Measuring the CKM angle γ

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Introduction

- The CKM Triangle
- Why γ ?
- Possible ways to measure γ
- Rare decays

Corollary CKM matrix describes quark-flavour changing

- decays described by quark-flavour transformations
- 3 up-type quarks, 3 down-type quarks
- Unitarity requirement imposes constraints on elements



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$$V_{
m CKM} = \left(egin{array}{ccc} V_{
m ud} & V_{
m us} & V_{
m ub} \ V_{
m cd} & V_{
m cs} & V_{
m cb} \ V_{
m td} & V_{
m ts} & V_{
m tb} \end{array}
ight)$$



Corollary CKM matrix describes quark-flavour changing

Example

- decays described by quark-flavour transformations
- 3 up-type quarks, 3 down-type quarks
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$$V_{\mathrm{CKM}} = \left(egin{array}{cc} V_{\mathrm{ud}} & V_{\mathrm{us}} & V_{\mathrm{ub}} \ V_{\mathrm{cd}} & V_{\mathrm{cs}} & V_{\mathrm{cb}} \ V_{\mathrm{td}} & V_{\mathrm{ts}} & V_{\mathrm{tb}} \end{array}
ight)$$

Unitarity yields ($V_{\text{CKM}} \cdot V_{\text{CKM}}^* = \mathbb{1}$)

Unitarity of the CKM matrix

Conclusion The CKM Triangle:

$$V_{\rm ud} V_{\rm ub}^* + V_{\rm cd} V_{\rm cb}^* + V_{\rm td} V_{\rm tb}^* = 0$$





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More triangle exist, *e.g.* for B_s physics:

$$V_{\rm ud} V_{\rm us}^* + V_{\rm cd} V_{\rm cs}^* + V_{\rm td} V_{\rm ts}^* = 0$$



The 3 Angles

$$\begin{aligned} \bullet & \alpha = \operatorname{Arg}\left[-\frac{V_{d}V_{b}^{*}}{V_{dd}V_{b}^{*}}\right] \\ \bullet & \beta = \operatorname{Arg}\left[-\frac{V_{cd}V_{cb}^{*}}{V_{cd}V_{b}^{*}}\right] \\ \bullet & \beta = \operatorname{Arg}\left[-\frac{V_{od}V_{cb}^{*}}{V_{cd}V_{cb}^{*}}\right] \end{aligned}$$

$$egin{aligned} eta_{\mathcal{S}} &= \mathrm{Arg}\left[-rac{V_{\mathrm{ts}}V_{\mathrm{tb}}^{*}}{V_{\mathrm{cs}}V_{\mathrm{cb}}^{*}}
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$$\beta_{s} = \operatorname{Arg}\left[-\frac{V_{\rm ts}V_{\rm ts}}{V_{\rm cs}V_{\rm ct}^{*}}\right]$$
$$\beta_{K} = \operatorname{Arg}\left[-\frac{V_{\rm cs}V_{\rm ct}^{*}}{V_{\rm us}V_{\rm ut}^{*}}\right]$$



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$\mathbf{B}_{u}^{\pm} \to \pi^{\pm}(\pi^{\pm} + \pi^{\mp})$

- Small branching fraction
- Measure γ through mixing of resonances w/o CP phase
- Direct CP-violating asymmetries

- $\mathcal{B} = (16.2 \pm 1.2 \pm 0.9) \cdot 10^{-6}$
- $(\pi^+ + \pi^-)$ resonance: $\rho^0, f_0, \chi_{c0} \dots$
- access to cp-violation in interference



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- $(\pi^+ + \pi^-)$ resonance: $\rho^0, f_0, \chi_{c0} \dots$
- access to cp-violation in interference
- \longrightarrow interference between (ρ^0 , f_0) and (χ_{c0}) : sensitivity to CP.



Feynmann Diagram



- $M(\chi_{c0}) = 3414.76 \pm 0.35 \text{ GeV}$
- χ_{c0} is a cc̄-state

A Little Complication

Apply common known Dalitz-plot analysis technique.

