

EFT methods for new physics searches in the Electroweak sector

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Outline

- 1 Setting the framework
- 2 HEFT in a nutshell
- 3 Key aspects of Electroweak physics
- 4 Case example @ tree-level: a heavy Higgs triplet
- 5 Case example @ loop-level: a heavy Higgs singlet
- 6 Summary & References
- 7 Bonus track - Nondecoupling

Whys and wherefores

WHY NEW PHYSICS?

- **Voice from Experiments:** $\Delta_{xxH} \sim \mathcal{O}(10\%)$ & EXP-TH mismatches
- **Voice from Theory:** naturalness, DM, vacuum stability, ν 's, Baryogenesis ...
- Too many **misteries** still surrounding EWSB \Leftrightarrow $V(H)$ describes, rather than explains
- The **Higgs portal** to new physics: $\mathcal{O}(\Phi^\dagger \Phi)$ (new physics)

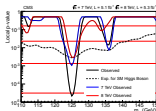
We are building on the evidence

... of

THE GENUINE ONE?

Or rather

an almost perfect copy?



Whys and wherefores

WHY ELECTROWEAK PRECISION?

APPARENT (*RESONANT*)

NEW PHYSICS

SUBTLE (*INDIRECT*)

- LHC searches have so far come up empty-handed \Rightarrow a gap between Λ_{new} and Λ_{EW} .
- Expected accuracies $\mathcal{O}(1 - 0.1)\%$ for precision Higgs and EW observables
- **Promising** window to indirectly exploring the BSM theory space
- *E.g. assuming $\Lambda_{\text{new}} \sim \text{TeV}$, we should be able to learn about the parameters of an underlying New Physics (NP) model by performing global fits to precision measurements, e.g. setting bounds on Λ_{new} .*

WHY EFFECTIVE THEORIES?



Fundamental:

New Physics is *SHY*...



Hierarchy of scales:

$$\Lambda_{\text{new}} \gg \Lambda_{\text{EW}}$$



Practical

- A TH-EXP *Lingua Franca*
- Minimizes theory input & computational effort
- Maximizes **model independence & scale separation**

Our mindset

EW scale Λ_{IR}
Light fields ϕ
low masses $m = \{m_W, m_Z, m_H\} \sim \mathcal{O}(\Lambda_{\text{EW}})$
Low-scale Lagrangian: $\mathcal{L}_{\text{IR}}(\phi; g, m) \equiv \mathcal{L}_{\text{SM}}$

Renormalization Group

High scale Λ_{new}
Heavy fields Ψ
Large masses $M_\Psi \sim \mathcal{O}(\Lambda_{\text{new}})$
High-scale Lagrangian: $\mathcal{L}_{\text{UV}}(\Psi, \phi; g, g_{\text{UV}}, m, M_\Psi) =$
 $\mathcal{L}_{\text{SM}}(\phi, g, m) + \mathcal{L}_{\text{new}}(\Psi, g_{\text{UV}}, M_\Psi) + \mathcal{L}_{\text{New-SM}}$





EFT basics

Building blocks

$$\mathcal{L}_{\text{eff}}(\phi, m) = \mathcal{L}_{\text{SM}} + \sum_{d=5}^{\infty} \sum_{i_d} \frac{C_{i_d}^{(d)}(\{g_{UV}\}, \mu)}{\Lambda^{d-4}} \mathcal{O}_{i_d}^{(d)}(\phi)$$

- Short-distance physics: is averaged \Rightarrow
 - ♣ Effective coupling strength c_i encoding $\{g_{UV}\}$ through **matching**

$$\Gamma_{\text{full}}^{\text{1PI}}[\phi](\{g_{UV}\}, \mu = \Lambda) = \Gamma_{\text{EFT}}^{\text{1PI}}[\phi](\{c_i\}, \mu = \Lambda)$$

- Long-distance physics: \Rightarrow
 - ♣ light-field local interactions parametrized by $\hat{\mathcal{O}}_{d>4}$
(Remainder of nonlocal interactions mediated by heavy-field exchanges)
 - ♣ $d > 4$ operators dependent on ϕ
 - ♣ $d > 4$ operators fixed by **light field symmetries**

TOP-DOWN CONSTRUCTION

INTEGRATING OUT

TRUNCATING

MATCHING

EFTs as a double expansion: ♣ Perturbative coupling ♣ Ratio of scales

HEFT operators: a closer look

LINEAR (decoupling) EFT: \mathcal{L}_{SM} and $\{\phi\}$ include H_{SM} explicitly

♣ Higgs fields only

$$\mathcal{O}_H = \frac{1}{2}(\partial^\mu |H|^2)^2, \quad \mathcal{O}_T = \frac{1}{2}(H^\dagger \overleftrightarrow{D}_\mu H)^2, \quad \mathcal{O}_r = |H|^2 |D_\mu H|^2, \quad \mathcal{O}_6 = \lambda |H|^6.$$

♣ Higgs & weak bosons - CP-even

$$\begin{aligned} \mathcal{O}_{BB} &= g'^2 |H|^2 B_{\mu\nu} B^{\mu\nu}, & \mathcal{O}_{GG} &= g_s^2 |H|^2 G_{\mu\nu}^A G^{A\mu\nu}, \\ \mathcal{O}_{HW} &= ig(D^\mu H)^\dagger \sigma^a (D^\nu H) W_{\mu\nu}^a, & \mathcal{O}_{HB} &= ig'(D^\mu H)^\dagger (D^\nu H) B_{\mu\nu}, \\ \mathcal{O}_{3W} &= \frac{1}{3!} g \epsilon_{abc} W_\mu^{a\nu} W_{\nu\rho}^b W^{c\rho\mu}, & \mathcal{O}_{3G} &= \frac{1}{3!} g_s f_{ABC} G_\mu^{A\nu} G_{\nu\rho}^B G^{C\rho\mu}, \end{aligned}$$

♣ Higgs & weak bosons - CP-odd

$$\begin{aligned} \mathcal{O}_{B\tilde{B}} &= g'^2 |H|^2 B_{\mu\nu} \tilde{B}^{\mu\nu}, & \mathcal{O}_{G\tilde{G}} &= g_s^2 |H|^2 G_{\mu\nu}^A \tilde{G}^{A\mu\nu}, \\ \mathcal{O}_{H\tilde{W}} &= ig(D^\mu H)^\dagger \sigma^a (D^\nu H) \tilde{W}_{\mu\nu}^a, & \mathcal{O}_{H\tilde{B}} &= ig'(D^\mu H)^\dagger (D^\nu H) \tilde{B}_{\mu\nu}, \\ \mathcal{O}_{3\tilde{W}} &= \frac{1}{3!} g \epsilon_{abc} \tilde{W}_\mu^{a\nu} W_{\nu\rho}^b W^{c\rho\mu}, & \mathcal{O}_{3\tilde{G}} &= \frac{1}{3!} g_s f_{ABC} \tilde{G}_\mu^{A\nu} G_{\nu\rho}^B G^{C\rho\mu}, \end{aligned}$$

All these operators originate whenever **heavy fields** directly **couple** only to the SM **gauge** fields and a SM-like **Higgs**. There are referred to as **universal** or **oblique**, as they universally affect all quarks and leptons via fermion couplings to the SM gauge fields.

