Measuring the CKM angle $\gamma$

J. Blouw

Physikalisches Institut, Universitaet Heidelberg

Tagungsstaette, Neckarzimmern, March 28-30, 2007
Introduction

- The CKM Triangle
- Why $\gamma$?
- Possible ways to measure $\gamma$
- Rare decays
Corollary

CKM matrix describes quark-flavour changing

Example

1. decays described by quark-flavour transformations
2. 3 up-type quarks, 3 down-type quarks
3. Unitarity requirement imposes constraints on elements
Particle Decays

Corollary

*CKM matrix describes quark-flavour changing*

Example

1. Decays described by quark-flavour transformations
2. 3 up-type quarks, 3 down-type quarks
3. Unitarity requirement imposes constraints on elements
Corollary

CKM matrix describes quark-flavour changing

Example

1. decays described by quark-flavour transformations
2. 3 up-type quarks, 3 down-type quarks
3. Unitarity requirement imposes constraints on elements
Introduction

Particle Decays

Corollary

**CKM matrix describes quark-flavour changing**

Example

1. decays described by quark-flavour transformations
2. 3 up-type quarks, 3 down-type quarks
3. Unitarity requirement imposes constraints on elements
Particle Decays

Corollary

*CKM matrix describes quark-flavour changing*

Example

1. Decays described by quark-flavour transformations
2. 3 up-type quarks, 3 down-type quarks
3. Unitarity requirement imposes constraints on elements

\[
V_{\text{CKM}} = \begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix}
\]
Corollary

*CKM matrix describes quark-flavour changing*

Example

1. decays described by quark-flavour transformations
2. 3 up-type quarks, 3 down-type quarks
3. Unitarity requirement imposes constraints on elements

\[ V_{\text{CKM}} = \begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix} \]

Unitarity yields \((V_{\text{CKM}} \cdot V_{\text{CKM}}^\ast = \mathbb{1})\)
Unitarity of the CKM matrix

Conclusion

The CKM Triangle:

\[ V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0 \]

- \( B_d^0 \rightarrow \pi^+ \pi^- \)
- \( B_d^0 \rightarrow \rho \pi \)
- \( B_s^0 \rightarrow D_s \pi \)
- \( B_d^0 \rightarrow D K^* \)
- \( B_s^0 \rightarrow D_s K \)
- \( B_d^0 \rightarrow D^* \pi, 3\pi \)
- \( B_d^0 \rightarrow J/\psi K_s^0 \)
Unitarity of the CKM matrix

Conclusion

*The CKM Triangle:*

\[ V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0 \]

More triangle exist, *e.g.* for $B_s$ physics:

\[ V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^* = 0 \]
Angles of the Triangle(s)

The 3 Angles

1. \( \alpha = \text{Arg} \left[ -\frac{V_{td} V_{tb}^*}{V_{ud} V_{ub}^*} \right] \)

2. \( \beta = \text{Arg} \left[ -\frac{V_{cd} V_{cb}^*}{V_{td} V_{tb}^*} \right] \)

3. \( \beta = \text{Arg} \left[ -\frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*} \right] \)

More Angles

4. \( \beta_s = \text{Arg} \left[ -\frac{V_{ts} V_{tb}^*}{V_{cs} V_{cb}^*} \right] \)

5. \( \beta_K = \text{Arg} \left[ -\frac{V_{cs} V_{cd}^*}{V_{us} V_{ud}^*} \right] \)
Angles of the Triangle(s)

The 3 Angles

1. \( \alpha = \text{Arg} \left[ -\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*} \right] \)

2. \( \beta = \text{Arg} \left[ -\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*} \right] \)

3. \( \beta = \text{Arg} \left[ -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} \right] \)

More Angles

4. \( \beta_s = \text{Arg} \left[ -\frac{V_{ts}V_{tb}^*}{V_{cs}V_{cb}^*} \right] \)

5. \( \beta_K = \text{Arg} \left[ -\frac{V_{cs}V_{cd}^*}{V_{us}V_{ud}^*} \right] \)
Angles of the Triangle(s)

