Quarks in the SM: $SU(2)_L \times U(1)_Y$
Peculiarities of Flavour in the Standard Model
Rare Decays: Small CKM Elements
Rare Decays: Loop Suppressed FCNC Decays

Rare $B$ Decays and all that ...

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Preliminary Remarks

- **Rare Decays:** Transitions are suppressed by either
  - Small (combinations of) CKM Matrix elements or
  - Loop factors $1/(16\pi^2)$ or
  - both

![Diagram showing quark distribution and decay pathways](image)
Outline of the event:

- Part 1: Rare decays in the Standard Model (TM)
- Part 2: Effective Hamiltonian (TF)
Rare $B$ Decays

Introduction and Standard Model

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Outline Part 1

1. Quarks in the SM: $SU(2)_L \times U(1)_Y$
   - Symmetries and Quantum Numbers
   - Quark Mixing and CKM Matrix

2. Peculiarities of Flavour in the Standard Model
   - Peculiarities of SM CP / Flavour

3. Rare Decays: Small CKM Elements

4. Rare Decays: Loop Suppressd FCNC Decays
Gauge Structure of the Standard Model

I assume a few things to be known:

- The Standard Model is a gauge theory based on
  \[ SU(3)_{QCD} \otimes SU(2)_{Weak} \otimes U(1)_{Hypercharge} \]
- Eight gluons, three weak gauge bosons, one photon
- Matter (quarks and leptons):
  Multiplets of the gauge group \( \rightarrow \) Quantum numbers
- Spontaneous Symmetry Breaking:
  Introduction of scalar fields
- Massless Goldstone Modes:
  Higgs Mechanism:
  \( \phi \rightarrow \) longitudinal modes of gauge bosons:
  \( \phi \sim \partial_\mu W^\mu \)
Matter Fields: Quarks

- **Left Handed Quarks:**
  - $SU(3)_C$ Triplets, $SU(2)_L$ Doublets
  
  \[
  Q_1 = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \quad Q_2 = \begin{pmatrix} c_L \\ s_L \end{pmatrix} \quad Q_3 = \begin{pmatrix} t_L \\ b_L \end{pmatrix}
  \]

  $SU(2)_L$ will be gauged

- **Right Handed Quarks:**
  - $SU(3)_C$ Triplets, $SU(2)_R$ Doublets
  
  \[
  q_1 = \begin{pmatrix} u_R \\ d_R \end{pmatrix} \quad q_2 = \begin{pmatrix} c_R \\ s_R \end{pmatrix} \quad q_3 = \begin{pmatrix} t_R \\ b_R \end{pmatrix}
  \]

  $SU(2)_R$ introduced “artificially”
Quantum Numbers

- **Hypercharge**

  \[ Y = T_{3,R} + \frac{1}{2}(B - L) \]

- **Charge**

  \[ q = T_{3,L} + Y = T_{3,L} + T_{3,R} + \frac{1}{2}(B - L) \]
Higgs Fields: Standard Model

- Single $SU(2)$ Doublet: Two Complex Fields

\[ \Phi = \begin{pmatrix} \phi^+ \\ \phi_0 \end{pmatrix} \]

- Charge Conjugate Field is also an $SU(2)$ Doublet

\[ \tilde{\Phi} = (i\tau_2)\Phi^* = \begin{pmatrix} \phi_0^* \\ -\phi_- = -\phi_+^* \end{pmatrix} \]

- It is useful to gather these into a $2 \times 2$ matrix

\[ H = \begin{pmatrix} \phi_0^* & \phi^+ \\ -\phi_- & \phi_0 \end{pmatrix} \]
• Transformation Properties: \( L \in SU(2)_L \):
  \[
  \Phi \rightarrow L\Phi \quad \tilde{\Phi} \rightarrow L\tilde{\Phi}
  \]

