Amplitude Analyses

B Workshop Neckarzimmern

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Why Amplitude Analyses?

- QM is intrinsically complex:

  Wave functions/transition amplitudes etc: $\psi = a\, e^{i\alpha}$. Observable: $|\psi|^2$.

  Only half the information. How do I get the rest?

- Note that the rest is very interesting - CP violation in the SM comes from phases!

- Answer: Interference effects:

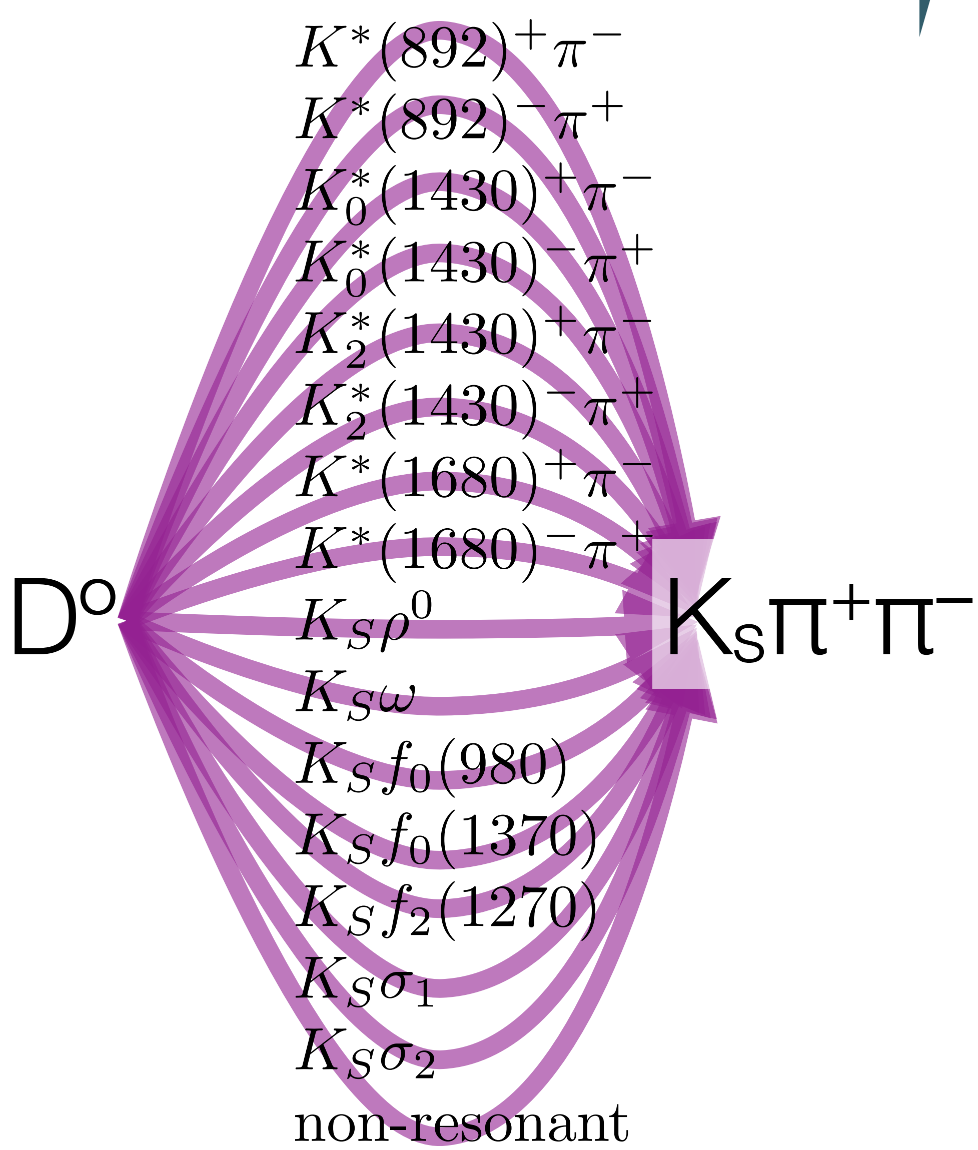
  $\psi_{total} \quad = \ a\, e^{i\alpha} + b\, e^{i\beta} + \ldots$
  $|\psi_{total}|^2 = \ |a\, e^{i\alpha} + b\, e^{i\beta} + \ldots| = a^2 + b^2 + 2ab \cos(\alpha - \beta) + \ldots$

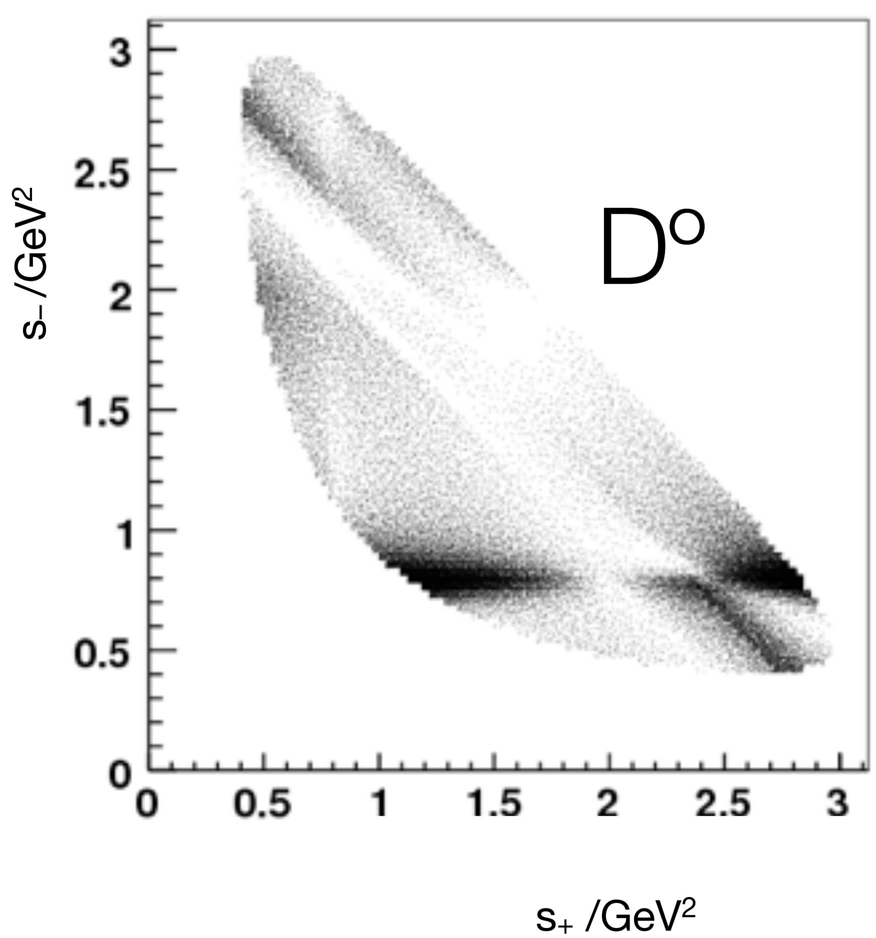
Dalitz plot analyses - lots of interfering amplitudes!

Many interfering decay paths contribute to the same final state

Described by a sum of complex amplitudes

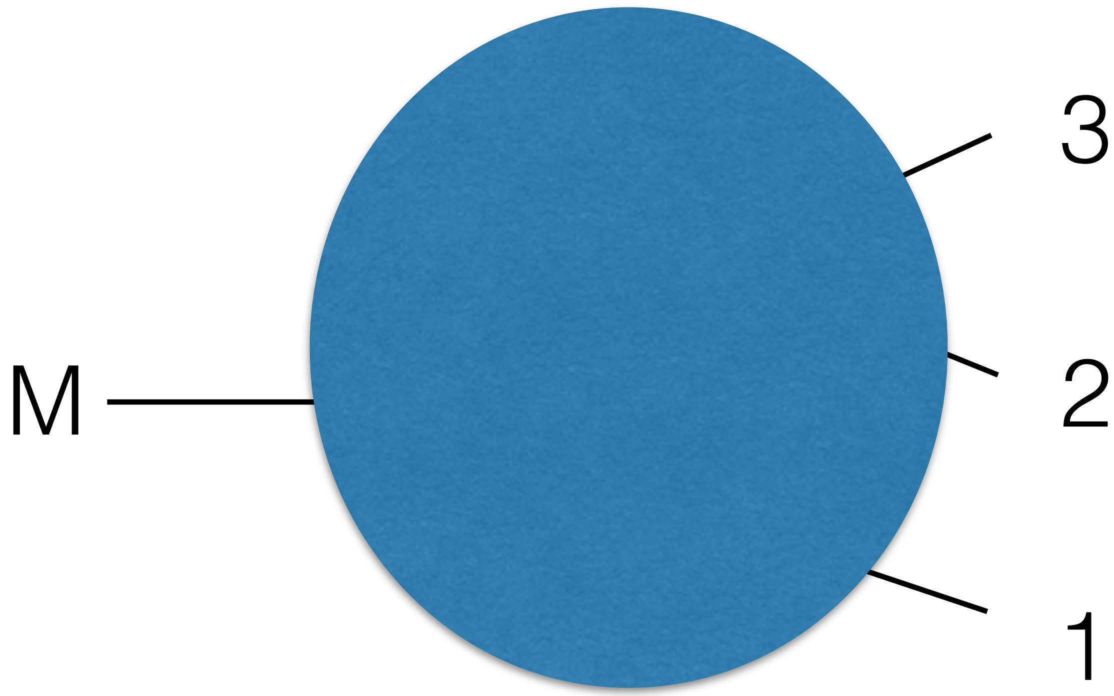
$$A(s_+, s_-) = \sum_k a_k(s_+, s_-)\, e^{i\phi_k(s_+, s_-)}$$





$|A(s_+, s_-)|^2$ represented in a Dalitz plot

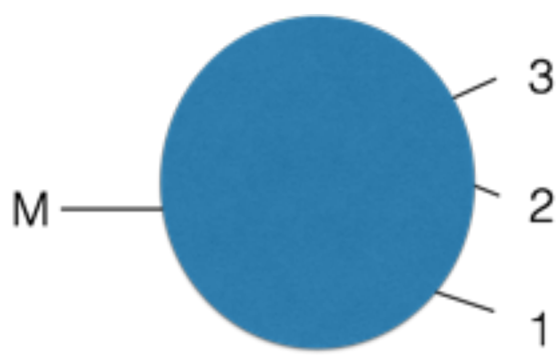
3 body decays

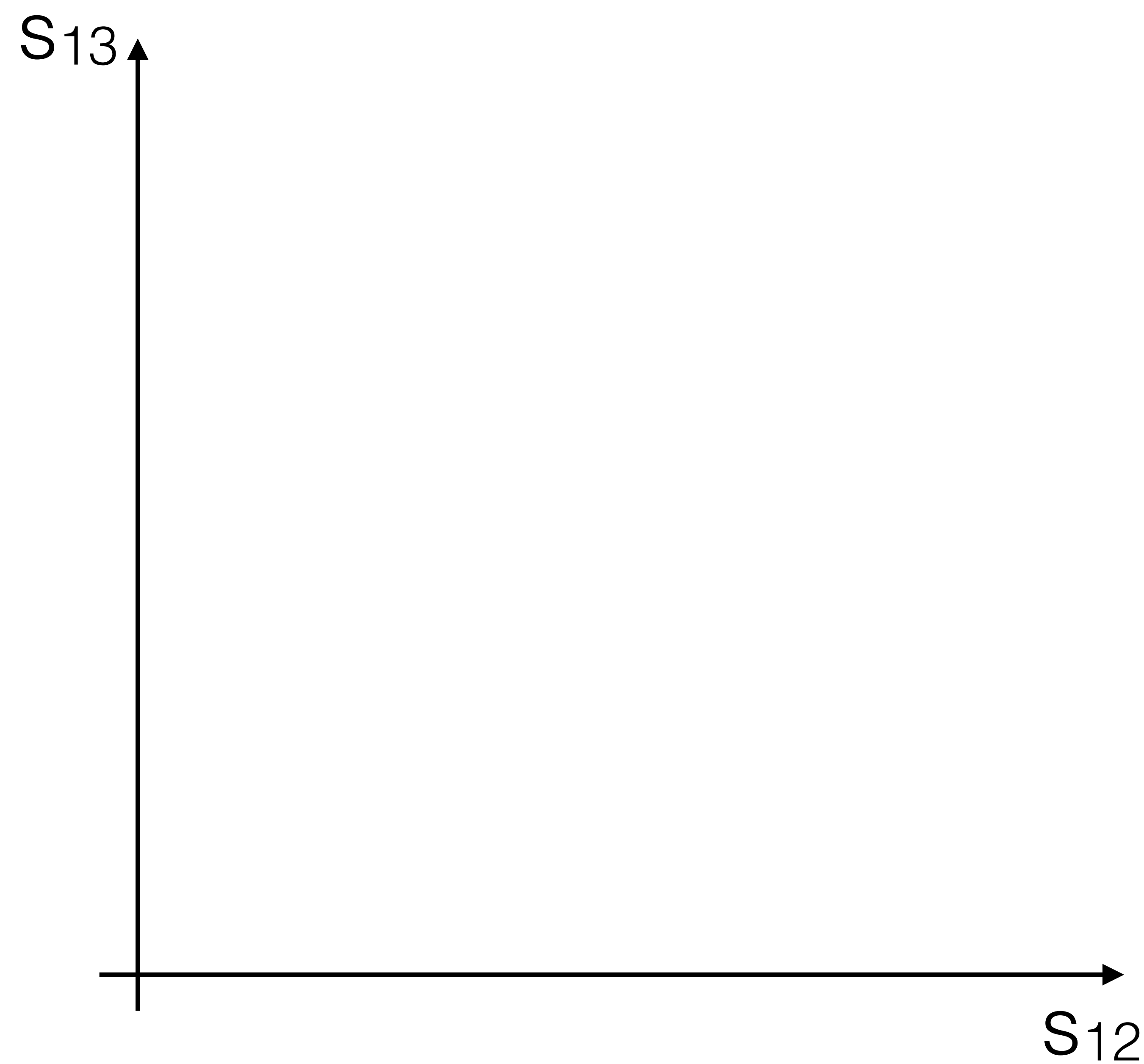


$$
\begin{aligned}
d\Gamma &= |\mathcal{M}_{fi}|^2 d\Phi \\
&= |\mathcal{M}_{fi}|^2 \left| \frac{\partial \Phi}{\partial(s_{12}, s_{13})} \right| ds_{12} ds_{13} \\
&= \frac{1}{(2\pi)^2\, 32 M^3} |\mathcal{M}_{fi}|^2 ds_{12} ds_{13}
\end{aligned}
$$

$$
s_{ij} \equiv (p_i + p_j)^2 \equiv m_{ij}^2
$$

3-body phase space





$s_{ij} = (p_i + p_j)^2 = m_{ij}^2$

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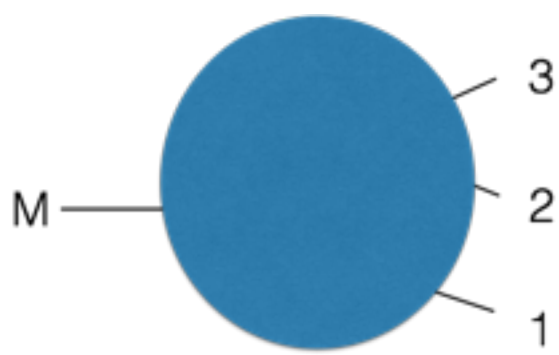

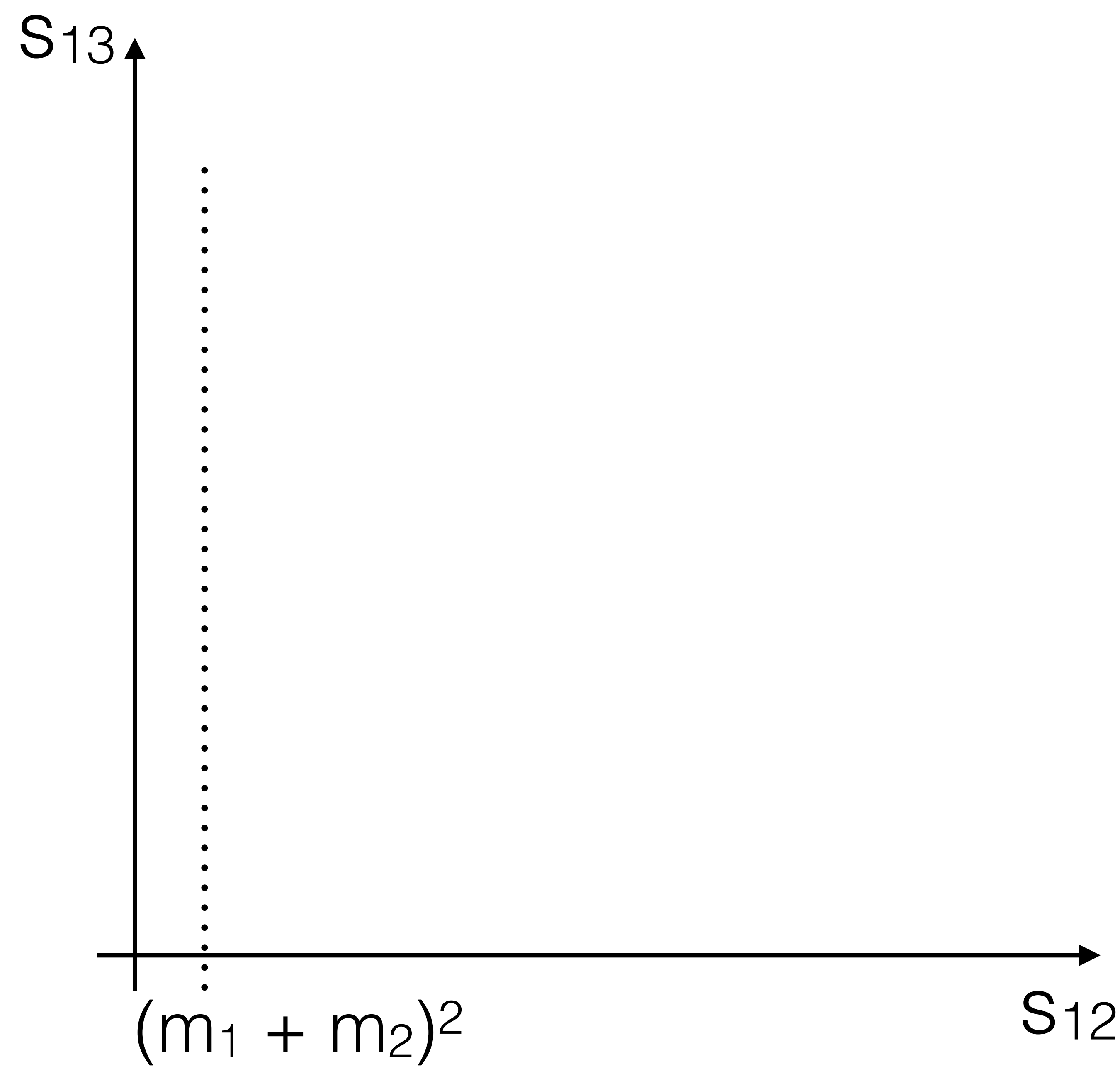

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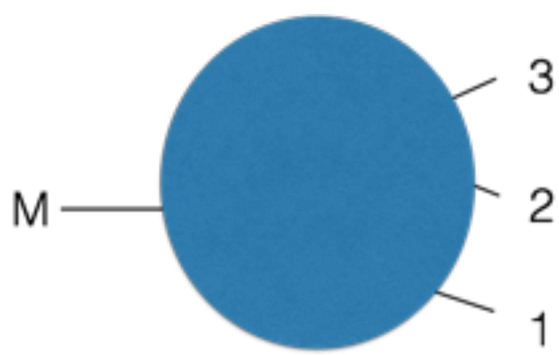

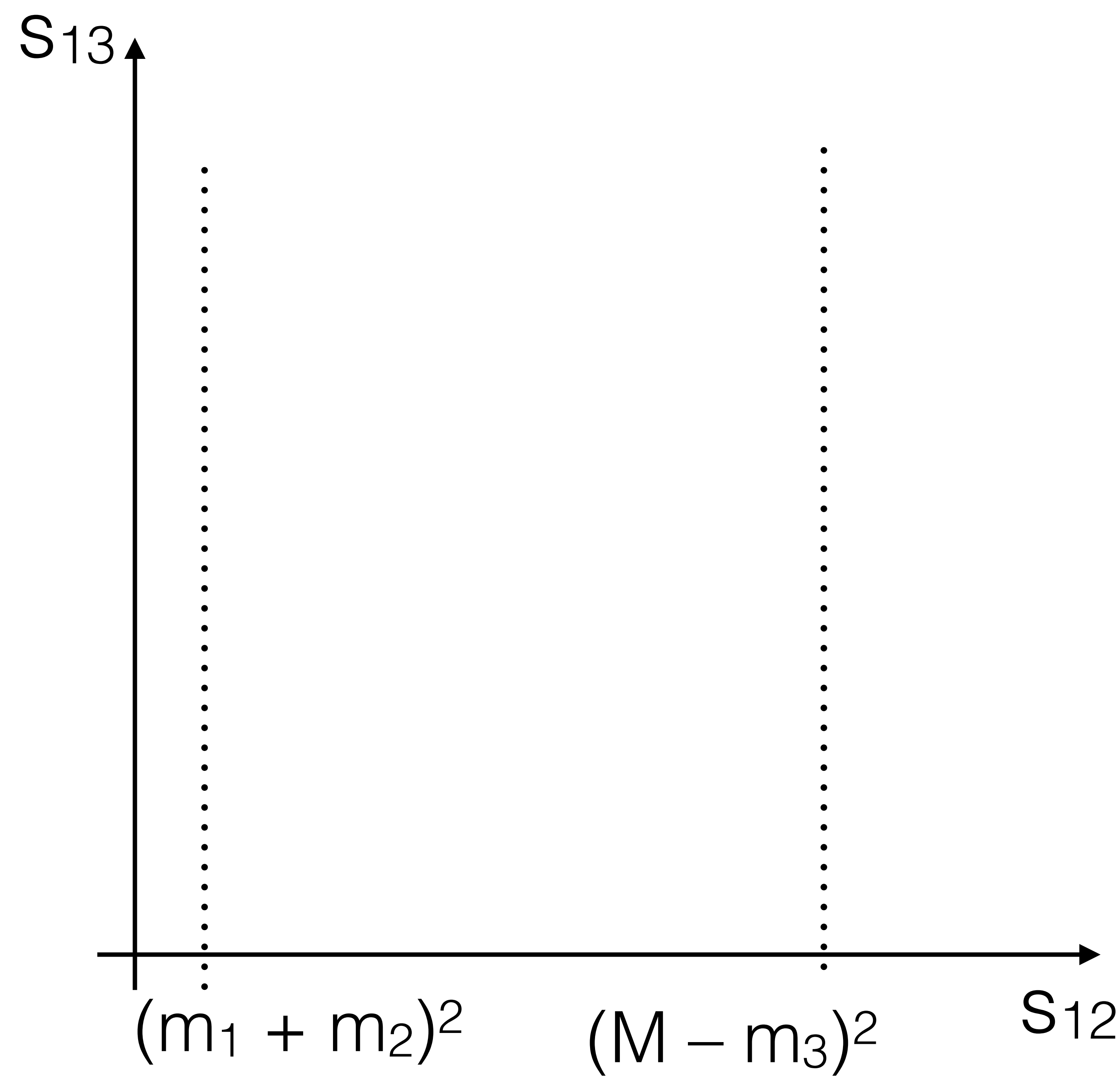

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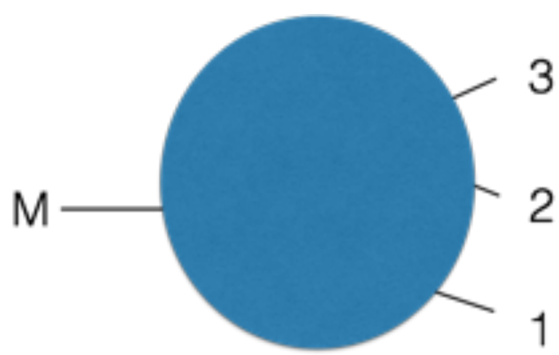

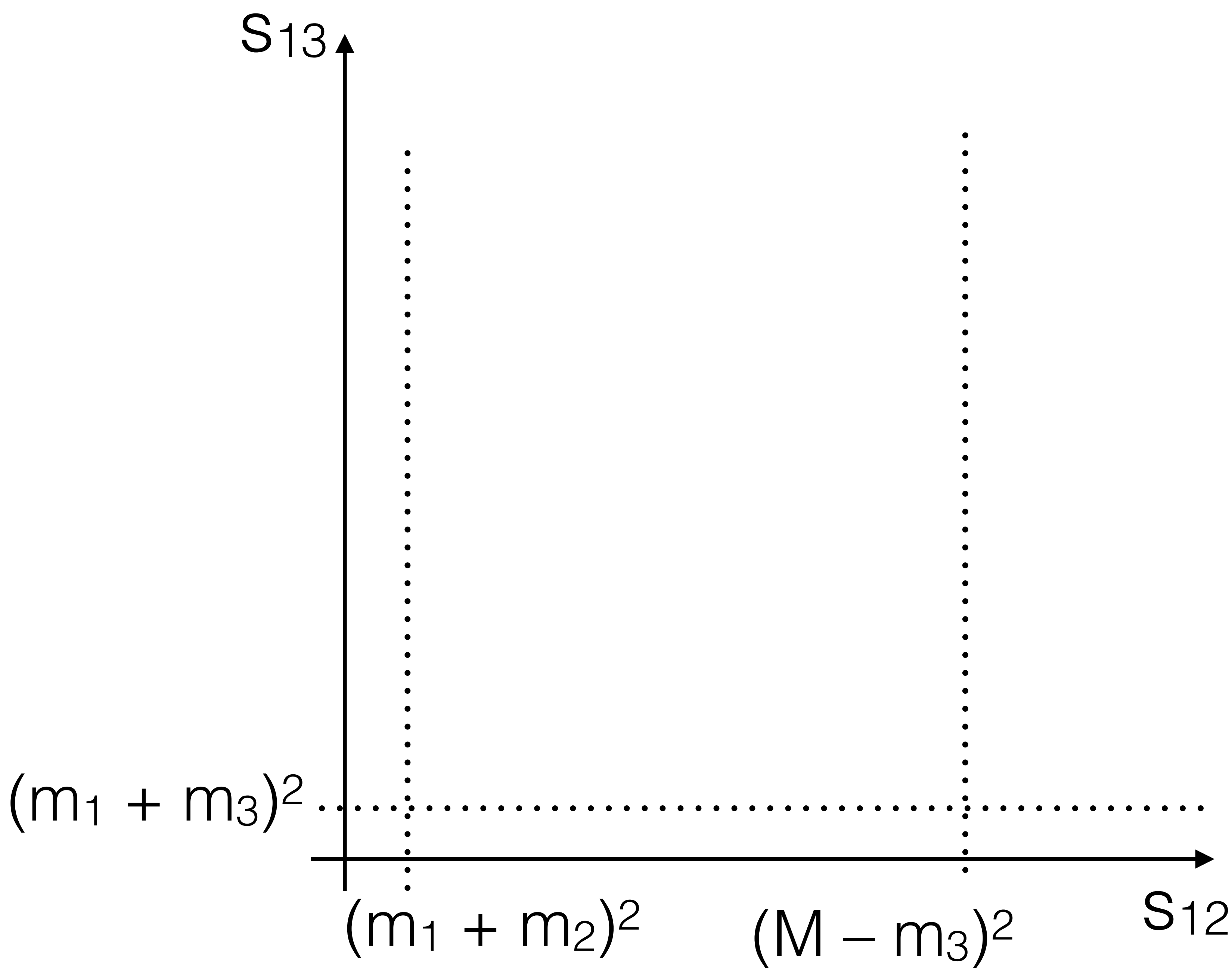

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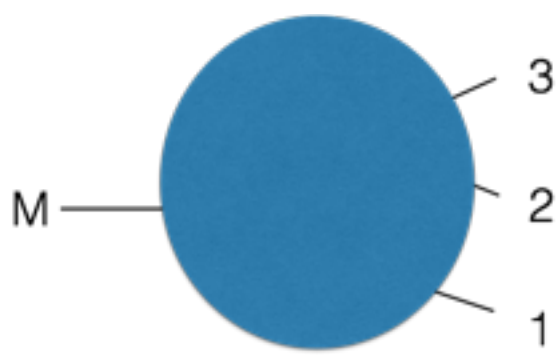

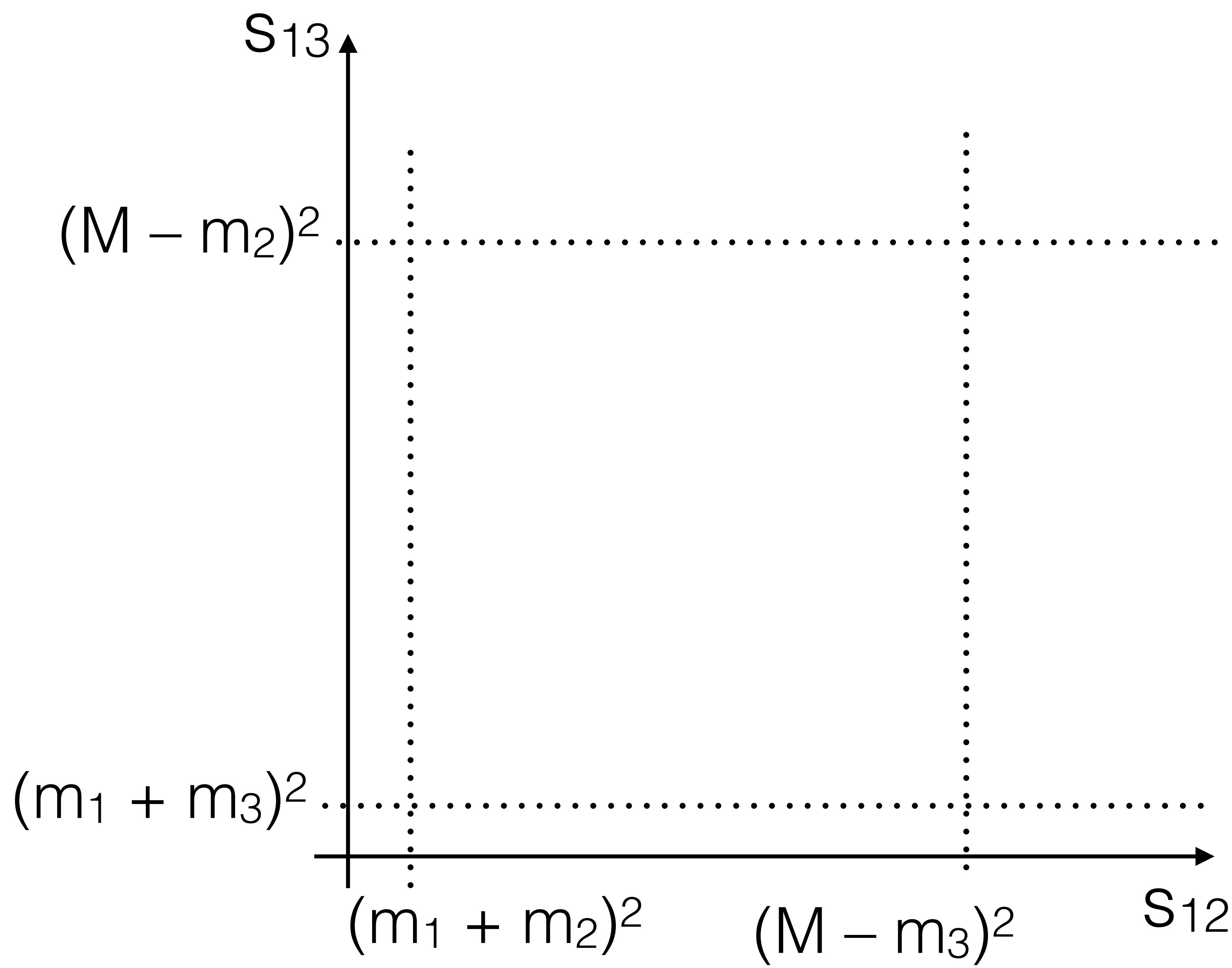

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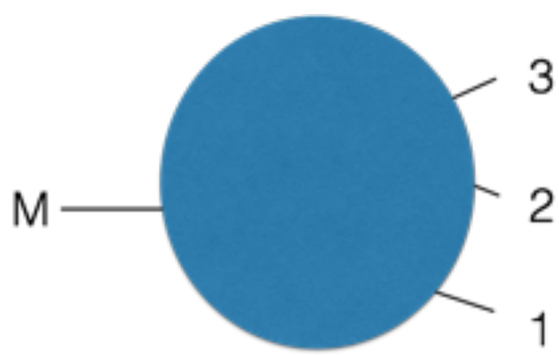
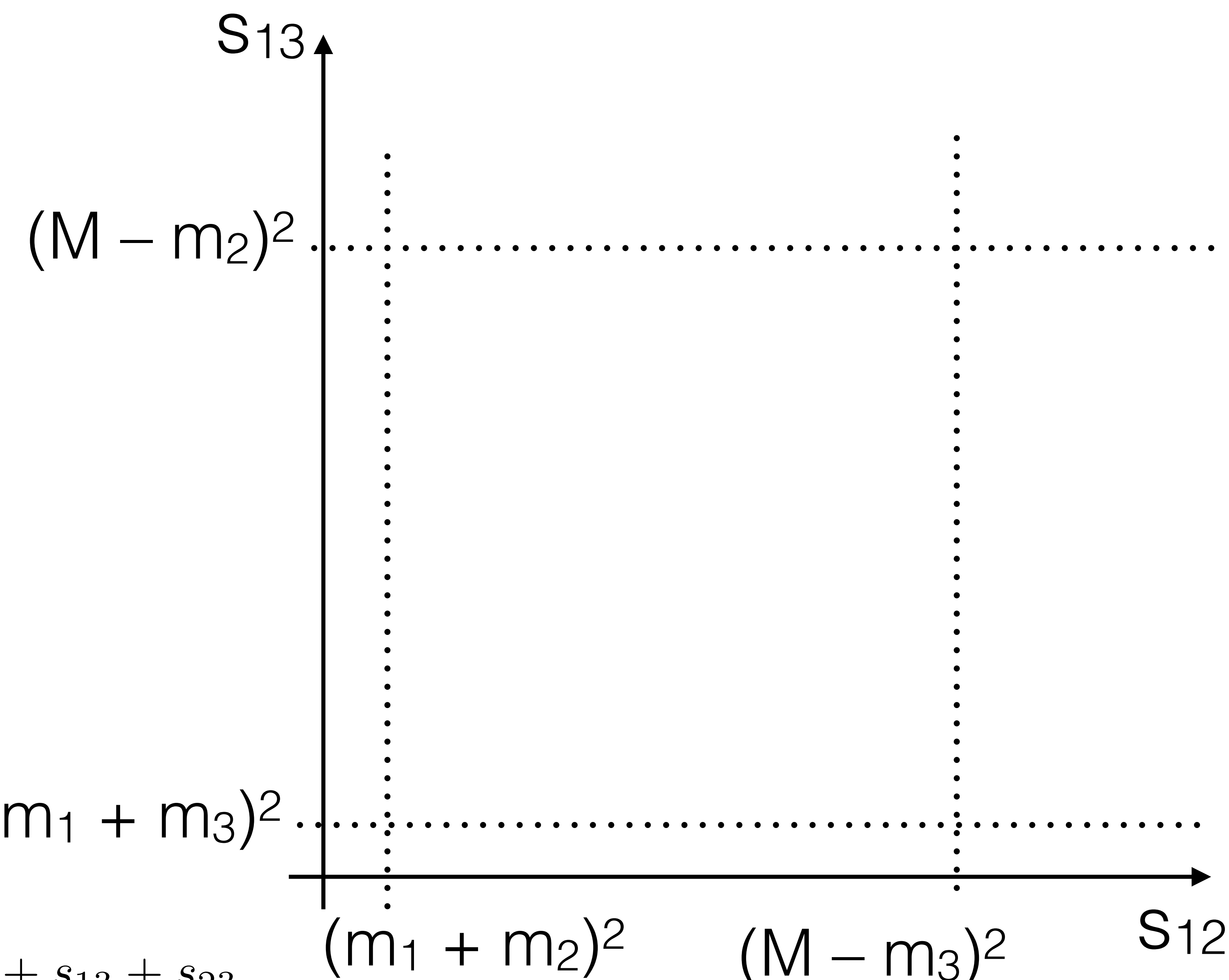
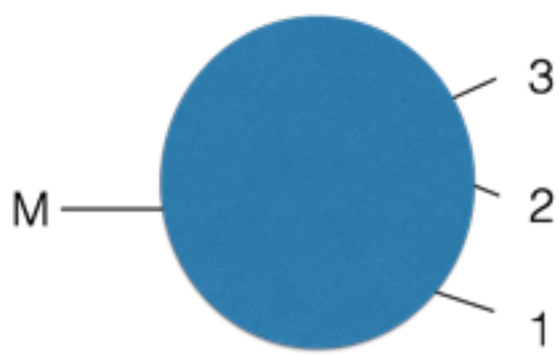


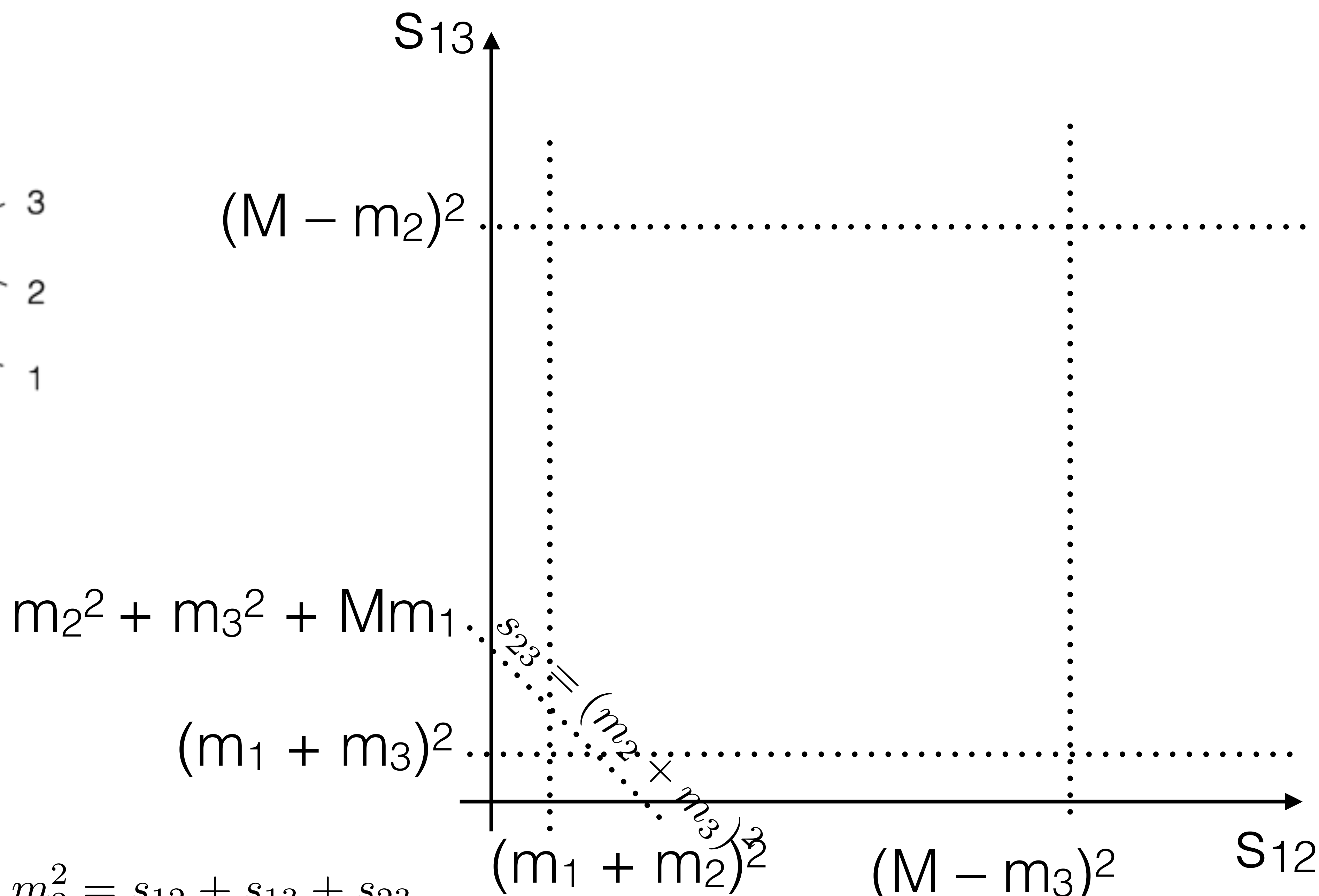


$$M^2 + m_1^2 + m_2^2 + m_3^2 = s_{12} + s_{13} + s_{23}$$

$$s_{ij} = (p_i + p_j)^2 = m_{ij}^2$$

3-body phase space





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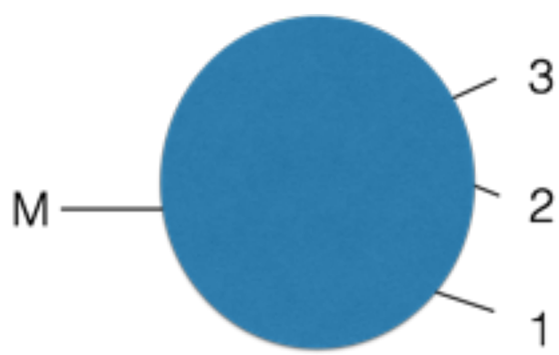

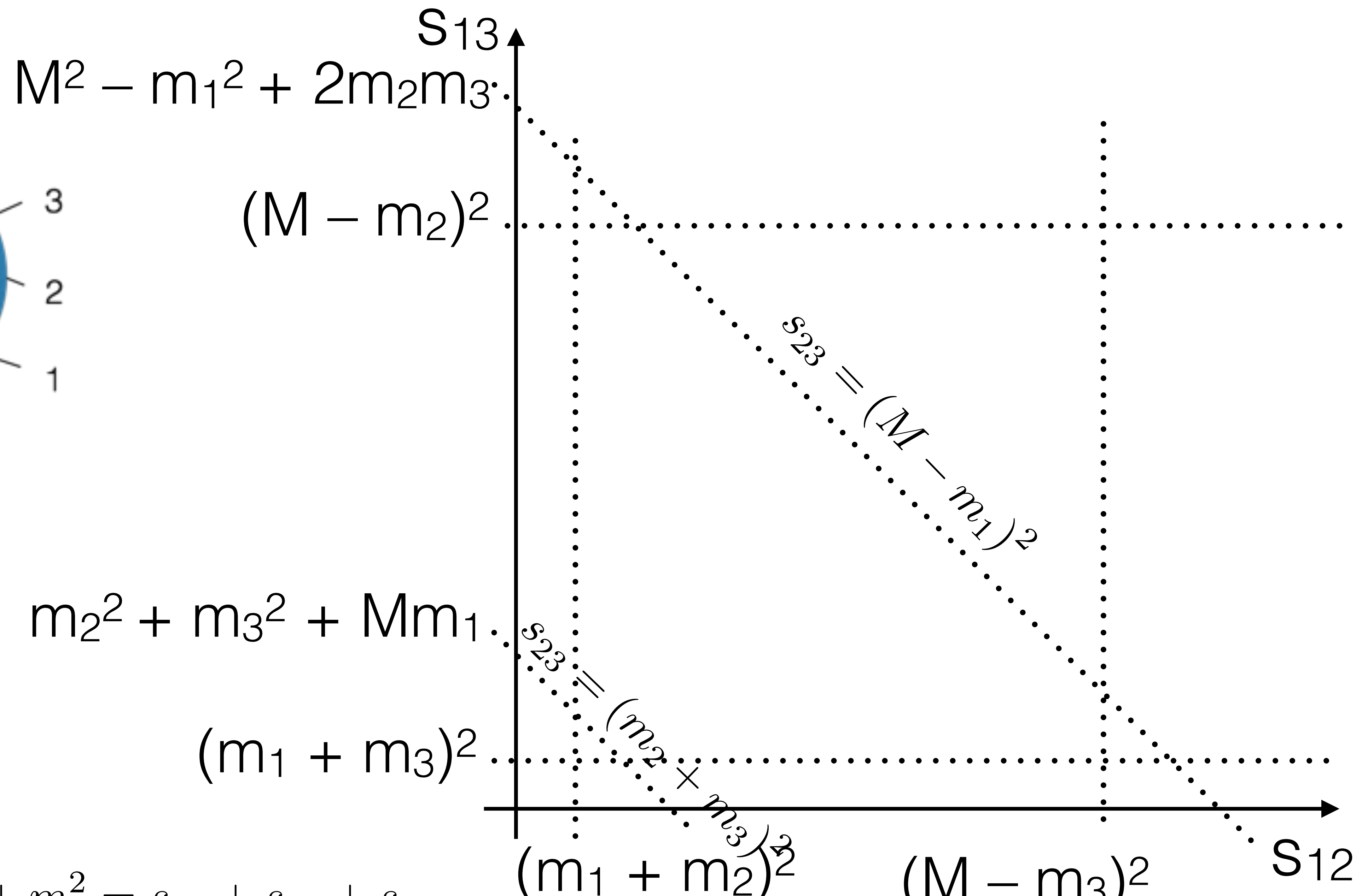

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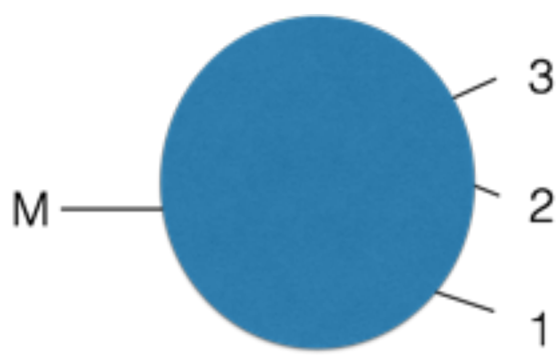

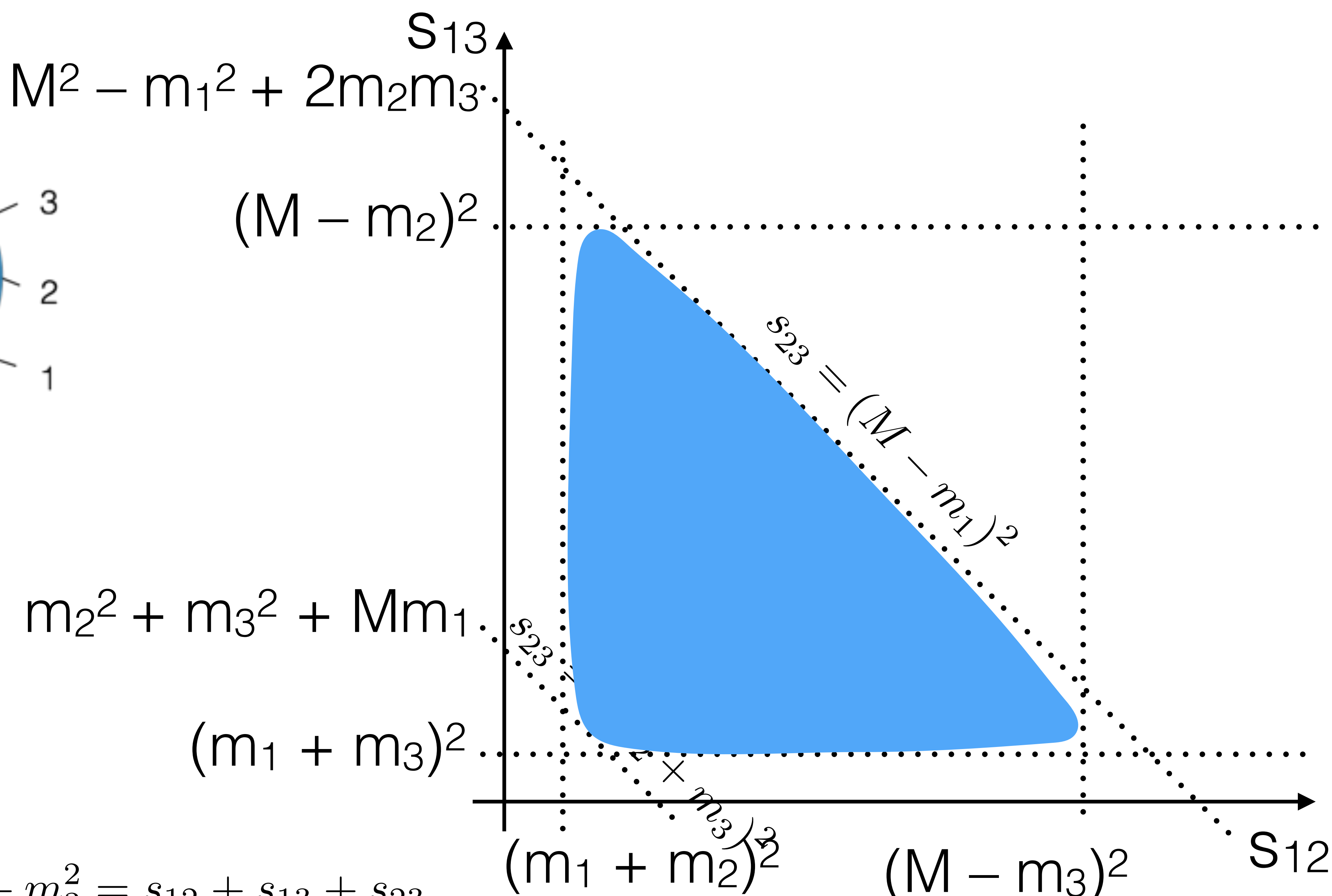

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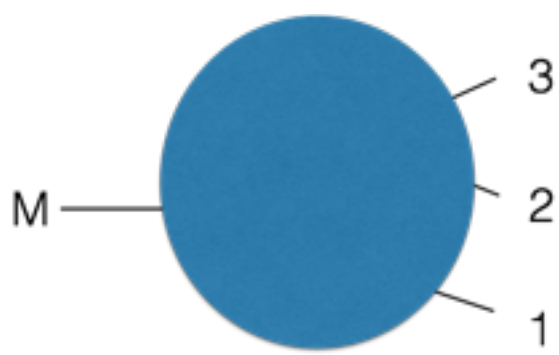
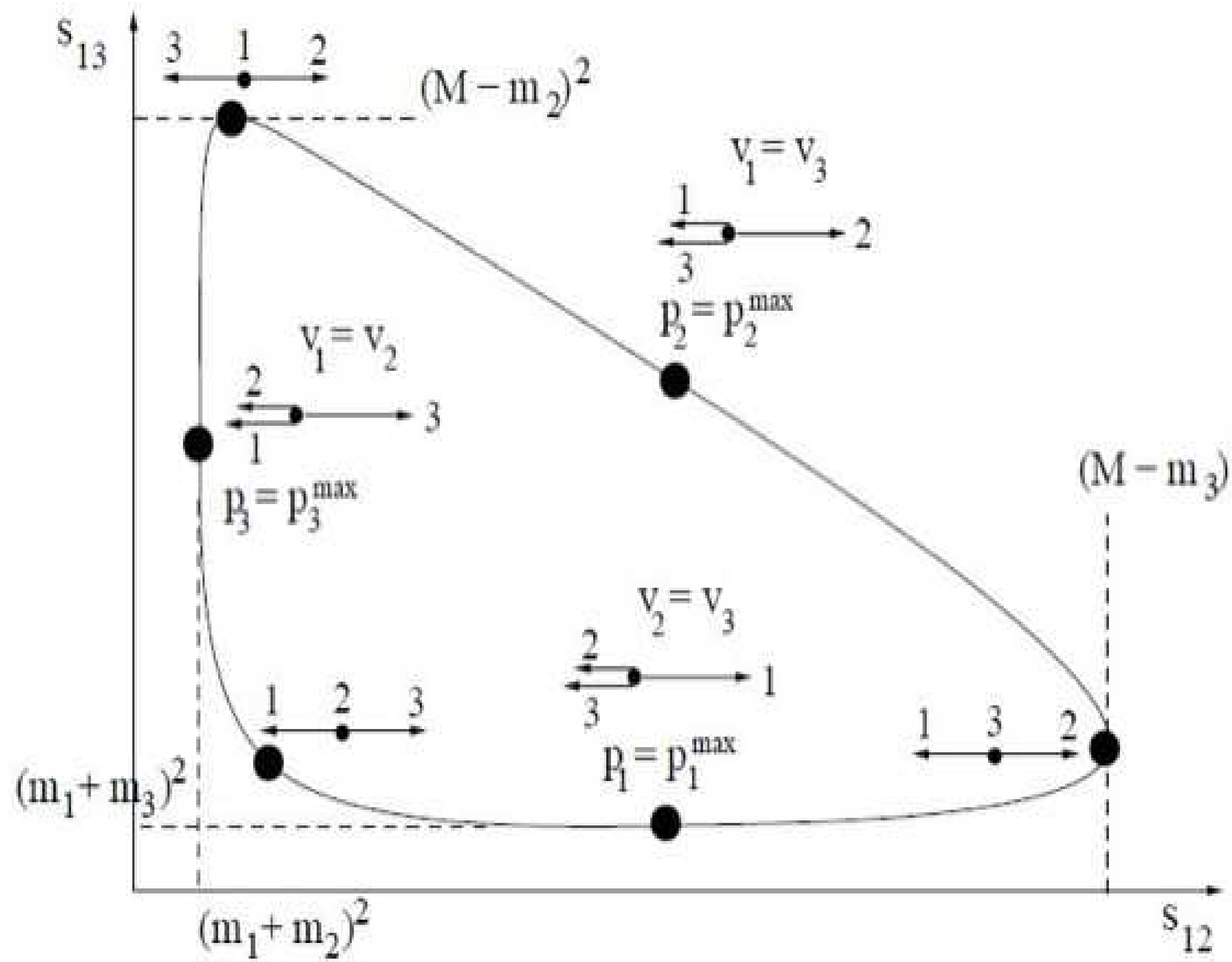
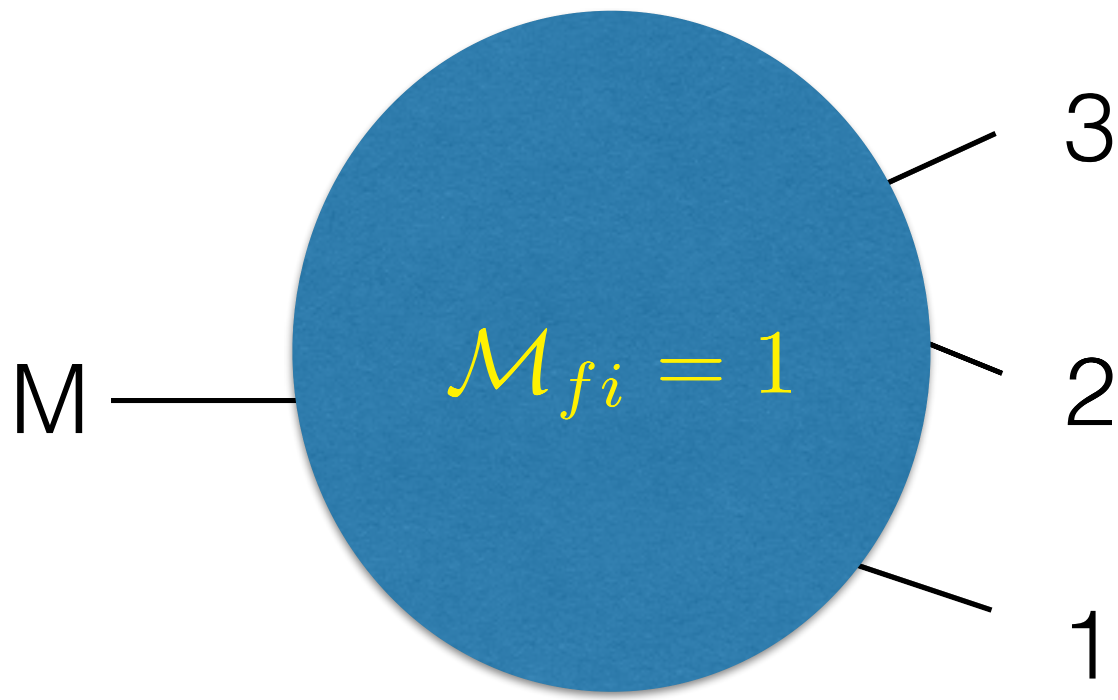
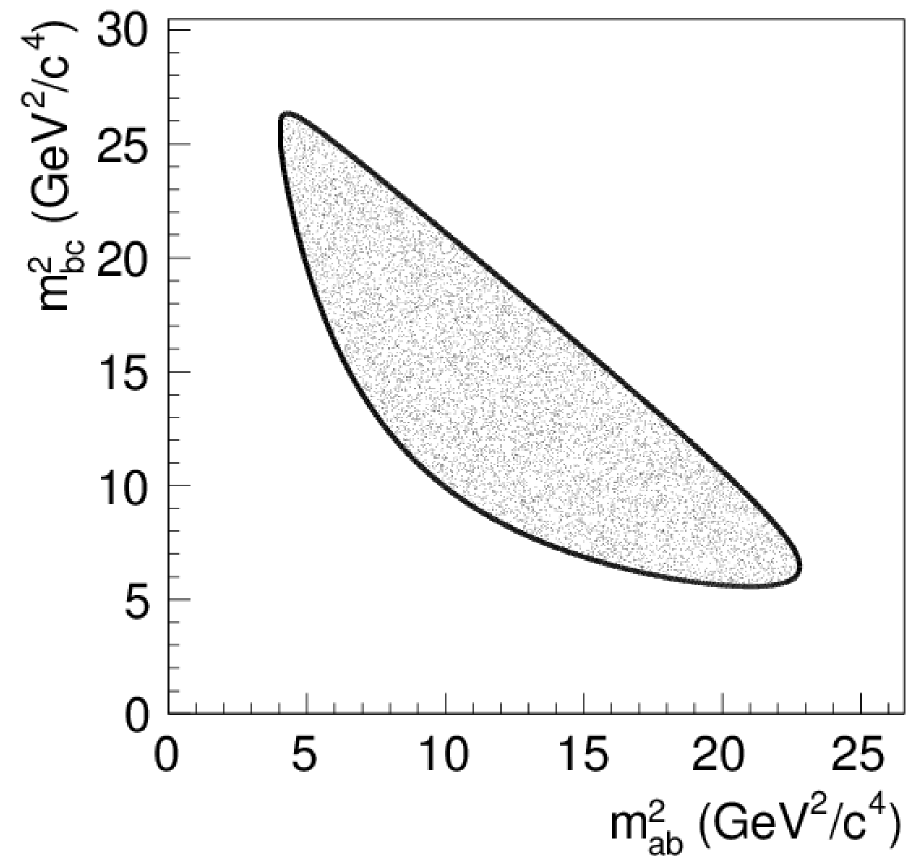




$s_{ij} = (p_i + p_j)^2 = m_{ij}^2$

What happens if nothing happens

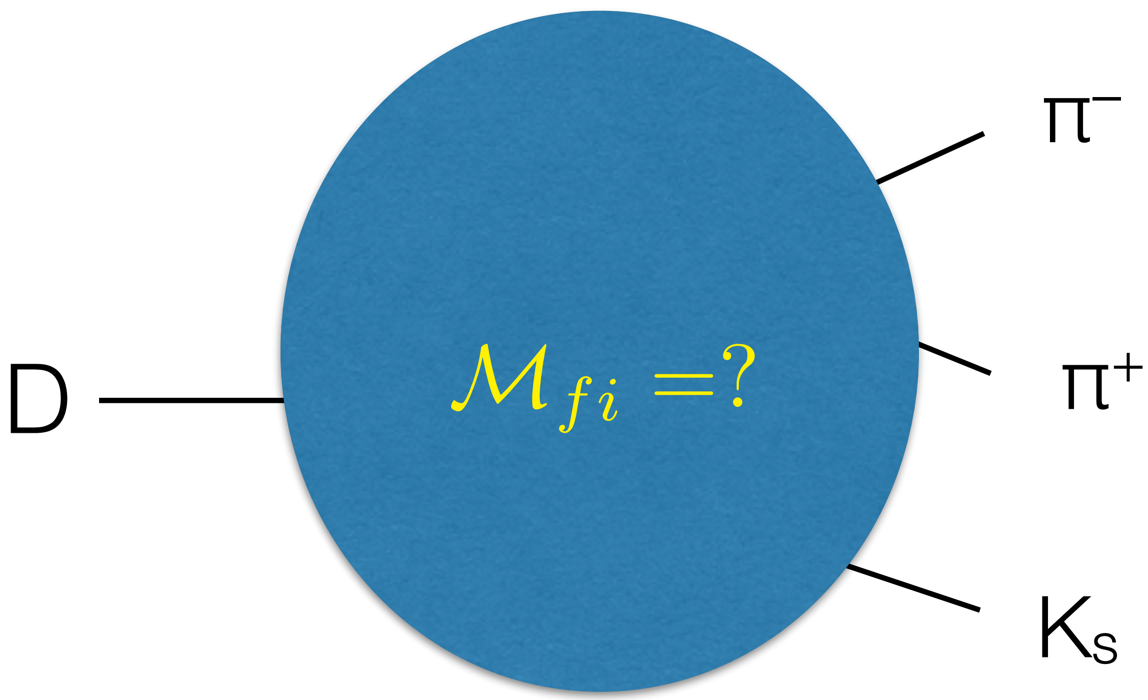




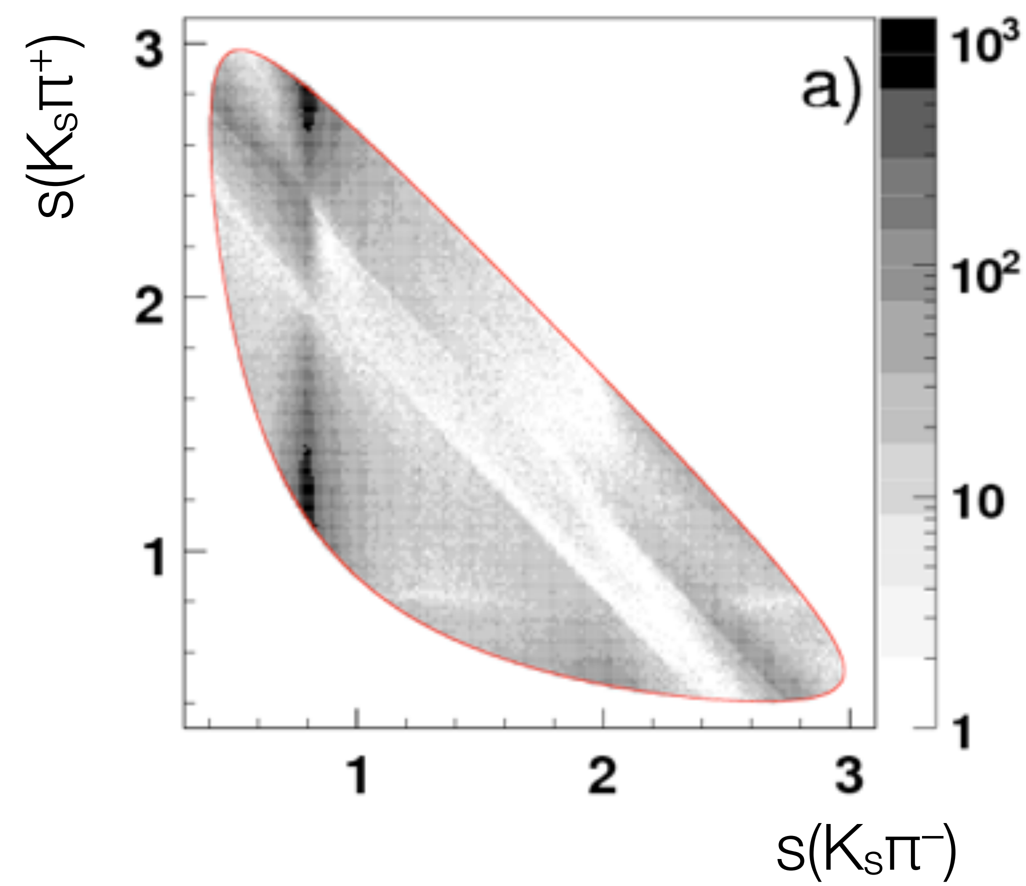
$$s_{ij} \equiv (p_i + p_j)^2 \equiv m^2_{ij}$$

$$d\Gamma = \frac{1}{(2\pi)^2\, 32 M^3} |\cancel{\mathcal{M}}_{fi}|^2 ds_{12} ds_{13}$$

What really happens



D→K_Sπ⁺π⁻



BaBar Phys. Rev. Lett. 105, 081803 (2010).

$$s_{ij} \equiv (p_i + p_j)^2 \equiv m_{ij}^2$$

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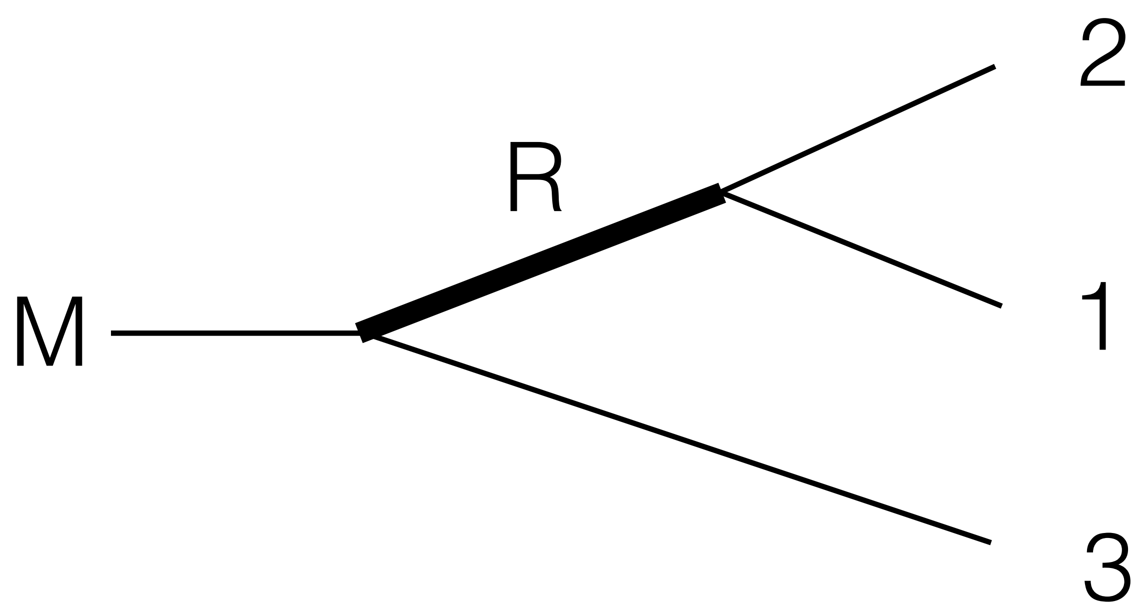
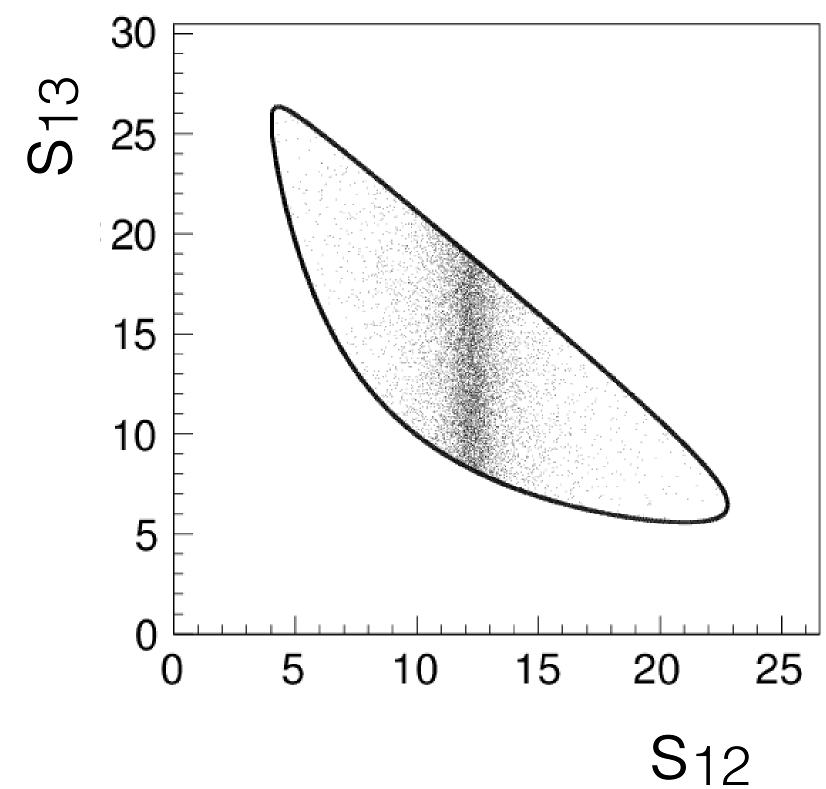
What happens if one thing happens





