

Neckarzimmern B physics workshop 2017

Effective theories for flavour within & beyond the SM

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Outline

1 EFT for weak decays

- Introduction
- Example: $\text{BR}(B_s \rightarrow \mu^+ \mu^-)$ in the SM

2 Applications to BSM phenomenology

- Example: $B_s \rightarrow \mu^+ \mu^-$ beyond the SM
- Standard Model as EFT
- Model-independent analysis of $B_s \rightarrow \mu^+ \mu^-$
- $B_s \rightarrow \mu^+ \mu^-$ in specific NP models

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2 Applications to BSM phenomenology

Multiple scales

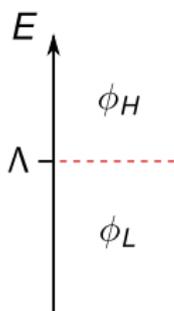
Weak decays always involve physics at vastly disparate energy scales, e.g.

- ▶ non-perturbative QCD interactions describing the hadrons,
 $\Lambda_{\text{QCD}} \sim 0.2 \text{ GeV}$
- ▶ b quark mass $\sim 4 \text{ GeV}$
- ▶ mass of the W mediating FCNCs $\sim 80 \text{ GeV}$
- ▶ top quark mass $\sim 170 \text{ GeV}$
- ▶ new heavy particles in loops $\sim \text{TeV?}$

Such a multitude of scales can only be tackled with a powerful tool:
Effective field theory

Effective field theory

We want to study physics at energies much lower than some scale Λ in a theory where particles lighter and heavier than Λ are present.



To this end, we can replace the complicated Lagrangian of the “full” theory by an **effective Lagrangian** containing only the light fields and a series of local operators built out of the light fields

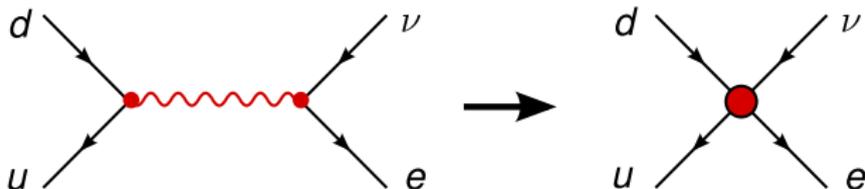
$$\mathcal{L}(\varphi_L, \varphi_H) \rightarrow \mathcal{L}(\varphi_L) + \mathcal{L}_{\text{eff}} = \mathcal{L}(\varphi_L) + \sum_i C_i Q_i(\varphi_L)$$

This expansion is called the **operator product expansion**

Example: modern view of Fermi theory

In Fermi's model of β decay, the full weak Lagrangian (that he didn't know of course) is effectively replaced by the low-energy (QED) Lagrangian plus a single operator

$$\mathcal{L}_{\text{ew}} \rightarrow \mathcal{L}_{\text{QED}} + \frac{G_F}{\sqrt{2}} (\bar{u}d)(e\bar{\nu})$$



Local operator \equiv effective vertex!

More about the OPE

$$\mathcal{L}_{\text{eff}} = \sum_i C_i Q_i(\varphi_L)$$

- ▶ the local operators have mass dimension > 4 , i.e. they are **non-renormalizable**
- ▶ operators with dimension $4 + n$ contribute with strength $(E/\Lambda)^n$ to a process with energy E ; thus, the OPE can be **truncated** at some dimension d and typically a small number of operators is important
- ▶ C_i are called **Wilson coefficients** \equiv effective coupling constants
- ▶ $\mathcal{H}_{\text{eff}} \equiv -\mathcal{L}_{\text{eff}}$

Weak effective Hamiltonian

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} \sum_i \xi_{\text{CKM}}^i C_i Q_i$$

- ▶ we only have to consider operators up to **dimension 6**
- ▶ since flavour-change is always mediated by the W boson, one can factor out the Fermi constant $\frac{4G_F}{\sqrt{2}} = \frac{g^2}{2m_W^2} \Rightarrow$ the WC of dimension-6 operators are **dimensionless**
- ▶ factoring out the CKM elements, the WC are **real** in the SM
- ▶ the amplitude of a weak decay takes the generic form

$$A(i \rightarrow f) = \langle f | \mathcal{H}_{\text{eff}} | i \rangle = \frac{4G_F}{\sqrt{2}} \sum_i \xi_{\text{CKM}}^i C_i(\mu) \langle f | Q_i(\mu) | i \rangle$$

Calculating Wilson coefficients: matching

- ▶ The values of the effective coupling constant should be such that amplitudes in the effective theory reproduce the ones in the full theory.

$$\mathcal{L}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} V_{ud} C Q$$

$$Q = (\bar{u}_L \gamma^\mu d_L)(\bar{e}_L \gamma_\mu \nu_L)$$

- ▶ Requiring the amplitudes to coincide, one finds

$$\mathcal{A}_{\text{full}} = \frac{g^2}{2m_W^2} V_{ud} \langle Q \rangle \stackrel{!}{=} \frac{4G_F}{\sqrt{2}} V_{ud} C \langle Q \rangle = \mathcal{A}_{\text{eff}}$$

$$\Rightarrow C = 1$$

- ▶ This process is called **matching**

Detour: renormalization

In a QFT, infinities in calculations have to be removed by **renormalizing** bare parameters in the Lagrangian, e.g. in QCD

$$G_{0\mu}^a = \sqrt{Z_3} G_\mu^a \quad q_0 = \sqrt{Z_q} q \quad g_{0s} = Z_g g_s \mu^\varepsilon \quad m_0 = Z_m m$$

- ▶ 0: unrenormalized = bare fields/parameters
- ▶ Z_i : renormalization constants
- ▶ μ : renormalization scale

Dimensional regularization + minimal subtraction: only poles in $\varepsilon = 2 - d/2$ subtracted

$$Z_i = \frac{\alpha_s}{4\pi} \frac{a_{1i}}{\varepsilon} + O(\alpha_s^2)$$

Renormalization scale

- ▶ g_s and m (in fact, all couplings in a QFT) become μ -dependent
- ▶ μ dependent terms are renormalization scheme dependent
- ▶ physical **observables** have to be μ -**independent**
- ▶ values of the parameters at different scales are connected by **renormalization group equations (RGE)**, e.g.

$$\frac{dg_s(\mu)}{d \ln \mu} = \beta(g_s(\mu)), \quad \frac{dm(\mu)}{d \ln \mu} = -\gamma_m(g_s(\mu)) m(\mu)$$

Operator renormalization

- ▶ Also Q_i have to be renormalized:

$$Q_i^0 = Z_{ij} Q_j$$

- ▶ C_i become scale-dependent
- ▶ The scale dependence is cancelled by the scale dependence of the matrix element

$$A(i \rightarrow f) = \frac{G_F}{\sqrt{2}} \sum_i \xi_{\text{CKM}}^i C_i(\mu) \langle f | Q_i(\mu) | i \rangle$$

Renormalizing non-renormalizable op.s?