• Transformation Properties: \( R \in SU(2)_R \):
  \[
  \begin{pmatrix}
  \phi_0 \\
  \phi_-
  \end{pmatrix}
  \rightarrow R
  \begin{pmatrix}
  \phi_0 \\
  \phi_-
  \end{pmatrix}
  \begin{pmatrix}
  \phi_+ \\
  -\phi_0^* 
  \end{pmatrix}
  \rightarrow R
  \begin{pmatrix}
  \phi_+ \\
  -\phi_0^*
  \end{pmatrix}
  \]

• In total:
  \[
  H \rightarrow LHR^\dagger \quad (\text{remember } Q \rightarrow LQ \quad q \rightarrow Rq)
  \]

• Hypercharges
  \[
  Y\Phi = -\Phi \quad Y\tilde{\Phi} = \tilde{\Phi} \quad YH = -HT_{3,R}
  \]
Gauge Interactions

- $SU(3)_{color}$ is gauged (not relevant for us now)
- $SU(2)_L$ is gauged Three $W^\mu_\alpha$ Bosons
- Hypercharge is gauged One $B^\mu$ Boson
- Recipe: Replace the ordinary derivative in the kinetic terms by the covariant one

$$\partial^\mu \rightarrow D^\mu = \partial^\mu - igT_L, a W^\mu_\alpha - iYB^\mu$$

+QCD interactions

- Weinberg rotation between $W^\mu_3$ and $B^\mu$ ···
- I assume you have heard the rest of the story ... 
- This is not relevant for the phenomenon of masses and mixing !
**Structure of the Standard Model**

- Start out from an $SU(2)_L \times SU(2)_R$ symmetric case:
- Kinetic Term for Quarks and Higgs ($i$: Generation)

$$\mathcal{L}_{\text{kin}} = \sum_i \left[ \bar{Q}_i \partial \bar{Q}_i + \bar{q}_i \partial \bar{q}_i \right] + \frac{1}{2} \text{Tr} \left[ (\partial_\mu H)^\dagger (\partial^\mu H) \right]$$

- Potential for the Higgs field

$$V = V(H) = V(\text{Tr} \left[ H^\dagger H \right])$$

- Interaction between Quarks and Higgs

$$\mathcal{L}_I = - \sum_{ij} y_{ij} \bar{Q}_i H q_j + \text{h.c.}$$

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Rare $B$ Decays and all that ...
\( y_{ij} \) can be made diagonal: Any Matrix \( y \) can be diagonalized by a Bi-Unitary Transformation:

\[ y = U^\dagger y_{\text{diag}} W \]

Thus

\[ \mathcal{L}_I = - \sum_{ijk} \bar{Q}_i (U^\dagger)_{ik} y_k W_{kj} Hq_j + \text{h.c.} \]

Rotation of \( Q_i \) and \( q_j \):

\[ Q' = UQ \quad q' = Wq \]

This has no effect on the kinetic term:

\( y_{ij} = y_i \delta_{ij} \) is the general case!

\[ \mathcal{L}_I = - \sum_i y_i \bar{Q}_i Hq_i + \text{h.c.} \]
Sponaneous Symmetry Breaking

- The Higgs Potential is (Renormalizability):

\[ V = \kappa \left( \text{Tr} \left[ H^\dagger H \right] \right) + \lambda \left( \text{Tr} \left[ H^\dagger H \right] \right)^2 \]

- For \( \kappa < 0 \) we have SSB:

\[ H \text{ acquires a Vacuum Expectation Value (VEV)} \]

\[ \text{Tr} \left[ \langle H^\dagger \rangle \langle H \rangle \right] = -\frac{\kappa}{2\lambda} > 0 \]

- Choice of the VEV

\[ < \text{Re} \phi_0 >= v \text{ or } < H > = v \begin{pmatrix} 1_{2x2} \end{pmatrix} \]
Three massless fields: $\phi_+, \phi_-, \text{Im}\phi_0$:
Goldstone Bosons

$\phi_0 \rightarrow \nu + \phi'_0$: One massive field

Higgs Mechanism: The massless scalars become the longitudinal modes of the massive vector bosons:
* $\phi_\pm \sim \partial^\mu W^\pm_\mu$
* $\text{Im}\phi_0 \sim \partial^\mu Z_\mu$

$\phi'_0$: Physical Higgs Boson
The Quarks become massive:

\[ \mathcal{L}_I = - \sum_i y_i v \bar{Q}_i q_i + \text{h.c.} + \cdots \]

We have \( \bar{Q}_1 q_1 = \bar{u}_L u_R + \bar{d}_L d_R \) etc.