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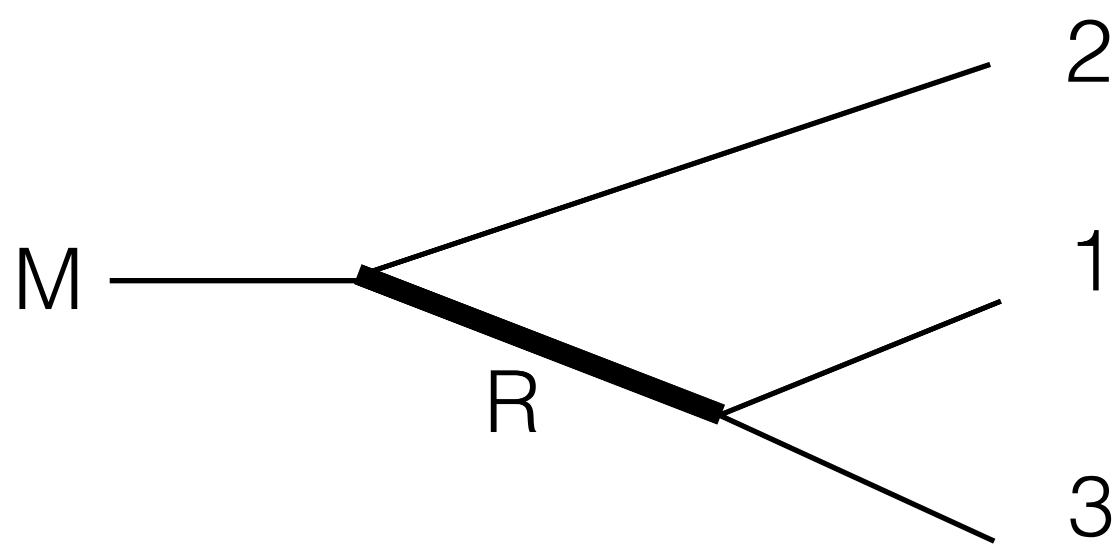

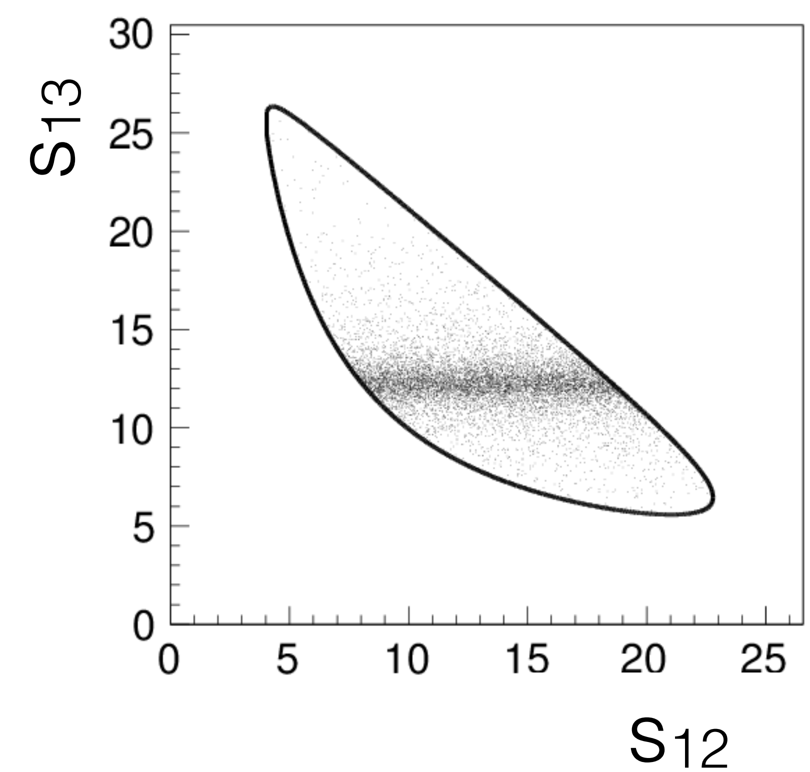

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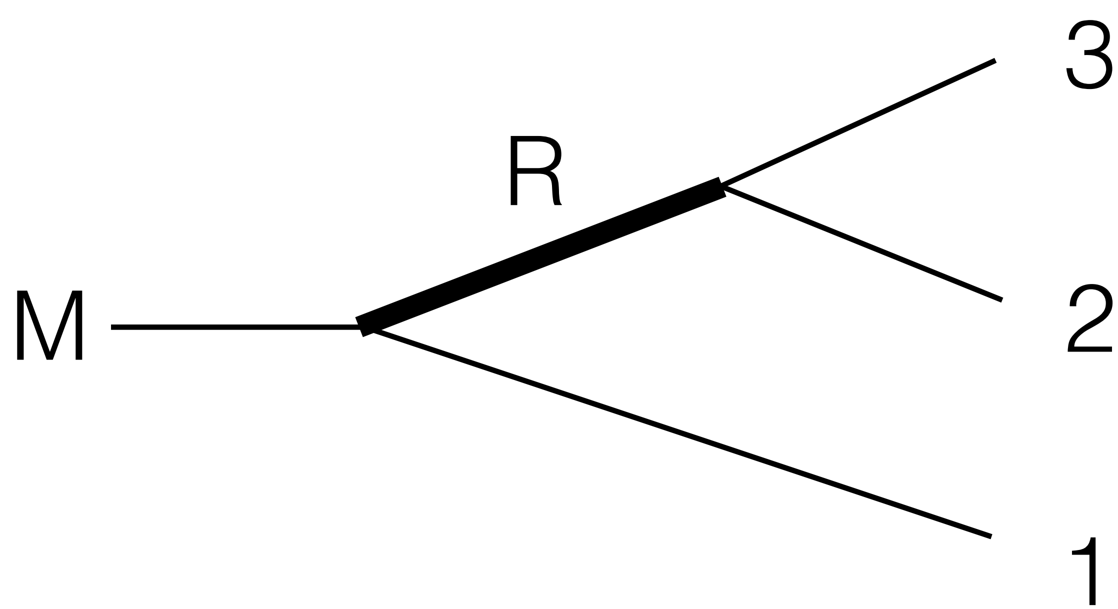

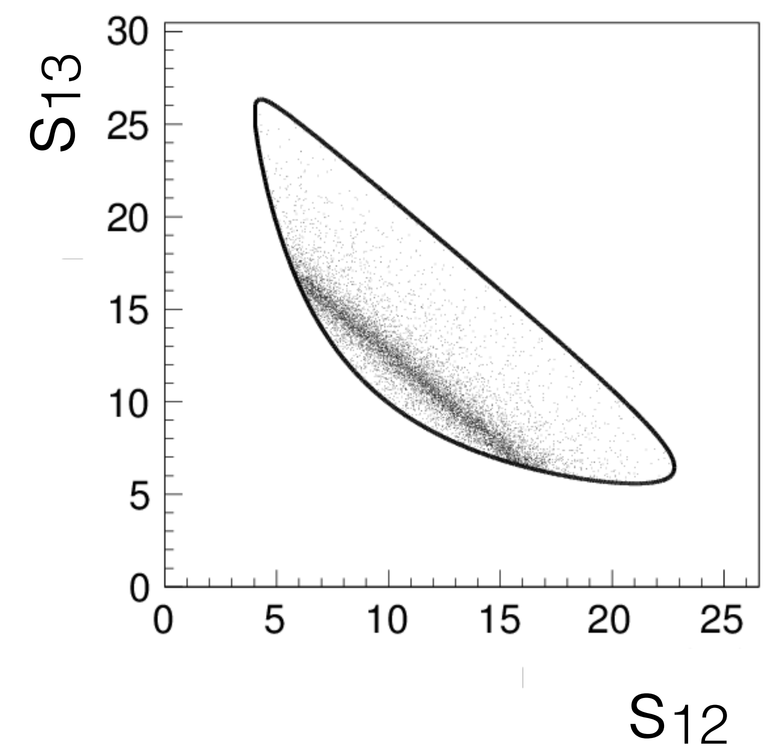

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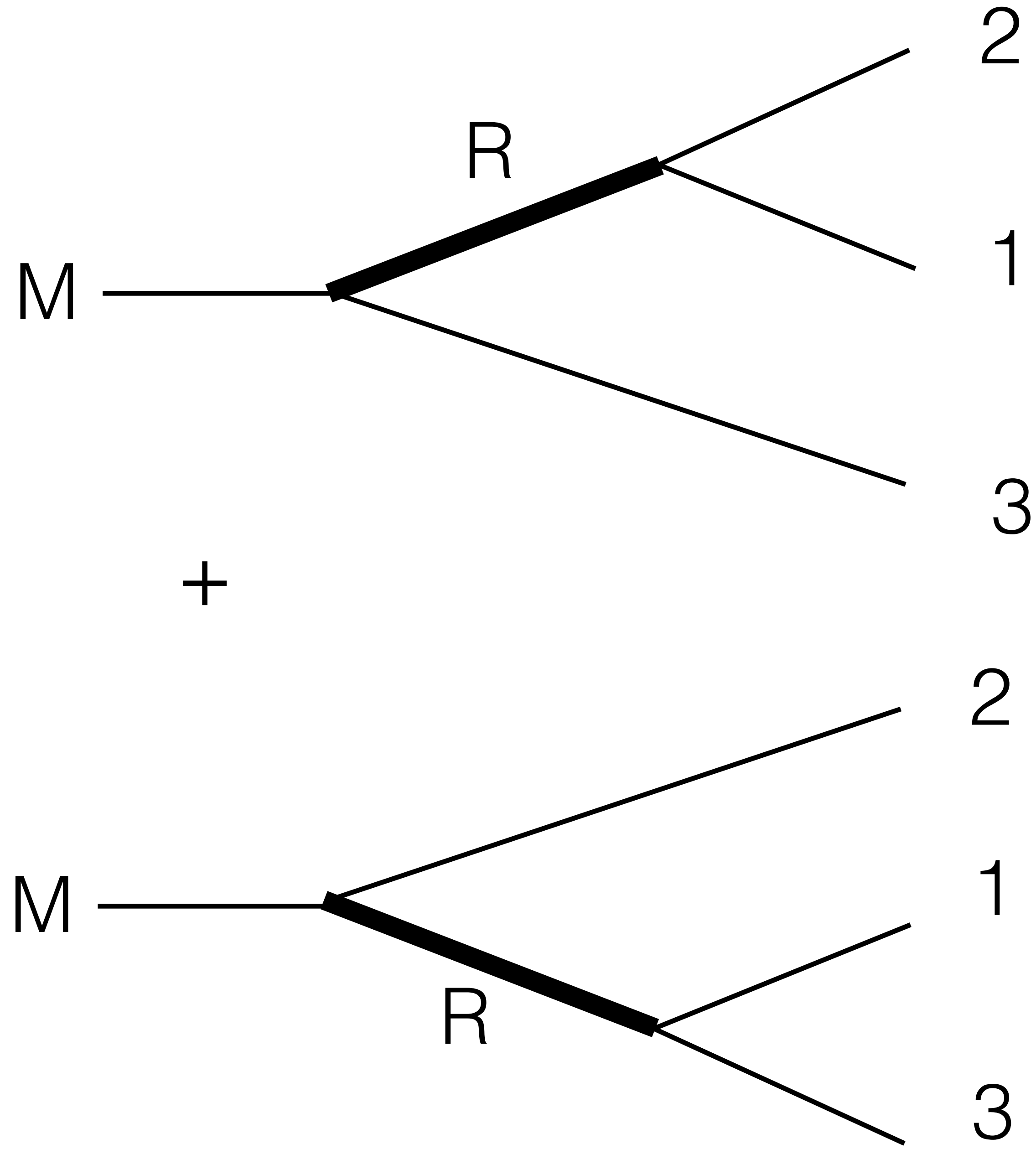
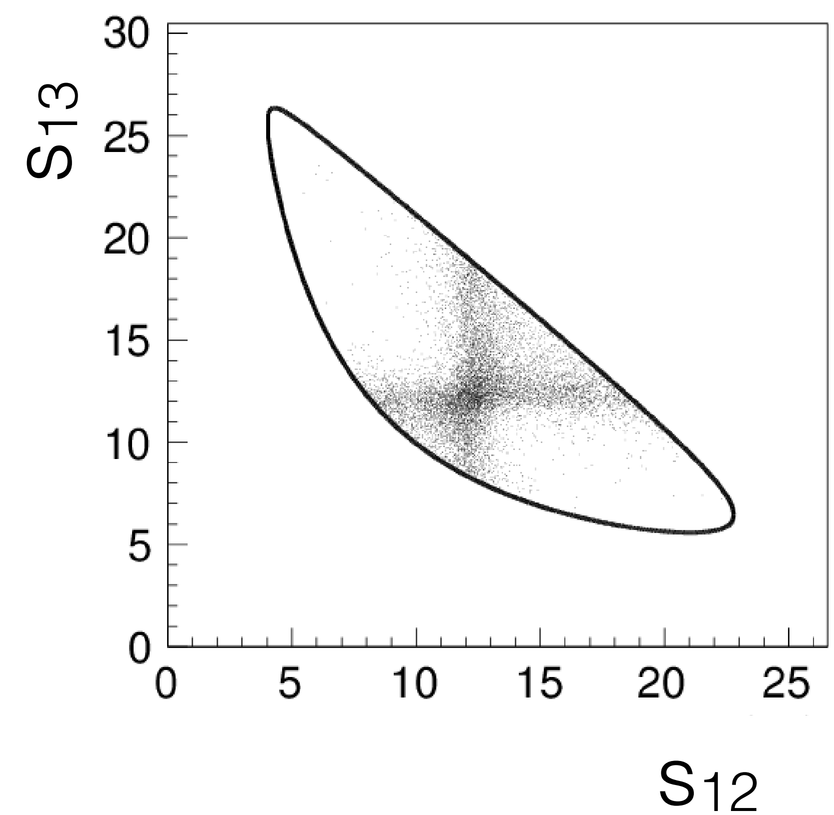
What happens if two things happens





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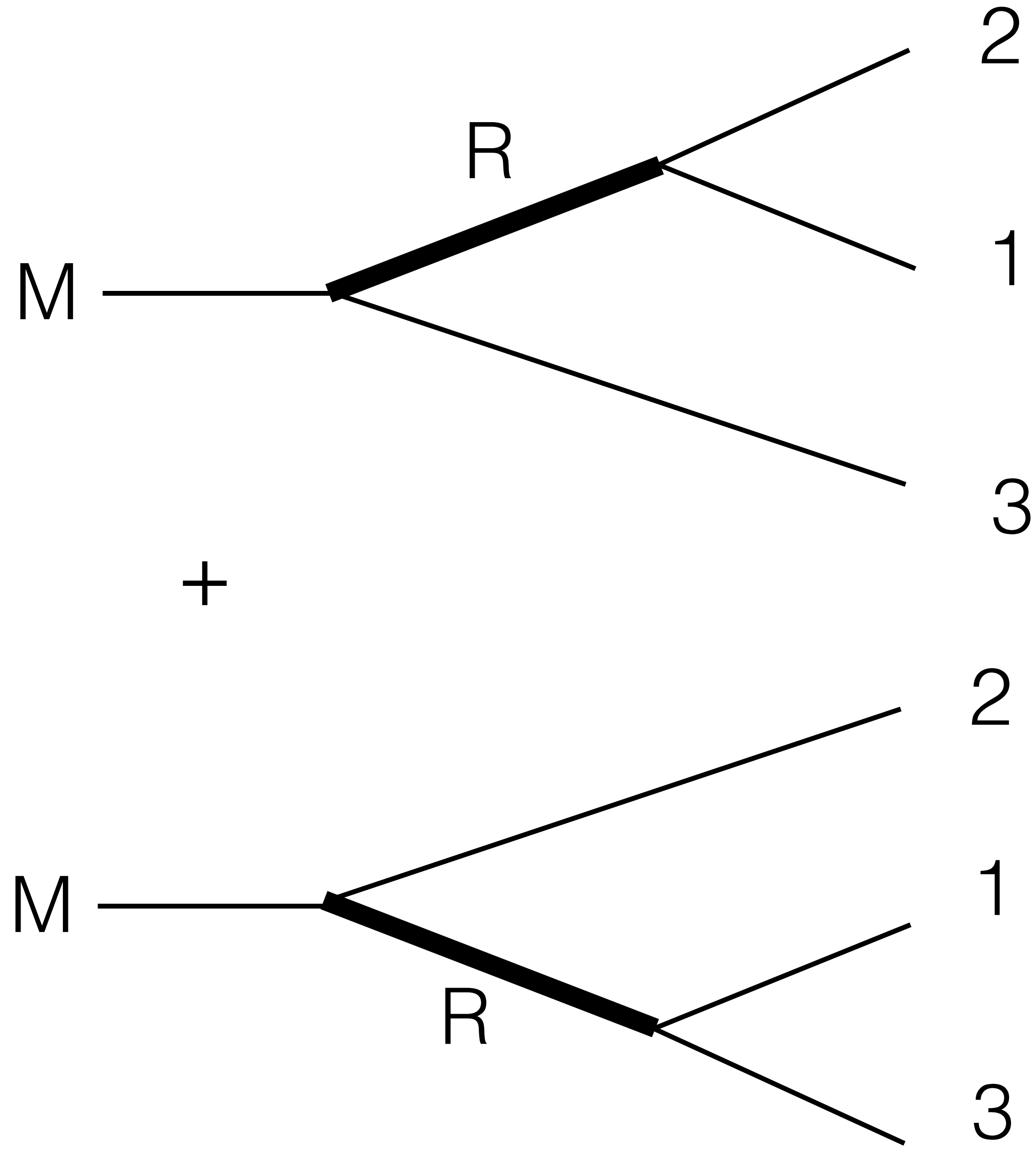
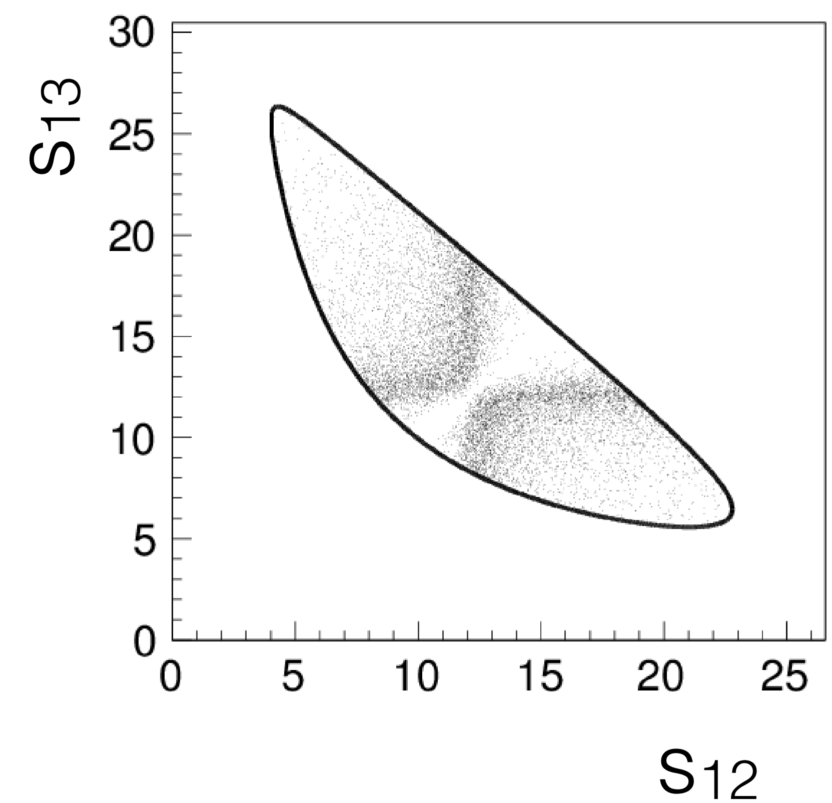
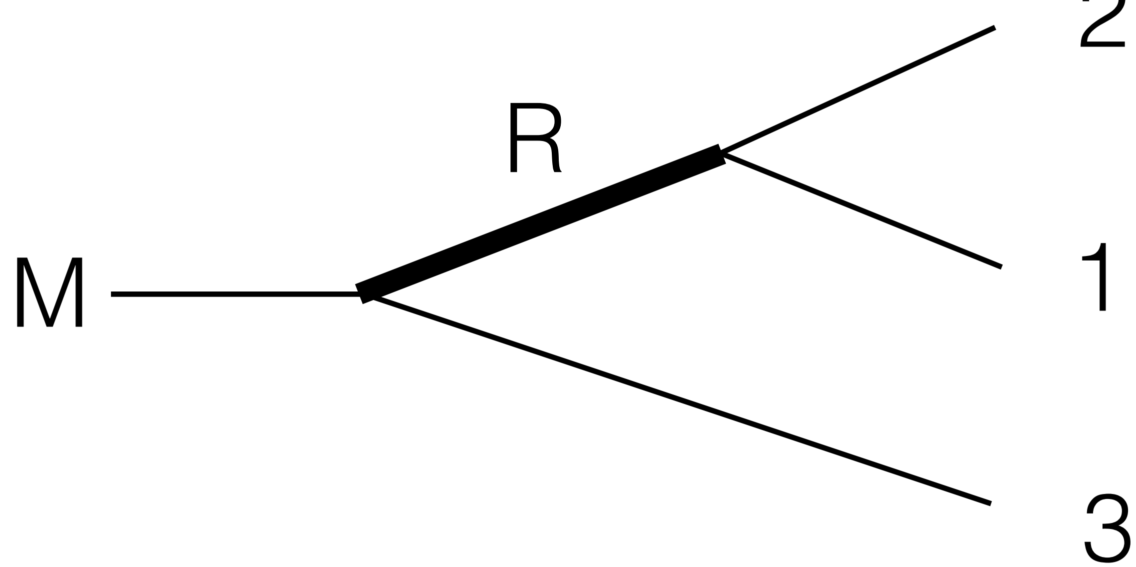
What happens if two things happens

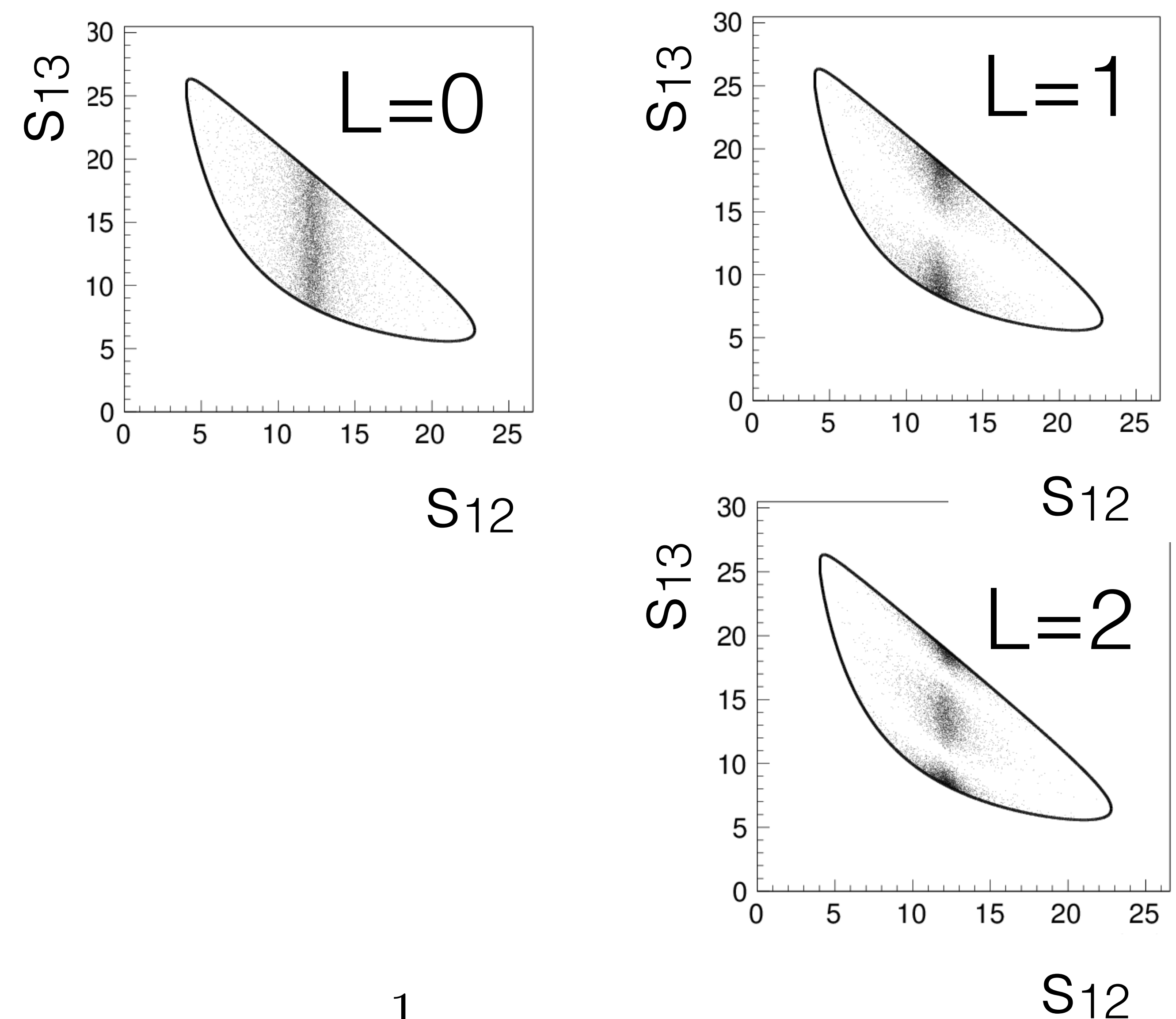




$$d\Gamma = \frac{1}{(2\pi)^2\, 32M^3}|\mathcal{M}_{fi}|^2 ds_{12}ds_{13}$$

What happens if something with spin happens





$$s_{ij} \equiv (p_i + p_j)^2 \equiv m_{ij}^2$$

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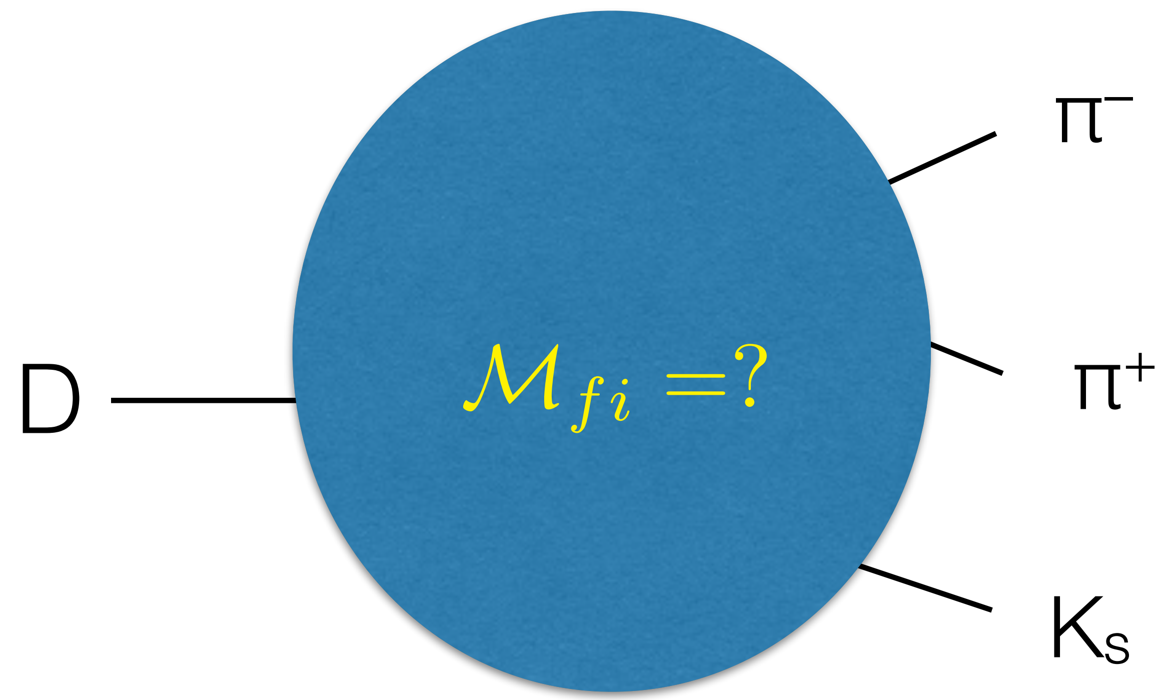
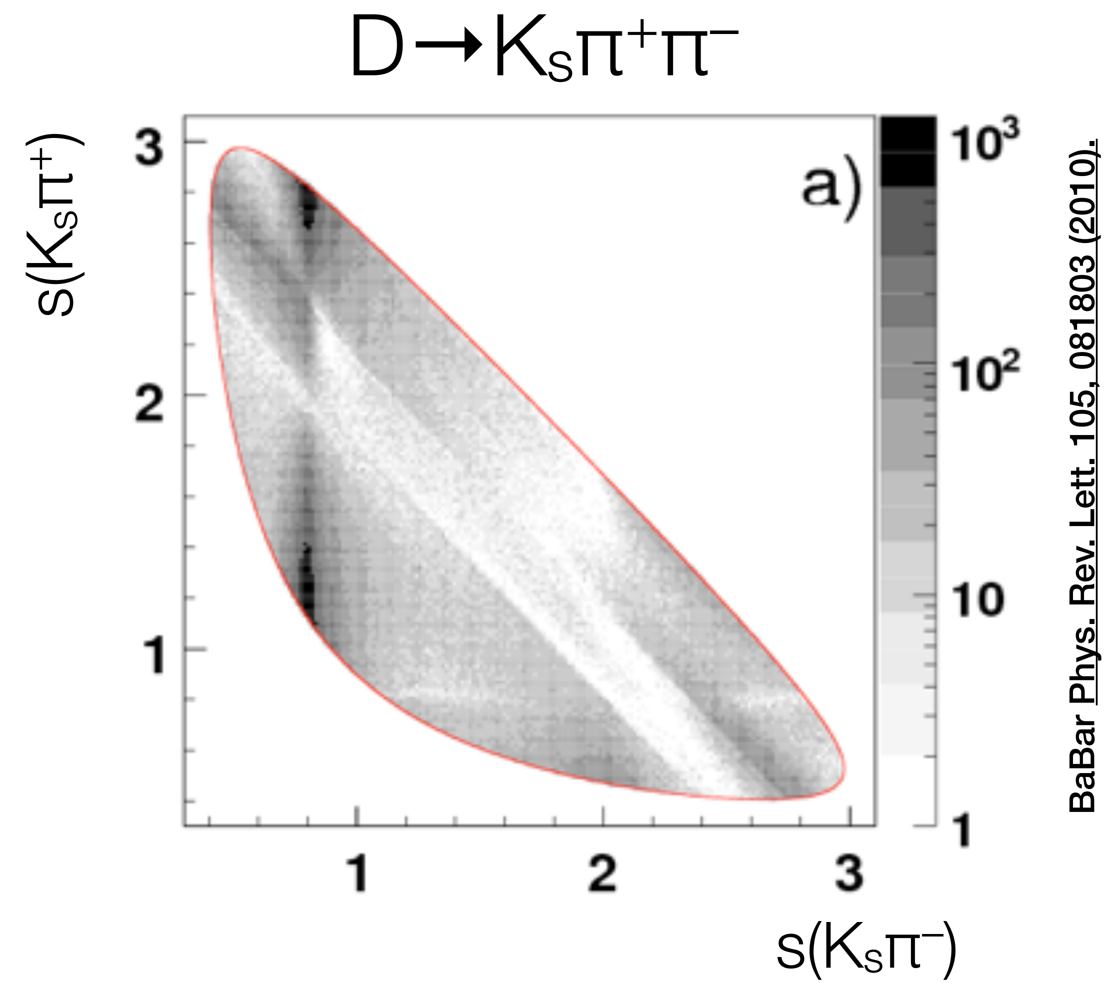
Real dalitz plots





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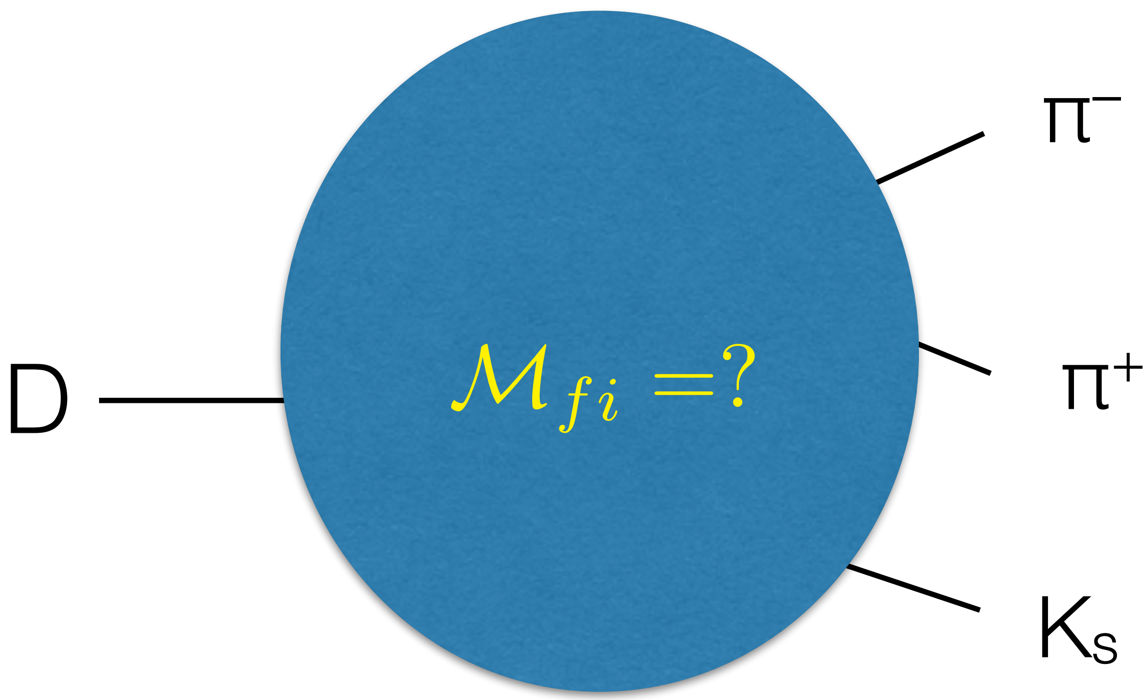
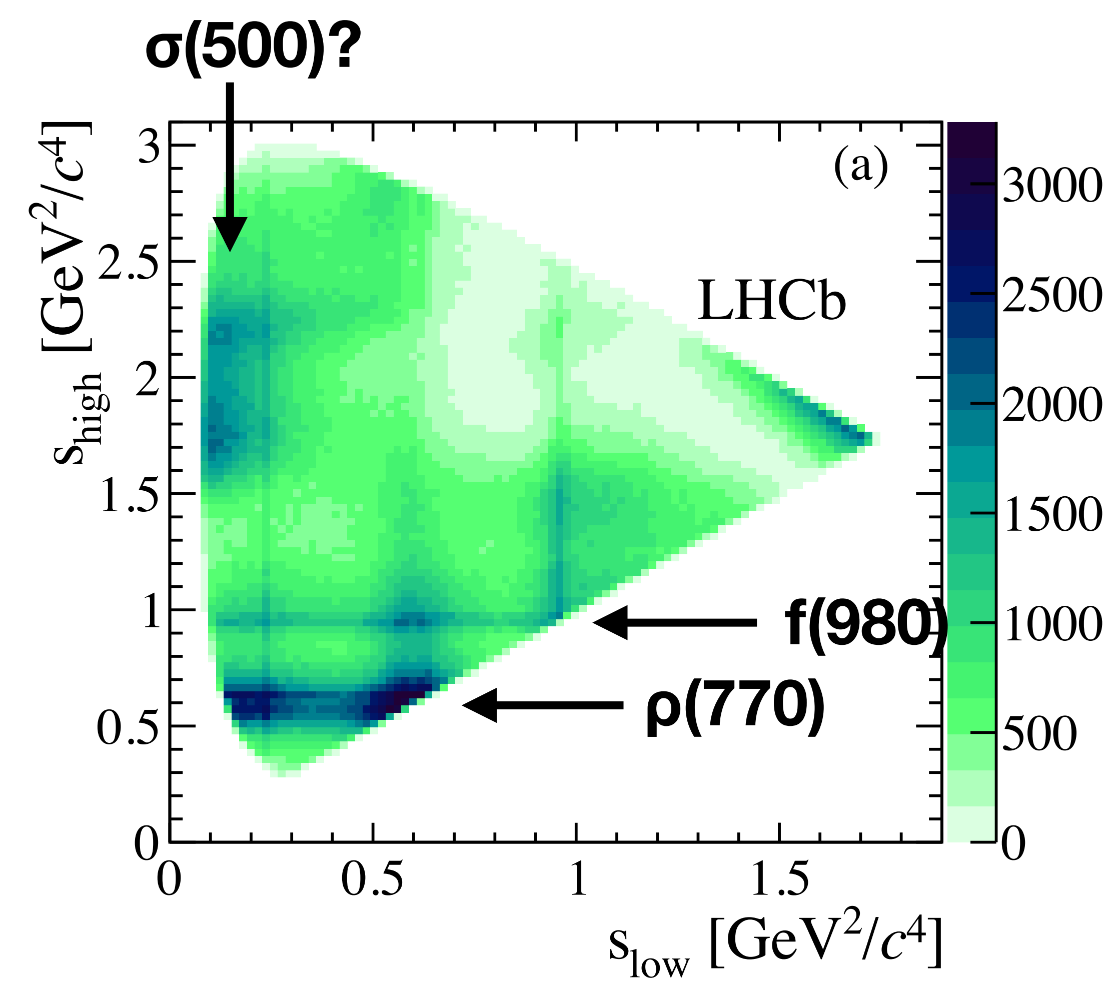
Real Dalitz pots



2.4M $D^\pm \to \pi^\pm \pi^\mp \pi^\pm$ decays (LHCb)



Phys. Lett. B728 (2014) 585

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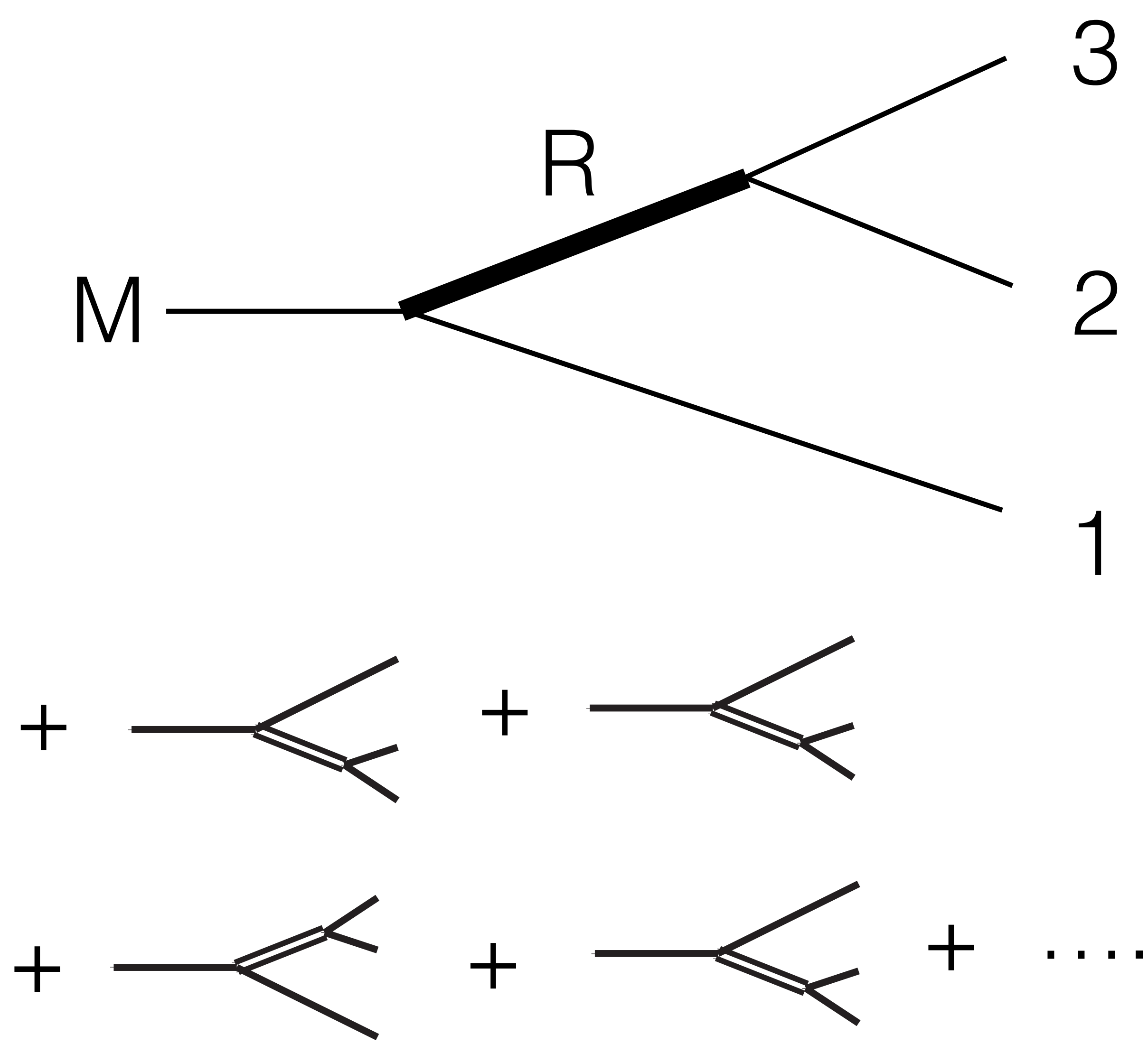
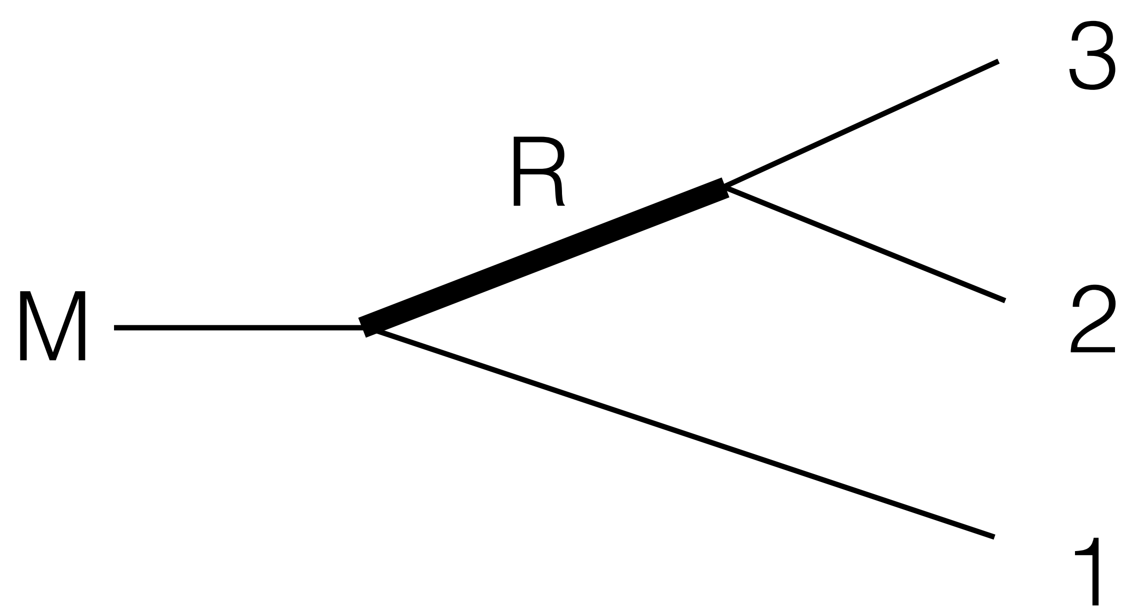
Calculating amplitudes



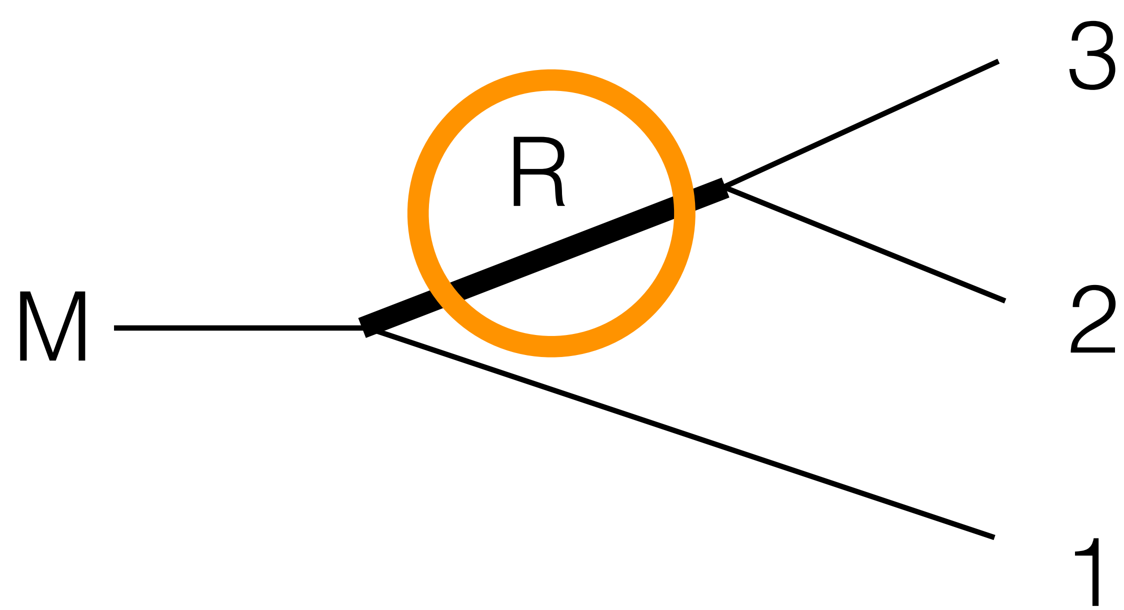
- Let us assume(!) that the full amplitude can be calculated as the sum of essentially independent two body processes.
- Doing this results in the so-called "isobar" model.

Calculating amplitudes

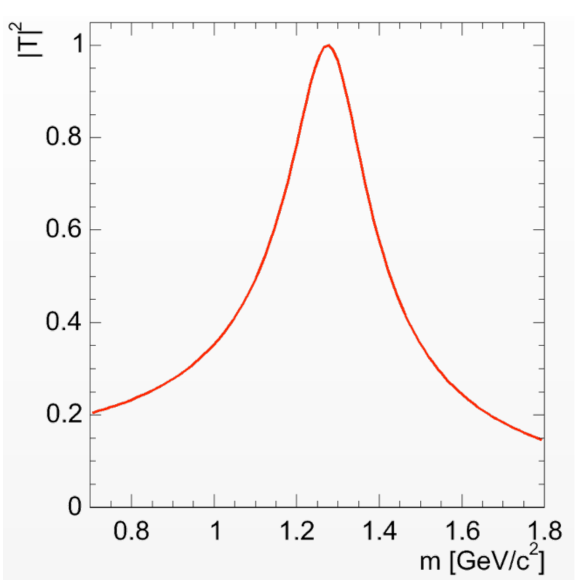


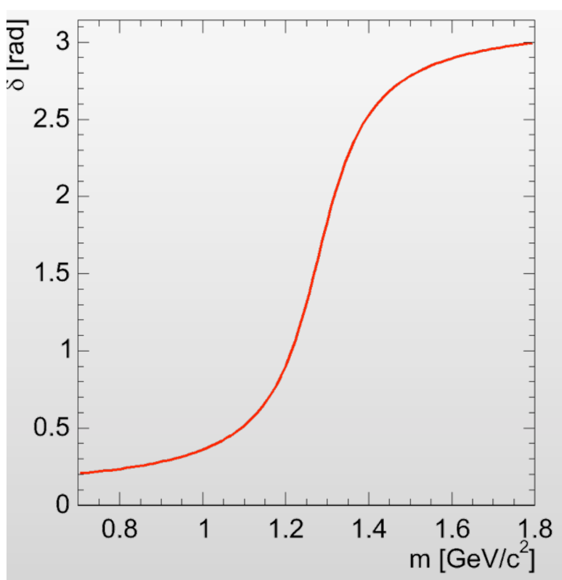
- We don't know anything about the strong interaction dynamics.
- As a first approximation, we treat each particle as point particle.
- We want a Lorentz-invariant matrix element…

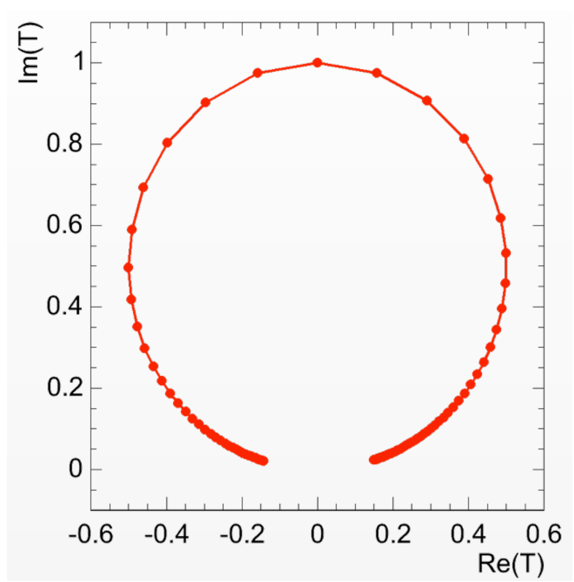
Calculating amplitudes



$$\frac{1}{s_{23} - m_R^2 - i m_R \Gamma}$$





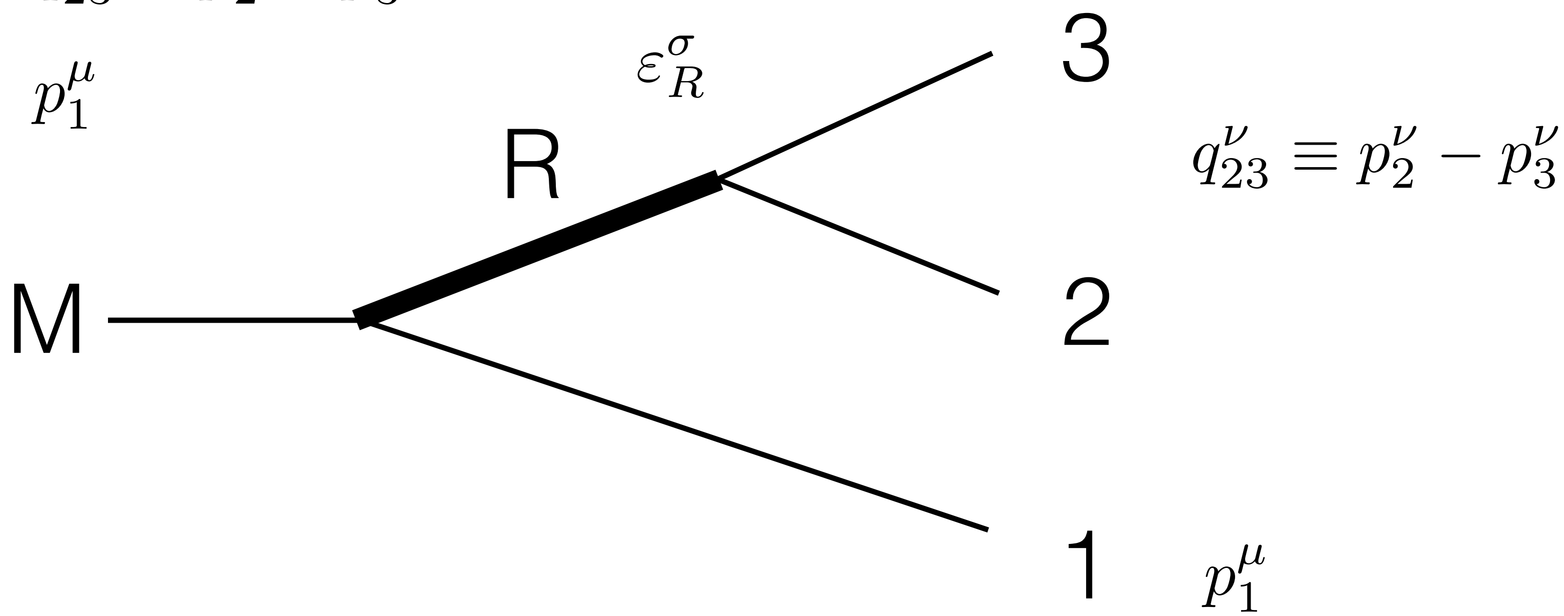


Calculating the amplitudes

ε_R^σ say R has spin 1 (e.g. K*(892), ρ(770) etc)

$q_{23}^\nu \equiv p_2^\nu - p_3^\nu$

p_1^μ



Calculating the amplitudes

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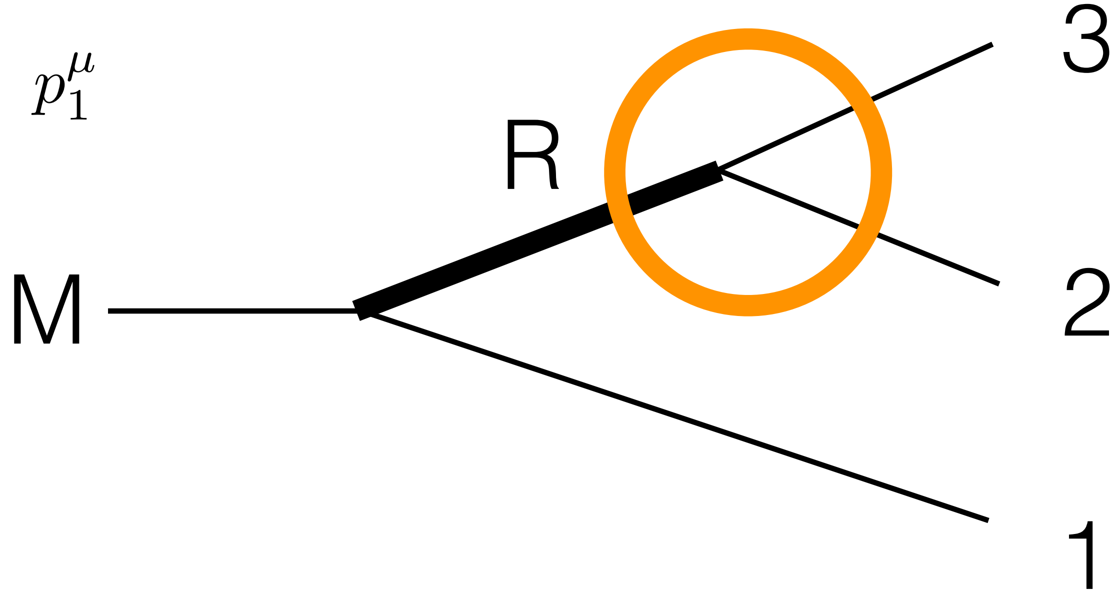

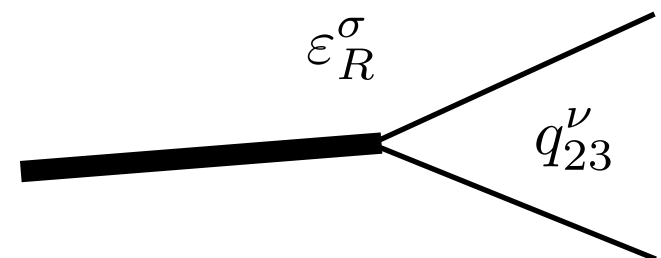

$$\frac{1}{s_{23} - m_R^2 - i m_R \Gamma}$$

Calculating the amplitudes

ε_R^{σ}

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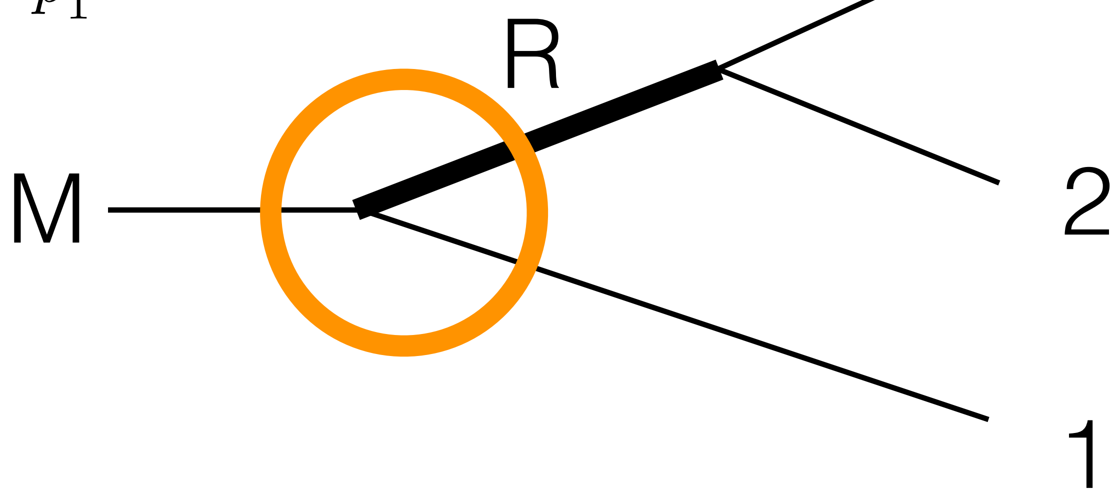
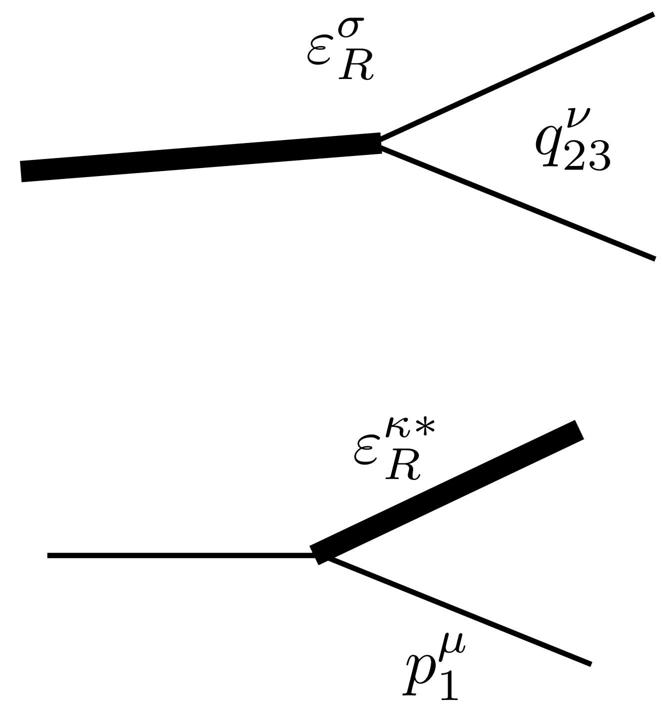
p_1^{μ}





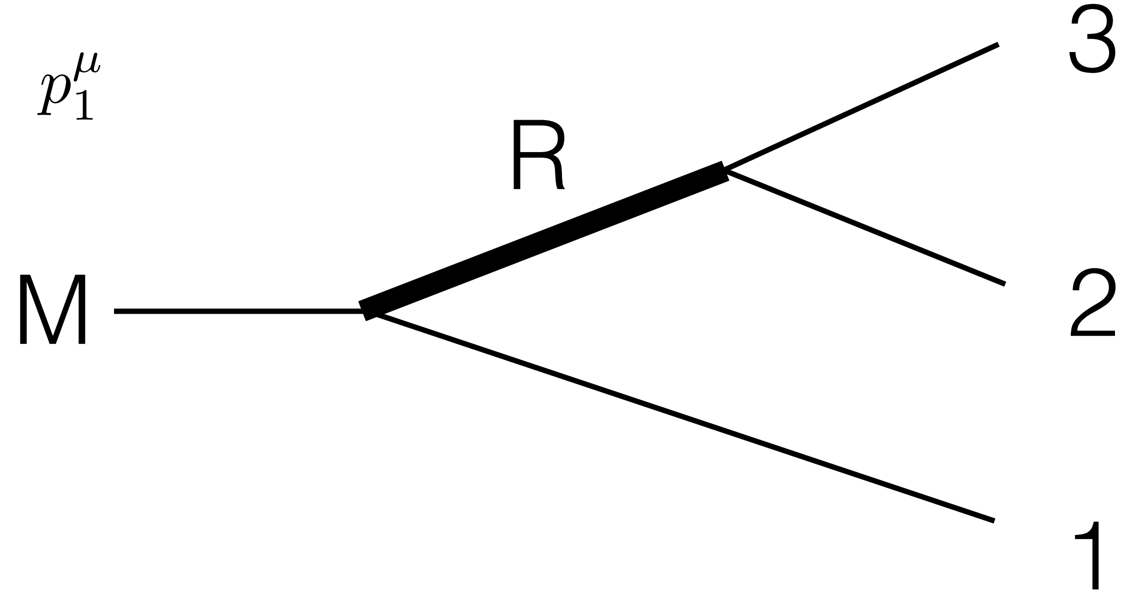
$$p_{1\,\mu} \quad \varepsilon_R^{\mu *} \frac{1}{s_{23} - m_R^2 - i m_R \Gamma} \varepsilon_R^{\nu} \quad q_{23\,\nu}$$

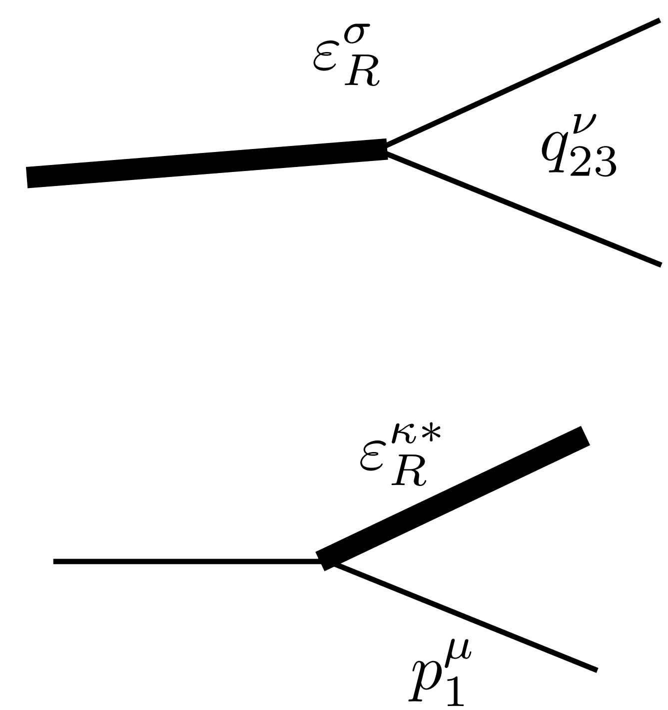
Calculating the amplitudes

ε_R^{σ}

$q_{23}^{\nu} \equiv p_2^{\nu} - p_3^{\nu}$

p_1^{μ}





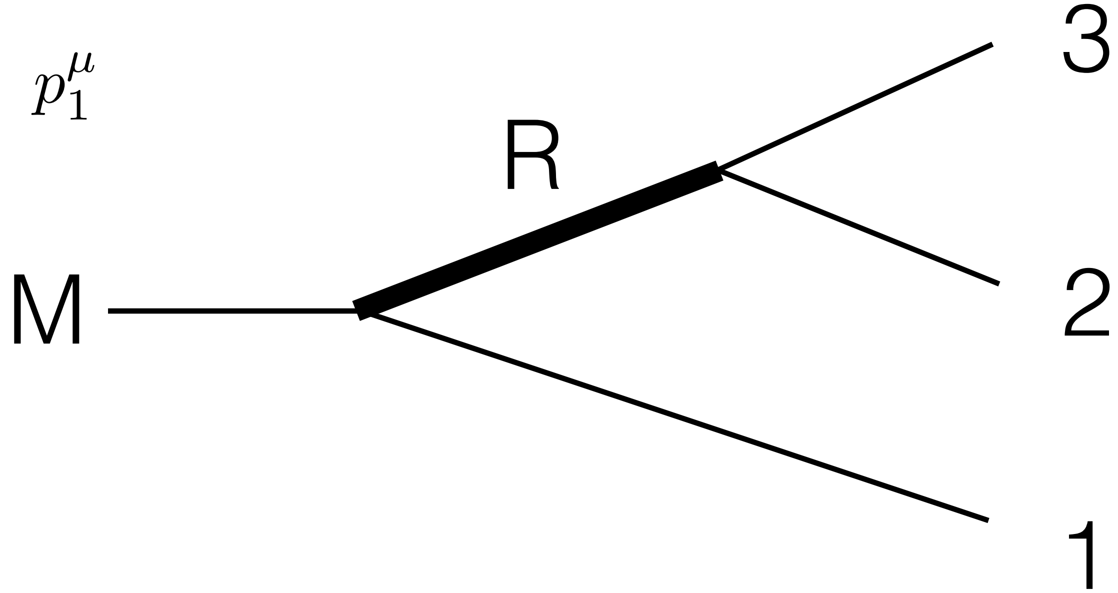
$$\sum_{\text{all } \lambda} \quad p_{1\,\mu} \;\; \varepsilon_R^{\lambda\mu *} \frac{1}{s_{23} - m_R^2 - i m_R \Gamma} \varepsilon_R^{\lambda\nu} \;\; q_{23\,\nu}$$

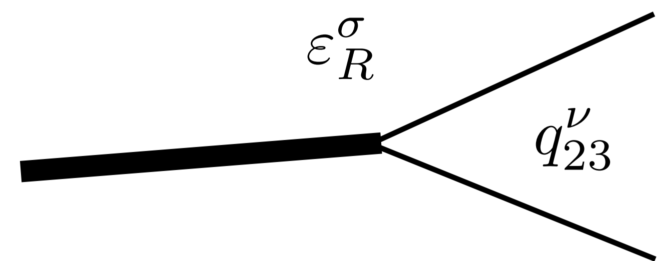
Calculating the amplitudes

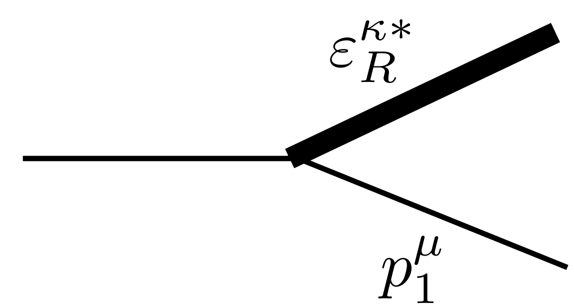
ε_R^σ

$q_{23}^\nu \equiv p_2^\nu - p_3^\nu$

p_1^μ







$$\sum_{\text{all } \lambda} \varepsilon_R^{\lambda\mu *} \varepsilon_R^{\lambda\nu} = -g^{\mu\nu} + \frac{p_R^\mu p_R^\nu}{p_R^2}$$

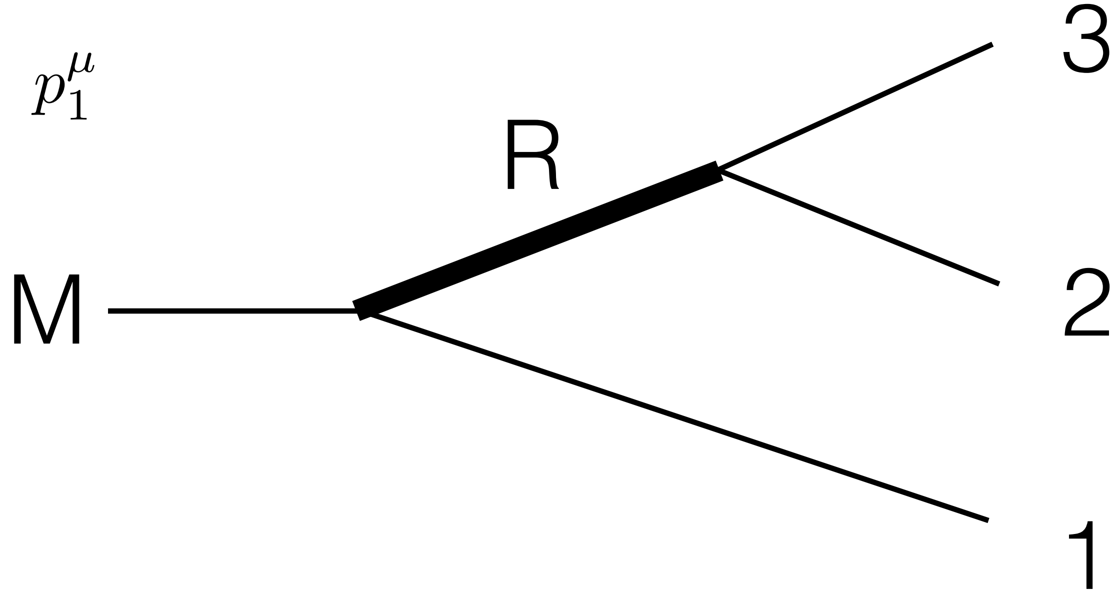
$$\sum_{\text{all } \lambda} \quad p_{1\,\mu} \;\; \varepsilon_R^{\lambda\mu *} \frac{1}{s_{23} - m_R^2 - i m_R \Gamma} \varepsilon_R^{\lambda\nu} \;\; q_{23\,\nu}$$