- ▶ In a **renormalizable** theory, all infinities can be removed to all orders in perturbation theory by a finite number of counterterms
- ▶ In a **non-renormalizable** theory, infinities have to be removed at any order and the number of subtraction terms is infinite
- ▶ In the OPE, once we renormalize the (finite number of) operators of a given dimension, the higher-dimensional ones are still divergent, but we don't care since they are **not relevant for low-energy physics**

Modern view of renormalization: the low-energy limit of every EFT is a renormalizable theory

RGE for the Wilson coefficients

- ▶ Recall our multi-scale problem: which renormalization scale μ to choose for $C_i(\mu)$?
- ▶ C_i obey a RGE

$$\frac{d}{d \ln \mu} C_j(\mu) = \sum_i C_i(\mu) \gamma_{ij}(\mu)$$
$$\Rightarrow C_j(\mu_1) = U_{ji}(\mu_1, \mu_2) C_i(\mu_2)$$

- ▶ we calculate (match) C_i at a high scale where QCD is perturbative and use the RGE to evolve it down to the appropriate scale
- ▶ by “running” the RGEs, we are in effect running through a series of EFTs where μ playing the role of the scale Λ

Generic weak decay amplitude

$$A(i \rightarrow f) = \frac{4G_F}{\sqrt{2}} \sum_i \xi_{\text{CKM}}^i C_i(m_W) U(\mu_I, m_W) \langle f | Q_i(\mu_I) | i \rangle$$

CKM factors

short-distance

QCD corrections

hadronic matrix element

– perturbative –

non-perturbative

– indep. of external states –

specific for ext. state

sensitive to NP

– independent of NP –

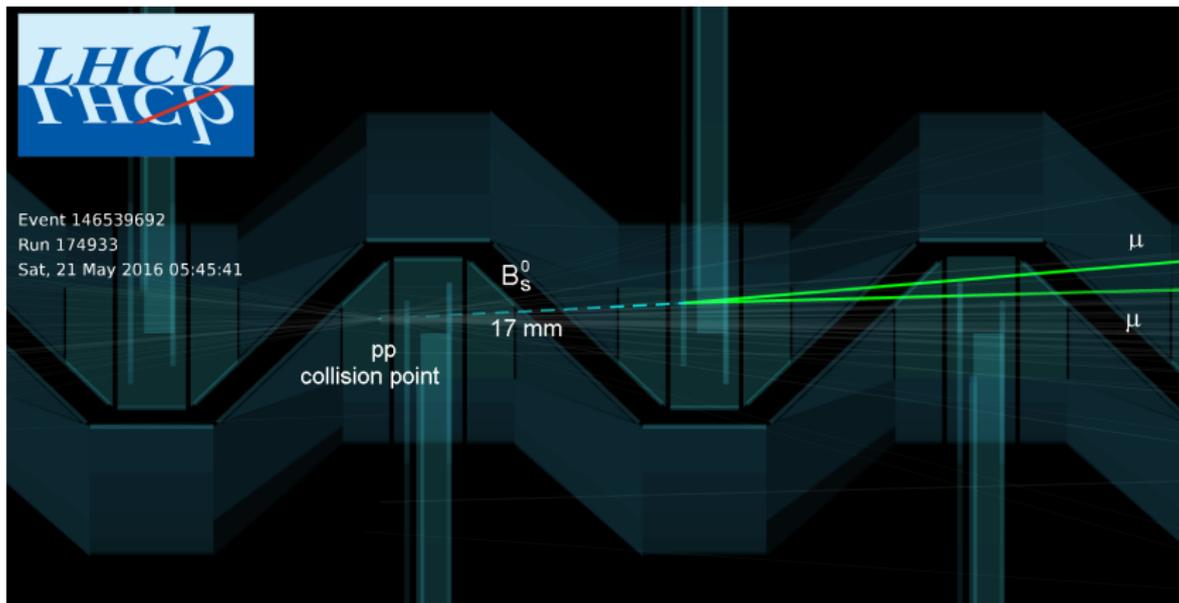
The OPE has achieved a **separation of scales**

1 EFT for weak decays

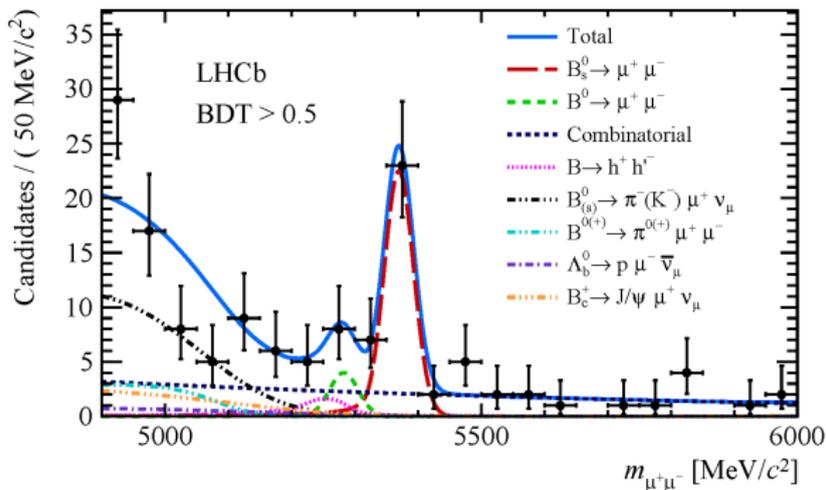
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$B_s \rightarrow \mu^+ \mu^-$ LHCb Run 2



$$\text{BR}(B_s \rightarrow \mu^+ \mu^-) = (3.0 \pm 0.6_{-0.2}^{+0.3}) \times 10^{-9} \quad \text{Aaij et al. 1703.05747}$$

What does this imply for the SM; for NP models?

