$$B \to K^* \mu^+ \mu^-$$

Theory Point of View 2009

1. The Standard Model and its high energy frontier
2. Calculating the $B \to K^* \mu^+ \mu^-$ decay amplitude
3. All the fun we could have with enough data (angular observables)

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The Standard Model of Particle Physics

renormalizable quantum field theory + local symmetry

$$SU(3)_C \times SU(2)_L \times U(1)_Y \to SU(3)_C \times U(1)_{em}$$

$$\mathcal{L} = -\frac{1}{4}F^2 + \bar{\psi} i \not{D} \psi + \frac{1}{2}(D\Phi)^2 - \bar{\psi} Y \psi \Phi + \mu^2 \Phi^2 - \lambda \Phi^4$$

put 3 generations for the fermions $\psi \to \psi_i$ and $i = 1, 2, 3$.

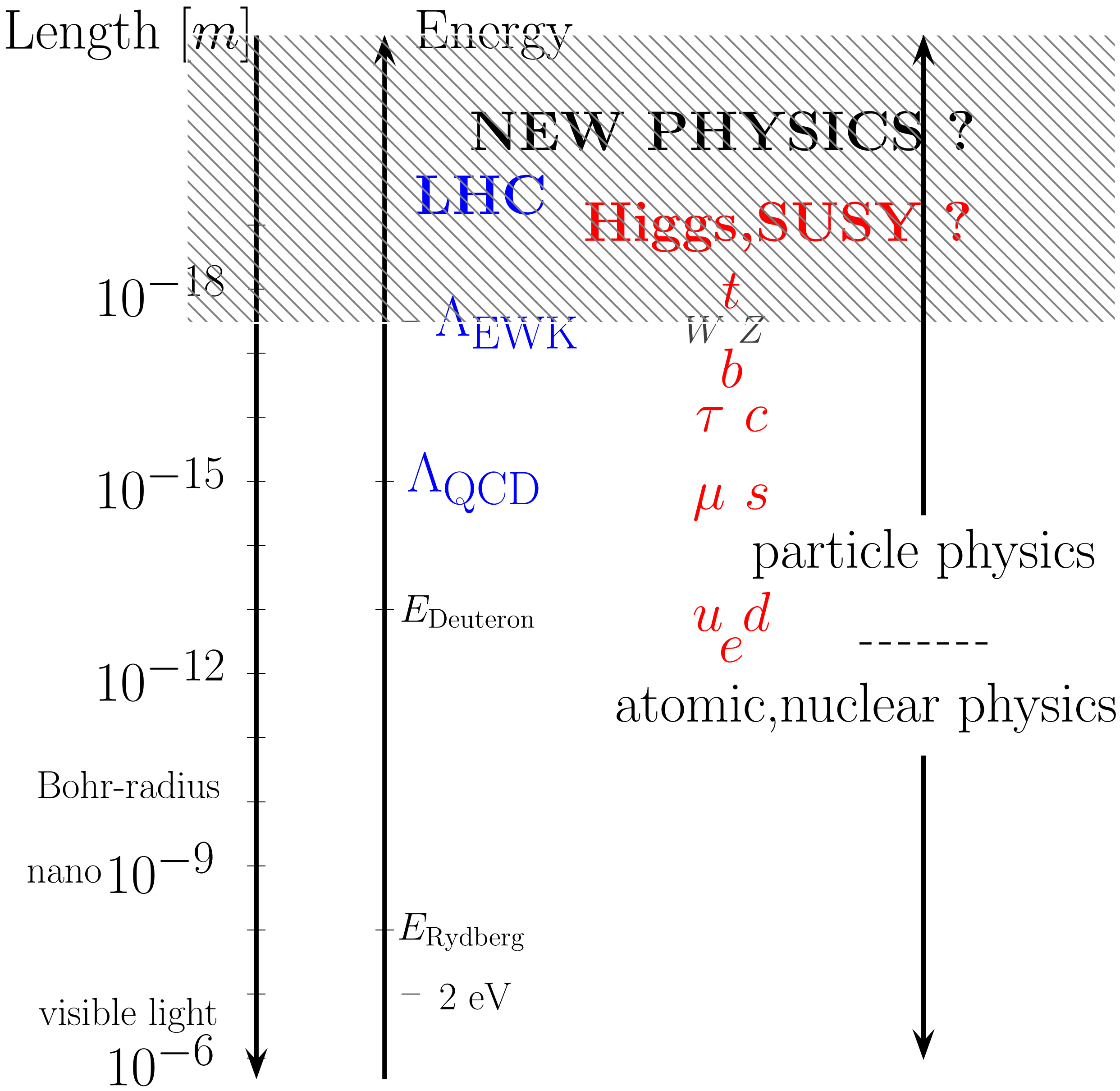
quarks: $Q_L(3, 2, 1/6)_i$, $D_R(\bar{3}, 1, -1/3)_i$, $U_R(\bar{3}, 1, 2/3)_i$

leptons: $L_L(1, 2, -1/2)_i$, $E_R(1, 1, -1)_i$

$$\begin{pmatrix} u \\ d \end{pmatrix}, \begin{pmatrix} c \\ s \end{pmatrix}, \begin{pmatrix} t \\ b \end{pmatrix} \qquad \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}, \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}, \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}$$

ok up to $\mathcal{O}(100)$ GeV ($\sim 10^{-18} m$)