Thus

\[ \mathcal{L}_{mass} = - m_u (\bar{u}u + \bar{d}d) - m_c (\bar{c}c + \bar{s}s) - m_t (\bar{t}t + \bar{b}b) \]

This is not (yet) what we want ...

We still have too much symmetry!
Custodial $SU(2)$

- Symmetry of the Higgs Sector in the Standard Model:

$$SU(2)_L \otimes SU(2)_R \xrightarrow{SSB} SU(2)_{L+R} = SU(2)_C$$

- Note that we cannot have explicit breaking of $SU(2)_R$ in the Higgs sector:

$$\text{Tr} \left[ H_{\tau i} H^\dagger \right] = 0$$

- $SU(2)_C$: Custodial Symmetry!
  $\rightarrow$ Extra Symmetry in the Higgs sector!

- This is more than needed: Only $U(1)_Y$ is needed

- $U(1)_Y$ will be related to the $\tau_3$ direction of $SU(2)_R$
Consequences of $SU(2)_C$:

- Relation between charged and neutral currents: $\rho$ parameter
- Masses of $W^\pm$ and of $Z^0$ are equal
- Up- and Down-type quark masses are equal in each family
- No mixing occurs among the families

$SU(2)_C$ is broken by:

- Yukawa Couplings
- Gauging only the Hypercharge

$$Y = T_3^{(R)} + \frac{1}{2}(B - L)$$
Breaking $SU(2)_C$: Yukawa Couplings

- Explicit breaking of $SU(2)_C$ by Yukawa Couplings:

$$\mathcal{L}'_I = - \sum_{ij} y'_{ij} \bar{Q}_i H (2T_{3,R}) q_j + \text{h.c.}$$

- Effect of this term:
  - Introduces a splitting between up- and down quark masses
  - Introduces mixing between different families
  - Affects the $\rho$ parameter

- Total Yukawa Coupling term:

$$\mathcal{L}_I + \mathcal{L}'_I = - \sum_{ij} \bar{Q}_i H (y_i \delta_{ij} + 2 T_{3,R} y'_{ij}) q_j + \text{h.c.}$$
Use the projections

\[ P_\pm = \frac{1}{2} \pm T_{3,R} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \text{ or } \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \]

Up quark Yukawa couplings:

\[ \mathcal{L}_{\text{mass}}^u = - \sum_{ij} \bar{Q}_i H (y_i \delta_{ij} + y'_{ij}) P_+ q_j + \text{ h.c.} \]

Down quark Yukawa couplings:

\[ \mathcal{L}_{\text{mass}}^d = - \sum_{ij} \bar{Q}_i H (y_i \delta_{ij} - y'_{ij}) P_- q_j + \text{ h.c.} \]

\[ \rightarrow \text{ mass terms, once } \text{Re} \phi_0 \rightarrow \nu \]
More compact notation

\[ \mathcal{U}_{L/R} = \begin{bmatrix} u_{L/R} \\ c_{L/R} \\ t_{L/R} \end{bmatrix} \quad \mathcal{D}_{L/R} = \begin{bmatrix} d_{L/R} \\ s_{L/R} \\ b_{L/R} \end{bmatrix} \]

Mass Term for Up-type quarks

\[ \mathcal{L}_u^\text{mass} = -v \, \bar{\mathcal{U}}_L Y_u \mathcal{U}_R + \text{h.c.} \]

with \( Y^u = (y + y') \)