HEFT Lagrangian

HEFT parametrization SILH basis Giudice, Grojean, Pomarol, Ratazzi [07]

$$\begin{aligned}
\mathcal{L}_{\text{EFT}} = & \mathcal{L}_{\text{SM}} + \frac{\bar{c}_H}{2v^2} \partial^\mu (\phi^\dagger \phi) \partial_\mu (\phi^\dagger \phi) + \frac{\bar{c}_T}{2v^2} (\phi^\dagger \overleftrightarrow{D}^\mu \phi) (\phi^\dagger \overleftrightarrow{D}_\mu \phi) - \frac{\bar{c}_6 \lambda}{v^2} (\phi^\dagger \phi)^3 \\
& + \frac{ig\bar{c}_W}{2m_W^2} (\phi^\dagger \sigma^k \overleftrightarrow{D}^\mu \phi) D^\nu W^k_{\mu\nu} + \frac{ig'\bar{c}_B}{2m_W^2} (\phi^\dagger \overleftrightarrow{D}^\mu \phi) \partial^\nu B_{\mu\nu} + \dots \\
& + \frac{ig\bar{c}_{HW}}{m_W^2} (D^\mu \phi^\dagger) \sigma^k (D^\nu \phi) W^k_{\mu\nu} + \frac{ig'\bar{c}_{HB}}{m_W^2} (D^\mu \phi^\dagger) (D^\nu \phi) B_{\mu\nu} \\
& + \frac{g'^2 \bar{c}_\gamma}{m_W^2} (\phi^\dagger \phi) B_{\mu\nu} B^{\mu\nu} + \frac{g_s^2 \bar{c}_g}{m_W^2} (\phi^\dagger \phi) G^A_{\mu\nu} G^{\mu\nu A} \\
& - \left[\frac{\bar{c}_u}{v^2} y_u (\phi^\dagger \phi) (\phi^\dagger \cdot \bar{Q}_L) u_R + \frac{\bar{c}_d}{v^2} y_d (\phi^\dagger \phi) (\phi \bar{Q}_L) d_R + \frac{\bar{c}_\ell}{v^2} y_\ell (\phi^\dagger \phi) (\phi \bar{L}_L) \ell_R + \text{h.c.} \right].
\end{aligned}$$

$$\begin{aligned}
\mathcal{L}_{WW} = & W_\mu^+ \left(\partial^4 g^{\mu\nu} - \partial^2 \partial^\mu \partial^\nu \right) W_\nu^- \left(-\frac{1}{\Lambda^2} c_{2W} \right) \\
& + W_\mu^+ \left(-\partial^2 g^{\mu\nu} + \partial^\mu \partial^\nu \right) W_\nu^- \frac{2m_W^2}{\Lambda^2} (4c_{WW} + c_W) \\
& + m_W^2 W_\mu^+ W^{-\mu} \frac{v^2}{\Lambda^2} c_R - W^{-\mu} (\partial_\mu \partial_\nu) W^{+\nu} \frac{m_W^2}{\Lambda^2} c_D
\end{aligned}$$

The two BSM portrays

Given a **UV model** with heavy BSM states; \Rightarrow how do these affect **H & EW** observables?

UV-complete picture

- **IDEAS??**
- **IDEAS??**



EFT picture

- **IDEAS??**
- **IDEAS??**

The two BSM portrays

Given a **UV model** with heavy BSM states; \Rightarrow how do these affect **H & EW** observables?

UV-complete picture

- Retains the complete BSM field content & interactions
- Fully accurate - to a given loop order
- Case-specific calculations - error-prone, uneasy to automate/generalize
- Better accuracy irrelevant for many practical purposes/realistic scenarios

EFT picture

- Model independence - minimizes theory input
- Universal computational effort
- Limited accuracy to both loop & $\mathcal{O}(\Lambda_{\text{new}}^{-n})$ expansions
- Delicate cases require careful treatment (*light states, unitarization, nondecoupling*)

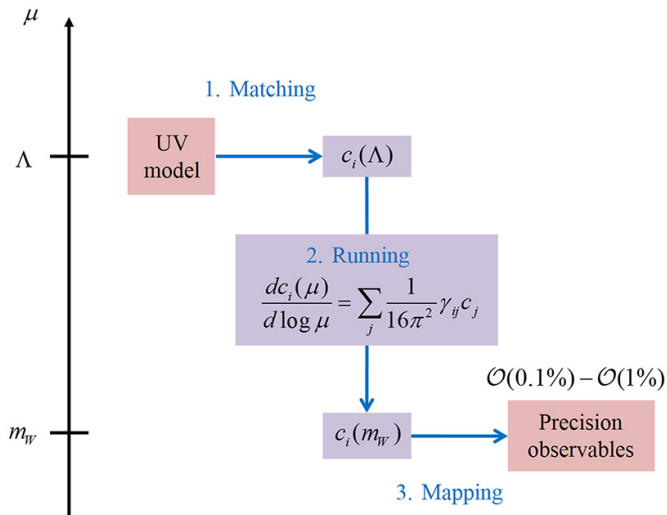
♣ Expected exp. precision: $\mathcal{O}(0.001)$

♣ EFT VS full-model error: $\mathcal{O}(\Lambda_{\text{EW}}/\Lambda_{\text{new}})^{2+n}$

Linear d6 EFT is a reliable tool for probing weakly coupled, $\Lambda_{\text{new}} \sim \mathcal{O}(\text{few})$ TeV new physics.

HEFT in use

Connecting **UV models** to **EW observables** in the EFT:



Taken from [Henning, Lu, Murayama arXiv:1604.01019](#)

HEFT in use

Connecting **UV models** to **EW observables** in the EFT:

🔥 **MATCHING:**

- A number of **1PI light-field correlation functions** $\Gamma[\{\phi_{SM}\}]^{1PI}$ are computed (at a given loop order) in terms of both UV-complete model parameters and EFT Wilson coefficients.
- Techniques available: **Feynman-diagrams**, **path integrals**
- The EFT results are expanded in powers of $\Lambda_{EW}/\Lambda_{new}$
- One identifies

$$\Gamma_{full}^{1PI} = \Gamma(\{g_j\}, \{\Psi, \phi_{SM}\}, \mu = \Lambda_{new})$$

with

$$\Gamma_{EFT}^{1PI} = \Gamma(\{c_j\}, \{\phi_{SM}\}, \mu = \Lambda_{new})$$

at the high-energy scale Λ_{new} , order by order in the perturbative (loop) and small parameter (ratio of scales) expansions.