$B_s \rightarrow \mu^+ \mu^-$ branching ratio in the SM

$$\begin{aligned}\text{BR}(B_s \rightarrow \mu^+ \mu^-) &= \Gamma(B_s \rightarrow \mu^+ \mu^-) / \Gamma(B_s \rightarrow \text{anything}) \\ &= \tau_{B_s} \Gamma(B_s \rightarrow \mu^+ \mu^-) \\ &= \tau_{B_s} \Phi(m_{B_s}, m_\mu) |\langle \mu\mu | A | B_s \rangle|^2\end{aligned}$$

- ▶ $\tau_{B_s} = 1/\Gamma_s$ – lifetime
- ▶ Φ – phase space
- ▶ A – amplitude

$$\Phi(m_{B_s}, m_\mu) = \frac{1}{16\pi} \frac{1}{m_{B_s}} \sqrt{1 - \frac{4m_\mu^2}{m_{B_s}^2}}$$

$B_s \rightarrow \mu^+ \mu^-$ amplitude in the EFT

$$\langle \mu\mu | A | B_s \rangle = i \langle \mu\mu | C_{10} O_{10} | B_s \rangle + O(m_b^2/m_W^2)$$

- ▶ $O_{10} = (\bar{s}_L \gamma^\mu b_L)(\bar{\mu} \gamma_\mu \gamma_5 \mu)$ – semi-leptonic axial vector operator
- ▶ C_{10} – Wilson coefficient

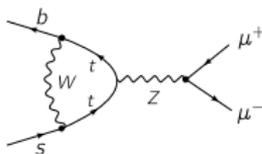
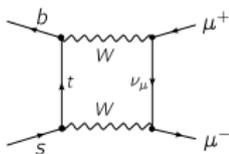
$$\langle \mu\mu | C_{10} O_{10} | B_s \rangle = C_{10} \langle 0 | \bar{s}_L \gamma^\mu b_L | B_s \rangle (\bar{\mu} \gamma_\mu \gamma_5 \mu)$$

- ▶ $\langle 0 | \bar{s}_L \gamma^\mu b_L | B_s \rangle$ – hadronic matrix element

$$\langle 0 | \bar{s}_L \gamma^\mu b_L | B_s \rangle = \frac{1}{2} \langle 0 | \bar{s} \gamma^\mu b | B_s \rangle - \frac{1}{2} \langle 0 | \bar{s} \gamma^\mu \gamma_5 b | B_s \rangle = 0 - \frac{1}{2} i f_{B_s} p^\mu$$

- ▶ f_{B_s} decay constant

$B_s \rightarrow \mu^+ \mu^-$: matching



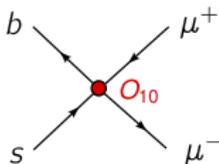
“Partonic” amplitude in the full theory

$$\mathcal{A}_{\text{full}} = \frac{4G_F}{\sqrt{2}} \frac{\alpha}{4\pi} \frac{1}{s_W^2} V_{tb} V_{ts}^* Y(x_t) (\bar{s}_L \gamma^\mu b_L) (\bar{\mu} \gamma_\mu \gamma_5 \mu)$$

- ▶ G_F – Fermi constant
- ▶ V_{tq} – CKM elements
- ▶ $x_t = m_t^2/m_W^2$
- ▶ Y – Inami-Lim function

$$Y(x_t) = Y_0(x_t) \left[1 + O(\alpha_s) + O(\alpha_s^2) + O(\alpha_{em}) + \dots \right]$$

$B_s \rightarrow \mu^+ \mu^-$: matching



“Partonic” amplitude in the effective theory

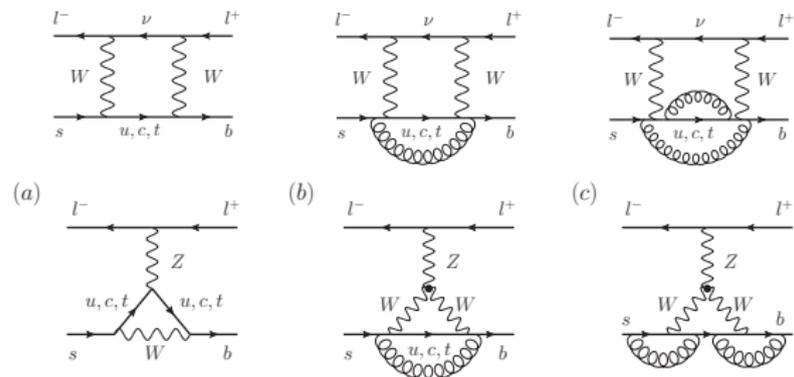
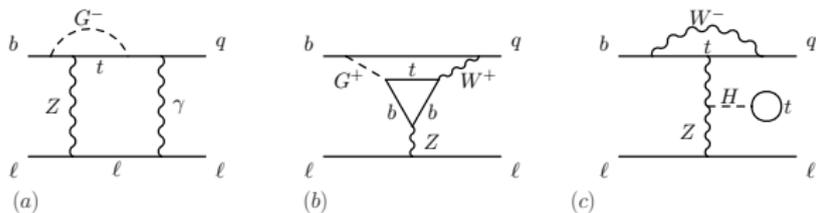
$$\mathcal{H}_{\text{eff}} = -C_{10} O_{10}$$

$$\mathcal{A}_{\text{eff}} = -C_{10} (\bar{s}_L \gamma^\mu b_L) (\bar{\mu} \gamma_\mu \gamma_5 \mu)$$

Matching

$$\mathcal{A}_{\text{full}} \stackrel{!}{=} \mathcal{A}_{\text{eff}} \Rightarrow C_{10} = -\frac{4G_F}{\sqrt{2}} \frac{\alpha}{4\pi} V_{tb} V_{ts}^* \frac{1}{S_W^2} Y(x_t)$$