The 3 Angles

1. \( \alpha = \text{Arg} \left[ -\frac{V_{td} V_{tb}^*}{V_{ud} V_{ub}^*} \right] \)

2. \( \beta = \text{Arg} \left[ -\frac{V_{cd} V_{cb}^*}{V_{td} V_{tb}^*} \right] \)

3. \( \beta = \text{Arg} \left[ -\frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*} \right] \)

More Angles

4. \( \beta_s = \text{Arg} \left[ -\frac{V_{ts} V_{tb}^*}{V_{cs} V_{cb}^*} \right] \)

5. \( \beta_K = \text{Arg} \left[ -\frac{V_{cs} V_{cd}^*}{V_{us} V_{ud}^*} \right] \)
Angles of the Triangle(s)

The 3 Angles

1. \( \alpha = \text{Arg} \left[ -\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*} \right] \)

2. \( \beta = \text{Arg} \left[ -\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*} \right] \)

3. \( \beta = \text{Arg} \left[ -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} \right] \)

More Angles

1. \( \beta_s = \text{Arg} \left[ -\frac{V_{ts}V_{tb}^*}{V_{cs}V_{cb}^*} \right] \)

2. \( \beta_K = \text{Arg} \left[ -\frac{V_{cs}V_{cd}^*}{V_{us}V_{ud}^*} \right] \)
Angles of the Triangle(s)

The 3 Angles

1. \( \alpha = \text{Arg} \left[ -\frac{V_{td}}{V_{ud}} \frac{V_{tb}^{*}}{V_{ub}^{*}} \right] \)
2. \( \beta = \text{Arg} \left[ -\frac{V_{cd}}{V_{td}} \frac{V_{cb}^{*}}{V_{tb}^{*}} \right] \)
3. \( \beta = \text{Arg} \left[ -\frac{V_{ud}}{V_{cd}} \frac{V_{ub}^{*}}{V_{cb}^{*}} \right] \)

More Angles

1. \( \beta_s = \text{Arg} \left[ -\frac{V_{ts}}{V_{cs}} \frac{V_{tb}^{*}}{V_{cb}^{*}} \right] \)
2. \( \beta_K = \text{Arg} \left[ -\frac{V_{cs}}{V_{us}} \frac{V_{cd}^{*}}{V_{ud}^{*}} \right] \)
Direct Measurement of $\gamma$

$B_u^{\pm} \rightarrow \pi^{\pm}(\pi^{\pm} + \pi^{\mp})$

1. Small branching fraction
2. Measure $\gamma$ through mixing of resonances w/o CP phase
3. Direct CP-violating asymmetries

$\mathcal{B} = (16.2 \pm 1.2 \pm 0.9) \cdot 10^{-6}$

$\left(\pi^+ + \pi^-\right)$ resonance: $\rho^0, f_0, \chi_{c0} \ldots$

access to cp-violation in interference
Direct Measurement of $\gamma$

1. Small branching fraction
2. Measure $\gamma$ through mixing of resonances w/o CP phase
3. Direct CP-violating asymmetries

$B_{u}^{\pm} \rightarrow \pi^{\pm}(\pi^{\pm} + \pi^{\mp})$

$B = (16.2 \pm 1.2 \pm 0.9) \cdot 10^{-6}$

$(\pi^{+} + \pi^{-})$ resonance:
- $\rho^{0}$, $f_{0}$, $\chi_{c0}$ ...
- Access to CP-violation in interference
Direct Measurement of $\gamma$

$B_u^\pm \rightarrow \pi^\pm (\pi^\pm + \pi^{\mp})$

1. Small branching fraction
2. Measure $\gamma$ through mixing of resonances w/o CP phase
3. Direct CP-violating asymmetries

$\mathcal{B} = (16.2 \pm 1.2 \pm 0.9) \cdot 10^{-6}$

$(\pi^+ + \pi^-)$– resonance: $\rho^0, f_0, \chi_{c0} \ldots$

access to cp-violation in interference
Introduction

Direct Measurement of $\gamma$

$B_u^\pm \rightarrow \pi^\pm (\pi^\pm + \pi^\mp)$

1. Small branching fraction
2. Measure $\gamma$ through mixing of resonances w/o CP phase
3. Direct CP-violating asymmetries

