Semi-Leptonic Theory

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Levels of complexity in B decays

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- Two-Body Non-leptonic: QCD Factorization (QCD-F)
- Multi-Body Non-Leptonic: To be developed

Make Use of the fact that $\alpha_s(m_b) \ll 1$

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- Exclusive semileptonic: $F^{B\to M}(q^2)$
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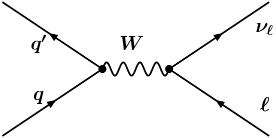
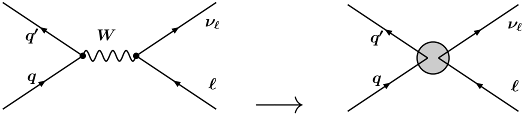
Semi-Leptonic Quark Transitions

- In the Standard Model:
  $q \to q'\ell\bar{\nu}$ or $b \to u\ell\bar{\nu}$ with $q = b$ and $q = c, u$



- The W is much heavier than the b quark:

$$\langle 0 | T[W_\mu(x) W_\nu^*(0)] | 0 \rangle \sim \frac{1}{M_W^2} \delta^4(x)$$



Effective Hamiltonian

It is useful to define the up and down quark fields

$$\mathcal{U}_{L/R} = \begin{bmatrix} u_{L/R} \\ c_{L/R} \\ t_{L/R} \end{bmatrix} \quad \mathcal{D}_{L/R} = \begin{bmatrix} d_{L/R} \\ s_{L/R} \\ b_{L/R} \end{bmatrix}$$

For the semi-leptonic effective Hamiltonian we get

$$H_{\rm eff}^{(sl)} = \frac{4G_F}{\sqrt{2}} \left(\bar{\mathcal{U}}_L \gamma_\mu V_{CKM} \mathcal{D}_L\right) \times \left(\bar{e}_L \gamma_\mu \bar{\nu}_{e,\rm L} + \bar{\mu}_L \gamma_\mu \bar{\nu}_{\mu,\rm L} + \bar{\tau}_L \gamma_\mu \bar{\nu}_{\tau,\rm L}\right) + \text{h.c.} \; ,$$

with

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \quad G_F = \frac{g^2}{2\sqrt{2} M_W^2}$$

For the cases at hand: $(\ell = e, \mu, \tau)$

$$H_{\rm eff} = \frac{4G_F}{\sqrt{2}} V_{cb} (\bar{c}_L \gamma_\mu b_L)(\bar{\ell}_L \gamma^\mu \nu_{\ell,L})$$
$$H_{\rm eff} = \frac{4G_F}{\sqrt{2}} V_{ub} (\bar{u}_L \gamma_\mu b_L)(\bar{\ell}_L \gamma^\mu \nu_{\ell,L})$$

- This is correct up to term of order m_b^2/M_W^2
- Largest QED corrections are of order (Sirlin)

$$\frac{\alpha}{\pi} \log(M_W^2/m_b^2)$$

- No QCD corrections of order

$$\frac{\alpha_s}{\pi} \log(M_W^2/m_b^2)$$

- ... unlike in non-leptonic decays.

Exclusive Decays

Matrix Elements and Form Factors

- Leptonic part is as in the textbook
- Hadronic Matrix Elements: In general parametrized by scalar functions ("Form Factors") of $q^2 = (p_B - p')^2$
- For a pseudoscalar final state $P(p_P)$ $(p' = b_P)$

$$\langle P(p_P)|\bar{q}\gamma^\mu b|B(p_B)\rangle = f_+(q^2)\left(p_B^\mu + p_P^\mu - \frac{m_B^2 - m_P^2}{q^2} q^\mu\right) + f_0(q^2)\frac{m_B^2 - m_P^2}{q^2} q^\mu$$

$$\langle P(p_P)|\bar{q}\gamma^\mu\gamma_5 b|B(p_B)\rangle = 0$$

- Vector final state $V(p_V, \epsilon)$ with polarization ϵ $(p' = p_V)$

$$\langle V(p_V, \epsilon)|\bar{q}\gamma^\mu b|B(p_B)\rangle = V(q^2)\, \varepsilon^{\mu\sigma}{}_{\nu\rho}\epsilon^*_\sigma \frac{2p_B^\nu p_V^\rho}{m_B + m_V},$$

$$\langle V(p_V, \epsilon)|\bar{q}\gamma^\mu \gamma^5 b|B(p_B)\rangle = i\epsilon^*_\nu \left[A_0(q^2)\, \frac{2m_V q^\mu q^\nu}{q^2} \right.$$

$$\left. + A_1(q^2)\,(m_B + m_V)\eta^{\mu\nu} - A_2(q^2)\, \frac{(p_B + p_V)_\sigma q^\nu}{m_B + m_V} \eta^{\mu\sigma} \right],$$

with $\epsilon p_V = 0$, and $\eta^{\mu\nu} = g^{\mu\nu} - \dfrac{q^\mu q^\nu}{q^2}$

- For massless leptons, f_0 and A_0 don't contribute

$B \to \pi\ell\bar{\nu}$

Straightforward calculation ($|\vec{p}_\pi| = \frac{1}{2}\sqrt{M_B^2 - 4m\pi^2}$)

$$\frac{d\Gamma(\bar{B}^0 \to \pi^+\ell^-\nu)}{dq^2} = \frac{G_F^2|V_{ub}|^2}{24\pi^3}|\vec{p}_\pi|^3|f_{B\pi}^+(q^2)|^2$$

What do we know about the form factor?