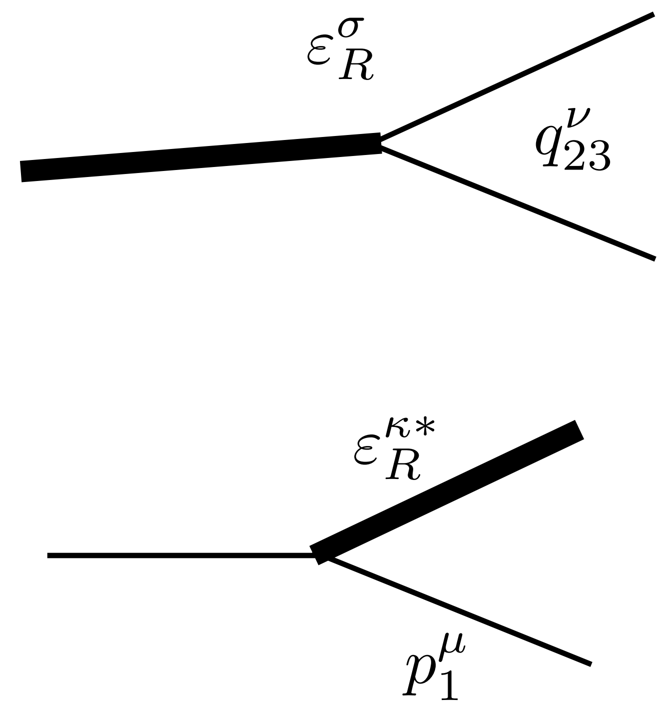
Calculating the amplitudes

ε_R^σ

$q_{23}^\nu \equiv p_2^\nu - p_3^\nu$

p_1^μ





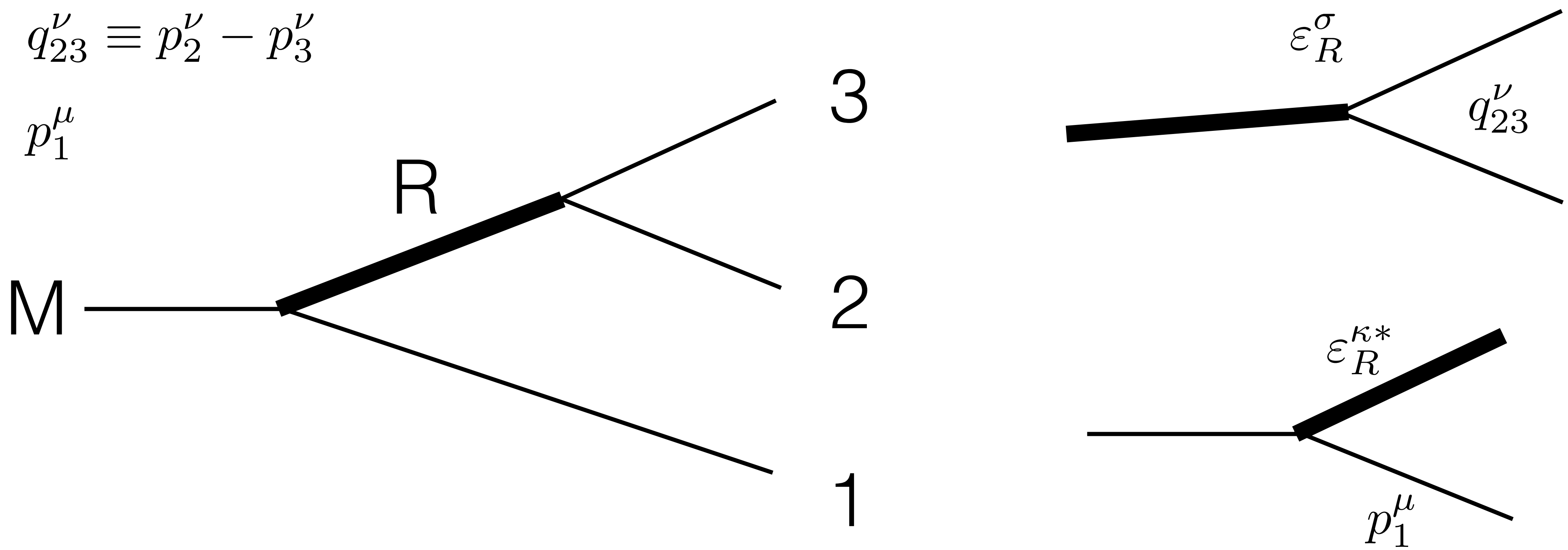
$$p_{1\,\mu} \quad \frac{-g^{\mu\nu} + \frac{p_R^\mu p_R^\nu}{p_R^2}}{s_{23} - m_R^2 - i m_R \Gamma} \quad q_{23\,\nu}$$

Calculating the amplitudes

ε_R^{σ}

$q_{23}^{\nu} \equiv p_2^{\nu} - p_3^{\nu}$

p_1^{μ}



$$p_{1\,\mu}\,\frac{-g^{\mu\nu}+\frac{p_R^{\mu}p_R^{\nu}}{p_R^2}}{s_{23}-m_R^2-i m_R \Gamma}\,q_{23\,\nu}$$

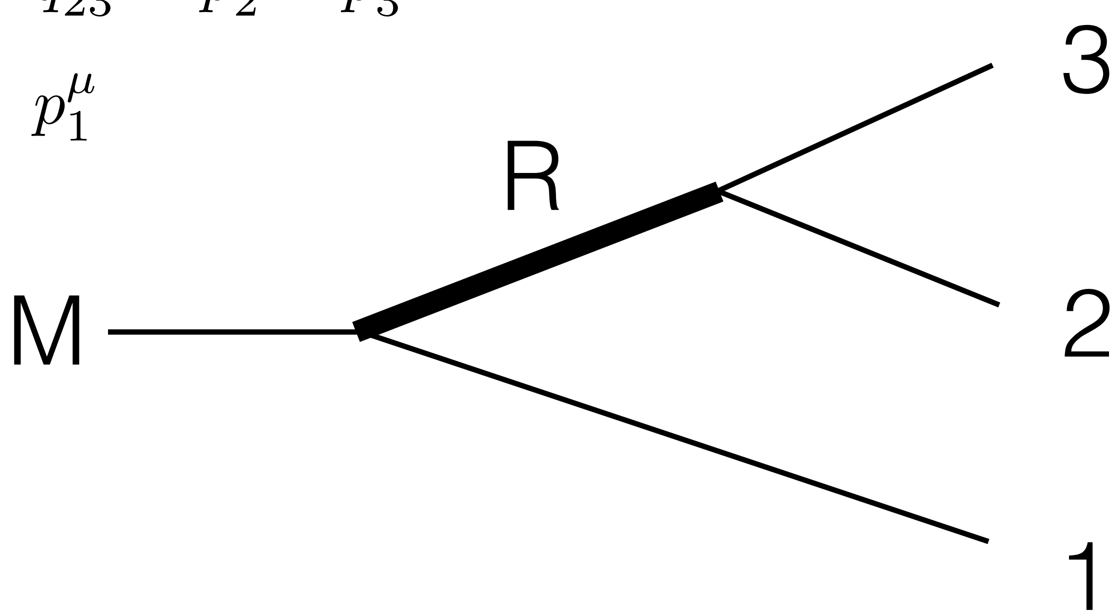
spin factor

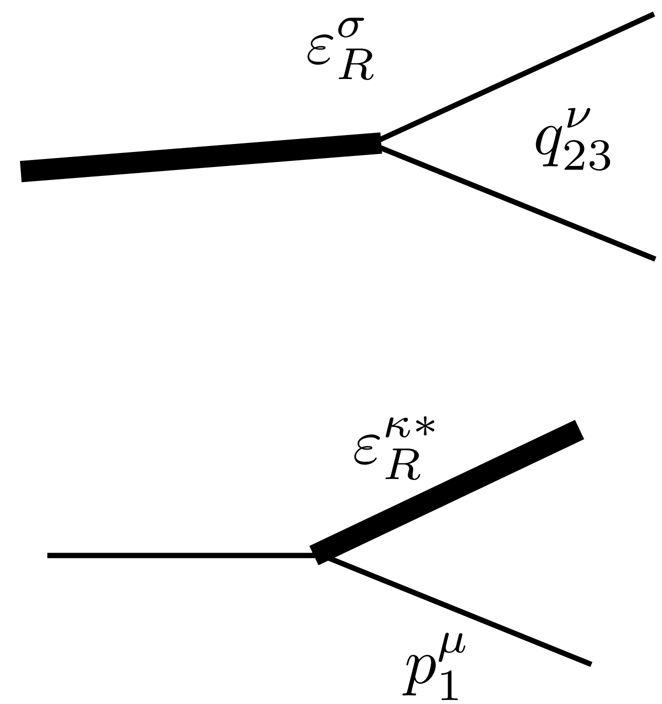
Calculating the amplitudes

ε_R^{σ}

$q_{23}^{\nu} \equiv p_2^{\nu} - p_3^{\nu}$

p_1^{μ}





$$\boxed{\left(-p_1 \cdot q_{23} + \frac{(p_1 \cdot p_R)(q_{23} \cdot p_R)}{p_R^2} \right)} \frac{1}{s_{23} - m_R^2 - i m_R \Gamma}$$

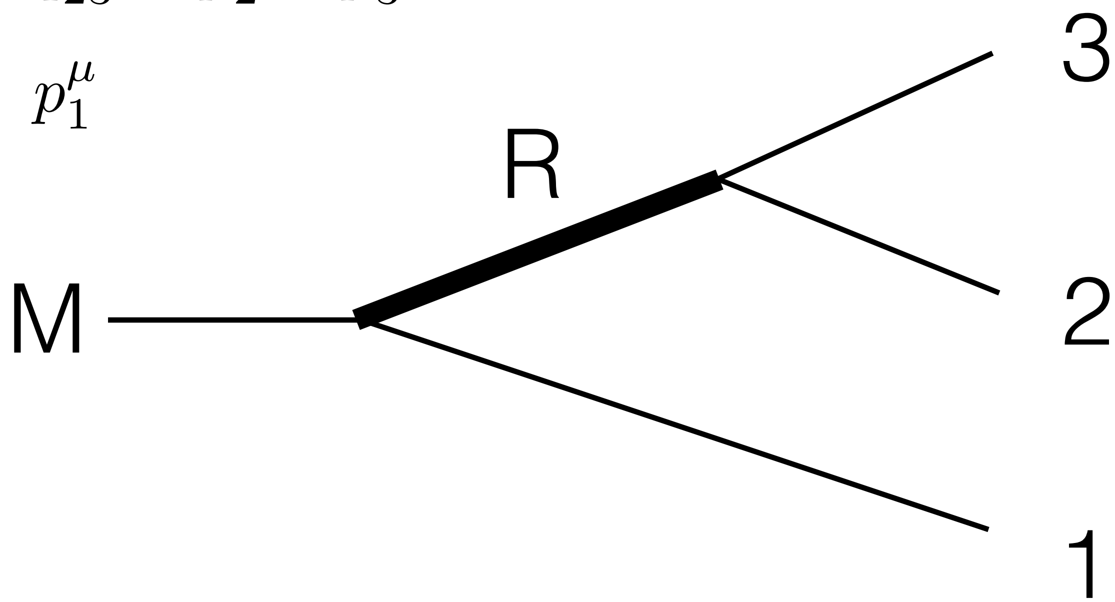
spin factor

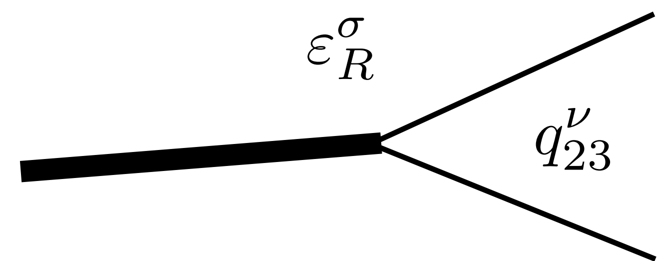
Calculating the amplitudes

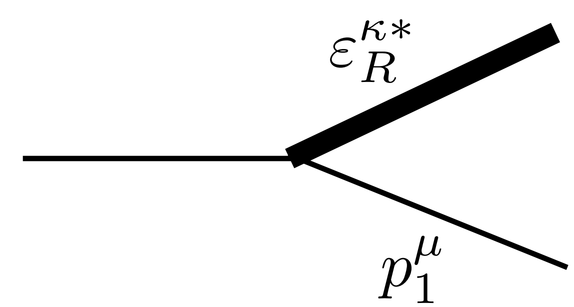
ε_R^{σ}

$q_{23}^{\nu} \equiv p_2^{\nu} - p_3^{\nu}$

p_1^{μ}







Express in terms of s_{ij} if you wish, using $p_i \cdot p_j = s_{ij} - m_i^2 - m_2^2$

$$\boxed{\left(-p_1 \cdot q_{23} + \frac{(p_1 \cdot p_R)(q_{23} \cdot p_R)}{p_R^2}\right)} \frac{1}{s_{23} - m_R^2 - i m_R \Gamma}$$

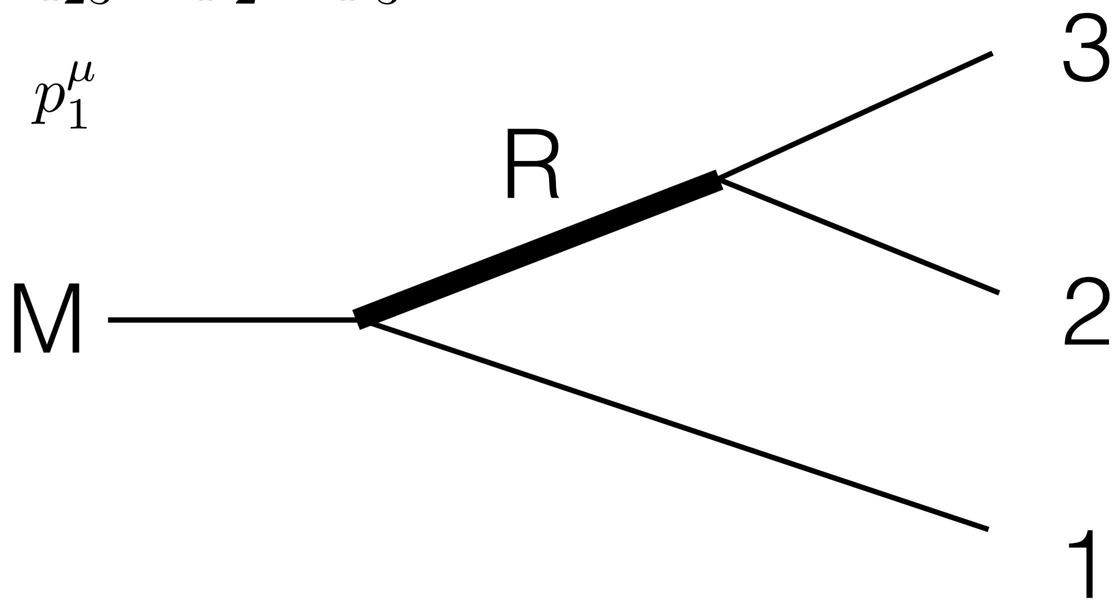
spin factor

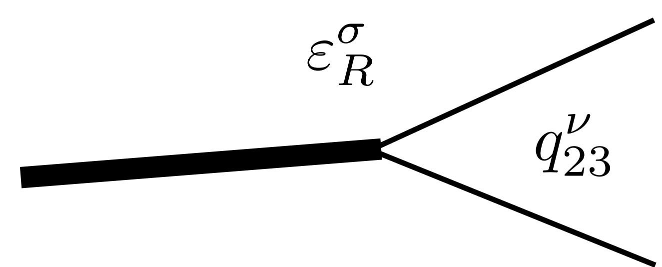
Calculating the amplitudes

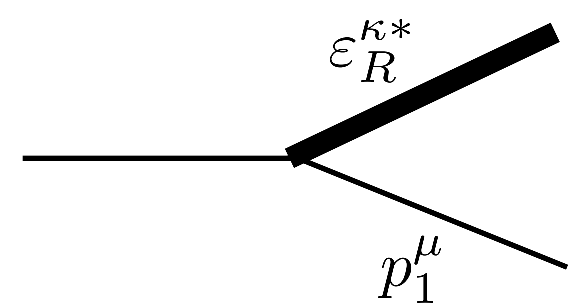
ε_R^{σ}

$q_{23}^{\nu} \equiv p_2^{\nu} - p_3^{\nu}$

p_1^{μ}







$$p_{1\,\mu} \quad \frac{-g^{\mu\nu} + \frac{p_R^{\mu} p_R^{\nu}}{p_R^2}}{s_{23} - m_R^2 - i m_R \Gamma} \quad q_{23\,\nu}$$

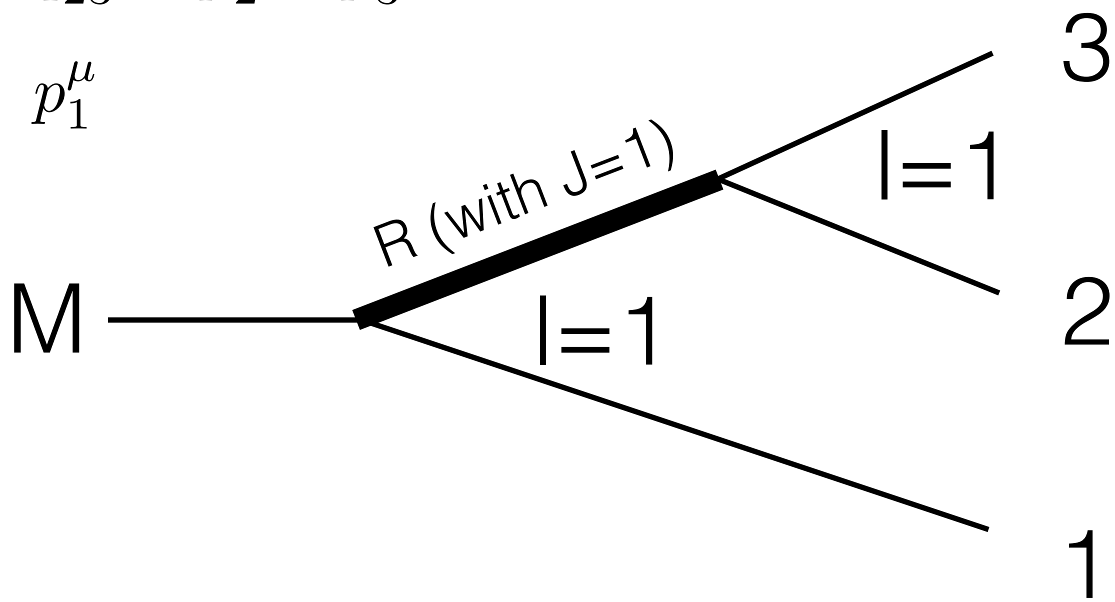
Calculating the amplitudes

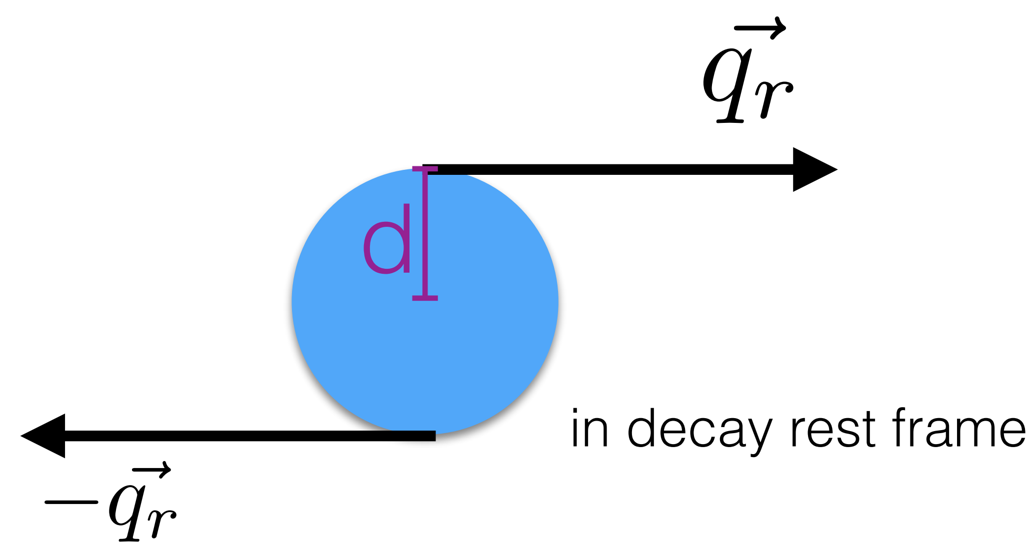
Angular Momenta
require momenta

ε_R^σ

$q_{23}^\nu \equiv p_2^\nu - p_3^\nu$

p_1^μ





$\vec{L} = 2\,\vec{d} \times \vec{q_r}$ classical mechanics

$L = \sqrt{l(l+1)}$ QM

$$p_{1\,\mu} \quad \frac{-g^{\mu\nu} + \frac{p_R^\mu p_R^\nu}{p_R^2}}{s_{23} - m_R^2 - i m_R \Gamma} \quad q_{23\,\nu}$$

Blatt Weisskopf Penetration Factors

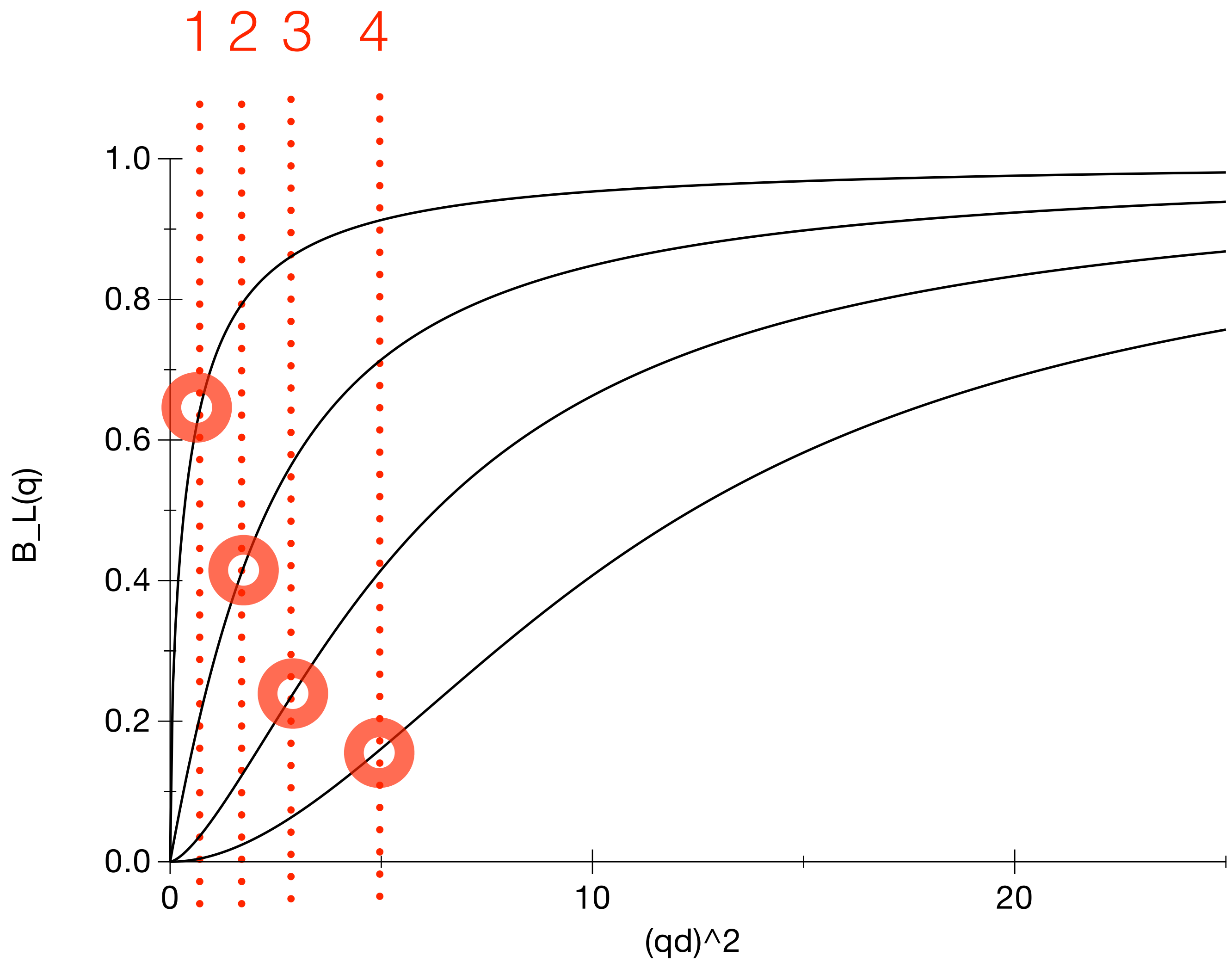
L	$B_L(q)$	$B'_L(q, q_0)$
0	1	1
1	$\sqrt{\frac{2z}{1+z}}$	$\sqrt{\frac{1+z_0}{1+z}}$
2	$\sqrt{\frac{13z^2}{(z-3)^2+9z}}$	$\sqrt{\frac{(z_0-3)^2+9z_0}{(z-3)^2+9z}}$
	where $z = (\|q\|\, d)^2$	and $z_0 = (\|q_0\|\, d)^2$

classical mechanics:
L = 2 qd

QM:
$L^2 = l(l+1)$

Blatt Weisskopf Penetration Factors

$1/4\ L^2 = 1/4\ l(l+1)$ for $l = \ldots$



classical mechanics:
$L = 2\ qd$

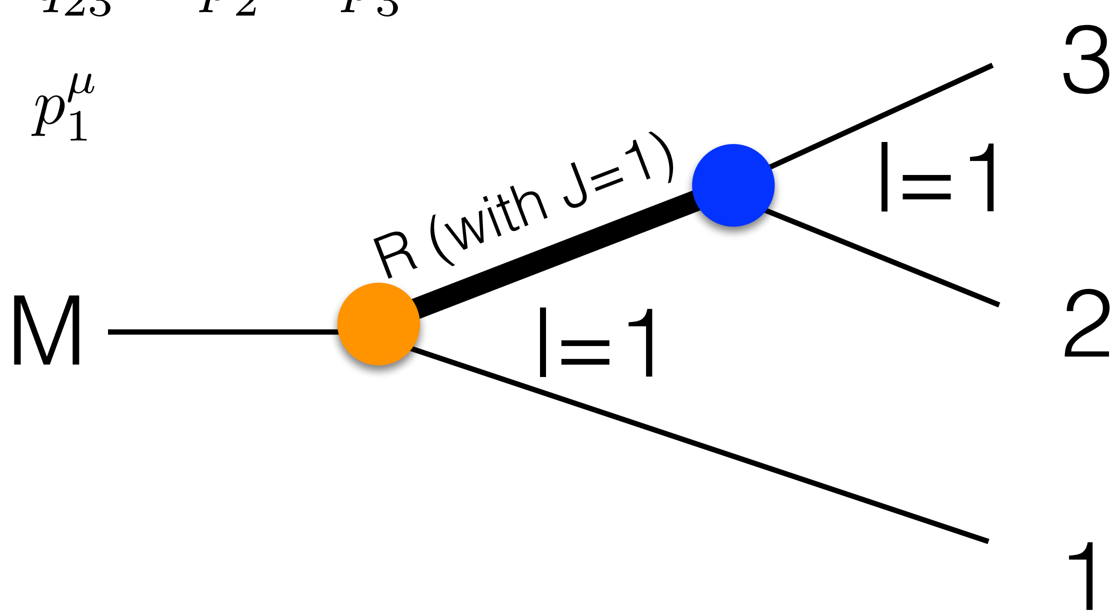
QM:
$L^2 = l(l+1)$

Calculating the amplitudes

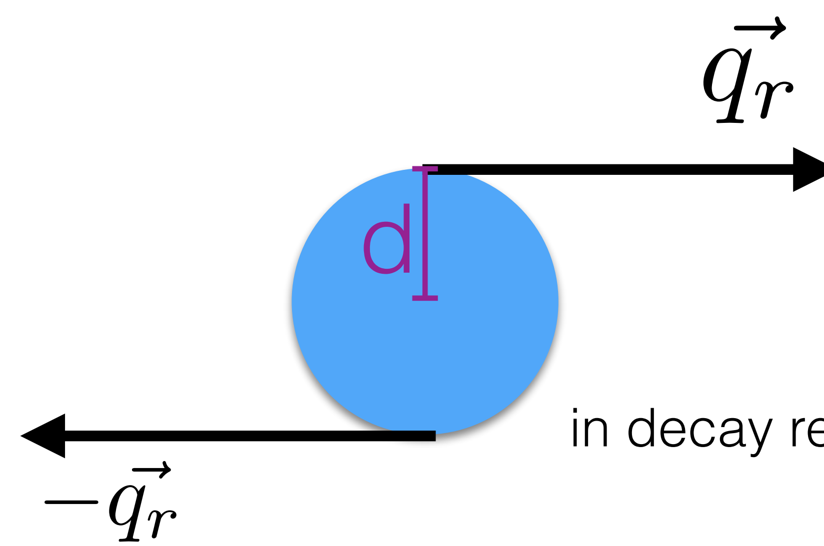
ε_R^{σ}

$q_{23}^{\nu} \equiv p_2^{\nu} - p_3^{\nu}$

p_1^{μ}



Angular Momenta require momenta



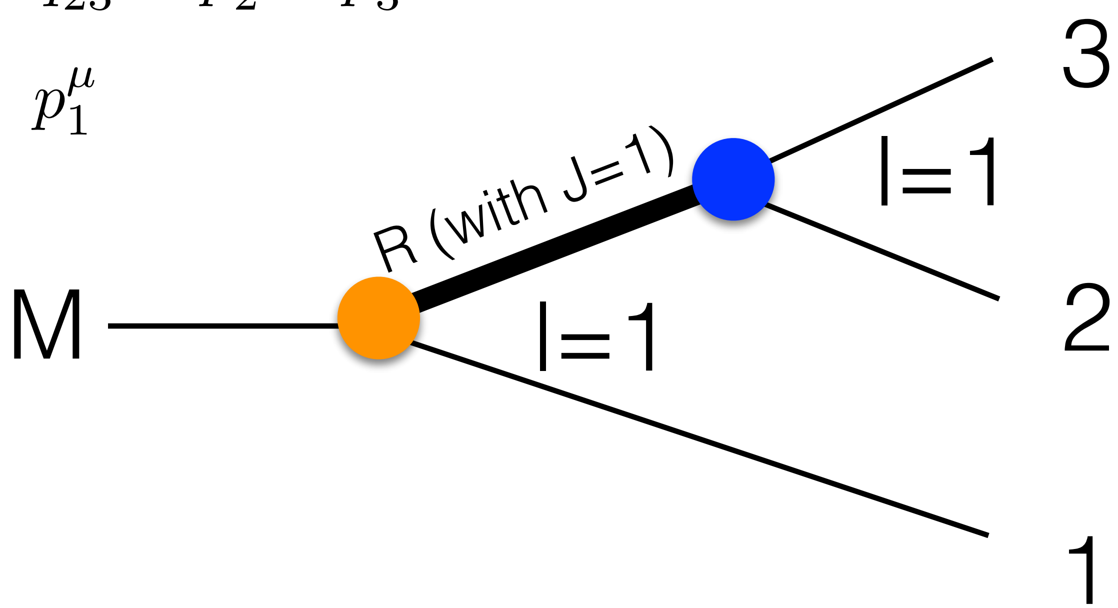
$$p_{1\,\mu}\; B_L(q_{rM}, d_M)\; \frac{-g^{\mu\nu} + \frac{p_R^{\mu} p_R^{\nu}}{p_R^2}}{s_{23} - m_R^2 - i m_R \Gamma}\; B_L(q_{rR}, d_R)\; q_{23\,\nu}$$

Calculating the amplitudes

ε_R^σ

$q_{23}^\nu \equiv p_2^\nu - p_3^\nu$

p_1^μ



- Width Γ = rate, depends on phase space = 2q/m.
  break-up momentum
- Rate also depends on B_L.

$$p_{1\,\mu}\; B_L(q_{rM}, d_M)\; \frac{-g^{\mu\nu} + \frac{p_R^\mu p_R^\nu}{p_R^2}}{s_{23} - m_R^2 - i m_R \Gamma}\; B_L(q_{rR}, d_R)\; q_{23\,\nu}$$

Calculating the amplitudes

ε_R^{σ}

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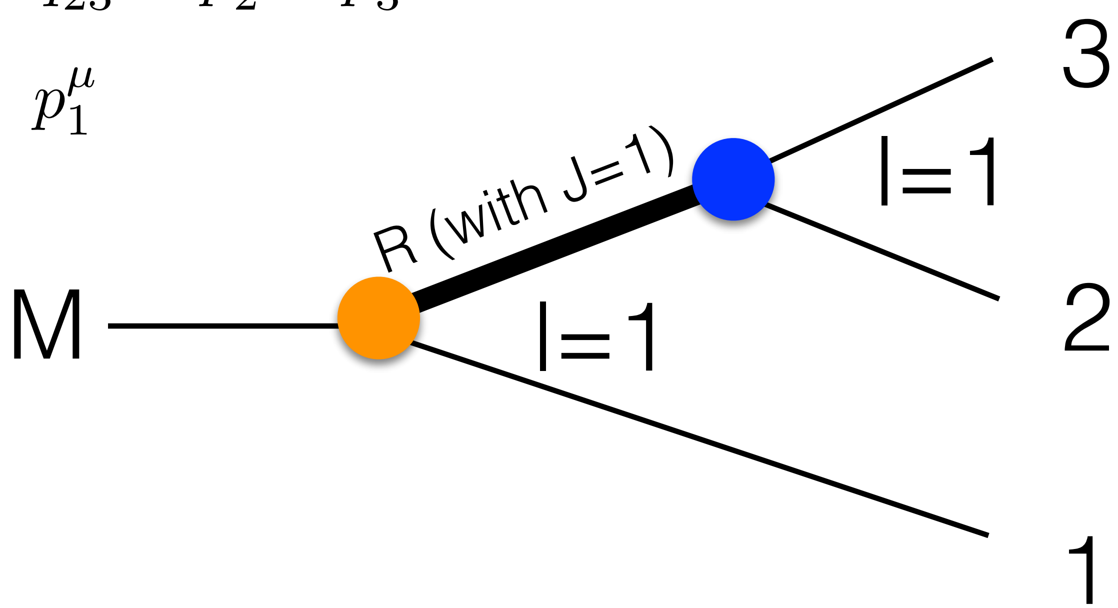

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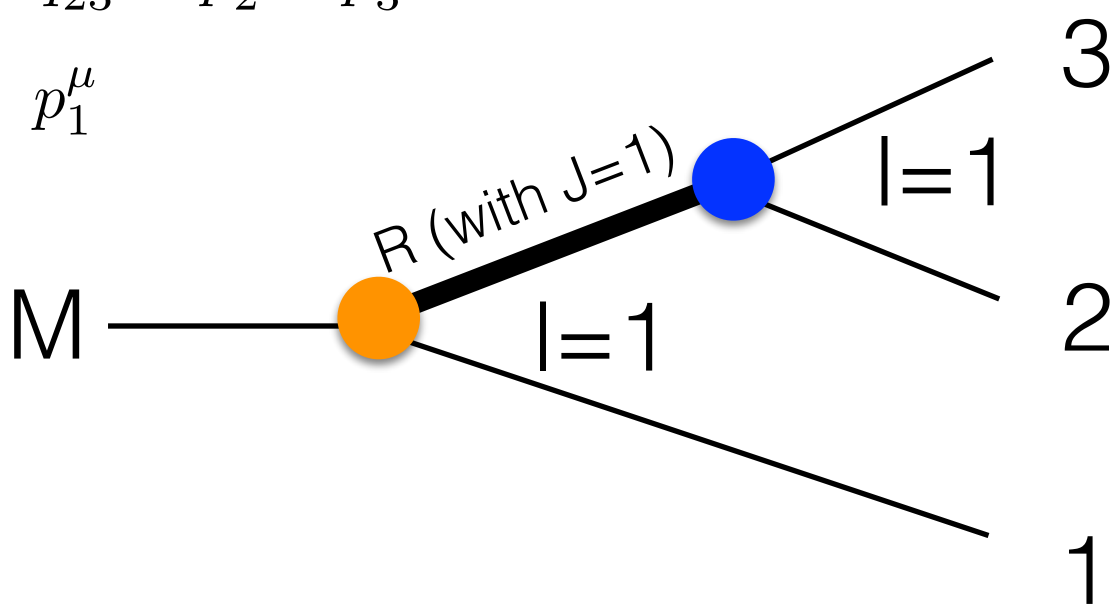
Calculating the amplitudes

ε_R^σ

$q_{23}^\nu \equiv p_2^\nu - p_3^\nu$

p_1^μ



- Width $\color{red}{\Gamma}$ = rate, depends on phase space = 2q/m.

  break-up momentum

- Rate also depends on $\color{blue}{B_L}$.

$$\color{red}{\Gamma(m_{23})} = \Gamma_0 \frac{(q_{23}/m_{23})\, B_L(q_{23})}{(q_0/m_R)\;\; B_L(q_0)}$$

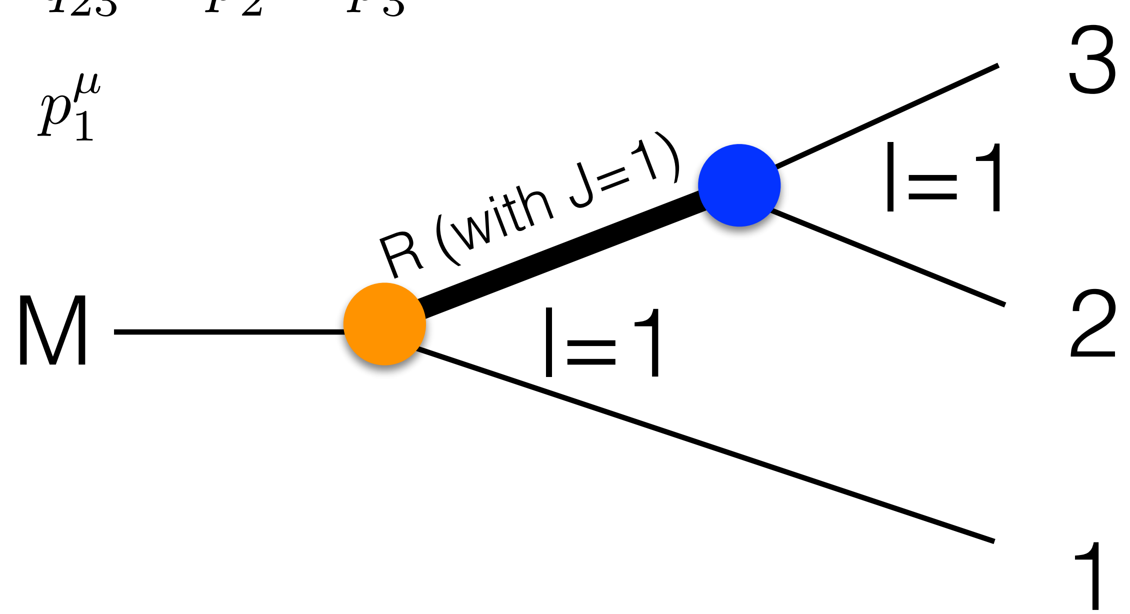
$$p_{1\,\mu}\, \color{orange}{B_L(q_{rM}, d_M)}\, \frac{-g^{\mu\nu} + \frac{p_R^\mu p_R^\nu}{p_R^2}}{s_{23} - m_R^2 - i m_R \color{red}{\Gamma(m_{23})}}\, \color{blue}{B_L(q_{rR}, d_R)}\, q_{23\,\nu}$$

Calculating the amplitudes

ε_R^σ

$q_{23}^\nu \equiv p_2^\nu - p_3^\nu$

p_1^μ



- Width Γ = rate, depends on phase space = 2q/m.

break-up momentum

- Rate also depends on B_L.

reconstructed mass $m_{23} \equiv \sqrt{s_{23}}$

break-up momentum in restframe of decaying resonance

$$\Gamma(m_{23}) = \Gamma_0 \frac{(q_{23}/m_{23})\, B_L(q_{23})}{(q_0/m_R)\;\, B_L(q_0)}$$

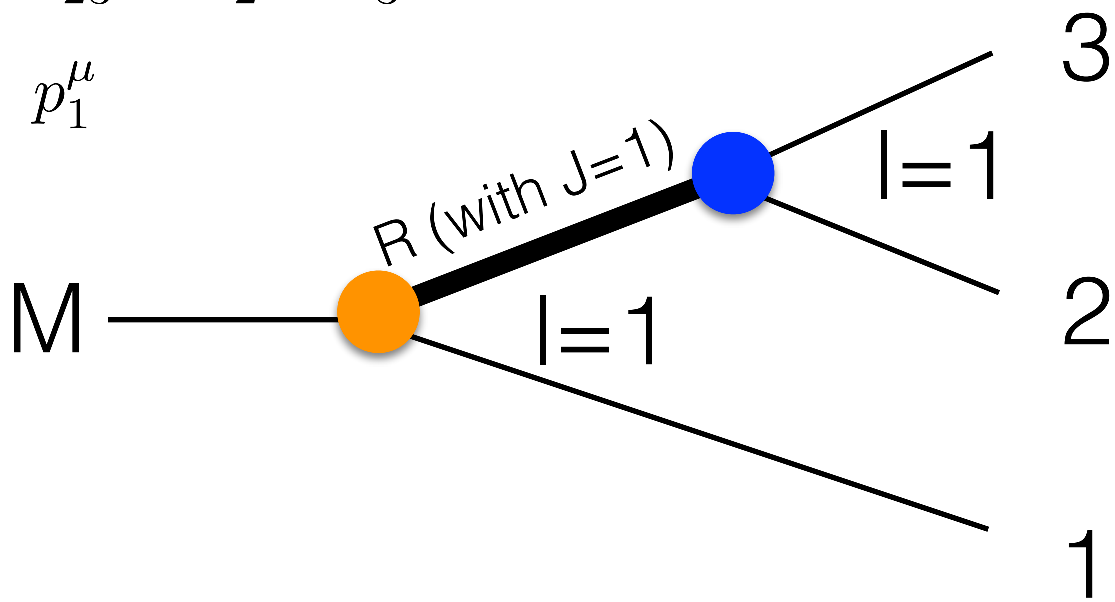
$$p_{1\,\mu}\; B_L(q_{rM}, d_M)\; \frac{-g^{\mu\nu} + \frac{p_R^\mu p_R^\nu}{p_R^2}}{s_{23} - m_R^2 - i m_R \Gamma(m_{23})}\; B_L(q_{rR}, d_R)\; q_{23\,\nu}$$

Calculating the amplitudes

ε_R^σ

$q_{23}^\nu \equiv p_2^\nu - p_3^\nu$

p_1^μ



- Width Γ = rate, depends on phase space = 2q/m.

break-up momentum

- Rate also depends on B_L.

reconstructed mass $m_{23} \equiv \sqrt{s_{23}}$

centrifugal barrier factor

break-up momentum in restframe of decaying resonance

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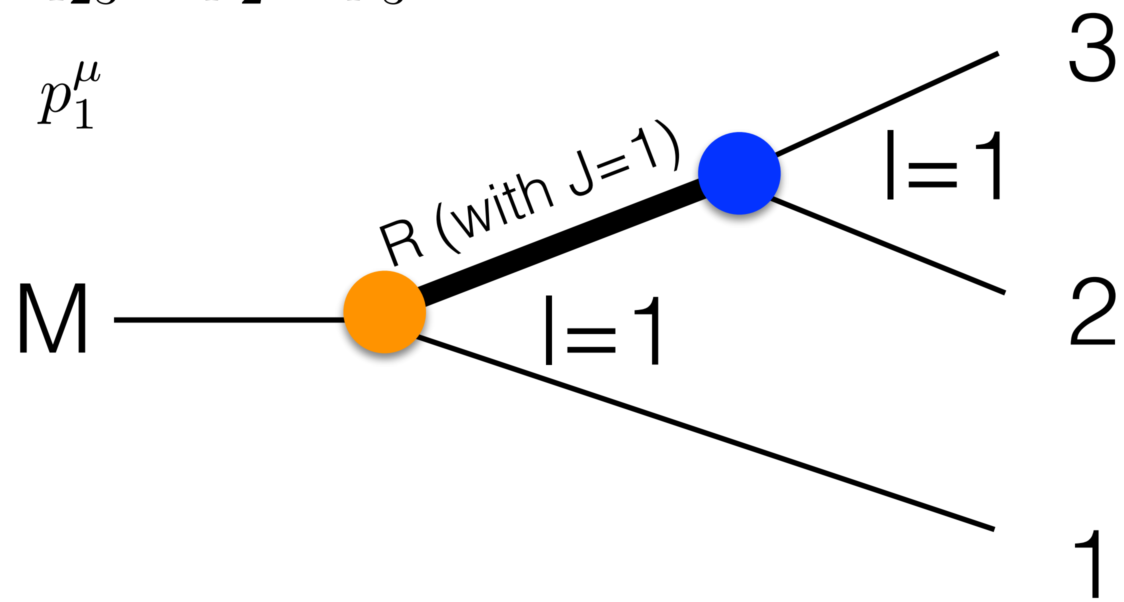
$$p_{1\,\mu}\ B_L(q_{rM}, d_M)\ \frac{-g^{\mu\nu} + \frac{p_R^\mu p_R^\nu}{p_R^2}}{s_{23} - m_R^2 - i m_R \Gamma(m_{23})}\ B_L(q_{rR}, d_R)\ q_{23\,\nu}$$

Calculating the amplitudes

ε_R^σ

$q_{23}^\nu \equiv p_2^\nu - p_3^\nu$

p_1^μ



- Width Γ = rate, depends on phase space = 2q/m.

break-up momentum

- Rate also depends on B_L.

reconstructed mass $m_{23} \equiv \sqrt{s_{23}}$

centrifugal barrier factor

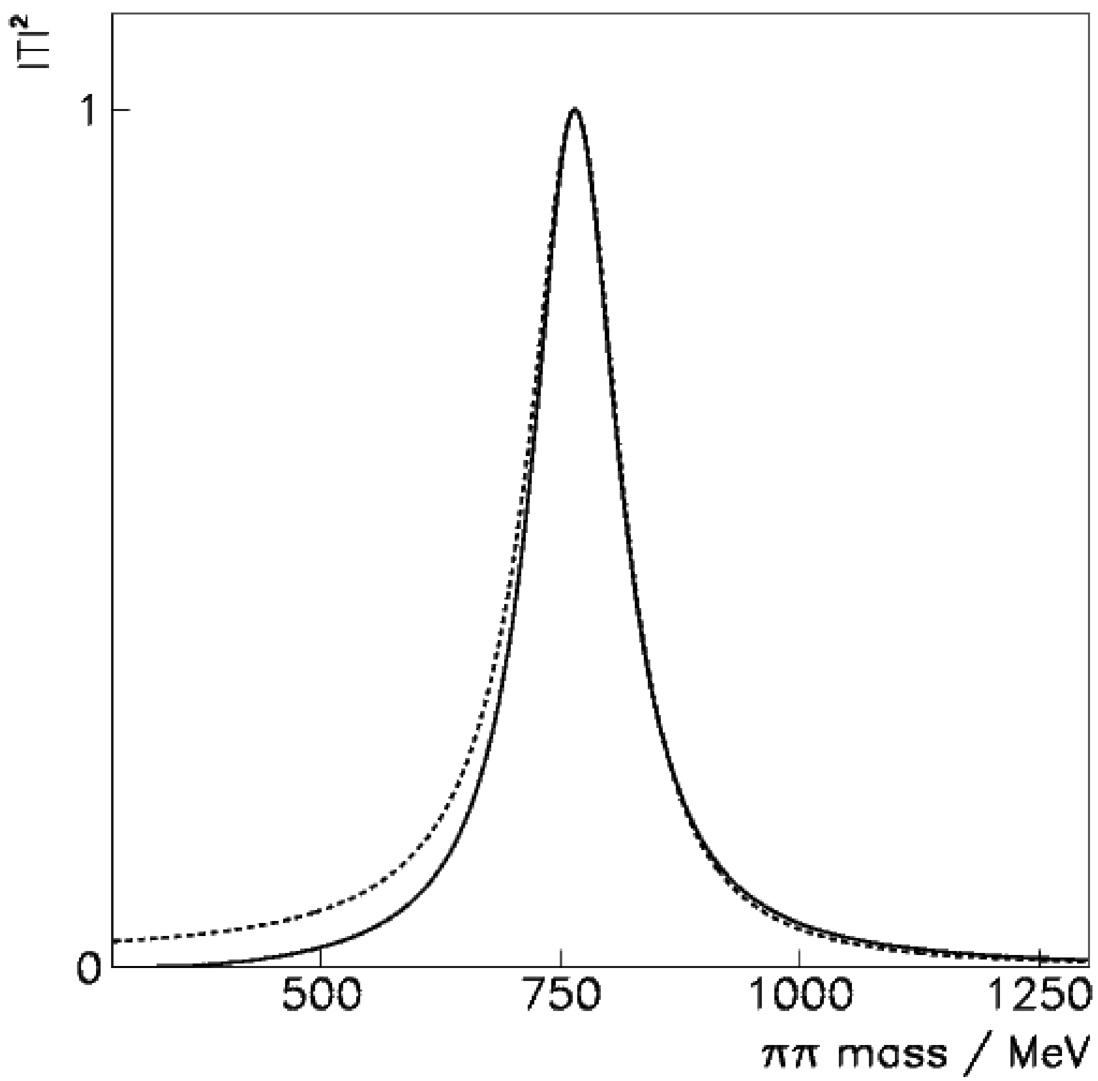
break-up momentum in restframe of decaying resonance

the same as numerator, but calculated for “nominal” (peak) resonance mass.

$$\Gamma(m_{23}) = \Gamma_0 \frac{(q_{23}/m_{23})\; B_L(q_{23})}{(q_0/m_R)\;\; B_L(q_0)}$$

$$p_{1\,\mu}\; B_L(q_{rM}, d_M)\; \frac{-g^{\mu\nu} + \frac{p_R^\mu p_R^\nu}{p_R^2}}{s_{23} - m_R^2 - i m_R \Gamma(m_{23})}\; B_L(q_{rR}, d_R)\; q_{23\,\nu}$$

Mass dependent width (ignoring ang. mom)

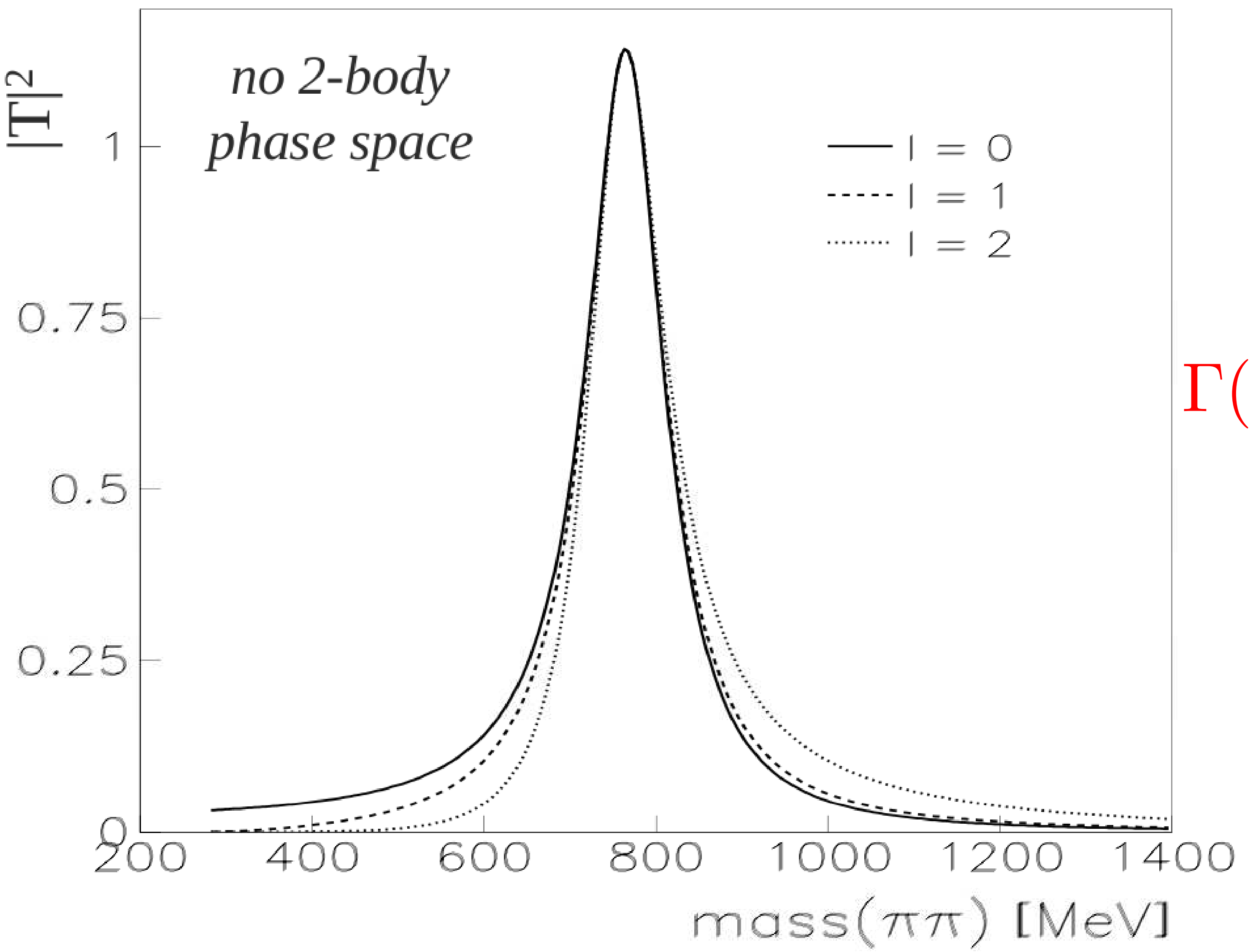


dashed: fixed width

solid: mass dependent width

$$\Gamma(m_{23}) = \Gamma_0 \frac{(q_{23}/m_{23})}{(q_0/m_R)} \frac{B_L(q_{23})}{B_L(q_0)}$$

Breit Wigner with angular momentum effects (only)



$$\Gamma(m_{23}) = \Gamma_0 \frac{(q_{23}/m_{23})}{(q_0/m_R)} \frac{B_L(q_{23})}{B_L(q_0)}$$

Amplitude Model

$$A_R = p_{1\,\mu}\, B_L(q_{rM}, d_M)\, \frac{-g^{\mu\nu} + \frac{p_R^\mu p_R^\nu}{p_R^2}}{s_{23} - m_R^2 - i m_R \Gamma(m_{23})}\, B_L(q_{rR}, d_R)\, q_{23\,\nu}$$

$$\mathcal{M}_{fi} = \sum_R c_R e^{i\theta_R} A_R(s_{12}, s_{23})$$

sensitivity to phases is one of the key reasons amplitude analyses are so interesting.

$$P(s_{12}, s_{23}) = \frac{|\mathcal{M}_{fi}|^2 \left|\frac{d\Phi}{ds_{12}\, ds_{23}}\right|}{\int |\mathcal{M}_{fi}|^2 \left|\frac{d\Phi}{ds_{12}\, ds_{23}}\right| ds_{12}\, ds_{23}}$$

$$= \frac{|\mathcal{M}_{fi}|^2}{\int\limits_{\text{within kin boundary}} |\mathcal{M}_{fi}|^2\, ds_{12}\, ds_{23}}$$

Fitters frequently used at LHCb:
MINT (esp for >3 body)
Laura++
GooFit-based fitter
and others.

Amplitude Model

$$\mathcal{M}_{fi} = \sum_R c_R e^{i\theta_R} A_R(s_{12}, s_{23})$$

example: **CDF: PHYSICAL REVIEW D 86, 032007 (2012)**

Resonance	a	δ [°]	Fit fractions [%]
$K^*(892)^\pm$	1.911 ± 0.012	132.1 ± 0.7	61.80 ± 0.31
$K_0^*(1430)^\pm$	2.093 ± 0.065	54.2 ± 1.9	6.25 ± 0.25
$K_2^*(1430)^\pm$	0.986 ± 0.034	308.6 ± 2.1	1.28 ± 0.08
$K^*(1410)^\pm$	1.092 ± 0.069	155.9 ± 2.8	1.07 ± 0.10
$\rho(770)$	1	0	18.85 ± 0.18
$\omega(782)$	0.038 ± 0.002	107.9 ± 2.3	0.46 ± 0.05
$f_0(980)$	0.476 ± 0.016	182.8 ± 1.3	4.91 ± 0.19
$f_2(1270)$	1.713 ± 0.048	329.9 ± 1.6	1.95 ± 0.10
$f_0(1370)$	0.342 ± 0.021	109.3 ± 3.1	0.57 ± 0.05
$\rho(1450)$	0.709 ± 0.043	8.7 ± 2.7	0.41 ± 0.04
$f_0(600)$	1.134 ± 0.041	201.0 ± 2.9	7.02 ± 0.30
σ_2	0.282 ± 0.023	16.2 ± 9.0	0.33 ± 0.04
$K^*(892)^\pm$(DCS)	0.137 ± 0.007	317.6 ± 2.8	0.32 ± 0.03
$K_0^*(1430)^\pm$(DCS)	0.439 ± 0.035	156.1 ± 4.9	0.28 ± 0.04
$K_2^*(1430)^\pm$(DCS)	0.291 ± 0.034	213.5 ± 6.1	0.11 ± 0.03
Nonresonant	1.797 ± 0.147	94.0 ± 5.3	1.64 ± 0.27
Sum			107.25 ± 0.65

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Amplitude Model

$$\mathcal{M}_{fi} = \sum_R c_R e^{i\theta_R} A_R(s_{12}, s_{23}) \ + a_0 e^{i\theta_0}$$

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Amplitude Model

$$\mathcal{M}_{fi} = \sum_R c_R e^{i\theta_R} A_R(s_{12}, s_{23}) \ + a_0 e^{i\theta_0}$$

$$FF_R = \frac{\int \left| c_R e^{i\theta_R} A_R(s_{12}, s_{23}) \right|^2 ds_{12} ds_{23}}{\int \left| \sum_j c_j e^{i\theta_j} A_j(s_{12}, s_{23}) \right|^2 ds_{12} ds_{23}}$$

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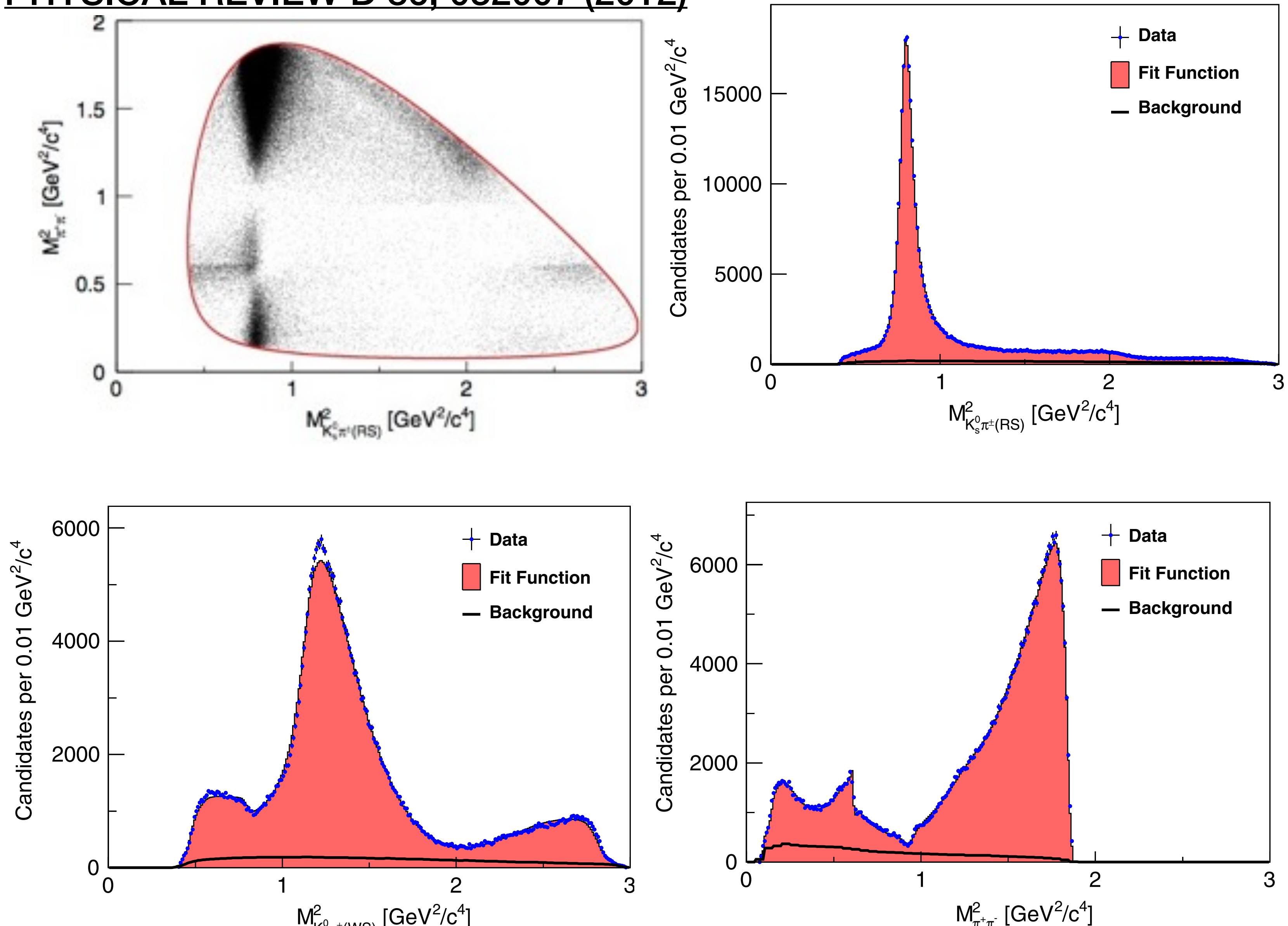
$K_0^*(1430)^\pm$	2.093 ± 0.065	54.2 ± 1.9	6.25 ± 0.25
$K_2^*(1430)^\pm$	0.986 ± 0.034	308.6 ± 2.1	1.28 ± 0.08
$K^*(1410)^\pm$	1.092 ± 0.069	155.9 ± 2.8	1.07 ± 0.10
$\rho(770)$	1	0	18.85 ± 0.18
$\omega(782)$	0.038 ± 0.002	107.9 ± 2.3	0.46 ± 0.05
$f_0(980)$	0.476 ± 0.016	182.8 ± 1.3	4.91 ± 0.19
$f_2(1270)$	1.713 ± 0.048	329.9 ± 1.6	1.95 ± 0.10
$f_0(1370)$	0.342 ± 0.021	109.3 ± 3.1	0.57 ± 0.05
$\rho(1450)$	0.709 ± 0.043	8.7 ± 2.7	0.41 ± 0.04
$f_0(600)$	1.134 ± 0.041	201.0 ± 2.9	7.02 ± 0.30
σ_2	0.282 ± 0.023	16.2 ± 9.0	0.33 ± 0.04
$K^*(892)^\pm$(DCS)	0.137 ± 0.007	317.6 ± 2.8	0.32 ± 0.03
$K_0^*(1430)^\pm$(DCS)	0.439 ± 0.035	156.1 ± 4.9	0.28 ± 0.04
$K_2^*(1430)^\pm$(DCS)	0.291 ± 0.034	213.5 ± 6.1	0.11 ± 0.03
Nonresonant	1.797 ± 0.147	94.0 ± 5.3	1.64 ± 0.27
Sum			107.25 ± 0.65

Amplitude Model

$$\mathcal{M}_{fi} = \sum_R {\color{red}c_R} e^{i{\color{red}\theta_R}} A_R(s_{12}, s_{23})$$

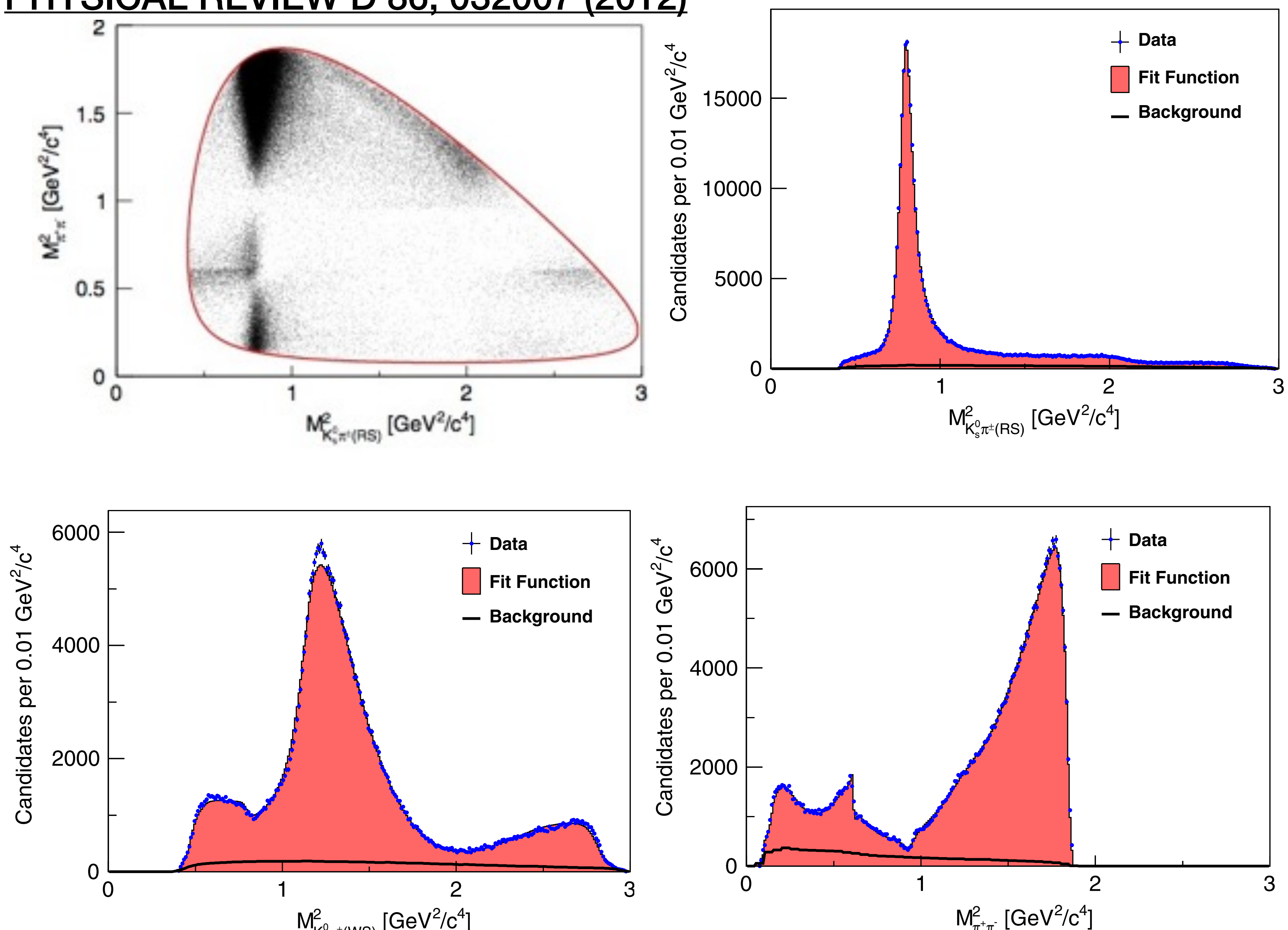
example: **CDF: PHYSICAL REVIEW D 86, 032007 (2012)**