- $B = (16.2 \pm 1.2 \pm 0.9) \cdot 10^{-6}$
- $(\pi^+ + \pi^-)$ resonance: $\rho^0, f_0, \chi_{c0} \ldots$
- Access to cp-violation in interference

J. Blouw

Measuring the CKM angle $\gamma$
Direct Measurement of $\gamma$

$B_u^{\pm} \rightarrow \pi^{\pm}(\pi^{\pm} + \pi^{\mp})$

1. Small branching fraction
2. Measure $\gamma$ through mixing of resonances w/o CP phase
3. Direct CP-violating asymmetries

$B = (16.2 \pm 1.2 \pm 0.9) \cdot 10^{-6}$

$\pi^+ + \pi^-$ resonance: $\rho^0, f_0, \chi_c 0 \ldots$

access to cp-violation in interference

$\rightarrow$ interference between $(\rho^0, f_0)$ and $(\chi_c 0)$: sensitivity to CP.
\[ M(\chi_{c0}) = 3414.76 \pm 0.35 \text{ GeV} \]

\( \chi_{c0} \) is a c\bar{c}-state
Apply common known Dalitz-plot analysis technique.

- need fit function for each type of resonance:
  1. \(a_i\): unknown (!!) parameter, amplitude fraction
  2. \(\theta_i\): unknown parameter, phase(…)
  3. \(F_i\): amplitudes from resonances included in fit

1. \(B^+\) yields \(\theta_i = \delta_i + \phi_i\)
2. \(B^-\) yields \(\bar{\theta}_i = \delta_i - \phi_i\)
Apply common known Dalitz-plot analysis technique.

need fit function for each type of resonance:

1. $a_i$: unknown (!!) parameter, amplitude fraction
2. $\theta_i$: unknown parameter, phase(...)
3. $F_i$: amplitudes from resonances included in fit

$B^+$ yields $\theta_i = \delta_i + \phi_i$

$B^-$ yields $\bar{\theta}_i = \delta_i - \phi_i$
Apply common known Dalitz-plot analysis technique.
need fit function for each type of resonance:

$$\mathcal{F}_{B^+ \to \pi^+ \pi^- \pi^+}(s_1, s_2) = |\sum_i a_i e^{i\theta_i} F_i(s_1, s_2)|$$

$$s_{12} = (p_1^\mu + p_2^\mu) \cdot (p_1^\mu + p_2^\mu)$$

$$s_{23} = (p_2^\mu + p_3^\mu) \cdot (p_2^\mu + p_3^\mu)$$

1. $a_i$: unknown (!!) parameter, amplitude fraction
2. $\theta_i$: unknown parameter, phase(...)
3. $F_i$: amplitudes from resonances included in fit

$B^+ \text{ yields } \theta_i = \delta_i + \phi_i$

$B^- \text{ yields } \bar{\theta}_i = \delta_i - \phi_i$
Apply common known Dalitz-plot analysis technique.

- need fit function for each type of resonance:

\[
F_{B^+ \rightarrow \pi^+ \pi^- \pi^+}(s_1, s_2) = \left| \sum_i a_i e^{i\theta_i} F_i(s_1, s_2) \right|
\]

\[
s_{12} = (p_{1\mu} + p_{2\mu}) \cdot (p_{1\mu} + p_{2\mu})
\]

\[
s_{23} = (p_{2\mu} + p_{3\mu}) \cdot (p_{2\mu} + p_{3\mu})
\]

1. \(a_i\): unknown (!!) parameter, amplitude fraction
2. \(\theta_i\): unknown parameter, phase(...)
3. \(F_i\): amplitudes from resonances included in fit

1. \(B^+\) yields \(\theta_i = \delta_i + \phi_i\)
2. \(B^-\) yields \(\bar{\theta}_i = \delta_i - \phi_i\)
Apply common known Dalitz-plot analysis technique.

