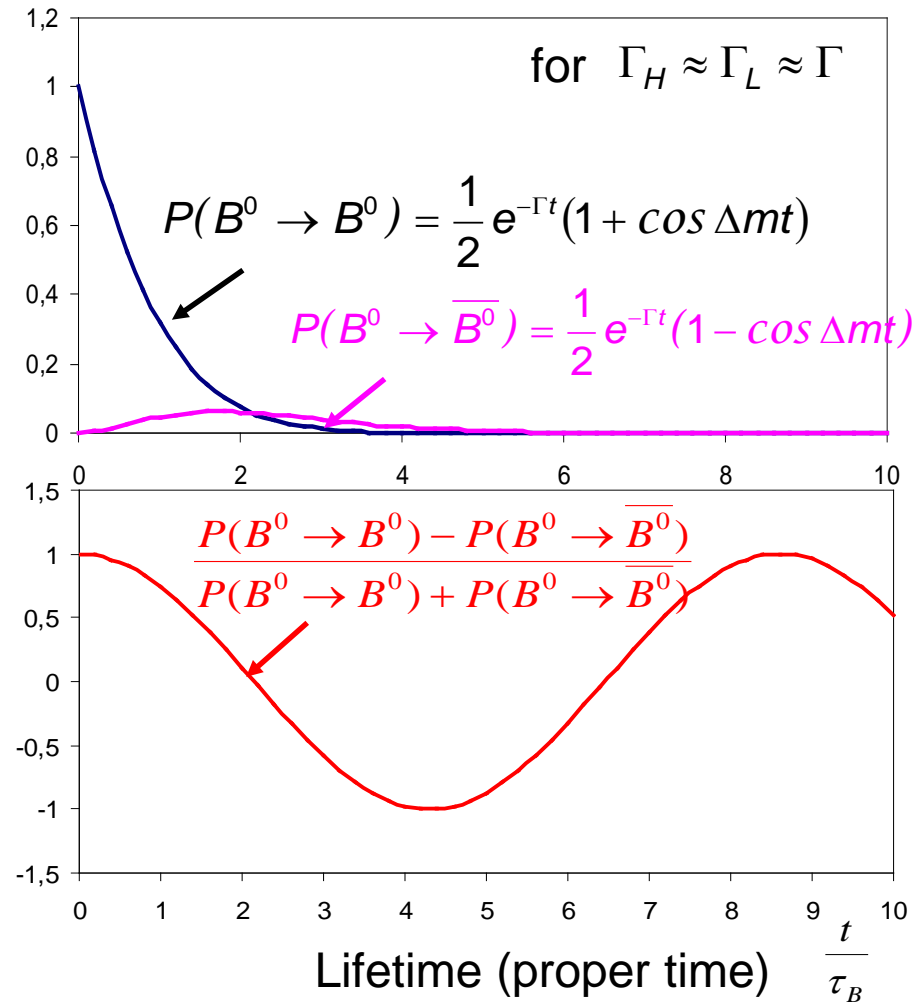
$$\mathcal{P}(B^0 \rightarrow B^0, t) = \left| \langle B^0 | B^0(t) \rangle \right|^2 = \frac{e^{-\Gamma t}}{2} (1 + \cos(\Delta m t))$$

$$\mathcal{P}(B^0 \rightarrow \bar{B}^0, t) = \left| \langle B^0 | \bar{B}^0(t) \rangle \right|^2 = \frac{e^{-\Gamma t}}{2} \left| \frac{q}{p} \right|^2 (1 - \cos(\Delta m t))$$

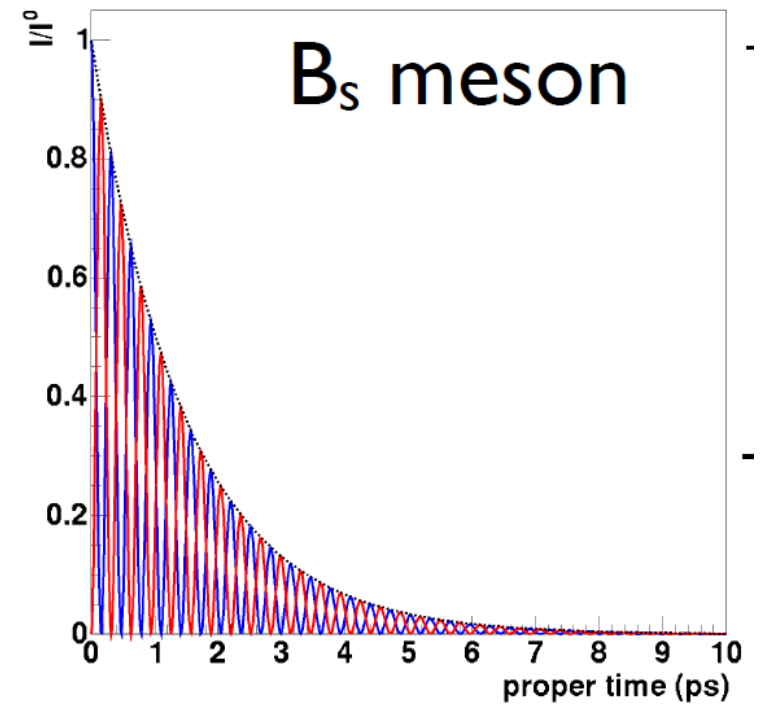
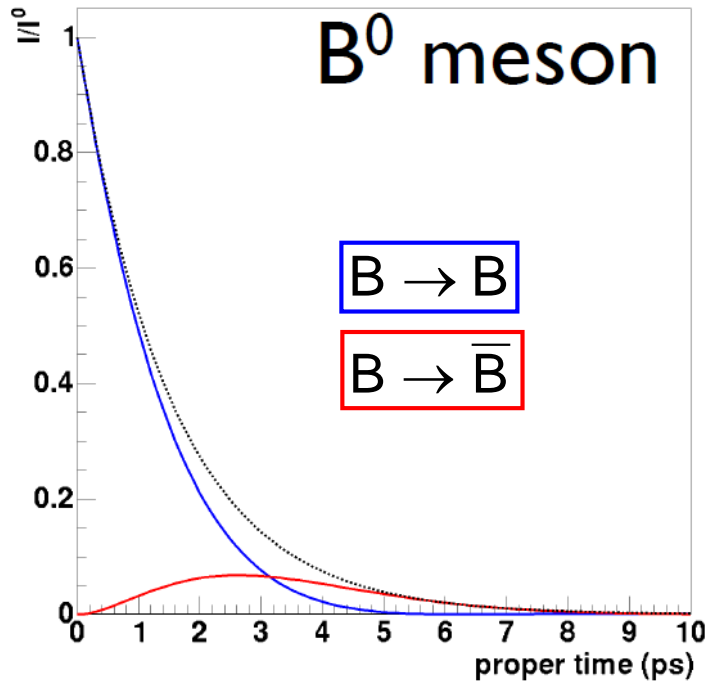
Mixing asymmetry:

$$A(t) = \frac{\text{unmixed}(t) - \text{mixed}(t)}{\text{unmixed}(t) + \text{mixed}(t)} = \cos(\Delta m t) \quad \text{If } |q/p| = 1$$

# Time dependent mixing asymmetry



# B meson mixing

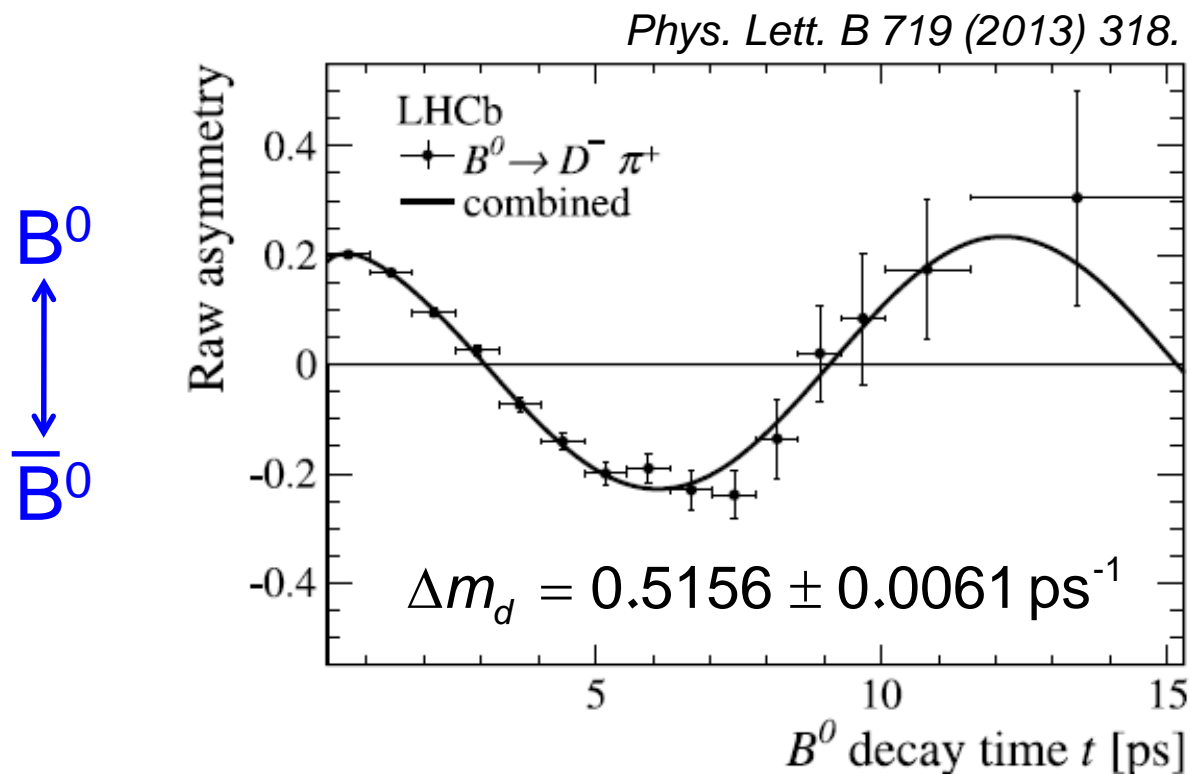


$$M_{12,q} = \frac{G_F^2}{12\pi^2} (V_{tq}^* V_{tb})^2 M_W^2 S_0(x_t) B_{B_q} f_{B_q}^2 M_{B_q} \hat{\eta}_B$$

$$\frac{\Delta m_d}{\Delta m_s} \approx \frac{|V_{td}|^2}{|V_{ts}|^2} \approx \frac{\lambda^6}{\lambda^4} = \lambda^2 \approx 0.04$$