- need fit function for each type of resonance:
- *a_i*: unkown (!!) parameter, amplitude fraction
- ² θ_i : unkown parameter, phase(...)
- [®] *F_i*: amplitudes from resonances included in fit
- 1) B^+ yields $\theta_i = \delta_i + \phi_i$
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$$\theta_i = \delta_i + \phi_i$$

- \rightarrow need another decay to disentangle sum of phases
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→ need another decay to disentangle sum of phases • B^+ yields $\theta_i = \delta_i + \phi_i$ • B^- yields $\bar{\theta}_i = \delta_i - \phi_i$



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$b\bar{b}$ production at BaBar





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reconstruct B[±]

- which π belongs to which?
- through which resonance did the $\pi's$ go?
- Example 1: BaBar analysis:
- 3 identical particles in final state
- use Dalitz-plot analysis
- $\ \, \odot \ \, \Delta E = E_{\rm B}^* \sqrt{s}/2,$
- ⁴ $m_{\rm ES} = \sqrt{(s/2 + p_i \cdot p_B)^2/E_i^2 p_{\rm B}^2}$
- 6 2 independent variables



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(Babar: Amplitude Analysis of the Decay $B^{\pm} \rightarrow \pi^{\pm}\pi^{\pm}\pi^{\mp}$, hep-ex/0507025)

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$e^+ + e^- ightarrow \Upsilon(4\mathrm{S}) ightarrow (bar{b})$: at $\sqrt(s) = 10.5~\mathrm{GeV}$

- 1 Use kinematics of $(b\bar{b})$ production at resonance
- (Compare with LHCb: $E(b\bar{b})$ and $p(b\bar{b})$ not a priori known!)
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- ⁴ construct "invariant" masses of 2 combinations of π 's:



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 $s_{13} = m_{\pi_1^+\pi_2}^2$
 $s_{23} = m_{\pi_2^+\pi_2}^2$

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Dalitz Plot Analysis



Example:
$$D^0 o K_s^0 \pi^+ \pi^-$$



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- Plot invariant masses
- Why observe structure in Dalitz plot?
- What did we expect?
 - *R_k*: mass distribution
 - T_k : angular-dependent amplitude
 - B R_k usual a Breit-Wigner

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$$T_k^{(0)} = 1, \ T_k^{(1)} = -2\vec{p}\cdot\vec{q}$$

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- ² Resonances from $(\pi^+ + K_s^0 = K^{*+}(892))$



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J. Blouw

Dalitz Plot Analysis

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Dalitz: $B^{\pm} \rightarrow \pi^{\mp} \pi^{\pm} \pi^{\pm}$

Usual Dalitz plot



Note: resonances occur at bounderies...

Rescale co-ordinates:



Empty space from charm vetoes



Dalitz from BB background

$B\bar{B}$ background: usual co-ordinates



Holes are from charm vetoes

re-scaled co-ordinates:





Results Mass projections

Projection from B^+ onto $m(\pi^+\pi^-)$

Projection from B⁻ onto $m(\pi^-\pi^-)$



J. Blouw

Results Mass projections

Project from B⁻ onto $m(\pi^-\pi^-)$

Projection from B^+ onto $m(\pi^+\pi^+)$



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9 $B^+ \to \pi^{\pm}\pi^{\pm}\pi^{\mp}$: (16.2 ± 1.2 ± 0.9) × 10⁻⁶ 2 $\rho^0(770)\pi^{\pm}(\rho^0 \to \pi^+\pi^-)$: (8.8 ± 1.0 ± 0.6) × 10⁻⁶ 3 $\rho^0(1450)\pi^{\pm}(\rho^0 \to \pi^+\pi^-)$: (1.0 ± 0.6 ± 0.4) × 10⁻⁶ 4 $f_0(980)\pi^{\pm}(f_0 \to \pi^+\pi^-)$: (1.2 ± 0.6 ± 0.5) × 10⁻⁶ 5 $\chi_{c0}\pi^{\pm}(\chi_{c0} \to \pi^+\pi^-)$: < 0.3 at 90% CL



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Conclusion No χ_{c0} signal(!)


Results & Conclusions

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Conclusion

No χ_{c0} signal(!) \Rightarrow CP-violation measurement not possible!!!