Exploring Physics at Highest Energies



Quarks mix and change flavor.

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \simeq \begin{pmatrix} 1 & \lambda & \lambda^3 \\ -\lambda & 1 & \lambda^2 \\ -\lambda^3 & -\lambda^2 & 1 \end{pmatrix} ; \quad \lambda \simeq 0.22$$

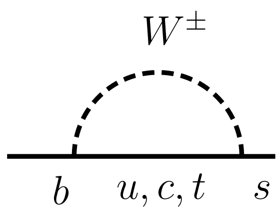
3 generations $=$ 10 parameters in flavor $\&$ CP sector: 6 masses, 3 angles and 1 phase in CKM-matrix unitary, complex, hierarchical, known

$|V_{us}| = 0.2257(21), \quad |V_{cb}| = 41.6 \pm 0.6 \cdot 10^{-3}, \quad |V_{ub}| = 4.31 \pm 0.3 \cdot 10^{-3}$
β(measured) $= (21.23^{+1.03}_{-0.99})^\circ$

-The third generation is decoupled from the first two.

-The CP violating phase is order one.

PS: enormous progress from B-factories over past decade. PPS: still improving precision generic SM $b \to s$ amplitude



quantum loop effect $\mathcal{A}(b \to s) = \underbrace{V_{ub}V_{us}^*}_{\mathcal{O}(\lambda^4)} A_u + \underbrace{V_{cb}V_{cs}^*}_{\mathcal{O}(\lambda^2)} A_c + \underbrace{V_{tb}V_{ts}^*}_{\mathcal{O}(\lambda^2)} A_t$

with unitarity $VV^\dagger = 1$:

$$\mathcal{A}(b \to s) = V_{tb}V_{ts}^*(A_t - A_c) + V_{ub}V_{us}^*(A_u - A_c) = V_{tb}V_{ts}^*(A_t - A_c) + \mathcal{O}(\lambda^4)$$

GIM inactive $\frac{m_t^2 - m_c^2}{m_W^2} \sim \mathcal{O}(1)$ b-FCNC sensitivity to EWK scale

FCNC predictions for SM ... imply smoking guns for NP:

– $c \to u$, top FCNCs: GIM and CKM suppressed

– direct CP violation $b \to s$ small: $|\mathcal{A}(b \to s)| = |\mathcal{A}(\bar{b} \to \bar{s})|(1 + \mathcal{O}(\lambda^2))$

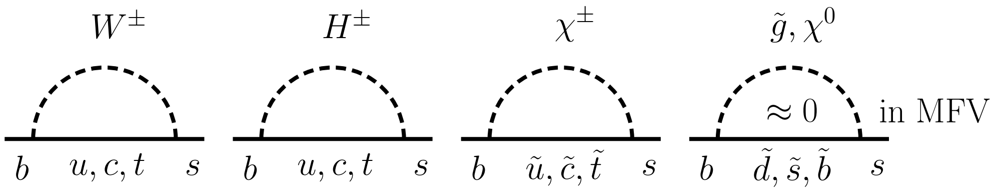
SM tests with indirect processes/FCNC

indirect loop processes: $\Lambda \gtrsim m_W$ scale of New Physics

$$\mathcal{L}_{eff} = \sum_i c_i^{(n)} \frac{O_i^{(n)}}{\Lambda^n} \qquad c_i^{(n)} \leftrightarrow f(\underbrace{m_j, g_l, \ldots}_{colliders}; \underbrace{\varphi_{\rm CKM}, \varphi_m, \delta_n}_{flavorphysics})$$

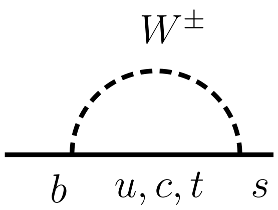
no competition from large SM tree contributions

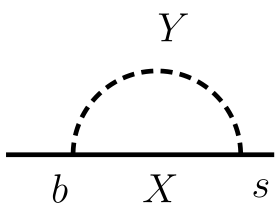
FCNC: sensitivity to SM, NP phases φ, flavor-breaking couplings δ



BSM-FCNC: offset rates, distort spectra, induce CP-asymmetries, V+A currents, ..