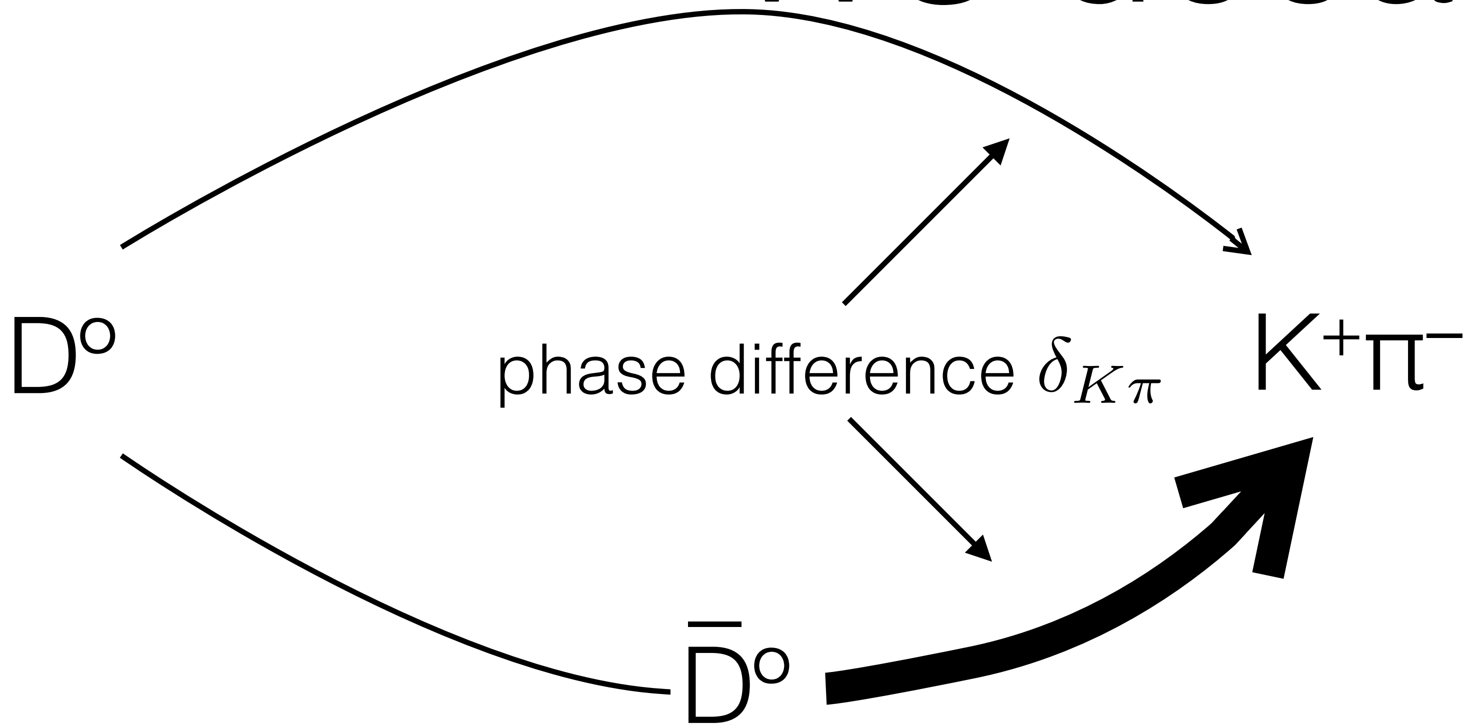
Amplitude Model

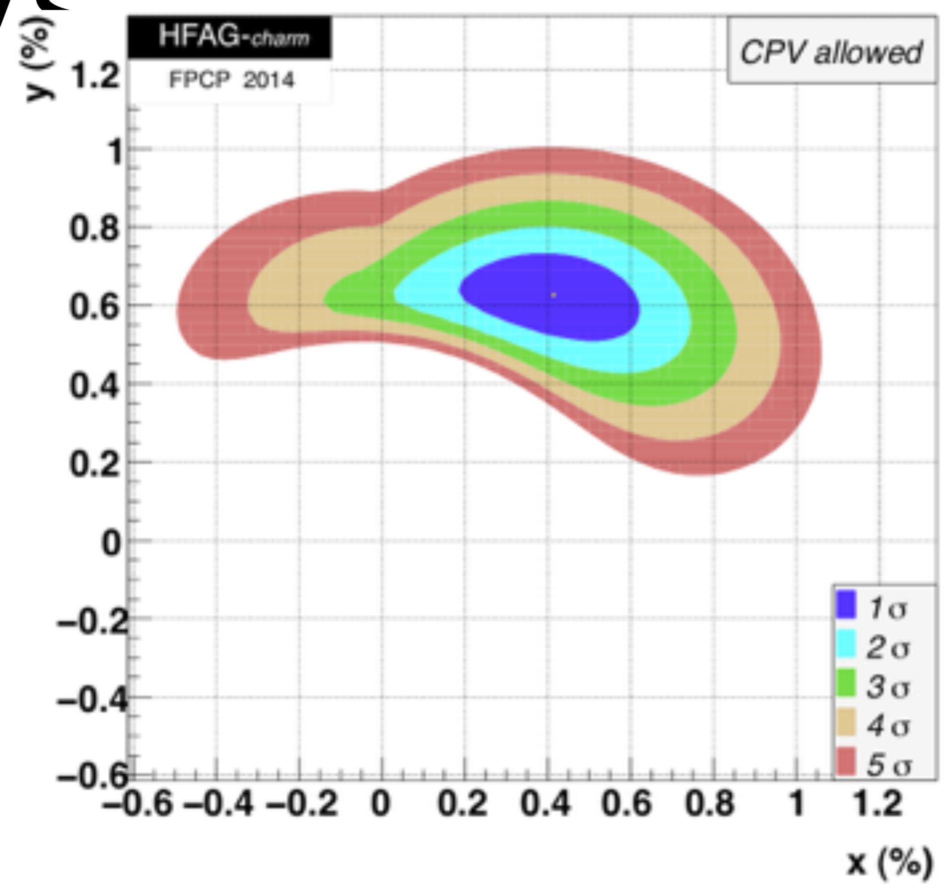
$$\mathcal{M}_{fi} = \sum_R c_R e^{i\theta_R} A_R(s_{12}, s_{23}) \quad + a_0 e^{i\theta_0}$$

example: **CDF: PHYSICAL REVIEW D 86, 032007 (2012)**



Mixing formalism for 2-body WS decays



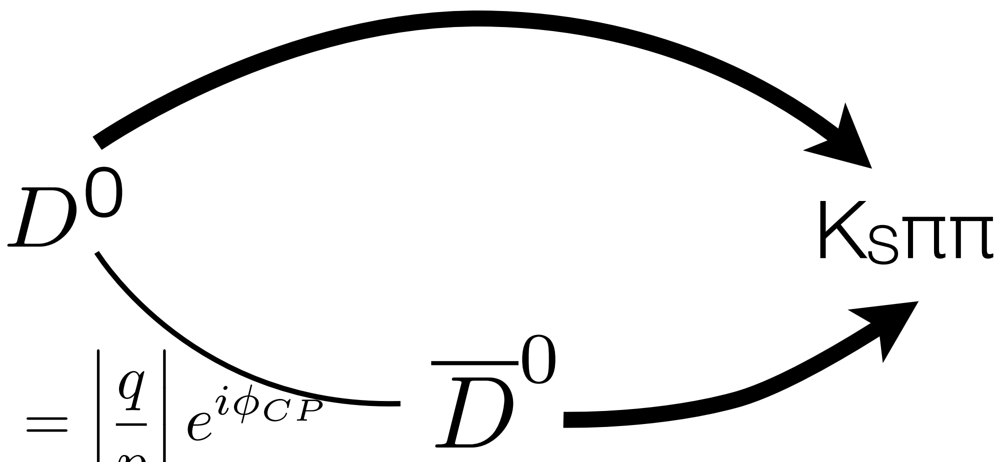


$$\frac{\Gamma(D^0 \to K^+\pi^-)}{\Gamma(D^0 \to K^-\pi^+)}(t) \approx \left(r_D^{K\pi}\right)^2 + r_D^{K\pi} y'_{K\pi}\Gamma t + \frac{x'^2_{K\pi} + y'^2_{K\pi}}{4}(\Gamma t)^2$$

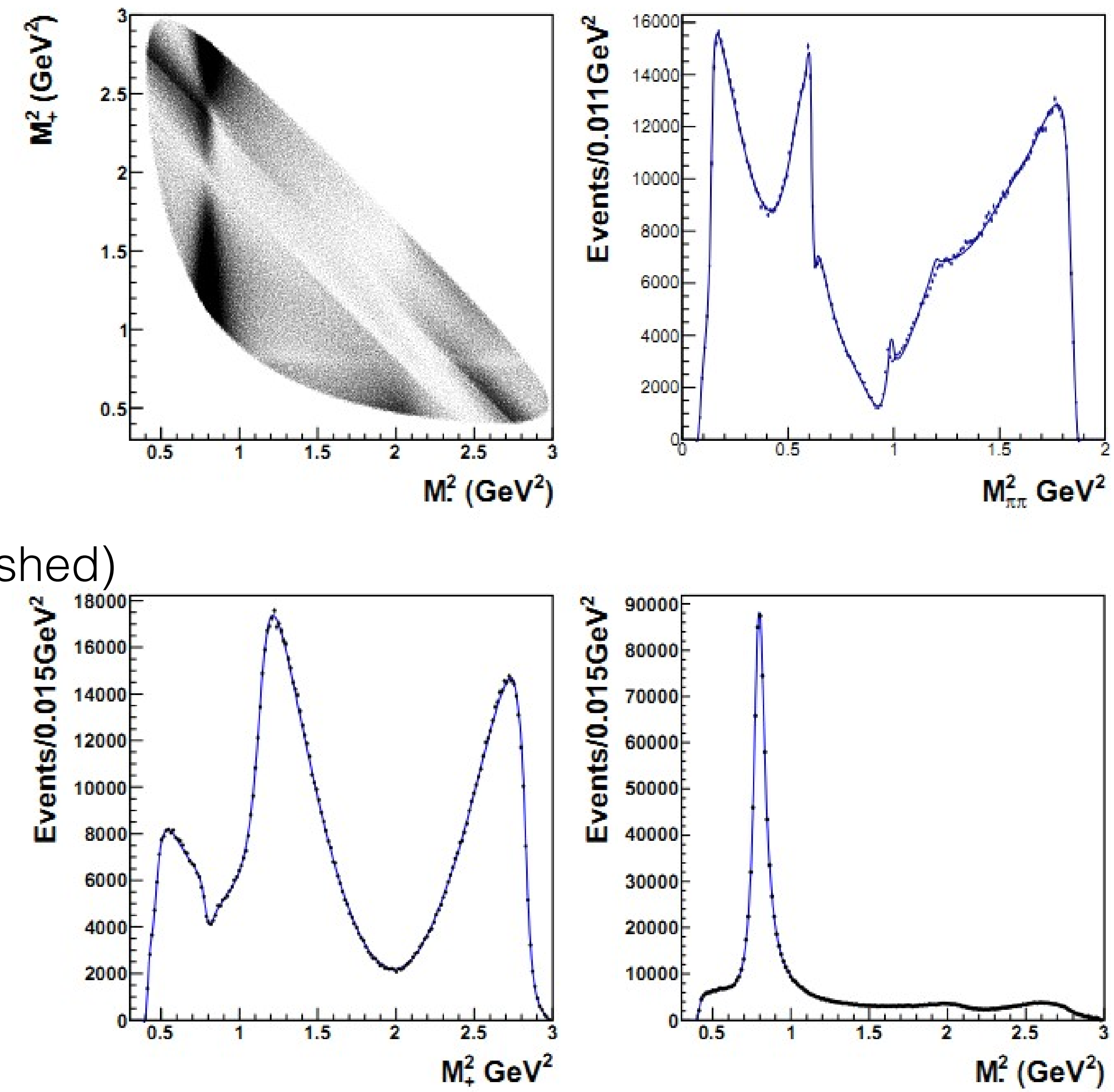
where $\begin{pmatrix} x'_{K\pi} \\ y'_{K\pi} \end{pmatrix} = \begin{pmatrix} \cos\delta_{K\pi} & \sin\delta_{K\pi} \\ \cos\delta_{K\pi} & -\sin\delta_{K\pi} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$

Time-dependent CPV $D^0 \rightarrow K_S\pi\pi$

see also previous result: Phys. Rev. Lett. 99, 131803 (2007).



$$\frac{q}{p} = \left|\frac{q}{p}\right| e^{i\phi_{CP}}$$

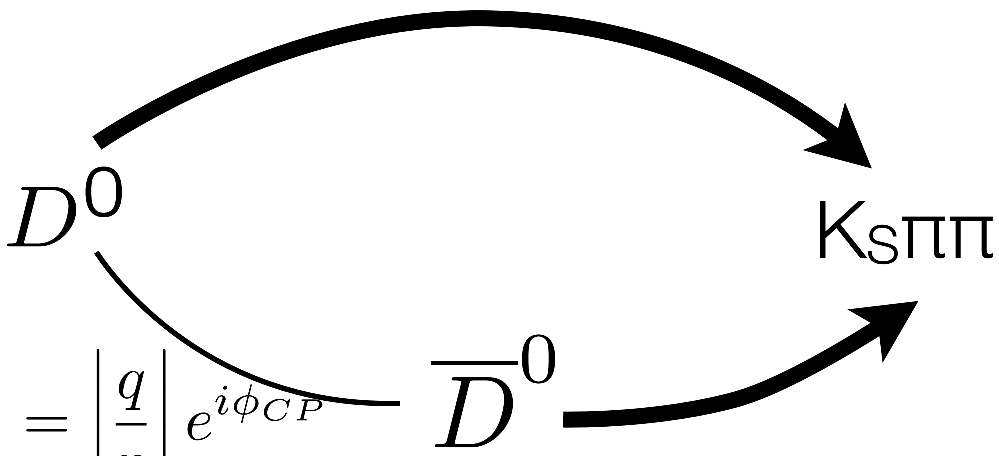


(Belle preliminary)(by now published)

Fit case	Parameter	Fit new result
No CPV	$x(\%)$	$0.56 \pm 0.19^{+0.03+0.06}_{-0.09-0.09}$
	$y(\%)$	$0.30 \pm 0.15^{+0.04+0.03}_{-0.05-0.06}$
No dCPV	$\lvert q/p \rvert$	$0.90^{+0.16+0.05+0.06}_{-0.15-0.04-0.05}$
	$\arg q/p(^o)$	$-6 \pm 11^{+3+3}_{-3-4}$

see also BaBar Phys. Rev. Lett. 105, 081803 (2010) and CLEO-c Phys. Rev. D 72, 012001 (2005).

Time-dependent CPV $D^o \rightarrow K_S\pi\pi$



$$\frac{q}{p} = \left|\frac{q}{p}\right| e^{i\phi_{CP}}$$

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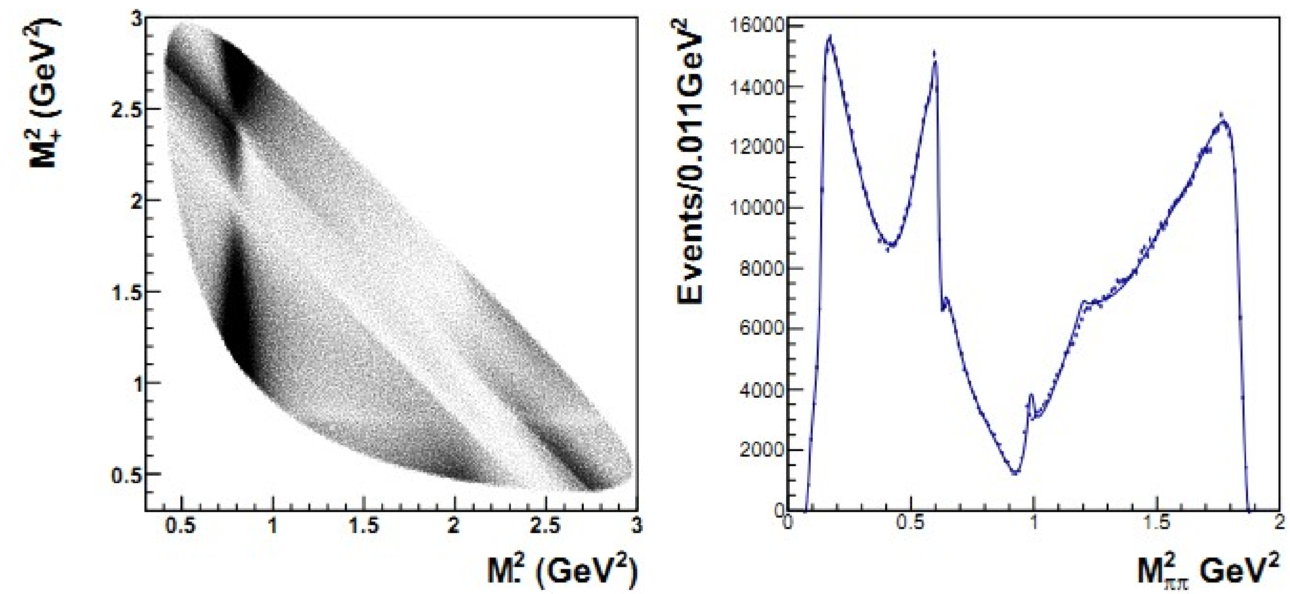

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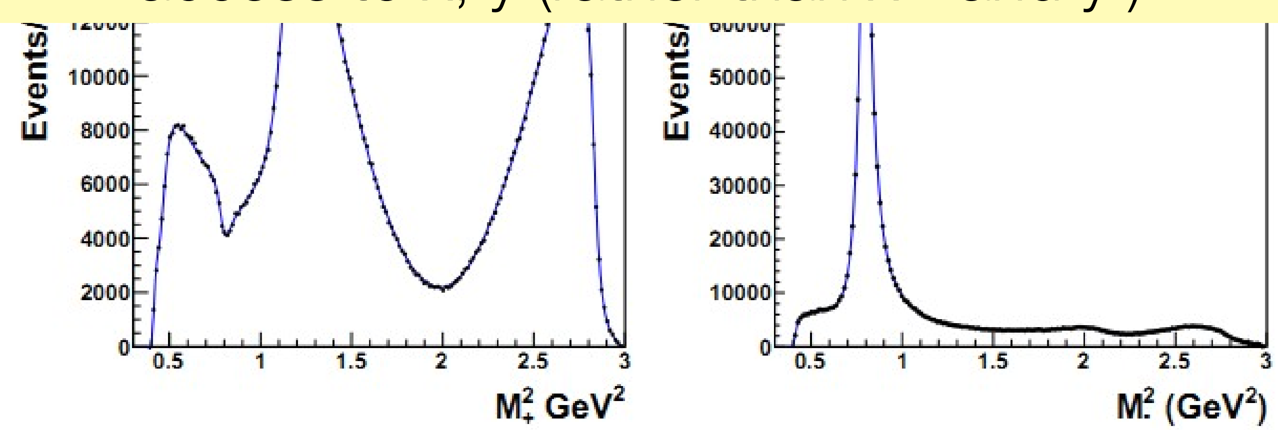
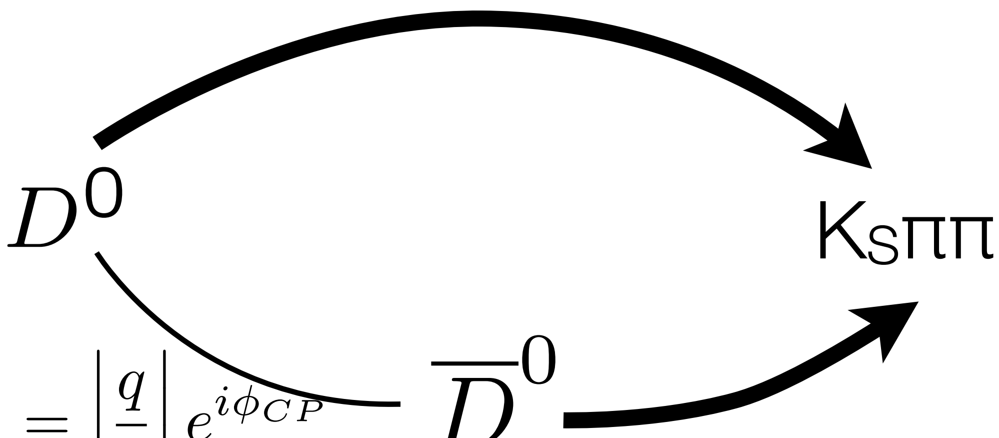
Magic of Dalitz plot (sensitivity to phases) gives access to x, y (rather than x'^2 and y')



see also BaBar Phys. Rev. Lett. 105, 081803 (2010) and CLEO-c Phys. Rev. D 72, 012001 (2005).

Time-dependent CPV $D^o \to K_S\pi\pi$



$$\frac{q}{p} = \left|\frac{q}{p}\right| e^{i\phi_{CP}}$$

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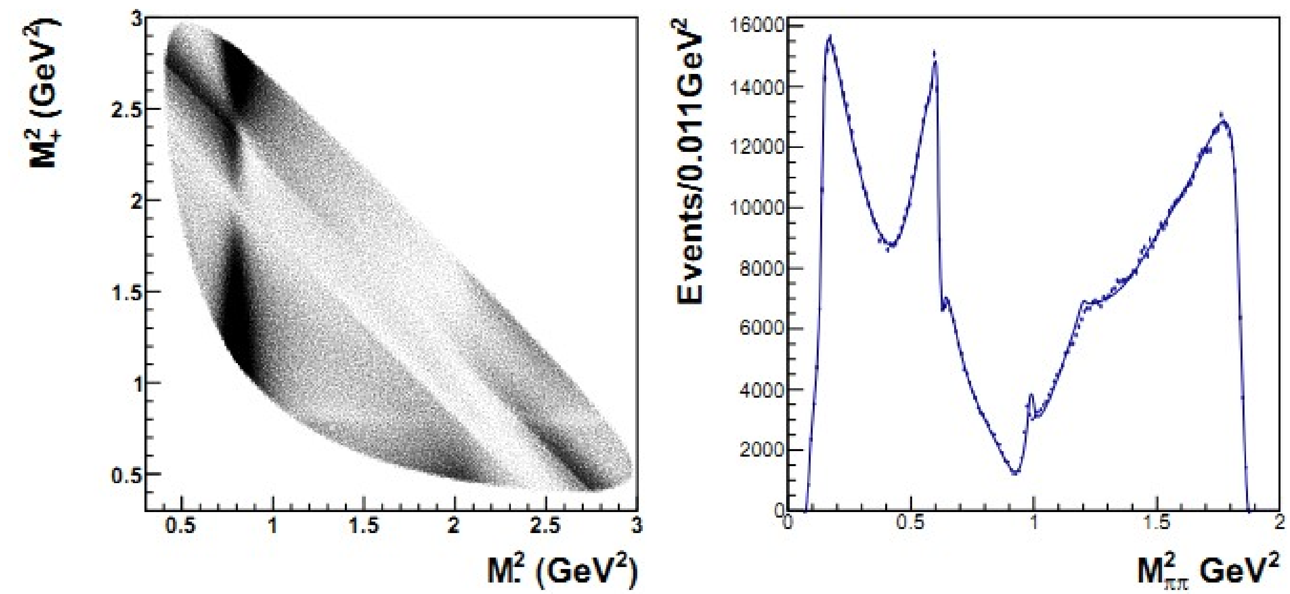

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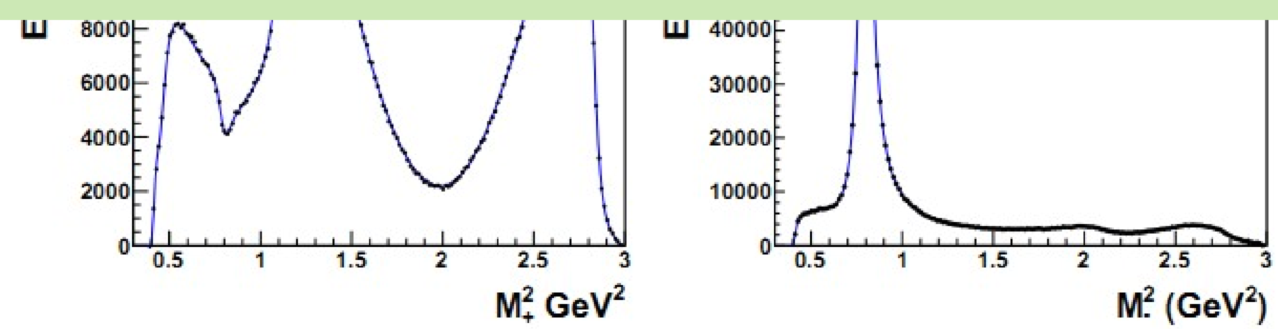
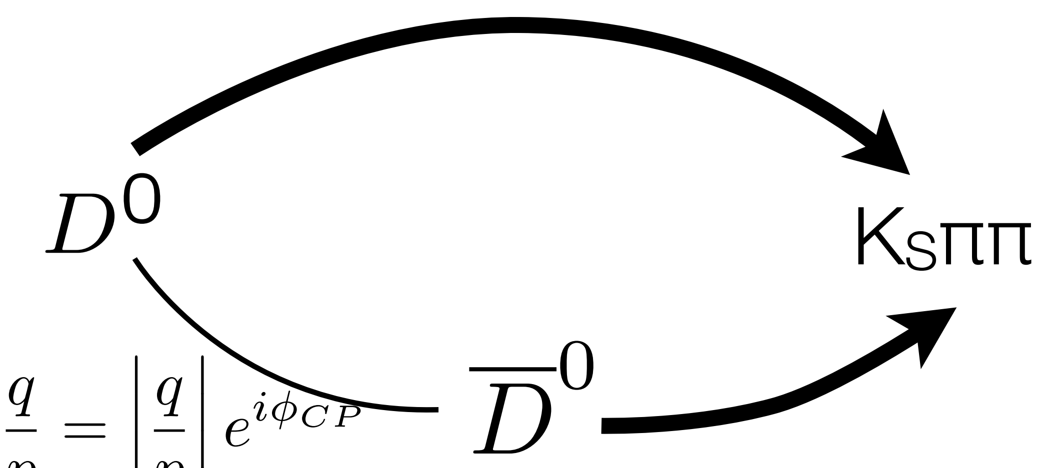
Magic of Dalitz plot (sensitivity to phases) gives access to x, y (rather than x'^{2} and y')

No evidence of CP violation



see also BaBar Phys. Rev. Lett. 105, 081803 (2010) and CLEO-c Phys. Rev. D 72, 012001 (2005).

Time-dependent CPV D° → $K_S\pi\pi$



$$\frac{q}{p} = \left|\frac{q}{p}\right| e^{i\phi_{CP}}$$

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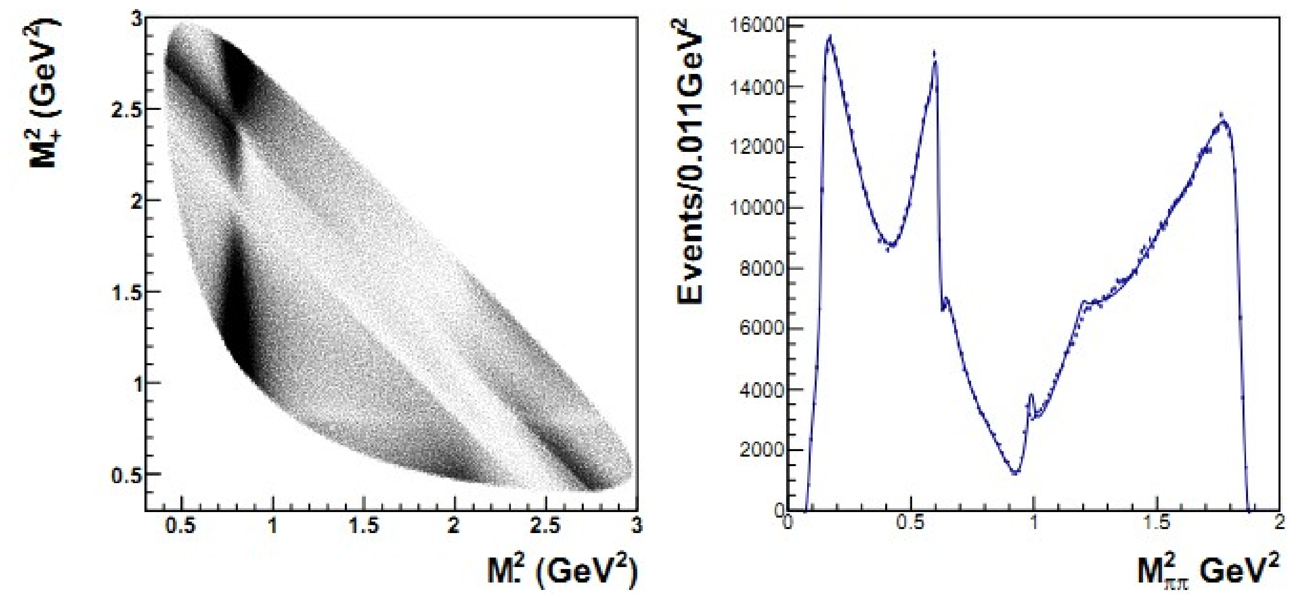

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	$\arg q/p (^o)$	$-6 \pm 11^{+3}_{-3}{}^{+3}_{-4}$

Magic of Dalitz plot (sensitivity to phases) gives access to x, y (rather than x'^{2} and y')

No evidence of CP violation

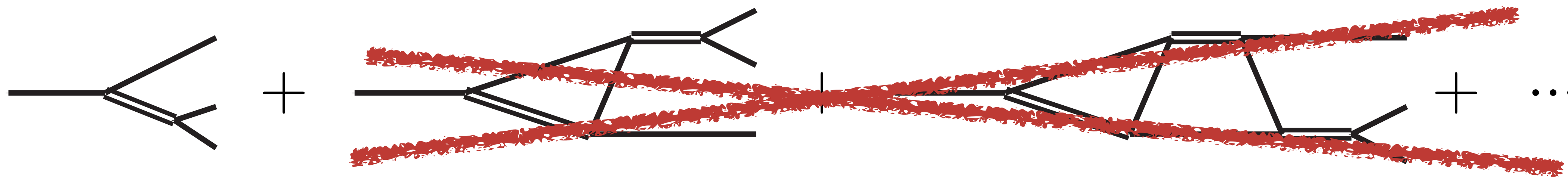
Significant systematic uncertainty from amplitude model dependence. (Could be limiting with future LHCb/upgrade statistics.)

M_+^2 GeV^2 M_-^2 (GeV^2)

see also BaBar Phys. Rev. Lett. 105, 081803 (2010) and CLEO-c Phys. Rev. D 72, 012001 (2005).

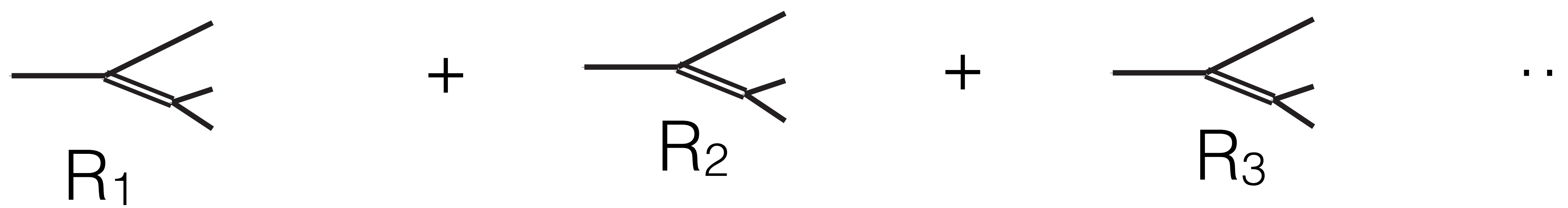
“Isobar” Model

- “Isobar”: Describe decay as series of 2-body processes.



- Usually: each resonance described by Breit Wigner lineshape (or similar) times factors accounting for spin.
- Popular amongst experimentalists, less so amongst theorists: violates unitarity. But not much as long as resonances are reasonably narrow, don’t overlap too much.
- General consensus: Isobar OK for P, D wave, but problematic for S-wave.Alternatives exist, e.g. K-matrix formalism, which respects unitarity.

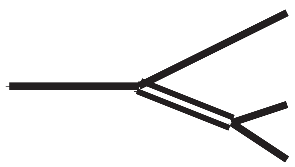
Isobar Model with sum of Breit Wigners



$$\frac{1}{s_{12} - m_1^2 - i m_1 \Gamma_1(s_{12})} + \frac{1}{s_{12} - m_2^2 - i m_2 \Gamma_2(s_{12})} + \frac{1}{s_{12} - m_3^2 - i m_3 \Gamma_3(s_{12})} \cdots$$

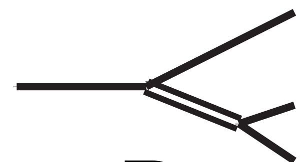
- Single resonance well described by Breit Wigner
- Overlapping resonances not so. Theoretically problematic: violates unitarity. From a practical point of view problematic as you might get the wrong phase motion.

Isobar Model with sum of Breit Wigners



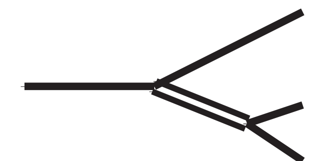
R_1

$+$



R_2

$+$



R_3

$\cdots$

$$\frac{1}{s_{12} - m_1^2 - i m_1 \Gamma_1(s_{12})} + \frac{1}{s_{12} - m_2^2 - i m_2 \Gamma_2(s_{12})} + \frac{1}{s_{12} - m_3^2 - i m_3 \Gamma_3(s_{12})} \cdots$$

4 resonances

32 resonances

Flatté Formula

- Consider $f_0(980)$ (width $\Gamma \approx$ 40-100 MeV). Decays to $\pi\,\pi$ and KK. To KK only above ~987.4 MeV.
- The availability of the KK final state above 987.4 MeV increases the phase space and thus the width above this threshold.
- Need to take this into account even if I only look at $f_0(980) \rightarrow \pi\pi$.

$$\Gamma_{f_0}(s) = \Gamma_\pi(s) + \Gamma_K(s).$$

$$\Gamma_\pi(s) = g_\pi \sqrt{s/4 - m_\pi^2},$$

$$\Gamma_K(s) = \frac{g_K}{2}\left(\sqrt{s/4 - m_{K^+}^2} + \sqrt{s/4 - m_{K^0}^2}\right)$$

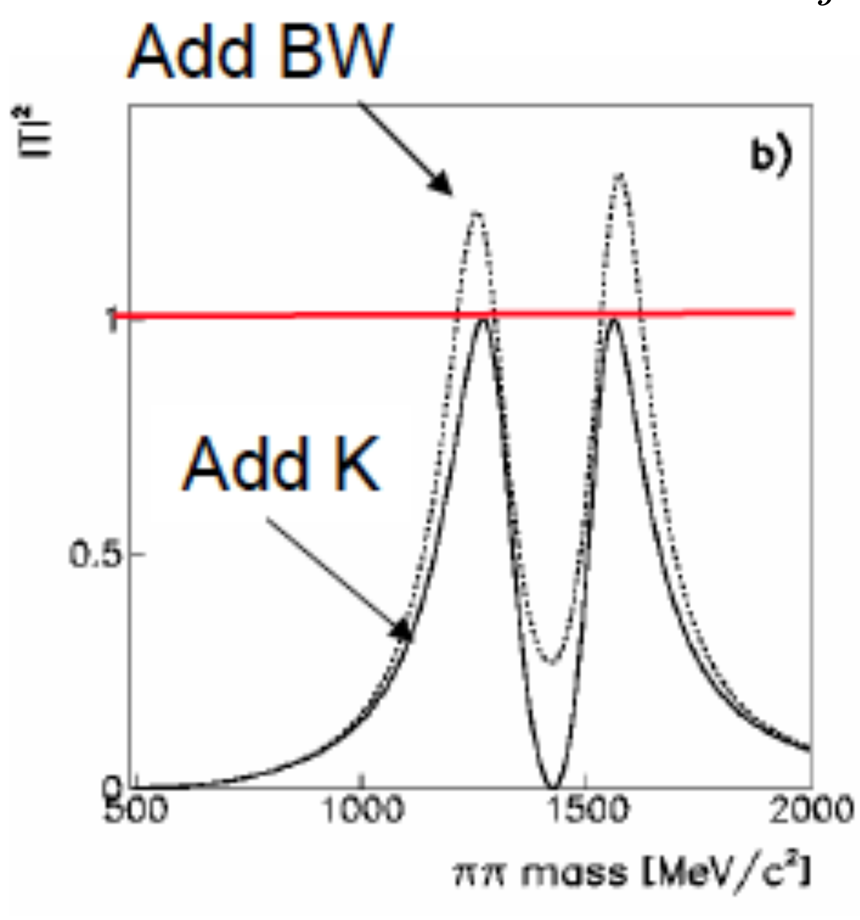
K-matrix

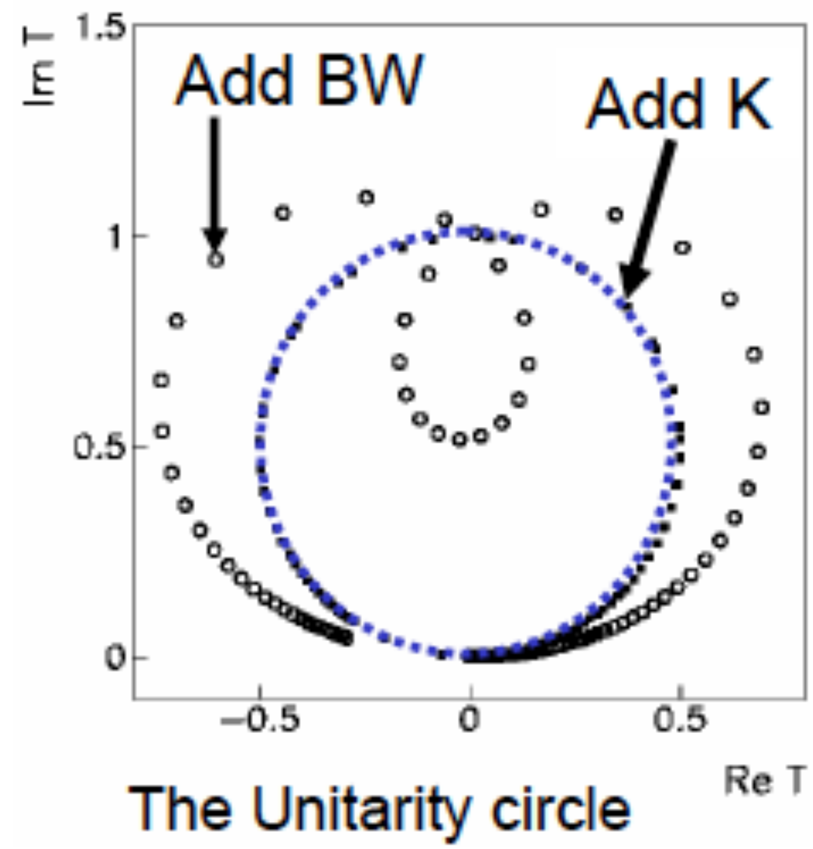
$$S_{fi} = \langle f|S|i\rangle = I + 2iT$$
$$T = K(I - iK)^{-1}$$
$$K_{ij} = \sum_{\alpha} \frac{\sqrt{m_\alpha \Gamma_{\alpha i}}\sqrt{m_\alpha \Gamma_{\alpha j}}}{m_\alpha^2 - m^2}$$

- For single channel: Reproduces Breit Wigner
- For single resonance that can decay to different final state: Reproduces Flatté.

K-matrix

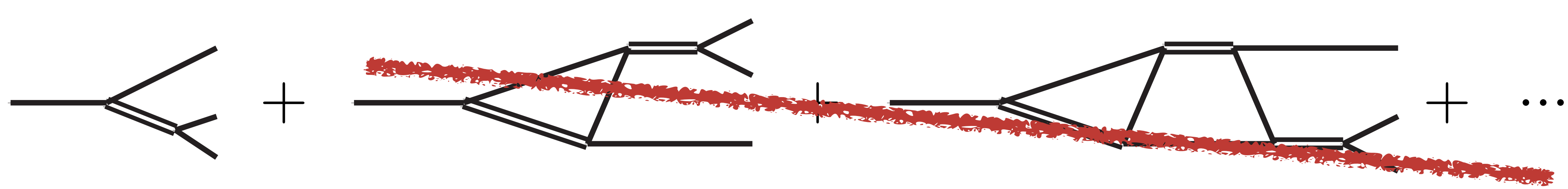
$$S_{fi} = \langle f|S|i\rangle = I + 2iT$$
$$T = K(I - iK)^{-1}$$
$$K_{ij} = \sum_{\alpha} \frac{\sqrt{m_\alpha \Gamma_{\alpha i}}\sqrt{m_\alpha \Gamma_{\alpha j}}}{m_\alpha^2 - m^2}$$





K-matrix

- Note that the K-matrix approach is still an approximation.
- While it ensures unitarity (by construction), it is not completely theoretically sound/motivated (and violates analyticity).
- And it does not in any way address this:



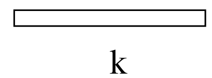
What theorists think of all this

(a few slides from a recent LHCb Amplitude Analysis Workshop with experimentalists and theorists)

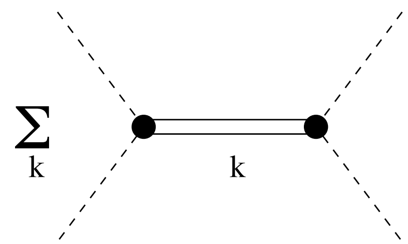
Christoph Hanhart

Modeling hadron physics

Standard treatment: sum of Breit-Wigners

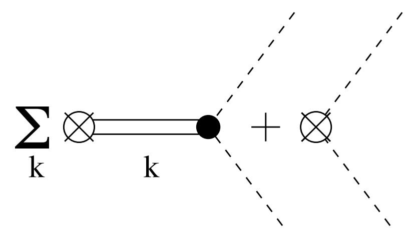
Propagator: $iG_k(s) =$  $= i/(s - M_k^2 + iM_k\Gamma_k)$

Scattering:



$= \sum_k ig_k^2 G_k(s)$

Production:



$= \left(\sum_k ig_k G_k(s)\alpha_k\right) + i\beta$

Problems:

→ Wrong threshold behavior (cured by $\Gamma = \Gamma(s)$)

→ Violates unitarity ⟶ wrong phase motion

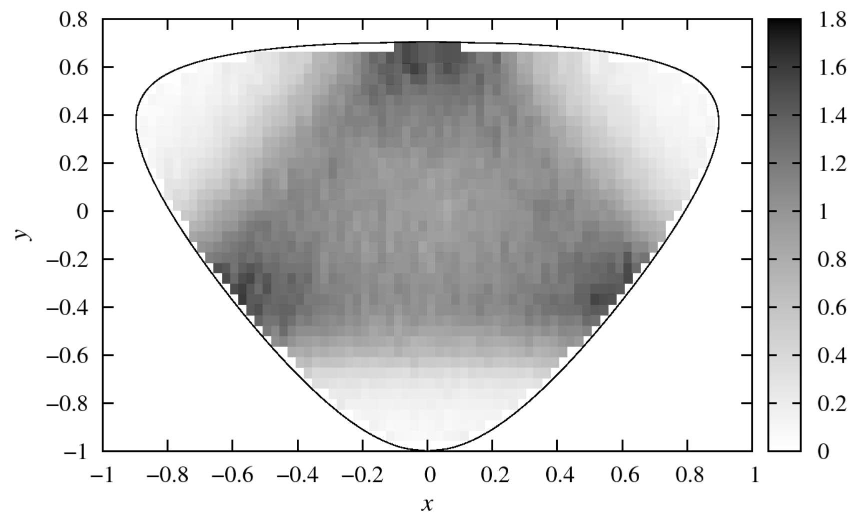
→ Parameters reaction dependent
only pole positions and resides universal!

Sum of Breit Wigners

3-body Dalitz plot (theory)

Bastian Kubis

A simple Dalitz plot: $\phi \to 3\pi$



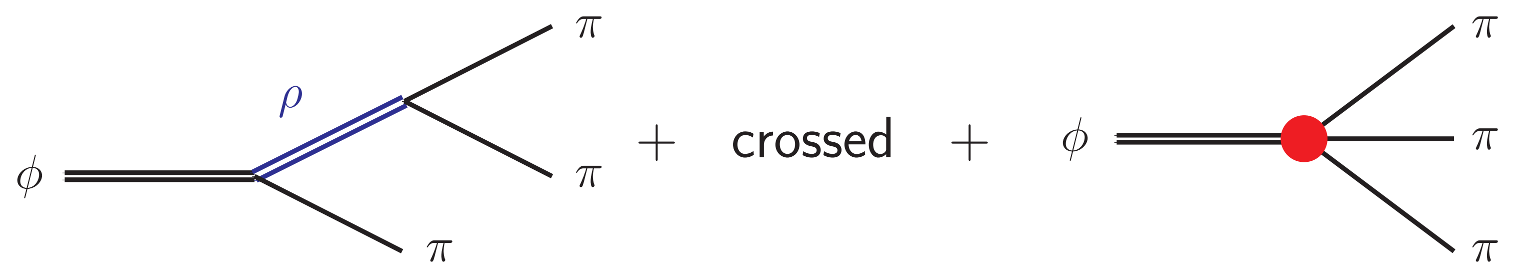
- 2×10^6 events in 1834 bins

KLOE 2003

- analyzed in terms of:

  sum of 3 Breit–Wigners ($\rho^\pm$, ρ^0)

\+ constant background term



Problem:

$\longrightarrow$ unitarity fixes Im/Re parts

$\longrightarrow$ adding a contact term destroys this relation

Sum of Breit Wigners with non-resonant term

Last Judgement (Detail) by Fra Angelico

Factorising the form factor into universal and reaction-specific parts

Christoph Hanhart

$$\boxed{F(s) = P(s)\Omega(s)}$$

$\rightarrow$ $\Omega(s)$ is universal and fixed in elastic regime **(Omnès function)**

$\rightarrow$ $P(s)$ reaction specific and contains e.g.

- $\triangleright$ higher thresholds
- $\triangleright$ inelastic resonances

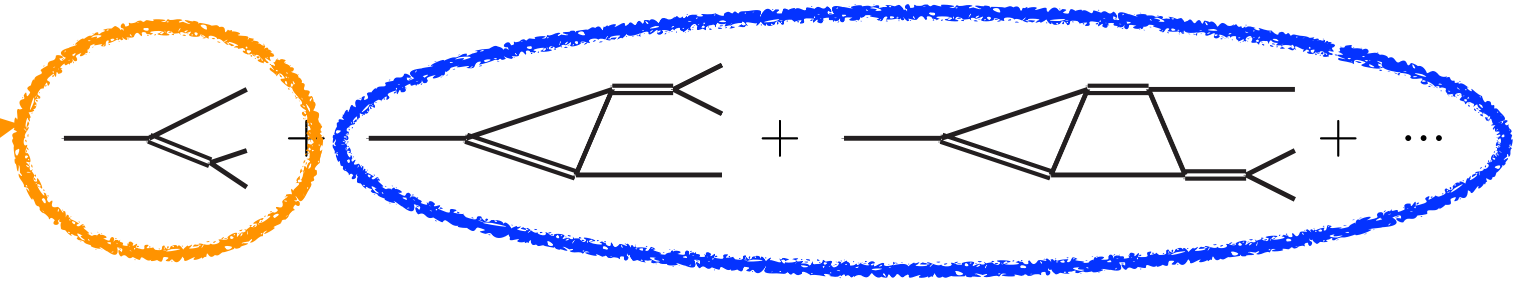
3-body Dalitz plot (theory)

Bastian Kubis

takes into account this

$$\mathcal{F}(s) = a\Omega(s)\left\{1+\frac{s}{\pi}\int_{4M_\pi^2}^{\infty}\frac{ds'}{s'}\frac{\sin\delta_1^1(s')\hat{\mathcal{F}}(s')}{|\Omega(s')|(s'-s)}\right\}$$

Omnès takes into account just this



3-body Dalitz plot (theory)

Bastian Kubis

calculable (but interaction-dependent)

$$\mathcal{F}(s) = \Omega(s)\left\{ a + b\, s + \frac{s^2}{\pi}\int_{4M_\pi^2}^{\infty} \frac{ds'}{s'^2} \frac{\sin\delta_1^1(s')\hat{\mathcal{F}}(s')}{|\Omega(s')|(s'-s)} \right\}$$

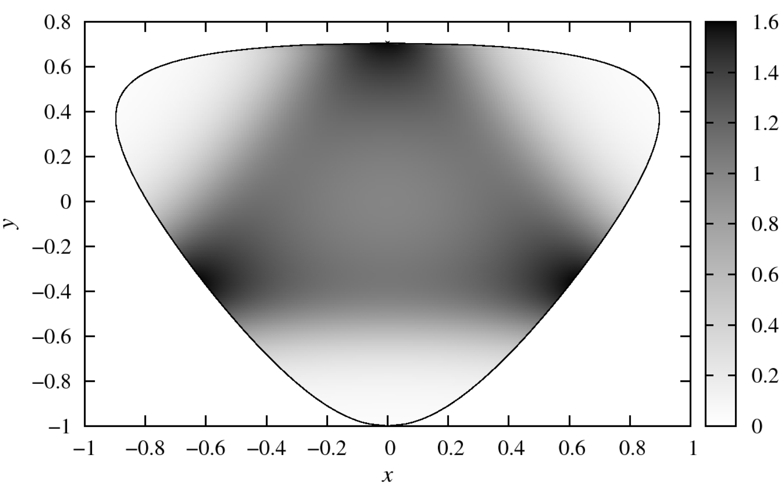
fit to data

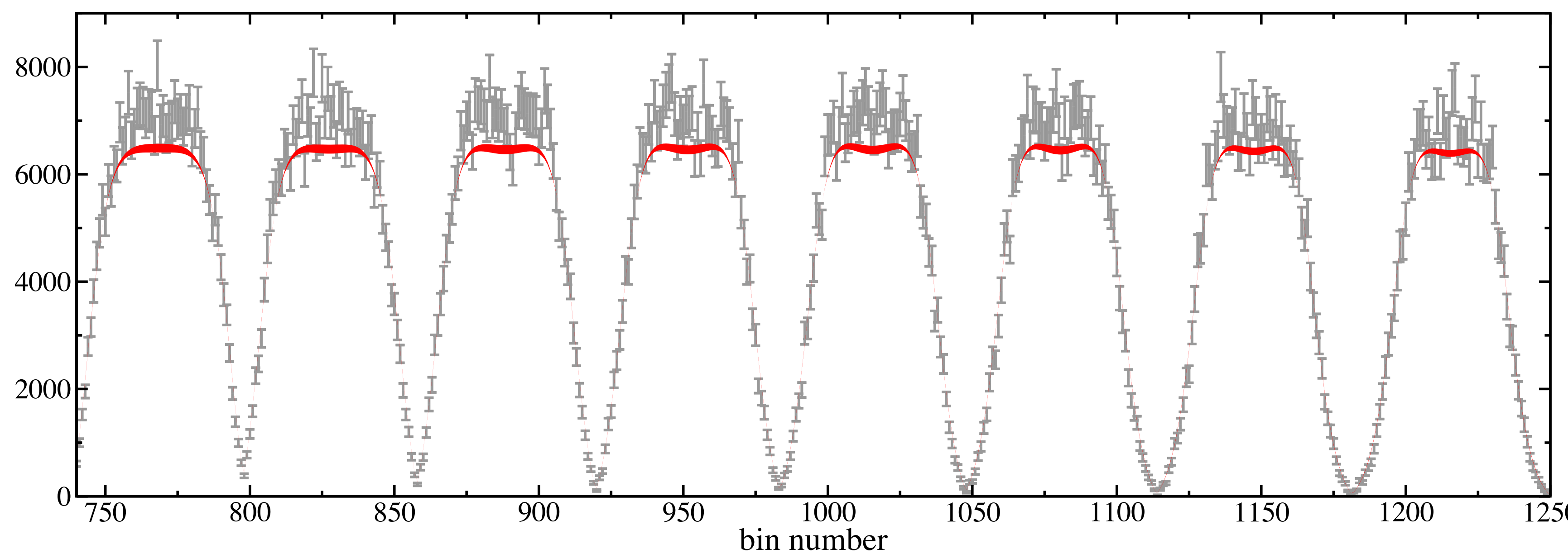
Formalism applied to $\phi \rightarrow \pi\pi\pi^0$

Bastian Kubis

Experimental comparison to $\phi \to 3\pi$

- successive slices through Dalitz plot: Niecknig, BK, Schneider 2012





χ^2/ndof	$1.7 \ldots 2.1$

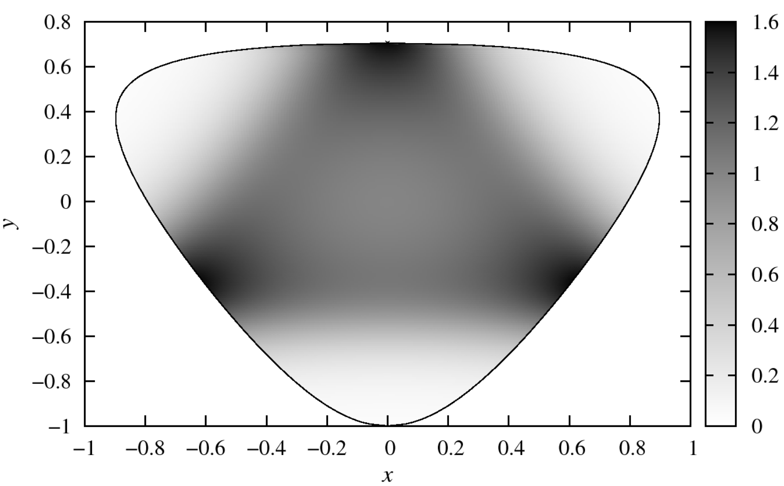
$\longrightarrow$ pairwise interaction only (with correct $\pi\pi$ scattering phase)

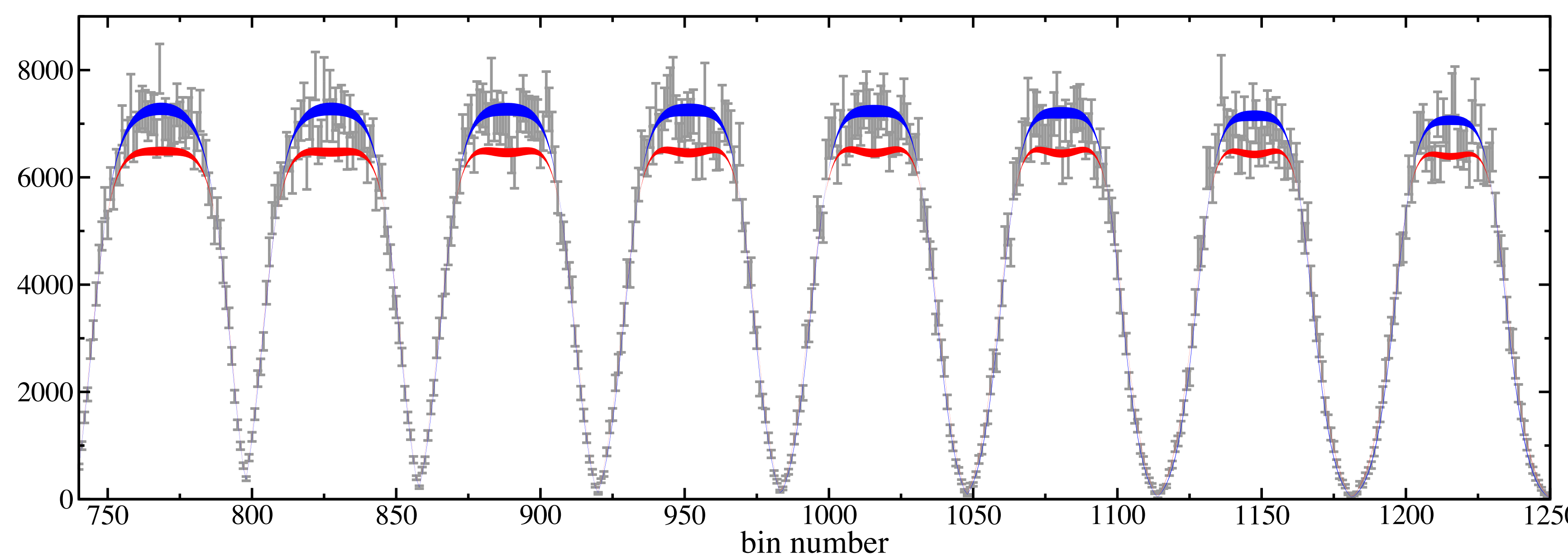
Formalism applied to ϕ→πππ°

Bastian Kubis

Experimental comparison to $\phi \to 3\pi$

- successive slices through Dalitz plot: Niecknig, BK, Schneider 2012





χ^2/ndof	$1.7 \ldots 2.1$	$1.2 \ldots 1.5$

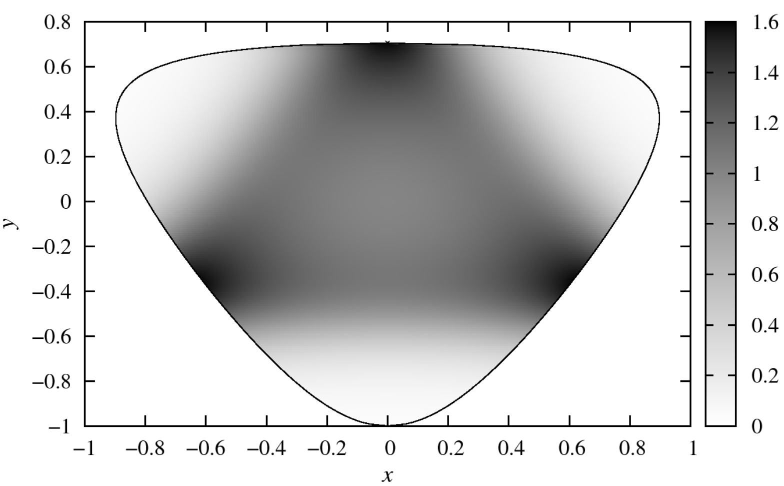
$\longrightarrow$ full 3-particle rescattering, only overall normalization adjustable

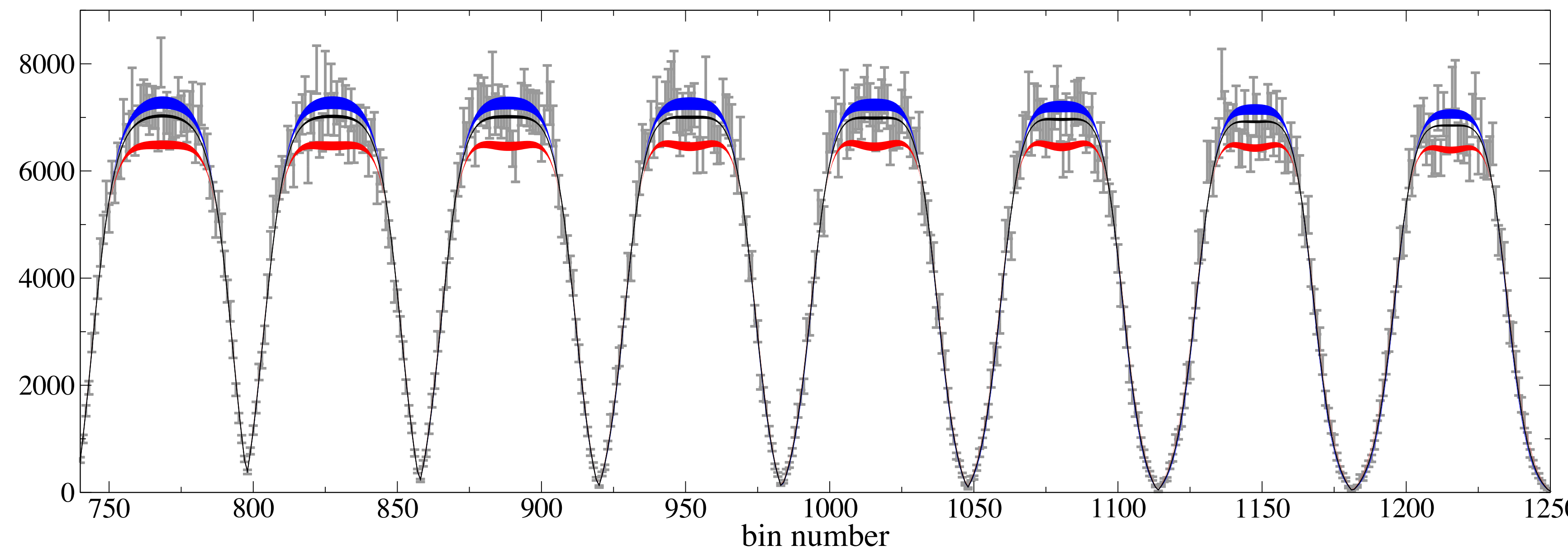
Formalism applied to ϕ→πππ°

Bastian Kubis

Experimental comparison to $\phi \to 3\pi$

- successive slices through Dalitz plot: Niecknig, BK, Schneider 2012





χ^2/ndof	$1.7\ldots2.1$	$1.2\ldots1.5$	1.0

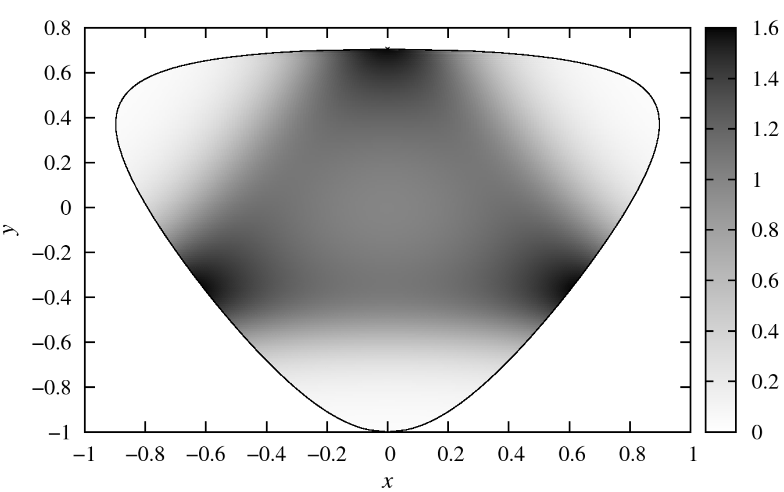
$\longrightarrow$ full 3-particle rescattering, 2 adjustable parameters (additional "subtraction constant" to suppress inelastic effects)

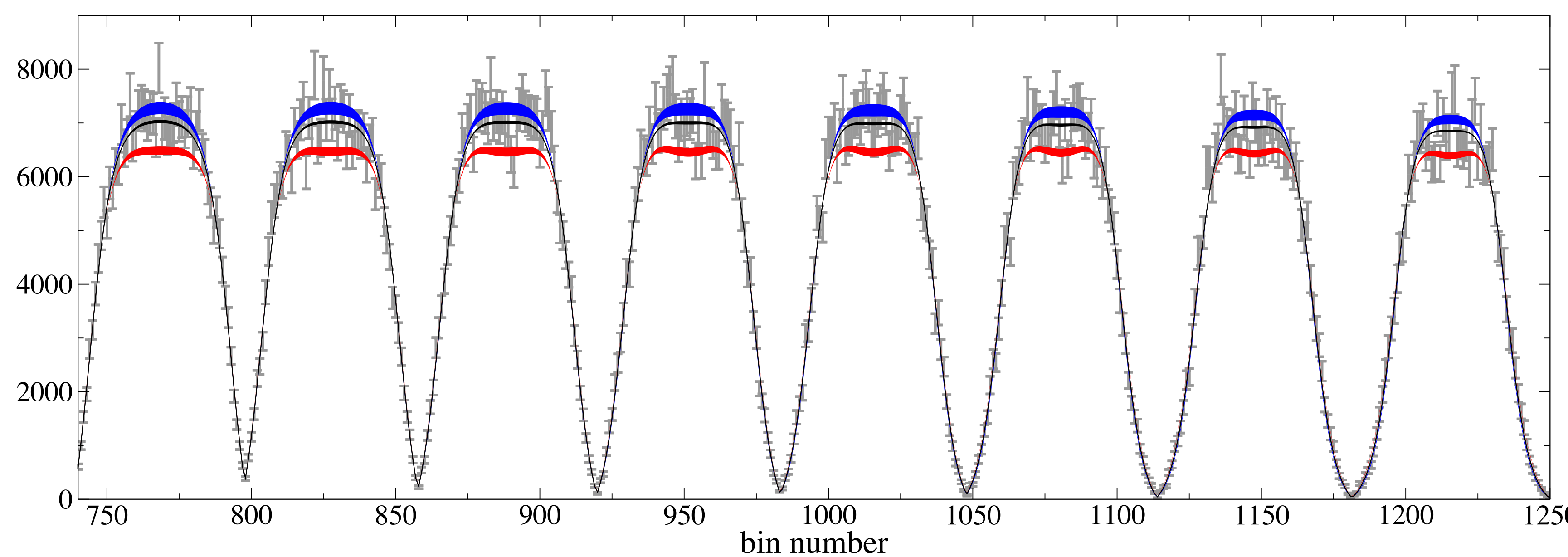
Formalism applied to $\phi \rightarrow \pi\pi\pi^0$

Bastian Kubis

Experimental comparison to $\phi \to 3\pi$

- successive slices through Dalitz plot: Niecknig, BK, Schneider 2012



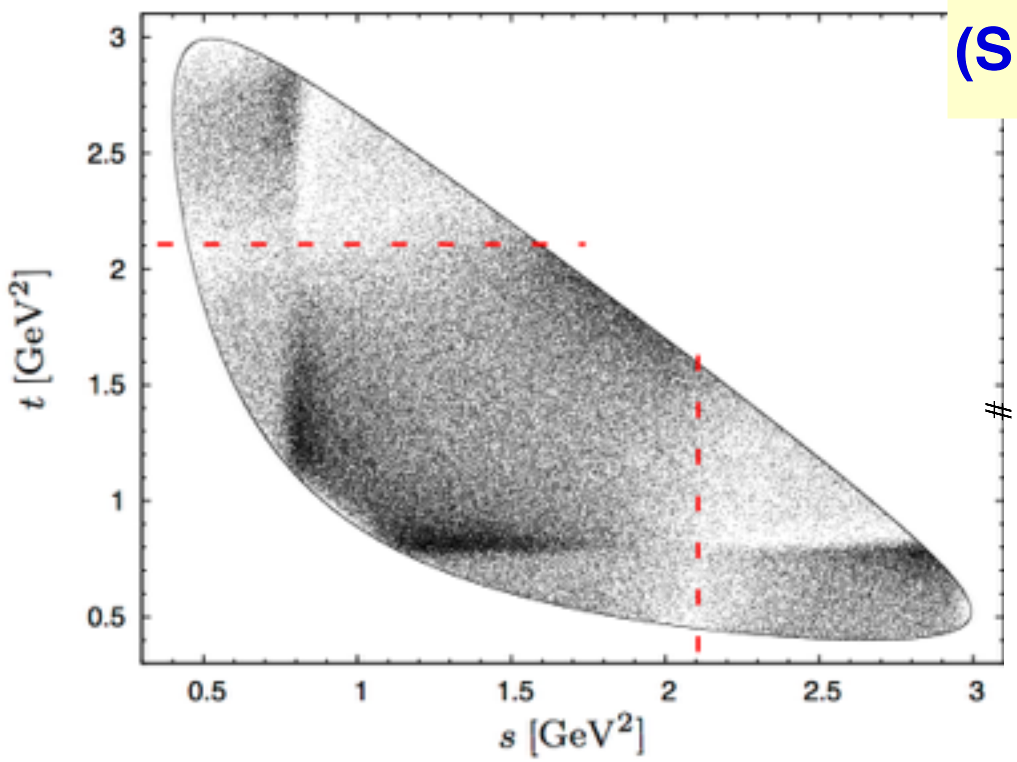


χ^2/ndof	$1.7 \ldots 2.1$	$1.2 \ldots 1.5$	1.0

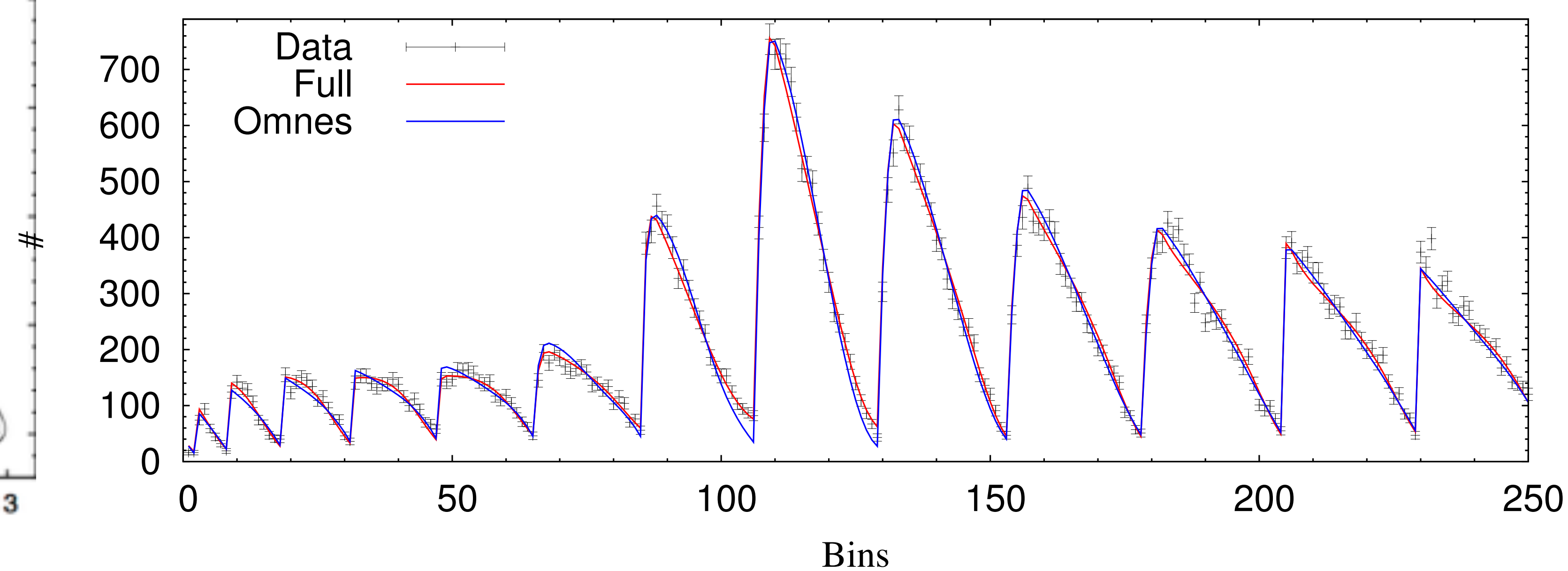
- perfect fit respecting analyticity and unitarity possible
- contact term emulates neglected rescattering effects
- no need for "background" — inseparable from "resonance"

Formalism applied to D→ππK

Bastian Kubis



(Slices through) Dalitz plot $D^+ \to \pi^+\pi^+K^-$



Fit limited to
M(Kπ) < M(η') + M(K) ≈ 1.45GeV
elastic approximation breaks down beyond.