- Lattice QCD: $q^2 \sim (M_B - m_\pi)^2$
- QCD Sum rules estimates: $q^2 \sim 0$
- Interpolation between these regions.
- Form Factor Bounds from Analyticity and Unitarity

Form Factor Parametrizations

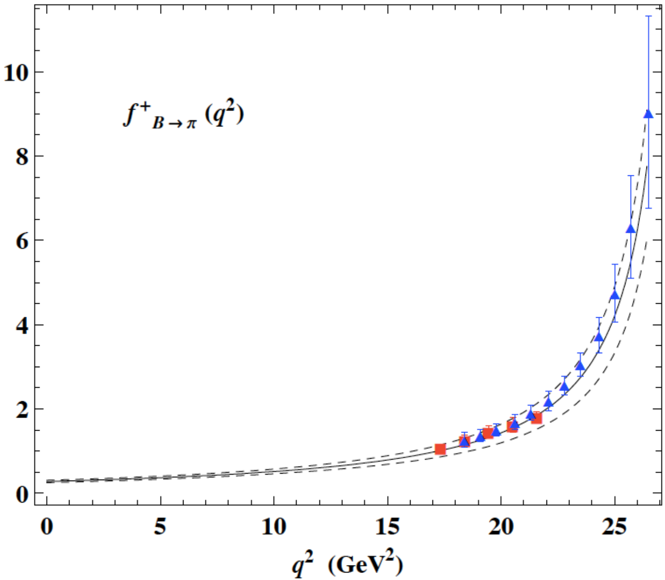
Aside form models, one uses the z parametrization

(Bourrely, Caprini Lellouch)

$$z(q^2, t_0) = \frac{\sqrt{(M_B + m_\pi)^2 - q^2} - \sqrt{(M_B + m_\pi)^2 - t_0}}{\sqrt{(M_B + m_\pi)^2 - q^2} + \sqrt{(M_B + m_\pi)^2 - t_0}}$$

$$f^+_{B\pi}(q^2) = \frac{1}{1 - q^2/M^2_{B^*}} \sum_{k=0}^{K} b_k(t_0)(z(q^2, t_0))^k$$

very few terms (~ 2) in this expansion are sufficient for the interpolation


$f^+_{B\to\pi}(q^2)$
10
8
6
4
2
0
0
5
10
15
20
25
q^2 (GeV^2)

$B \to \rho\ell\bar{\nu}$

Only a few remarks:

- Can be computed in terms of the form factors V, A_1, A_2
- Not much is known about these form factors: Models
- Lattice as well as QCD SR fail, since the ρ is not a stable particle
- Study instead $B \to \pi\pi\ell\bar{\nu}$ (Faller et al.)

Decomposition into Form Factors:

$$\langle \pi^+(k_1)\pi^-(k_2)|\bar{u}\gamma^\mu b|B^-(p)\rangle = iF_\perp \frac{1}{\sqrt{k^2}}\, \bar{q}^\mu_{(\perp)}$$

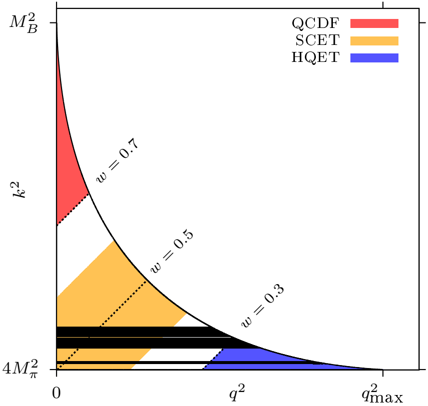
$$\langle \pi^+(k_1)\pi^-(k_2)|\bar{u}\gamma^\mu\gamma_5 b|B^-(p)\rangle = -F_t \frac{q^\mu}{\sqrt{q^2}}$$

$$+F_0 \frac{2\sqrt{q^2}}{\sqrt{\lambda}}\, k^\mu_{(0)} + F_{||} \frac{1}{\sqrt{k^2}}\, \bar{k}^\mu_{(||)} \,.$$

Form factors depend on the variables

$$k = k_1 + k_2, \quad \bar{k} = k_1 - k_2 : \quad q^2,\ k^2,\ q\cdot\bar{k}$$

$$w = (v \cdot k)/M_B$$



... work in progress

$B \to D\ell\bar{\nu}$ and $B \to D^*\ell\bar{\nu}$

- Useful Tool: Heavy Quark Limit = $1/m_Q$ Expansion
- Static Limit: $m_b,\ m_c \to \infty$ with fixed (four)velocity

$$v_Q = \frac{p_Q}{m_Q}, \qquad Q = b, c$$

- In this limit we have

$$\left.\begin{array}{l} m_{Hadron} = m_Q \\ p_{Hadron} = p_Q \end{array}\right\} v_{Hadron} = v_Q$$

- For $m_Q \to \infty$ the heavy quark does not feel any recoil from the light quarks and gluons (Cannon Ball)
- This is like the H-atom in Quantum Mechanics I!