Penguins and effective theory





add $A = \gamma, g, Z, h^0$, ... BSM neutral ="A"-penguin

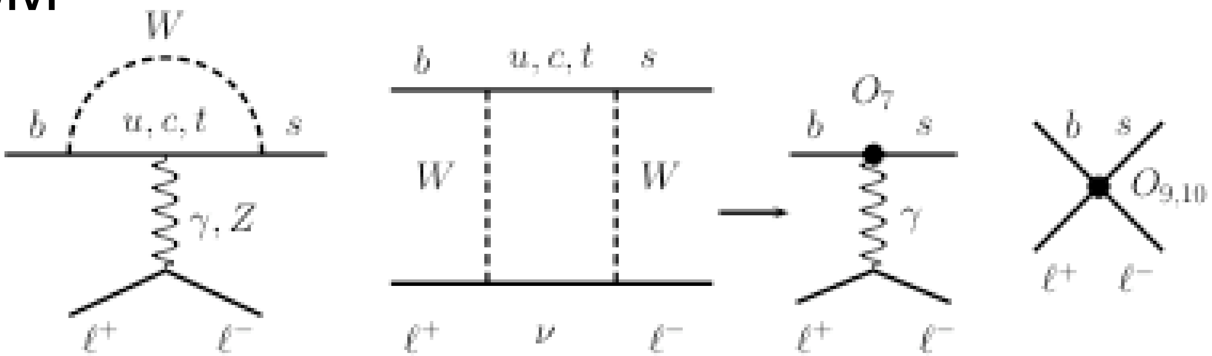
construction of weak low energy effective theory valid $\mu \lesssim \mu_W = \Lambda$

$$\mathcal{L}_{\rm eff} = \sum_i c_i(\mu) \frac{O_i(\mu)}{\Lambda^2} + \mathcal{O}(\frac{p^4}{\Lambda^4})$$

O_i: dim 6 operators out of light degrees of freedom originate from penguins and boxes

c_i: Wilson coefficients: contain info on high scales $\gtrsim \mu_W$ e.g., hep-ph/9806471
$c_i(m_W)$: matching of effective onto full theory. RG-running
c_i: known up to NNLO in SM for QCD, and NLO for EWK corr.

diagrams in SM



$$\mathcal{H}_{\text{eff}} = -4\frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum C_i(\mu) O_i(\mu)$$

dipole operators $O_7 \propto \bar{s}_L \sigma_{\mu\nu} b_R F^{\mu\nu}$ $O_8 \propto \bar{s}_L \sigma_{\mu\nu} b_R G^{\mu\nu}$

4-Fermi operators $O_9 \propto (\bar{s}_L \gamma_\mu b_L)(\bar{\ell}\gamma^\mu \ell)$ $O_{10} \propto (\bar{s}_L \gamma_\mu b_L)(\bar{\ell}\gamma^\mu \gamma_5 \ell)$

NP in Wilson coefficients $C_i = C_i^{SM} + C_i^{NP}$ or new operators

model-independent analysis: Br's, $A_{CP}, A_{FB} = f(C_i) \to$ fit! hep-ph/9408213

Brief penguin summary & prospects

- Penguin bounds: (at $\mu \simeq m_b$, assuming no BSM operators)

$$bsZ : |C_{10}| \lesssim (1-2)|C_{10}|_{\rm SM} \, , \quad bs\gamma : C_7 \simeq C_{7\,\rm SM} \, , \quad bsg : |C_8| \lesssim 5|C_8|_{\rm SM} \, .$$

- Todays best bound on MSSM Higgs-penguins from Tevatron $\mathcal{B}(B_s \to \mu^+\mu^-)$
- Tools in penguin-physics: multi-observable analyses and fits and SM-null tests.
- $b \to d$ beginning to be probed, MFV-link with $b \to s$ to come: CP-phases and helicity.

How to calculate $A(B \to K^* \mu\mu)$

1. choose model, such as SM, MSSM etc. This is your ”full” theory.

2. Calculate the low energy effects ”Wilson coefficitents” of this full theory within a ”generalized Fermi-theory”, the effective theory, H_{eff}.

3. Take the matrix element $A(B \to K^* \mu\mu) = \langle K^* \mu\mu | H_{eff} | B \rangle$. This needs input from non-pertubative QCD: form factors etc.