Mass Term for down-type quarks

\[ \mathcal{L}_d^\text{mass} = -v \, \bar{\mathcal{D}}_L Y^d \mathcal{D}_R + \text{h.c.} \]

with \( Y^d = (y - y') \)
Mass matrices:

\[ M^u = \nu Y^u \quad M^d = \nu Y^d \]

In general non-diagonal: Diagonalization by a bi-unitary transformation:

\[ M = U^\dagger M_{\text{diag}} W \]

New basis for the quark fields

\[ \mathcal{L}_{\text{mass}}^u = -\bar{U}_L U^{u,\dagger} M_{\text{diag}}^u W^u U_R + \text{h.c.} \]

and

\[ \mathcal{L}_{\text{mass}}^d = -\bar{D}_L U^{d,\dagger} M_{\text{diag}}^d W^d D_R + \text{h.c.} \]
Quark Mixing: The CKM Matrix

- Effect of the basis transformation:
  - Mass matrices become diagonal
  - Interaction with $\text{Re} \phi_0$ (= Physical Higgs Boson) becomes diagonal!
  - Interaction with $\text{Im} \phi_0$ (= $Z_0$) becomes diagonal!

\[
\mathcal{L}_{\text{Re} \phi_0} = -\text{Re} \phi_0 [U_L Y^u U_R + D_L Y^d D_R]
\]
\[
\mathcal{L}_{\text{Im} \phi_0} = -\text{Im} \phi_0 [U_L Y^u U_R - D_L Y^d D_R]
\]

- NO FLAVOUR CHANGING NEUTRAL CURRENTS (at tree level in the Standard Model)

- $\rightarrow$ GIM Mechanism
Effect on the charged current ONLY:
Interaction with $\phi_-$:

$$\sum_{ij} \bar{Q}_i (y_i \delta_{ij} + y'_{ij}) \phi_- \tau^- P_+ q_j + \text{h.c.}$$

$$= D_L Y^u U_R \phi_- + \text{h.c.}$$

$$= \bar{D}_L \bar{U}^d,\dagger \left( U^d U^u,\dagger \right) Y_{\text{diag}} W^u U_R \phi_- + \text{h.c.}$$

In the charged currents flavour mixing occurs!

Parametrized through the Cabbibo-Kobayashi-Maskawa Matrix:

$$V_{\text{CKM}} = U^d U^{u,\dagger}$$
Properties of the CKM Matrix

- \( V_{CKM} \) is unitary (by our construction)
- Number of parameters for \( n \) families
  - Unitary \( n \times n \) matrix: \( n^2 \) real parameters
  - Freedom to rephase the \( 2n \) quark fields: \( 2n - 1 \) relative phases
- \( n^2 - 2n + 1 = (n - 1)^2 \) real parameters
  * \((n - 1)(n - 2)/2\) are phases
  * \(n(n - 1)/2\) are angles
- Phases are sources of \( CP \) violation
- \( n = 2 \): One angle, no phase \( \rightarrow \) no \( CP \) violation
- \( n = 3 \): Three angles, one phase
- \( n = 4 \): Six angles, three phases
CKM Basics

- Three Euler angles $\theta_{ij}$
  
  $U_{12} = \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad U_{13} = \begin{bmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{bmatrix}, \quad U_{23} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix}$

- Single phase $\delta$:
  
  $U_\delta = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\delta_{13}} \end{bmatrix}$.

- PDG CKM Parametrization:
  
  $V_{\text{CKM}} = U_{23} U_\delta^\dagger U_{13} U_\delta U_{12}$

- Large Phases in $V_{ub} = |V_{ub}| e^{-i\gamma} = s_{13} e^{-i\delta_{13}}$ and $V_{td} = |V_{td}| e^{i\beta}$
CKM Unitarity Relations

\[ V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \]