- Omnès fit: $\chi^2/\mathrm{ndof} \approx 1.42$
  ("isobar model" + non-resonant background waves)
- full dispersive solution: $\chi^2/\mathrm{ndof} \approx 1.11$
  $\longrightarrow$ visible improvement similar to $\phi \to 3\pi$
- full fit in terms of 7 complex subtraction constants
  (–1 phase, –1 overall normalisation)

7 fit parameters

Niecknig, BK *in progress*

Summary / Open questions

Bastian Kubis

Dalitz plot analyses

- rigorous using modern phase shift input
- allow to understand ad-hoc "background"

  non-resonant contribution
- ideal demonstration case: $\phi \to 3\pi$ (elastic, one partial wave)
- implementation: + linear combination of basis functions

  – basis functions different for each decay

Open questions / problems

- inelastic effects
  - ▷ we understand $I = 0$ S-wave $\pi\pi \leftrightarrow K\bar{K} \leftrightarrow f_0(980)$

    $\longrightarrow$ may attempt $D \to 3\pi \,/\, \pi K\bar{K}$
  - ▷ how to parametrise "small" inelastic effects ($\eta' K$ in πK)?
- complex subtractions — can we understand imaginary parts?
- uncertainties in πK phase shifts? can we learn about them?
- high-energy extensions ($B \to 3h$ Dalitz plots??)

Das Model

The Model

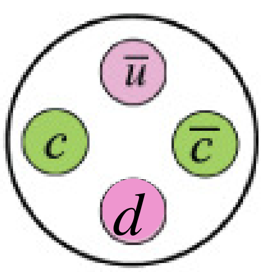
- There are, for most cases we care about, no theoretically sound amplitude models…
- However, there are “good enough” models. What’s good enough depends on the purpose.
- So what to do? Suggest a mix of….
  - model-independent approaches
  - “good enough” models of various levels of sophistication
  - improve models (there is - and that’s fairly new - real, tangible, progress!)

A few recent applications of amplitude analyses.

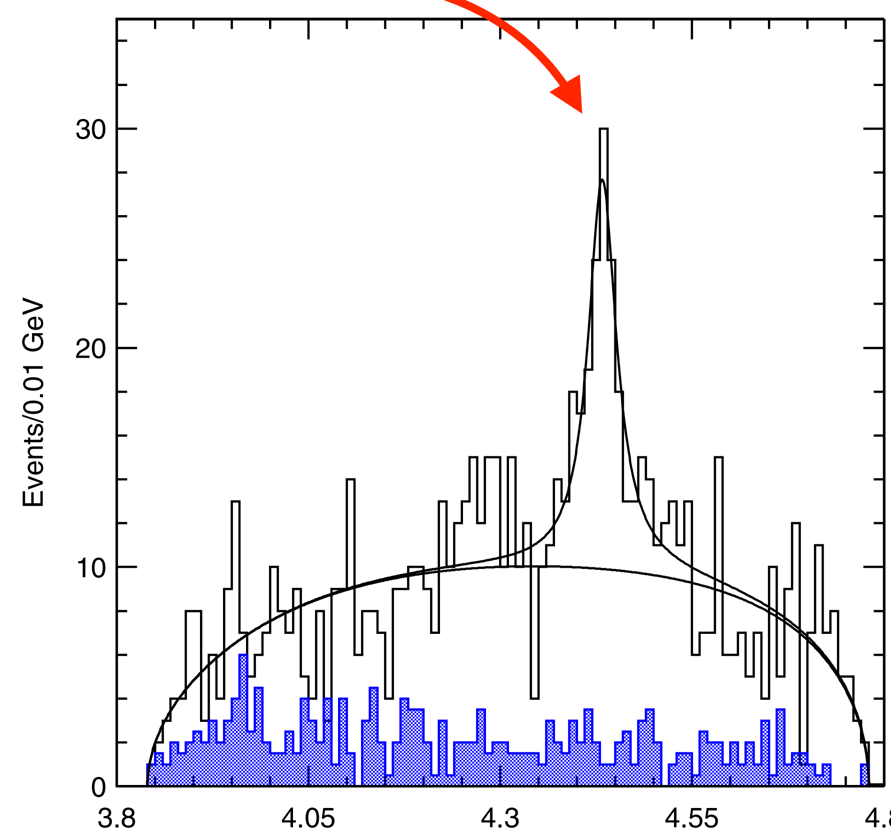
The Z(4430) question:

Is this peak in the $\psi(2S)\pi^-$ invariant mass, seen first by BELLE in 2008 when analysing $B \to \psi(2S)\pi^- K^+$, really a resonance?

Big thing - charged 4-quark state



The problem is that this is just the 1-D projection of a 4-D distribution…



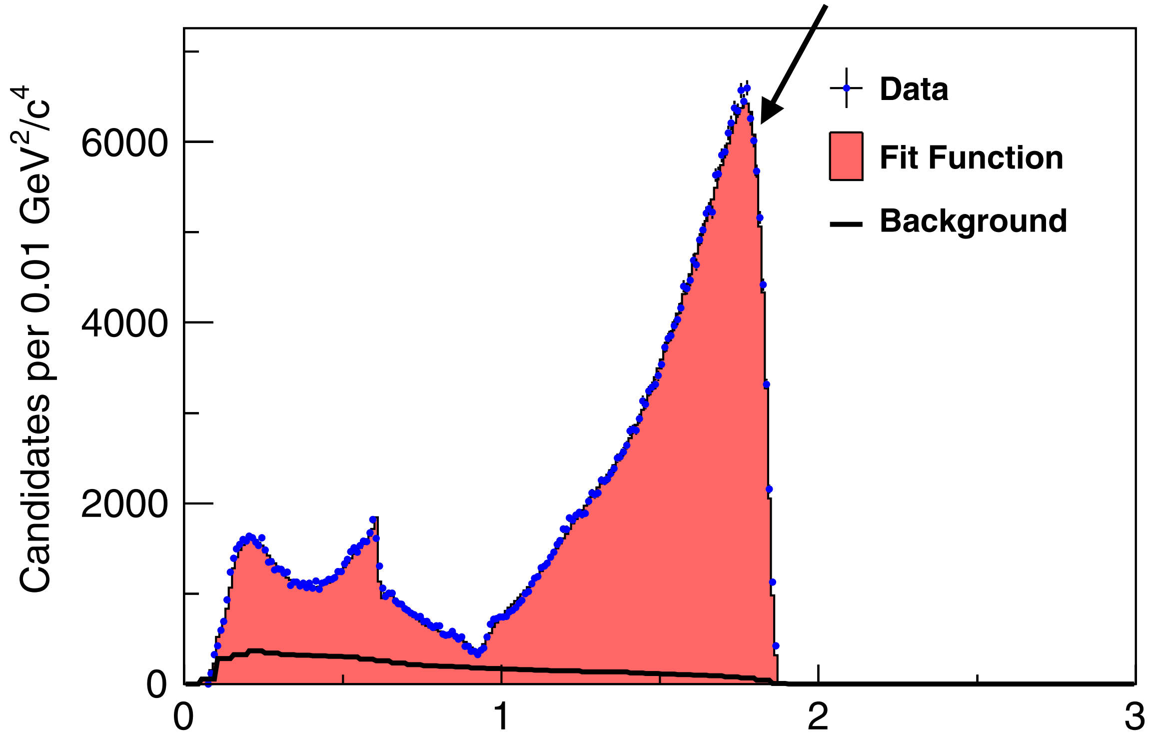
$m(\psi(2S)\ \pi)$/GeV

BELLE, Phys. Rev. Lett. 100 (2008) 142001, arXiv:0708.1790.

The 2-D illustration of this 4-D question

$D^o \rightarrow K_S\pi\pi$

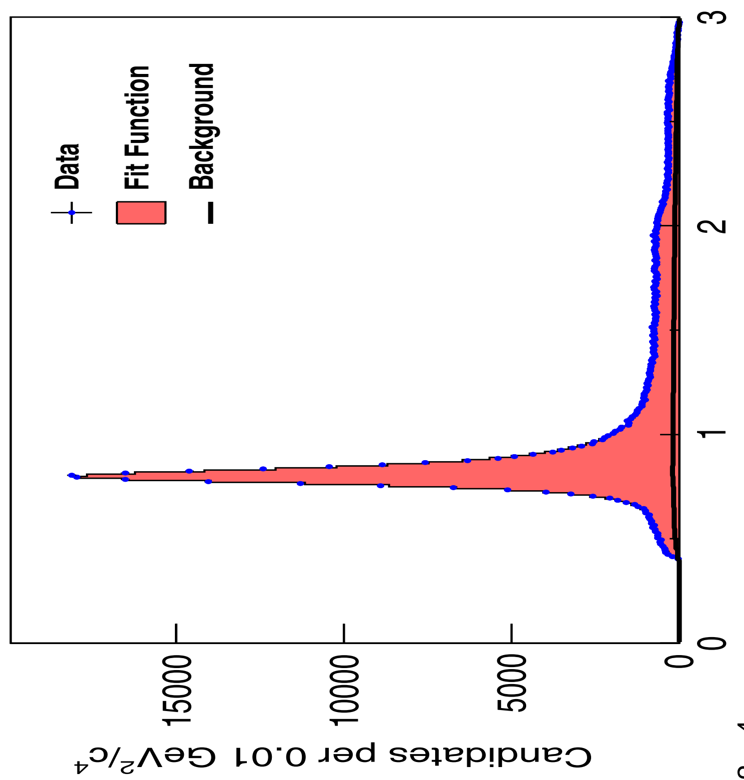
ππ resonance near 2GeV^2?

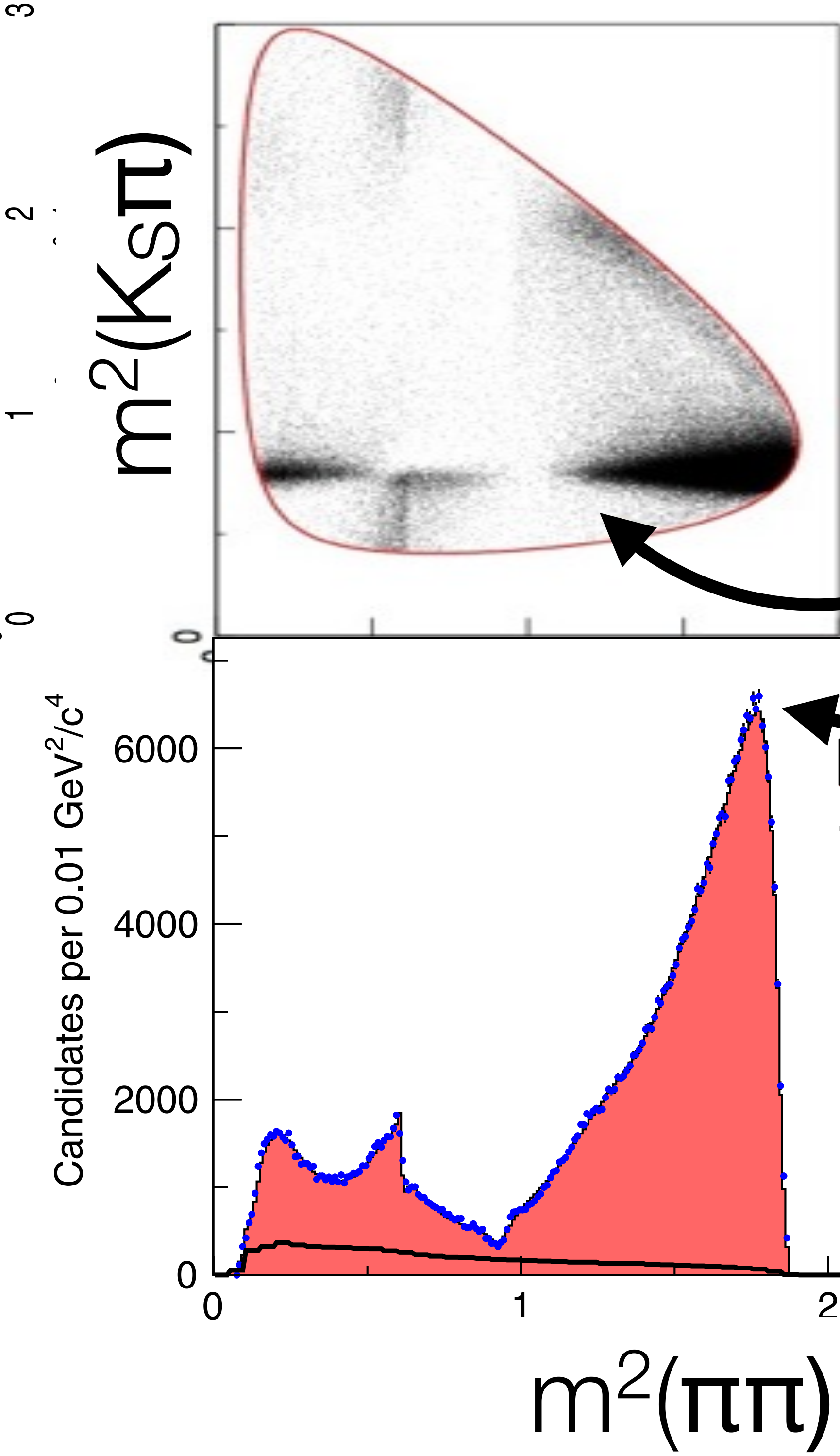


$m^2(\pi\pi)/\text{GeV}^2$

CDF PHYSICAL REVIEW D 86, 032007 (2012) (no claim of any such thing is made in this paper, it's a paper about CPV in charm).

The 2-D illustration of this 4-D question





Structure due to angular distribution in $D \rightarrow K^*(K_S\pi)\pi$

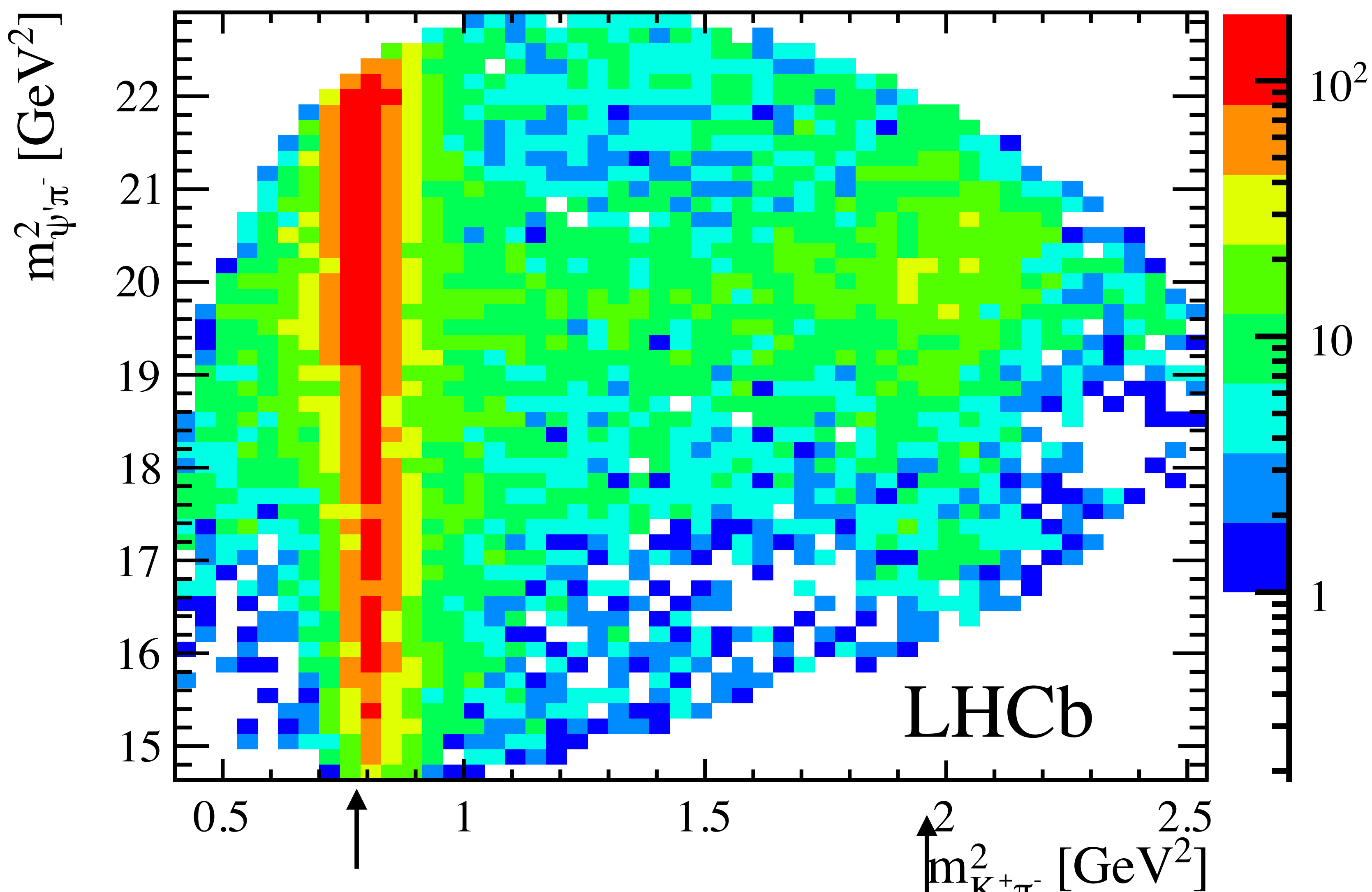
Not a (new) $\pi\pi$ resonance

$D^o \rightarrow K_S\pi\pi$

CDF PHYSICAL REVIEW D 86, 032007 (2012) (no claim of any such thing is made in this paper, it's a paper about CPV in charm).

$Z(4430) \rightarrow \psi(2S)\pi^-$ in $B \rightarrow \psi(2S)\pi^- K^+$?

$Z(4430)? \rightarrow$



$K^*(893)$ $K^*_2(1430)$

plus helicity angles

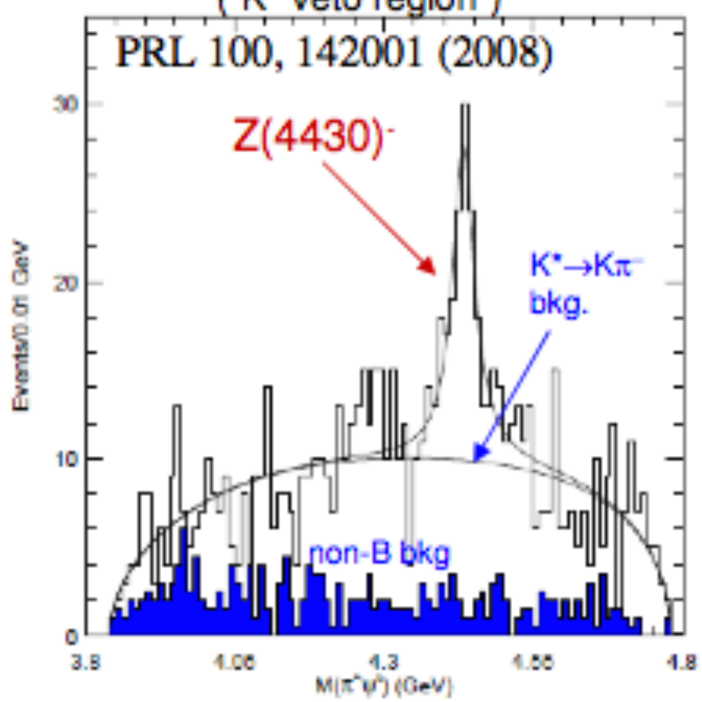
…and many other K^* resonances

Z(4430)→ψ(2S)π⁻ in B→ψ(2S)π⁻ K⁺?

Belle 2008

1D

("K* veto region")



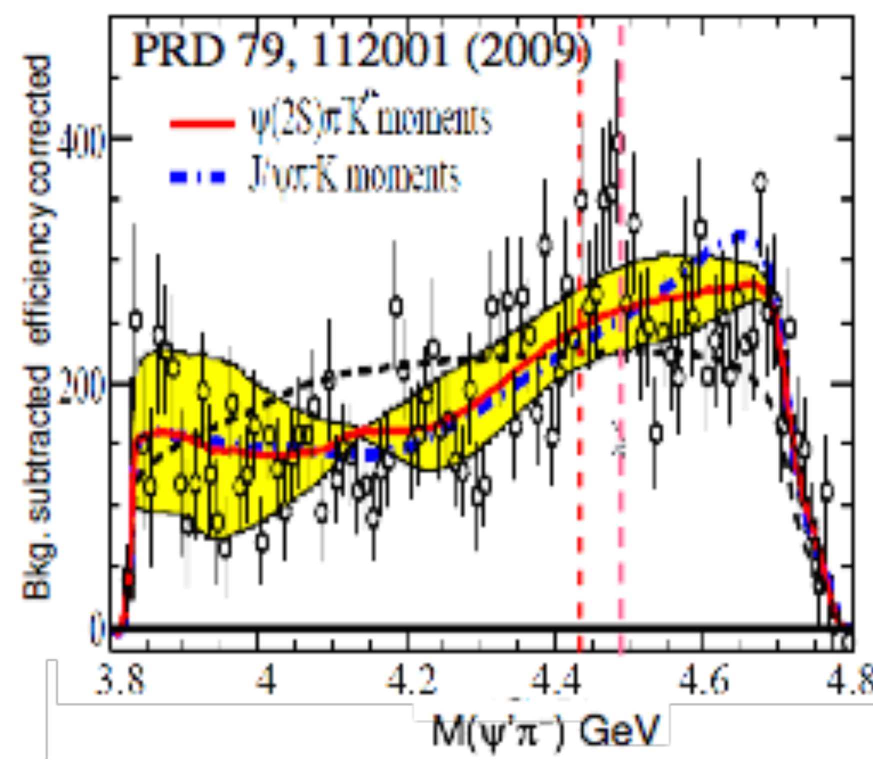
$M(Z) = 4433 \pm 4 \pm 2$ MeV

$\Gamma(Z) = 45^{+18}_{-13}{}^{+30}_{-13}$ MeV

significance 6.5σ

BaBar 2009

model-independent

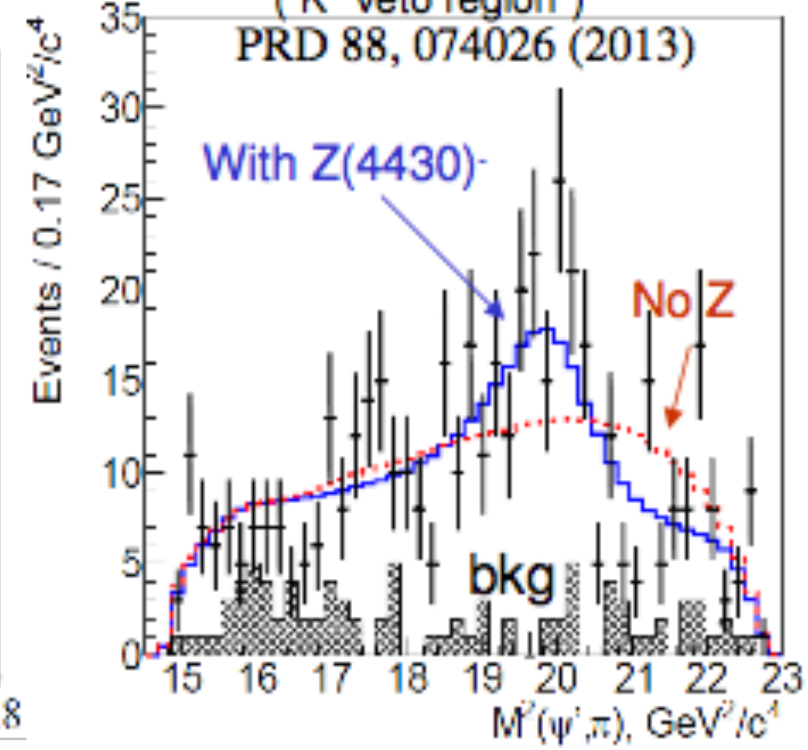


Takes Kπ mass distribution as is (no K* model) and attempts to reproduce structure in ψ(2S)π⁻. from angular momentum effects. Works within statistics.

Belle 2013

4D

("K* veto region")



$M(Z) = 4485^{+22}_{-22}{}^{+28}_{-11}$ MeV

$\Gamma(Z) = 200^{+41}_{-46}{}^{+26}_{-35}$ MeV

6.4σ (5.6σ with sys.)

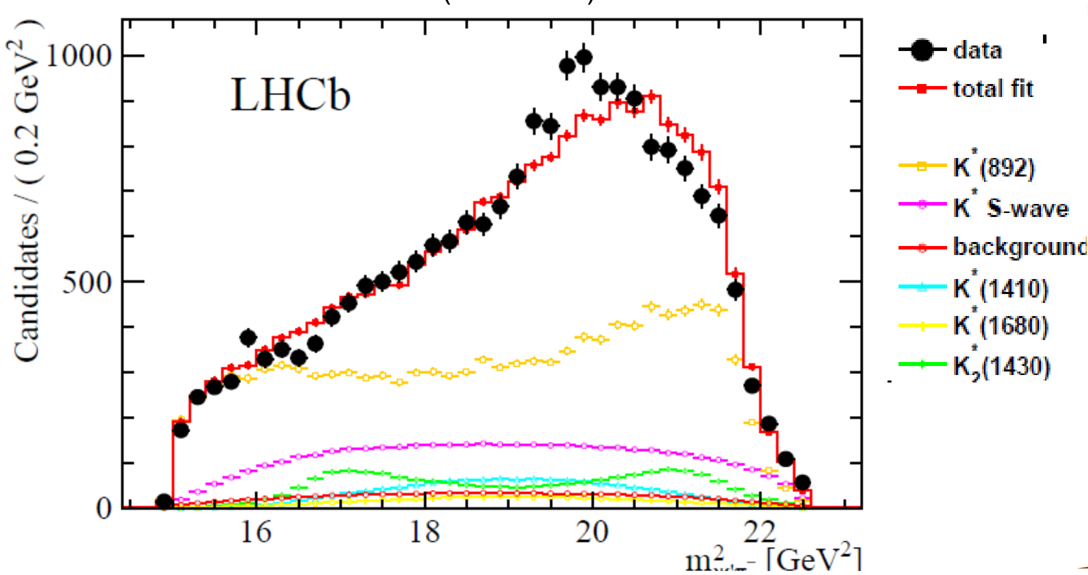
$J^P=1^+$ preferred by $>3.4\sigma$

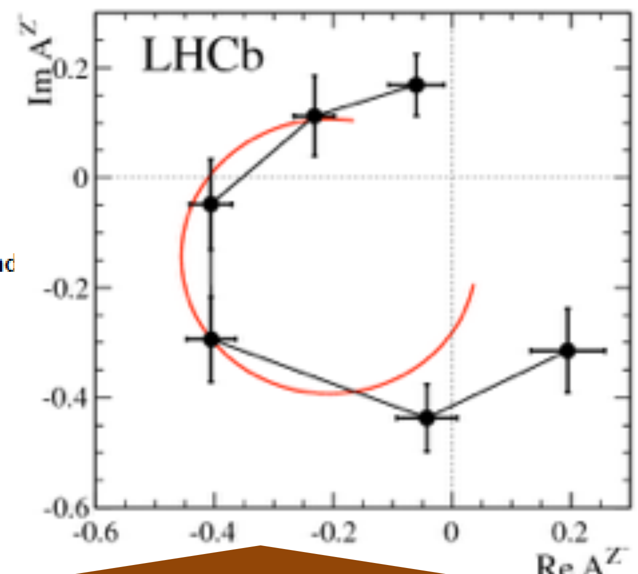
LHCb's evidence for the Z(4430) in B→ψ(2S)π⁻K⁺

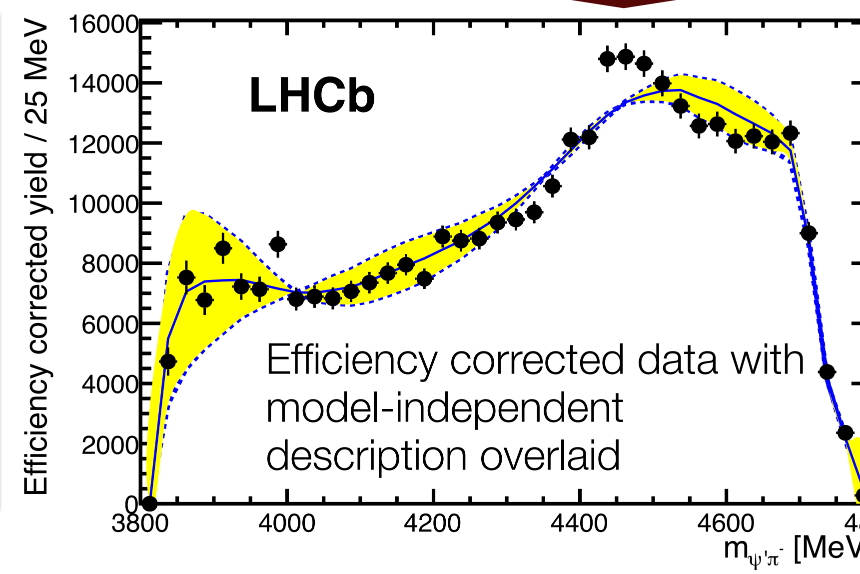
Amplitude fit:
>13.9σ in amplitude fit for Z(4430) (and >9.7σ for 1⁺ relative other J^P assignments)

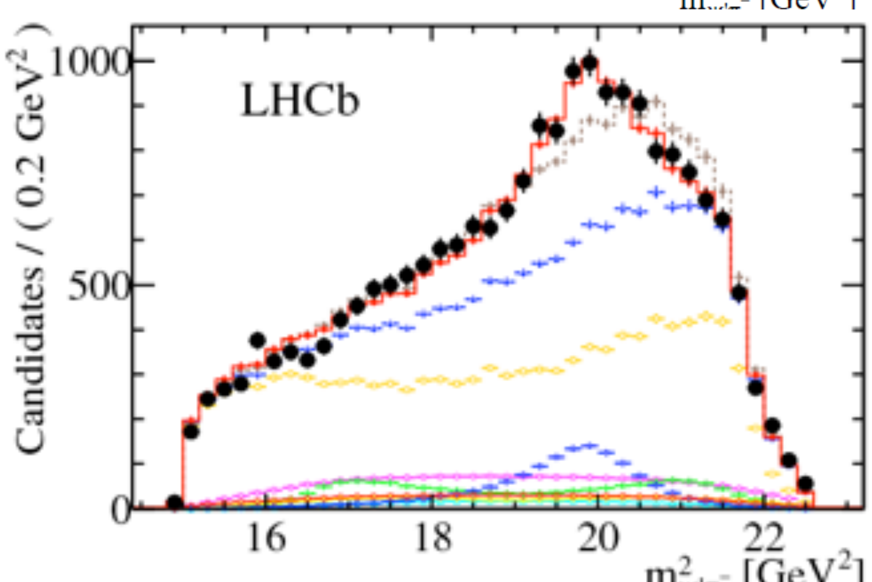
Model-independent
Model-indep. description of K* resonances (w/o Z) incompatible with data, clear excess in Z(4430) region

(“all data”)









Phase Motion
Fit where K* amplitudes are allowed to float, but Z amplitude is described model-independently by complex numbers in 6 bins of m(ψ(2S)π) confirms resonance-like phase motion

Tetraquark candidate travels the world:

LHCb confirms existence of exotic hadrons

How CERN's Discovery of Exotic Particles May Affect Astrophysics
by BRIAN KOBERLEIN *on* APRIL 10, 2014

大型强子对撞机捕获到神秘粒子Z (4430)
或许成为物质形式 "四夸克态" 存在的有力证据

2014/04/13 15:46
LHCb実験を行っている国際研究チームが、4個のクォークが結合した粒子である「Z(4430)⁻」を合成したと発表した。Z(4430) としては、初発見から7年目にしてようやく別の研究チームが存在を立証した事になる。

นักฟิสิกส์ยืนยันพบฮาดรอนสองคว้ากสองแอนตีคว้าก
WRITTEN BY NATTY_SCI ON APRIL 13, 2014. POSTED IN ฟิสิกส์, วิทยาศาสตร์
ล่าสุด เครื่อง LHCb ได้มีการศึกษาอีกครั้งและใช้ข้อมูลอนุภาคจากเครื่องโดยตรงมาวิเคราะห์ แต่นำเอาเทคนิคการวิเคราะห์ของศูนย์ปฏิบัติการวิจัยเบลล์และ BaBar มาใช้ ศาสตราจารย์ชวาร์นิคกี้และทีมงานได้ยืนยันแล้วว่า Z(4430) นั้นมีอยู่จริง และ exotic hadron ก็มีอยู่จริงด้วย

Nowa forma materii: potwierdzono istnienie egzotycznych hadronów
13-04-2014 13:08 TO TRZECI RODZAJ HADRONÓW, DOTYCHCZAS WYRÓŻNIANO BARIONY I MEZONY

"המובהקות לאותות של Z (4430) מדהימה – לפחות 13.9 סיגמה – דבר המאשר את קיומו של מצב זה" אמר דובר LHCb פיירולואיג'י קמפנה. "ניתוח ה- LHCb חשף את הטבע המהדהד של המבנים הנצפים, והוכיח כי זהו באמת חלקיק, ולא תכונה מיוחדת של הנתונים".

Confirmada l'existència d'una nova partícula subatòmica

Эксперимент LHCb окончательно доказал реальность экзотического мезона Z(4430)

PISTOLA FUMANTE DI UNA PARTICELLA A QUATTRO QUARK

LHCb kinnitas tetrakvargi olemasolu

LHC Beauty Tangkap Z (4430) Mungkin Tetraquark

SU professors test boundaries of 'new physics' with discovery of four-quark hadron
Physicist Tomasz Skwarnicki confirms existence of exotic hadron with two quarks, two anti-quarks
Apr 10, 2014 | Article by: Rob Enslin

Mystisk partikel udfordrer fysikernes kvarkmodel

Các nhà nghiên cứu tại LHC xác nhận sự tồn tại của hạt Tetraquark: tổ hợp tạo thành từ 4 quark
Thảo luận trong 'Khoa học' bắt đầu bởi ndminhduc, 15/4/14.

De LHCb heeft 't bevestigd: er bestaan exotische hadronen
10 APRIL 2014 DOOR ARIE NOUWEN • REAGEER

ISNA
خبرگزاری دانشجویان ایران، ایسنا
Iranian Students' News Agency
اخبار خوب، مفید و امید بخش مخابره کنید. مقام معظم رهبری

تاکنون کشف ذره (4430)Z در سال 2007 بشدت جنجال برانگیز بود و فیزیکدانان بر سر موجودیت یا عدم موجودیت آن اختلاف نظر داشتند

تائید کنونی ذره با استفاده از آشکارساز LHCb ماورای هرگونه تردید منطقی موجود است.

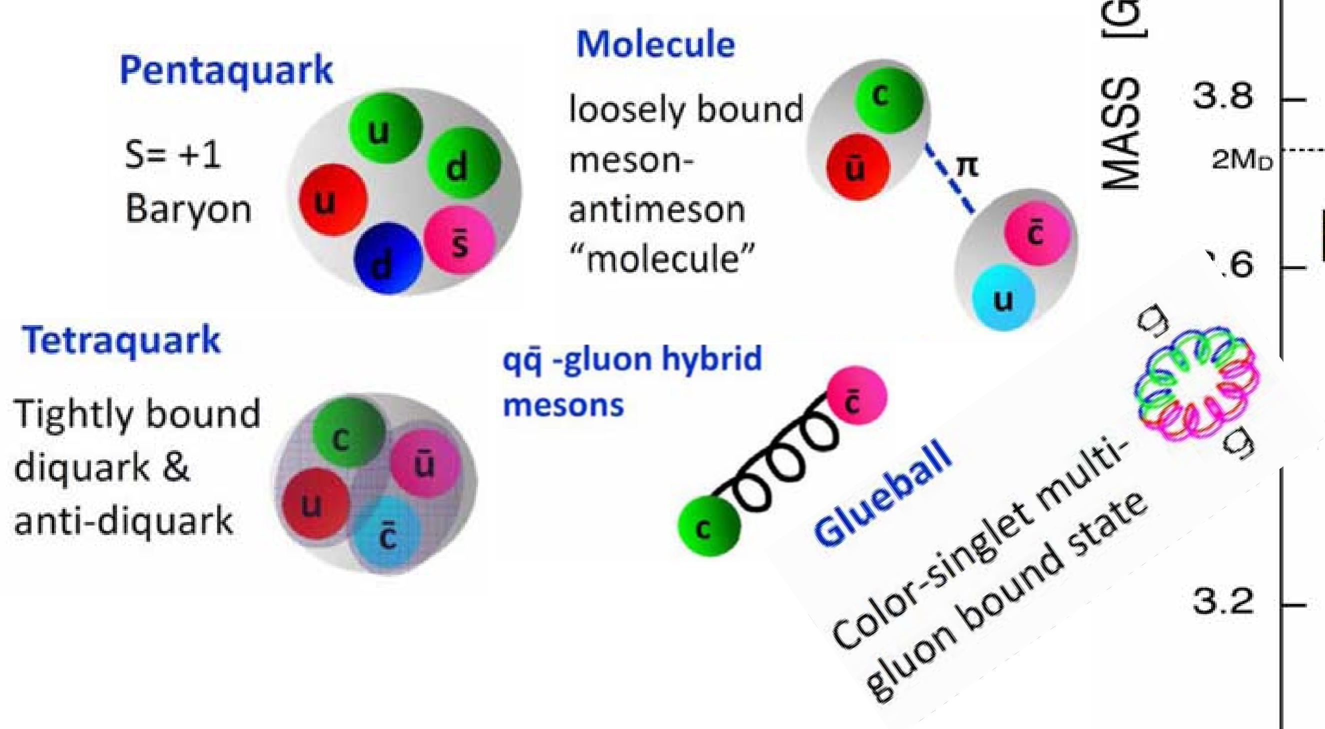
LHCb confirma la existencia de la partícula Z(4430) formada por cuatro quarks

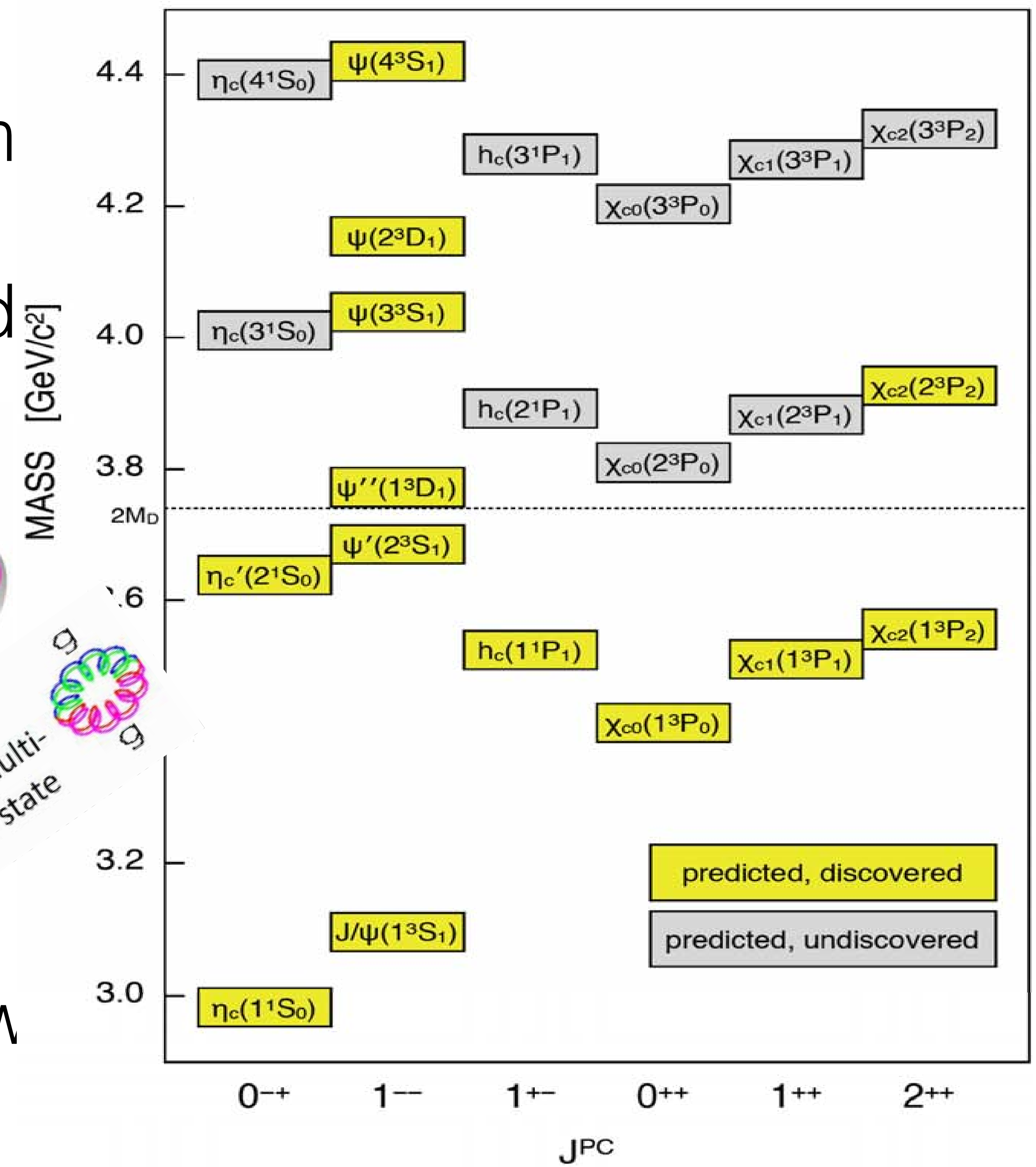
Παρασκευή, 11 Απριλίου 2014

Ο LHCb επιβεβαιώνει την ύπαρξη εξωτικού σωματιδίου, LHCb confirms existence of exotic hadrons

XYZ like states

- Plenty of new charm
- What are they? (And
- and how/where do w

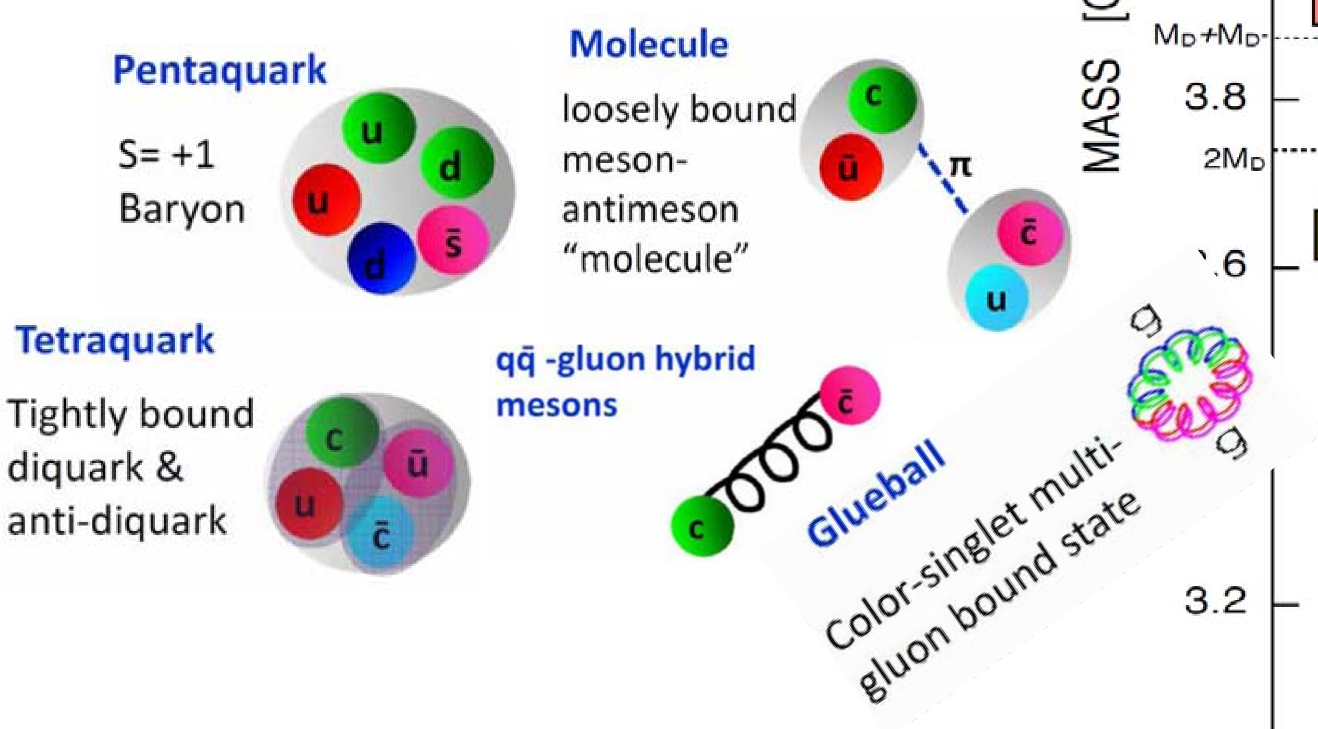


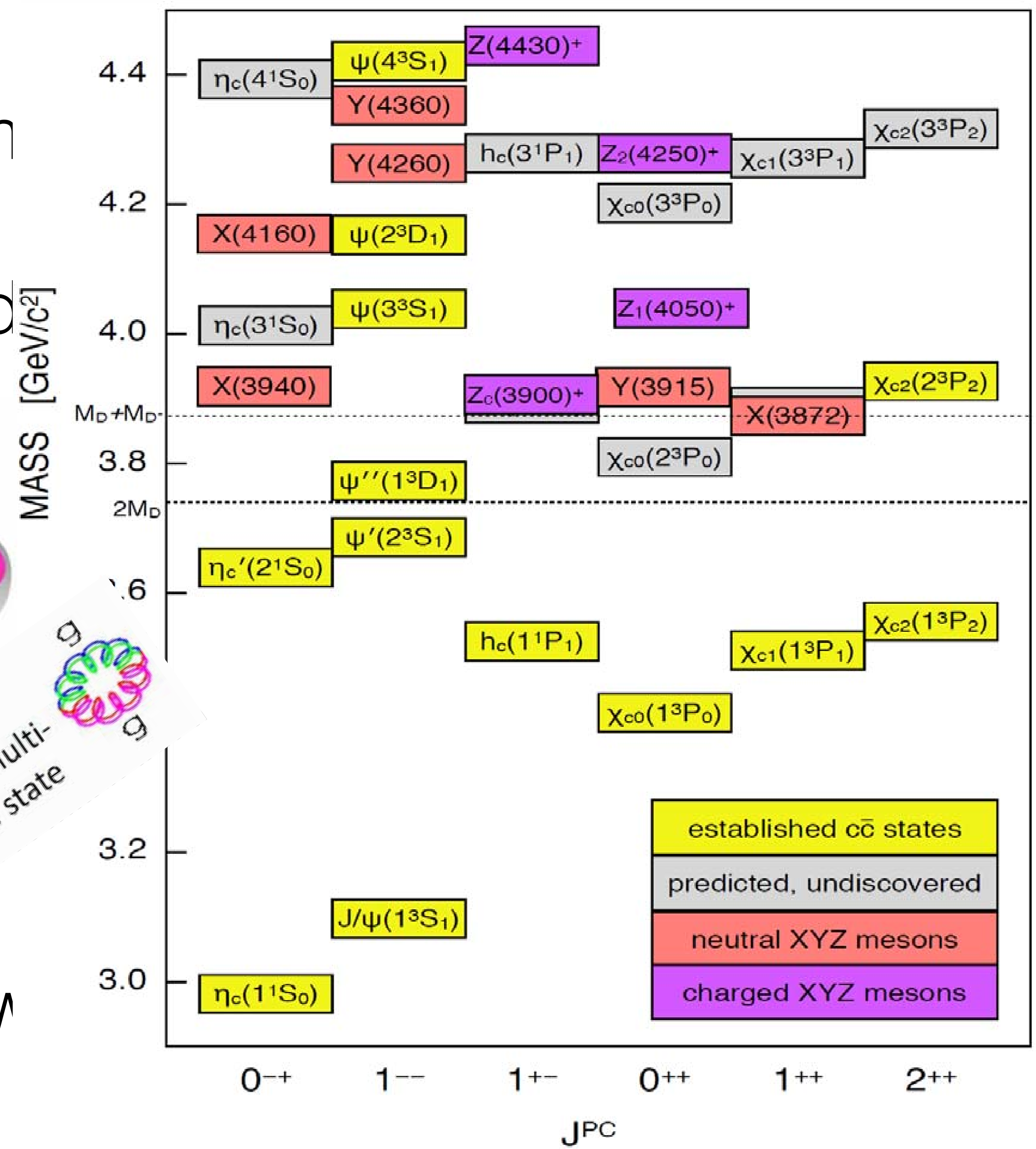


Diagrams and many results from Chengping Shen's talk at B2TIP Workshop, 28-31 October 2014, KEK

XYZ like states

- Plenty of new charm
- What are they? (And





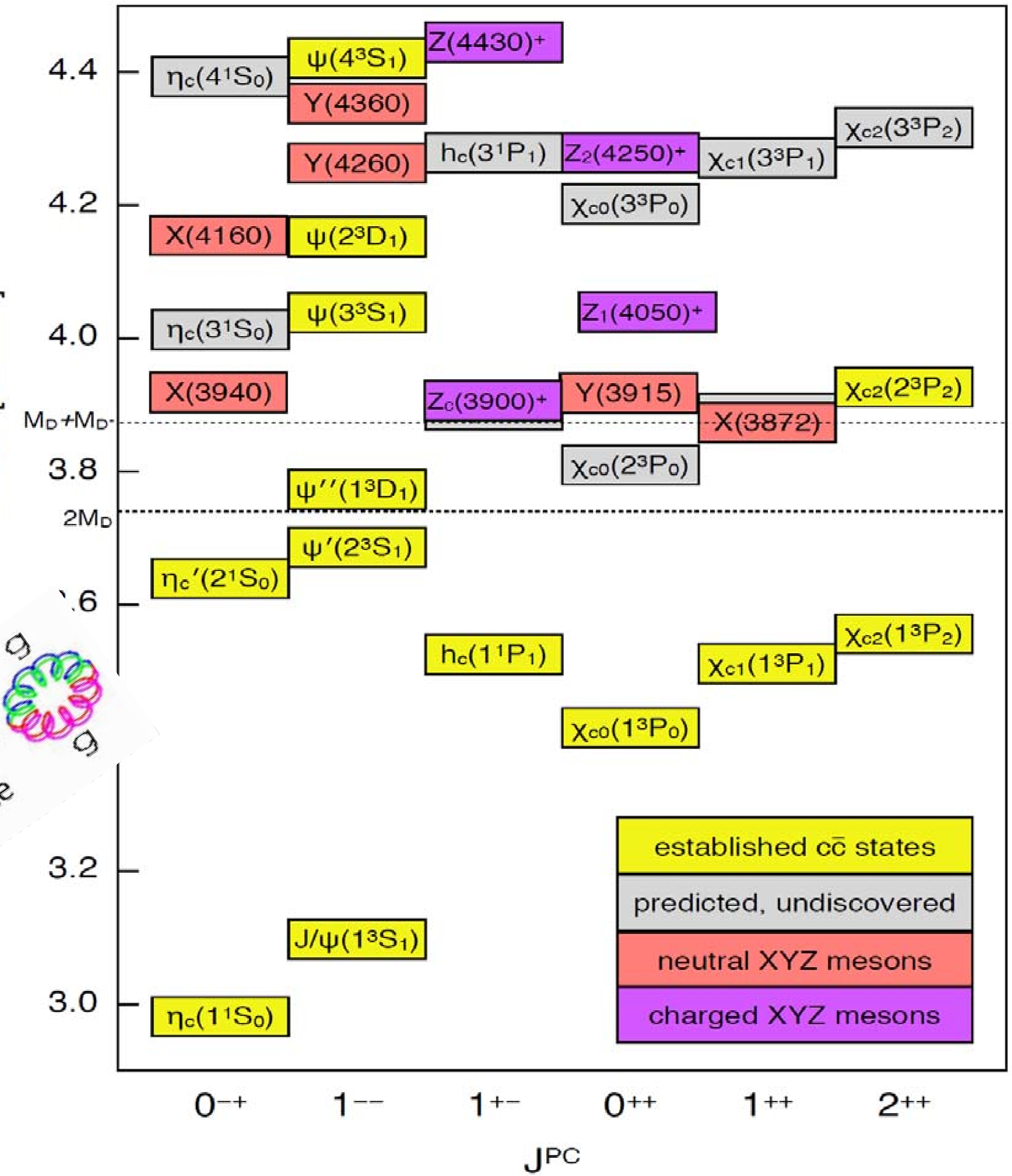
- and how/where do w

Diagrams and many results from Chengping Shen's talk at B2TIP Workshop, 28-31 October 2014, KEK

XYZ like states

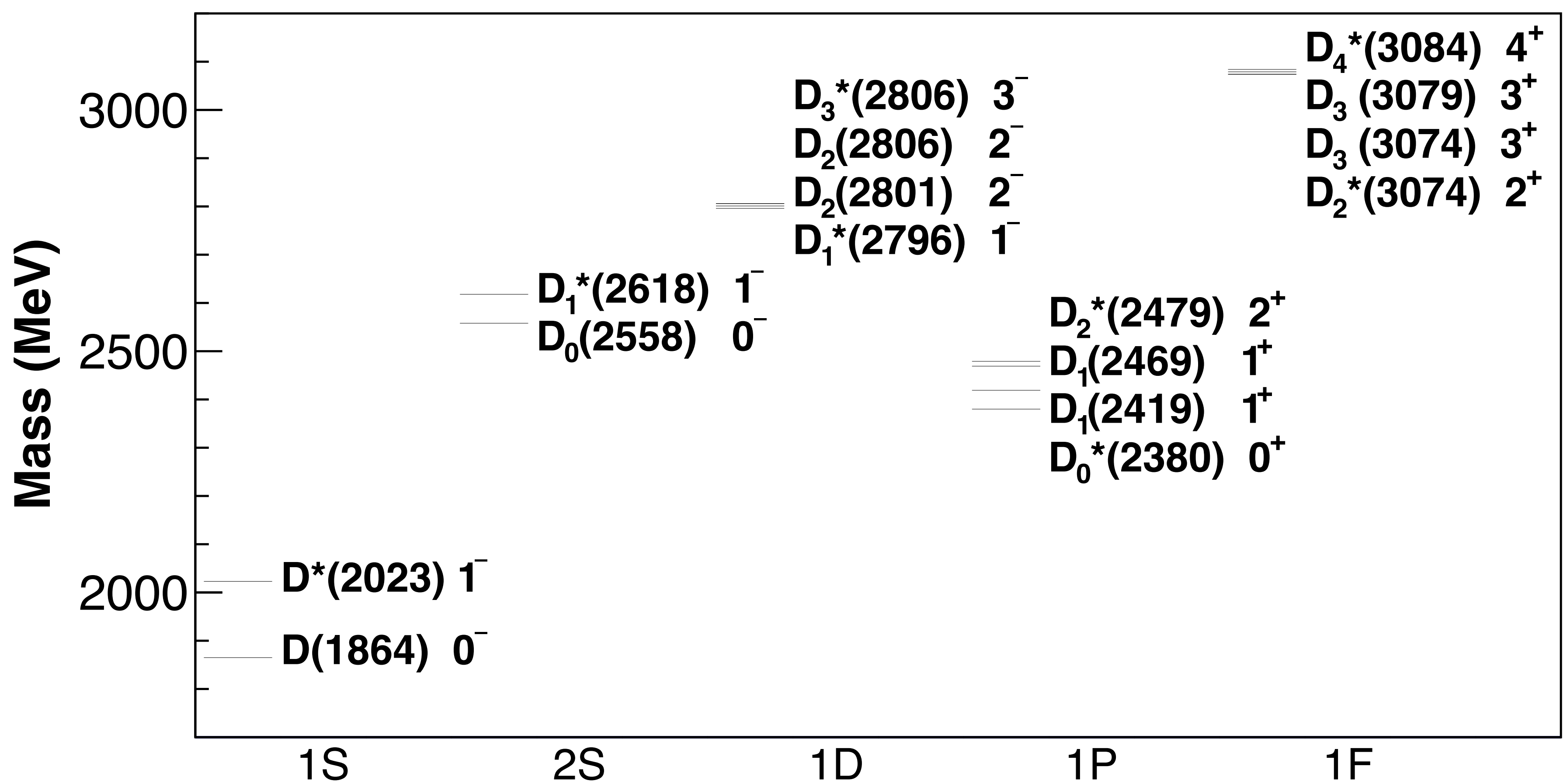
XYZ papers published in 2013 and 2014 (incomplete list)
X(3872)
LHCb: PRL 110, 222001 (2013)
BES III: Phys. Rev. Lett. 112, 092001 (2014)
BELLE: Phys. Rev. Lett. 110 252002 (2013)
Y(4008, 4260, 4360, 4660)
BES III Phys. Rev. Lett. 110, 252001 (2013)
Z(3900, 4020, 4200, 4430)
BELLE: Phys. Rev. Lett. 110 252002 (2013)
BELLE: Phys. Rev. D 89, 072015 (2014)
LHCb (2014): Phys.Rev.Lett. 112 (2014) 222002
BELLE (2014): Phys.Rev. D88 (2013) 074026
(no) Zcs
BES III Phys. Rev. Lett. 111, 242001 (2013)
BaBar: PRD 89, 111103(R) (2014)
BES III: PRL 111, 032001 (2013)X(3823)

- and how/where do w



Diagrams and many results from Chengping Shen's talk at B2TIP Workshop, 28-31 October 2014, KEK

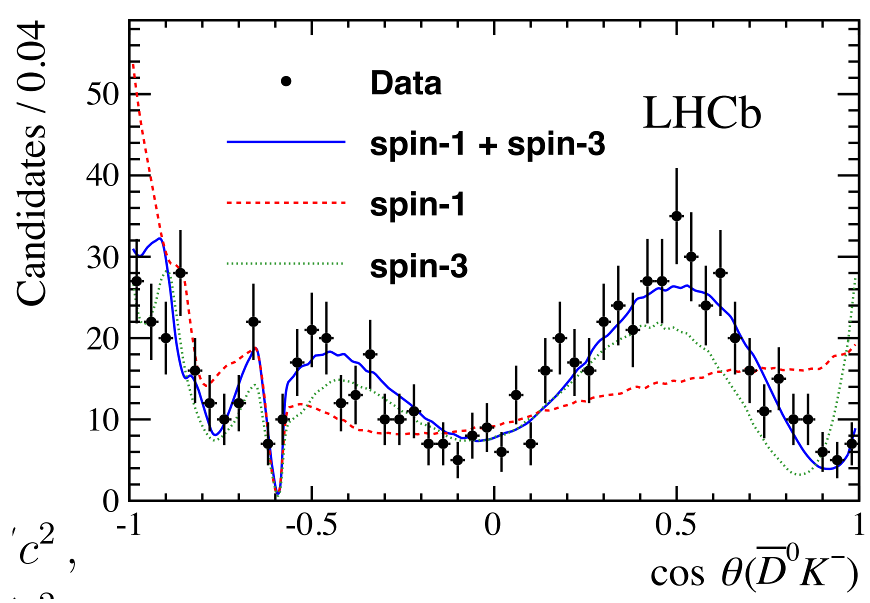
Spectroscopy



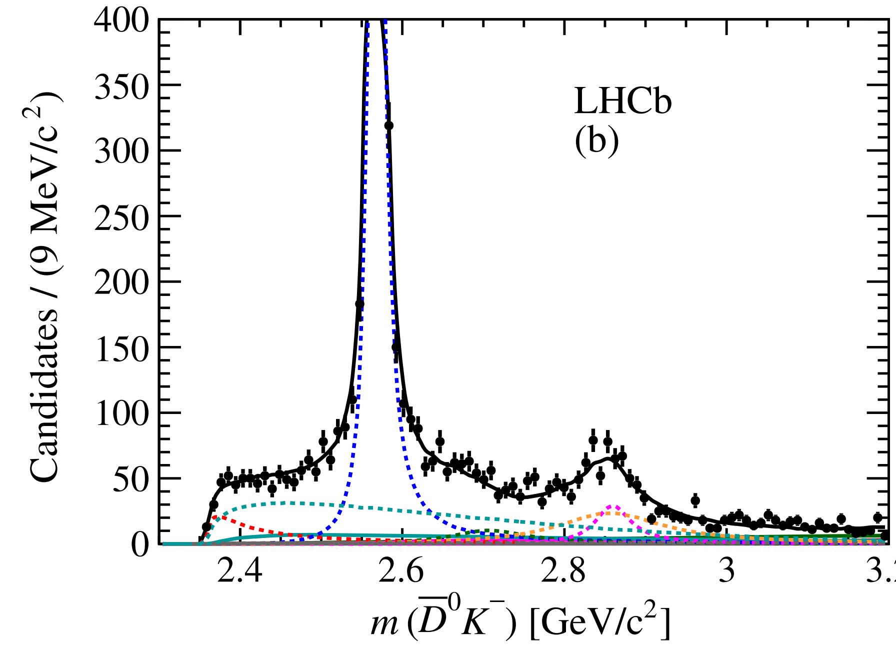
$B_s \to \bar{D}K^-\pi^+$

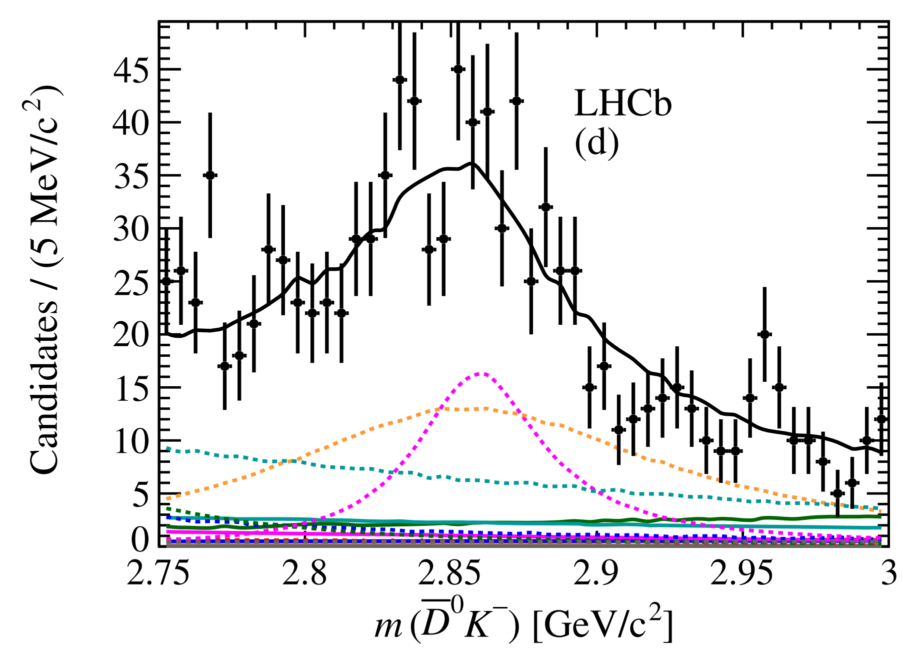
- Amongst many new results: The $D^*_{sJ}(2860)$ does exist - not only once, but twice:

  $B \to \bar{D}K^-\pi^+$ Dalitz plot analysis finds two particles in the same mass region, one with spin 1, one with spin 3.



DK spectra in $B \to \bar{D}K^-\pi^+$ at LHCb (Phys.Rev. D90 (2014) 072003)





$B_s \to J/\psi\pi\pi$ CP content

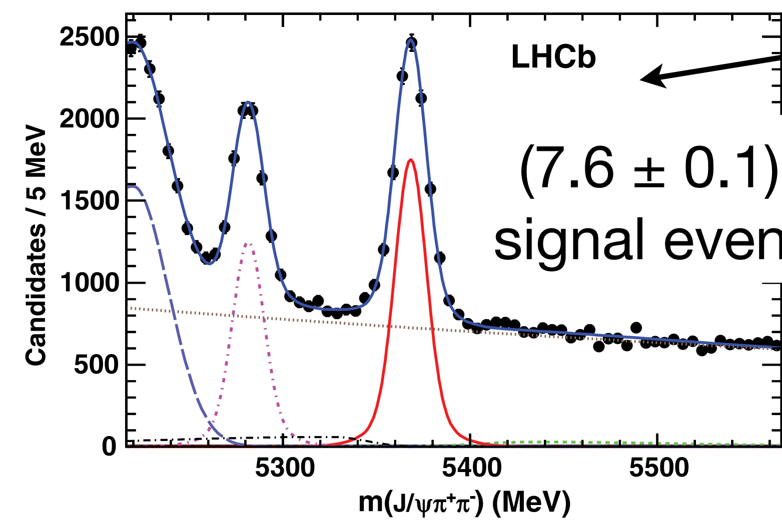
PRD 86, 052006 (2012)

- Amplitude analysis to evaluate the CP content of $B_s \to J/\psi\pi\pi$
- 4-dimensional analysis: 2 masses, 2 helicity angles.
- Result:

Resonance	Normalized fraction (%)
$f_0(980)$	69.7 ± 2.3
$f_0(1370)$	21.2 ± 2.7
non-resonant $\pi^+\pi^-$	8.4 ± 1.5
$f_2(1270)$, $\Lambda = 0$	0.49 ± 0.16
$f_2(1270)$, $\|\Lambda\| = 1$	0.21 ± 0.65

- Nearly all (>97.7% at 95 C.L.) CP-odd
- ⇒ No need for angular analysis to extract ϕ_s!