In full QCD, there are 7 form factors in $B \to K^*$:
$A_0, A_1, A_2, V, T_1, T_2, T_3$. see, e.g., ABHH,hep-ph/9910221
This simplifies for low dilepton mass to just 2: $\xi_\perp, \xi_{||}$. e.g., BFS hep-ph/0412400

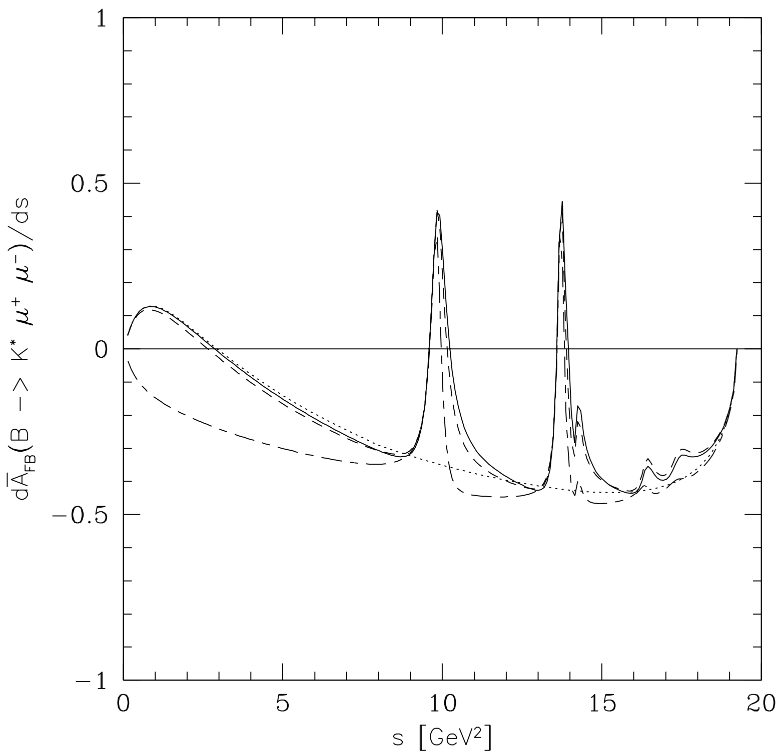
4. Work out your observables/distributions.

5. Employ cuts: Remove huge BGD from $B \to V_{cc} K^* \to \mu\mu K^*$; $V_{cc} = J/\Psi, \Psi', ..$ by cuts in dilepton invariant mass.

$d^2\Gamma/dq^2 d\cos\Theta$; note: $\cos\Theta(\bar{B}l^+) = -\cos\Theta(\bar{B}l^-)$

A_{FB}: # forward - # backward ℓ^+ in dilepton CMS w.r.t. $\bar{B}$ (CP-odd)

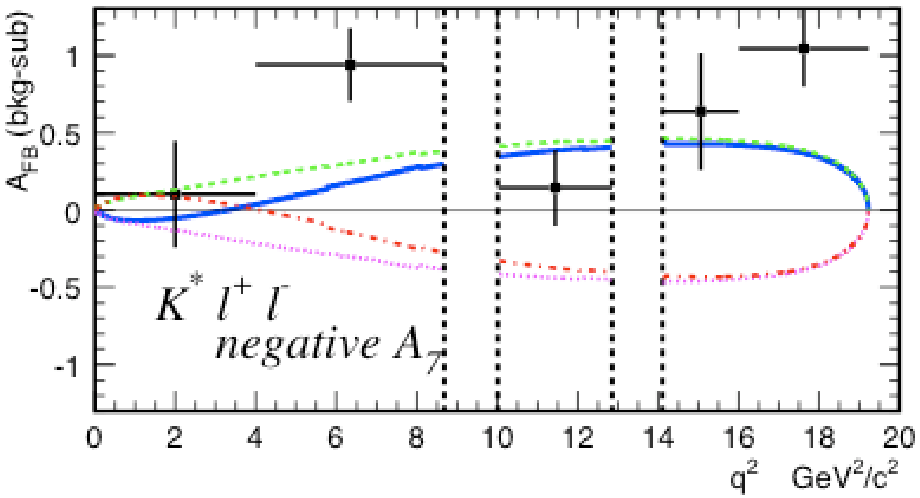
$$A_{FB}(\hat{s}) \equiv \int d\cos\Theta \text{sign}(\cos\Theta) \frac{d\Gamma}{d\hat{s} d\cos\Theta} \sim -\text{Re}\left[C_{10}^*(C_7^{\text{eff}} + \beta(\hat{s}) C_9^{\text{eff}})\right]$$