- Off diagonal zeros of \( V_{CKM}^\dagger V_{CKM} = V_{CKM} V_{CKM}^\dagger = 1 \):
  \[ V_{ub} V_{ud}^* + V_{cb} V_{cd}^* + V_{tb} V_{td}^* = 0 \]
  \[ V_{ub} V_{us}^* + V_{cb} V_{cs}^* + V_{tb} V_{ts}^* = 0 \]
  \[ V_{us} V_{ud}^* + V_{cs} V_{cd}^* + V_{ts} V_{td}^* = 0 \]

- \( V_{CKM}^\dagger V_{CKM} = 1 \):
  \[ V_{ud} V_{td}^* + V_{us} V_{ts}^* + V_{ub} V_{tb}^* = 0 \]
  \[ V_{ud} V_{cd}^* + V_{us} V_{cs}^* + V_{ub} V_{cb}^* = 0 \]
  \[ V_{cd} V_{td}^* + V_{cs} V_{ts}^* + V_{cb} V_{tb}^* = 0 \]
Wolfenstein Parametrization of CKM

- Diagonal CKM matrix elements are almost unity
- CKM matrix elements decrease as we move off the diagonal
- Wolfenstein Parametrization:

\[
V_{CKM} = \begin{pmatrix}
1 - \frac{\lambda^2}{2} & \frac{\lambda}{2} & \frac{\lambda^3 A (\rho - i\eta)}{2} \\
-\lambda & 1 - \frac{\lambda^2}{2} & \frac{\lambda^2 A}{2} \\
\lambda^3 A (1 - \rho - i\eta) & -\lambda^2 A & 1
\end{pmatrix}
\]

- Expansion in \( \lambda \approx 0.22 \) up to \( \lambda^3 \)
- \( A, \rho, \eta \) of order unity
Unitarity Triangle(s)

- The unitarity relations:
  Sum of three complex numbers = 0

- Triangles in the complex plane

- Only two out of the six unitarity relations involve terms of the same order in $\lambda$:

\[
V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0
\]
\[
V_{ud}^* V_{td} + V_{us}^* V_{ts} + V_{ub}^* V_{tb} = 0
\]

- Both correspond to

\[
A\lambda^3(\rho + i\eta - 1 + 1 - \rho - i\eta) = 0
\]

- This is THE unitarity triangle ...
• Definition of the CKM angles $\alpha$, $\beta$ and $\gamma$
• To leading order Wolfenstein:

$$V_{ub} = |V_{ub}| e^{-i\gamma} \quad V_{tb} = |V_{tb}| e^{-i\beta}$$

all other CKM matrix elements are real.

• $\delta \gamma$ is order $\lambda^5$
Quarks in the SM: $SU(2)_L \times U(1)_Y$

Peculiarities of Flavour in the Standard Model

Rare Decays: Small CKM Elements

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- Aerea of the Triangle(s): Measure of CP Violation
- Invariant measure of CP violation:

$$\text{Im}\Delta = \text{Im} V_{ud} V_{td}^* V_{tb} V_{ub}^* = c_{12}s_{12}c_{13}^2 s_{13} s_{23} c_{23} \sin \delta_{13}$$

- Maximal possible value $\delta_{\text{max}} = \frac{1}{6\sqrt{3}} \sim 0.1$

- CP Violation is a small effect:
  Measured value $\delta_{\text{exp}} \sim 0.0001$

- CP Violation vanishes in case of degeneracies: (Jarlskog)

$$J = \text{Det}([M_u, M_d])$$
$$= 2i\text{Im}\Delta (m_u - m_c)(m_u - m_t)(m_c - m_t)$$
$$\times (m_d - m_s)(m_d - m_b)(m_s - m_b)$$

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Symmetries and Quantum Numbers
Quark Mixing and CKM Matrix
Peculiarities of SM Flavour Mixing

- Hierarchical structure of the CKM matrix
- Quark Mass spectrum ist widely spread
  \[ m_u \sim 10 \text{ MeV to } m_t \sim 170 \text{ GeV} \]
- PMNS Matrix for lepton flavour mixing is not hierarchical
- Only the charged lepton masses are hierarchical
  \[ m_e \sim 0.5 \text{ MeV to } m_\tau \sim 1772 \text{ MeV} \]
- Up-type leptons \sim Neutrinos have very small masses
- (Enormous) Suppression of Flavour Changing Neutral Currents:
  \[ b \to s, \ c \to u, \ \tau \to \mu, \ \mu \to e, \ \nu_2 \to \nu_1 \]
Peculiarities of SM CP Violation