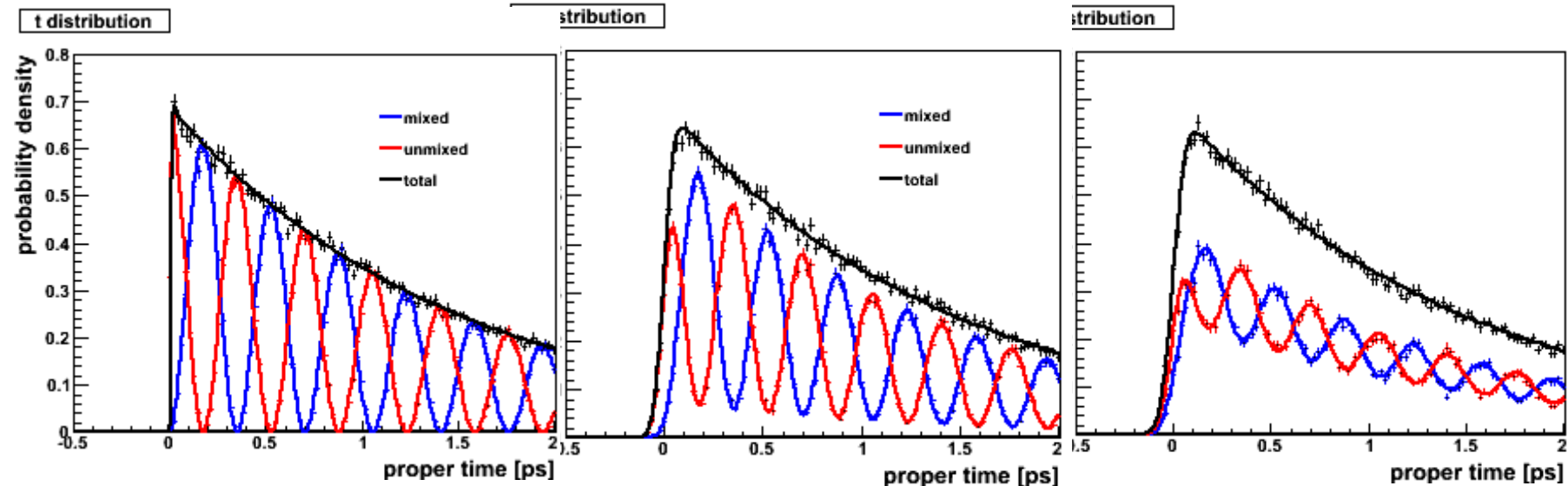
# B<sup>0</sup> Mixing \*)



Question: Why is oscillation not from +1 to -1?

Question: ARGUS (DESY) in 1987:  $m_{\text{top}} > 50 \text{ GeV}$ . Why???

# Detector effects on $B_s$ oscillation



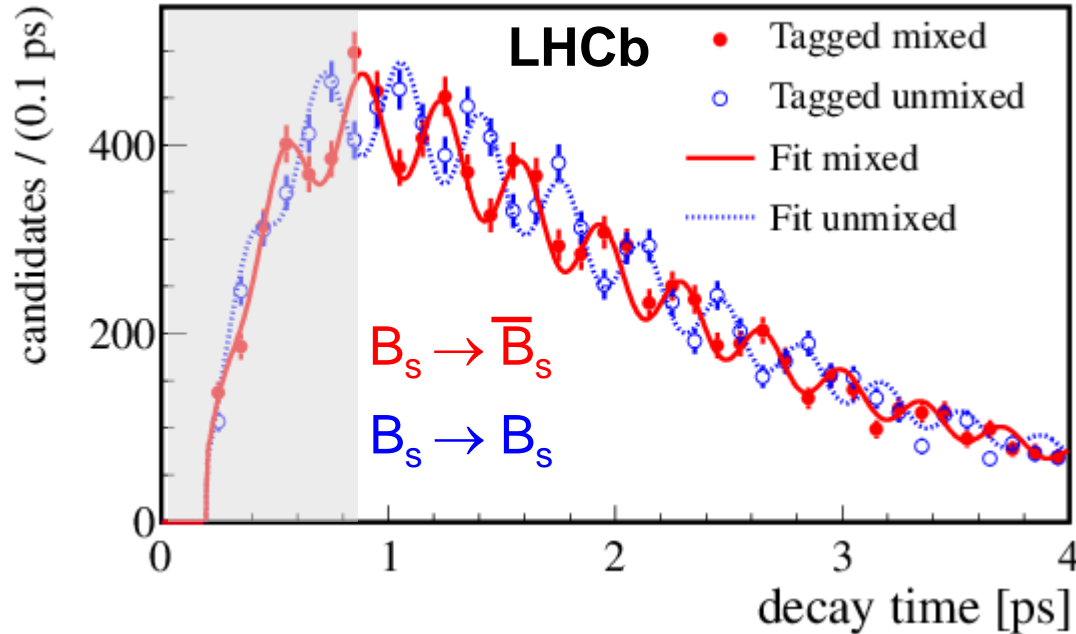
Finite time  
resolution: 44 fs



Realistic tagging

# B<sub>s</sub>-Mixing

*New J. Phys. 15 (2013) 053021*



$$\Delta m_s = 17.768 \pm 0.023 \pm 0.006 \text{ ps}^{-1}$$

Theorie (*M. Artuso et al., 2015*)

$$\Delta m_s = 18.3 \pm 2.7 \text{ ps}^{-1}$$

1 per mille  
(syst: z & p scale)

Precision tests of the Standard Model difficult:  
Hadronic uncertainties limit the precision of the theoretical prediction

# Parameters with better precision?

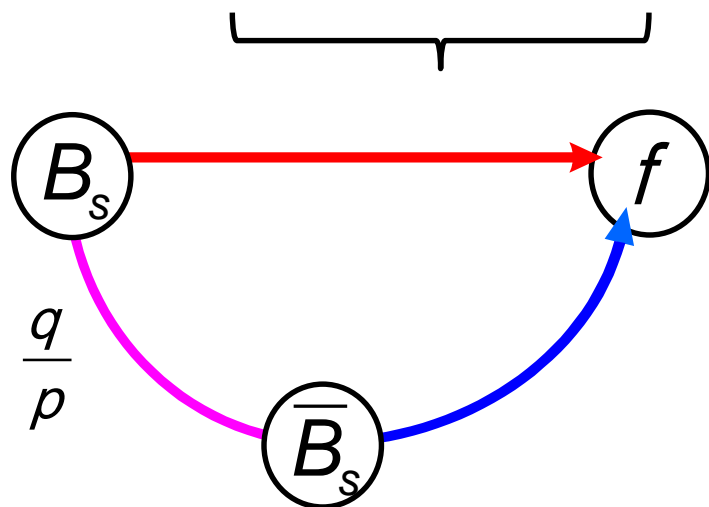
Phases have very small absolute theoretical uncertainties:

$$\phi_M = \arg(M_{12}) = \arg\left(\frac{q}{p}\right) \quad \text{Mixing phase}$$

Theory:

$$\phi_{M/\Gamma} = \arg\left(-\frac{M_{12}}{\Gamma_{12}}\right)$$

Theory:  $\phi_{M/\Gamma} = 0.0038 \pm 0.0010$



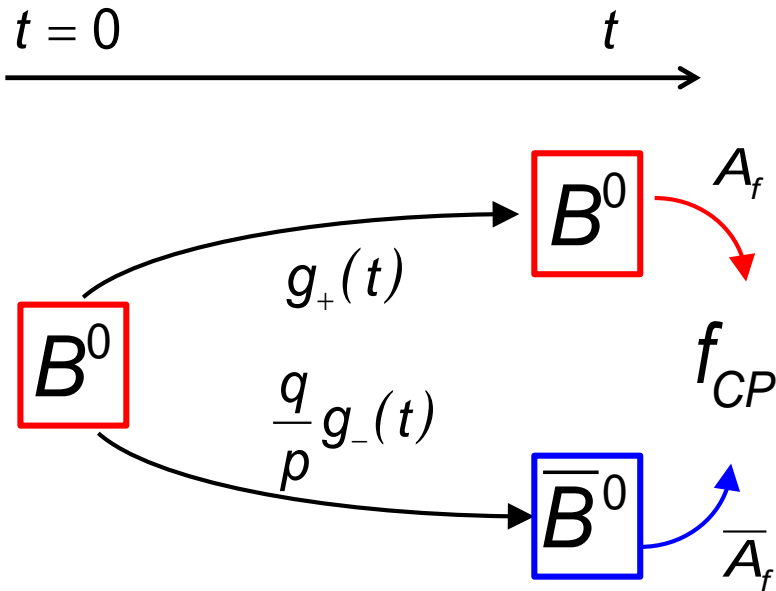
Time dependent CP-violation  
of  $B_s$  decaying to a CP eigenstate

$$P(B^0 \rightarrow \bar{B}^0) \neq P(\bar{B}^0 \rightarrow B^0)$$

CP-violation in mixing

# Interference between Mixing and Decay

*adapted from G. Raven*



$$g_+(t)A_f + \frac{q}{p}g_-(t)\bar{A}_f$$

$$g_+(t) = e^{-i(m-i\frac{\Gamma}{2})t} \left[ + \cosh \frac{\Delta\Gamma t}{4} \cos \frac{\Delta m t}{2} - i \sinh \frac{\Delta\Gamma t}{4} \sin \frac{\Delta m t}{2} \right]$$

$$g_-(t) = e^{-i(m-i\frac{\Gamma}{2})t} \left[ - \sinh \frac{\Delta\Gamma t}{4} \cos \frac{\Delta m t}{2} + i \cosh \frac{\Delta\Gamma t}{4} \sin \frac{\Delta m t}{2} \right]$$