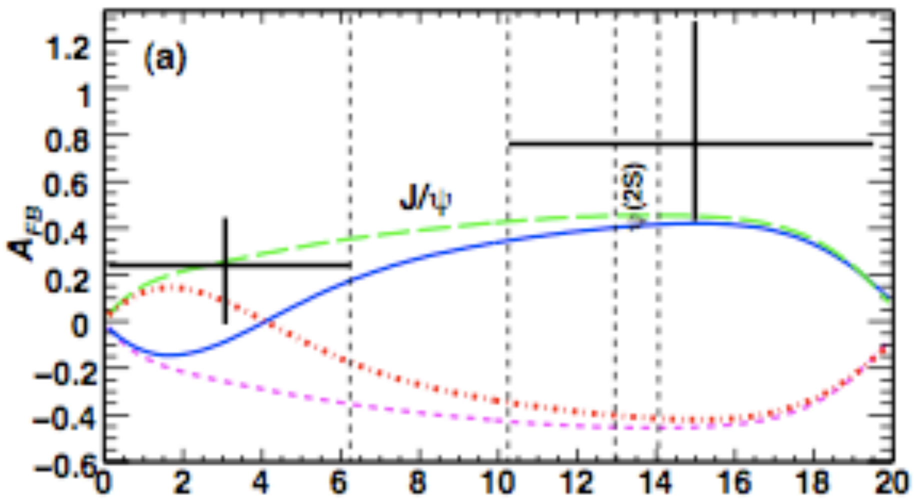


Exclusive $b \to (s,d)ll$ and New Physics Searches

There is no unique rigorous framework available to describe exclusive $b \to sll$ decays in the whole kinematically accessible range. Theoretically preferred region: low dilepton mass below J/Ψ (QCDF); low recoil region also calculable. CUTS are important!

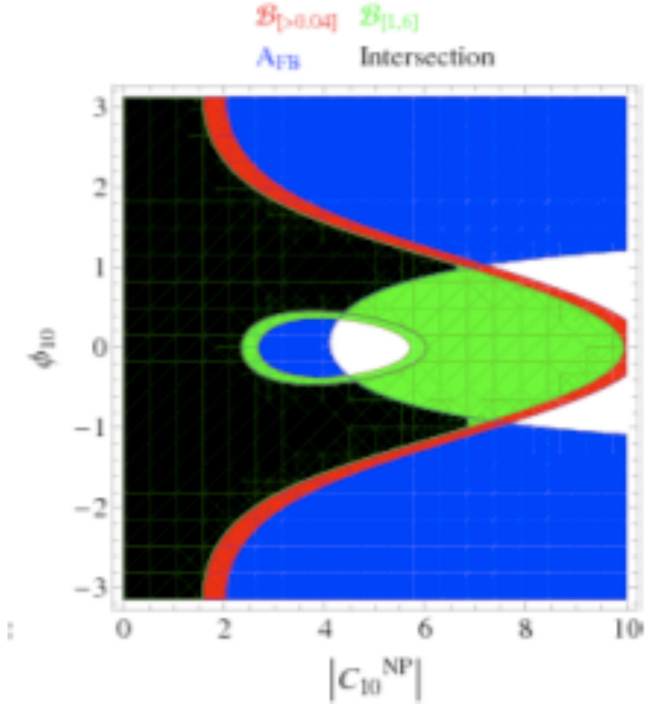
Whole q^2-region tests the SM; different regions are sensitive to different New Physics. blue: SM; green: no zero at lo $q^2 \to signC_7$; red/pink: sign A_{FB} at hi $q^2 \to signC_{10}$





Exclusive $b \to (s,d)ll$ and New Physics Searches

early data can help a lot – already sign determination useful



BaBar : $A_{FB}^{high\, q^2} = 0.76^{+0.52}_{-0.32} \pm 0.07$ arXiv: 0804.4412 hep-ex

Belle measurement of similar quality 0410006, 0603018; $|C_{10}^{NP}| - \Phi_{10}$ plot from 0805.2525 uses sign$A_{FB}^{hi\, q^2}$ is SM-like (blue: A_{FB}constraint; black allowed)

Full Angular Analysis in $B \to V(\to PP)\mu\mu$

full angular analysis hep-ph/9907386

$$d\Gamma^4 \sim J dq^2 d\cos\Theta_l d\cos\Theta_{K^*} d\Phi;\ J = \sum_{i=1}^{9} J_i(q^2) f(\Theta_l, \Theta_{K^*}, \Phi)$$

$\Gamma \sim J_1 - J_2/3$, $A_{FB} \sim J_6$, $A_T^{(2)} \sim J_3$ hep-ph/0502060

$B \to K^{(*)} ll$ CP observables in angular analysis Bobeth,GH,Piranishvili 0805.2525

CP-asymmetries $A_i \propto J_i - \bar{J}_i$: SM: all doubly Cabbibo-suppressed

A_3, A_9 vanish in SM by helicity conservation: sens. to RH currents
$A_3, A_9, (A_6)$ can be extracted from single-diff distribution in $\Phi(\Theta_l)$
A_7, A_8, A_9: T-odd: no strong phase suppression; O(1) with NP
A_5, A_6, A_8, A_9: CP-odd: can be extracted without tagging from $\Gamma + \bar{\Gamma}$
Difference between $B_d \to K^*$ and $B_s \to \Phi$ probes predom. B_s mixing ($\Delta\Gamma_s$ and phase); $A_{5,6,8,9}$ without flavor-taging and time-integrated !