Heavy Quark Symmetries

- The interaction of gluons is identical for all quarks
- Flavour enters QCD only through the mass terms
  - $m \to 0$: (Chiral) Flavour Symmetry (Isospin)
  - $m \to \infty$ Heavy Flavour Symmetry
  - Consider b and c heavy: Heavy Flavour SU(2)
- Coupling of the heavy quark spin to gluons:

$$H_{int} = \frac{g}{2m_Q} \bar{Q}(\vec{\sigma} \cdot \vec{B})Q \quad \xrightarrow{m_Q \to \infty} \quad 0$$

  - Spin Rotations become a symmetry
  - Heavy Quark Spin Symmetry: SU(2) Rotations
- Spin Flavour Symmetry Multiplets

- HQS imply a “Wigner Eckart Theorem”

$$\left\langle H^{(*)}(v)\right| Q_v \Gamma Q_{v'} \left|H^{(*)}(v')\right\rangle = C_\Gamma(v, v')\xi(v \cdot v')$$

with $H^{(*)}(v) = D^{(*)}(v)$ or $B^{(*)}(v)$

  - $C_\Gamma(v, v')$: Computable Clebsh Gordan Coefficient
  - $\xi(v \cdot v')$: Reduced Matrix Element
  - $\xi(v \cdot v')$: universal non-perturbative Form Faktor: Isgur Wise Funktion
  - Normalization of ξ at $v = v'$:

$$\xi(v \cdot v' = 1) = 1$$

Form Factors

Express the form factors in terms of velocities:

$$\frac{\langle D(v')|\bar{c}\gamma^\mu b|B(v)\rangle}{\sqrt{m_B m_D}} = h_+(w)\,(v+v')^\mu + h_-(w)\,(v-v')^\mu,$$

$$\frac{\langle D^*(v',\epsilon)|\bar{c}\gamma^\mu b|B(v)\rangle}{\sqrt{m_B m_{D^*}}} = h_V(w)\,\varepsilon^{\mu\nu\rho\sigma} v_\nu v'_\rho \epsilon^*_\sigma,$$

$$\frac{\langle D^*(v',\epsilon)|\bar{c}\gamma^\mu \gamma^5 b|B(v)\rangle}{\sqrt{m_B m_{D^*}}} = \\ i h_{A_1}(w)\,(1+w)\epsilon^{*\mu} - i\left[h_{A_2}(w) v^\mu + h_{A_3}(w) v'^\mu\right](\epsilon^* \cdot v)$$

with $w = (v \cdot v')$

In the heavy quark limit:
$m_b \to \infty$ and $m_c \to \infty$ with m_c/m_b fixed

$$h_+(w) = h_V(w) = h_{A_1}(w) = h_{A_3}(w) = \xi(w)$$
$$h_-(w) = h_{A_2}(w) = 0$$

Only a single, normalized form factor!

Corrections to the HQL be discussed below.

$$\frac{d\Gamma^{B\to D\ell\bar{\nu}}}{dw} = \frac{G_F^2}{48\pi^3}|V_{cb}|^2(m_B+m_D)^2 m_D^3 (w^2-1)^{3/2}|\eta_{\rm ew}\mathcal{G}(w)|^2 ,$$

with

$$\mathcal{G}(w) = h_+(w) - \frac{m_B - m_D}{m_B + m_D} h_-(w)$$

- $\eta_{\rm ew} = 1.007$: electroweak corrections
- in the Heavy Quark Limit: $\mathcal{G}(w) = \xi(w)$
- Corrections at the normalization point:
  (without QCD perturbative corrections)
  - $h_+(1) = 1 + \mathcal{O}(1/m_c^2)$ (Luke's Theorem)
  - $h_-(1) = 0 + \mathcal{O}(1/m_c)$

Remark on Short Distance QCD Corrections

Perturbative QCD for on-shell quarks with equal velocities:

$$\langle c(v)|\bar{c}\gamma^\mu b|b(v)\rangle = 1 + \frac{2\alpha_s}{3\pi}\left[\frac{3m_b^2 + 2m_c m_b + 3m_c^2}{2(m_b^2 - m_c^2)} \ln\left(\frac{m_b}{m_c}\right) - 2\right]$$

$$\langle c(v)|\bar{c}\gamma^\mu \gamma_5 b|b(v)\rangle = 1 - \frac{\alpha_s}{\pi}\left[\frac{m_b + m_c}{m_b - m_c} \ln\left(\frac{m_c}{m_b}\right) + \frac{8}{3}\right]$$

Numerically (including also the known α_s^2 corrections):

$$\langle c(v)|\bar{c}\gamma^\mu b|b(v)\rangle = \eta_V = 1.022 \pm 0.004$$

$$\langle c(v)|\bar{c}\gamma^\mu \gamma_5 b|b(v)\rangle = \eta_A = 0.960 \pm 0.007$$

Estimates for $\mathcal{G}(w)$

- Estimate for the Normalization from the Lattice:
  $\mathcal{G}(1) = 1.074 \pm 0.024$ (Okamoto et al.)
- Estimate for the Normalization from continuum:
  $\mathcal{G}(1) = 1.04 \pm 0.02$ (Uraltsev)
- Extrapolation to $w \neq 1$: z-expansion
  (ρ_D: Slope Parameter)

$$z = \frac{\sqrt{w+1} - \sqrt{2}}{\sqrt{w+1} + \sqrt{2}}$$

$$\mathcal{G}(w) = \mathcal{G}(1) \quad \left[1 - 8\rho_D^2 z + (51.\rho_D^2 - 10.)z^2 - (252.\rho_D^2 - 84.)z^3\right]$$