Higher order corrections: full theory



Bobeth et al. 1311.1348, Hermann et al. 1311.1347

Recipe: how to predict $\text{BR}(B_s \rightarrow \mu^+ \mu^-)_{\text{SM}}$

$$\text{BR}(B_s \rightarrow \mu^+ \mu^-)_{\text{SM}} = \tau_{B_s} \frac{G_F^2}{\pi} \left(\frac{\alpha}{4\pi s_w^2} \right)^2 m_\mu^2 \sqrt{1 - \frac{4m_\mu^2}{m_{B_s}^2}} m_{B_s} f_{B_s}^2 |V_{tb} V_{ts}^*|^2 Y(x_t)^2$$

- Lifetime τ_{B_s} : take from experiment

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- ▶ Lifetime τ_{B_s} : take from experiment
- ▶ $G_F, \alpha, s_w, m_{B_s}^2, m_\mu$: take from PDG

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- ▶ $Y(x_t)$: include NNLO QCD and NLO EW corrections and RG evolution

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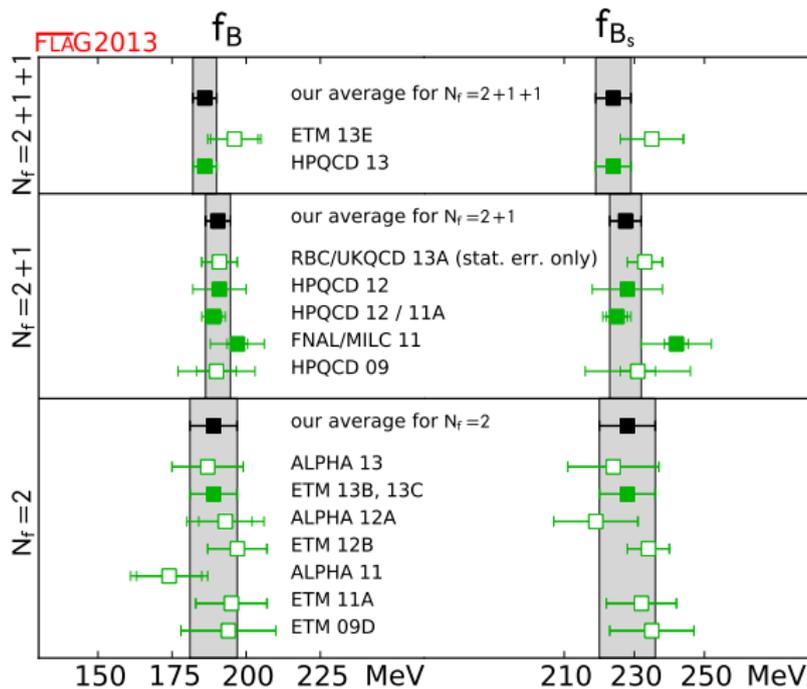
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- ▶ $f_{B_s}^2$: from lattice QCD

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- ▶ $f_{B_s}^2$: from lattice QCD
- ▶ $|V_{tb} V_{ts}^*|^2$: from experiment

Lattice determinations of f_{B_S}

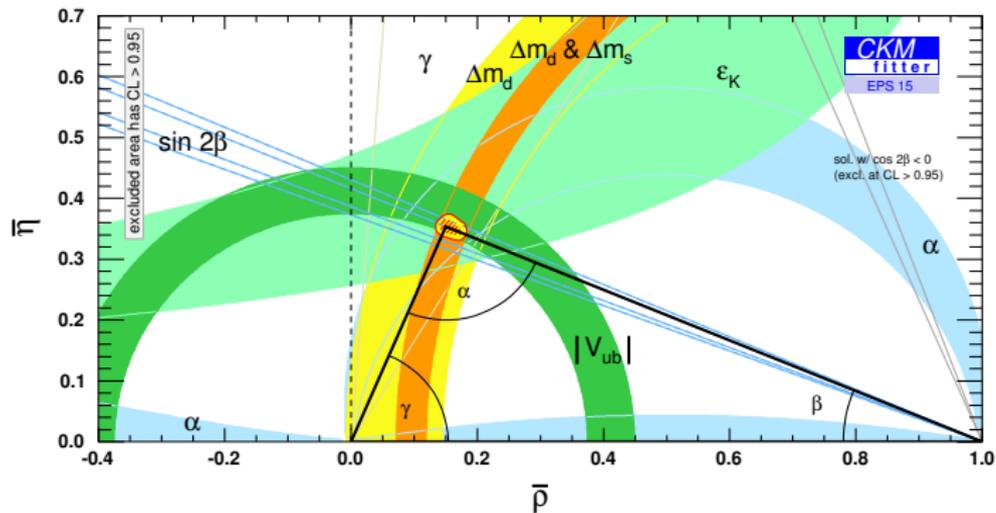


Determining $|V_{tb}V_{ts}^*|$

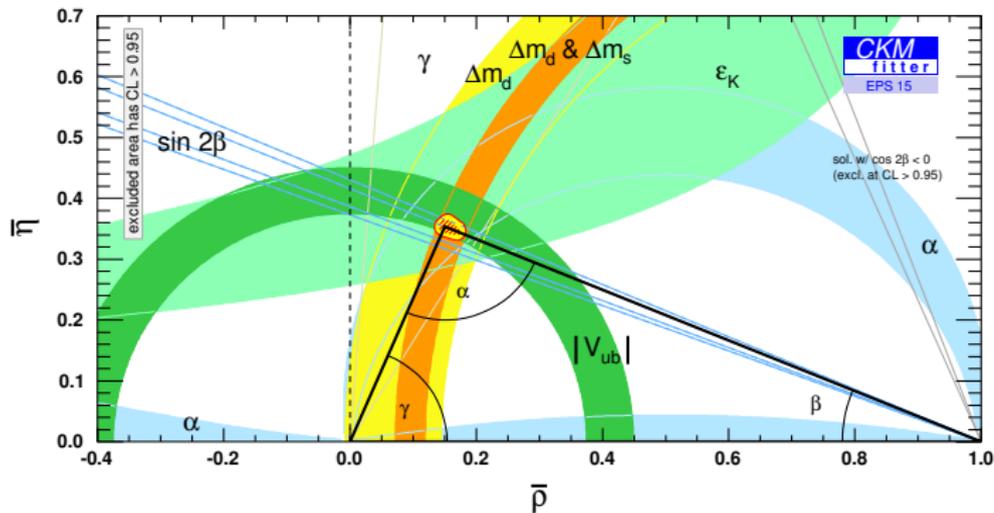
- ▶ There is no direct measurement of V_{ts}
- ▶ But CKM elements can be extracted from a global fit of the CKM matrix

$$|V_{tb}V_{ts}^*| = A\lambda^2 \left[1 + \lambda^2 \left(\bar{\rho} - \frac{1}{2} \right) \right] + O(\lambda^6)$$