- Doing so, the S-matrix elements in the effective and the UV-complete models are the same at $\mu = \Lambda_{new}$, up to higher orders in the two simultaneous power series.
- Thereby the (scale-dependent) Wilson coefficients at $\mu = \Lambda_{new}$ get fixed in terms of the full model parameters

HEFT in use


Connecting **UV models** to **EW observables** in the EFT:
 **RUNNING:**

- $c(\Lambda)$ are RG-evolved down to the EW scale.
- The RG flow of the Wilson coefficients is governed by the $\hat{\mathcal{O}}_{d>4}$ **anomalous dimensions** γ_{ij} :

$$c_i^{(0)} \rightarrow c_i = Z_{c_i, c_j} c_j = \left(1 + \frac{\delta Z_{c_i, c_j}}{2} \right) c_j$$

$$\delta Z_{c_i, c_j} \equiv \frac{\Gamma(1 + \epsilon)}{(4\pi)^2} \left(\frac{4\pi\mu^2}{\mu_R^2} \right)^\epsilon \frac{\gamma_{ij}}{\epsilon} \quad (+ \text{finite parts})$$

$$\gamma_{ij} \equiv (4\pi)^2 \frac{dc_j(c_i)}{d \log(\mu)} \Rightarrow c_i(\mu) = c_i(\Lambda_{\text{new}}) + \frac{1}{(4\pi)^2} \gamma_{ij} c_j(\Lambda_{\text{new}}) \log \left(\frac{\mu}{\Lambda_{\text{new}}} \right).$$

- The non-diagonal γ_{ij} implies both operator *running* and *mixing*.
- for TeV-scale BSM, $\log(\Lambda_{\text{new}}/\Lambda_{\text{EW}}) \sim \mathcal{O}(1)$  one-loop accuracy is enough

 **MAPPING:**

- EW observables are written in terms of $c_i(\Lambda_{\text{EW}})$ and **SM parameters**.

Custodial symmetry

Let us inspect the SM in the limit of absent hypercharge interaction ($g' \rightarrow 0$)

$$\mathcal{L}_{\text{Higgs}} = \frac{1}{2} \text{tr} (D_\mu \Omega^\dagger D^\mu \Omega) - V(\text{tr}(\Omega^\dagger \Omega)) \quad \text{where} \quad \boxed{\Omega \equiv (i\sigma_2 H^*, H)}$$

- ♠ $\mathcal{L}_{\text{Higgs}}$ is $SU(2)_L \otimes SU(2)_R$ -invariant if $\boxed{\Omega \rightarrow L\Omega R^\dagger}$
- ♠ Both the global $SU(2)_L$ and the accidental $SU(2)_R$ are **exact** in the **symmetric phase**, while separately **broken** by $\langle H \rangle = v$.
- ♠ However, $\mathcal{L}_{\text{Higgs}}$ is still invariant under $SU(2)_{L+R}$ ($\Omega \rightarrow L\Omega L^\dagger$) even when $\langle H \rangle = v$
- ♠ G^\pm, G^0 as **Goldstones** for $SU(2)_L \times SU(2)_R \rightarrow SU(2)_{L+R}$
- ♠ W^\pm, Z form a $SU(2)_{L+R}$ **triplet** \Rightarrow $\boxed{m_W \simeq m_Z}$ to all orders

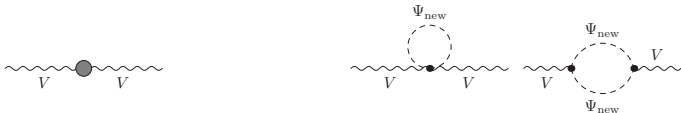
$$m_W^2 = \frac{1}{4} g^2 v^2 \quad m_Z^2 = \frac{1}{4} (g^2 + g'^2) v^2 \quad \frac{m_W^2}{m_Z^2} = \frac{g^2}{g'^2 + g^2} \rightarrow 1 \Big|_{g' \rightarrow 0}$$

$$\rho \equiv \frac{m_W^2}{m_Z^2 c_w^2} = \frac{\sum_i \left[T_i(T_i + 1) - \frac{1}{4} Y_i^2 \right] v_i^2}{\frac{1}{2} \sum_i Y_i^2 v_i^2} \quad (+\text{higher orders}) .$$

- ♠ Any number of **singlets & doublets** respects **custodial** symmetry at tree-level
- ♠ Loop-induced contributions naturally remain small.

Electroweak Precision Observables

- Virtual exchange of heavy particles as an indirect footprint of TeV-scale new physics



$$i \Pi_{\mu\nu}^{XY}(p^2) = i \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \Pi_T^{XY}(p^2) + i \frac{p_\mu p_\nu}{p^2} \Pi_L^{XY}(p^2) \equiv \int d^4x e^{-ipx} \langle J_X^\mu J_Y^\nu \rangle$$

$$\Pi^{XY}(p^2) = \underbrace{\Pi^{XY}(0)}_{\text{static part}} + p^2 \Pi'^{XY}(p^2)$$

- The QED Ward identities fix $\Pi_{\gamma\gamma}(0) = \Pi_{\gamma Z}(0) = 0$; another three combinations are absorbed in the definition of g, g' and v

- Up to p^2 order, the remaining combinations define the **Peskin-Takeuchi STU parameters**:

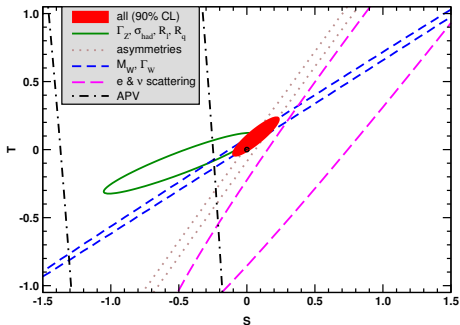
$$\alpha_{\text{em}} S = -4s_w^2 c_w^2 \Pi'_{3B}(0) = 4s_w c_w \left[s_w c_w \Pi'_{ZZ}(0) - s_w c_w \Pi'_{\gamma\gamma}(0) - (c_w^2 - s_w^2) \Pi'_{\gamma Z}(0) \right]$$

$$\alpha_{\text{em}} T = \frac{1}{m_W^2} [\Pi_{WW}(0) - \Pi_{33}(0)] = \frac{1}{m_W^2} \left[\Pi_{WW}(0) - c_w^2 \Pi_{ZZ}(0) - s_w^2 \Pi_{\gamma\gamma}(0) - 2c_w s_w \Pi_{\gamma Z} \right]$$

$$\alpha_{\text{em}} U = 4s_w^2 [\Pi'_{WW}(0) - \Pi'_{33}(0)] = \frac{1}{m_W^2} \left[\Pi'_{WW}(0) - c_w^2 \Pi'_{ZZ}(0) - s_w^2 \Pi'_{\gamma\gamma}(0) - 2c_w s_w \Pi'_{\gamma Z} \right]$$

Electroweak Precision: experimental perspective

- ♣ Experimentally, STU can be linked to a small subset of accurate data:
 - **Flavor-diagonal** (oblique) processes involving the weak gauge bosons (e.g. Z-pole observables)
 - **LEP2 e^+e^- data** at various \sqrt{S} between m_Z and 209 GeV
- ♣ Confronting BSM predictions to SM fits, STU can be used to **probe & constrain** the parameter space of new physics models.
- ♣ Either directly on **model-specific $\{g, M\}$** - or through **EFT Wilson coefficients**.