need fit function for each type of resonance:

\[
F\rightarrow\pi^+\pi^-\pi^+(s_1, s_2) = \left| \sum_i a_i e^{i\theta_i} F_i(s_1, s_2) \right|
\]

- \(a_i\): unknown (!!!) parameter, amplitude fraction
- \(\theta_i\): unknown parameter, phase(...)
- \(F_i\): amplitudes from resonances included in fit

\[
s_{12} = (p_1^\mu + p_2^\mu) \cdot (p_1^\mu + p_2^\mu)
\]

\[
s_{23} = (p_2^\mu + p_3^\mu) \cdot (p_2^\mu + p_3^\mu)
\]

- \(B^+\) yields \(\theta_i = \delta_i + \phi_i\)
- \(B^-\) yields \(\bar{\theta}_i = \delta_i - \phi_i\)
Apply common known Dalitz-plot analysis technique.

need fit function for each type of resonance:

\[ F_{B^+ \rightarrow \pi^+ \pi^- \pi^+}(s_1, s_2) = \left| \sum_i a_i e^{i\theta_i} F_i(s_1, s_2) \right| \]

\[ s_{12} = (p_{\mu 1}^\mu + p_{\mu 2}^\mu) \cdot (p_{\mu 1}^\mu + p_{\mu 2}^\mu) \]

\[ s_{23} = (p_{\mu 2}^\mu + p_{\mu 3}^\mu) \cdot (p_{\mu 2}^\mu + p_{\mu 3}^\mu) \]

1. \( a_i \): unknown (!!) parameter, amplitude fraction
2. \( \theta_i \): unknown parameter, phase(...)
3. \( F_i \): amplitudes from resonances included in fit

1. \( B^+ \) yields \( \theta_i = \delta_i + \phi_i \)
2. \( B^- \) yields \( \bar{\theta}_i = \delta_i - \phi_i \)
A Little Complication

- Apply common known Dalitz-plot analysis technique.
- Need fit function for each type of resonance:

\[
\mathcal{F}_{B^+ \rightarrow \pi^+ \pi^- \pi^+}(s_1, s_2) = \left| \sum_i a_i e^{i\theta_i} F_i(s_1, s_2) \right|
\]

\[
s_{12} = (p_{1\mu} + p_{2\mu}) \cdot (p_{1\mu} + p_{2\mu})
\]

\[
s_{23} = (p_{2\mu} + p_{3\mu}) \cdot (p_{2\mu} + p_{3\mu})
\]

1. \(a_i\): unknown (!!) parameter, amplitude fraction
2. \(\theta_i\): unknown parameter, phase(...)
3. \(F_i\): amplitudes from resonances included in fit

Split phase in \(cp\)-conserving & \(cp\)-violating part:

\[
\theta_i = \delta_i + \phi_i
\]

→ need another decay to disentangle sum of phases

1. \(B^+\) yields \(\theta_i = \delta_i + \phi_i\)
2. \(B^-\) yields \(\bar{\theta}_i = \delta_i - \phi_i\)
Apply common known Dalitz-plot analysis technique.

need fit function for each type of resonance:

\[
\mathcal{F}_{B^+\rightarrow\pi^+\pi^-\pi^+}(s_1, s_2) = \left| \sum_i a_i e^{i\theta_i} F_i(s_1, s_2) \right|
\]

\[
s_{12} = (p_{1\mu}^\mu + p_{2\mu}^\mu) \cdot (p_{1\mu}^\mu + p_{2\mu}^\mu)
\]

\[
s_{23} = (p_{2\mu}^\mu + p_{3\mu}^\mu) \cdot (p_{2\mu}^\mu + p_{3\mu}^\mu)
\]

1. \(a_i\): unknown (!!) parameter, amplitude fraction
2. \(\theta_i\): unknown parameter, phase(...)
3. \(F_i\): amplitudes from resonances included in fit

Split phase in \(cp\)-conserving & \(cp\)-violating part:

\[
\theta_i = \delta_i + \phi_i
\]