For $\bar{B} = (b\bar{q})$ decays:

$$J(q^2, \theta_l, \theta_{K^*}, \phi) = J_1^s \sin^2\theta_{K^*} + J_1^c \cos^2\theta_{K^*} + (J_2^s \sin^2\theta_{K^*} + J_2^c \cos^2\theta_{K^*}) \cos 2\theta_l$$
$$+ J_3 \sin^2\theta_{K^*} \sin^2\theta_l \cos 2\phi + J_4 \sin 2\theta_{K^*} \sin 2\theta_l \cos\phi + J_5 \sin 2\theta_{K^*} \sin\theta_l \cos\phi$$
$$+ J_6 \sin^2\theta_{K^*} \cos\theta_l + J_7 \sin 2\theta_{K^*} \sin\theta_l \sin\phi$$
$$+ J_8 \sin 2\theta_{K^*} \sin 2\theta_l \sin\phi + J_9 \sin^2\theta_{K^*} \sin^2\theta_l \sin 2\phi, \quad (2.3)$$

$J_i = J_i(q^2)$, $q = p_{l^+} + p_{l^-}$; J_i are functions of transversity amplitudes.

Θ_l: angle between l^- and $\bar{B}$ in dilepton CMS (warning: different conventions in literature)
Θ_{K^*}: angle between K and $\bar{B}$ in K^*-cms
Φ: angle between normals of the $K\pi$ and l^+l^- plane

For CP-conjugate B decays: $J_{1,2,3,4,7} \to \bar{J}_{1,2,3,4,7}$, $J_{5,6,8,9} \to -\bar{J}_{5,6,8,9}$

T-odd versus T-even CP asymmetries

Here, what is meant by T is the naive T transformation, not T-reversal ! Under naive T, the momenta and spins of all particles are flipped, but initial and final state are not interchanged.

φ_W; weak, CP-violating phase; φ_S: strong, CP-conserving phase

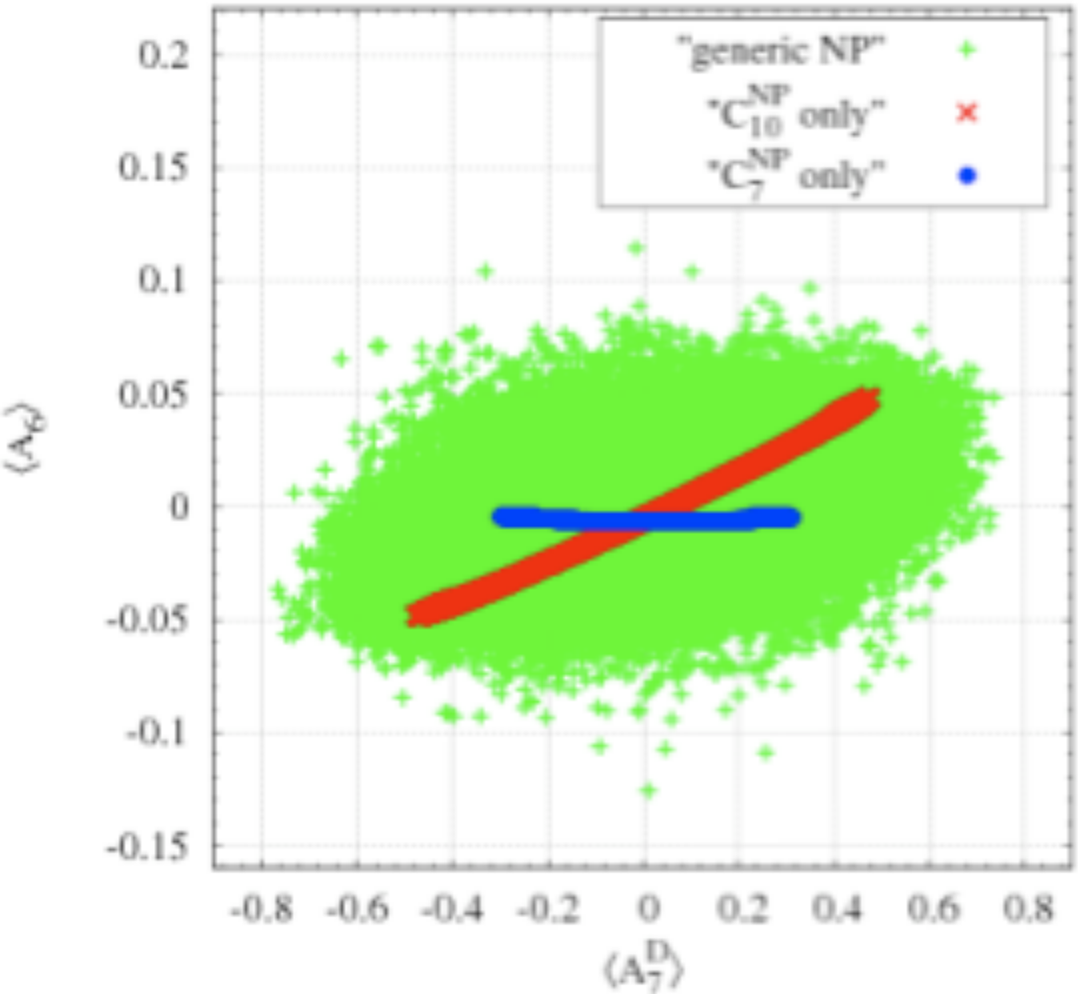
T-even CP asymmetries: $\propto \sin\varphi_W \sin\varphi_S$: small if QCD gives us only small strong phases despite a possible O(1) NP phase.