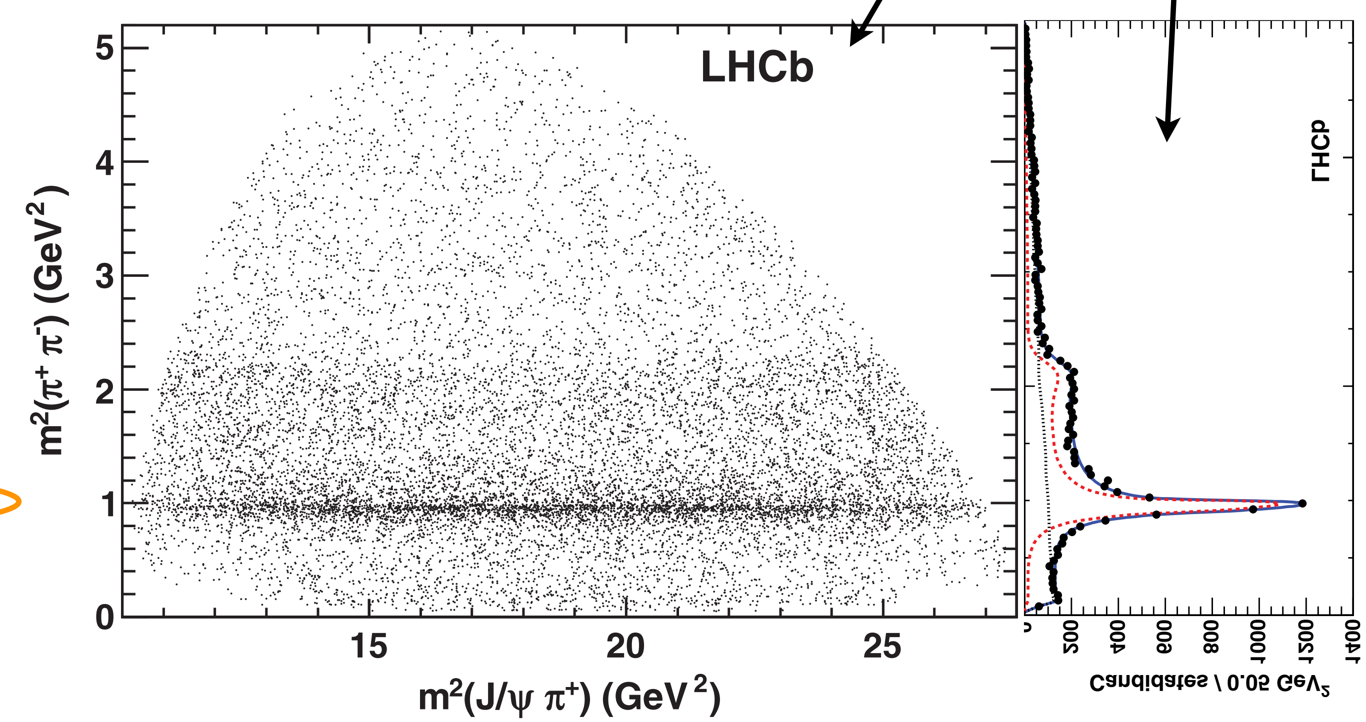
Reconstructed B_s mass



(7.6 ± 0.1)k signal events

Dalitz Plot

$m^2(\pi\pi)$



(see also arXiv:1302.1213 for an amplitude analysis of $B_s \to J/\psi KK$)

Model-independent check

PRD 86, 052006 (2012)

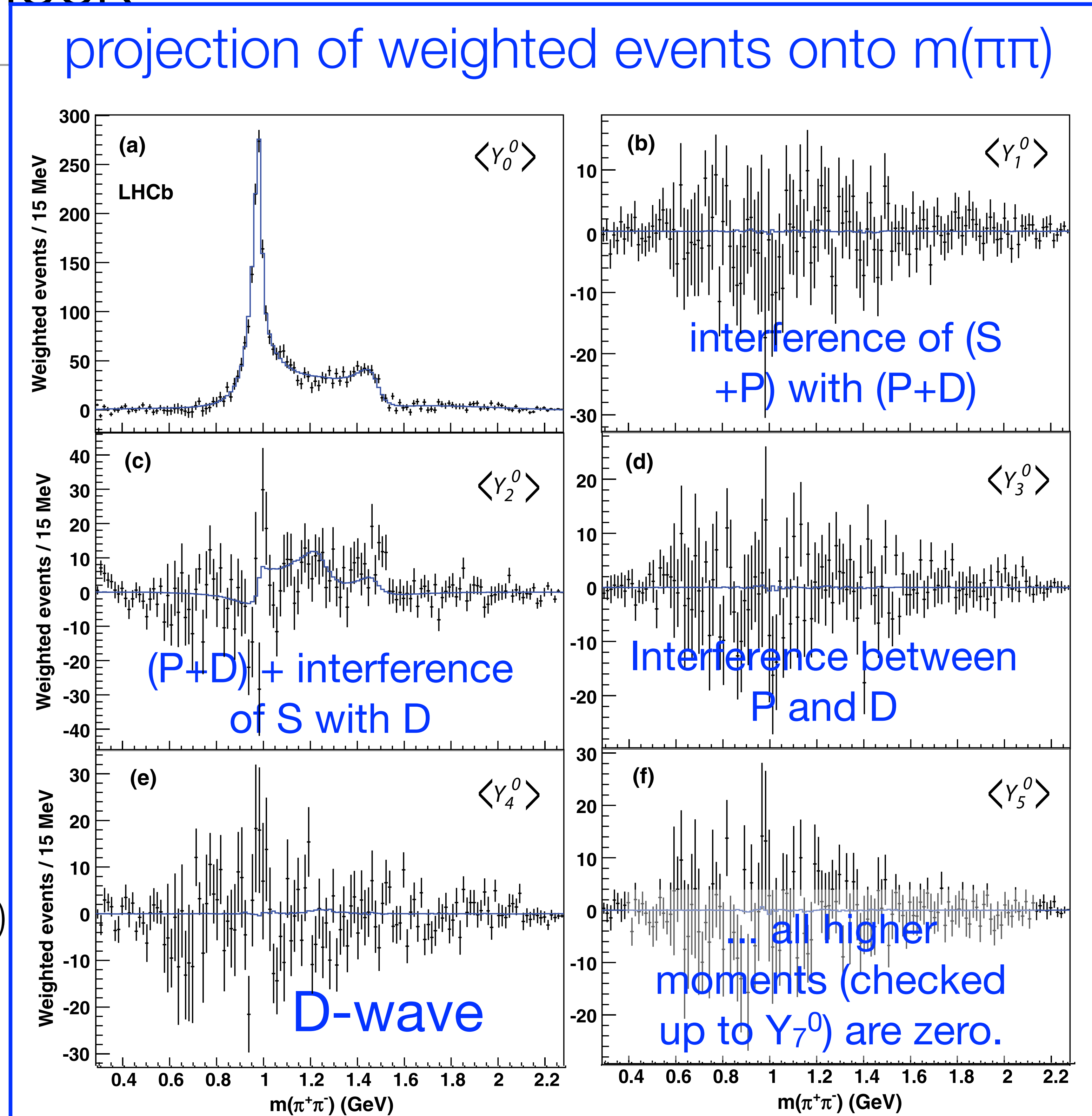
- Decay rate can be expressed in terms of spherical harmonics

$$\frac{d\Gamma}{d(\cos\theta)} = a_l^m Y_l^m(\cos\theta)$$

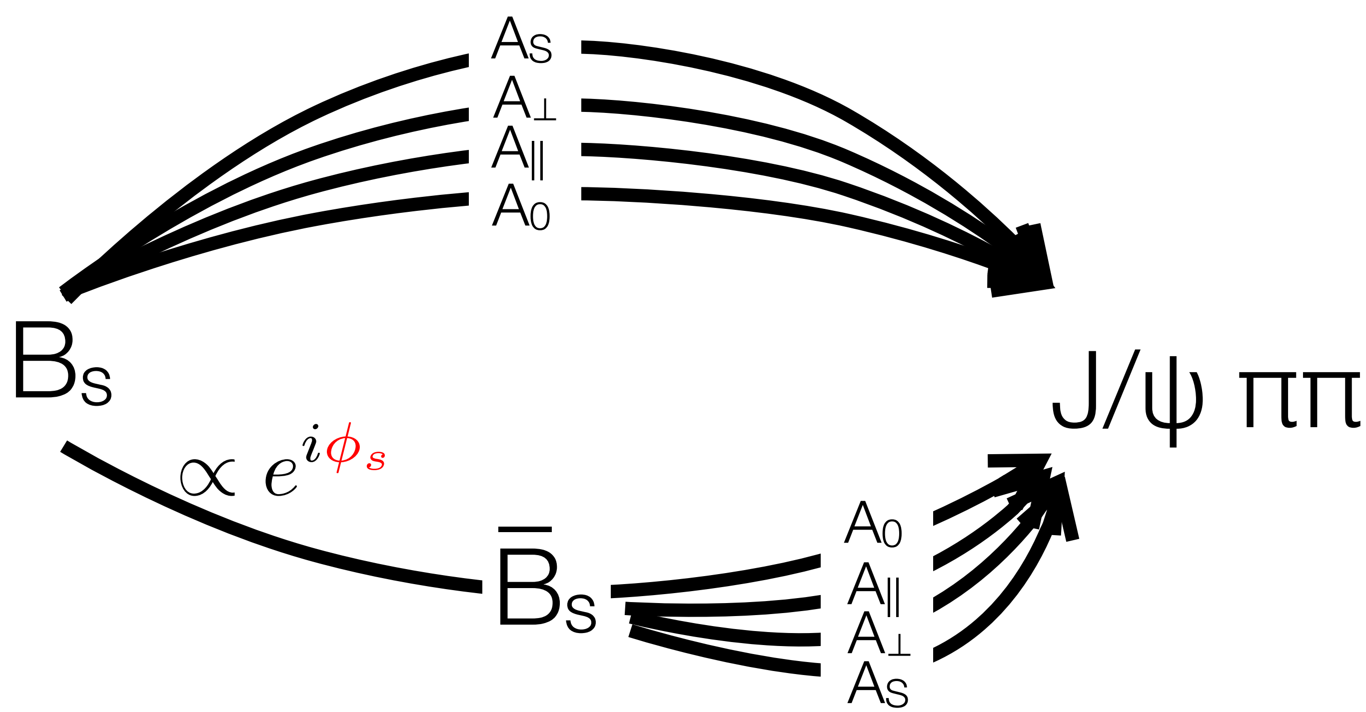
- These can be related to different S, P, D amplitude components.
- To project out a given component:

$$a_l^m = \int Y_l^m(\cos\theta)\,\frac{d\Gamma}{d(\cos\theta)}\,d(\cos\theta) \approx \sum_{\text{events}} Y_l^m(\cos\theta_i)$$

= sum of weighted events

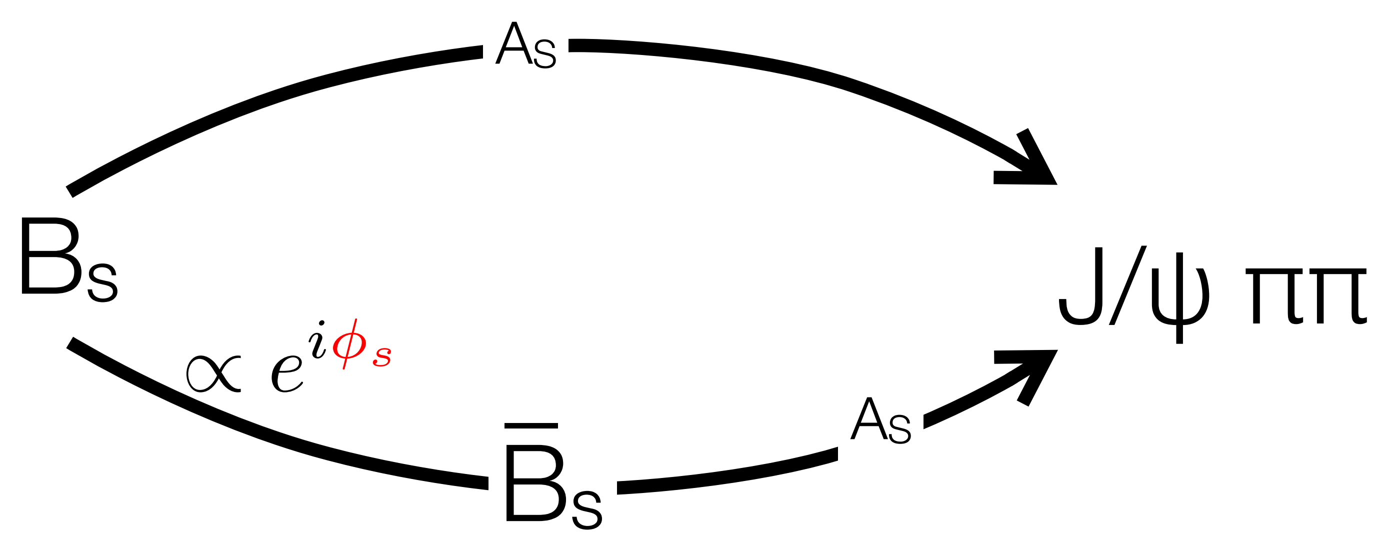


$B_s \to J/\psi\, \pi\pi$ for ϕ_s



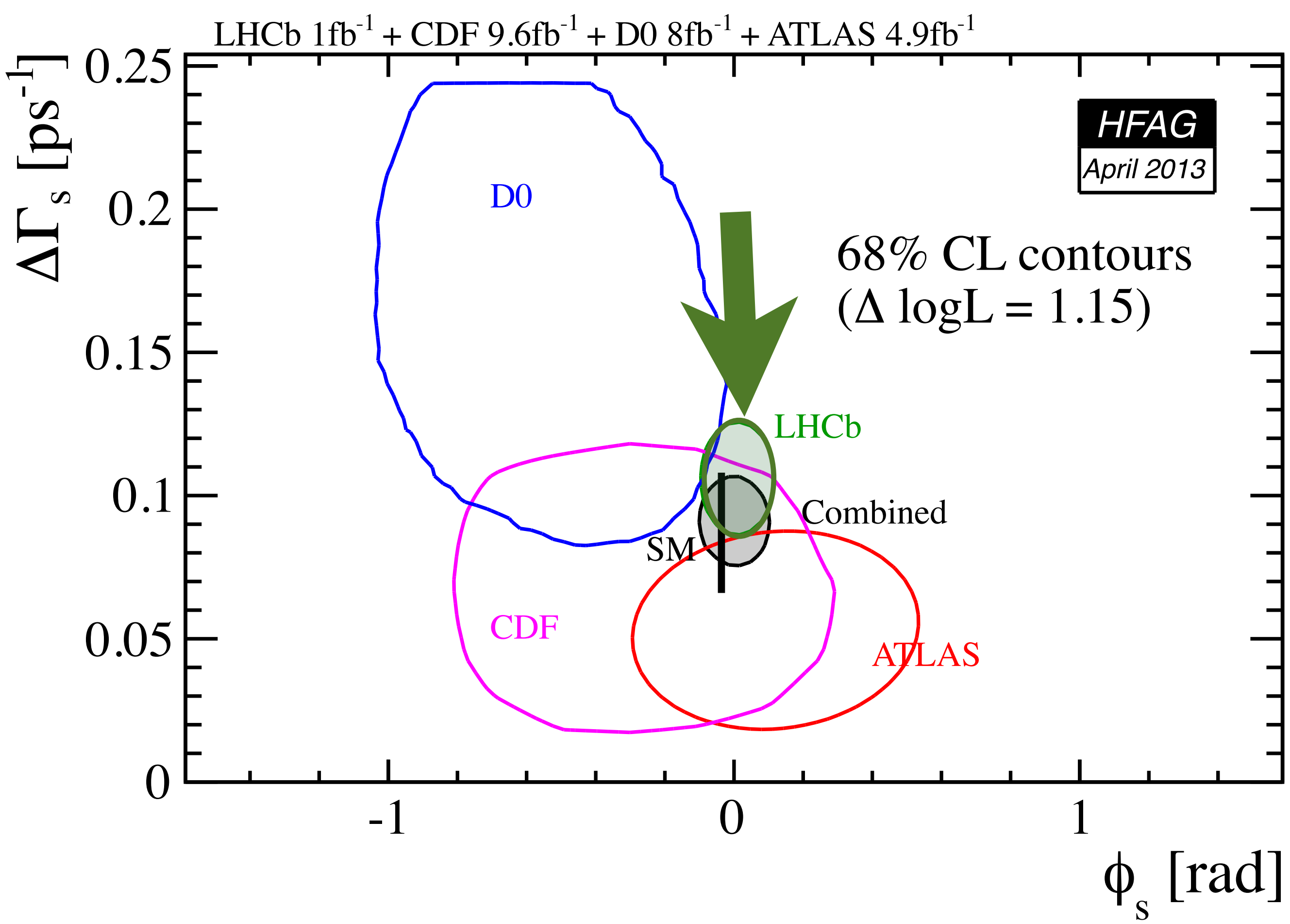
k	$f_k(\theta_\mu, \theta_K, \varphi_h)$	N_k	a_k	b_k	c_k	d_k
1	$2\cos^2\theta_K \sin^2\theta_\mu$	$\lvert A_0\rvert^2$	1	$\boxed{D}$	$\boxed{C}$	$\boxed{-S}$
2	$\sin^2\theta_K \left(1 - \sin^2\theta_\mu \cos^2\varphi_h\right)$	$\lvert A_\parallel\rvert^2$	1	$\boxed{D}$	$\boxed{C}$	$\boxed{-S}$
3	$\sin^2\theta_K \left(1 - \sin^2\theta_\mu \sin^2\varphi_h\right)$	$\lvert A_\perp\rvert^2$	1	$\boxed{-D}$	$\boxed{C}$	$\boxed{S}$
4	$\sin^2\theta_K \sin^2\theta_\mu \sin 2\varphi_h$	$\lvert A_\parallel A_\perp\rvert$	$\boxed{C}\sin(\delta_\perp - \delta_\parallel)$	$\boxed{S}\cos(\delta_\perp - \delta_\parallel)$	$\sin(\delta_\perp - \delta_\parallel)$	$\boxed{D}\cos(\delta_\perp - \delta_\parallel)$
5	$\frac{1}{2}\sqrt{2}\sin 2\theta_K \sin 2\theta_\mu \cos\varphi_h$	$\lvert A_0 A_\parallel\rvert$	$\cos(\delta_\parallel - \delta_0)$	$\boxed{D}\cos(\delta_\parallel - \delta_0)$	$\boxed{C}\cos(\delta_\parallel - \delta_0)$	$\boxed{-S}\cos(\delta_\parallel - \delta_0)$
6	$-\frac{1}{2}\sqrt{2}\sin 2\theta_K \sin 2\theta_\mu \sin\varphi_h$	$\lvert A_0 A_\perp\rvert$	$\boxed{C}\sin(\delta_\perp - \delta_0)$	$\boxed{S}\cos(\delta_\perp - \delta_0)$	$\sin(\delta_\perp - \delta_0)$	$\boxed{D}\cos(\delta_\perp - \delta_0)$
7	$\frac{2}{3}\sin^2\theta_\mu$	$\lvert A_S\rvert^2$	1	$\boxed{-D}$	$\boxed{C}$	$\boxed{S}$
8	$\frac{1}{3}\sqrt{6}\sin\theta_K \sin 2\theta_\mu \cos\varphi_h$	$\lvert A_S A_\parallel\rvert$	$\boxed{C}\cos(\delta_\parallel - \delta_S)$	$\boxed{S}\sin(\delta_\parallel - \delta_S)$	$\cos(\delta_\parallel - \delta_S)$	$\boxed{D}\sin(\delta_\parallel - \delta_S)$
9	$-\frac{1}{3}\sqrt{6}\sin\theta_K \sin 2\theta_\mu \sin\varphi_h$	$\lvert A_S A_\perp\rvert$	$\sin(\delta_\perp - \delta_S)$	$\boxed{-D}\sin(\delta_\perp - \delta_S)$	$\boxed{C}\sin(\delta_\perp - \delta_S)$	$\boxed{S}\sin(\delta_\perp - \delta_S)$
10	$\frac{4}{3}\sqrt{3}\cos\theta_K \sin^2\theta_\mu$	$\lvert A_S A_0\rvert$	$\boxed{C}\cos(\delta_0 - \delta_S)$	$\boxed{S}\sin(\delta_0 - \delta_S)$	$\cos(\delta_0 - \delta_S)$	$\boxed{D}\sin(\delta_0 - \delta_S)$

$B_s \to J/\psi\, \pi\pi$ for ϕ_s



k	$f_k(\theta_\mu, \theta_K, \varphi_h)$	N_k	a_k	b_k	c_k	d_k
1	$2\cos^2\theta_K \sin^2\theta_\mu$	$\lvert A_0\rvert^2$	1	D	C	$-S$
2	$\sin^2\theta_K\left(1-\sin^2\theta_\mu\cos^2\varphi_h\right)$	$\lvert A_\parallel\rvert^2$	1	D	C	$-S$
3	$\sin^2\theta_K\left(1-\sin^2\theta_\mu\sin^2\varphi_h\right)$	$\lvert A_\perp\rvert^2$	1	$-D$	C	S
4	$\sin^2\theta_K\sin^2\theta_\mu\sin 2\varphi_h$	$\lvert A_\parallel A_\perp\rvert$	$\boxed{C}\sin(\delta_\perp-\delta_\parallel)$	$\boxed{S}\cos(\delta_\perp-\delta_\parallel)$	$\sin(\delta_\perp-\delta_\parallel)$	$\boxed{D}\cos(\delta_\perp-\delta_\parallel)$
5	$\frac{1}{2}\sqrt{2}\sin 2\theta_K\sin 2\theta_\mu\cos\varphi_h$	$\lvert A_0 A_\parallel\rvert$	$\cos(\delta_\parallel-\delta_0)$	$\boxed{D}\cos(\delta_\parallel-\delta_0)$	$\boxed{C}\cos(\delta_\parallel-\delta_0)$	$\boxed{-S}\cos(\delta_\parallel-\delta_0)$
6	$-\frac{1}{2}\sqrt{2}\sin 2\theta_K\sin 2\theta_\mu\sin\varphi_h$	$\lvert A_0 A_\perp\rvert$	$\boxed{C}\sin(\delta_\perp-\delta_0)$	$\boxed{S}\cos(\delta_\perp-\delta_0)$	$\sin(\delta_\perp-\delta_0)$	$\boxed{D}\cos(\delta_\perp-\delta_0)$
7	$\frac{2}{3}\sin^2\theta_\mu$	$\lvert A_S\rvert^2$	1	$\boxed{-D}$	$\boxed{C}$	$\boxed{S}$
8	$\frac{1}{3}\sqrt{6}\sin\theta_K\sin 2\theta_\mu\cos\varphi_h$	$\lvert A_S A_\parallel\rvert$	$\boxed{C}\cos(\delta_\parallel-\delta_S)$	$\boxed{S}\sin(\delta_\parallel-\delta_S)$	$\cos(\delta_\parallel-\delta_S)$	$\boxed{D}\sin(\delta_\parallel-\delta_S)$
9	$-\frac{1}{3}\sqrt{6}\sin\theta_K\sin 2\theta_\mu\sin\varphi_h$	$\lvert A_S A_\perp\rvert$	$\sin(\delta_\perp-\delta_S)$	$\boxed{-D}\sin(\delta_\perp-\delta_S)$	$\boxed{C}\sin(\delta_\perp-\delta_S)$	$\boxed{S}\sin(\delta_\perp-\delta_S)$
10	$\frac{4}{3}\sqrt{3}\cos\theta_K\sin^2\theta_\mu$	$\lvert A_S A_0\rvert$	$\boxed{C}\cos(\delta_0-\delta_S)$	$\boxed{S}\sin(\delta_0-\delta_S)$	$\cos(\delta_0-\delta_S)$	$\boxed{D}\sin(\delta_0-\delta_S)$

Combined $B_s \to J/\psi$ KK and $B_s \to J/\psi\ \pi\pi$ for ϕ_s



arXiv:1304.2600 (2013)

supersedes previous results:
Phys. Rev. Lett. 108 (2012) 101803
Physics Letters B 713 (2012) 378

ϕ_s very sensitive to NP. But no NP effects seen, yet...

$\Delta\Gamma_s$ less sensitive to NP ($\propto \cos(\phi^{new})$), but impressive validation of HQE calculation.

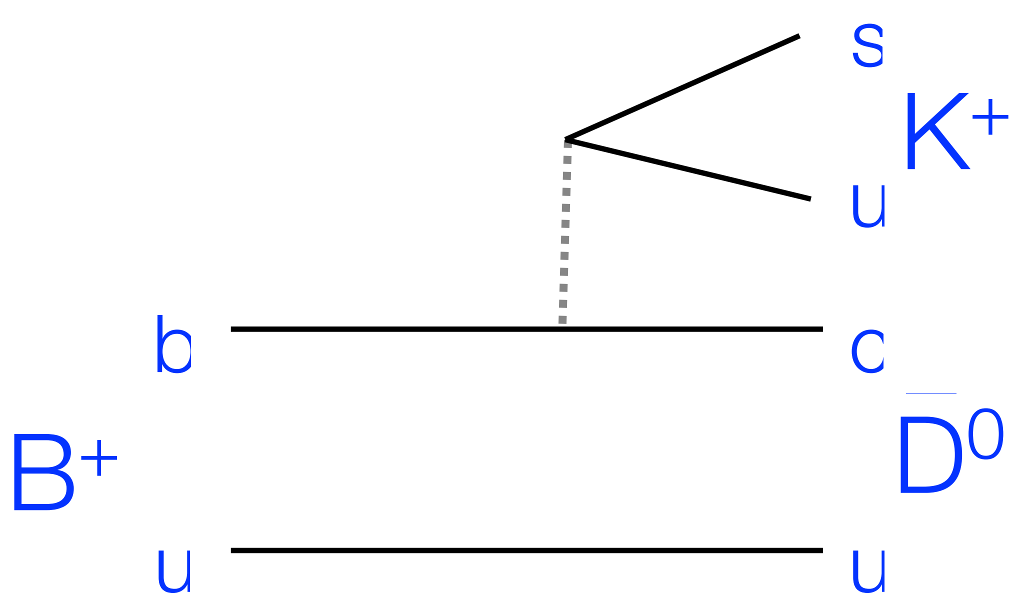
SM: $\phi_s^{SM} = -0.036 \pm 0.002$ rad

LHCb:

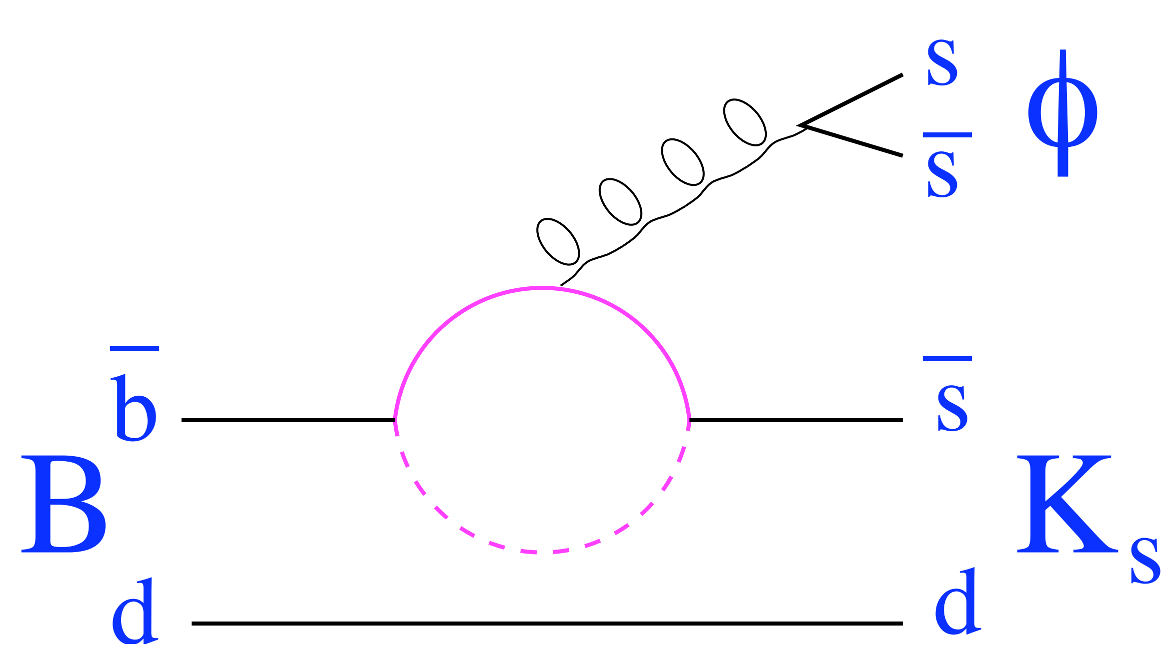
$$\phi_s = 0.07 \pm 0.09\ (\text{stat}) \pm 0.01\ (\text{syst})\ \text{rad},$$
$$\Gamma_s \equiv (\Gamma_L + \Gamma_H)/2 = 0.663 \pm 0.005\ (\text{stat}) \pm 0.006\ (\text{syst})\ \text{ps}^{-1}$$
$$\Delta\Gamma_s \equiv \Gamma_L - \Gamma_H = 0.100 \pm 0.016\ (\text{stat}) \pm 0.003\ (\text{syst})\ \text{ps}^{-1}$$

Loops vs Trees

- Expect no New Physics in Trees

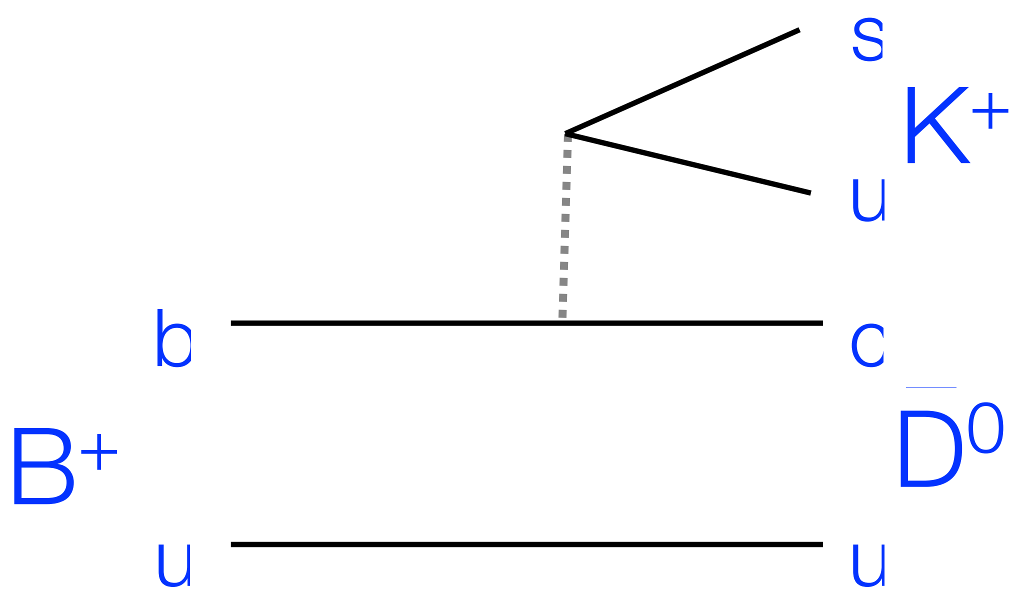


- New Physics in loops?

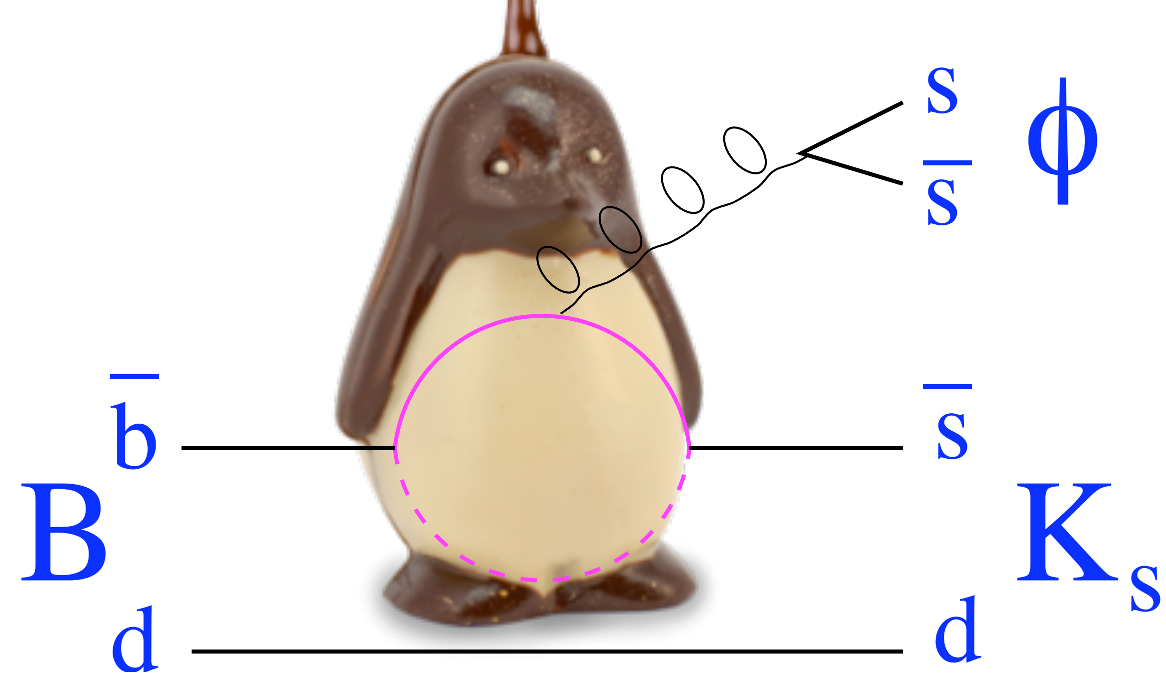


Loops vs Trees

- Expect no New Physics in Trees



- New Physics in loops?



Can penguins be bad?

http://youtu.be/5IjmOSFtoJc

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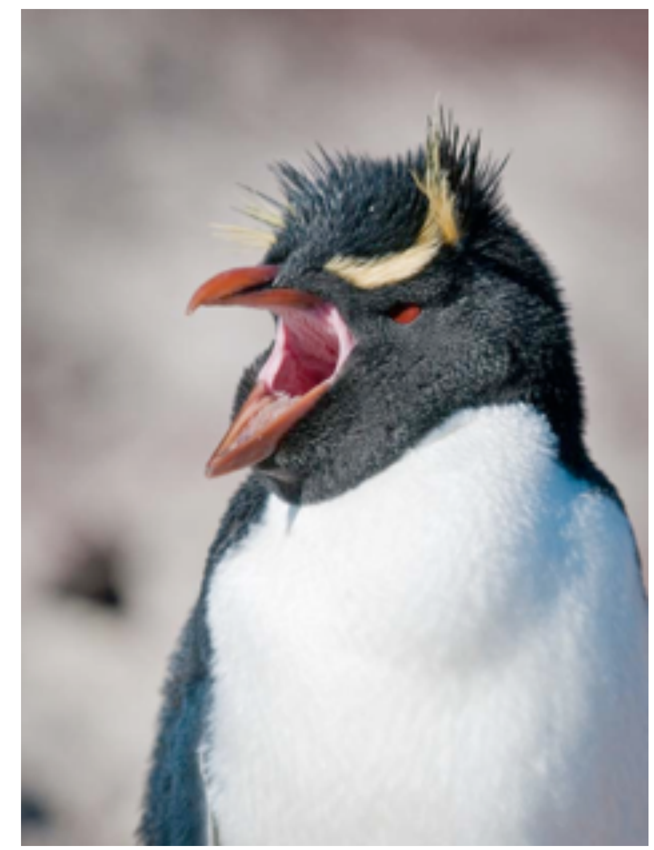
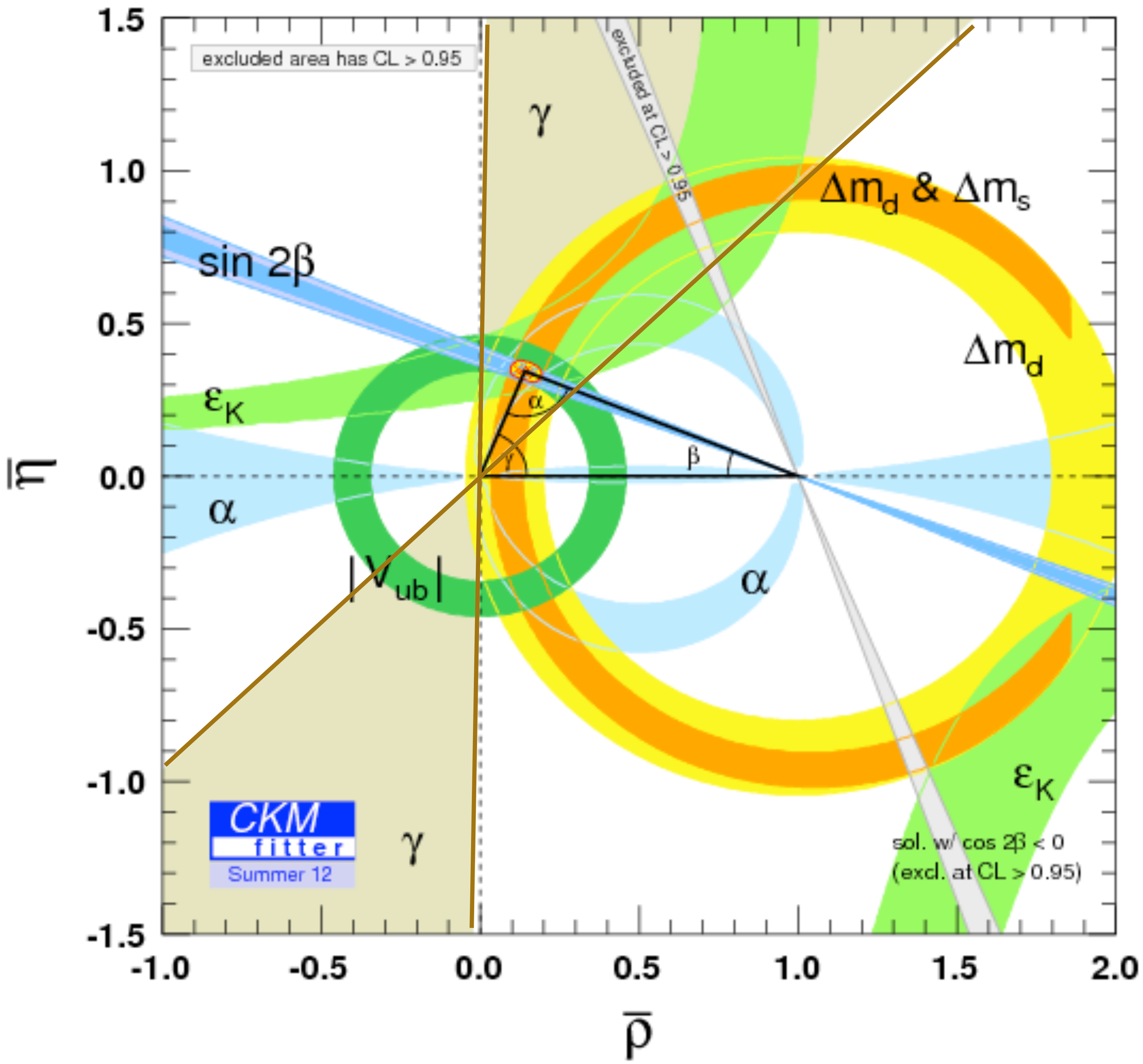


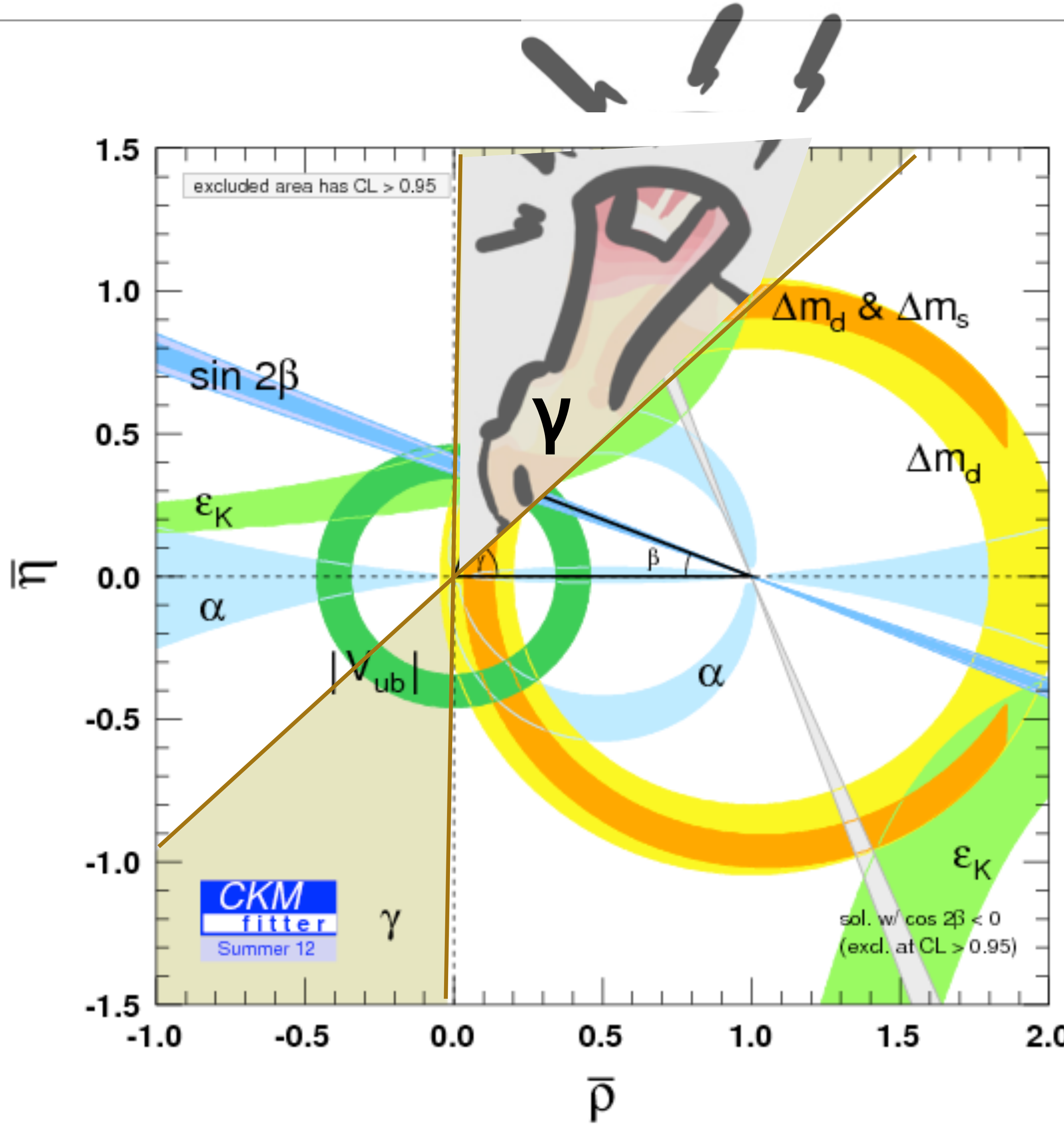
They can.

Measuring γ



excluded area has CL > 0.95
excluded at CL > 0.95
γ
Δm_d & Δm_s
sin 2β
Δm_d
ε_K
α
β
γ
α
$|V_{ub}|$
α
ε_K
CKM fitter
Summer 12
γ
sol. w/ cos 2β < 0
(excl. at CL > 0.95)
$\bar{\eta}$
$\bar{\rho}$
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
-1.0
-0.5
0.0
0.5
1.0
1.5
2.0

Measuring γ



excluded area has CL > 0.95
sin 2β
γ
Δm_d & Δm_s
Δm_d
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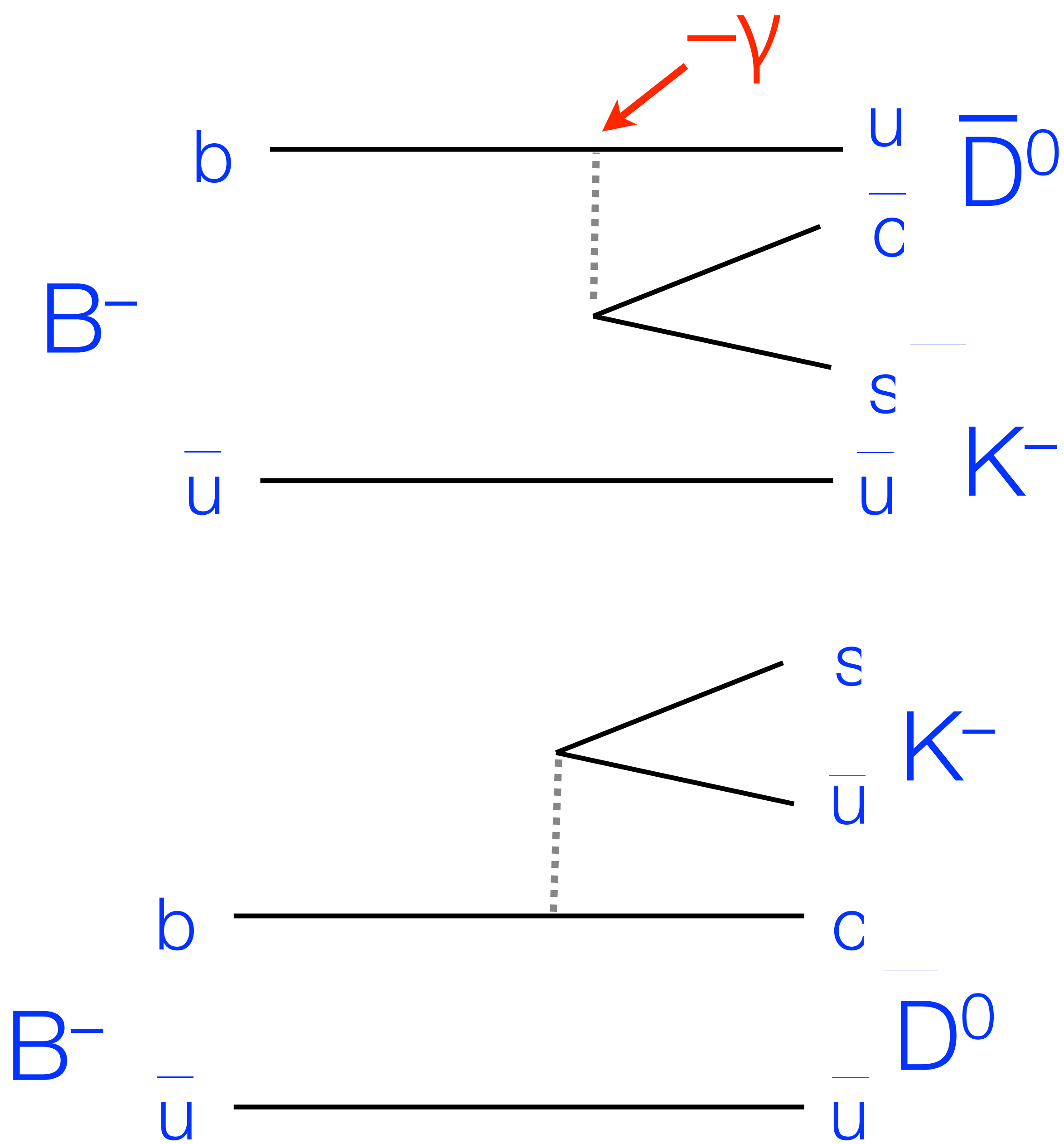
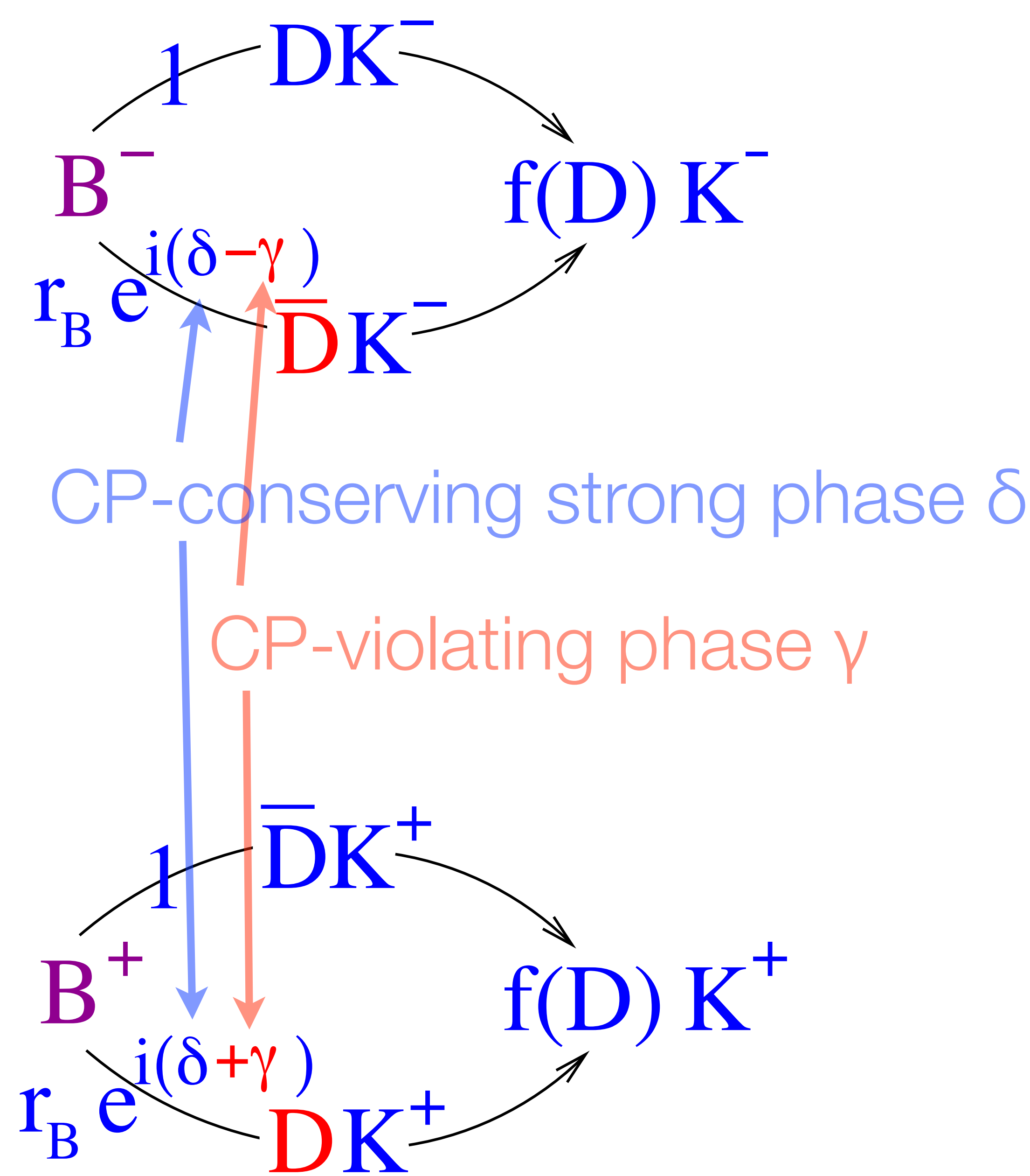
$\bar{\eta}$
$\bar{\rho}$
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
2.0

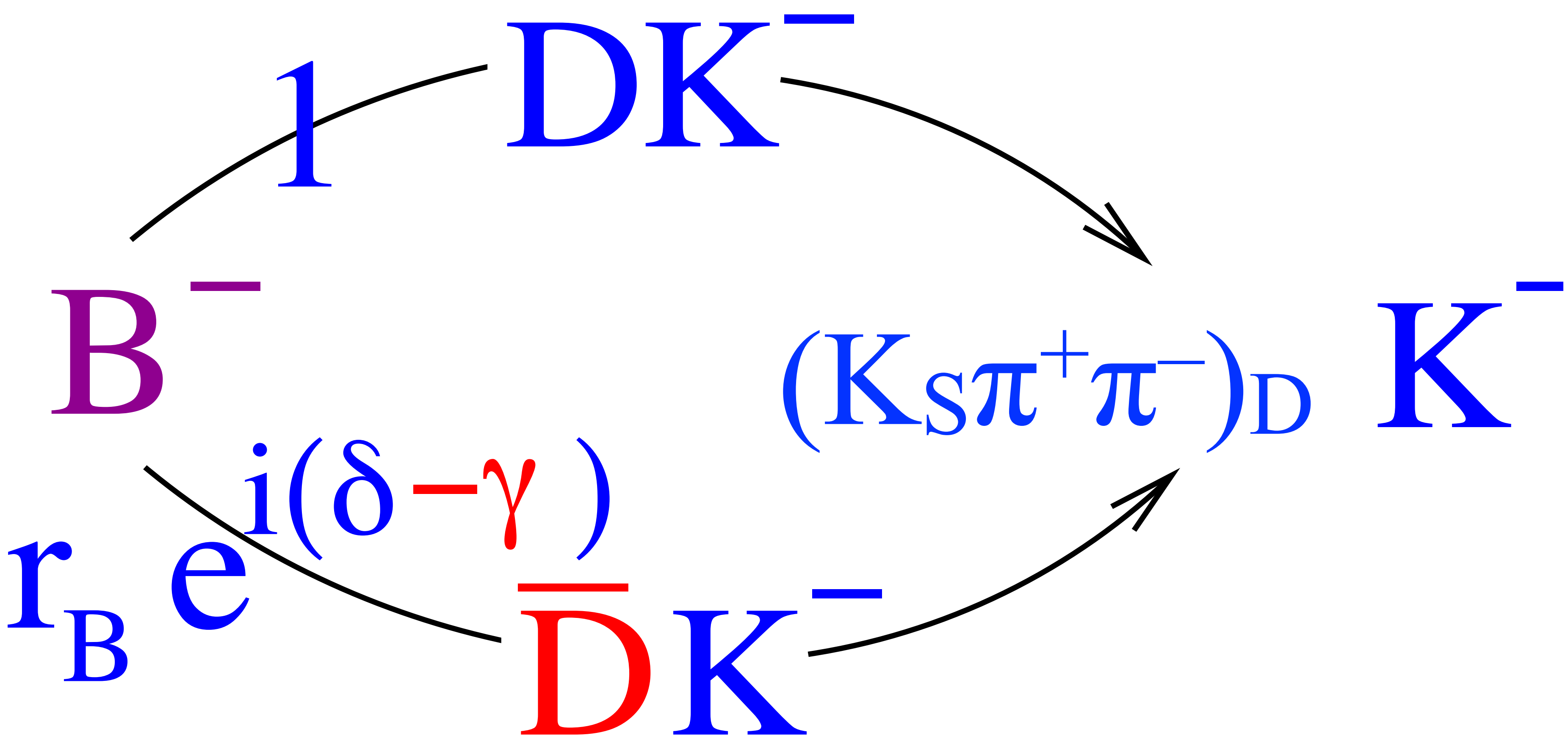
$B^{\pm} \rightarrow DK^{\pm}$





Gronau, Wyler Phys.Lett.B265:172-176,1991, (GLW), Gronau, London Phys.Lett.B253:483-488,1991 (GLW) Atwood, Dunietz and Soni Phys.Rev.Lett. 78 (1997) 3257-3260 (ADS) Giri, Grossman, Soffer and Zupan Phys.Rev. D68 (2003) 054018 Belle Collaboration Phys.Rev. D70 (2004) 072003

CP violation is an interference effect



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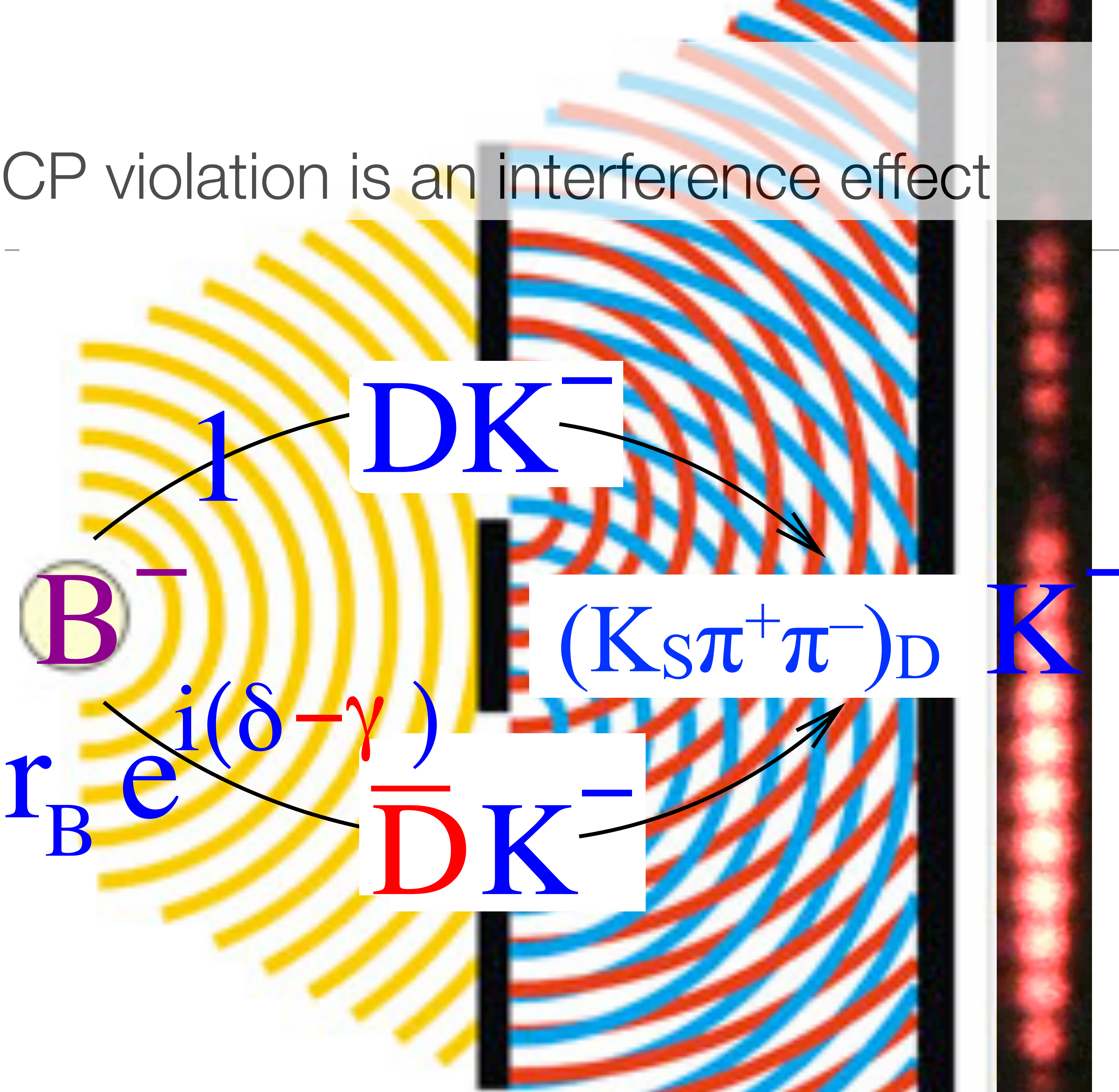

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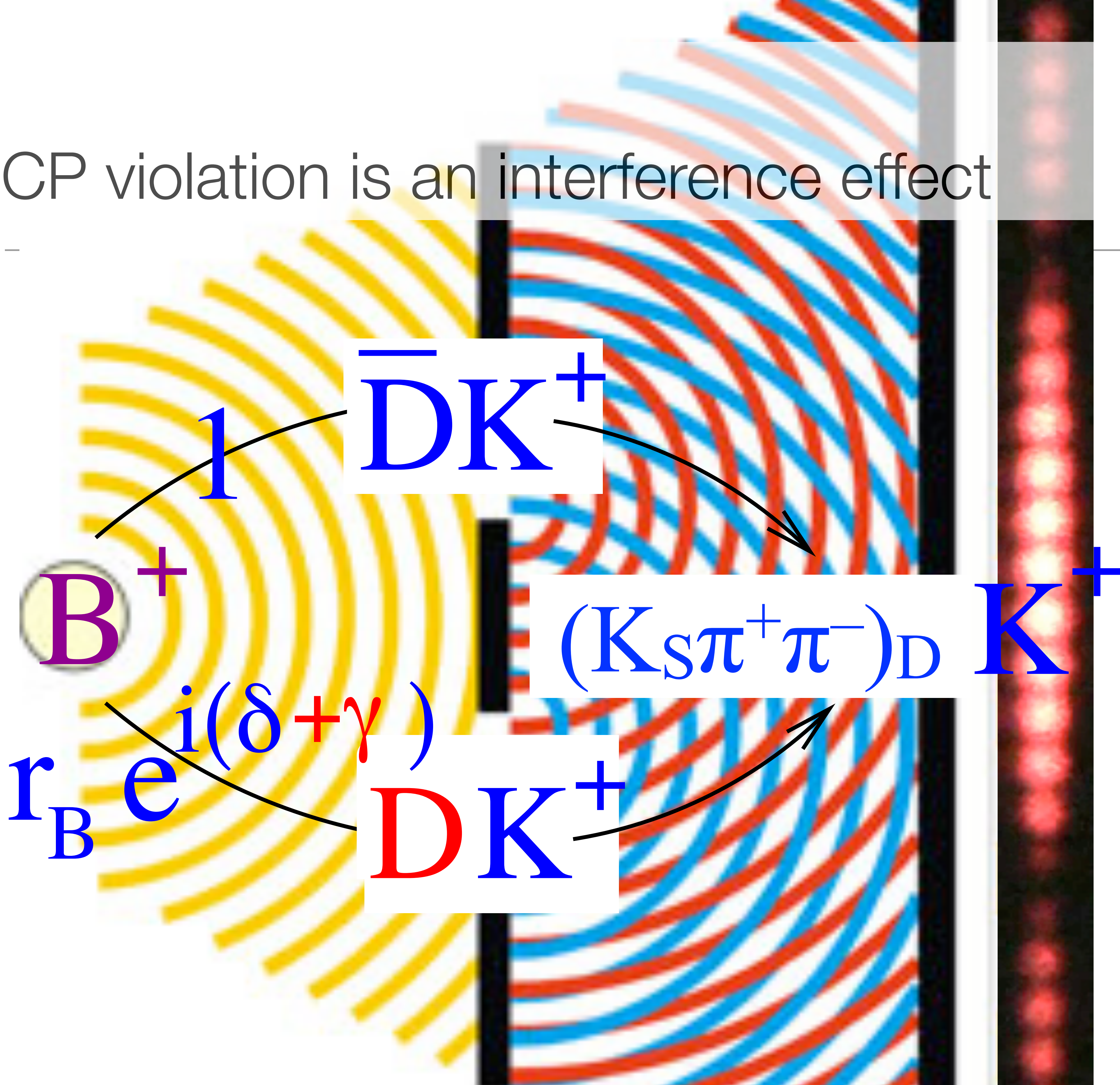

Gronau, Wyler Phys.Lett.B265:172-176,1991, (GLW), Gronau, London Phys.Lett.B253:483-488,1991 (GLW) Atwood, Dunietz and Soni Phys.Rev.Lett. 78 (1997) 3257-3260 (ADS) Giri, Grossman, Soffer and Zupan Phys.Rev. D68 (2003) 054018 Belle Collaboration Phys.Rev. D70 (2004) 072003

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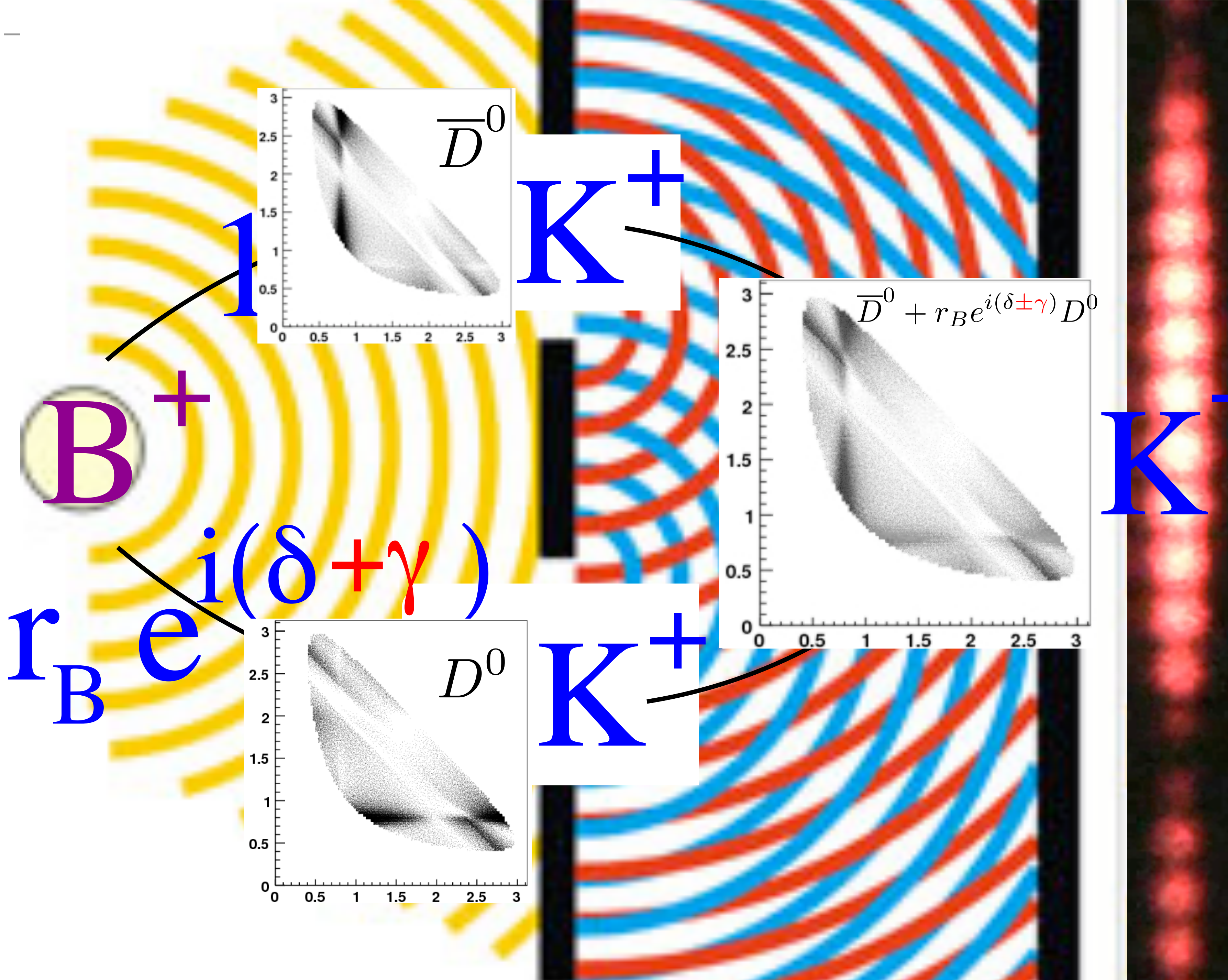


- For D→3-body decays, the interference takes place in an abstract 2-D space (Dalitz plot)
- Analysing the Dalitz plot of the D decay, in D's that come from B±'s, gives access to γ

Gronau, Wyler Phys.Lett.B265:172-176,1991, (GLW), Gronau, London Phys.Lett.B253:483-488,1991 (GLW) Atwood, Dunietz and Soni Phys.Rev.Lett. 78 (1997) 3257-3260 (ADS) Giri, Grossman, Soffer and Zupan Phys.Rev. D68 (2003) 054018 Belle Collaboration Phys.Rev. D70 (2004) 072003

Multi-Generational Flavour Physics

Edward V. Brewer (1883 – 1971)

Multi-Generational Flavour Physics

Edward V. Brewer (1883 – 1971)

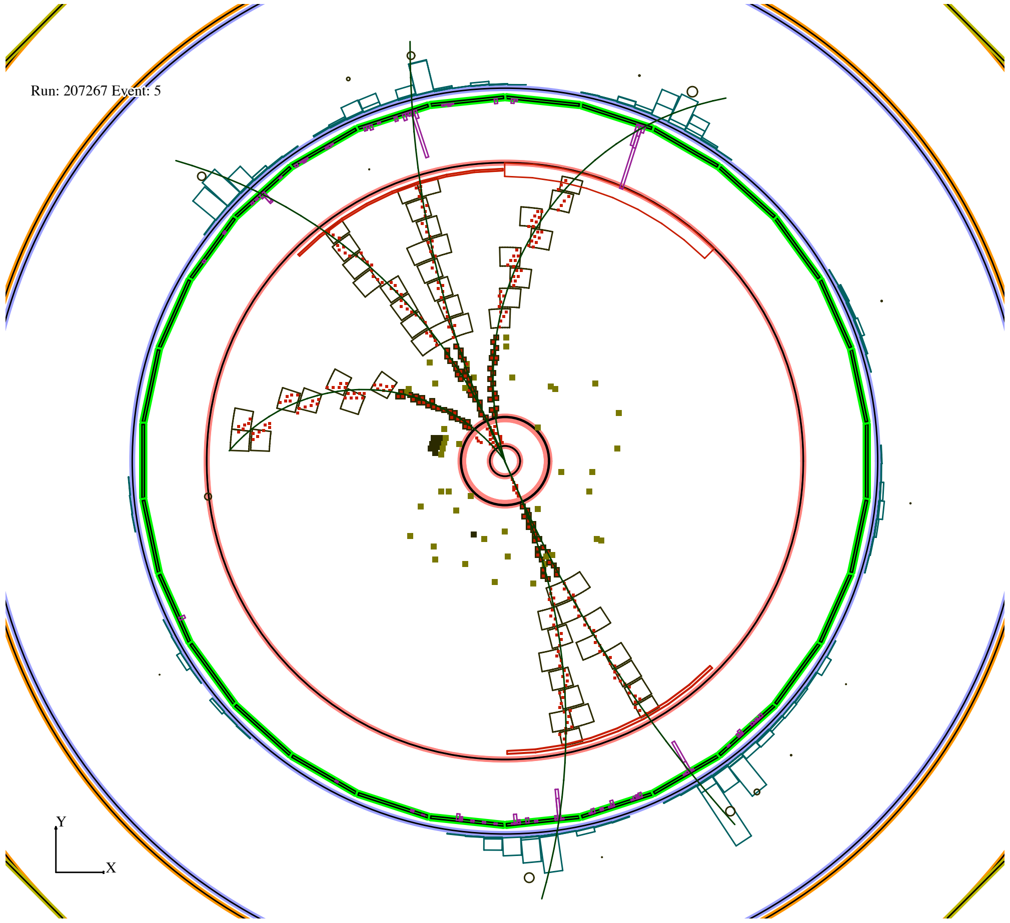
Regrettably, CLEO recently deceased - but her data live on.