- Strong CP remains mysterious
- Flavour diagonal CP Violation is well hidden:
  e.g electric dipole moment of the neutron:
  At least three loops (Shabalin)

\[ d_e \sim e \frac{\alpha_s}{\pi} \frac{G_F^2}{(16\pi^2)^2} \frac{m_t^2}{M_W^2} \text{Im} \Delta \mu^3 \]
\[ \sim 10^{-32} \text{e cm} \quad \text{with } \mu \sim 0.3 \text{GeV} \]

\[ d_{\text{exp}} \leq 3.0 \times 10^{-26} \text{e cm} \]
Pattern of mixing and mixing induced CP violation determined by GIM: Tiny effects in the up quark sector

- $\Delta C = 2$ is very small
- Mixing with third generation is small: charm physics basically “two family”
- $\rightarrow$ CP violation in charm is small in the SM

Fully consistent with particle physics observations

... but inconsistent with matter-antimatter asymmetry
Our Understanding of Flavour is unsatisfactory:

- 22 (out of 27) free Parameters of the SM originate from the Yukawa Sector (including Lepton Mixing)
- Why is the CKM Matrix hierarchical?
- Why is CKM so different from the PMNS?
- Why are the quark masses (except the top mass) so small compared with the electroweak VEV?
- Why do we have three families?

Why is CP Violation in Flavour-diagonal Processes not observed? (e.g. z.B. electric dipolmoments of electron and neutron)

Where is the CP violation needed to explain the matter-antimatter asymmetry of the Universe?
Rare Decays: Small CKM Elements

- Focus on Non-leptonic Decays
- Small CKM Elements: Non-Charmed Decays
Roadmap of Bottom Decays (R. Fleischer)

\[ B \rightarrow \pi \pi \text{ (isospin)}, \quad B \rightarrow \rho \pi, \quad B \rightarrow \rho \rho \]

\[ R_b \ (b \rightarrow u, c \ell \bar{\nu}_\ell) \]

\[ R_t \ (B_q^0 - \bar{B}_q^0 \text{ mixing}) \]

\[ B \rightarrow \pi K \text{ (penguins)} \]

\[ B_u^\pm \rightarrow K^\pm D \]
\[ B_d^+ \rightarrow K^{*0} D \]
\[ B_c^\pm \rightarrow D_s^\pm D \] only trees

\[ B_d \rightarrow \psi K_S \ (B_s \rightarrow \psi \phi : \phi_s \approx 0) \]

\[ B_d \rightarrow \phi K_S \text{ (pure penguin)} \]

\[ B_d \rightarrow D^{(*)\pm} K^\mp : \gamma + 2\beta \] only trees
\[ B_s \rightarrow D_s^{\pm} K^\mp : \gamma + \phi_s \] only trees
Interplay between “Tree” and ”Penguin”

- Tree $\sim |V_{ub}|^2 |V_{ud}|^2 \sim \lambda^3$
- Penguin $\sim |V_{tb}|^2 |V_{td}|^2 \sim \lambda^3$
- Expect sizable direct CP violation
Isospin Relations in $B \rightarrow \pi\pi$

- Isospin relation: $\frac{1}{\sqrt{2}} A^{+-} + A^{00} = A^{+0}$
- In combination with

  $\frac{1}{\sqrt{2}} A^{+-} = Te^{i\gamma} + Pe^{-i\beta}$ \quad $T$ : tree amplitude

  $A^{00} = Ce^{i\gamma} - Pe^{-i\beta}$ \quad $P$ : penguin amplitude

  $A^{+0} = (C + T)e^{i\gamma}$ \quad $C$ : Color suppressed amplitude

  gives a good handle on $\alpha$ (Gronau, London, Wyler)