# Master Formula for t-dependent CPV

$$\Gamma(B^0 \rightarrow f)(t) = |A_f|^2 \left(1 + |\lambda_f|^2\right) \frac{e^{-\Gamma t}}{2} \left\{ \cosh\left(\frac{\Delta\Gamma}{2} t\right) + D_f \sinh\left(\frac{\Delta\Gamma}{2} t\right) + C_f \cos(\Delta m t) - S_f \sin(\Delta m t) \right\}$$

$$\lambda_f = \frac{q}{p} \frac{\bar{A}_f}{A_f} \quad D_f = \frac{2\Re A_f}{1 + |\lambda_f|^2} \quad C_f = \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2} \quad S_f = \frac{2\Im A_f}{1 + |\lambda_f|^2}$$

$$\Gamma(\bar{B}^0 \rightarrow f)(t) = |A_f|^2 \left(1 + |\lambda_f|^2\right) \frac{e^{-\Gamma t}}{2} \left\{ \cosh\left(\frac{\Delta\Gamma}{2} t\right) + D_f \sinh\left(\frac{\Delta\Gamma}{2} t\right) - C_f \cos(\Delta m t) + S_f \sin(\Delta m t) \right\}$$

# Master Formula for t-dependent CPV

$$A_{CP}(t) = \frac{\Gamma(B^0 \rightarrow f)(t) - \Gamma(\bar{B}^0 \rightarrow f)(t)}{\Gamma(B^0 \rightarrow f)(t) + \Gamma(\bar{B}^0 \rightarrow f)(t)} = \frac{2C_f \cos(\Delta m t) - 2S_f \sin(\Delta m t)}{2 \cosh\left(\frac{\Delta\Gamma}{2} t\right) + 2D_f \sinh\left(\frac{\Delta\Gamma}{2} t\right)}$$

$$\lambda_f = \frac{q}{p} \frac{\bar{A}_f}{A_f} \quad D_f = \frac{2\Re A_f}{1 + |\lambda_f|^2} \quad C_f = \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2} \quad S_f = \frac{2\Im A_f}{1 + |\lambda_f|^2}$$

Time-dependent CPV even if  $|q/p| = 1$  (i.e. no CPV in mixing) and  $\bar{A}_f / A_f = 1$  (no direct CPV) if

$$|\lambda_f| = 1 \quad \text{and} \quad \arg\left(\frac{q}{p} \frac{\bar{A}_f}{A_f}\right) = \phi_{weak} \neq 0$$

# Time-dependent CP-Asymmetry $\Delta\Gamma \approx 0$

*adapted from G. Raven*

$t = 0$	$t$	Rate
$B^0$	$\rightarrow f_{CP}$	$\propto e^{-\Gamma t} [1 + \sin(\phi_{\text{weak}}) \sin(\Delta m t)]$
$\bar{B}^0$	$\rightarrow f_{CP}$	$\propto e^{-\Gamma t} [1 - \sin(\phi_{\text{weak}}) \sin(\Delta m t)]$

$$\begin{aligned} A_{CP}(t) &= \frac{\Gamma(B^0 \rightarrow f)(t) - \Gamma(\bar{B}^0 \rightarrow f)(t)}{\Gamma(B^0 \rightarrow f)(t) + \Gamma(\bar{B}^0 \rightarrow f)(t)} = \frac{2C_f \cos(\Delta m t) - 2S_f \sin(\Delta m t)}{2 \cosh\left(\frac{\Delta\Gamma}{2} t\right) + 2D_f \sinh\left(\frac{\Delta\Gamma}{2} t\right)} \\ &= -S_f \sin(\Delta m t) = -\sin \phi_{\text{weak}} \sin(\Delta m t) \end{aligned}$$

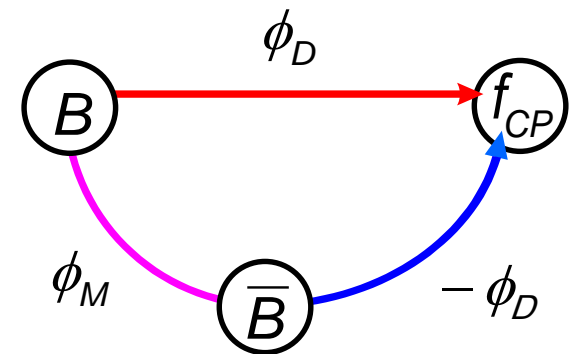


# Time-dependent CP Asymmetry $\Delta\Gamma \neq 0$

$$\begin{aligned} \mathcal{A}_{CP}(t) &\equiv \frac{\Gamma(\overline{B}^0 \rightarrow f_{CP}) - \Gamma(B^0 \rightarrow f_{CP})}{\Gamma(\overline{B}^0 \rightarrow f_{CP}) + \Gamma(B^0 \rightarrow f_{CP})} \\ &= \frac{-\Im\lambda_f \sin \Delta m t}{\cosh \frac{1}{2}\Delta\Gamma t + \Re\lambda_f \sinh \frac{1}{2}\Delta\Gamma t} \\ &\approx -\sin \phi_{weak} \sin(\Delta m t) \end{aligned}$$

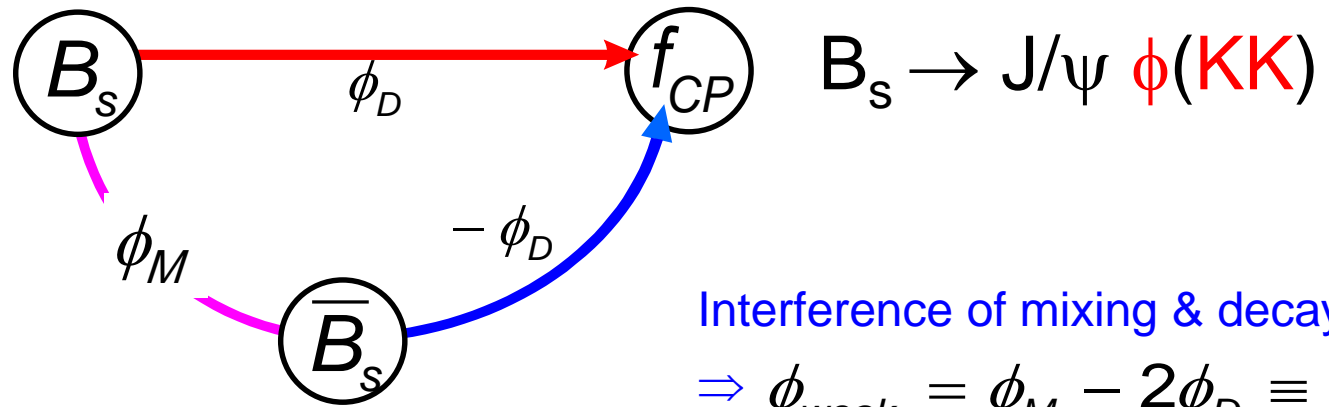
$$|\lambda_f| = \left| \frac{q \bar{A}_f}{p A_f} \right| = 1$$

Measurement of **time dependent CP** asymmetry of a process  $B^0 \rightarrow f_{CP}$  measures the phase difference  $\phi_{weak}$  between the two paths:



$$\phi_{weak} = \phi_M - 2\phi_{weak}$$

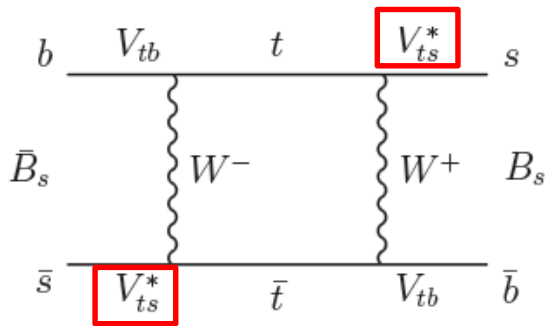
# Measuring the $B_s$ mixing phase



$$\phi_M = \arg(M_{12})$$

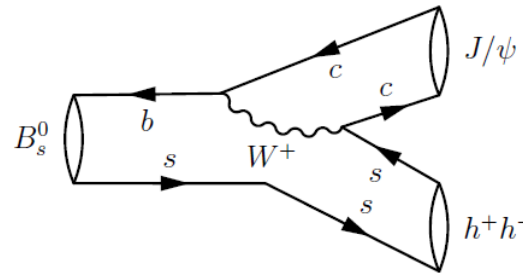
$$= \arg\left(\frac{q}{p}\right)$$

## Standard Model:



$$\phi_M \approx 2 \arg(V_{ts}) \approx -2\beta_s$$

$$V_{ts} = |V_{ts}| e^{-i\beta_s}$$



+ small penguin pollution

$$\phi_D^{SM} = -2 \arg(V_{cs} V_{cb}^*) \approx 0$$

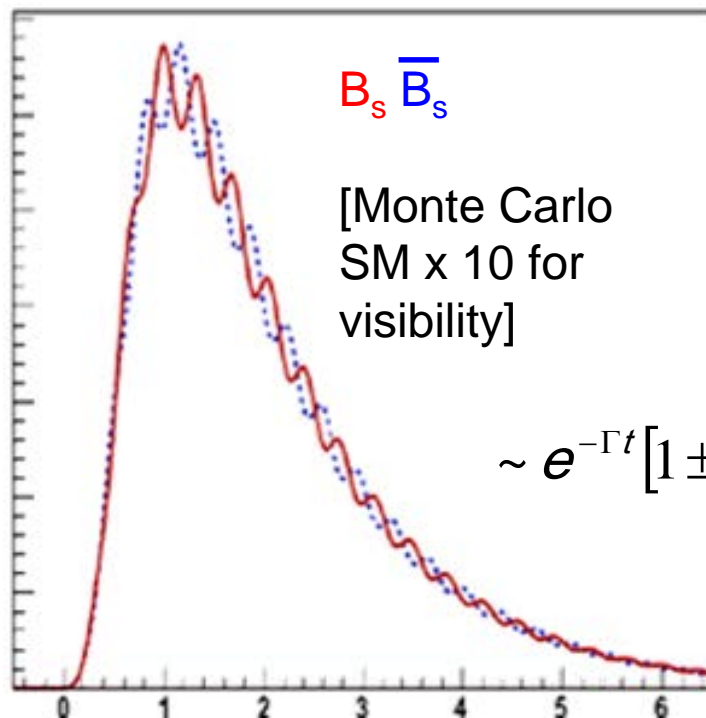
$$\phi_{weak,s}^{SM} = -0.0364 \pm 0.0016 \text{ rad (CKMFitter)}$$