M. Baak *et al.* [Gfitter Group] arXiv:1407.3792 [hep-ph]

Electroweak Precision: EFT perspective

STU in the EFT follow from the **c_i -modified weak boson vacuum polarization**:

$$i\Pi_{WW}^{\mu\nu}(p^2) = i \left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2} \right) \cdot \left[p^4 \left(-\frac{1}{\Lambda^2} c_{2W} \right) + p^2 \frac{2m_W^2}{\Lambda^2} (4c_{WW} + c_W) \right. \\ \left. + m_W^2 \frac{v^2}{\Lambda^2} c_R \right] + i \frac{p^\mu p^\nu}{p^2} \cdot \left(p^2 \frac{m_W^2}{\Lambda^2} c_D + m_W^2 \frac{v^2}{\Lambda^2} c_R \right),$$

$$i\Pi_{ZZ}^{\mu\nu}(p^2) = i \left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2} \right) \cdot \left\{ p^4 \left[-\frac{1}{\Lambda^2} (c_Z^2 c_{2W} + c_{2B} s_Z^2) \right] \right. \\ \left. + p^2 \frac{2m_Z^2}{\Lambda^2} \left[4 (c_Z^4 c_{WW} + s_Z^4 c_{BB} + c_Z^2 s_Z^2 c_{WB}) + (c_Z^2 c_W + s_Z^2 c_B) \right] \right. \\ \left. + m_Z^2 \frac{v^2}{\Lambda^2} (-2c_T + c_R) \right\} + i \frac{p^\mu p^\nu}{p^2} \cdot m_Z^2 \frac{v^2}{\Lambda^2} (-2c_T + c_R),$$

$$\alpha_{\text{em}} S = \frac{4s_w^2 c_w^2 m_Z^2}{\Lambda^2} (4c_{WB} + c_W + c_B);$$

$$\alpha_{\text{em}} T = \frac{v^2}{\Lambda^2} c_T$$

$$U = 0$$

UV complete model

Lagrangian & field content

$$\Phi: \mathbf{2}_{\frac{1}{2}}$$

$$\Delta: \mathbf{3}_0$$

$$\mathcal{L} \supset \frac{1}{2} (D_\mu \Delta^a)^2 - \frac{1}{2} M_\Delta^2 (\Delta^a)^2 + \kappa v_\Delta \Phi^\dagger \sigma^a \Phi \Delta^a - \eta (\Phi^\dagger \Phi) (\Delta^a)^2$$

$$D_\mu \Delta^c = (\partial_\mu + i g t_a W^a) \Delta^c$$

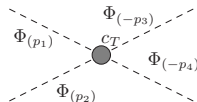
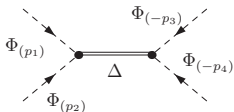
$$t_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad t_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad t_3 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$\Phi = \begin{pmatrix} \phi^\pm \\ \frac{v + \phi^0 + i\chi^0}{\sqrt{2}} \end{pmatrix} \quad \Delta = \begin{pmatrix} \Delta^+ \\ \Delta^0 \\ -\Delta^- \end{pmatrix}$$

- ♣ Simplest $\rho \neq 1$ extended Higgs sector
- ♣ Benchmark for exotics (H^{++})

Chen, Dawson, Jackson arXiv:0809.4185 [hep-ph]

Tree-level matching - diagrammatic calculation



EFT result

♠ Feynman rule for \hat{O}_T : $i\mathcal{L} = i\frac{c_T}{2\Lambda^2} [\Phi^\dagger \overleftrightarrow{D}^\mu \Phi] [\Phi^\dagger \overleftrightarrow{D}_\mu \Phi] + \text{Fourier Transform}$

♠ Field expansion in normal modes: $\Phi_k = \int \frac{d^4 p}{(2\pi)^4} [a_{pk} e^{-ip_k x} + a_{pk}^\dagger e^{ip_k x}]$

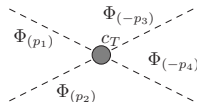
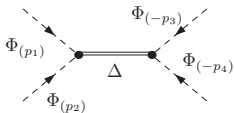
♠ 1PI tree-level amplitude

$$i\mathcal{M}_{\Phi_2^\dagger \Phi_1 \rightarrow \Phi_4^\dagger \Phi_3}^{\text{EFT}} = \frac{\delta^4}{\delta \Phi_4^\dagger \delta \Phi_3^\dagger \delta \Phi_2 \delta \Phi_1} \left\{ \frac{ic_T}{2} [\Phi_2^\dagger(-ip_1) \Phi_1 - \Phi_2^\dagger(ip_2) \Phi_1] [\Phi_4^\dagger(-ip_3) \Phi_3 - \Phi_4^\dagger(ip_4) \Phi_3] + \text{perm} \right\}$$

$$= \frac{-ic_T}{2} [(p_1 \cdot p_2) + (p_3 \cdot p_4)] + \text{perm} = \boxed{-2ic_T (p_1 \cdot p_2)}$$

♣ Momentum conservation $\Rightarrow \boxed{p_1 + p_2 + p_3 + p_4 = 0}$

Tree-level matching - diagrammatic calculation



Full model result

♣ tree-level amplitude describing the s-channel Δ exchange:

$$\begin{aligned}
 i\mathcal{M}_{\Phi_2^\dagger \Phi_1 \rightarrow \Phi_4^\dagger \Phi_3}^{\text{full}} &= -i\kappa^2 v_\Delta^2 \left[\frac{1}{(p_1 + p_2)^2 - M_\Delta^2} \right] \\
 &= \frac{-i\kappa^2 v_\Delta^2}{M_\Delta^2} \left[1 + \frac{(p_1 + p_2)^2}{M_\Delta^2} \right] + \mathcal{O}(M_\Delta^{-4}) = \boxed{\frac{-2i\kappa^2 v_\Delta^2}{M_\Delta^4} (p_1 \cdot p_2)}.
 \end{aligned}$$