PS: this is exaclty what happens at low dilepton mass in $B \to K^{(*)} ll$ decays where QCDF predicts small φ_S

T-odd CP asymmetries: $\propto \sin\varphi_W \cos\varphi_S$ maximal for vanishing strong phase

T-odd versus T-even CP asymmetries

Both A_7 and A_6 are sensitive to Z-penguins ($\sim C_{10}$) Fig. from 0805.2525

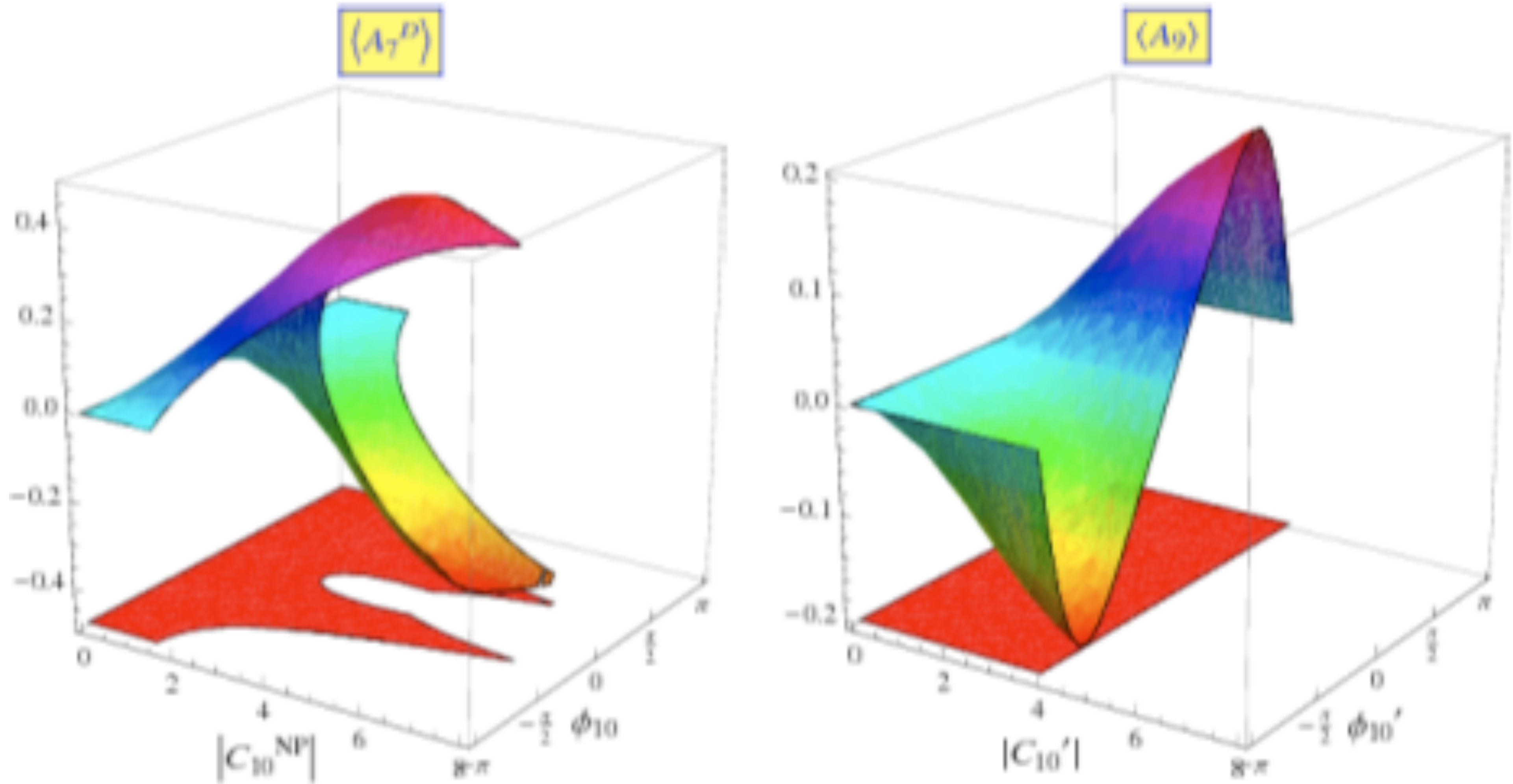


T-odd versus T-even CP asymmetries

A_7, A_8, A_9 are T-odd and can be order one with NP Tab. from 0805.2525

	generic NP	$C_{10}^{\rm NP}$ only	$C_{10}^{\prime\rm NP}$ only	$C_9^{\rm NP}$ only
$\langle A_{\rm CP}\rangle$	$[-0.12, 0.10]$	$[3, 8]\cdot 10^{-3}$	SM-like	$[-0.02, 0.02]$
$\langle A_3\rangle$	$[-0.08, 0.08]$	SM-like	SM-like	SM-like
$\langle A_4^D\rangle$	$[-0.04, 0.04]$	$[-4, -1]\cdot 10^{-3}$	$[-3, -1]\cdot 10^{-3}$	$[-0.01, 0.01]$
$\langle A_5^D\rangle$	$[-0.07, 0.07]$	$[-0.04, 0.04]$	$[-0.02, 0.04]$	$[5, 9]\cdot 10^{-3}$
$\langle A_6\rangle$	$[-0.13, 0.11]$	$[-0.05, 0.05]$	$[-9, -3]\cdot 10^{-3}$	SM-like
$\langle A_7^D\rangle$	$[-0.76, 0.76]$	$[-0.48, 0.48]$	$[-0.38, 0.38]$	SM-like
$\langle A_8^D\rangle$	$[-0.48, 0.48]$	$[2, 7]\cdot 10^{-3}$	$[-0.28, 0.28]$	$[-0.17, 0.17]$
$\langle A_9\rangle$	$[-0.62, 0.60]$	SM-like	$[-0.20, 0.20]$	SM-like
$\mathcal{B}(\bar B_s \to \bar\mu\mu)$	$< 1.4\cdot 10^{-8}$	$< 6.3\cdot 10^{-9}$	$< 1.3\cdot 10^{-8}$	SM

A_7, A_8, A_9 are T-odd and can be order one with NP Fig. from 0805.2525