(Caprini, Lellouch, Neubert)

$$\frac{d\Gamma^{B\to D^*\ell\bar{\nu}}}{dw} = \frac{G_F^2 m_B^5}{48\pi^3}|V_{cb}|^2(w^2-1)^{1/2}P(w)|\eta_{\rm ew}\mathcal{F}(w)|^2$$

with the phase space function

$$P(w) = r^3(1-r)^2(w+1)^2\left(1+\frac{4w}{w+1}\frac{1-2rw+r^2}{(1-r)^2}\right).$$

with $r = m_{D^*}/m_B$ and

$$P(w)|\mathcal{F}(w)|^2 = |h_{A_1}(w)|^2\left\{2\frac{r^2-2rw+1}{(1-r)^2}\left[1+\frac{w-1}{w+1}R_1^2(w)\right] + \left[1+\frac{w-1}{1-r}(1-R_2(w))\right]^2\right\}$$

and

$$R_1(w) = \frac{h_V(w)}{h_{A_1}(w)}, \quad R_2(w) = \frac{h_{A_3}(w)+r\,h_{A_2}(w)}{h_{A_1}(w)}$$

- In the heavy mass limit: $R_1(w) = R_2(w) = 1$
- In the heavy mass limit: $\mathcal{F}(1) = 1$
- Corrections to the normalization are known:
  - Lattice QCD: $\mathcal{F}(1) = 0.902 \pm 0.017$ (Bailey et al.)
  - QCD Sum rules: $\mathcal{F}(1) = 0.86 \pm 0.03$ (Gambino et al.)
- Extrapolation to $w \neq 1$: z-Expansion

$$\mathcal{F}(w) = \mathcal{F}(1) \times \left[1 - 8\rho^2_{\mathrm{A1}} z + (53.\rho^2_{\mathrm{A1}} - 15.)z^2 - (231.\rho^2_{\mathrm{A1}} - 91.)z^3\right]$$

ρ_{A1} is the slope parameter of the form factor h_{A_1}

- Exptrapolation of R_1 and R_2 with QCD sum rules

Implications for V_{cb}

- V_{cb} determination proceeds via extrapolation to $w = 1$
- Theoretical uncertainties are at the level of 3%
- ... with a perspective of improvement from Lattice
- However ...

Numbers from Jochen ...

$B \to D^{**}\ell\bar{\nu}$ = Orbitally excited states

- $B \to D$ and $B \to D^*$ exhaust about 75% of the inclusive $b \to c$ rate
- Aside from non-resonant $B \to D\pi$:
  Decays into D^{**} states
- ... mainly the orbitally excited states

Make use of Heavy Quark Symmetry:

- Spin Symmetry Doublets of orbitally excited states, labelled by the total j of the light degrees of freedom:

$$\begin{pmatrix} |D(0^+)\rangle \\ |D(1^+)\rangle \end{pmatrix} \quad j = 1/2 \qquad \text{and} \qquad \begin{pmatrix} |D^*(1^+)\rangle \\ |D^*(2^+)\rangle \end{pmatrix} \quad j = 3/2$$

- Masses in the $m_c \to \infty$ limit:

$$M(D(0^+)) = M(D(1^+)) = m_c + \bar{\Lambda}_{1/2}$$
$$M(D^*(1^+)) = M(D^*(2^+)) = m_c + \bar{\Lambda}_{3/2}$$

- $\bar{\Lambda}_{3/2} - \bar{\Lambda}_{1/2}$ does not scale with m_c!
- Each Doublet as a new Isgur Wise Function: $\tau_{1/2}(w)$ and $\tau_{3/2}(w)$

- Corrections to the HQL are sizable
- Theory is underway ...
- Experiment: Jochen ...

$B \to D^{**} \ell \bar{\nu}$

Channel	GI	VD	CCCN	ISGW
$m_c \to \infty$				
$\mathcal{B}(B^- \to D(0^+)\ell\bar{\nu})$	$4.7 \cdot 10^{-4}$	$1.8 \cdot 10^{-4}$	$3.7 \cdot 10^{-5}$	$1.0 \cdot 10^{-3}$
$\mathcal{B}(B^- \to D(1^+)\ell\bar{\nu})$	$6.4 \cdot 10^{-4}$	$2.5 \cdot 10^{-4}$	$4.9 \cdot 10^{-5}$	$1.4 \cdot 10^{-3}$
$\mathcal{B}(B^- \to D^*(1^+)\ell\bar{\nu})$	$4.4 \cdot 10^{-3}$	$2.9 \cdot 10^{-3}$	$4.0 \cdot 10^{-3}$	$4.7 \cdot 10^{-3}$
$\mathcal{B}(B^- \to D^*(2^+)\ell\bar{\nu})$	$7.4 \cdot 10^{-3}$	$4.9 \cdot 10^{-3}$	$6.7 \cdot 10^{-3}$	$8.0 \cdot 10^{-3}$
$\mathcal{B}(B^- \to D^{**}\ell\bar{\nu})$	1.3%	0.82%	1.1%	1.5%
$\frac{\mathcal{B}(B^- \to D^*(1^+)\ell\bar{\nu})}{\mathcal{B}(B^- \to D(1^+)\ell\bar{\nu})}$	6.9	11	80	3.4
m_c finite				
$\mathcal{B}(B^- \to D_L\ell\bar{\nu})$	$3.0 \cdot 10^{-3}$	$2.1 \cdot 10^{-3}$	$3.0 \cdot 10^{-3}$	$3.2 \cdot 10^{-3}$
$\mathcal{B}(B^- \to D_H\ell\bar{\nu})$	$2.3 \cdot 10^{-3}$	$1.3 \cdot 10^{-3}$	$1.3 \cdot 10^{-3}$	$3.1 \cdot 10^{-3}$
$\frac{\mathcal{B}(B^- \to D_L\ell\bar{\nu})}{\mathcal{B}(B^- \to D_H\ell\bar{\nu})}$	1.3	1.6	2.3	1.0

(R. Klein et al.)