→ need another decay to disentangle sum of phases

1. \(B^+\) yields \(\theta_i = \delta_i + \phi_i\)
2. \(B^-\) yields \(\bar{\theta}_i = \delta_i - \phi_i\)
Apply common known Dalitz-plot analysis technique. 

need fit function for each type of resonance:

\[ F_{B^+ \rightarrow \pi^+ \pi^- \pi^+}(s_1, s_2) = \left| \sum_i a_i e^{i\theta_i} F_i(s_1, s_2) \right| \]

\[ s_{12} = (p_1^\mu + p_2^\mu) \cdot (p_1^\mu + p_2^\mu) \]

\[ s_{23} = (p_2^\mu + p_3^\mu) \cdot (p_2^\mu + p_3^\mu) \]

1. \( a_i \): unknown (!!) parameter, amplitude fraction
2. \( \theta_i \): unknown parameter, phase(...)
3. \( F_i \): amplitudes from resonances included in fit

Split phase in \text{cp-conserving} & \text{cp-violating} part:

\[ \theta_i = \delta_i + \phi_i \]

\[ \rightarrow \text{need another decay to disentangle sum of phases} \]

1. \( B^+ \) yields \( \theta_i = \delta_i + \phi_i \)
2. \( B^- \) yields \( \bar{\theta}_i = \delta_i - \phi_i \)
**Introduction**

$b\bar{b}$ production at BaBar

**PEP-II at SLAC in Kalifornia**

Electron-Positron-Collisions:

- $e^{-}$: 9.0 GeV  
- $e^{+}$: 3.1 GeV

- Two storage rings
- Boost
- Center of mass energy: 10.58 GeV
  - Corresponds to $Y(4S)$ resonance

**B-Factory**

$e^{+} e^{-} \rightarrow B \bar{B}$:  
\[ \sigma = 1.05 \text{ nb} \]

... but also Tau-Factory

$e^{+} e^{-} \rightarrow \tau \tau$:
\[ \sigma = 0.89 \text{ nb} \]
$b\bar{b}$ production at BaBar

**BABAR Detector**

- Muon/Hadron Detector
- Magnet Coil
- Electron/Photon Detector
- Cherenkov Detector
- Tracking Chamber
- Support Tube
- Vertex Detector
How to Analyse?

- reconstruct $B^\pm$
- which $\pi$ belongs to which?
- through which resonance did the $\pi'$s go?

Example 1: BaBar analysis:

1. 3 identical particles in final state
2. use Dalitz-plot analysis
3. $\Delta E = E_B^* - \sqrt{s}/2,$
4. $m_{ES} = \sqrt{(s/2 + p_i \cdot p_B)^2/E_i^2 - p_B^2}$
5. 2 independent variables

J. Blouw
Measuring the CKM angle $\gamma$
How to Analyse?

- reconstruct $B^{\pm}$
- which $\pi$ belongs to which?
- through which resonance did the $\pi'$s go?

Example 1: BaBar analysis:

1. 3 identical particles in final state
2. use Dalitz-plot analysis
3. $\Delta E = E_B^* - \sqrt{s}/2$,
4. $m_{ES} = \sqrt{\left(s/2 + p_i \cdot p_B\right)^2 / E_i^2 - p_B^2}$
5. 2 independent variables
How to Analyse?

- reconstruct $B^\pm$
- which $\pi$ belongs to which?
- through which resonance did the $\pi'$s go?

Example 1: BaBar analysis:

1. 3 identical particles in final state
2. use Dalitz-plot analysis
3. $\Delta E = E_B^* - \sqrt{s}/2,$
4. $m_{ES} = \sqrt{(s/2 + p_i \cdot p_B)^2 / E_i^2 - p_B^2}$
5. 2 independent variables
How to Analyse?

- reconstruct $B^\pm$
- which $\pi$ belongs to which?
- through which resonance did the $\pi'$s go?