$B \to Kll$, $l = e, \mu$ angular analysis Bobeth,GH,Piranishvili 0709.4174

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\Theta_l} = \frac{3}{4}(1 - F_H^l)(1 - \cos^2\Theta_l) + F_H^l/2 + A_{FB}^l \cos\Theta_l$$

information in F_H^l and A_{FB}^l beyond $d\Gamma^l/dq^2$ (in general: lepton flavor dependence)

F_H^l CAN be correlated with $R_K = \mathcal{B}(B \to K\mu\mu)/\mathcal{B}(B \to Kee)$

In SM: $R_K - 1$, F_H^l and A_{FB}^l (and $\mathcal{B}(B \to ll)$) are suppressed by lepton mass.

Probe of Higgs-exchanges, lepto-quarks, R-parity violation etc.

Model-independently w. scalar/tensor couplings (for low q^2):
$|A_{FB}^e| < 13\%$, $|A_{FB}^\mu| < 15\%$, $R_K - 1 = O(1)$, $F_H^{e,\mu} < O(0.5)$

summary

- (Quark)flavor physics $=$ investigations on generational structure of quarks and quark partners. Studies can be done directly and indirectly. Indirectly, rare, FCNC $b \to s l \bar{l}$ decays are useful.
- While the Br is observed, and first data on A_{FB} etc are available, $B \to K^{(*)} l \bar{l}$ has great potential to test the SM, search for NP and if dicovered, classify it (CP, right-handed currents, Higgs effects,..).
- Theorists would like to have everything measured: the full angular distributions, and final states $l = e, \mu, \tau$ and ν (Z-penguins).
  $Br(B \to K^{(*)} \mu\mu)/Br(B \to K^{(*)} ee)$ tests lepton non-universality.
  Also $b \to d$.

some theory references

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