- GI: Godfrey, Isgur (1985);
- VD: Veseli, Dunietz (1996);
- CCCN: Cea, Colangelo, Cosmai, Nardulli (1991);
- ISGW: Isgur, Scora, Grinstein, Wise (1989)

$$\frac{\mathcal{B}(B^- \to D^*(1^+)\ell\nu)}{\mathcal{B}(B^- \to D(1^+)\ell\nu)} \approx 2.2$$

Inclusive Decays

Heavy Quark Expansion

Heavy Quark Expansion = Operator Product Expansion
(Chay, Georgi, Bigi, Shifman, Uraltsev, Vainstain, Manohar. Wise, Neubert, M,...)

$$\Gamma \propto \sum_X (2\pi)^4 \delta^4(P_B - P_X) |\langle X | \mathcal{H}_{eff} | B(v) \rangle|^2$$

$$= \int d^4x \, \langle B(v) | \mathcal{H}_{eff}(x) \mathcal{H}^\dagger_{eff}(0) | B(v) \rangle$$

$$= 2 \, \mathrm{Im} \int d^4x \, \langle B(v) | T\{\mathcal{H}_{eff}(x) \mathcal{H}^\dagger_{eff}(0)\} | B(v) \rangle$$

$$= 2 \, \mathrm{Im} \int d^4x \, e^{-im_b v \cdot x} \langle B(v) | T\{\widetilde{\mathcal{H}}_{eff}(x) \widetilde{\mathcal{H}}^\dagger_{eff}(0)\} | B(v) \rangle$$

- Last step: $b(x) = b_v(x) \exp(-im_v vx)$, corresponding to $p_b = m_b v + k$
  Expansion in the residual momentum k

- Perform an "OPE": m_b is much larger than any scale appearing in the matrix element

$$\int d^4x e^{-im_b vx} T\{\widetilde{\mathcal{H}}_{eff}(x)\widetilde{\mathcal{H}}^\dagger_{eff}(0)\} = \sum_{n=0}^{\infty} \left(\frac{1}{2m_Q}\right)^n C_{n+3}(\mu)\mathcal{O}_{n+3}$$

$\to$ The rate for $B \to X_c \ell \bar{\nu}_\ell$ can be written as

$$\Gamma = \Gamma_0 + \frac{1}{m_Q}\Gamma_1 + \frac{1}{m_Q^2}\Gamma_2 + \frac{1}{m_Q^3}\Gamma_3 + \cdots$$

- The Γ_i are power series in $\alpha_s(m_Q)$:
  $\to$ Perturbation theory!
- Works also for differential rates!

- Γ_0 is the decay of a free quark ("Parton Model")
- Γ_1 vanishes due to Heavy Quark Symmetries
- Γ_2 is expressed in terms of two parameters

$$2M_H\mu_\pi^2 = -\langle H(v)|\bar{Q}_v(iD)^2 Q_v|H(v)\rangle$$
$$2M_H\mu_G^2 = \langle H(v)|\bar{Q}_v\sigma_{\mu\nu}(iD^\mu)(iD^\nu)Q_v|H(v)\rangle$$

μ_π: Kinetic energy and μ_G: Chromomagnetic moment

- Γ_3 two more parameters

$$2M_H\rho_D^3 = -\langle H(v)|\bar{Q}_v(iD_\mu)(ivD)(iD^\mu)Q_v|H(v)\rangle$$
$$2M_H\rho_{LS}^3 = \langle H(v)|\bar{Q}_v\sigma_{\mu\nu}(iD^\mu)(ivD)(iD^\nu)Q_v|H(v)\rangle$$

ρ_D: Darwin Term and ρ_{LS}: Spin-Orbit Term

- Γ_4 and Γ_5 have been computed Bigi, Uraltsev, Turczyk, TM, ...

Structure of the HQE

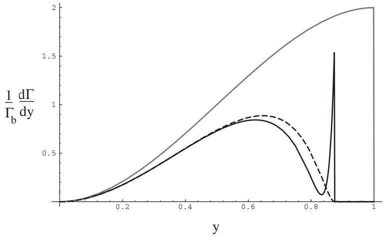
- Structure of the expansion (@ tree):

$$d\Gamma = d\Gamma_0 + \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^2 d\Gamma_2 + \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^3 d\Gamma_3 + \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^4 d\Gamma_4$$
$$+ d\Gamma_5 \left(a_0 \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^5 + a_2 \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^3 \left(\frac{\Lambda_{\rm QCD}}{m_c}\right)^2 \right)$$
$$+ \ldots + d\Gamma_7 \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^3 \left(\frac{\Lambda_{\rm QCD}}{m_c}\right)^4$$

- $d\Gamma_3 \propto \ln(m_c^2/m_b^2)$
- Power counting $m_c^2 \sim \Lambda_{\rm QCD} m_b$

Determination of the HQE Parameters

- m_b, m_c, μ_π, μ_G, ρ_D etc. are determined from data
- Spectra: Hadronic invariant mass, Charegd lepton energy, Hadronic Energy
- However: There are corners in Phase Space where the OPE breaks down