CLEO-c

$e^+e^- \rightarrow \psi(3770) \rightarrow D\overline{D}$

CLEAN-c

- Threshold production of correlated DD.
- Final state must be CP-even with L=1: D mesons must have opposite intrinsic CP.
- Final state is also flavour-neutral.
- That gives us access to both amplitude and phase across the Dalitz plot.

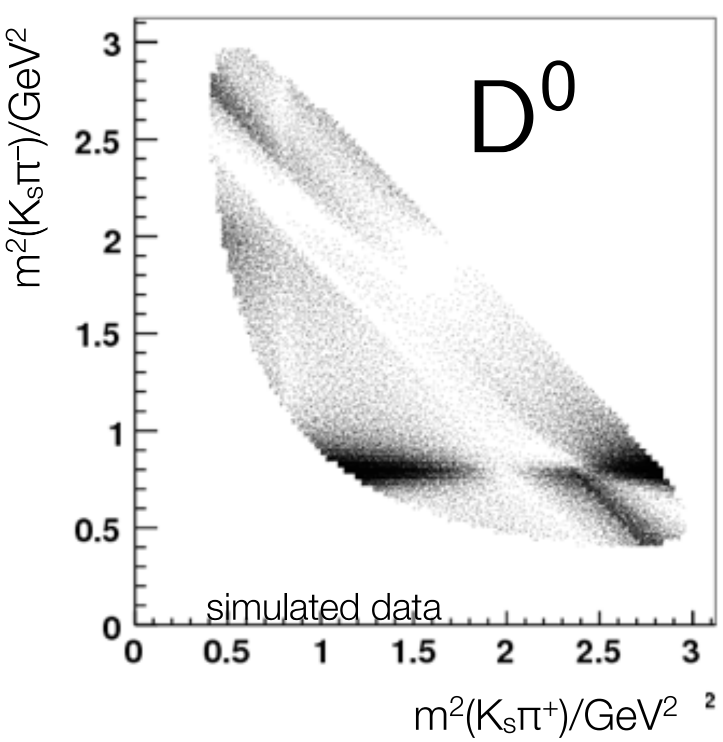


$$\psi(3770) \to D^0(K_S\pi^+\pi^-)\bar{D}^0(K^+\pi^-)$$

CP and flavour tagged D°

ψ'' → $\bar{D}_{flavour} \to K^+ e^- \nu_e$

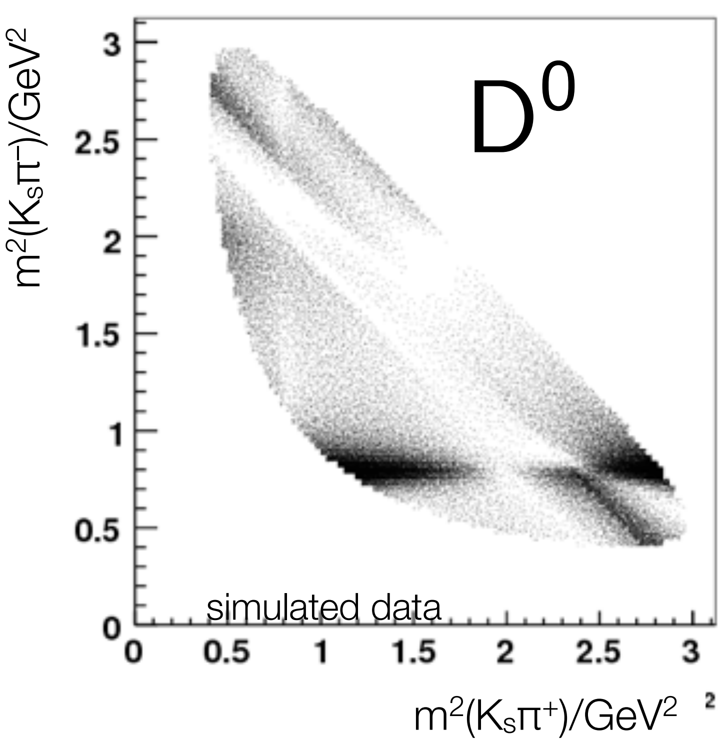
ψ'' → $D_{flavour} \to K_s \pi^+ \pi^-$



CP and flavour tagged D^o

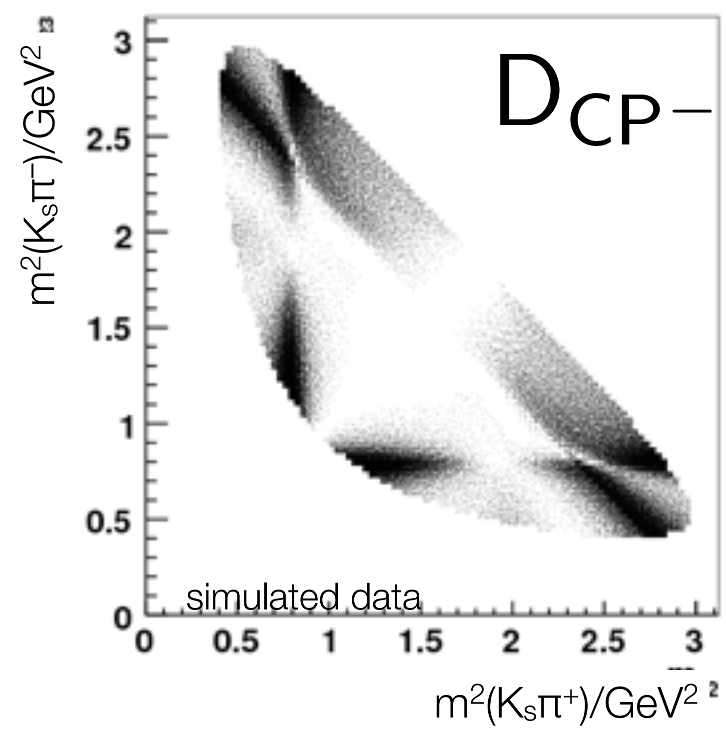
$\psi'' \rightarrow \bar{D}_{flavour} \rightarrow K^+ e^- \nu_e$

$\psi'' \rightarrow D_{flavour} \rightarrow K_S \pi^+ \pi^-$



$\psi'' \rightarrow D_{CP+} \rightarrow KK$

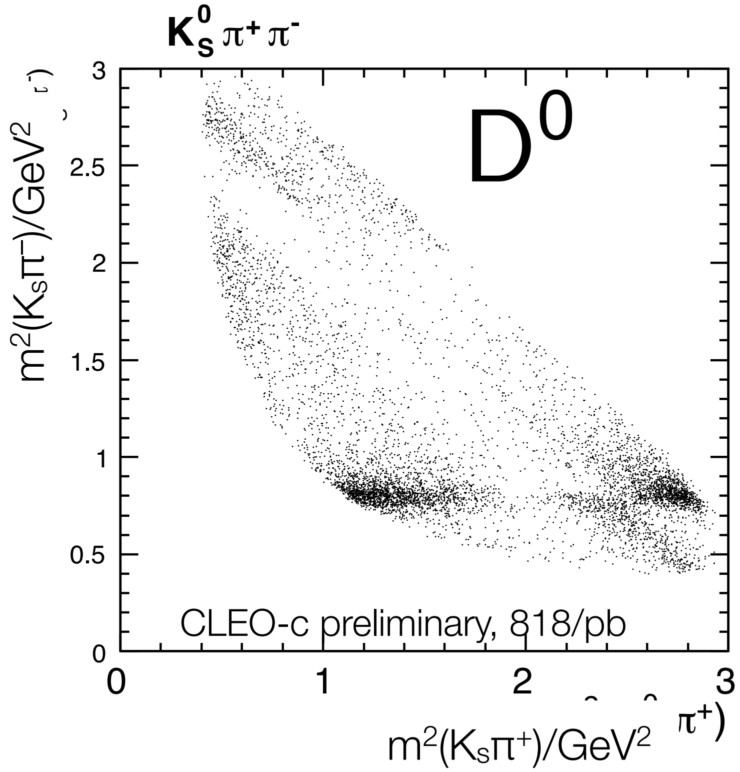
$\psi'' \rightarrow D_{CP-} \rightarrow K_S \pi^+ \pi^-$

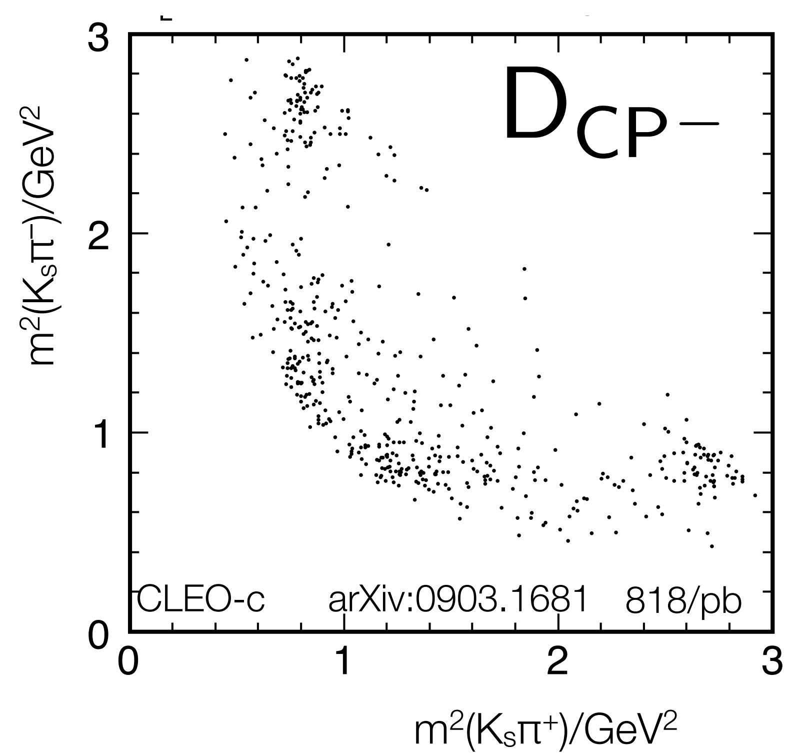


CP and flavour tagged D° at CLEO

$$\psi'' \rightarrow \bar{D}_{flavour} \rightarrow K^+ e^- \nu_e$$
$$\psi'' \rightarrow D_{flavour} \rightarrow K_s \pi^+ \pi^-$$

$$\psi'' \rightarrow D_{CP+} \rightarrow KK$$
$$\psi'' \rightarrow D_{CP-} \rightarrow K_s \pi^+ \pi^-$$





Model independent γ fit

Giri, Grossmann, Soffer, Zupan, Phys Rev D 68, 054018 (2003).

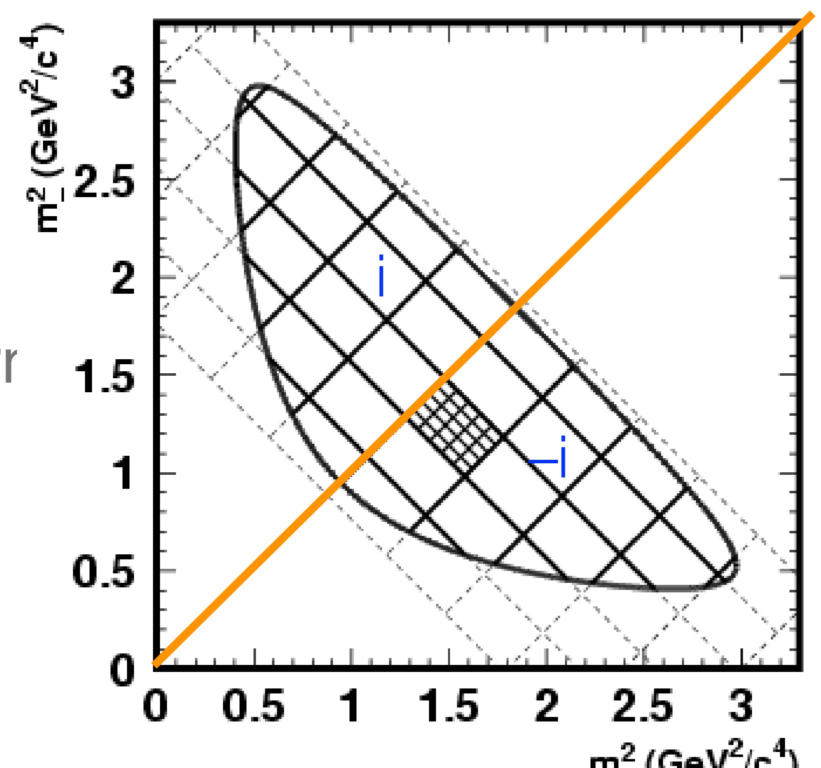
- Binned decay rate:

$\mathcal{T}_i$ known from flavour-specifc D decays (e.g. D*)

$$\Gamma\left(B^{\pm} \to D(K_s\pi^+\pi^-)K^{\pm}\right)_i =$$
$$\mathcal{T}_i + r_B^2\mathcal{T}_{-i} + 2r_B\sqrt{\mathcal{T}_i\mathcal{T}_{-i}}\left\{c_i\cos\left(\delta \pm \gamma\right) + s_i\sin\left(\delta \pm \gamma\right)\right\}$$

(weighted) average of $\cos(\delta_D)$ and $\sin(\delta_D)$ over bin i, where δ_D = phase difference between D→Ksππ and Dbar→Ksππ

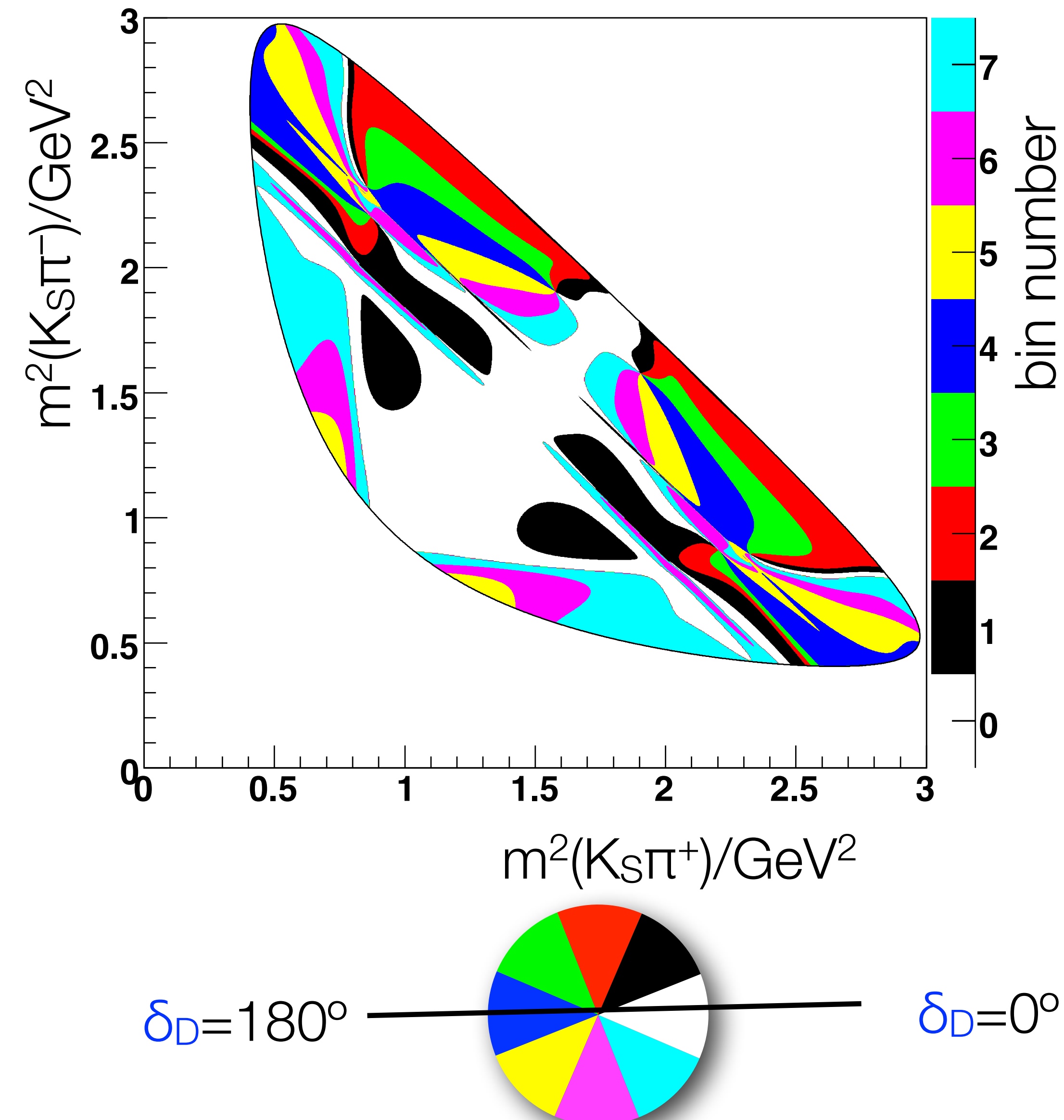
- Binning such that such that $c_i = c_{-i}$, $s_i = -s_{-i}$
- Distribution sensitive to c_i, s_i, r_B, δ and γ.
- To extract γ from realistic numbers of B events need extern input from CLEO's quantum-correlated DDbar pairs.



Optimal binning

- Best γ sensitivity if phase difference δ_D is as constant as possible over each bin[1].
- Plot shows CLEO-c's 8 bins, uniform in δ_D, (based on BaBar isobar model*).
- Choice of model will not bias result. (At worst a bad model would reduce the statistical precision of the result.)

Binning at CLEO-c based on BaBar model*



*model = BaBar PRL 95 (2005) 121802

[1] Bondar, Poluektov hep-ph/0703267v1 (2007)

LHCb model-independent γ from $B^\pm \to (K_S\pi\pi)_D K$ and $B^\pm \to (K_S KK)_D K$

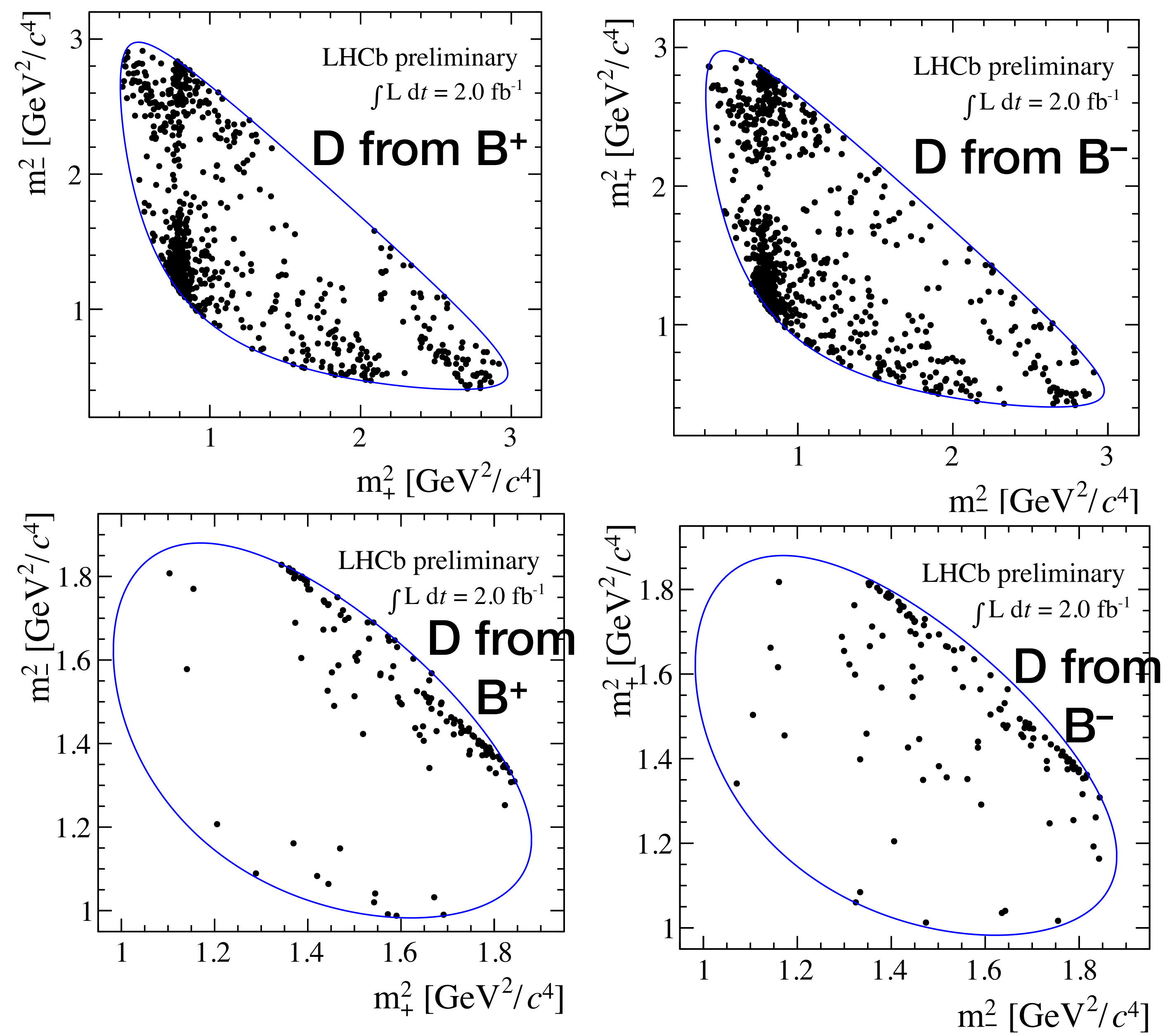
LHCb-CONF-2013-004
LHCb 2011 Result: Phys. Lett. B718 (2012) 43

- Binned, model-independent analysis using CLEO-c input.
  Phys. Rev. D 82 112006.
- Plots show LHCb 2012 data
- Result of combined analysis (2011 & 2012 data, $K_S\pi\pi$ & $K_S KK$):

$$\gamma = (57 \pm 16)^\circ$$

$$\delta_B = (124^{+15}_{-17})^\circ$$

$$r_B = (8.8^{+2.3}_{-2.4}) \times 10^{-2}$$



CLEO-c input:: Phys. Rev. D 82 112006.
Model-independent method: Giri, Grossmann, Soffer, Zupan, Phys Rev D 68, 054018 (2003).
Optimal binning: Bondar, Poluektov hep-ph/0703267v1 (2007)
BELLE's first model-independent γ measurement: PRD 85 (2012) 112014

LHCb's γ combination

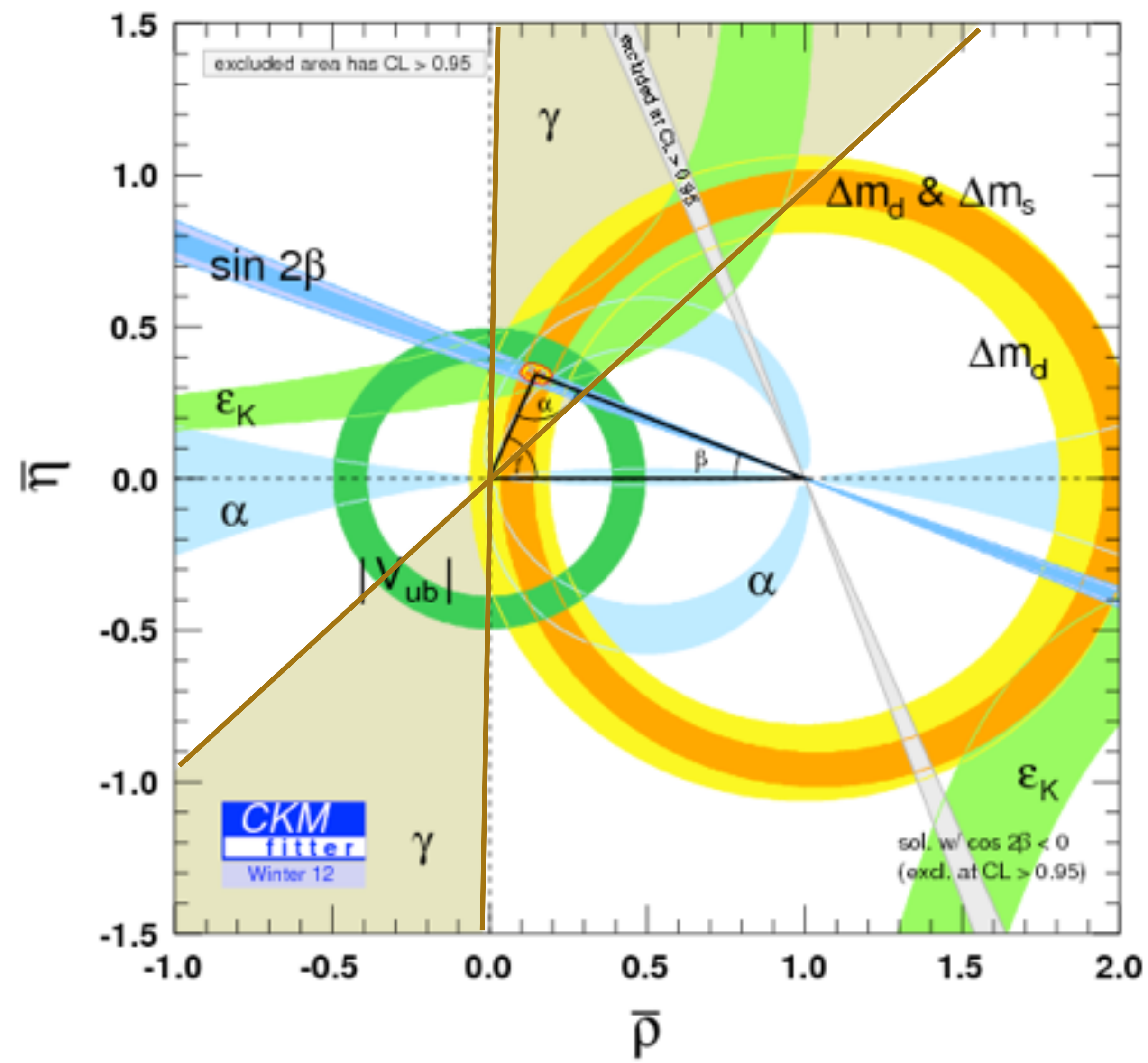
technique & 2011 data: Phys. Lett. B726 (2013) 151
2012 data: LHCb-CONF 2013-006)

- LHCb combines inputs from
  $B^\pm \to (hh')_D K^\pm$
  $B^\pm \to (K_S\pi\pi)_D K^\pm$
  $B^\pm \to (K_S KK)_D K^\pm$
  $B^\pm \to (K\pi\pi\pi)_D K^\pm$
- Result:

$$\gamma = (67.2 \pm 12)^o$$

- More channels available, including $B^\pm \to D\pi^\pm$, $B^0 \to DK^*$.
- Most recent addition: $B^\pm \to (K_S K\pi)_D K^\pm$ (see arXiv:1402.2982, 2014)

World averages by CKM Fitter



previous world average (Moriond 2012): $\gamma = 68^\circ \pm 12^\circ$

LHCb's γ combination

technique & 2011 data: Phys. Lett. B726 (2013) 151
2012 data: LHCb-CONF_2013-006)

- LHCb combines inputs from
  $B^{\pm}\to(hh')_D K^{\pm}$
  $B^{\pm}\to(K_S\pi\pi)_D K^{\pm}$
  $B^{\pm}\to(K_S KK)_D K^{\pm}$
  $B^{\pm}\to(K\pi\pi\pi)_D K^{\pm}$

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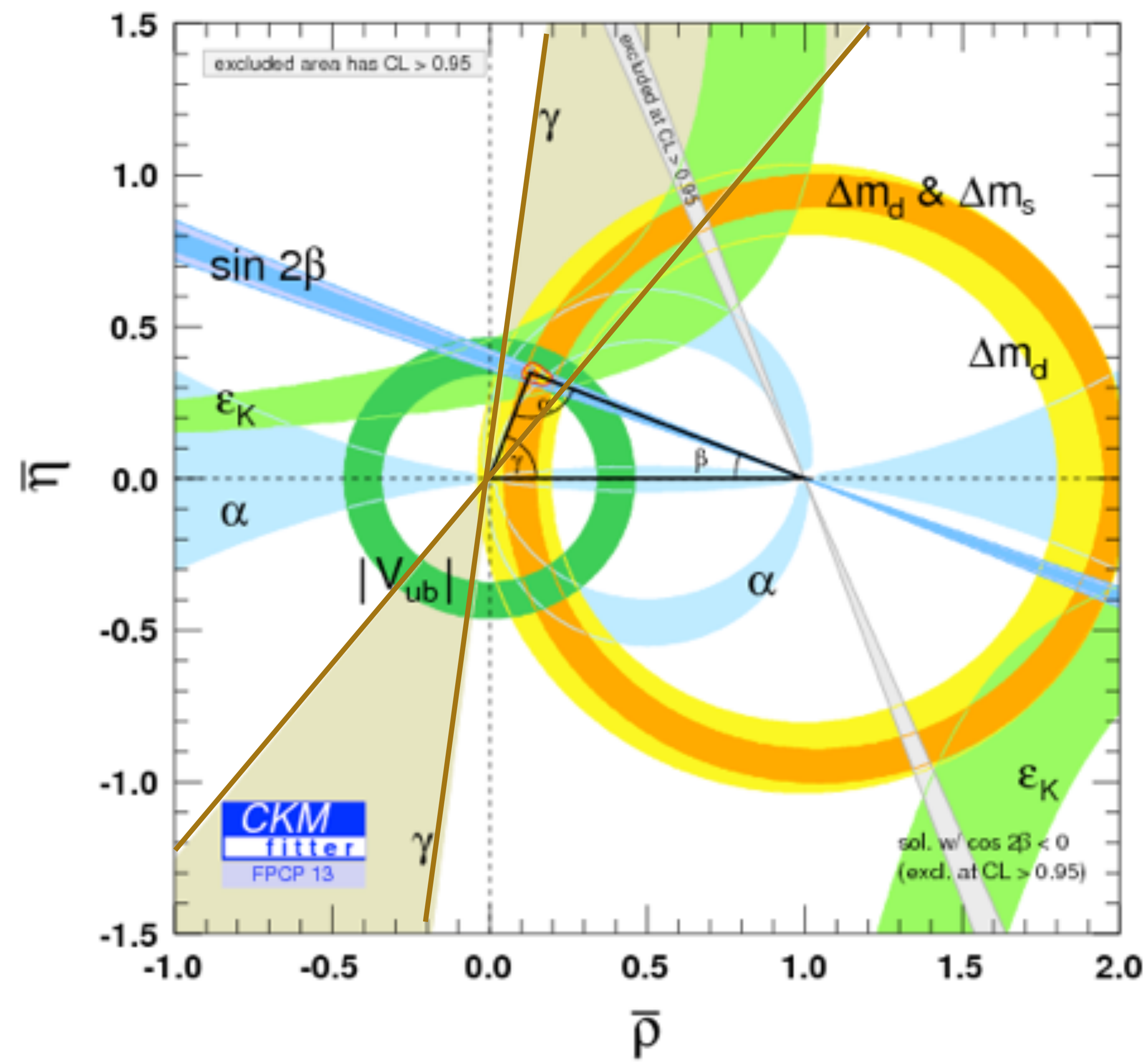
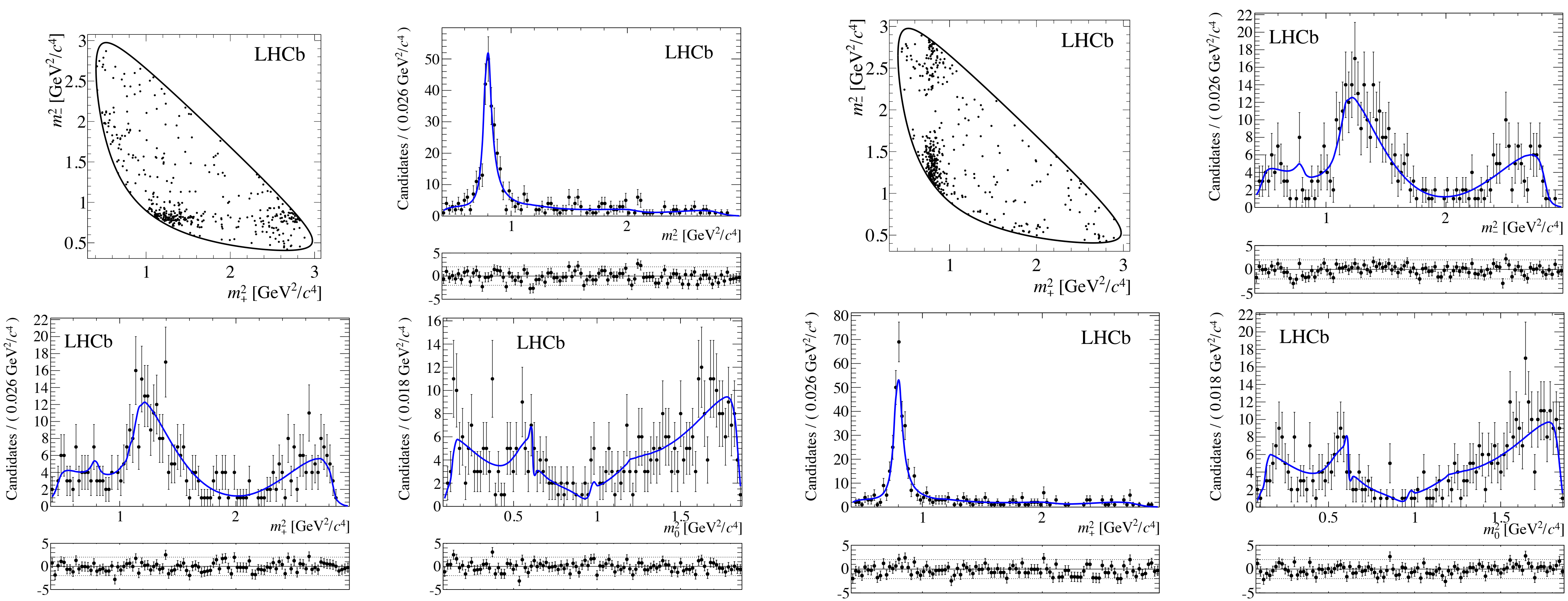


previous world average (Moriond 2012): $\gamma = 68^{\circ} \pm 12^{\circ}$

LHCb: γ=67.2°±12°

$$\gamma = 68^{\circ}{}^{+8.0^{\circ}}_{-8.5^{\circ}}$$

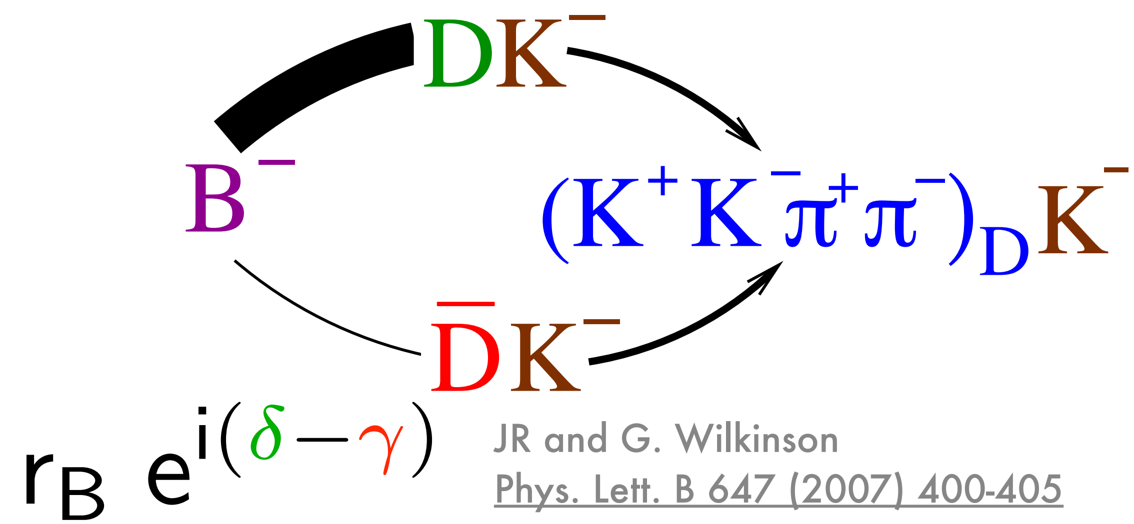
LHCb model-dependent γ from $B^{\pm}\to(K_S\pi\pi)_D K$



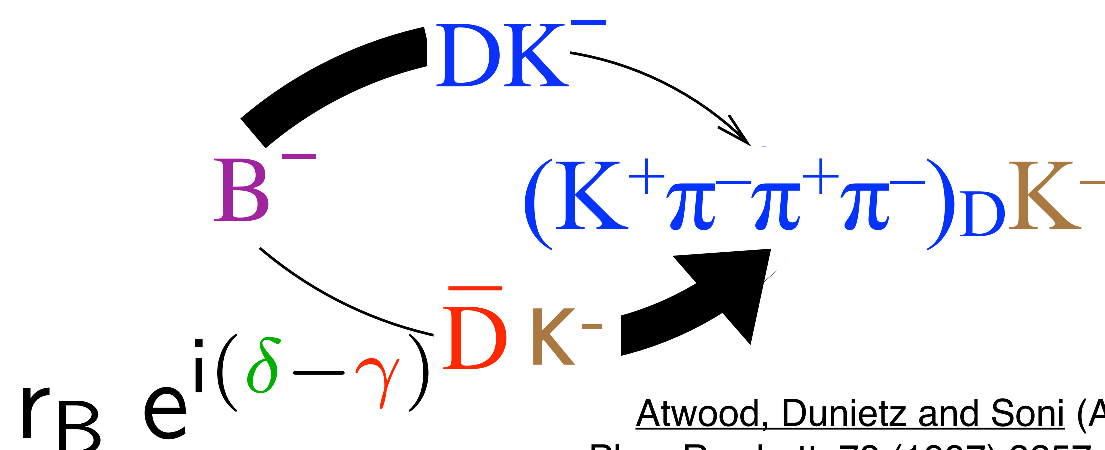
$$\gamma = (84^{+49}_{-42})^{\circ}$$

Why stop here

- Why stop at 3-body decays?
- 4-body amplitude analyses very promising for γ measurement at LHCb.
- Tricky… “Dalitz Plot” becomes 5-dimensional, phase space not flat, spin factors more complicated…



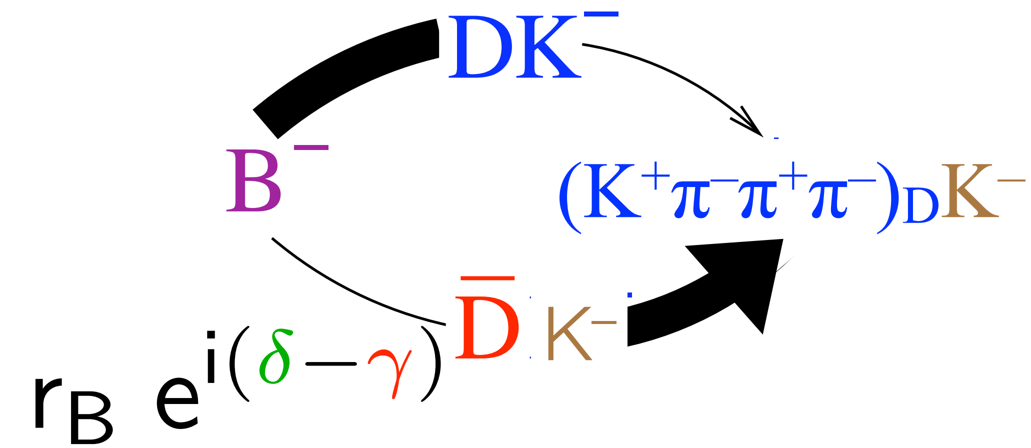
JR and G. Wilkinson
Phys. Lett. B 647 (2007) 400-405



Atwood, Dunietz and Soni (ADS), Phys.Rev.Lett. 78 (1997) 3257-3260

Atwood, Soni: Phys.Rev. D68 (2003) 033003

Coherence Factor Analysis of



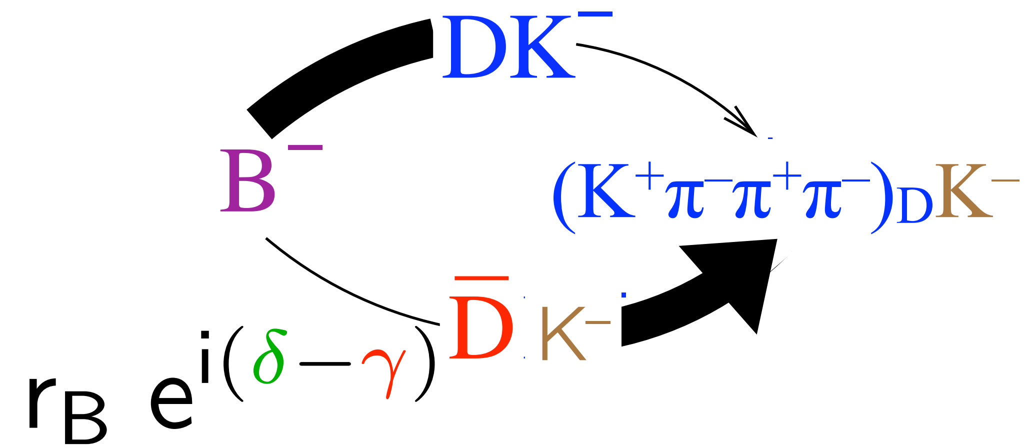
- Treat K3π like two-body decay with single effective strong phase δ_D.

- Complex coherence parameter $Z = c + i\,s = R\,e^{i\delta}$ with coherent factor $R < 1$.

$$\Gamma\left(\mathrm{B}^- \to \left(\mathrm{K}^+ 3\pi\right)_\mathrm{D} \mathrm{K}^-\right) \propto r_B^2 + \left(r_D^{K3\pi}\right)^2 + 2R_{K3\pi} r_B r_D^{K3\pi} \cdot \cos\left(\delta_B + \delta_D^{K3\pi} - \gamma\right)$$

$$r_B = \left|\frac{A(B^- \to \overline{D}^0 K^-)}{A(B^- \to D^0 K^-)}\right| \qquad\qquad r_D = \left|\frac{A(D^0 \to K^+\pi^-\pi^+\pi^-)}{A(\overline{D}^0 \to K^+\pi^-\pi^+\pi^-)}\right|$$

Coherence Factor Analysis of



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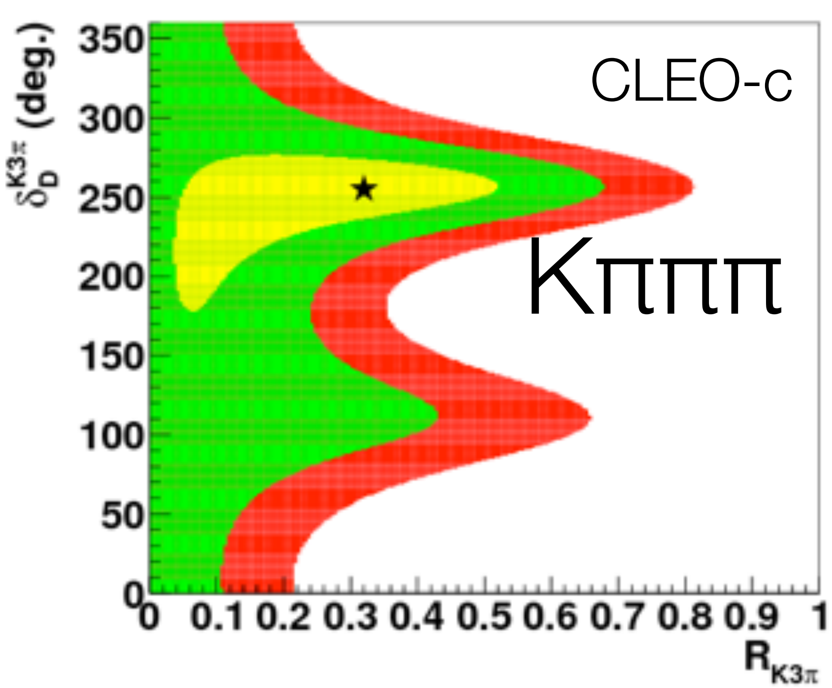
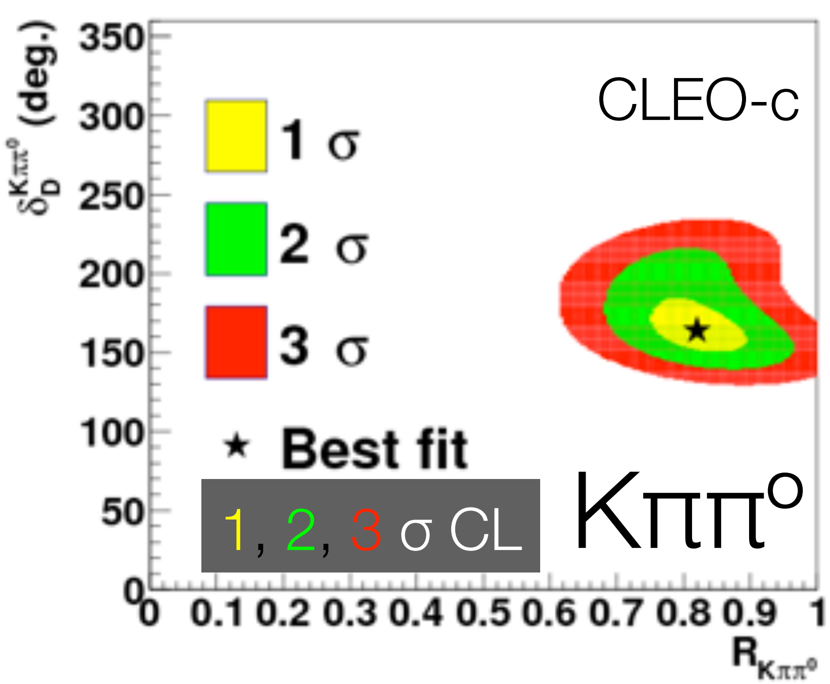
$$\Gamma\left(B^- \to \left(K^+3\pi\right)_D K^-\right) \propto r_B^2 + \left(r_D^{K3\pi}\right)^2 + 2R_{K3\pi} r_B r_D^{K3\pi} \cdot \cos\left(\delta_B + \delta_D^{K3\pi} - \gamma\right)$$

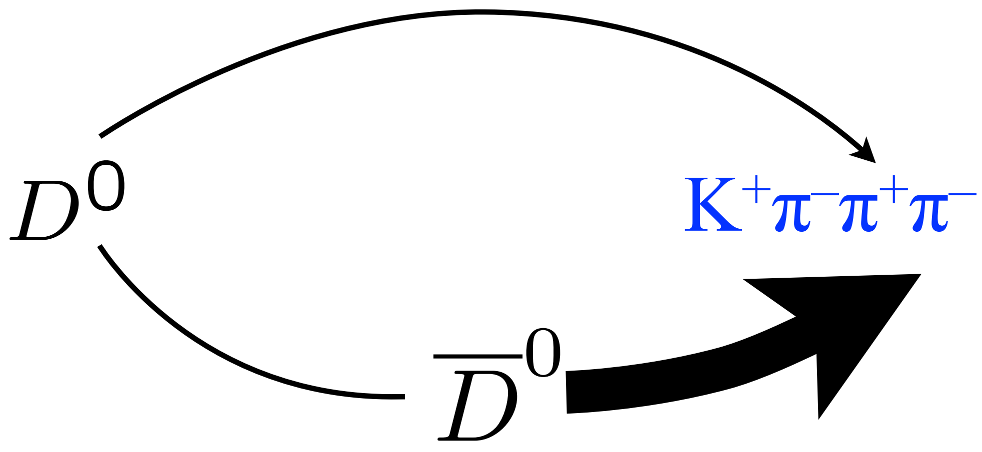
- CLEO-c used coherent ψ(3770)→DD events to measure R, δ_D for Kπππ and Kππ°.

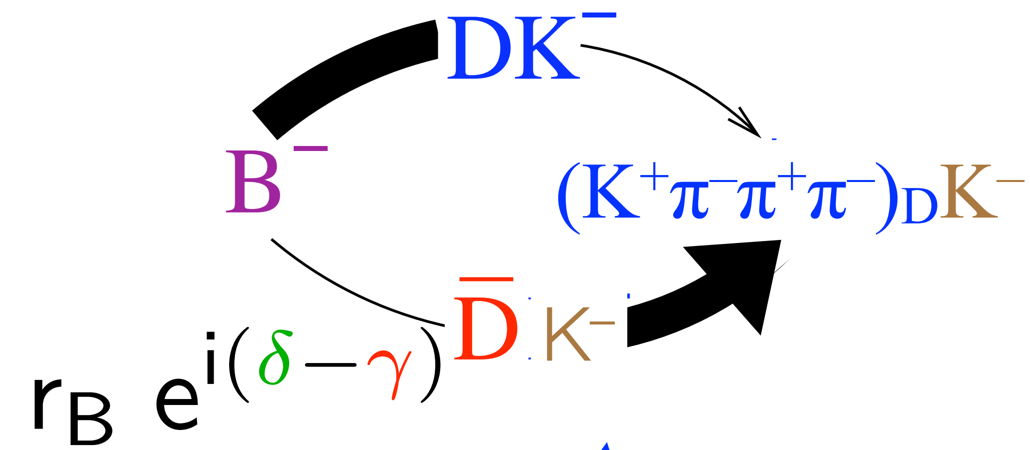




Theory:
Atwood, Soni: Phys.Rev. D68 (2003) 033003
CLEO-c input:
Phys.Rev.D80:031105,2009
Phys.Lett. B731 (2014) 197-203
LHCb CPV result:
Physics Letters B 723 (2013), 44

D^0 Mixing as input to γ from $B^\pm \to DK^\pm$



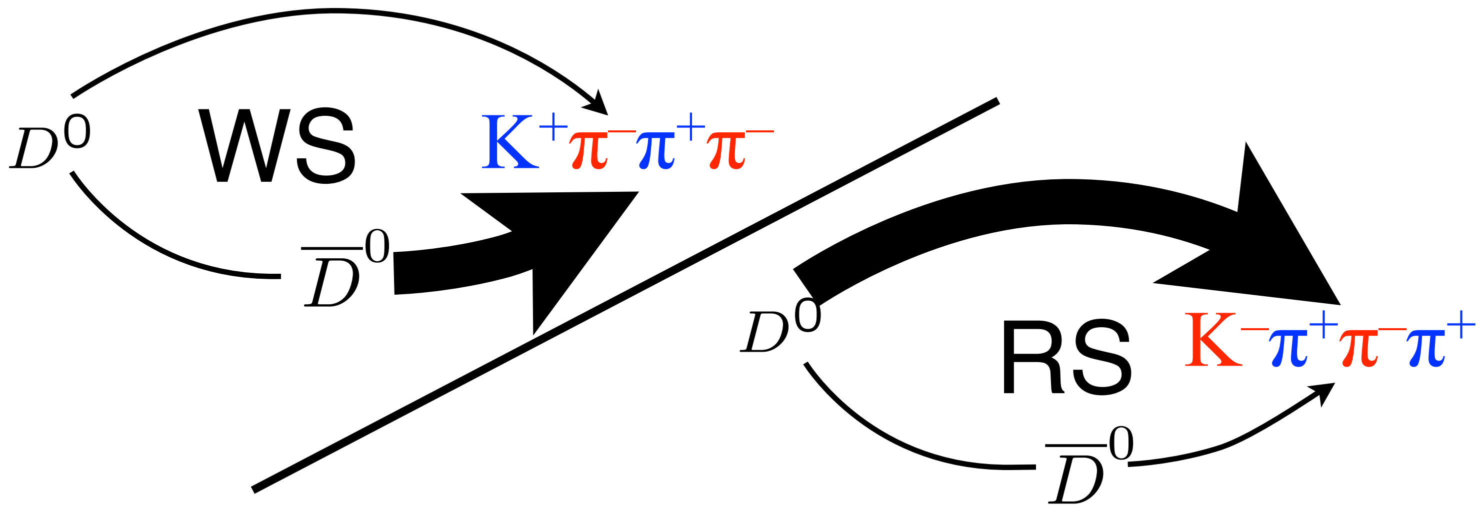


This process is sensitive to the same D-$\overline{\text{D}}$ interference effects that pollute this measurement.

Phys.Lett. B728 (2014) 296-302

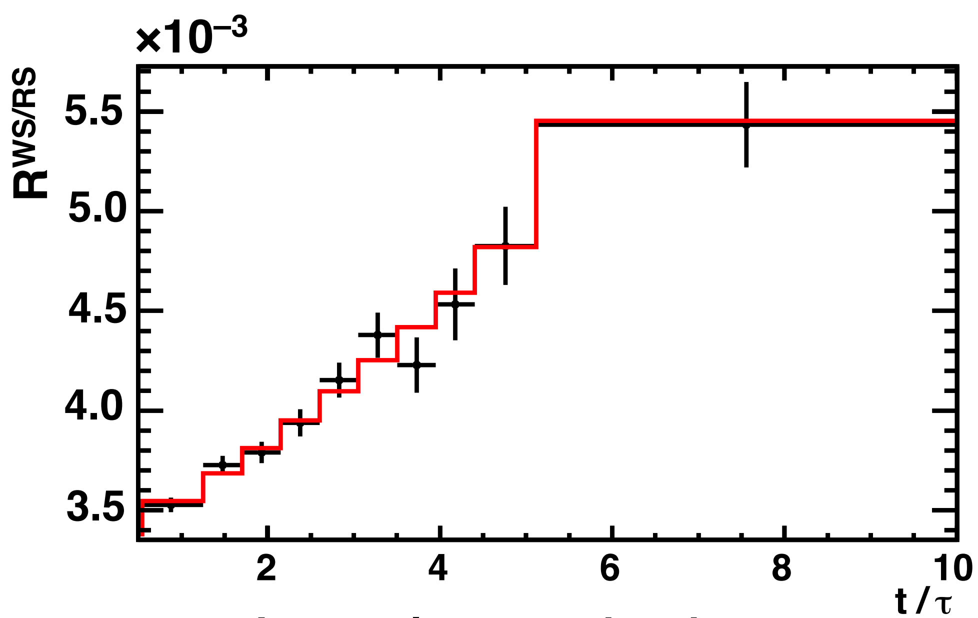
Phys.Lett. B728 (2014) 296-302

D° Mixing as input to γ from B±→DK±

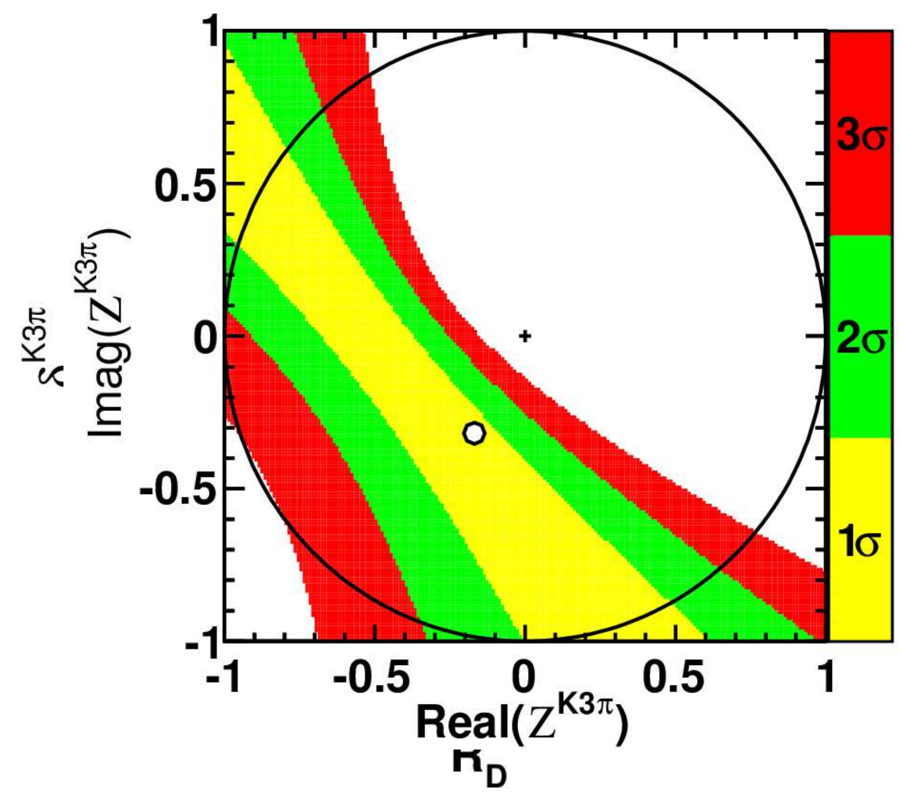


D $\overline{\text{D}}$ tagged with D*→Dπ

$$\frac{\Gamma(D^0 \to K^+3\pi)}{\Gamma(D^0 \to K^-3\pi)}(t) = {r_D^{K3\pi}}^2 + r_D^{K3\pi}\left(yReZ^{K3\pi} + xImZ^{K3\pi}\right)\Gamma t + \frac{x^2+y^2}{4}(\Gamma t)^2$$

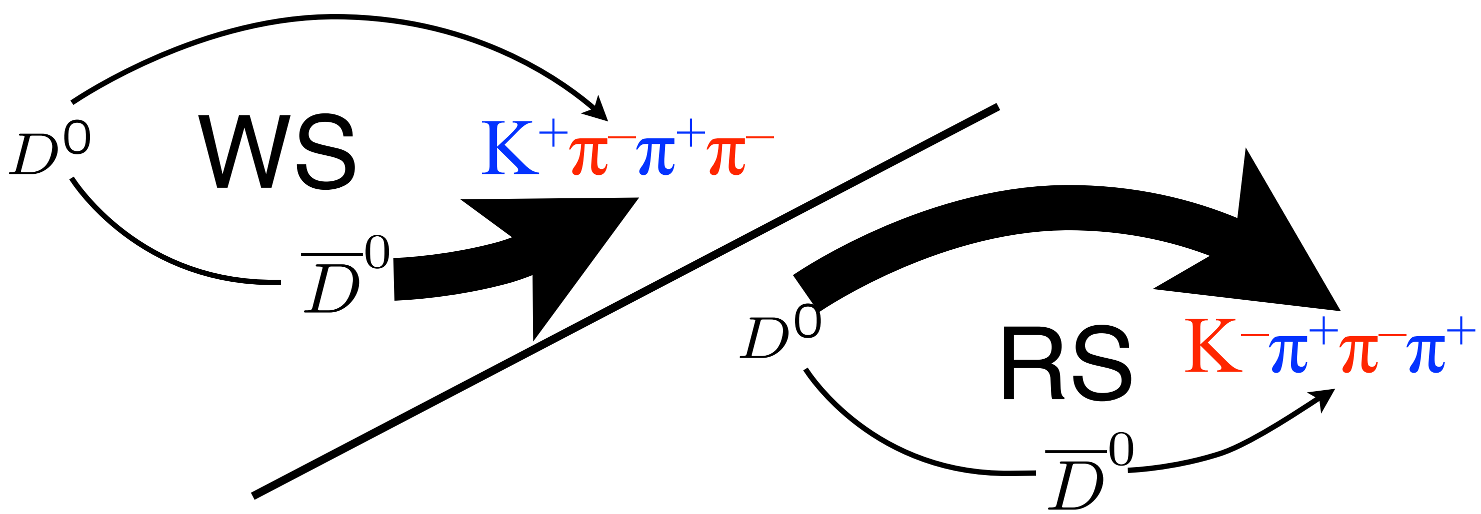


simulated data



Phys.Lett. B728 (2014) 296-302

D° Mixing as input to γ from B±→DK±



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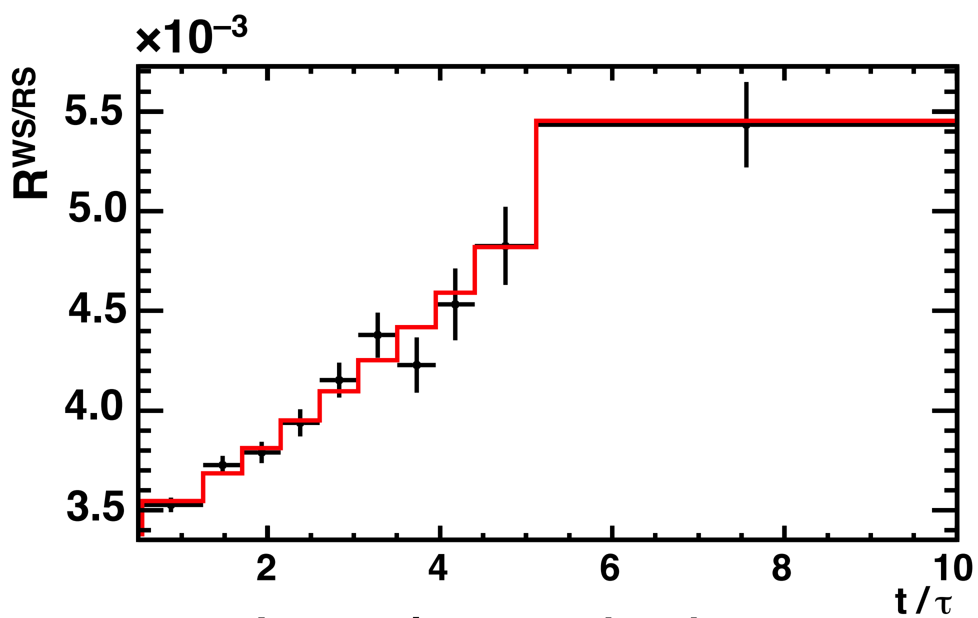
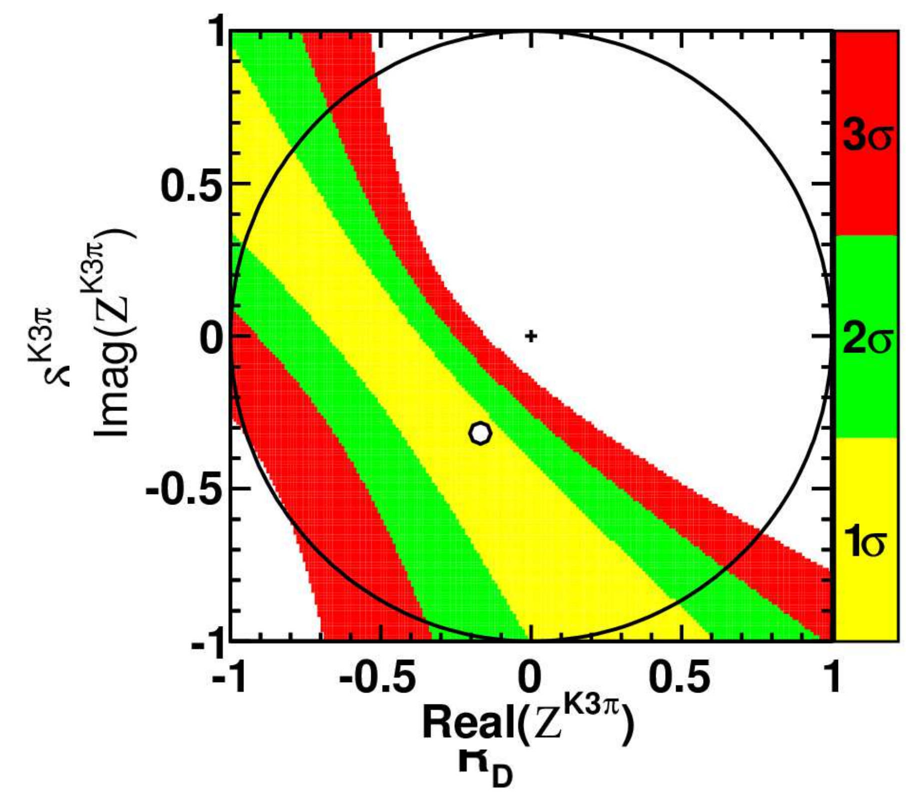
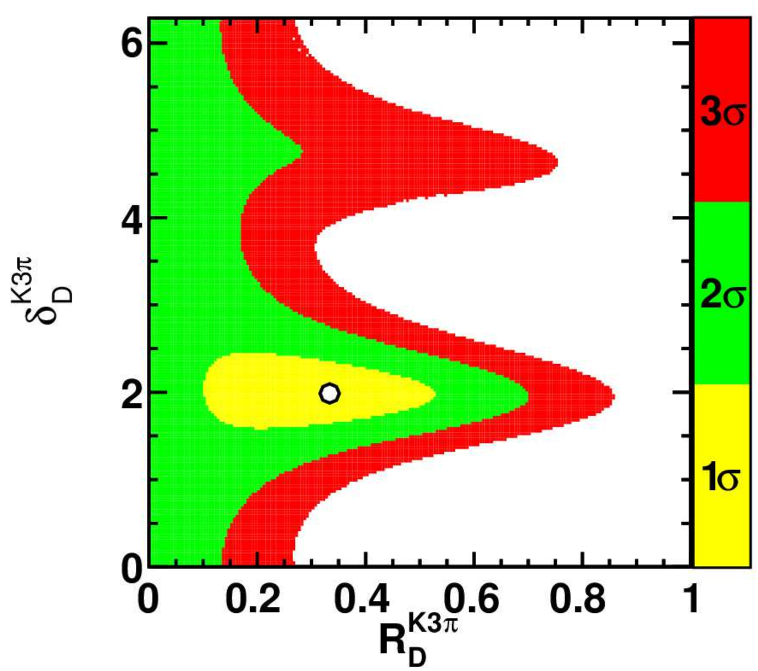


simulated data

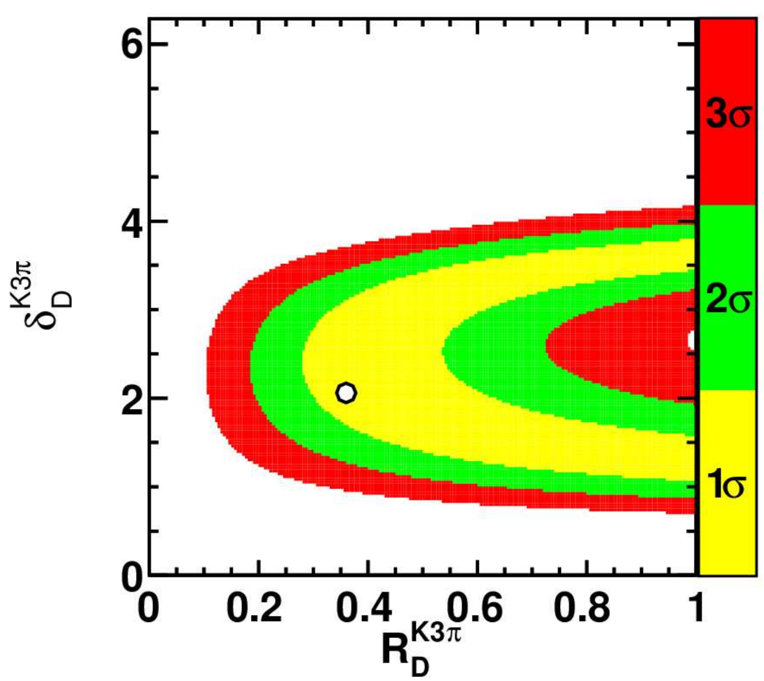


Unpublished, unofficial preview:
$D \rightarrow K^{-}\pi^{+}\pi^{-}\pi^{+}$ coherence factor from mixing at LHCb