$\rightarrow$  very small CPV

# Standard Model Expectation

Precise Standard Model prediction:

$$\phi_s^{SM} = -0.0364 \pm 0.0016 \text{ rad}$$

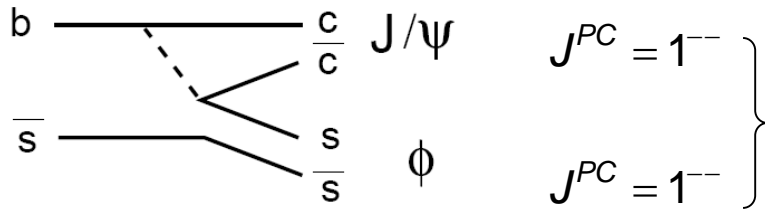


➔  $\phi_s$  small:  
expect very small CPV

# $B_s \rightarrow J/\psi (\mu\mu) \phi(KK)$

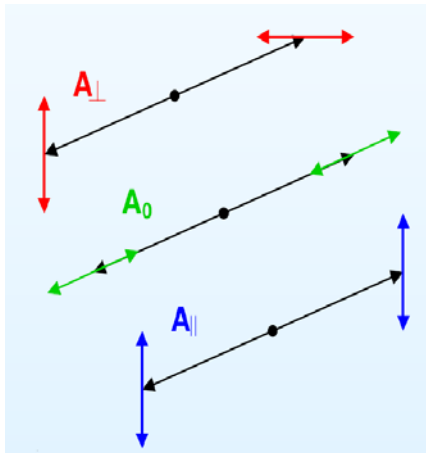
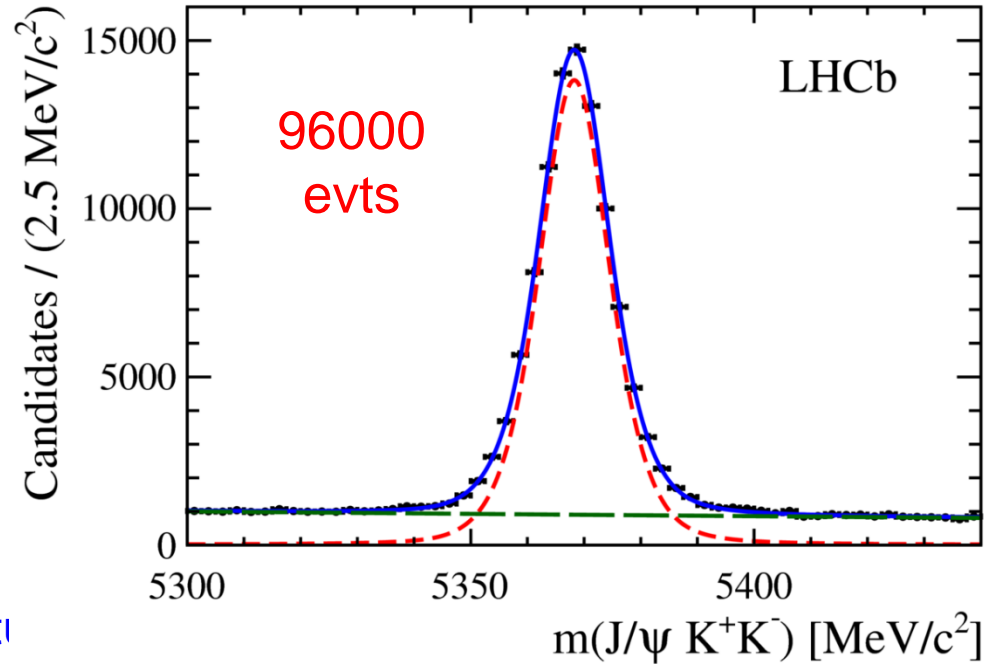
Phys. Rev. Lett. 114, 041801 (2015)

- experimentally clean
- VV final state:



$$CP(J/\psi\phi) = CP(J/\psi)CP(\phi)(-1)^L$$

( $L = 0, 1, 2 =$  relative orbital momentum)



3 different polarization amplitudes  
different relative orbital momentum

CP-odd ( $l = 1$ ):

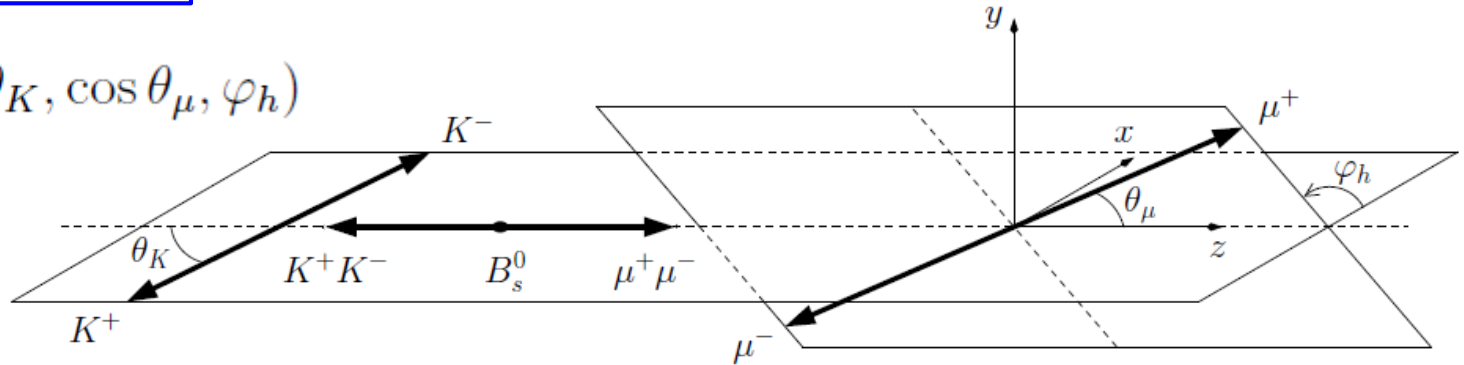
CP-even ( $l = 0, 2$ ):

angular analysis to disentangle CP even/odd state

# Angular dependent t distributions

Helicity frame

$$\Omega = (\cos \theta_K, \cos \theta_\mu, \varphi_h)$$



$B_s$

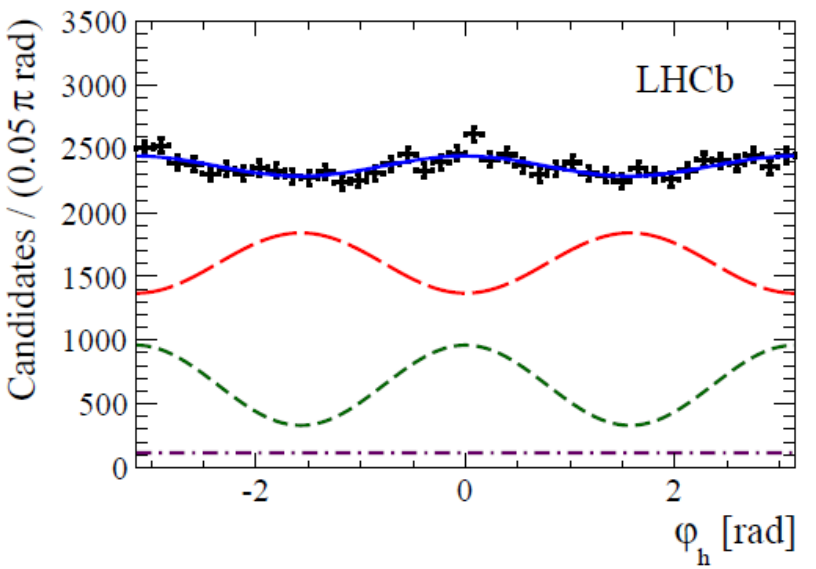
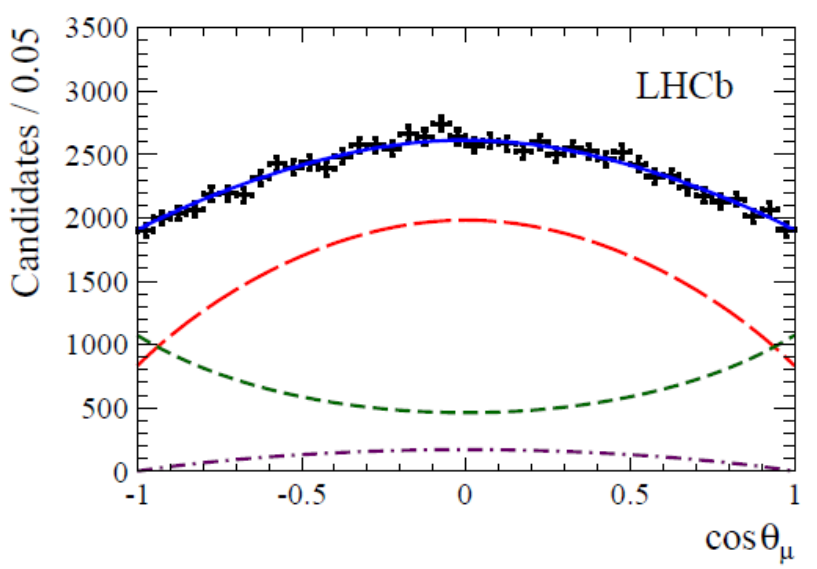
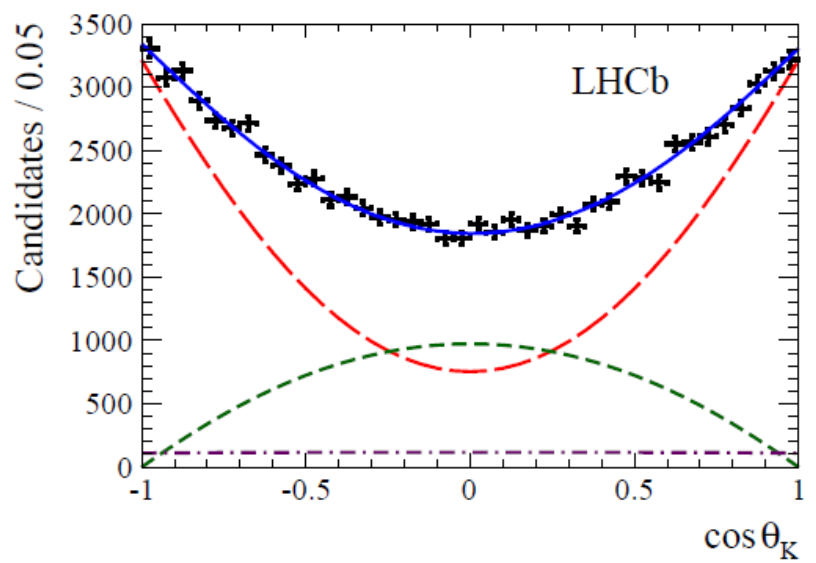
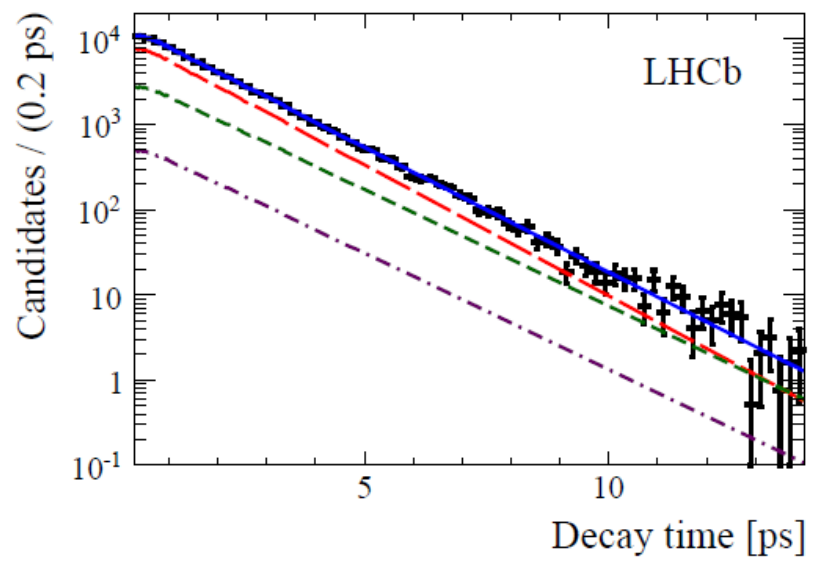
$$\frac{d^4\Gamma(B_s^0 \rightarrow J/\psi K^+ K^-)}{dt d\Omega} \propto \sum_{k=1}^{10} h_k(t) f_k(\Omega)$$