Moments of the spectra can be computed in the HQE

m_b^{kin}	$\overline{m}_c(3\,\text{GeV})$	μ_π^2	ρ_D^3	μ_G^2	ρ_{LS}^3	$\text{BR}_{c\ell\nu}$	$10^3\lvert V_{cb}\rvert$
4.553	0.987	0.465	0.170	0.332	-0.150	10.65	42.21
0.020	0.013	0.068	0.038	0.062	0.096	0.16	0.78

WITHOUT MASS CONSTRAINTS

$$m_b^{kin}(1\text{GeV}) - 0.85\,\overline{m}_c(3\text{GeV}) = 3.714 \pm 0.018\,\text{GeV}$$

Alberti, Healey, Nandi, Gambino arXiv 1411.6560

- Includes HQE parameters up to $1/m^3$ and full α_s/m_Q^2

QCD Corrections

For a massless final-state quark:

$$\Gamma_0 = \frac{G_F^2 |V_{cb}|^2}{192\pi^3} m_b^5 \left(1 + \frac{\alpha_s}{\pi} g_1 + \cdots\right)$$

What is the mass m_b?

- Start with the pole mass $m_b = m_b^{\mathrm{pole}}$
- This yields a large g_1
- Problem for a precision calculation!
- Switch to a "proper mass" m_b^{kin}:
  This has a perturbative relation to the pole mass

$$m_b^{\mathrm{kin}}(\mu) = m_b^{\mathrm{pole}} \left(1 + \frac{\alpha_s}{\pi} m_1(\mu) + \cdots\right)$$

- Insert this

$$\Gamma_0 = \frac{G_F^2 |V_{cb}|^2}{192\pi^3} (m_b^{\rm kin}(\mu))^5 \left(1 + \frac{\alpha_s}{\pi}(g_1 - m_1(\mu)) + \cdots \right)$$

- $m_b^{\rm kin}$ is much better known as the pole mass
- The perturbative series converges better: $|g_1 - m_1| \ll g_1$

$b \to c$ also depends on the charm mass

- The rate and the moments depend on

$$m_b^{\rm kin}(1\,{\rm GeV}) - a\, m_c \quad \text{with} \quad a \sim 0,7 - 0.8$$

Present state of the $b \to c$ semileptonic Calculations

- Tree level terms up to and including $1/m_b^5$ known
  Bigi, Zwicky, Uraltsev, Turczyk, TM, ...
- $\mathcal{O}(\alpha_s)$ and full $\mathcal{O}(\alpha_s^2)$ for the partonic rate known
  Melnikov, Czarnecki, Pak
- $\mathcal{O}(\alpha_s)$ for the full $1/m_b^2$ is known
  Becher, Boos, Lunghi, Gambino, Pivovarov, Rosenthal, Alberti
- In the pipeline:
  - α_s/m_b^3
  - Partial Resummations

We are getting at a TH-uncertainty of 1% in $V_{cb,incl}$!

Modified Heavy Quark Expansion: $B \to X_u \ell \bar{\nu}$

- Problem: Cuts needed to suppress charmed decays
- Forces us into corners of phase space, where the usual OPE breaks down
- Expansion parameter $\Lambda_{\text{QCD}}/(m_b - 2E_\ell)$
- Instead of HQE Parameters: Shape Functions $f(\omega)$

$$2M_B f(\omega) = \langle B(v)|\bar{b}_v \delta(\omega + i(n \cdot D))|B(v)\rangle$$

- Universal for all heavy-to-light decays
- Systematics: S$_{\text{oft}}$C$_{\text{ollinear}}$E$_{\text{ffective}}$T$_{\text{heory}}$ calculation
  - Several subleading shape functions
  - perturbative QCD corrections

Shape Functions

- Shape function vs. local OPE: Moment Expansion

$$f(\omega) = \delta(\omega) + \frac{\mu_\pi^2}{6m_b^2}\delta''(\omega) - \frac{\rho_D^3}{18m_b^3}\delta'''(\omega) + \cdots$$

- Perturbative “jetlike” contributions: Convolution

$$S(\omega, \mu) = \int dk \; C_0(\omega - k, \mu) f(k)$$

- Charged Lepton Energy Spectrum (H: hard QCD corrections)

$$\frac{d\Gamma}{dy} = \frac{G_F^2 |V_{ub}^2| m_b^5}{96\pi^3} \int d\omega \, \Theta(m_b(1-y) - \omega) H(\mu) S(\omega, \mu)$$

Approaches

- Obtaining the Shape functions:
  - From Comparison with $B \to X_s \gamma$
  - From the knowledge of (a few) moments
  - From modeling
- QCD based:
  - BLNP (Bosch, Lange, Neubert, Paz)
  - GGOU (Gambino, Giordano, Ossola, Uraltsev)
  - SIMBA (Tackmann, Tackmann, Lacker, Liegti, Stewart ...)
- QCD inspired:
  - Dressed Gluon Exponentiation (Andersen, Gardi)
  - Analytic Coupling (Aglietti et al.)
- Attempts to avoid the shape functions (Bauer Ligeti, Luke ...)