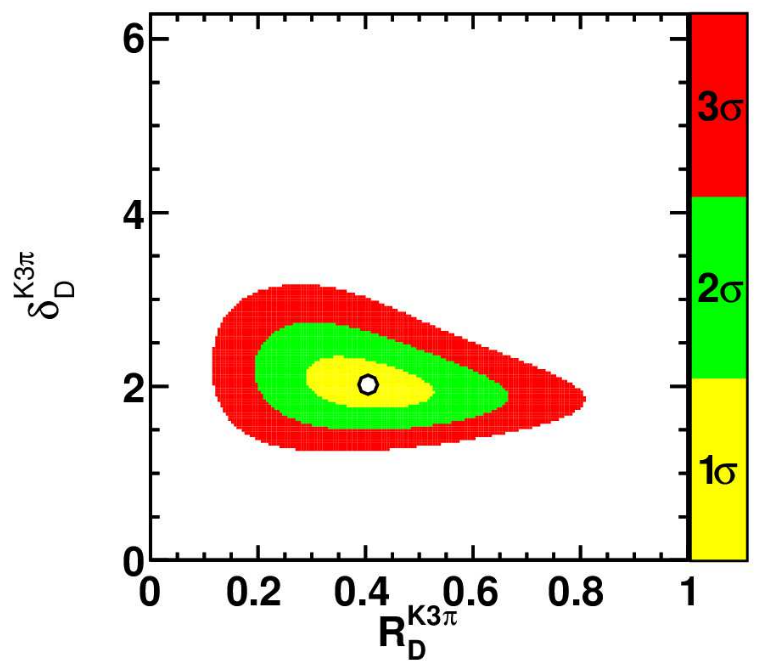
CLEO-c
(previous result)



Simulated
LHCb 1/fb



Both

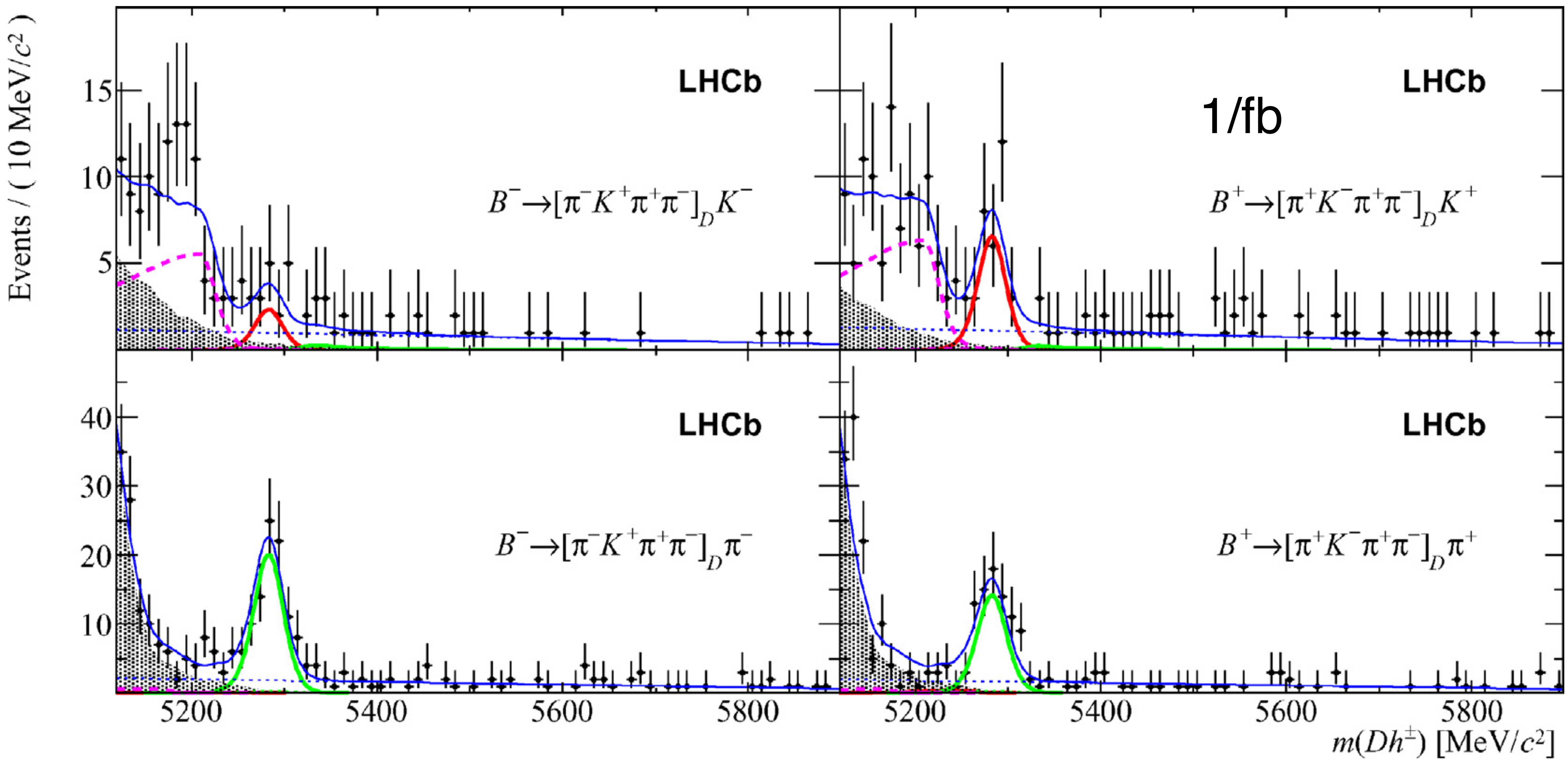


CLEO-c input: Phys.Rev.D80:031105,2009
Phys.Lett. B731 (2014) 197-203

D-mixing input: Phys.Lett. B728 (2014) 296-302

LHCb B$^\pm \to$D(Kπππ)K$^\pm$

Physics Letters B 723 (2013), pp. 44-53



A closer look at Z

$$\frac{\Gamma\left(B^- \to DK^-, D \to f\right)_\Omega}{\Gamma\left(B^- \to DK^-, D \to \bar{f}\right)_{\bar{\Omega}}} = r_{D,\Omega}^2 + r_B^2 + r_{D,\Omega} r_B \left|\mathcal{Z}_\Omega^f\right| \cos(\delta_B - \delta_\Omega^f - \gamma)$$

$$\mathcal{A}_\Omega \equiv \sqrt{\int\limits_\Omega |\langle f_\mathbf{p}|\hat{H}|D^0\rangle|^2 \left|\frac{\partial^n \phi}{\partial(p_1 \dots p_n)}\right| \mathrm{d}^n p}, \qquad \mathcal{B}_\Omega \equiv \sqrt{\int\limits_\Omega |\langle f_\mathbf{p}|\hat{H}|\overline{D}^0\rangle|^2 \left|\frac{\partial^n \phi}{\partial(p_1 \dots p_n)}\right| \mathrm{d}^n p}$$

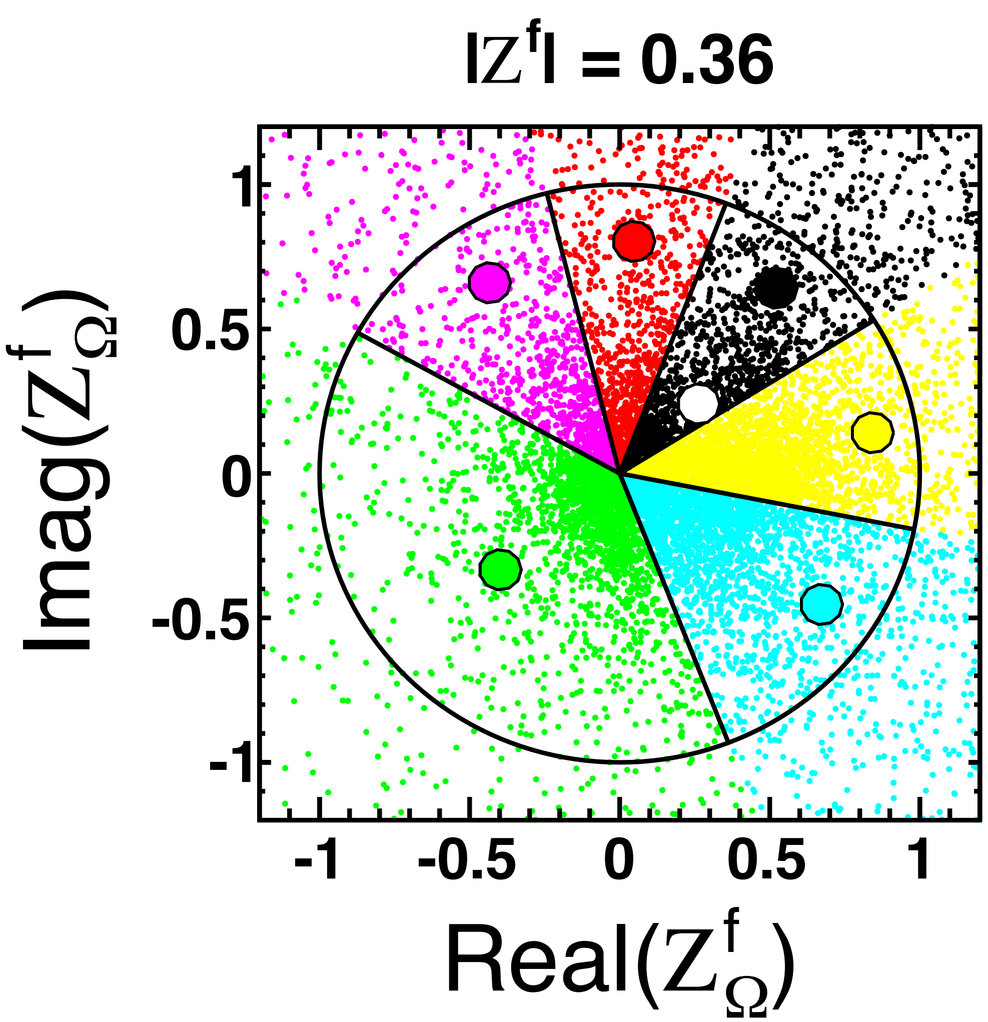
$$\mathcal{Z}_\Omega^f \equiv \frac{1}{\mathcal{A}_\Omega \mathcal{B}_\Omega} \int\limits_\Omega \langle f_\mathbf{p}|\hat{H}|D^0\rangle \langle f_\mathbf{p}|\hat{H}|\overline{D}^0\rangle^* \left|\frac{\partial^n \phi}{\partial(p_1 \dots p_n)}\right| \mathrm{d}^n p$$

amplitude of D (Dbar) going to 5-D phase space point p

Binning is good for you

arXiv:1412.7254

$$\frac{\Gamma\left(B^- \to DK^-, D \to f\right)_\Omega}{\Gamma\left(B^- \to DK^-, D \to \bar{f}\right)_{\bar{\Omega}}} = r_{D,\Omega}^2 + r_B^2 + r_{D,\Omega} r_B \left|\mathcal{Z}_\Omega^f\right| \cos(\delta_B - \delta_\Omega^f - \gamma)$$



$$\mathcal{Z}_\Omega^f \equiv \frac{1}{\mathcal{A}_\Omega \mathcal{B}_\Omega} \int_\Omega \langle f_{\mathbf{p}} | \hat{H} | D^0 \rangle \langle f_{\mathbf{p}} | \hat{H} | \overline{D}^0 \rangle^* \left| \frac{\partial^n \phi}{\partial(p_1 \ldots p_n)} \right| \mathrm{d}^n p$$

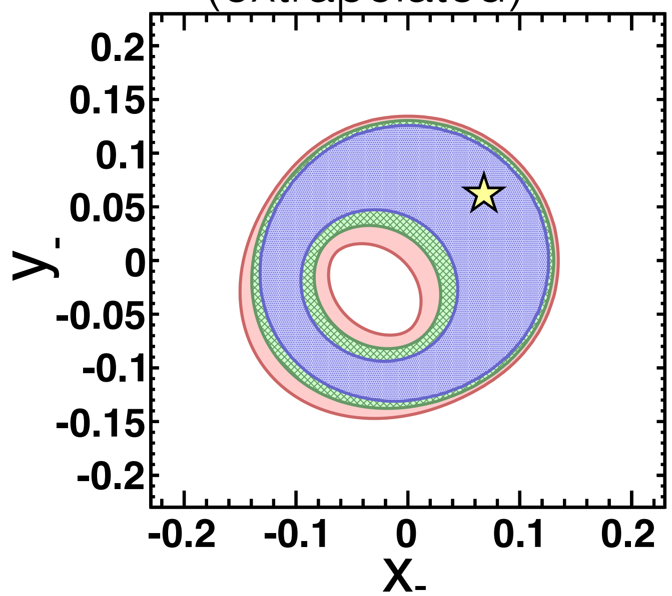
Mean |Z| increases if you bin in terms of the phase difference between D and Dbar amplitudes.

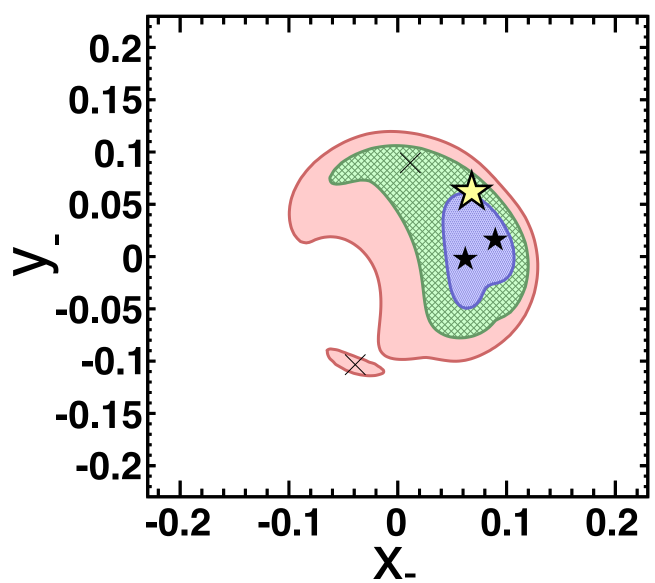
Turns out: if you have sufficiently many bins, you can extract γ model-independently, even w/o input from the charm threshold.

Gets even better if we divide the 5-D space into bins

BES III

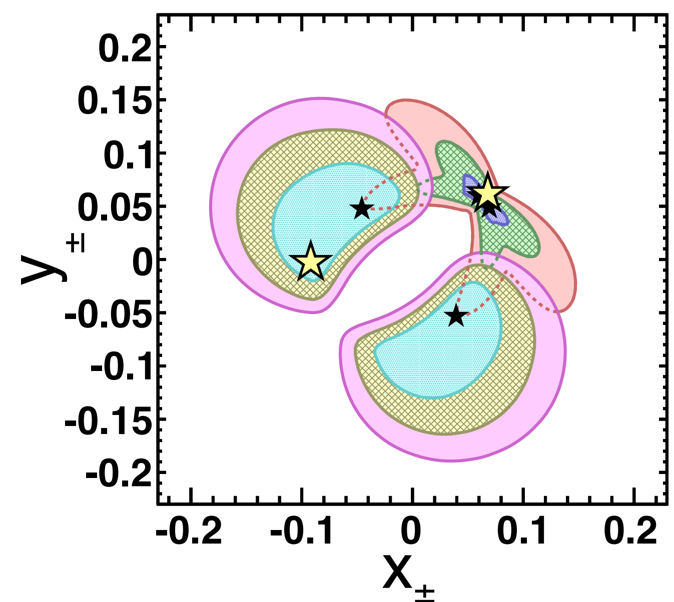
(extrapolated)





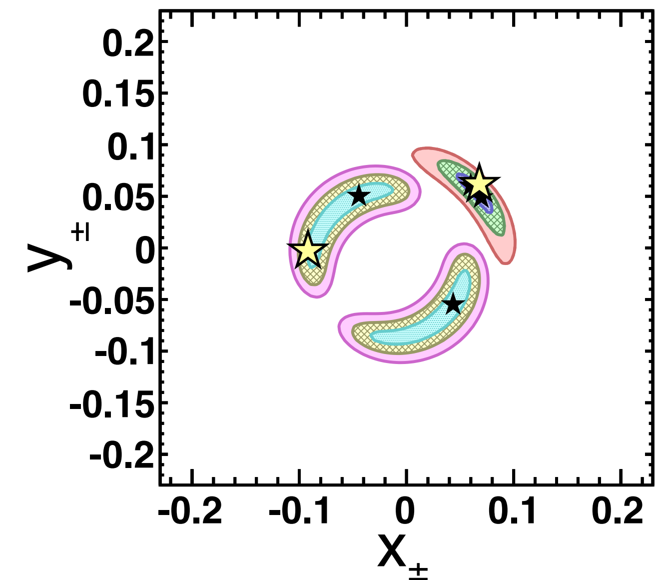
BES III - binned

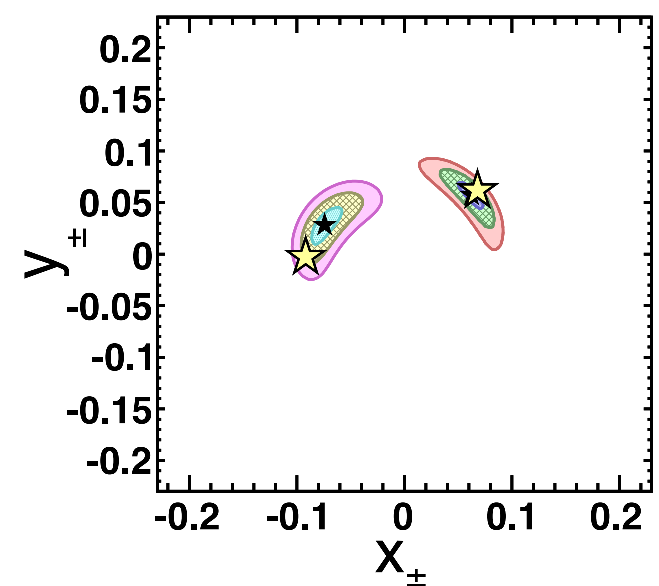
D-mixing (binned, LHCb run II statistics)



$$x_{\pm} = r_B \cos(\delta_B \pm \gamma)$$
$$y_{\pm} = r_B \sin(\delta_B \pm \gamma)$$

Combination

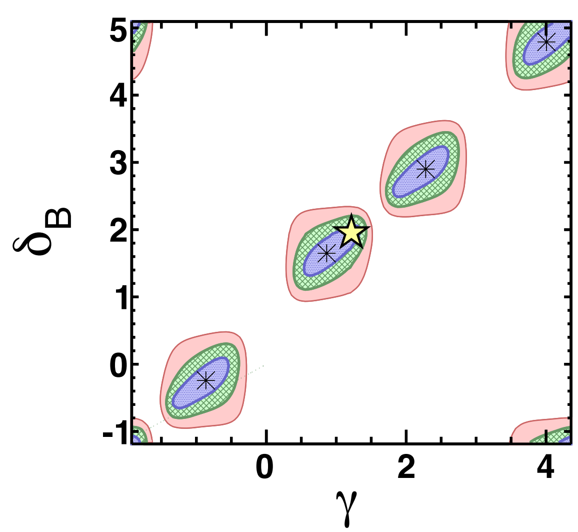




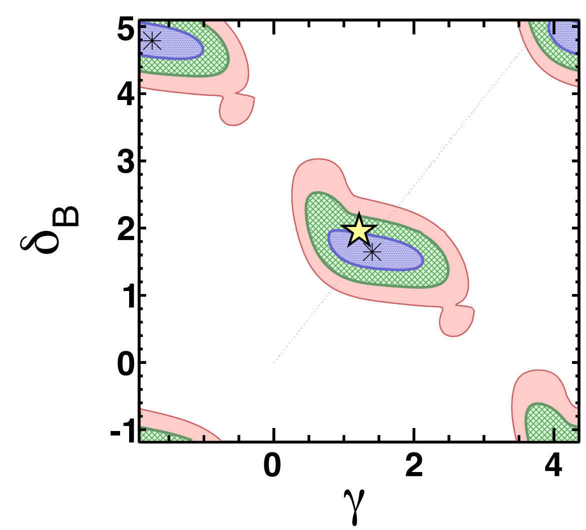
arXiv:1412.7254 (accepted by JHEP)

Gets even better if we divide the 5-D space into bins

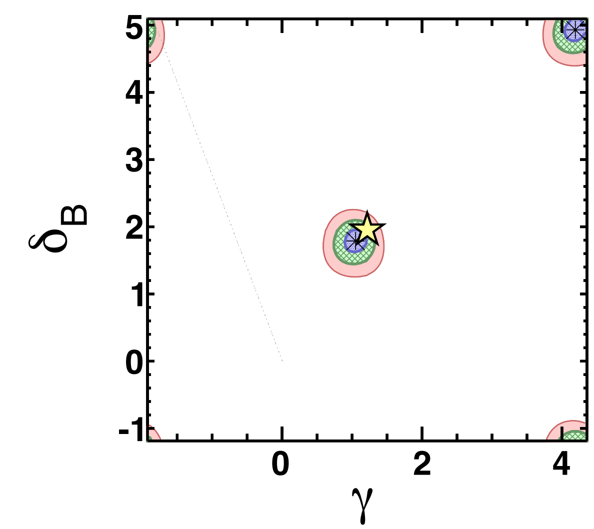
BES III - binned



D-mixing (binned, LHCb run II statistics)



Combination



(all simulated data)

arXiv:1412.7254 (accepted by JHEP)

Searches for CPV by comparing binned Dalitz plots

PhysRevD.84.112008

- Compare yields in CP-conjugate bins

$$S_{CP} = \frac{N_i - \alpha \overline{N}_i}{\sigma(N_i - \alpha \overline{N}_i)}$$

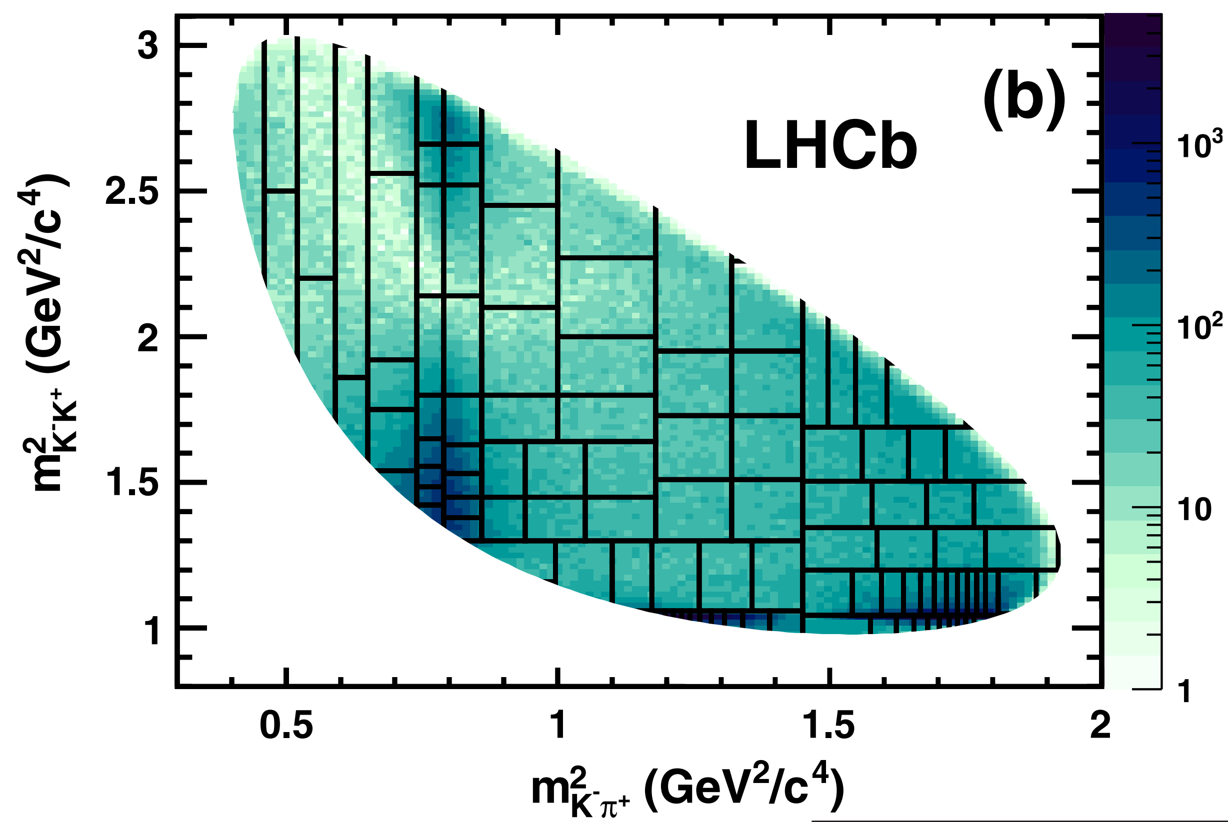
$$\alpha = \frac{N_{\text{total}}}{\bar{N}_{\text{total}}}$$

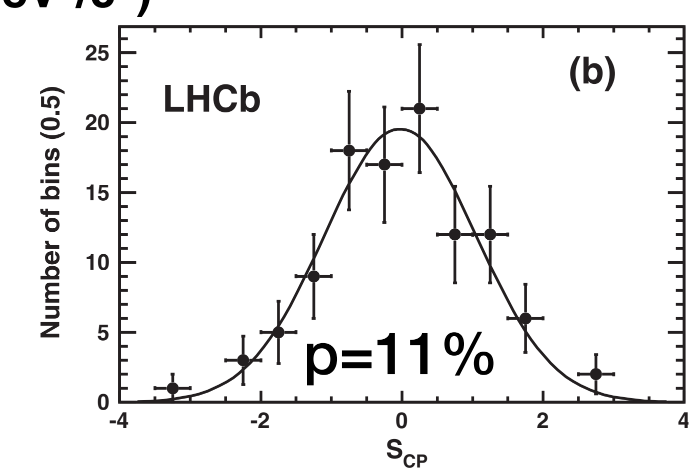
- Calculate p-value for no-CPV hypothesis based on

$$\chi^2 = \sum_i (S^i_{CP})^2$$

- Model independent. Many production and detection effects cancel.

330k $D^+ \rightarrow K^- K^+ \pi^+$ in 35/pb



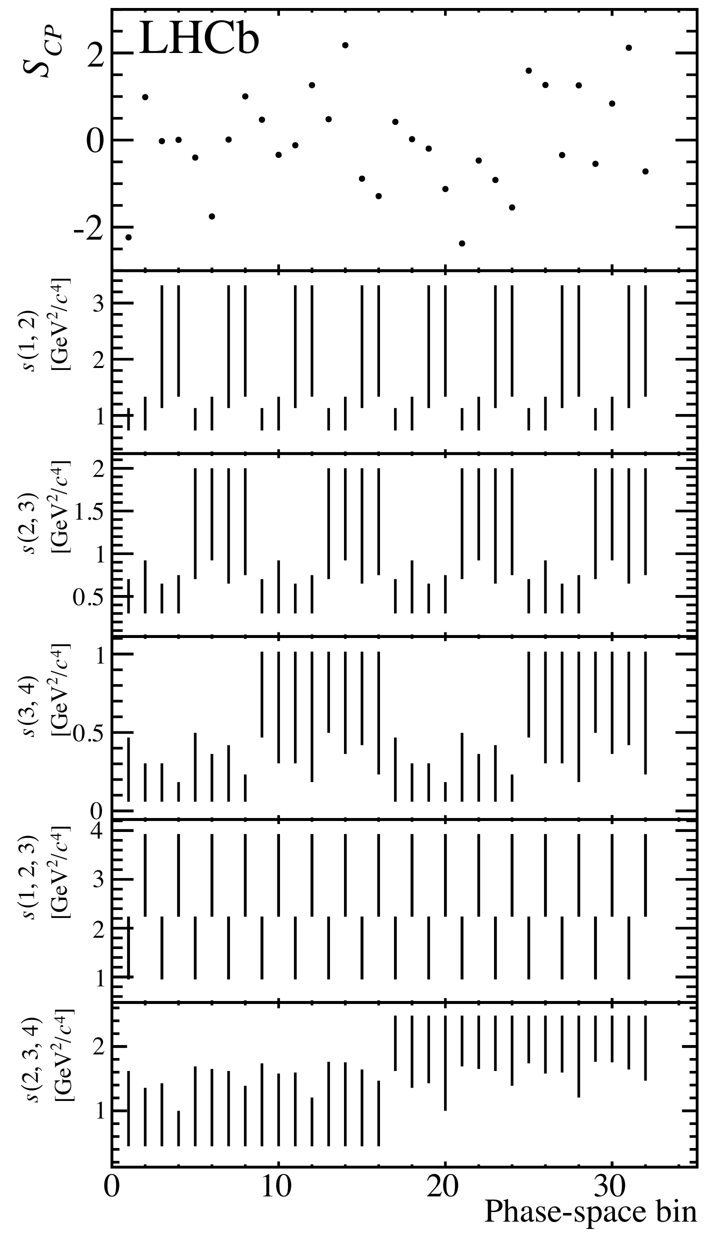


5-D binned analysis in $D^0 \to K^+K^-\pi^+\pi^-$, $D^0 \to \pi^+\pi^-\pi^+\pi^-$

LHCb 1fb^{-1} Phys.Lett. B726 (2013) 623-633

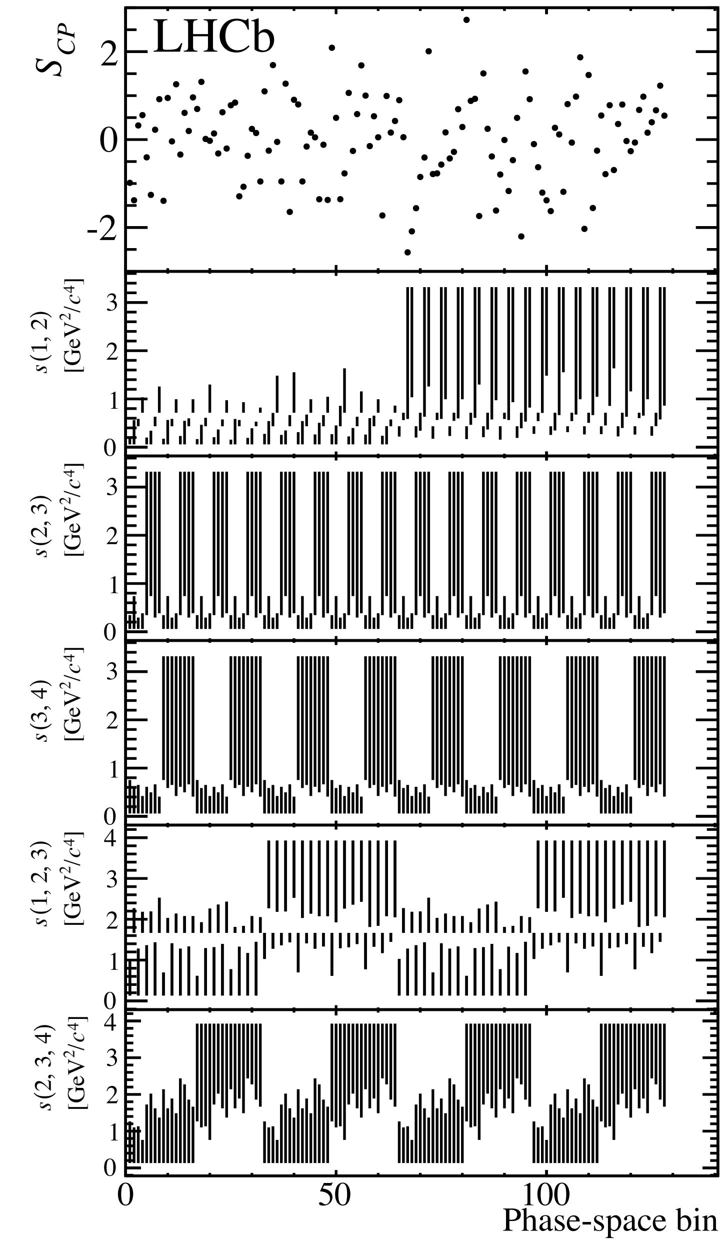
- Binning in 5-dimensional hyper-cuboids.
- Adaptive binning to ensure similar number of entries per bin.
- Plots show for each bin the range in invariant mass squared and S_{CP} value in that bin.

$D^0 \to K^-K^+\pi^-\pi^+$



p=9%

$D^0 \to \pi^-\pi^+\pi^-\pi^+$



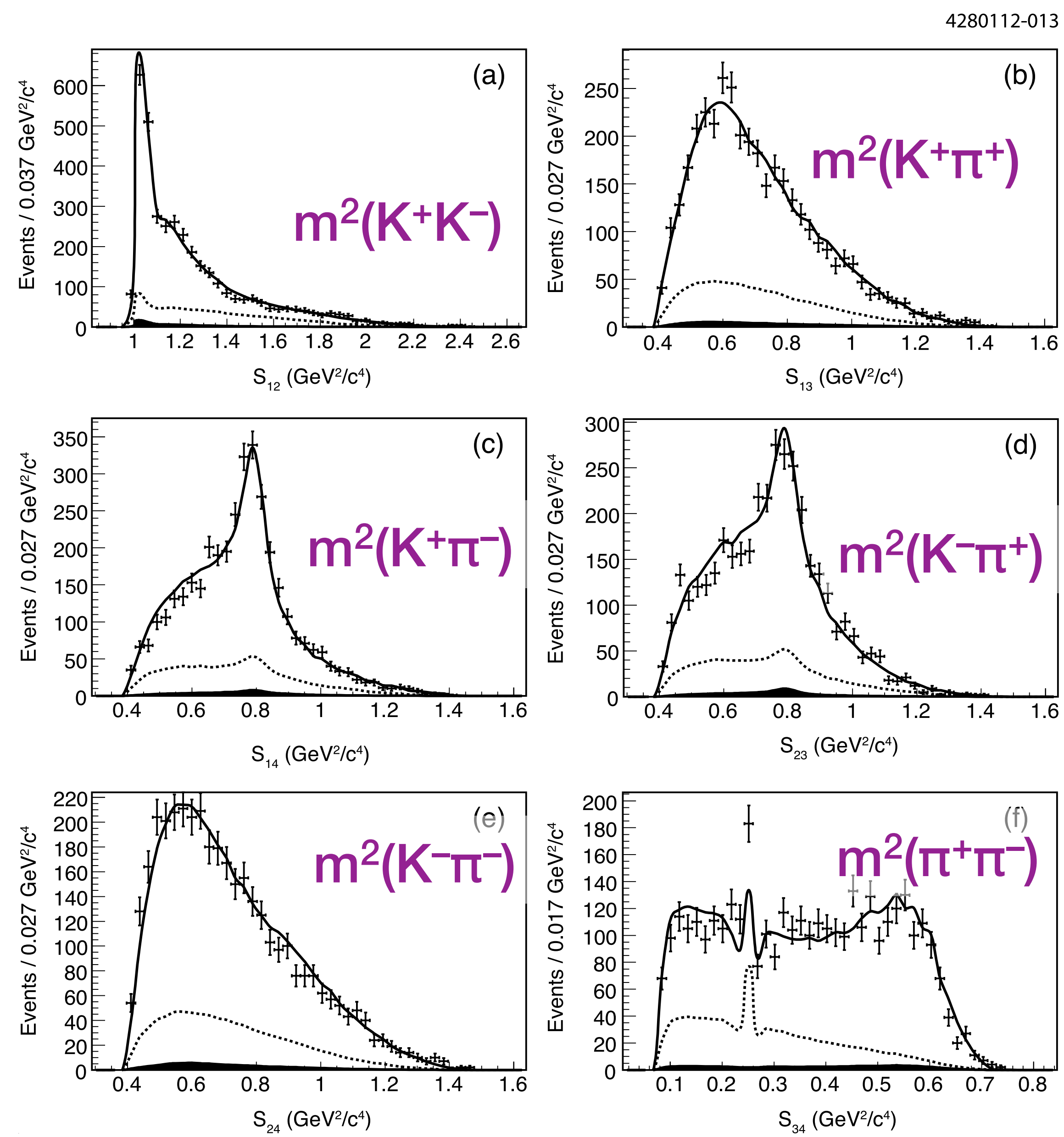
p=41%

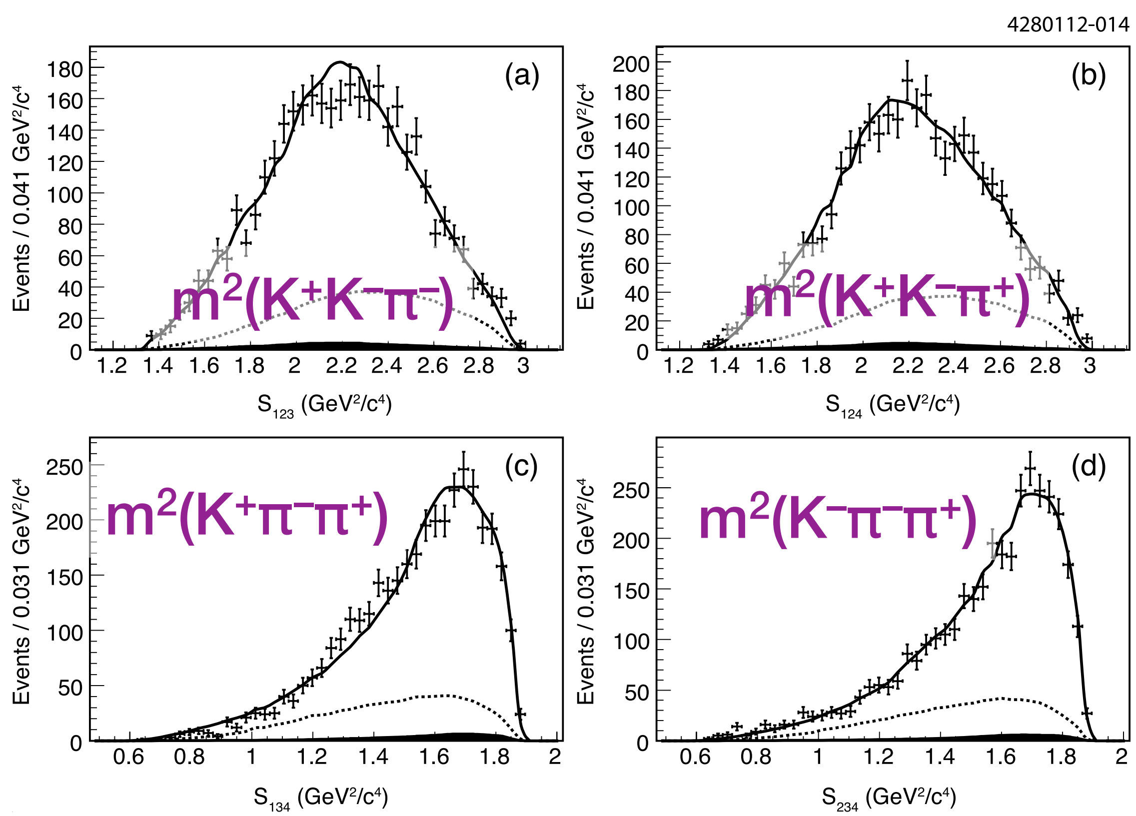
Model-dependent CPV search in $D^o \rightarrow K^+ K^- \pi^+ \pi^-$

CLEO: Phys.Rev. D85 122002 (2012)

1-D projections of 5-D amplitude fit

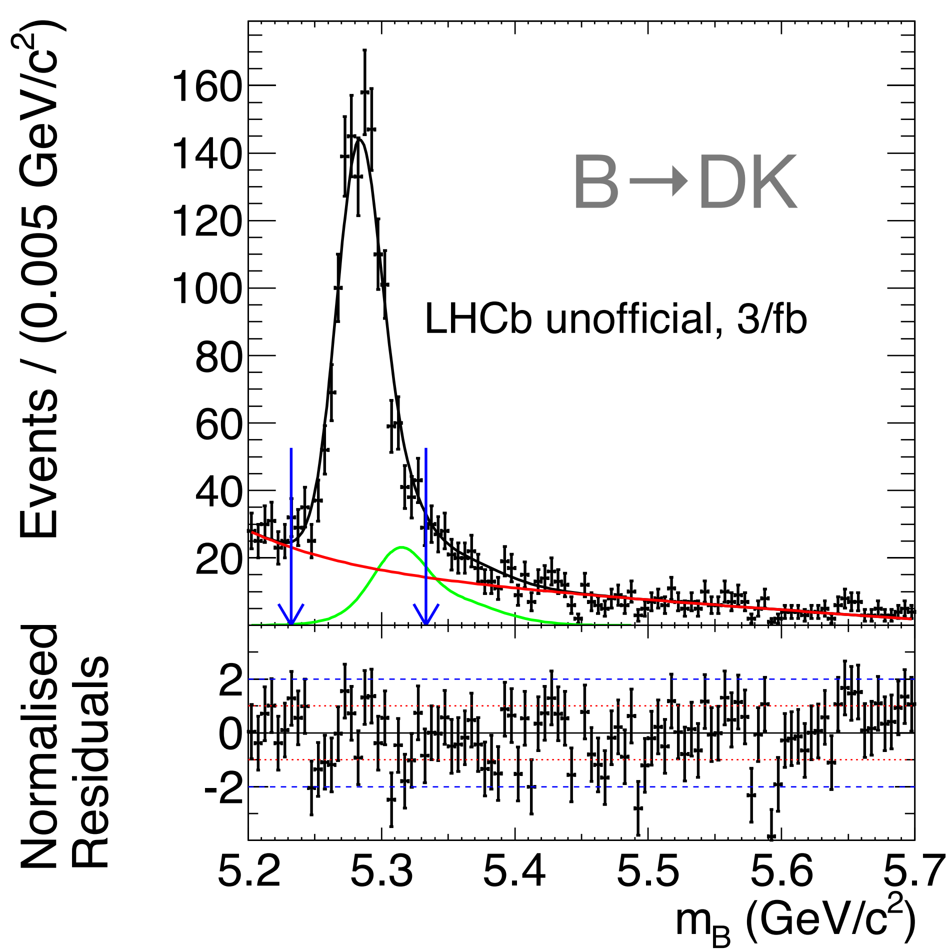
(D^o, D^obar combined, charge assignments in $m^2(K^+\pi^-)$ etc are for D^o and are reversed for D^obar)



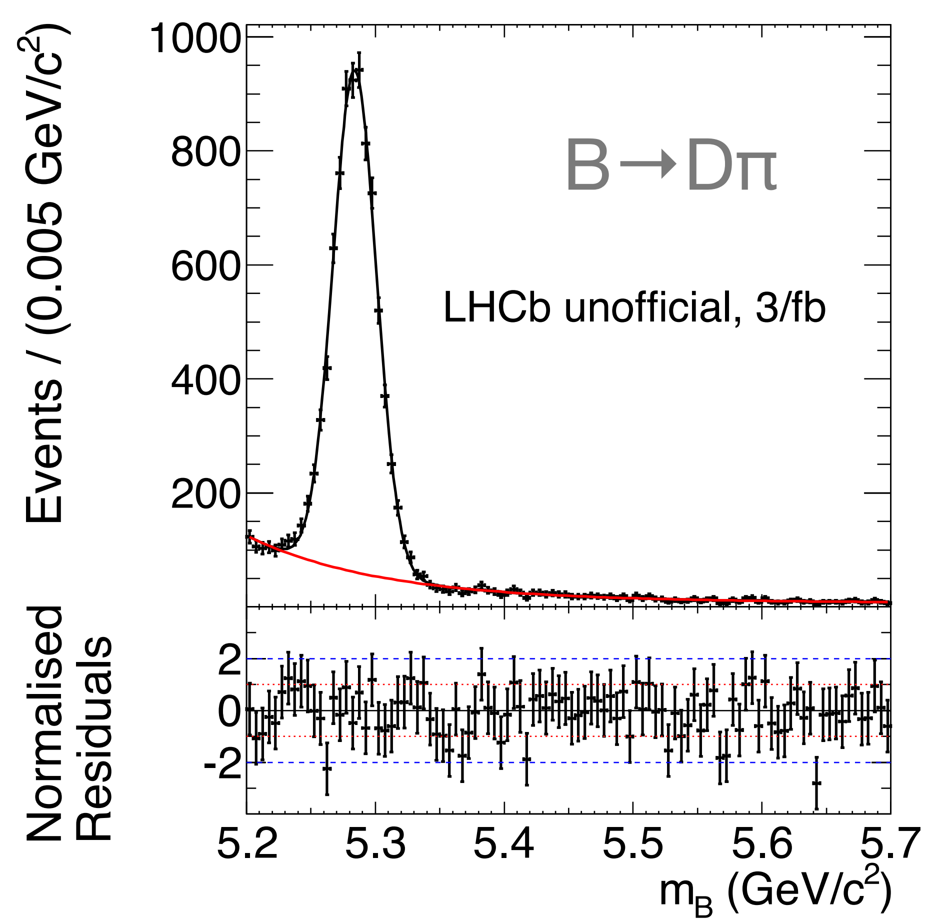


Towards γ with $B^{\pm} \rightarrow D(KK\pi\pi)K^{\pm}$

Signal



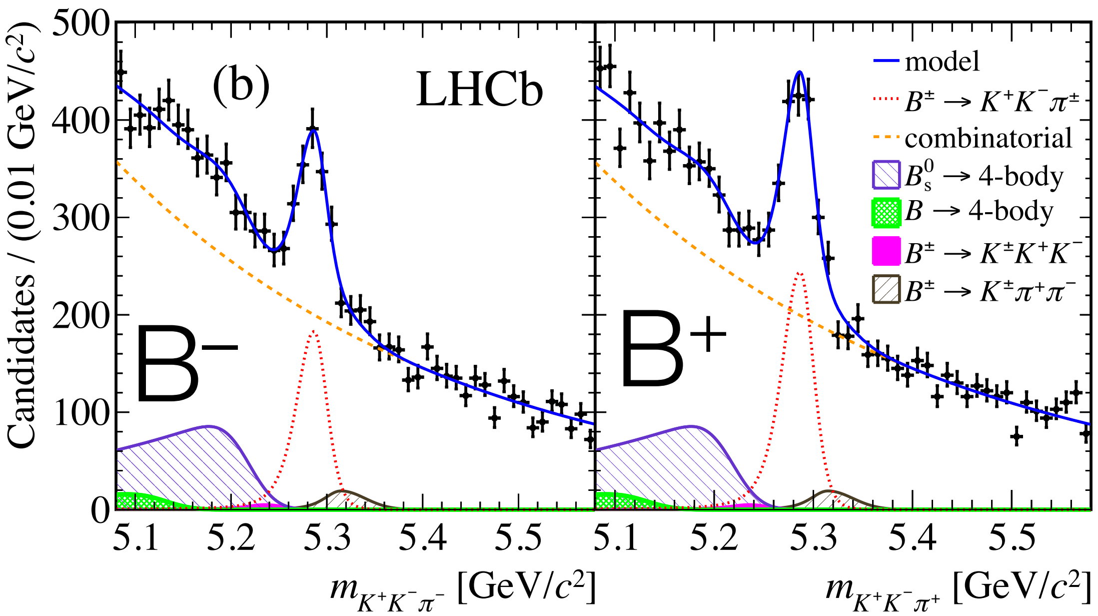
Control Channel



Nicole Skidmore & Jeremy Dalseno (Bristol)

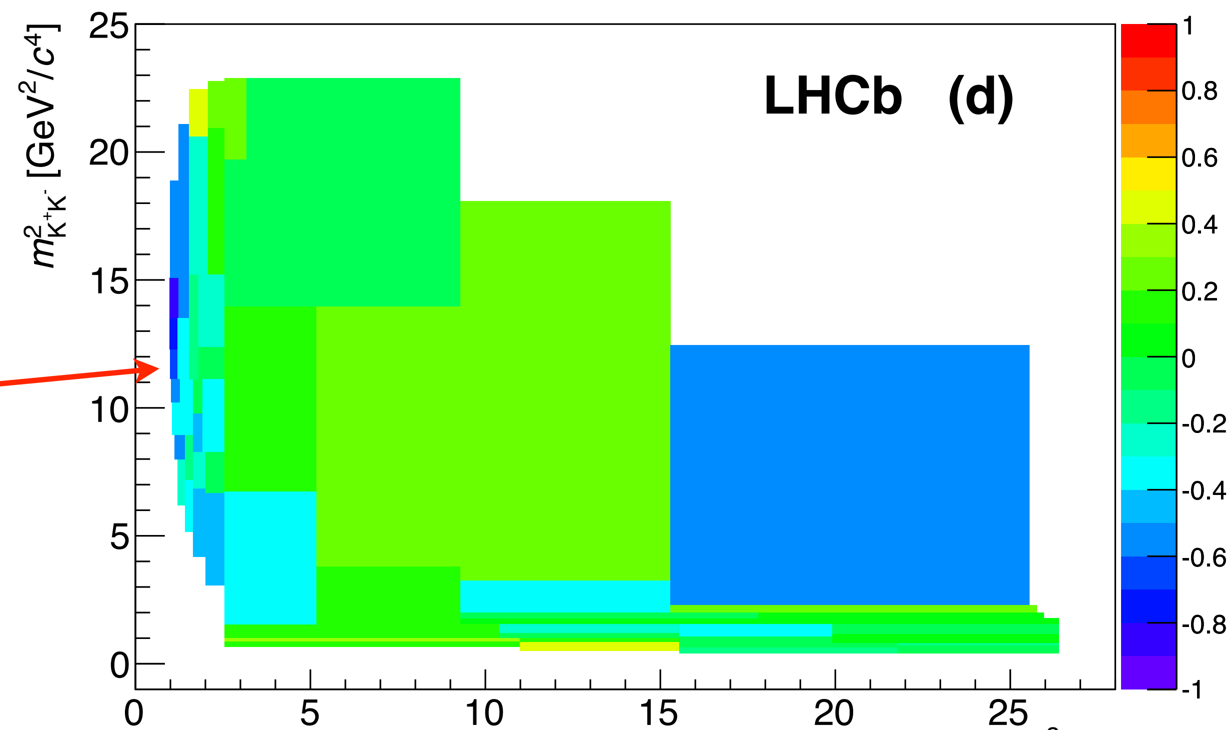
CPV in $B^{\pm} \to \pi^{\pm} K^{+} K^{-}$

Phys. Rev. Lett. 112, 011801 (2014)
Phys. Rev. Lett. 111, 101801 (2013)
3/fb update: arXiv:1501.06777 (2014)



local:

$$A_{CP\ \mathrm{bin}} = \frac{N_{\mathrm{bin}}(B^{-}) - N_{\mathrm{bin}}(B^{+})}{N_{\mathrm{bin}}(B^{-}) + N_{\mathrm{bin}}(B^{+})}$$



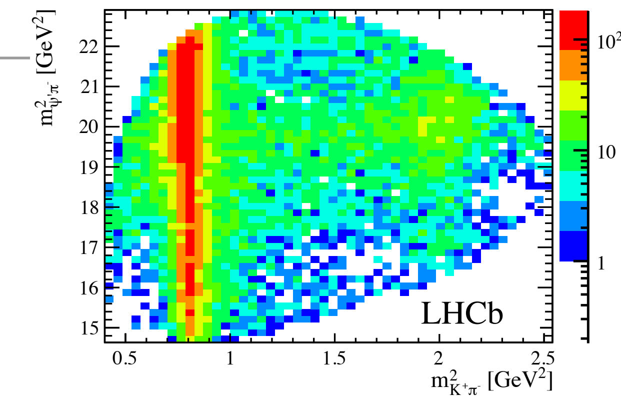
Large

local A_{CP} at low $m(KK)^2$, not associated to a resonance

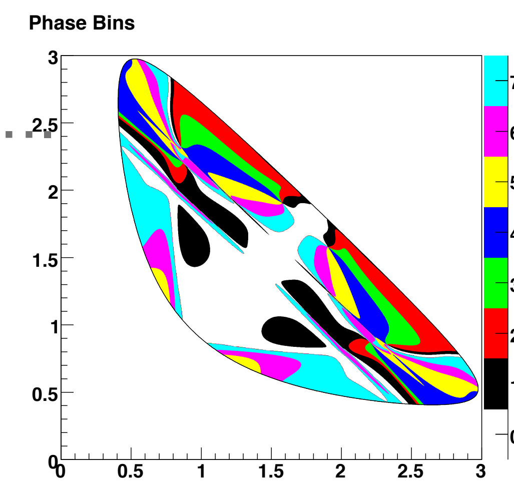
Also found large local CPV in low mass regions w/o clear association to known resonances in other $B^{\pm} \to hhh$ modes:

$B^{\pm} \to K^{\pm}\pi^{+}\pi^{-}$, $B^{\pm} \to K^{\pm}K^{+}K^{-}$, $B^{\pm} \to \pi^{\pm}\pi^{+}\pi^{-}$, $B^{\pm} \to \pi^{\pm}K^{+}K^{-}$

Conclusions



- Amplitude Analyses are a very powerful tool used at LHCb and elsewhere for wide variety of measurements, including
  - searching for new resonances and characterising them
  - precision CP violation and mixing measurements in charm and beauty



- They are not “just” Dalitz plots. Vectors in final state, 4 body analyses,…
- Most remarkable strength: unique sensitivity to phases.
- Most annoying weakness: theoretically not well understood. This is increasingly problematic with increasingly ginormous data samples.

- Theorists are making tangible progress on theoretically sound models.
- Future: improved models, model independent methods, pragmatic compromises.

Credits

Special thanks to Antimo Palano and Marco Pagapallo, from whose excellent talks I lifted a particularly large number of plots.

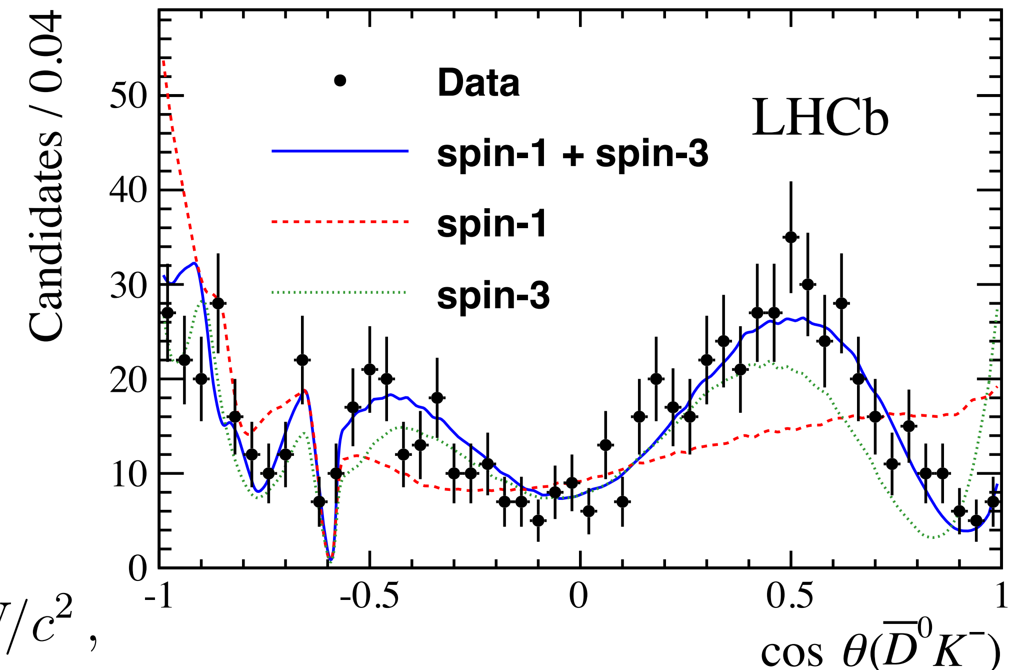
Backup

$B_s \to DK\pi$ at LHCb

(Phys.Rev. D90 (2014) 072003)

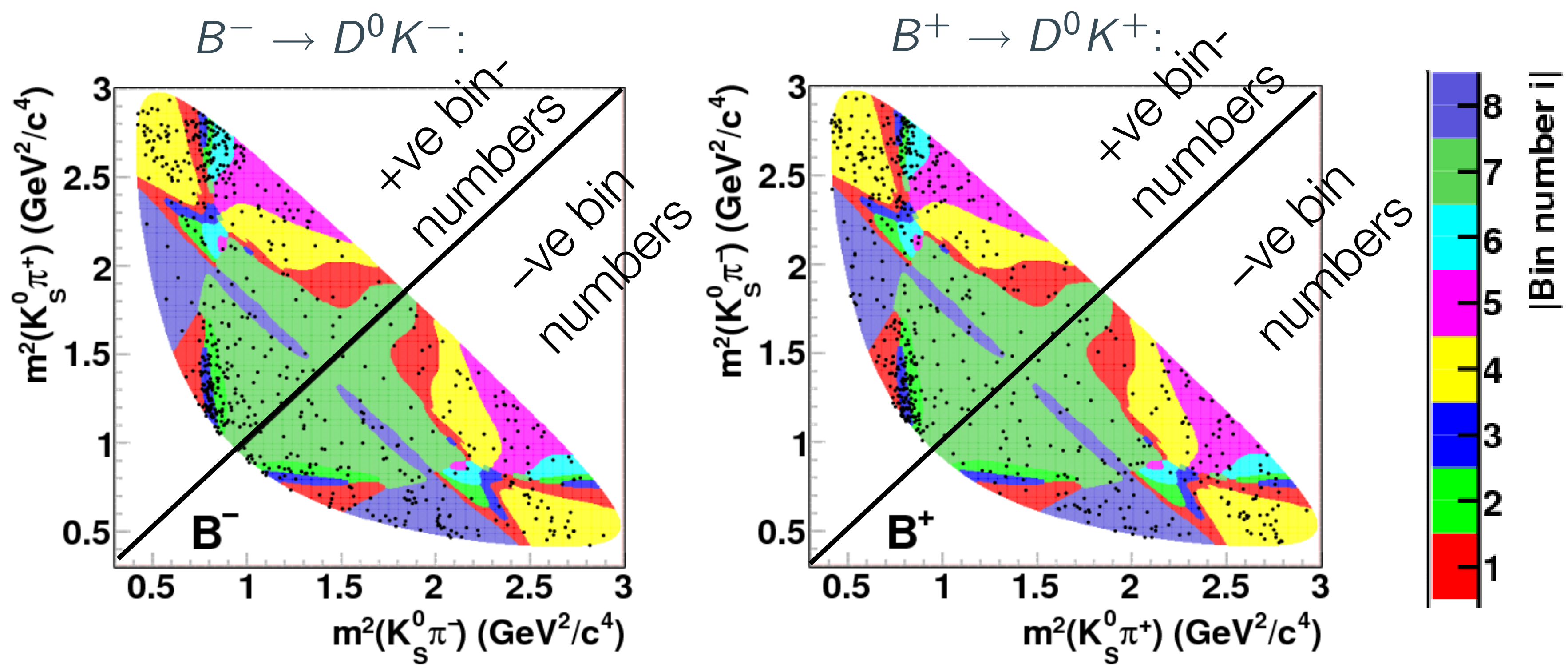
- Resolved the $D_{sJ}^*(2860)$ state into spin 1 and spin 3 states
  - Now part of a renaissance in D(s) spectroscopy (15 citations so far)

- Other results
  - Mass, width and spin of D_{s2}^*
  - Fit fractions
  - Branching fractions
  - Complex amplitudes

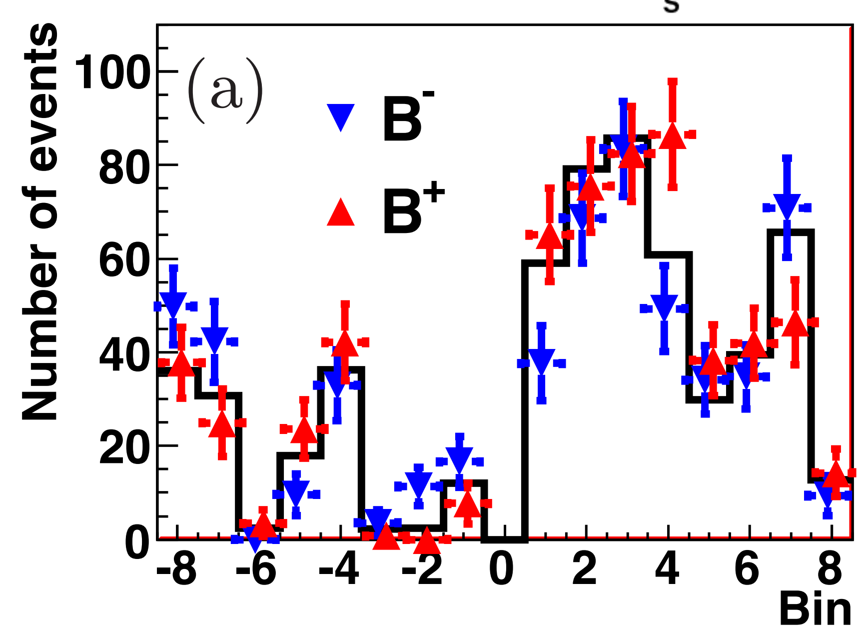


$$
\begin{aligned}
m(D_{s1}^*(2860)^-) &= 2859 \pm 12 \pm 6 \pm 23\,\mathrm{MeV}/c^2\,,\\
\Gamma(D_{s1}^*(2860)^-) &= 159 \pm 23 \pm 27 \pm 72\,\mathrm{MeV}/c^2\,,\\
m(D_{s3}^*(2860)^-) &= 2860.5 \pm 2.6 \pm 2.5 \pm 6.0\,\mathrm{MeV}/c^2\,,\\
\Gamma(D_{s3}^*(2860)^-) &= 53 \pm 7 \pm 4 \pm 6\,\mathrm{MeV}/c^2\,,
\end{aligned}
$$

First model-independent γ measurement (BELLE)



$$\gamma = (77.3^{+15.1}_{-14.9} \pm 4.2 \pm 4.3)^\circ$$
$$r_B = 0.145 \pm 0.030 \pm 0.011 \pm 0.011$$
$$\delta_B = (129.9 \pm 15.0 \pm 3.9 \pm 4.7)^\circ,$$



where the last uncertainty on γ of 4.3° the former model uncertainty of 8.9°

BELLE: **arXiv:1106.4046**. See also Anton Poluektov's talk at Moriond EW 2011 (from which I lifted several of the plots shown here): http://belle.kek.jp/belle/talks/moriondEW11/poluektov.pdf
CLEO-c input:Phys.Rev.D82:112006,2010.

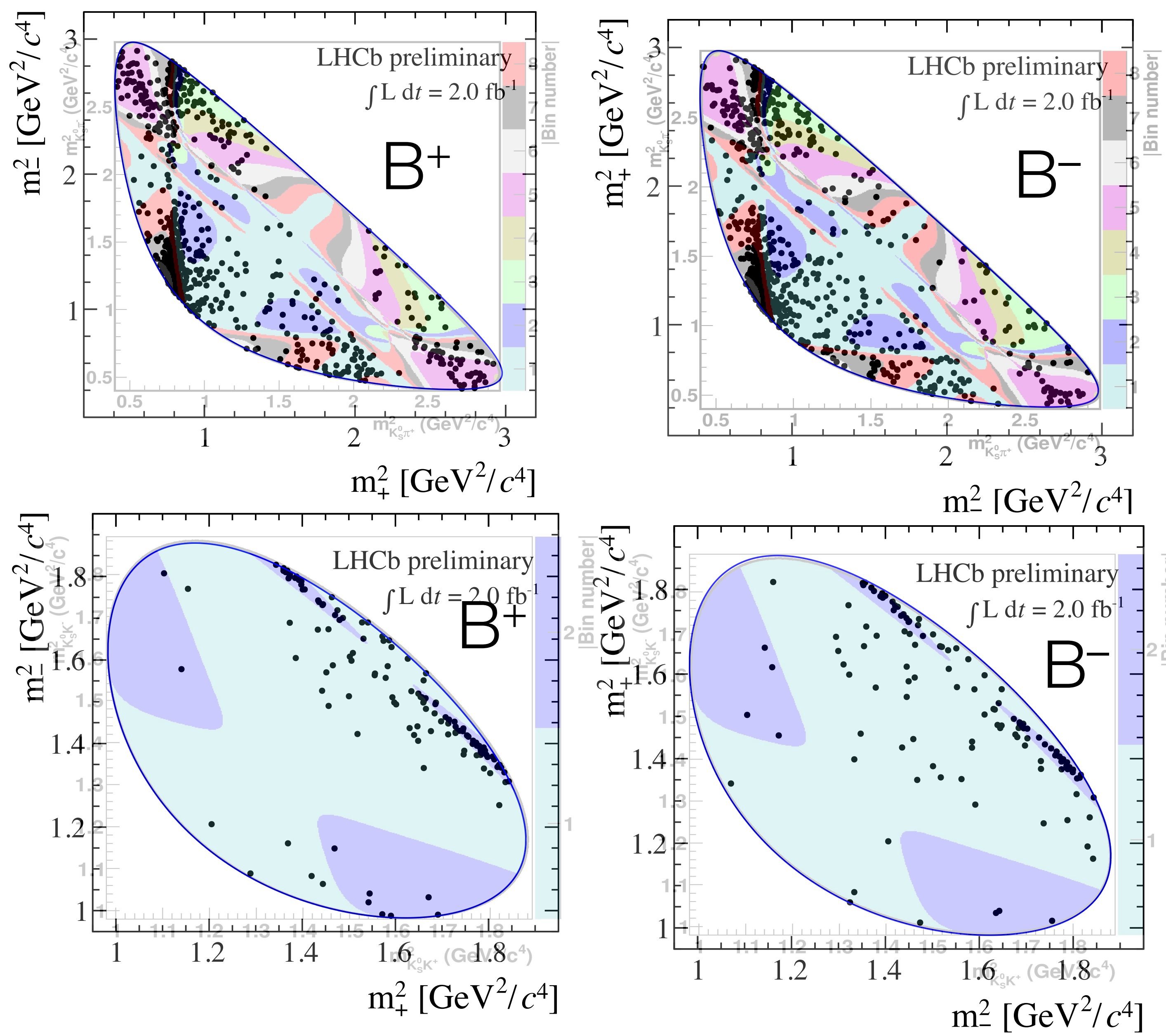
LHCb model-independent γ from $B^\pm \to (K_S\pi\pi)_D K$ and $B^\pm \to (K_S KK)_D K$

LHCb-CONF-2013-004

LHCb 2011 Result: Phys. Lett. B718 (2012) 43

- Binned, model-independent analysis using CLEO-c input.
  Phys. Rev. D 82 112006.
- Plots show LHCb 2012 data - the colours represent the bins, shaped to optimise sensitivity.
- Result of combined analysis (2011 & 2012 data, $K_S\pi\pi$ & K_SKK):

$$\gamma = (57 \pm 16)^\circ$$
$$\delta_B = (124^{+15}_{-17})^\circ$$
$$r_B = (8.8^{+2.3}_{-2.4}) \times 10^{-2}$$



CLEO-c input:: Phys. Rev. D 82 112006.
Model-independent method: Giri, Grossmann, Soffer, Zupan, Phys Rev D 68, 054018 (2003).
Optimal binning: Bondar, Poluektov hep-ph/0703267v1 (2007)
BELLE's first model-independent γ measurement: PRD 85 (2012) 112014

B->DK, D->3pi with BES & Mixing

LHCb scenario	D^0 mix?	charm threshold?	$\sigma(\gamma)$ [°]	$\sigma(\delta_B)$ [°]	$\sigma(r_B)$ $\times 10^2$	$\sigma(x_+)$ $\times 10^2$	$\sigma(y_+)$ $\times 10^2$	$\sigma(x_-)$ $\times 10^2$	$\sigma(y_-)$ $\times 10^2$
run I			26	47	1.6	8.7	9.1	8.8	8.2
run II	Y	none	22	29	1.4	7.6	6.9	4.5	4.0
upgr			15	14	0.17	4.7	5.2	0.56	0.98
run I			20	29	0.82	6.4	5.7	6.6	5.9
run II	Y	CLEO global	15	19	0.62	5.4	3.9	2.5	2.7
upgr			11	10	0.16	3.8	2.8	0.44	0.50
run I			19	25	0.78	6.4	5.5	6.5	5.8
run II	Y	BESIII global	14	18	0.57	5.4	3.9	2.4	2.7
upgr			9.0	8.2	0.15	3.7	2.7	0.43	0.48
run I			46	35	3.2	6.9	6.5	8.6	10
run II	N	CLEO binned	50	34	3.3	6.9	6.7	8.9	11
upgr			52	35	3.3	7.6	6.7	8.9	11
run I			40	24	2.6	4.1	5.0	5.7	6.2
run II	N	BESIII binned	34	17	2.5	3.6	4.1	5.0	5.1
upgr			39	14	2.9	3.9	4.1	4.3	5.6
run I			16	18	0.78	2.1	3.5	2.6	3.1
run II	Y	CLEO binned	12	13	0.53	1.7	3.1	1.7	2.0
upgr			7.8	7.2	0.15	1.1	2.6	0.40	0.46
run I			12	14	0.68	1.6	2.6	2.0	2.5
run II	Y	BESIII binned	8.6	9.6	0.47	0.90	2.1	1.5	1.5
upgr			4.1	3.9	0.14	0.53	1.3	0.35	0.38

arXiv:1412.7254

(accepted by JHEP)