♣ i) **Expand**; ii) **Truncate** to $d6$; iii) **Match**

$$i\mathcal{M}_{\Phi_2^\dagger \Phi_1 \rightarrow \Phi_4^\dagger \Phi_3}^{\text{full}} = i\mathcal{M}_{\Phi_2^\dagger \Phi_1 \rightarrow \Phi_4^\dagger \Phi_3}^{\text{EFT}}$$

\Rightarrow

$$c_T = \frac{\kappa^2 v_\Delta^2}{M_\Delta^2} \quad \text{with} \quad \Lambda = M_\Delta$$

Tree-level matching - Path Integral calculation

Functional formalism - general setup

- Starting from a generic UV model Lagrangian:

$$\mathcal{L}[\Psi, \Phi] = \mathcal{L}_{\text{SM}} + [\Psi^\dagger B(x) + \text{h.c.}] + \Psi^\dagger [-D^2 - M^2 - U(x)] \Psi + \mathcal{O}(\Psi^3)$$

$B(x), U(x)$: generic functions of the light fields $\{\Phi\}$

- Effective action:

$$e^{iS_{\text{eff}}[\Phi]} = \int [D\Psi] e^{iS_{\text{full}}[\Psi, \Phi]} = \int [D\Psi] e^{i \int d^4x \mathcal{L}_{\text{full}}[\Psi, \Phi]}$$

- $\Psi \rightarrow \Psi_c + \eta(x)$

$$\Rightarrow S_{\text{full}}[\Psi = \Psi_c + \eta, \Phi] = S[\Psi_c] + \left. \frac{1}{2} \frac{\delta^2 S}{\delta \Psi^2} \right|_{\Psi = \Psi_c} \eta^2 + \mathcal{O}(\eta^3)$$

- Classical solution:

$$\left. \frac{\delta S[\Psi, \Phi]}{\delta \Psi} \right|_{\Psi = \Psi_c} = 0 \Rightarrow [-D^2 - M^2 - U(x)] \Psi_c = -B(x) + \mathcal{O}(\Psi^2)$$

$$\begin{aligned} \Rightarrow \Psi_c &= -\frac{B(x)}{p^2 - M^2 - U(x)} = \left[1 - \left(\frac{p^2 - U(x)}{M^2} \right) \right]^{-1} \frac{B(x)}{M^2} = \\ &= \frac{B(x)}{M^2} + \frac{B^\dagger(x)}{M^2} [p^2 - U(x)] \frac{B(x)}{M^2} + \mathcal{O}(M^{-6}) \quad [\text{recall: } p_\mu \equiv iD_\mu] \end{aligned}$$

Tree-level matching - Path Integral calculation

Functional formalism - triplet model

♠ Triplet model Lagrangian:

$$\mathcal{L} = \mathcal{L}_{\text{sm}} + \frac{1}{2} (D_\mu \Delta^a)^2 - \frac{1}{2} M_\Delta^2 \Delta^2 + \underbrace{\kappa v_\Delta \Phi^\dagger \sigma^a \Phi}_{B(x)} \Delta^a - \Delta^{a*} \underbrace{\eta (\Phi^\dagger \Phi)}_{U(x)} \Delta^a,$$

♠ Euler-Lagrange Equation:

$$\frac{\delta \mathcal{L}}{\delta (\partial_\mu \Delta^a)} = \frac{\delta \mathcal{L}}{\delta \Delta^a} \Rightarrow \partial^2 \Delta^a = -M_\Delta^2 \Delta^a + \kappa v_\Delta (\Phi^\dagger \sigma^a \Phi) - 2\eta (\Phi^\dagger \Phi) \Delta^a$$

♠ Triplet scalar classical background: $\Delta_c^a = \frac{-\kappa v_\Delta \Phi^\dagger \sigma^a \Phi}{p^2 - M_\Delta^2 - 2\eta \Phi^\dagger \Phi}$

♣ Tree-level Effective Lagrangian:

$$\mathcal{L} \supset \frac{1}{2} \left(\frac{\kappa v_\Delta}{M_\Delta^2} \right)^2 \left[D_\mu (\Phi^\dagger \sigma^a \Phi) \right]^2 + \dots = \frac{1}{2} \left(\frac{\kappa v_\Delta}{M_\Delta^2} \right)^2 (2\hat{\mathcal{O}}_T + 4\mathcal{O}_D) + \dots$$

$$\Rightarrow \boxed{c_T = \frac{\kappa^2 v^2 v_\Delta^2}{M_\Delta^2}} \Rightarrow \boxed{\alpha_{\text{em}} T = \frac{\kappa^2 v^2 v_\Delta^2}{M_\Delta^4}} \quad \text{cf. 1412.1837 [hep-ph]}$$

UV complete model

Lagrangian & field content

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \partial^\mu S \partial_\mu S - \mu_s^2 S^2 - \lambda_2 S^4 - \lambda_3 \Phi^\dagger \Phi S^2$$

$$\Phi = \begin{pmatrix} G^+ \\ \frac{v + \phi_h + iG^0}{\sqrt{2}} \end{pmatrix}$$

$$S = \frac{v_s + \phi_s}{\sqrt{2}}$$

$$\begin{array}{ccc} m_h & m_H & \sin \alpha \\ \sin \alpha & v & \tan \beta \equiv \frac{v_s}{v} \end{array}$$

Salient parameter relations:

$$\sin^2 \alpha = \frac{m_h^2 - 2\lambda_1 v^2}{m_h^2 - m_H^2}; \quad \tan^2 \beta = \frac{v_s^2}{v^2} = \frac{m_h^2 + m_H^2 - 2\lambda_1 v^2}{2\lambda_2 v^2} \quad m_H^2 \approx 2\lambda_2 v_s^2$$

Interactions

$$g_{xxy} = g_{xxy}^{\text{SM}} (1 + \Delta_{xy}) \quad \text{with} \quad 1 + \Delta_{xy} = \begin{cases} \cos \alpha & y = h \\ \sin \alpha & y = H \end{cases}$$



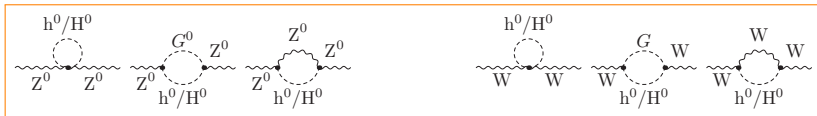
Motivations:

Simplified model $\rightarrow \mathcal{L}_{\text{UV}}$

EW baryogenesis

LHC searches

STU: full model prediction



$$\begin{aligned} \Pi_{WW}(0) = & -\frac{\alpha_{\text{em}} \sin^2 \alpha}{16\pi s_w^2} \left\{ 3m_W^2 \Delta_\epsilon - 4m_W^2 \log \frac{m_H^2}{\mu^2} + \frac{5m_W^2 - m_H^2}{2} \right. \\ & \left. + m_W^2 \log \frac{m_W^2}{\mu^2} - \frac{m_W^2}{m_H^2 - m_W^2} (4m_W^2 - m_H^2) \log \frac{m_H^2}{m_W^2} \right\} \end{aligned}$$

$$S \approx \frac{\sin^2 \alpha}{12\pi} \left(-\log \frac{m_h^2}{m_Z^2} + \log \frac{m_H^2}{m_Z^2} \right) \approx \frac{\lambda_3^2}{24\pi\lambda_2} \frac{v^2}{m_H^2} \log \frac{m_H^2}{m_h^2}$$

$$T = \frac{-3 \sin^2 \alpha}{16\pi s_w^2 m_W^2} \left(m_Z^2 \log \frac{m_H^2}{m_h^2} - m_W^2 \log \frac{m_H^2}{m_h^2} \right) \approx \frac{-3\lambda_3^2 v^2}{32\pi s_w^2 \lambda_2 m_W^2} \left(\frac{m_Z^2}{m_H^2} - \frac{m_W^2}{m_H^2} \right) \log \frac{m_H^2}{m_h^2}$$

Tree-level ffective Lagrangian

$$e^{iS_{\text{eff}}[\phi]} = \int [D\Phi] e^{iS[\phi, \Phi]} = \int [D\eta] e^{i(S[\phi, \Phi_{\text{class}}] + \mathcal{O}(\eta^2))}$$

$$\Phi(x) = \Phi_{\text{class}}(x) + \eta(x)$$

$$\left. \frac{\delta S}{\delta \Phi} \right|_{\Phi = \Phi_{\text{class}}} = 0$$

Singlet model EFT

$$\phi_{s, \text{class}} = -\frac{\lambda_3 v_s (\phi^\dagger \phi)}{-\partial^2 - \mu_s^2 + \mathcal{F}(\phi)} = -\frac{\lambda_3 v_s (\phi^\dagger \phi)}{\mu_s^2} + \mathcal{O}(\phi^3)$$

$$\mathcal{L}_{\text{eff}}^{\text{tree}} \supset \frac{\lambda_3^2}{4\lambda_2^2 v_s^2} \partial_\mu (\phi^\dagger \phi) \partial^\mu (\phi^\dagger \phi)$$

 \Leftrightarrow

$$\mathcal{L}_{\text{eff}} \supset \frac{\bar{c}_H}{2\Lambda^2} \partial_\mu (\phi^\dagger \phi) \partial^\mu (\phi^\dagger \phi)$$

$$\Lambda^2 = 2\lambda_2 v_s^2$$

$$\bar{c}_H = \frac{\lambda_3^2}{2\lambda_2}$$

Effective Lagrangian at one-loop

$$\begin{aligned}
 e^{iS_{\text{eff}}[\phi]} &= \int [D\Phi] e^{iS[\phi, \Phi]} = \int [D\eta] e^{i\left(S[\phi, \Phi_{\text{class}}] + \frac{1}{2} \frac{\delta^2 S}{\delta \Phi^2} \Big|_{\Phi=\Phi_{\text{class}}} \eta^2 + \mathcal{O}(\eta^3)\right)} \\
 &\approx e^{iS[\phi, \Phi_{\text{class}}]} \left[\det \left(-\frac{\delta^2 S}{\delta \Phi^2} \Big|_{\Phi=\Phi_{\text{class}}} \right) \right]^{-\frac{1}{2}} \approx e^{iS[\phi, \Phi_{\text{class}}] - \frac{1}{2} \text{Tr} \log \left(-\frac{\delta^2 S}{\delta \Phi^2} \Big|_{\Phi=\Phi_{\text{class}}} \right)}
 \end{aligned}$$

Functional methods available: [Henning, Lu, Murayama \['14,'16\]](#); [Ellis & al. \['16\]](#); [Zhang \['16\]](#);

$$\mathcal{L}_{\text{eff}}^{1\text{-loop}} \supset -\frac{1}{2(4\pi)^2 M^2} \left\{ \frac{1}{6} U^3 + \frac{1}{12} (P_\mu U)^2 \right\} = -\frac{\lambda_3^2}{24(4\pi)^2 \Lambda^2} \hat{\mathcal{O}}_H + \frac{1}{48(4\pi)^2 \Lambda^2} \left(\frac{\lambda_3^2}{\lambda_{\text{SM}}} \right) \hat{\mathcal{O}}_6$$



c_W, c_B, c_T ??

EFT Renormalization

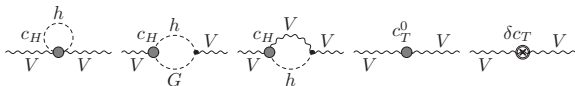
Renormalizing the Effective Lagrangian

$$\mathcal{L}_{\text{eff}}^{\text{bare}}(\{c_i^{\text{bare}}\}) \rightarrow \mathcal{L}_{\text{eff}}(\{c_i\}) + \delta\mathcal{L}_{\text{eff}}(\delta c_i)$$

$$Z_{c_i, c_j} = 1 + \delta Z_{ij}$$

$$\frac{dc_i}{d \log \mu} = \gamma_{ij} c_j$$

$$\delta Z_{c_i, c_j} \equiv \frac{\Gamma(1 + \epsilon)}{(4\pi)^2} \left(\frac{4\pi\mu^2}{\mu_R^2} \right)^\epsilon \frac{\gamma_{ij}}{\epsilon} \quad (+ \text{fin})$$

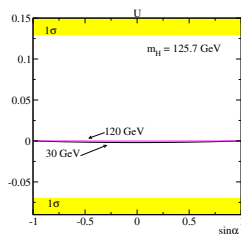
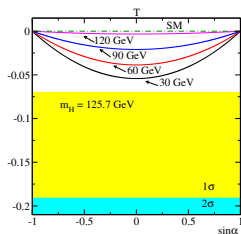
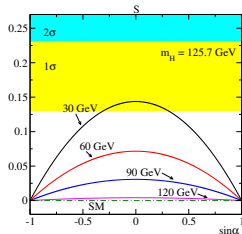
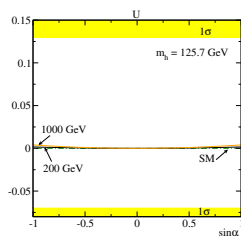
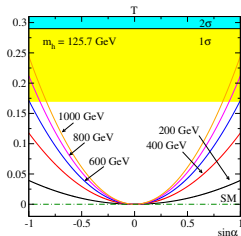
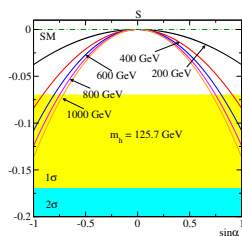


$$\begin{aligned} \Pi_{WW}(0) = \frac{\alpha_{\text{em}} \bar{c}_H}{16\pi s_w^2} & \left\{ 3m_W^2 \Delta_\epsilon - 4m_W^2 \log \frac{m_h^2}{\mu^2} + \frac{5m_W^2 - m_h^2}{2} + \right. \\ & \left. + m_W^2 \log \frac{m_W^2}{\mu^2} - \frac{m_W^2}{m_h^2 - m_W^2} (4m_W^2 - m_h^2) \log \frac{m_h^2}{m_W^2} \right\} + \Xi[\delta Z_{c_W, B, T}] \end{aligned}$$

$$\gamma_{\text{TH}} \equiv \gamma_{\text{H} \rightarrow \text{T}} = \frac{3e^2}{2c_w^2}$$

$$\frac{v^2}{\Lambda^2} c_T(m_Z) = -\frac{3\alpha_{\text{ew}} \tan^2 \theta_W}{(4\pi)^2} \left(\frac{\lambda_3^2 v^2}{2\lambda_2 (2\lambda_2 v_s^2)} \right) \log \left(\frac{2\lambda_2 v_s^2}{m_Z^2} \right)$$