Global CKM fits



Global CKM fits



Using the global fit result assumes that neutral meson mixing is free from physics BSM

Using tree-level CKM determinations

- ▶ $|V_{cb}|$ from inclusive & exclusive $b \rightarrow c\ell\nu$
- ▶ $|V_{ub}|$ from inclusive & exclusive $b \rightarrow u\ell\nu$
- ▶ $|V_{us}|$ from $K \rightarrow \pi\ell\nu$
- ▶ γ from $B \rightarrow DK$

$$|V_{tb}V_{ts}^*| = |V_{cb}| \left(1 - \frac{|V_{us}|^2}{2} + \frac{|V_{ub}|}{|V_{cb}|} |V_{us}| \cos \gamma \right) \approx |V_{cb}| (1 - 0.025 + 0.007)$$

Subtle issue: B_s lifetime difference

Due to B_s - \bar{B}_s mixing, there is a sizable lifetime difference between the two B_s mass eigenstates:

$$\tau_{B_s^L} = \Gamma_{B_s^L}^{-1} = 1.42 \text{ ps} \quad \tau_{B_s^H} = \Gamma_{B_s^H}^{-1} = 1.61 \text{ ps}$$

$$\tau_{B_s} = \Gamma_{B_s}^{-1} = \left[\frac{1}{2} \left(\Gamma_{B_s^L} + \Gamma_{B_s^H} \right) \right]^{-1}$$

Which lifetime should we use?

Time-dependent untagged decay rate

$$\Gamma(B_s(t) \rightarrow \mu^+ \mu^-) = R_H e^{-t/\tau_{B_s^H}} + R_L e^{-t/\tau_{B_s^L}}$$

So far, we have computed

$$\text{BR}(B_s \rightarrow \mu^+ \mu^-) = \frac{\tau_{B_s}}{2} \Gamma(B_s(t=0) \rightarrow \mu^+ \mu^-)$$

But experiments actually measure

$$\overline{\text{BR}}(B_s \rightarrow \mu^+ \mu^-) = \frac{1}{2} \int_0^\infty \Gamma(B_s(t) \rightarrow \mu^+ \mu^-) dt$$

It turns out that [De Bruyn et al. 1204.1737](#)

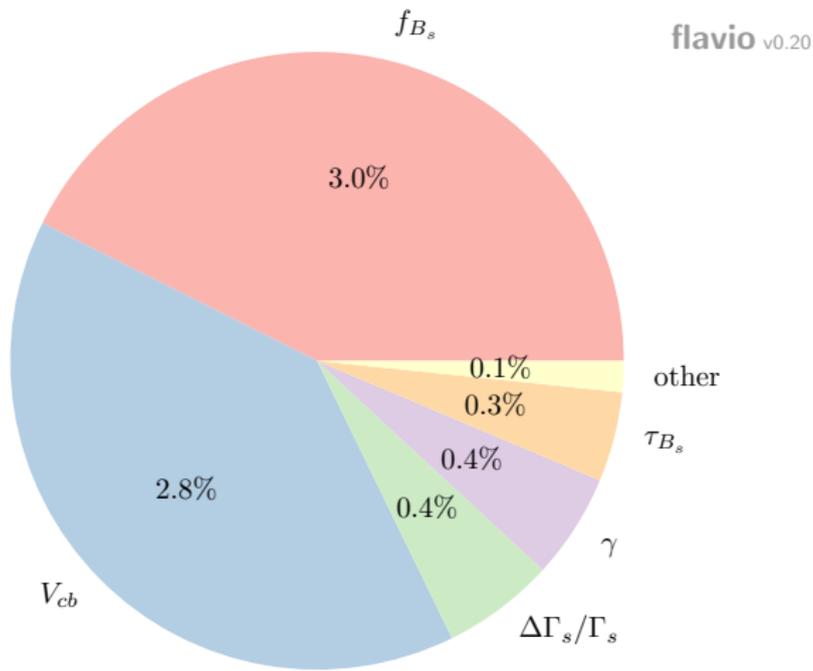
$$\frac{\overline{\text{BR}}(B_s \rightarrow \mu^+ \mu^-)}{\text{BR}(B_s \rightarrow \mu^+ \mu^-)} = \frac{\tau_{B_s^H}}{\tau_{B_s}}$$

Result: $B_s \rightarrow \mu^+ \mu^-$ SM vs. experiment

$$\overline{\text{BR}}(B_s \rightarrow \mu^+ \mu^-)_{\text{LHCb}} (3.0 \pm 0.6^{+0.3}_{-0.2}) \times 10^{-9}$$

$$\overline{\text{BR}}(B_s \rightarrow \mu^+ \mu^-)_{\text{SM}} = (3.59 \pm 0.18) \times 10^{-9}$$

BR($B_s \rightarrow \mu^+ \mu^-$) error budget



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Physics beyond the SM in $B_s \rightarrow \mu^+ \mu^-$

Assuming no new particles below 5 GeV, new physics does **not** affect

- ▶ Matrix element (f_{B_s})
- ▶ CKM extraction based on tree-level decays *
- ▶ QCD corrections
- ▶ Phase space