- $P$ is (expected to be) sizable

- Separate measurement of $B_d \rightarrow \pi^0\pi^0$ and $\bar{B}_d \rightarrow \pi^0\pi^0$
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$B \rightarrow \pi K$

$\text{B} \rightarrow \pi \text{K}$

$V^*_{us}$

$V^*_{cs}$

$V_{ub}$

$V_{cb}$

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Rare $B$ Decays and all that ...
Electroweak penguins can be significant!
Rare Decays: FCNC’s

Example: \( b \rightarrow s \gamma \)

\[
\mathcal{A}(b \rightarrow s \gamma) = V_{ub} V_{us}^* f(m_u) + V_{cb} V_{cs}^* f(m_c) + V_{tb} V_{ts}^* f(m_t)
\]

In case of degenerate masses up-type masses:

\[
\mathcal{A}(b \rightarrow s \gamma) = f(m) \left[ V_{ub} V_{us}^* + V_{cb} V_{cs}^* + V_{tb} V_{ts}^* \right] = 0
\]
Loop induced Processes: Sensitivity to “new physics”

Exclusive as well as inclusive decays of the form

\[ b \rightarrow s \gamma \text{ and } b \rightarrow s \ell \ell \]

\[ b \rightarrow s \ell \ell \text{ contains a lot of information through various observables} \]

- Lepton invariant mass
- Lepton energy spectra
- Forward Backward Asymmetries
- ...
Quarks in the SM: \( SU(2)_L \times U(1)_Y \)

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\[ B \rightarrow X_s \gamma \]

- Complete NNLO Calculation has been performed
- Theory Predictions: (from Flächer @ EPS07)
  - Misiak et al. \( \text{hep-ph/0609232} \)
    - \[ BF(B \rightarrow X_s \gamma) = (3.15 \pm 0.23) \times 10^{-4} \text{ for } E_{\gamma} > 1.6 \text{ GeV} \]
    - dedicated error analysis resulting in 7% error
  - Becher et al. \( \text{hep-ph/0610067} \)
    - \[ BF(B \rightarrow X_s \gamma) = (2.98 \pm 0.26) \times 10^{-4} \text{ for } E_{\gamma} > 1.6 \text{ GeV} \]
    - larger perturbative uncertainty resulting in 9% error
  - Andersen et al. \( \text{hep-ph/0609250} \)
    - \[ BF(B \rightarrow X_s \gamma) = (3.47 \pm 0.48) \times 10^{-4} \text{ for } E_{\gamma} > 1.6 \text{ GeV} \]
    - 11% uncertainty from variation of renormalisation scale
- HFAG Average: \( (3.55 \pm 0.26) \times 10^{-4} \)
In addition to the diagrams for $B \rightarrow X_s \ell^+ \ell^-$

- $s \rightarrow W \rightarrow u, c, l$
- $b \rightarrow W \rightarrow s, b$
- $\nu_{\ell}$
- $Z/\gamma \rightarrow c/u \rightarrow \ell$

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Rare $B$ Decays and all that ...
Doubly differential rate: \( \frac{d^2\Gamma}{(dM_{\ell\ell}^2 \, d(\cos \theta))} \)

Total rate \( R \) (or rate within cuts)

Forward backward Asymmetry \( A_{FB} \)

\[
A_{FB}(M_{\ell\ell}^2) = \left[ \int_{-1}^{0} d(\cos \theta) - \int_{0}^{-1} d(\cos \theta) \right] \frac{d^2\Gamma}{dM_{\ell\ell}^2 \, d(\cos \theta)}
\]

Observables depend on the Wilson Coefficients

→ Try to extract \( C_1 \ldots C_{10} \)
The lepton invariant mass spectrum of $B \rightarrow X_{s\ell\ell}$
The forward-backward of $B \rightarrow X_s \ell \ell$

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Rare $B$ Decays and all that ...
Extraction of Wilson Coefficients (Example)