Parameter space constraints



DLV, T. Robens arXiv:1406.1043 [hep-ph]

Take-home ideas

- Ongoing and future experiments may access **Higgs & EW precision physics** to $\mathcal{O}(0.1)\%$
- **Higgs & EW precision physics probe** & **constrain** BSM physics
- **EFTs** provide a **model-independent**, **renormalizable** handle the BSM theory landscape

The EFT approach to Higgs& EW precision:

- **Construct the EFT:** i) Integrate out ; ii) Select Operator Basis ; iii) Truncate ; iv) Match
- **Link scales:** $c_i(\Lambda_{UV}) \rightarrow c_i(\mu = \Lambda_{EW})$ through RG running & mixing
- **Map Coefficients onto Observables:** $(S, T, U) = \{(c_i)\}$
- **Compare to data, fit, constrain!** $c_i = c_i(\{g_{UV}\})$

Virtues & limitations

- Accurately reproduces full model predictions for **weakly-coupled** scenarios & **well-separated** scales
- **Challenged** by i) **strong** BSM-Higgs couplings ii) mass **splittings** iii) v -induced and/or light scales

Selected reading - to learn more

Lecture notes & pedagogical reviews

- C. G. Krause, *Higgs Effective Field Theories - Systematics and Applications*, arXiv:1610.08537 [hep-ph].
- B. Henning, X. Lu, H. Murayama, *How to use the Standard Model effective field theory*, arXiv:1412.1837; *One-loop Matching and Running with Covariant Derivative Expansion*, arXiv:1604.01019.
- W. Skiba, *TASI Lectures on Effective Field Theory and Precision Electroweak Measurements*, arXiv:1006.2142

HEFT: theory & applications

- J. Elias-Miró, J. R. Espinosa, E. Massó and A. Pomarol, *Higgs windows to new physics through $d=6$ operators: constraints and one-loop anomalous dimensions*, arXiv:1308.1879 [hep-ph].
- J. Brehmer, A. Freitas, DLV, T. Plehn, *Pushing Higgs Effective Theory to its limits* arXiv:1510.03443; A. Freitas, DLV, T. Plehn, *When matching matters: Loop effects in Higgs effective theory*, arXiv:1607.08251

Extended Higgs models

- **Higgs Triplet:** M. C. Chen, S. Dawson and C. B. Jackson, arXiv:0809.4185; Z. U. Khandker, D. Li and W. Skiba, arXiv:1201.4383 [hep-ph]
- **Higgs Singlet:** T. Robens, T. Stefaniak, arXiv:1501.02234; DLV, T. Robens, arXiv:1406.1043

A historical perspective

The notion of decoupling and the road towards EFTs



TIMEFRAME [’80s]

CONTEXT GUT model building

Kazama & Yao Phys. Rev. D 25 (1982) 1605

- ♣ *A central idea behind the unification of forces of vastly different strength is that such an apparent hierarchy arises not from the difference of the fundamental coupling constants of the theory but rather from that of the masses of the exchanged particles.*
 - ♣ *In constructing a viable unified theory, these heavy particles must be incorporated into the structure with due caution. Among the most important requirements are:*
 - 1 *that superheavy particles must effectively decouple at low energies;*
 - 2 *that correct effective light-particle theory must emerge at low energies;*
 - 3 *that the mass hierarchy, arranged at the tree level, should be stable against radiative corrections.*
- As we shall see, these requirements are deeply interrelated. None of them is trivial to satisfy.*

Key concepts: decoupling

HIGH SCALE Λ_{UV} **HEAVY FIELDS** Φ **LARGE MASSES** M $\hat{\mathcal{O}}(\Lambda_{UV})$ $\mathcal{L}_{UV}(\Phi, \phi; g, g^*, m, M)$



for $M \rightarrow \infty$

LOW SCALE Λ_{IR} **LIGHT FIELDS** ϕ **LOW MASSES** m $\hat{\mathcal{O}}(\Lambda_{IR})$ $\mathcal{L}_{IR}(\phi; g, m)$



DECOUPLING



$\hat{\mathcal{O}}(\Lambda_{IR})$ UV-insensitive for $M \rightarrow \infty$



heavy field effects undetectable for $M \rightarrow \infty$

A more formal definition

The property by which field theory amplitudes computed from \mathcal{L}_{IR} and \mathcal{L}_{UV} are coincident in the asymptotic heavy mass limit – up to at most a redefinition of the renormalizable parameters in \mathcal{L}_{IR}

Key concepts: nondecoupling

HIGH SCALE Λ_{UV} **HEAVY FIELDS** Φ **LARGE MASSES** M $\mathcal{O}(\Lambda_{UV})$ $\mathcal{L}_{UV}(\Phi, \phi; g, g^*, m, M)$



for $M \rightarrow \infty$

LOW SCALE Λ_{IR} **LIGHT FIELDS** ϕ **LOW MASSES** m $\mathcal{O}(\Lambda_{IR})$ $\mathcal{L}_{IR}(\phi; g, m)$



NONDECOUPLING



♣ $\mathcal{O}(\Lambda_{IR})$ are UV-sensitive for $M \rightarrow \infty$

♣ heavy field effects remain for $M \rightarrow \infty$

A more formal definition

Remainder of the UV-scale dynamics in low-energy observables for asymptotically large heavy field masses

QFT formalism: Appelquist–Carazzone

♠ The notion of heavy field decoupling - a first QFT formalization :

♣ K. Symanzik (1973), *Infrared singularities and small distance behavior analysis*

♣ T. Appelquist, J. Carazzone, (1975), *Infrared Singularities and Massive Fields*

Phys.Rev. **D11** (1975) 2856

We examine some problems associated with the low-momentum behavior of gauge theories and other renormalizable field theories. Our main interest is in the infrared structure of unbroken non-Abelian gauge theories and how this is affected by the presence of other heavy fields coupled to the massless gauge fields. It is shown in the context of a simple model of gauge mesons coupled to massive fermions that the heavy fields decouple at low momenta except for their contribution to renormalization effects.