Theo. uncertainty in $V_{ub,incl}$ is still (7 ... 10) %

Semi-Tauonic Decays

Inclusive $B \to X_c \tau \bar{\nu}$

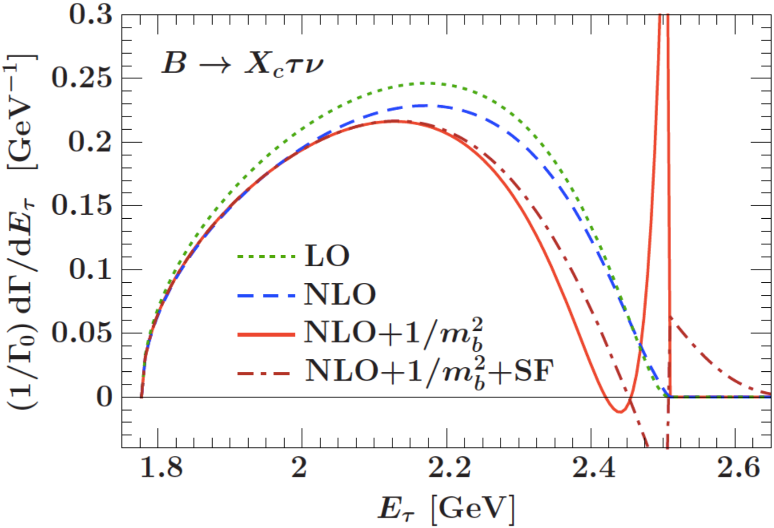
- One can use the HQE to compute $B \to X_c \tau \bar{\nu}$
(LIgeti, Tackmann)

$$\mathrm{Br}(B \to X_c \ell \bar{\nu}) = (2.42 \pm 0.06)\% \quad \text{(includes } \alpha_s \text{ and } 1/m_b^2\text{)}$$

- More precise calculations are under way (Shahriaran et al.)
- Measurement: (b-Admixture from LHC, LEP, Tevatron, SppS)

$$\mathrm{Br}(b \to X_\ell \bar{\nu}) = (2.41 \pm 0.23)\%$$

- seems fairly well under control


$B \to X_c \tau \nu$
LO
NLO
NLO+$1/m_b^2$
NLO+$1/m_b^2$+SF
0.3
0.25
0.2
0.15
0.1
0.05
0
1.8
2
2.2
2.4
2.6
E_τ [GeV]
$(1/\Gamma_0)\, d\Gamma/dE_\tau$ [GeV^{-1}]

Inclusive $B \to X_c \tau \bar{\nu}$

- For $B \to D^{(*)} \tau \bar{\nu}$ we have the general form:

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 \, |V_{cb}|^2 \, |\vec{p}_{D^{(*)}}| q^2}{96\pi^3 m_B^2} \left(1 - \frac{m_\tau^2}{q^2}\right)^2 \times \left[(|H_+|^2 + |H_-|^2 + |H_0|^2) \left(1 + \frac{m_\tau^2}{2q^2}\right) + \frac{3 m_\tau^2}{2q^2} |H_s|^2 \right]$$

- In particular: The rate depends on the form factors proportional to q_μ
- In the HQL: Known in terms of $\xi(w)$
- Additional factor m_τ^2 / m_B^2 in front of these form factots

- Based on this, we get the SM predictions

$$\mathrm{Br}(B \to D\tau\bar{\nu}) = (0.66 \pm 0.05)\%$$
$$\mathrm{Br}(B \to D^*\tau\bar{\nu}) = (1.43 \pm 0.05)\%$$

and

$$R(D) = \frac{\Gamma(B \to D\tau\bar{\nu})}{\Gamma(B \to D\ell\bar{\nu})} = 0.297 \pm 0.017$$
$$R(D^*) = \frac{\Gamma(B \to D^*\tau\bar{\nu})}{\Gamma(B \to D^*\ell\bar{\nu})} = 0.252 \pm 0.003$$

$B \to D\tau\bar{\nu}$ and $B \to D^*\tau\bar{\nu}$
exhaust about 86% of the inclusive rate.

Problem: The measured values are significantly larger and overshoot the inclusive rate

$$\mathrm{Br}(B \to D\tau\bar{\nu}) = (0.77 \pm 0.25)\%$$
$$\mathrm{Br}(B \to D^*\tau\bar{\nu}) = (2.1 \pm 0.4)\%$$

$$\mathrm{Br}(B \to D\tau\bar{\nu}) + \mathrm{Br}(B \to D^*\tau\bar{\nu}) = (2.87 \pm 0.47)\%$$

Λ_b Semi-Leptonics

$\Lambda_b \to \Lambda_c$ form factors

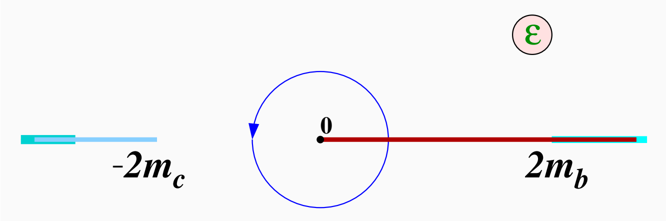
- Recent V_{ub} determination from LHCb based on $\Lambda_b \to p\ell\bar{\nu}$ is normalized to $\Lambda_b \to \Lambda_c\ell\bar{\nu}$
- Relies heavily on Lattice QCD Form Factors for both $\Lambda_b \to p$ and $\Lambda_b \to \Lambda_c$ (W. Detmold, C. Lehner, S. Meinel (2015))
- Can we say something using continuum methods?
- $\to$ Zero Recoil Sum Rule for $\Lambda_b \to \Lambda_c$ form factors