All “short-distance” physics enters through modified **Wilson coefficients**

* see however Brod et al. 1412.1446

All possible contributing operators ($D = 6$)

$$O_{10} = (\bar{s}_L \gamma^\mu b_L)(\bar{\mu} \gamma_\mu \gamma_5 \mu)$$

$$O'_{10} = (\bar{s}_R \gamma^\mu b_R)(\bar{\mu} \gamma_\mu \gamma_5 \mu)$$

$$O_S = m_b (\bar{s}_R b_L)(\bar{\mu} \mu)$$

$$O'_S = m_b (\bar{s}_L b_R)(\bar{\mu} \mu)$$

$$O_P = m_b (\bar{s}_R b_L)(\bar{\mu} \gamma_5 \mu)$$

$$O'_P = m_b (\bar{s}_L b_R)(\bar{\mu} \gamma_5 \mu)$$

- ▶ In the SM, $C'_{10} = C_S = C'_S = C_P = C'_P = 0$
- ▶ f_{B_s} remains the only required matrix element because

$$\langle 0 | \bar{s} \gamma_\mu \gamma_5 b | \bar{B}_s \rangle = i p^\mu f_{B_s}, \quad \langle 0 | \bar{s} \gamma_5 b | \bar{B}_s \rangle = -\frac{if_{B_s} m_{B_s}^2}{m_b + m_s},$$

- ▶ Other operators (tensor, dipole) have vanishing matrix elements

Branching ratio beyond the SM

$$\text{BR}(B_s \rightarrow \mu^+ \mu^-) = \text{BR}(B_s \rightarrow \mu^+ \mu^-)_{\text{SM}} \left[|A|^2 + |B|^2 \left(1 - \frac{4m_\mu^2}{m_{B_s}^2} \right) \right]$$

$$A = \frac{1}{C_{10}^{\text{SM}}} \left[(C_{10} - C'_{10}) + \frac{m_{B_s}^2}{2m_\mu} (C_P - C'_P) \right]$$

$$B = \frac{1}{C_{10}^{\text{SM}}} \left[\frac{m_{B_s}^2}{2m_\mu} (C_S - C'_S) \right]$$

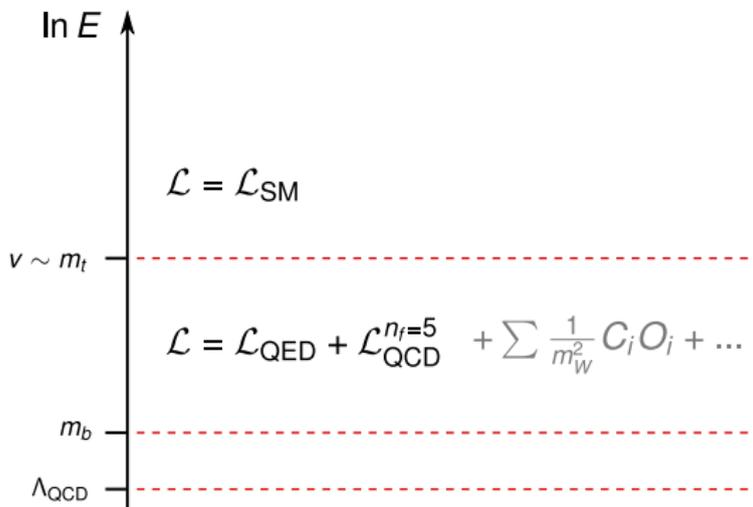
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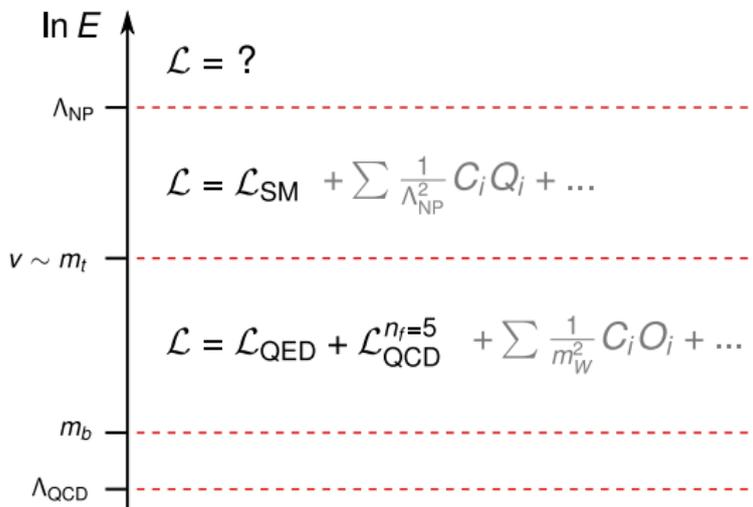
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Hierarchy of effective theories



- $m_b \ll v$: weak effective Hamiltonian

Hierarchy of effective theories



- ▶ $m_b \ll v$: weak effective Hamiltonian
- ▶ $v \ll \Lambda_{\text{NP}}$: “SMEFT”

SMEFT operators matching onto $O_{10}^{(\prime)}$

$$Q_{Hq}^{(1)} = (H^\dagger iD_\mu H) (\bar{q}_s \gamma^\mu q_b)$$

$$Q_{Hq}^{(3)} = H^\dagger iD'_\mu H (\bar{q}_s \tau^I \gamma^\mu q_b)$$

$$Q_{Hd} = (H^\dagger iD_\mu H) (\bar{s}_R \gamma^\mu b_R)$$

$$Q_{\ell q}^{(1)} = (\bar{\ell} \gamma_\mu \ell) (\bar{q}_s \gamma^\mu q_b),$$

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$$Q_{ed} = (\bar{l}_R \gamma_\mu l_R) (\bar{s}_R \gamma^\mu b_R),$$

$$Q_{\ell d} = (\bar{\ell} \gamma_\mu \ell) (\bar{s}_R \gamma^\mu b_R),$$

$$Q_{qe} = (\bar{q}_s \gamma_\mu q_b) (\bar{l}_R \gamma^\mu l_R)$$

$$C_{10}^{\text{NP}} = C_{qe} - C_{\ell q}^{(1)} - C_{\ell q}^{(3)} + (C_{Hq}^{(1)} + C_{Hq}^{(3)})$$

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For C_{10} , we have not gained anything