Generalized conditions:

- 1 **A separably renormalizable light sector**, which remains so at each possible stage of symmetry breaking
- 2 **Asymptotically weak couplings**

Kazama& Yao ['82]; Senjakovic & Sokorac ['80]

QFT formalism: Appelquist–Carazzone

♠, **Setup:** Simplified model of massless gauge bosons coupled to massive fermions:

$$\begin{aligned}
 \mathcal{L}_{\text{full}} &= \underbrace{-\frac{1}{4} F^{\mu\nu} F_{\mu\nu}}_{\text{light sector}} + \underbrace{i\bar{\psi}\not{D}\psi + M\bar{\psi}\psi}_{\text{heavy sector}} + \underbrace{\mathcal{L}_{\phi_\mu-\psi}}_{\text{light-heavy cross-talk}} \\
 \mathcal{L}_{\text{full}} &= \mathcal{L}_{\text{IR}(\phi, \{g\})} + \mathcal{L}_{\text{UV}(\phi, \{g, g^*, M\})} + \mathcal{L}_{\phi_\mu-\psi}
 \end{aligned}$$

$G^{(n)}(p_1, \dots, p_n; \mu)$ generic n-point light field Green's function computed from $\mathcal{L}_{\text{full}}$

♠ **Statement:** In the limit $\{p_i\} \ll \{M\}$,

$$\underbrace{G^{(n)}(p_1, \dots, p_n; \mu)}_{\text{from } \mathcal{L}_{\text{full}}} = Z^{n/2} \underbrace{\tilde{G}^{(n)}(p_1, \dots, p_n; \mu)}_{\text{from } \mathcal{L}_{\text{eff}}} + \mathcal{O}(1/M),$$

$\mathcal{L}_{\text{eff}} \equiv \mathcal{L}_{\text{IR}}(\{\tilde{g}_i\})$, with $\tilde{g}_i \equiv \tilde{g}_i(\{g_i, M\})$ and $Z \equiv Z(\{g_i, M\})$.

Proof keypoints:



SCALING



RENORMALIZABILITY

Nondecoupling in BSM Higgs

♠ An example with a colored scalar partner $\mathcal{L} \supset -\mu_S^2 |S|^2 - \lambda_s \Phi^\dagger \Phi |S|^2$,

$$m_S^2(v) = \frac{\lambda_s v^2}{2} + \mu_S^2$$

$$\mathcal{L}_{hSS} \supset \lambda_s v |S|^2 H$$

$$\lambda_s = \frac{2(m_S^2 - \mu_S^2)}{v^2}$$

$$\mathcal{L} \supset -\frac{1}{4} g_{hGG} G^{\mu\nu A} G_{\mu\nu}^A H$$

with

$$g_{hGG} = -\frac{g_s^2}{4\pi^2} \left[\sum_S \dim(r_L) C_{2,S}(r_C) \frac{\lambda_s v}{2m_S^2} A_s(\tau_S) \right]$$

→ 0 if $m_s \rightarrow \infty$ as $\mu_S \rightarrow \infty$ while $\lambda \sim \mathcal{O}(1)$

$$g_{hGG} \sim \frac{\lambda v^2}{m_S^2} \left\{ \begin{array}{l} \rightarrow 0 \\ \rightarrow \text{constant} \end{array} \right.$$

→ constant if $m_s \rightarrow \infty$ while $\mu_S \ll m_s$

♣ Non-decoupling contributions are connected to **LETs**

$$\mathcal{L} \supset \frac{\alpha_s}{12\pi v} \left[s C_2(r_C) \frac{\partial \log(m(v))}{\partial \log(v)} \right] H G^{\mu\nu A} G_{\mu\nu}^A$$

⇒ Φ as a background field to which the (dynamical) heavy fields couple as $M_\Phi = M_\Phi(v)$

The EFT reflect

♣ Back to History: Kazama & Yao Phys. Rev. D 25 (1982) 1605

♣ *It is clear in the case of theories without spontaneously broken symmetry one cannot talk about decoupling without the existence of an effective light-particle theory, because in its absence we cannot absorb the large mass effects by redefinition of the parameters of the light theory. These two concepts are, therefore, two sides of one and the same subject.*

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \sum_{i_D} \sum_{D>4} \frac{C_{i_D}^{(D)}}{\Lambda^{D-2}} \mathcal{O}_{i_D}^{(D)}$$

♣ **Linearly-realized EWSB:**

Passarino ['12]

- **Decoupling:** $C_{i_D}^{(D)} \sim \mathcal{O}(1)$ \Rightarrow power-counting obeys canonical dim.
- **Screening:** $C_{i_D}^{(D)} \sim \log(\Lambda)$
- **Non-decoupling:** $C_{i_D}^{(D)} \sim \mathcal{O}(\Lambda)$ spoils canonical power-counting

♣ **Strongly-coupled EWSB** \Leftrightarrow **Non-linear EFT** CCWZ formalism - EW_χ Lagrangian

\Rightarrow double expansion: chiral loops & mass dimensions

The HEFT reflect

♠ The plain (*linear*, $d6$) HEFT is jeopardized by ♠ v -induced scales & mass splittings

♠ **Non-unique choices** - *Let's try to be wise!*

♣ $\Lambda_{\text{matching}}$

♣ unbroken VS broken phase

♣ $d6$ or beyond?

♠ **Default matching**

- Matching scale: $\Lambda = M_{\text{heavy}}$
- Unbroken phase matching: ($M \gg \mu_{\text{EWSB}}$)
- $\mathcal{O}(\Lambda^{-2})$ truncation

⇒ **Hazards!**

- $M_{\text{phys}} \sim M_{\text{heavy}} \pm gv$ ⇒ spoilt scale separation & sizable mass splittings
- Large v -induced, eventually $d > 6$ contributions $\mathcal{O}^d \propto \mathcal{O}^{d=6} (\Phi^\dagger \Phi)^{d-6}$

♠ v -improved matching:

absorb $\mathcal{O} \left(\frac{v}{\Lambda} \right)^{d-6}$ terms into $\mathcal{L}_{\text{eff}}^{d=6}$.

- Matching scale $\Lambda = M_{\text{phys}}$
- Wilson coefficients written in terms of **mass-eigenstate & mixing**

$$\mathcal{O}_H : \frac{\lambda_3^2 v^2}{2\lambda_2 \Lambda^2} [\partial_\mu (\Phi^\dagger \Phi) \partial^2 \mu (\Phi^\dagger \Phi)] \quad \text{VS} \quad \frac{2(1 - \cos \alpha) v^2}{m_H^2} [\partial_\mu (\Phi^\dagger \Phi) \partial^\mu (\Phi^\dagger \Phi)]$$