$\bar{B}_s$

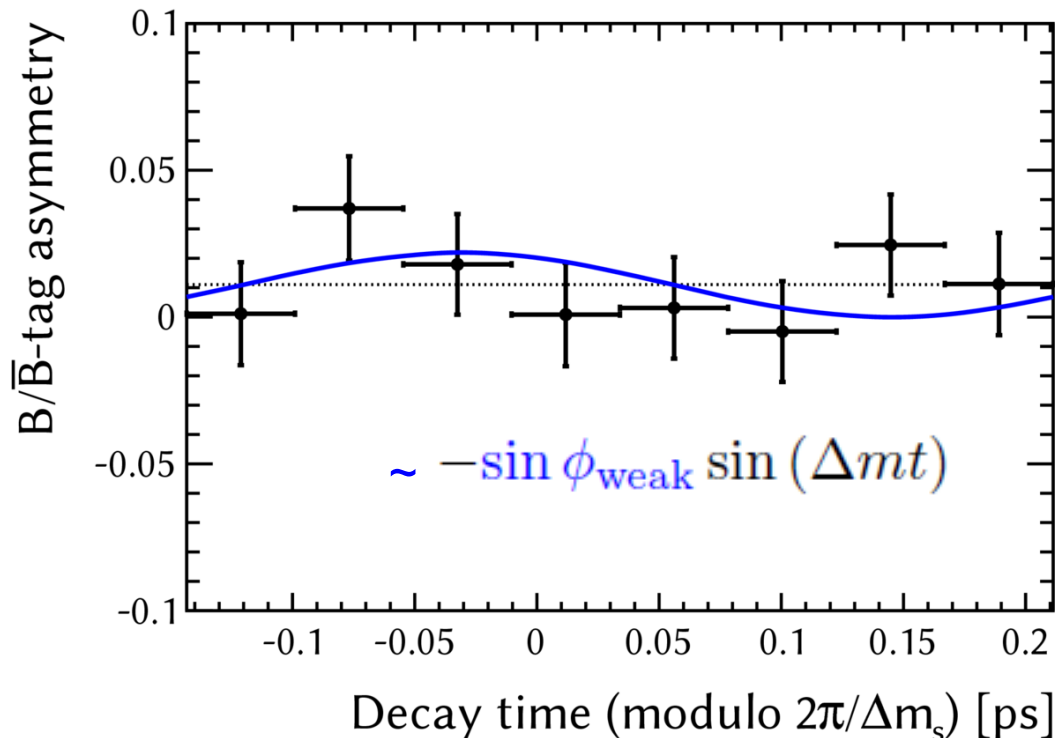
$$\frac{d^4\Gamma(\bar{B}_s^0 \rightarrow J/\psi K^+ K^-)}{dt d\Omega} \propto \sum_{k=1}^{10} \bar{h}_k(t) \bar{f}_k(\Omega)$$

# Decay time and decay angles

- - - CP-even   
 - - - CP-odd   
 - · - · S-wave



# Time-dependent CP Asymmetry for $B_s$



Phys. Rev. Lett. 114, 041801 (2015)

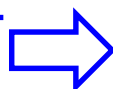
$$\phi_s = -0.058 \pm 0.049 \pm 0.006$$

$$\Gamma = 0.6603 \pm 0.0027 \pm 0.0015 \text{ ps}^{-1}$$

$$\Delta\Gamma = 0.0805 \pm 0.0091 \pm 0.0032 \text{ ps}^{-1}$$

$$|\lambda| = 0.964 \pm 0.019 \pm 0.007$$

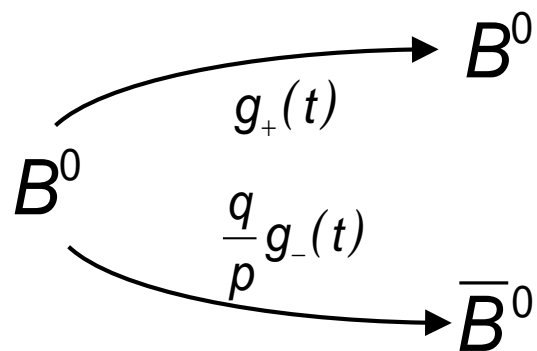
Consistent w/  $\lambda = 1$ :  
no CPV



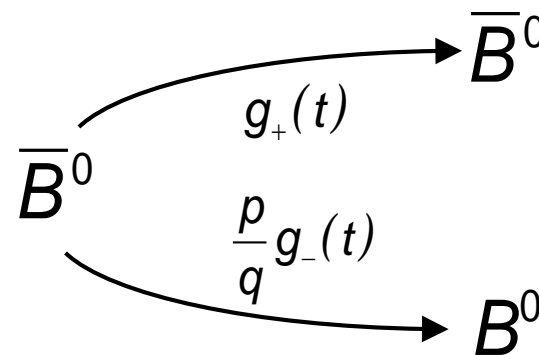
# CP Violation in B mixing

$$P(B_{d,s}^0 \rightarrow \overline{B}_{d,s}^0) \neq P(\overline{B}_{d,s}^0 \rightarrow B_{d,s}^0)$$

$t=0$   $\xrightarrow{\hspace{10em}}$   $t$



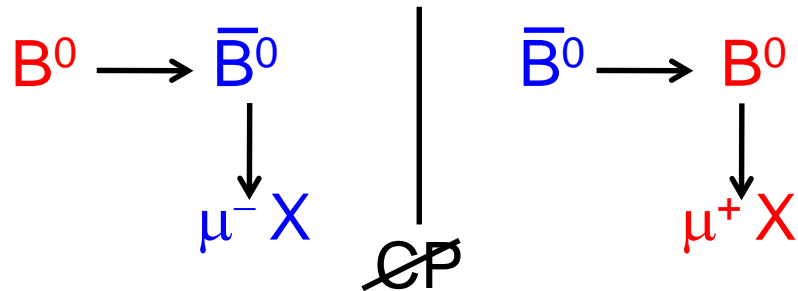
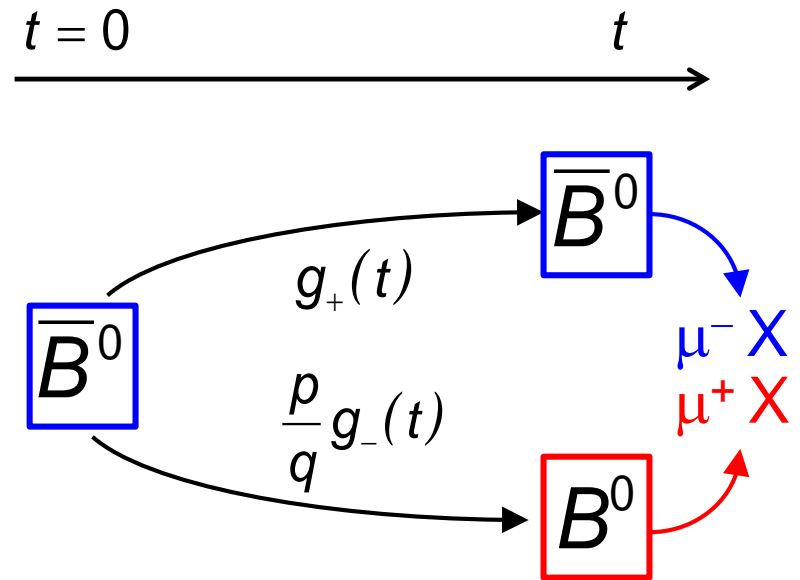
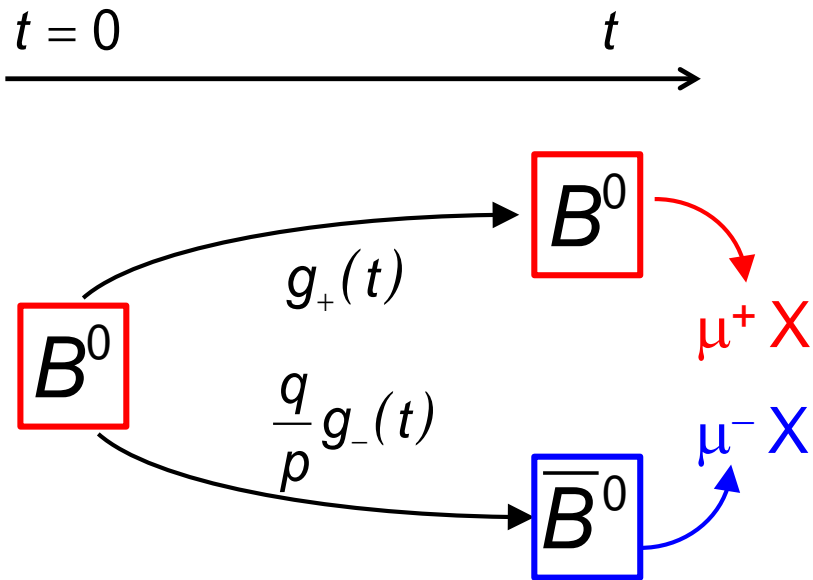
$t=0$   $\xrightarrow{\hspace{10em}}$   $t$