Zero Recoil Sum Rule

- Start from

$$T(\omega) = \frac{1}{3} \int d^4x \, e^{i\,(v\cdot x)\omega}$$
$$\langle \Lambda_b(P) | \mathcal{T}\{\bar{b}_v(x)\gamma_\mu\gamma_5 c_v(x)\, \bar{c}_v(0)\gamma^\mu\gamma_5 b_v(0)\} | \Lambda_b(P)\rangle$$

- compute the contour integral $I_0(\mu) = -\frac{1}{2\pi i} \oint\limits_{|\varepsilon|=\mu} T(\varepsilon)\,\mathrm{d}\varepsilon$



- Inserting a complete set of states:
  Lowest state is Λ_c
- Form factor definition: (T. Feldmann , M. Yip (2011))

$$\langle \Lambda_c(v', s')|\bar{c}\gamma_5\gamma_\mu b|\Lambda_b(v, s)\rangle = \bar{u}_{\Lambda_c}(v', s')\gamma_5 \left[g_0(w)(M_{\Lambda_b} + M_{\Lambda_c})\frac{q^\mu}{q^2} \right.$$
$$+ g_+(w)\frac{M_{\Lambda_b} - M_{\Lambda_c}}{s_-}\left(M_{\Lambda_b} v_\mu + M_{\Lambda_c} v'_\mu - (M^2_{\Lambda_b} - M^2_{\Lambda_c})\frac{q^\mu}{q^2}\right)$$
$$\left. + g_\perp(w)\left(\gamma_\mu - \frac{2M_{\Lambda_c}M_{\Lambda_b}}{s_+}(v_\mu - v'_\mu)\right)\right] u_{\Lambda_b}(v, s)$$

- Zero Recoil Sum Rule:

$$I_0(\epsilon_M) = \frac{1}{3}\left[2|g_\perp(1)|^2 + |g_+(1)|^2\right] + \text{ inelastic}$$
$$= \xi^{\text{pert}}(\epsilon_M, \mu) - \Delta_{1/m^2}(\epsilon_M, \mu) - \Delta_{1/m^3}(\epsilon_M, \mu) + \cdots$$

- Perturbative Contribution

($\mathcal{O}(\alpha_s)$, computed in Wilsonian Cut Off Scheme)

$$\xi^{\rm pert}(\epsilon_M = \mu = 0.75\,{\rm GeV}) = 0.970 \pm 0.02$$

- Nonperturbative Contributions

$$\Delta_{1/m^2} = \frac{\mu_\pi^2(\Lambda_b)}{4}\left(\frac{1}{m_c^2} + \frac{1}{m_b^2} + \frac{2}{3m_b m_c}\right)$$
$$\Delta_{1/m^3} = \frac{\rho_D^3(\Lambda_b)}{4m_c^3} + \frac{\rho_D^3(\Lambda_b)}{12m_b}\left(\frac{1}{m_c^2} + \frac{3}{m_b^2} + \frac{1}{m_b m_c}\right)$$

- Note $\mu_G = \rho_{LS} = 0$ and $\mu_\pi^2(\Lambda_b) \sim \mu_\pi^2(B)$

- ... or as an inequality

$$\frac{1}{3}\left[2|g_\perp(1)|^2 + |g_+(1)|^2\right] \leq \xi^{\text{pert}}(\epsilon_M,\mu) - \Delta_{1/m^2}(\epsilon_M,\mu) - \Delta_{1/m^3}(\epsilon_M,\mu) + \cdots$$

- Numerically we have (Preliminary)

$$\frac{1}{3}\left[2|g_\perp(1)|^2 + |g_+(1)|^2\right] \leq 0.86$$

- ... to be compared to the lattice number
(W. Detmold, C. Lehner, S. Meinel (2015))

$$\frac{1}{3}\left[2|g_\perp(1)|^2 + |g_+(1)|^2\right] = 0.824 \pm 0.020$$

Some Personal Conclusions

The V_{cb} "Problem"

$B \to D\ell\bar{\nu}$

- Exclusive V_{cb} from the lattice
  - Extrapolation to $w = 1$: $V_{cb} = (39.80 \pm 1.1) \times 10^{-3}$
  - Calculation of full $\mathcal{G}(w)$: $V_{cb} = (41.08 \pm 0.95) \times 10^{-3}$
- Continuum methods are close to the second value

$B \to D^*\ell\bar{\nu}$

- Extapolation, lattice $\mathcal{F}(1)$: $V_{cb} = (39.04 \pm 0.85) \times 10^{-3}$
- Extapolation, QCD-SR $\mathcal{F}(1)$: $V_{cb} = (41.3 \pm 1.1) \times 10^{-3}$

$B \to X_c\ell\bar{\nu}$

- From the HQE: $V_{cb} = (42.21 \pm 0.78) \times 10^{-3}$

The V_{ub} Problem

$B \to \pi \ell \bar{\nu}$

- Lattice $\otimes$ QCD Sum Rules: $V_{ub} = (3.72 \pm 0.19) \times 10^{-3}$

$B \to X_u \ell \bar{\nu}$

- Average over shape function parametrizations: $V_{ub} = (4.49 \pm 0.21) \times 10^{-3}$

Other decays:

- $B \to \tau \bar{\nu}$ is not yet precise enough
- Up to now, $\Lambda_b \to p \ell \bar{\nu}$ only measures V_{ub}/V_{cb}

This still looks like a problem