SMEFT operators matching onto $O_{S,P}^{(\prime)}$

$$Q_{\ell edq}^{ijkl} = (\bar{l}_a^i e^j)(\bar{d}^k q_a^l)$$

$$C_S = -C_P = C_{\ell edq}^{3323}$$

$$C'_S = C'_P = C_{\ell edq}^{3332}$$

SMEFT operators matching onto $O_{S,P}^{(l)}$

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$$C_S = -C_P = C_{\ell edq}^{3323}$$

$$C'_S = C'_P = C_{\ell edq}^{3332}$$

- ▶ At dimension 6 in the SMEFT, there are **only 2** independent scalar/pseudoscalar operators (as opposed to 4 in the low-energy EFT).
- ▶ The SM gauge symmetries **restrict** the form of scalar NP contributions (valid if $\Lambda_{\text{NP}} \gg v$)

Alonso et al. 1407.7044

1 EFT for weak decays

- Introduction
- Example: $\text{BR}(B_s \rightarrow \mu^+ \mu^-)$ in the SM

2 Applications to BSM phenomenology

- Example: $B_s \rightarrow \mu^+ \mu^-$ beyond the SM
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- Model-independent analysis of $B_s \rightarrow \mu^+ \mu^-$
- $B_s \rightarrow \mu^+ \mu^-$ in specific NP models

Fitting the Wilson coefficients

- ▶ We can obtain model-independent constraints on new physics by considering the x^2 function

$$x^2(C_i) = \frac{(x(C_i) - x_{\text{exp}})^2}{\sigma_{\text{exp}}^2 + \sigma_{\text{th}}^2}$$

where $x = \overline{\text{BR}}(B_s \rightarrow \mu^+ \mu^-)$ and C_i are the Wilson coefficients.

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$$x^2(C) - x^2(C^*) < 1 \quad (< 4)$$

where C^* is the value that minimizes x^2 .

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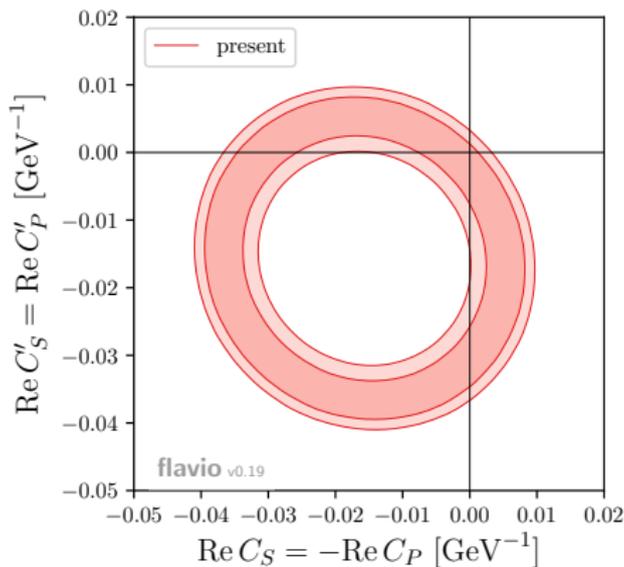
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where C^* is the value that minimizes x^2 .

- ▶ For two coefficients, the 1σ (2σ) regions are given by

$$x^2(\vec{C}) - x^2(\vec{C}^*) < 2.3 \quad (< 6)$$

Fit results



Constraint on real part of scalar operators imposing SMEFT relation

Advertisement break: flavio

- ▶ A Python package for flavour phenomenology in the SM & beyond
 - ▶ repository: <http://github.com/flav-io/flavio>
 - ▶ documentation: <http://flav-io.github.io>
- ▶ Features
 - ▶ SM predictions with uncertainties
 - ▶ NP predictions for arbitrary Wilson coefficients
 - ▶ Fitting SM parameters and Wilson coefficients to data
 - ▶ Plotting library to visualize fit results

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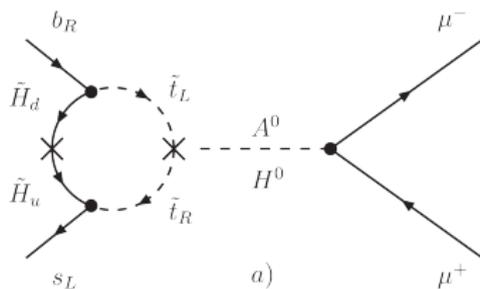
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Minimal Supersymmetric SM (MSSM)

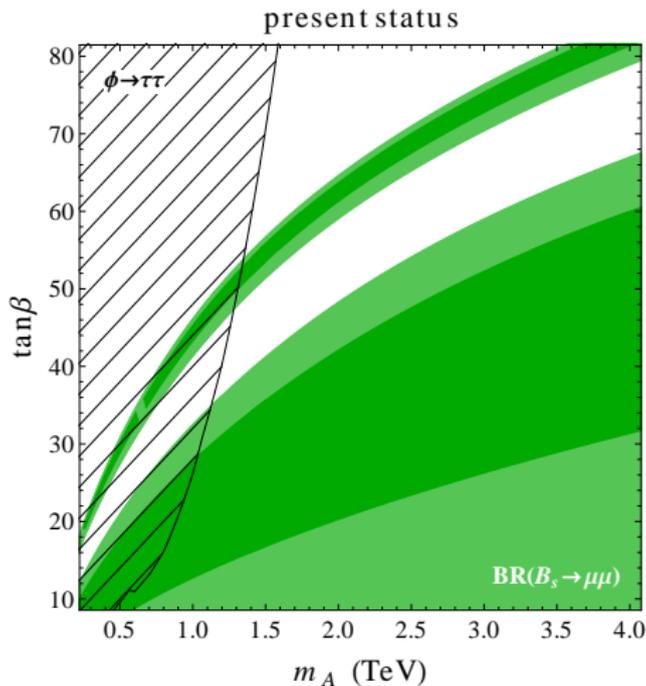
Even for a degenerate spectrum: Higgsino contribution

$$C_S \approx -C_P \approx \frac{G_F^2 m_t^2}{8\pi^2} \frac{m_\mu}{m_A^2} \frac{A_t \mu \tan^3 \beta}{m_{\tilde{t}}^2} f\left(\frac{\mu^2}{m_{\tilde{t}}^2}\right)$$



MSSM with Minimal Flavour Violation: $C'_{S,P} = 0$, $C_S \approx -C_P \in \mathbb{R}$

Constraints on parameter space



- ▶ $B_s \rightarrow \mu^+ \mu^-$ is complementary to Higgs physics ($H/A \rightarrow \tau^+ \tau^-$)
- ▶ Two disjoint solutions corresponding to different overall signs of the amplitude. How to disentangle?