CP violation if  $\left| \frac{q}{p} \right| \neq 1$

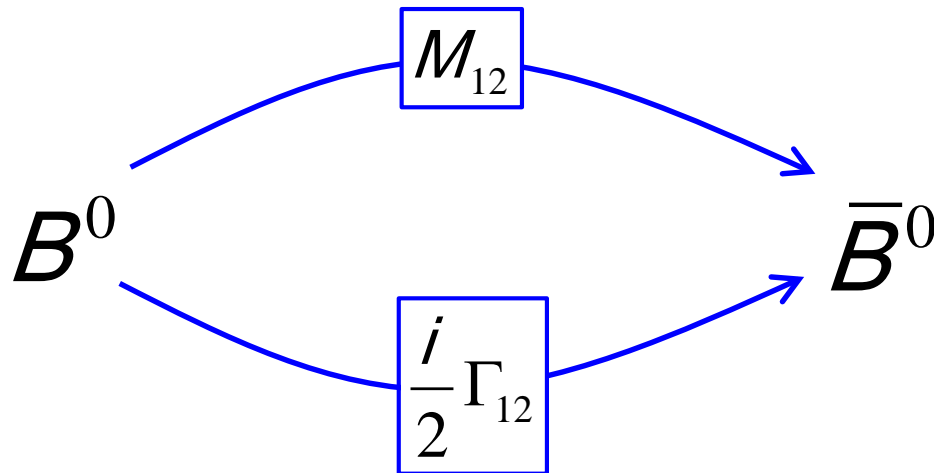


# Semileptonic CP asymmetry



Question: Which amplitudes interfere?

# Interference-Effect



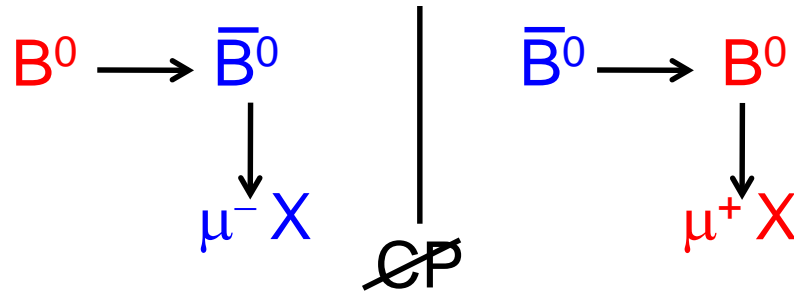
Weak phase difference:

$$\phi_{M/\Gamma} = \arg\left(-\frac{M_{12}}{\Gamma_{12}}\right)$$

In case of CPV in mixing:  $\left|\frac{q}{p}\right| = \left|\frac{1-\varepsilon}{1+\varepsilon}\right| \neq 1$  with  $\varepsilon = \frac{p-q}{p+q}$  complex

Physical states ( $B_H, B_L$ ) are not any longer pure CP states.

# Time integrated asymmetry



$$a_{sl}^q \equiv \frac{\Gamma(\bar{B}_q^0 \rightarrow B_q^0 \rightarrow \mu^+ X) - \Gamma(B_q^0 \rightarrow \bar{B}_q^0 \rightarrow \mu^- X)}{\Gamma(\bar{B}_q^0 \rightarrow B_q^0 \rightarrow \mu^+ X) + \Gamma(B_q^0 \rightarrow \bar{B}_q^0 \rightarrow \mu^- X)}, \quad q = d, s$$

$$= \frac{|p/q|^2 - |q/p|^2}{|p/q|^2 + |q/p|^2} = \frac{1 - |q/p|^4}{1 + |q/p|^4} \approx \frac{\Delta\Gamma}{\Delta m} \tan \phi_{M/\Gamma} \quad \phi_{M/\Gamma} = \arg\left(-\frac{M_{12}}{\Gamma_{12}}\right)$$

$$a_{fs}^{d,SM} = (-4.5 \pm 0.8) \cdot 10^{-4} \quad a_{fs}^{s,SM} = (2.11 \pm 0.36) \cdot 10^{-5}$$

*A. Lenz and U. Nierste*

# LHCb measurement of $a_{SL}$

- Tagging of the initial state reduces the statistical power drastically
- A untagged analysis is possible, reduction of stat. power only by factor 2. However this requires an excellent knowledge of the production asym.

$$A_P = \frac{\mathcal{P}(B^0) - \mathcal{P}(\bar{B}^0)}{\mathcal{P}(B^0) + \mathcal{P}(\bar{B}^0)}$$

- Moreover one needs to know the detection asymmetry for the final state

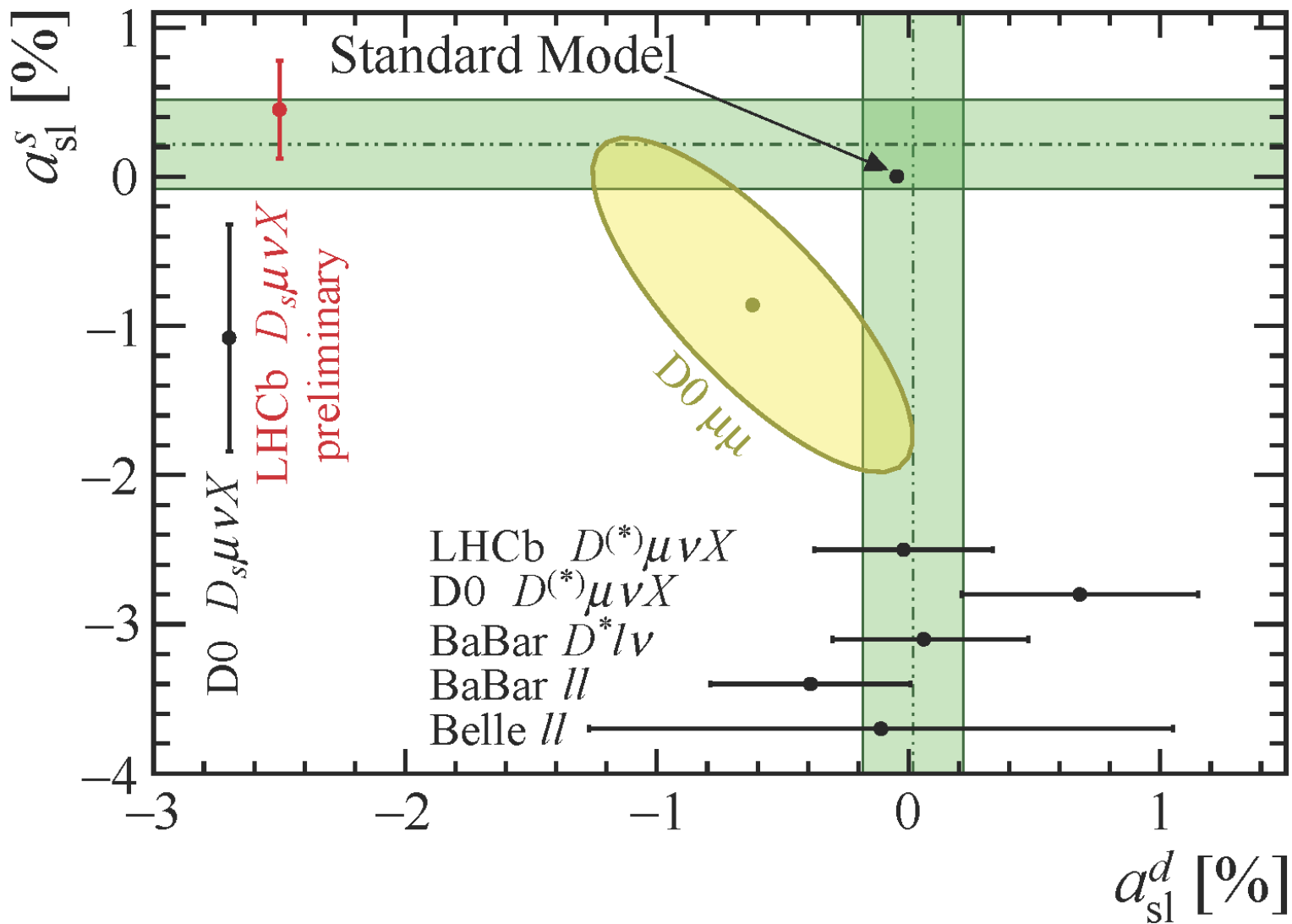
$$A_D = \frac{\varepsilon(f) - \varepsilon(\bar{f})}{\varepsilon(f) + \varepsilon(\bar{f})}$$

- Knowing the detection asymmetry, the production and semi-leptonic asymmetries can be determined in a **time dependent analysis**:

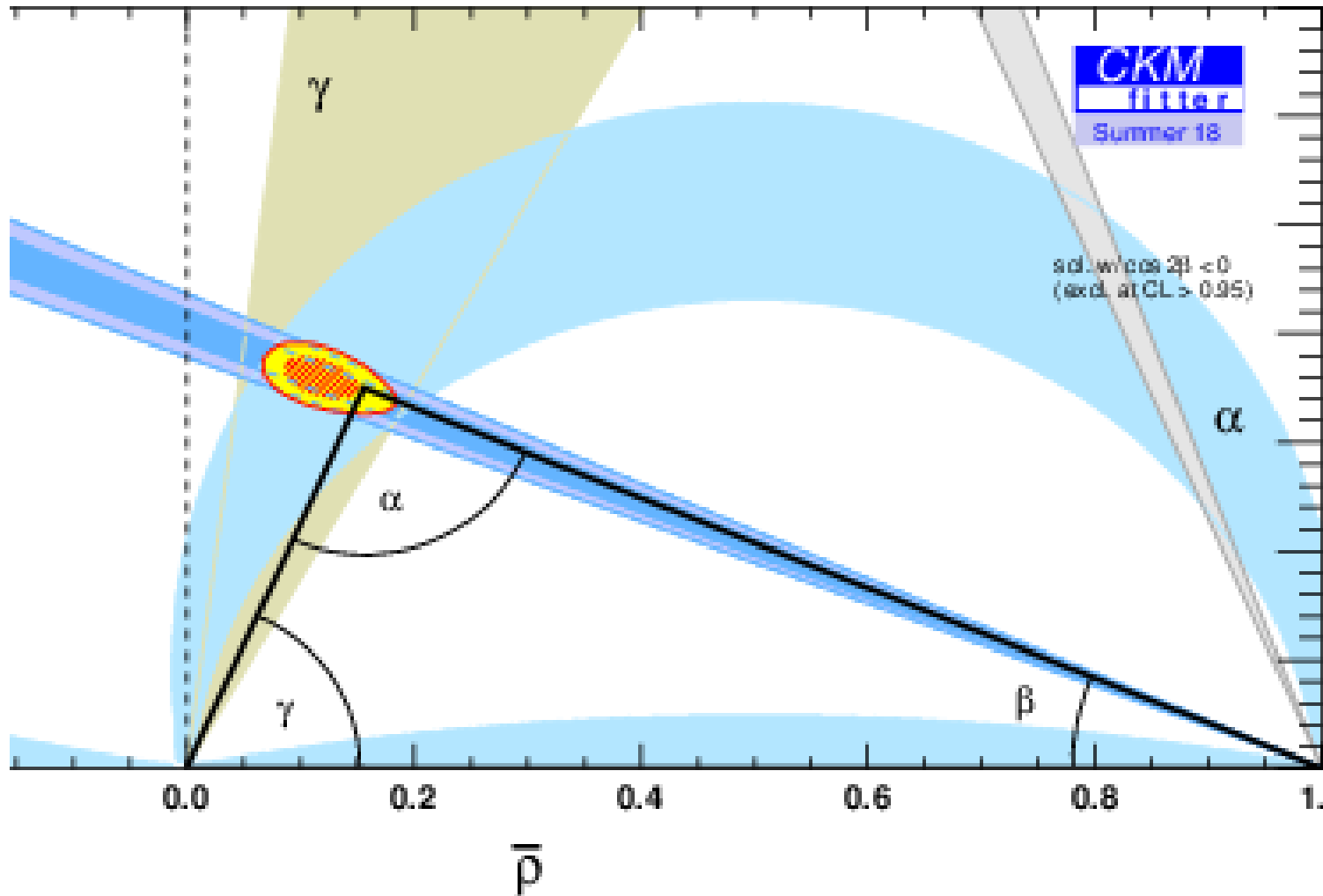
$$A_{\text{meas}}(t) = \frac{N(f, t) - N(\bar{f}, t)}{N(f, t) + N(\bar{f}, t)} \approx A_D + \frac{a_{sl}^d}{2} + \left( A_P - \frac{a_{sl}^d}{2} \right) \cos(\Delta m_d t)$$

- Due to the fast oscillation, the production asymmetry for  $B_s$  mesons is washed out and no time dependent measurement is necessary.

# Experimental Status



# Direct CP Violation & CKM angle $\gamma$



# Direct CP Violation & CKM angle $\gamma$

- CP Violation in mixing
- CP Violation through interference between decay and mixing

} Indirect CPV

- CP violation in decay

$$\left| \begin{array}{c} A_f \\ \text{---} B^0 \end{array} \rightarrow f \right|^2 \neq \left| \begin{array}{c} \bar{A}_f \\ \text{---} \bar{B}^0 \end{array} \rightarrow \bar{f} \right|^2$$

$$P(B \rightarrow f) \neq P(\bar{B} \rightarrow \bar{f})$$

(time integrated)

} direct CPV

# CP Violation in meson decays

CKM phase do not lead easily to measurable CPV asymmetries.

To observe CP violation needs at least two amplitudes with different weak (sign flip under CP) and different strong (invariant under CP) amplitudes:

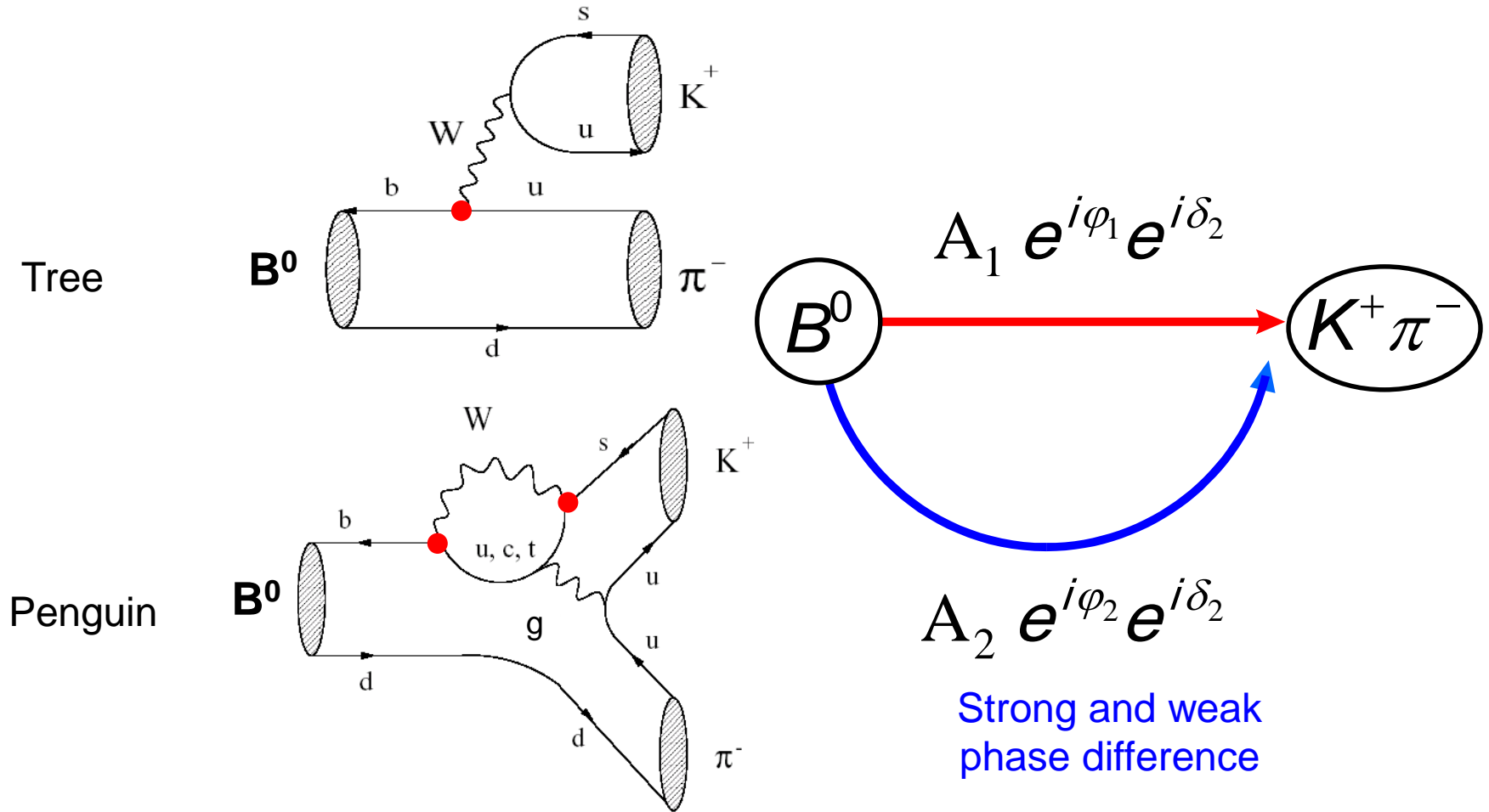
$$\mathcal{A}(B \rightarrow f) = A_1 e^{i\phi_1} e^{i\delta_1} + A_2 e^{i\phi_2} e^{i\delta_2}$$

$$\bar{\mathcal{A}}(\bar{B} \rightarrow \bar{f}) = A_1 e^{-i\phi_1} e^{i\delta_1} + A_2 e^{-i\phi_2} e^{i\delta_2}$$

$$|\bar{\mathcal{A}}|^2 - |\mathcal{A}|^2 = 4 A_1 A_2 \sin(\phi_1 - \phi_2) \sin(\delta_1 - \delta_2)$$