The $B_s \rightarrow \mu^+ \mu^-$ time-dependent rate

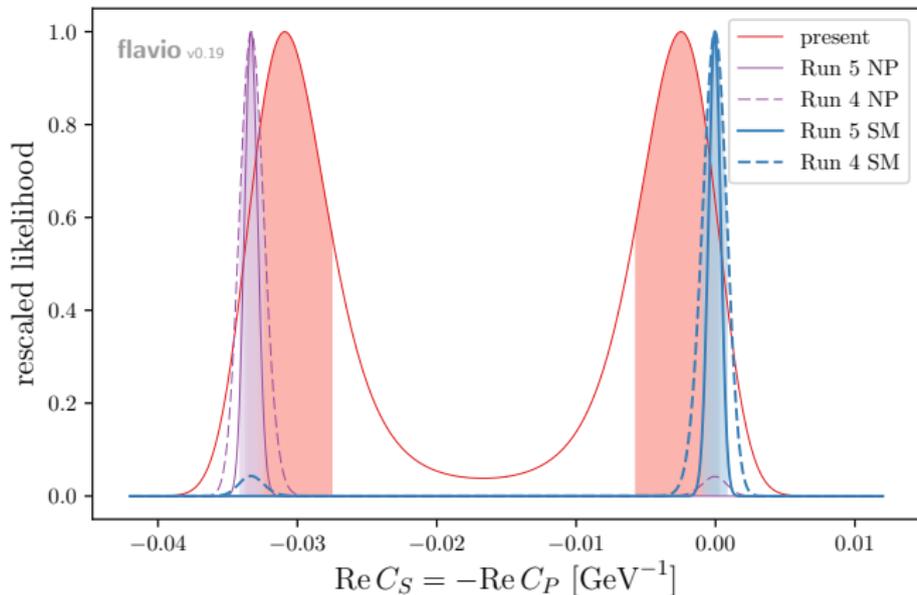
The non-zero $B_s^H - B_s^L$ lifetime difference, $y_s = \frac{\Delta\Gamma_s}{2\Gamma_s} \approx 7\%$ gives access to an additional observable

$$\Gamma(B_s(t) \rightarrow \mu^+ \mu^-) + \Gamma(\bar{B}_s(t) \rightarrow \mu^+ \mu^-) \propto \left[\cosh\left(\frac{y_s t}{\tau_{B_s}}\right) + A_{\Delta\Gamma} \sinh\left(\frac{y_s t}{\tau_{B_s}}\right) \right] \times e^{-t/\tau_{B_s}}$$

Mass-eigenstate rate asymmetry $A_{\Delta\Gamma}$

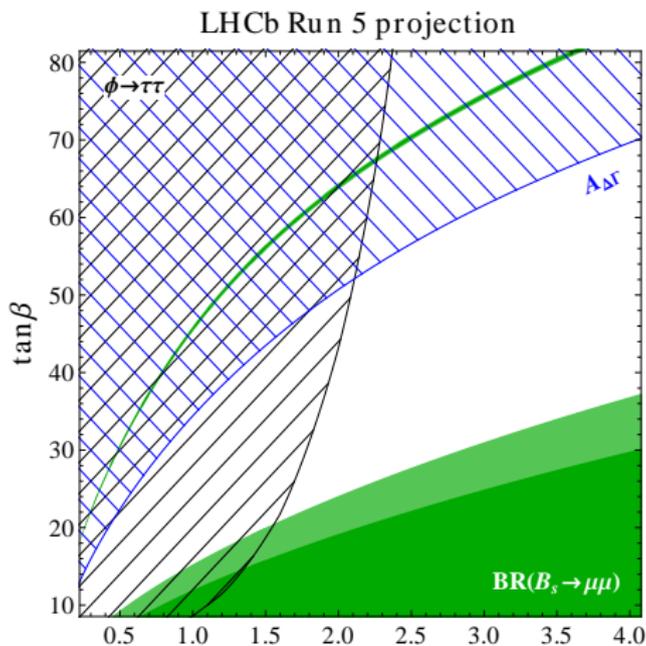
cf. Fleischer 0802.2882

Impact of future measurement of $A_{\Delta\Gamma}$



Constraint on real part of C_S imposing SMEFT relation [Altmannshofer et al. 1702.05498](#)

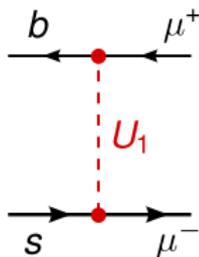
Future constraints on MSSM parameter space



- $A_{\Delta\Gamma}$ can exclude (or confirm??) second solution

Example 2: Leptoquarks

$$\mathcal{L}_{U_1} = \hat{\lambda}_L^{ij} \left(\bar{Q}_L^i \gamma_\mu L_L^j \right) U_1^\mu + \hat{\lambda}_R^{ij} \left(\bar{d}_R^i \gamma_\mu e_R^j \right) U_1^\mu + \text{h.c.}$$



$$C_9^{\text{NP}}$$

$$C_9'$$

$$C_{10}^{\text{NP}}$$

$$C_{10}'$$

$$C_S = -C_P$$

$$C_S' = C_P'$$

$$-\frac{1}{2} \mathcal{N} \lambda_L^{s\mu} \lambda_L^{b\mu*}$$

$$-\frac{1}{2} \mathcal{N} \lambda_R^{s\mu} \lambda_R^{b\mu*}$$

$$\frac{1}{2} \mathcal{N} \lambda_L^{s\mu} \lambda_L^{b\mu*}$$

$$-\frac{1}{2} \mathcal{N} \lambda_R^{s\mu} \lambda_R^{b\mu*}$$

$$\mathcal{N} \lambda_L^{s\mu} \lambda_R^{b\mu*} m_b^{-1}$$

$$\mathcal{N} \lambda_R^{s\mu} \lambda_L^{b\mu*} m_b^{-1}$$

Constraint on LQ parameter space

Scenario with $C'_S = C'_P = 0$