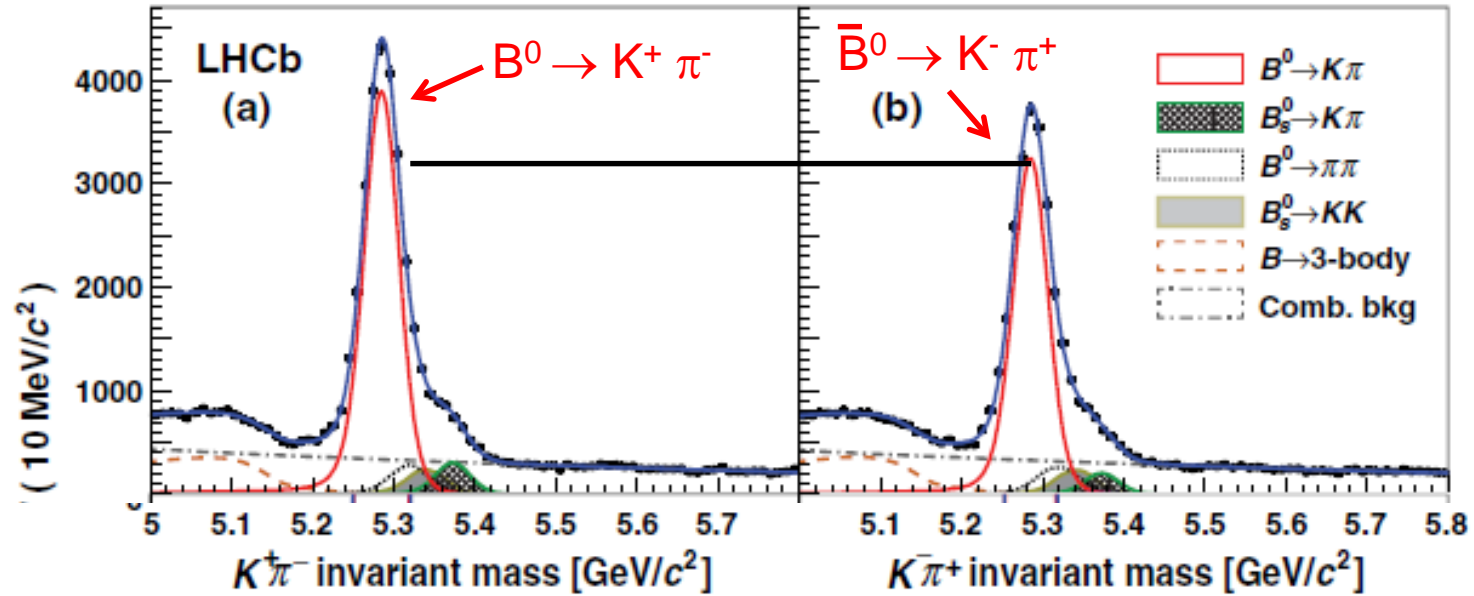
# Direct CP Violation in $B \rightarrow K\pi$



CP Asymmetrie  $|\bar{A}|^2 - |A|^2 = 4|A_1||A_2|\sin(\Delta\varphi)\sin(\Delta\delta)$  Strong phase difficult to predict

# Direct CP asymmetries for $B_{d,s}^0 \rightarrow K\pi$

PRL 110, 221601 (2013)



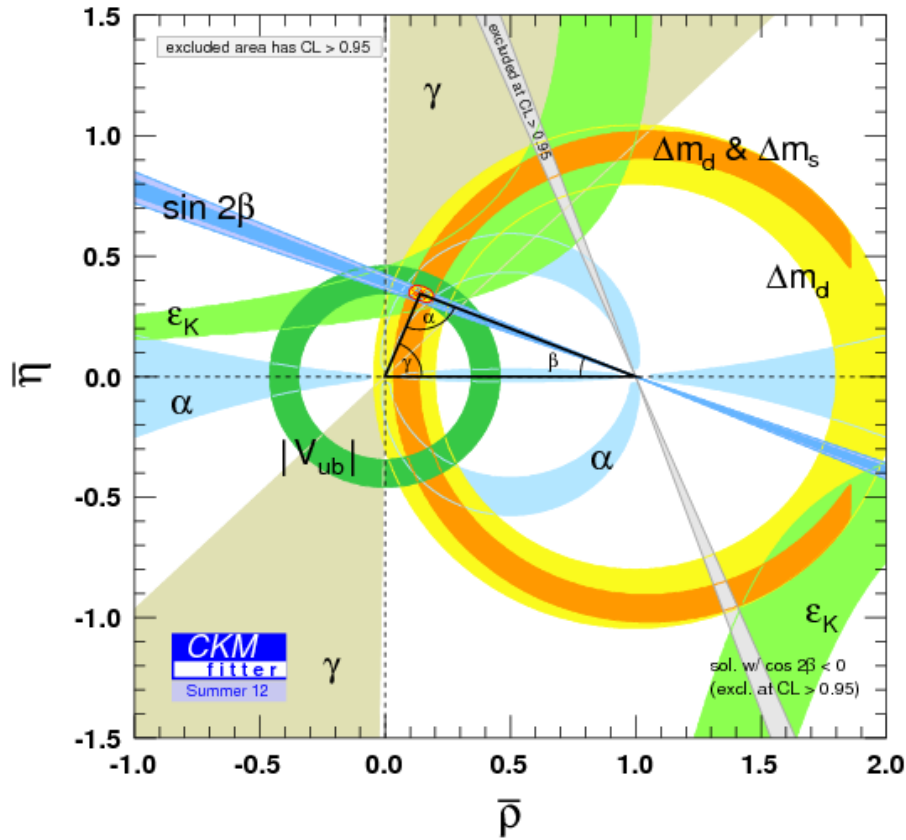
$$A_{CP}(B \rightarrow f) = \frac{\Gamma(\bar{B} \rightarrow \bar{f}) - \Gamma(B \rightarrow f)}{\Gamma(\bar{B} \rightarrow \bar{f}) + \Gamma(B \rightarrow f)}$$

Correction for  
detection / production  
asymmetry

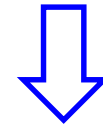
$$A_{CP}(B^0 \rightarrow K^+ \pi^-) = -0.080 \pm 0.007 \text{ (stat)} \pm 0.003 \text{ (syst)} \quad [10.5\sigma]$$

$$A_{CP}(B_s^0 \rightarrow K^- \pi^+) = 0.27 \pm 0.04 \text{ (stat)} \pm 0.01 \text{ (syst)}. \quad [6.5\sigma]$$

# CKM Angle $\gamma$



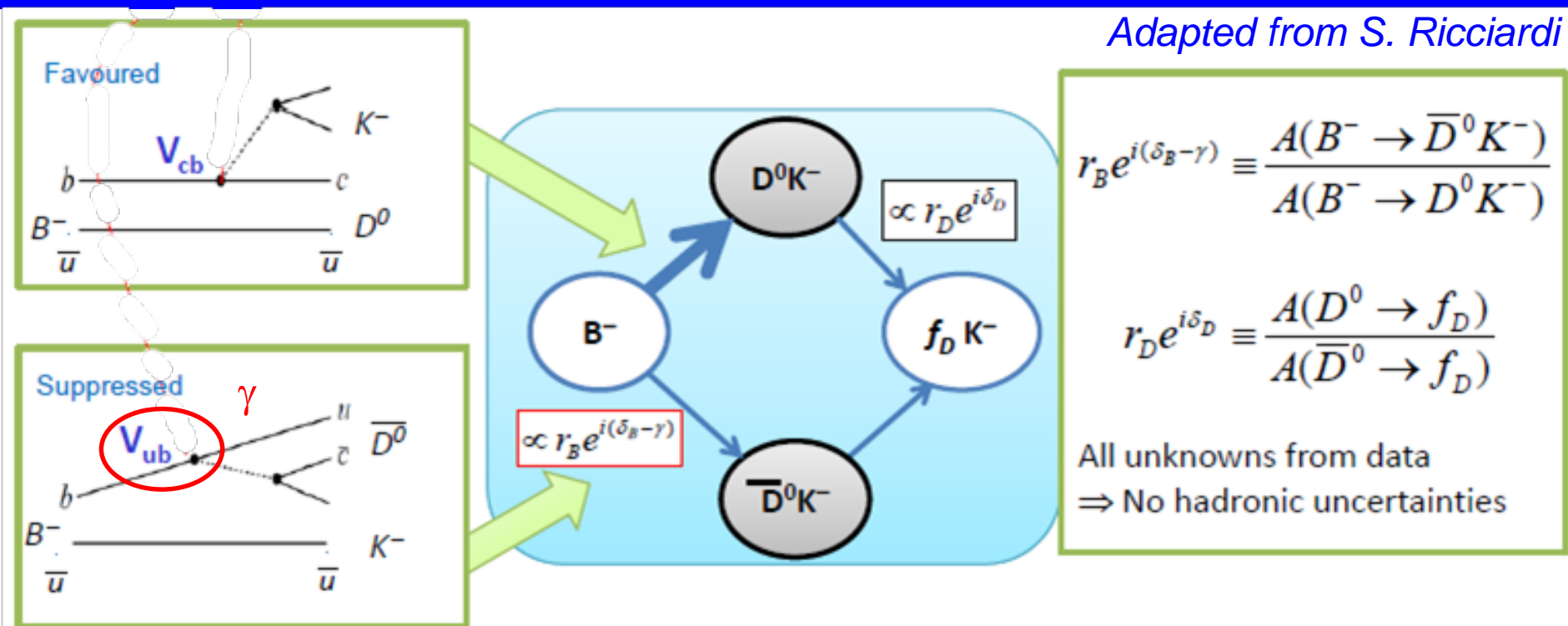
$$\gamma = \arg \left( -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} \right)$$



Exploit direct CPV in  
B $\rightarrow$ DK decays

# Sensitivity of $B \rightarrow DK$ decays to $\gamma$

Adapted from S. Ricciardi



Gronau, London, Wyler (GLW)

$f_D = KK, \pi\pi$  (CP state)

Atwood, Dunietz, Soni (ADS)

$f_D = K^+\pi^-$  and  $\pi^+K^-$

Giri, Grossman,  
Soffer, Zupan  
(GGSZ)

Self conjugated  
Dalitz modes

LHCb

- $B^\pm \rightarrow D(KK) K^\pm$
- $B^\pm \rightarrow D(\pi\pi) K^\pm$
- $B^\pm \rightarrow D(KK) \pi^\pm$
- $B^\pm \rightarrow D(\pi\pi) \pi^\pm$

LHCb

- $B^\pm \rightarrow D(\pi^+K^-) K^\pm$
- $B^\pm \rightarrow D(\pi^+K^-) \pi^\pm$
- $B^\pm \rightarrow D(K_s K^+\pi^-) \pi^\pm$
- $B^\pm \rightarrow D(K_s K^+\pi^-) K^\pm$