Example 1: BaBar analysis:

1. 3 identical particles in final state
2. use Dalitz-plot analysis
3. $\Delta E = E_B^* - \sqrt{s}/2$, 
4. $m_{ES} = \sqrt{(s/2 + p_i \cdot p_B)^2 / E_i^2 - p_B^2}$ 
5. 2 independent variables
How to Analyse?

- reconstruct $B^{\pm}$
- which $\pi$ belongs to which?
- through which resonance did the $\pi^\prime$s go?
- Example 1: BaBar analysis:

(Babar: Amplitude Analysis of the Decay $B^{\pm} \rightarrow \pi^{\pm} \pi^{\pm} \pi^{\mp}$, hep-ex/0507025)

1. 3 identical particles in final state
2. use Dalitz-plot analysis
3. $\Delta E = E_B^* - \sqrt{s}/2$
4. $m_{ES} = \sqrt{(s/2 + p_i \cdot p_B)^2/E_i^2 - p_B^2}$
5. 2 independent variables
How to Analyse?

- reconstruct $B^{\pm}$
- which $\pi$ belongs to which?
- through which resonance did the $\pi'$s go?
- Example 1: BaBar analysis:

1. 3 identical particles in final state
2. use Dalitz-plot analysis
3. $\Delta E = E_B^* - \sqrt{s}/2,$
4. $m_{ES} = \sqrt{(s/2 + p_i \cdot p_B)^2 / E_i^2 - p_B^2}$
5. 2 independent variables
How to Analyse?

- reconstruct $B^\pm$
- which $\pi$ belongs to which?
- through which resonance did the $\pi'$s go?
- Example 1: BaBar analysis:

1. 3 identical particles in final state
2. use Dalitz-plot analysis
3. $\Delta E = E_B^* - \sqrt{s}/2,$
4. $m_{ES} = \sqrt{(s/2 + p_i \cdot p_B)^2/E_i^2 - p_B^2}$
5. 2 independent variables
Introduction

How to Analyse?

- reconstruct $B^\pm$
- which $\pi$ belongs to which?
- through which resonance did the $\pi'$s go?

Example 1: BaBar analysis:

1. 3 identical particles in final state
2. use Dalitz-plot analysis
3. $\Delta E = E_B^* - \sqrt{s}/2,$
4. $m_{ES} = \sqrt{(s/2 + p_i \cdot p_B)^2/E_i^2 - p_B^2}$
5. 2 independent variables
How to Analyse?

- reconstruct $B^{\pm}$
- which $\pi$ belongs to which?
- through which resonance did the $\pi'$s go?
- Example 1: BaBar analysis:

1. 3 identical particles in final state
2. use Dalitz-plot analysis
3. $\Delta E = E_B^* - \sqrt{s}/2$,
4. $m_{ES} = \sqrt{(s/2 + p_i \cdot p_B)^2 / E_i^2 - p_B^2}$
5. 2 independent variables
How to Analyse?

- reconstruct $B^\pm$
- which $\pi$ belongs to which?
- through which resonance did the $\pi'$s go?
- Example 1: BaBar analysis:

1. 3 identical particles in final state
2. use Dalitz-plot analysis
3. $\Delta E = E_B^* - \sqrt{s}/2,$
4. $m_{ES} = \sqrt{(s/2 + p_i \cdot p_B)^2 / E_i^2 - p_B^2}$
5. 2 independent variables
Analysis Techniques

\[ e^+ + e^- \rightarrow \gamma(4S) \rightarrow (b\bar{b}) : \text{at } \sqrt{s} = 10.5 \text{ GeV} \]

1. Use kinematics of \((b\bar{b})\) production at resonance
2. (Compare with LHCb: \(E(b\bar{b})\) and \(p(b\bar{b})\) not \textit{a priori} known!)
3. Kinematic constraints from \(b\bar{b}\) production help to suppress background
4. Construct “invariant” masses of 2 combinations of \(\pi\)'s:
Analysis Techniques

\[ e^+ + e^- \rightarrow \Upsilon(4S) \rightarrow (b\bar{b}) : \text{at } \sqrt{s} = 10.5 \text{ GeV} \]

1. Use kinematics of \((b\bar{b})\) production at resonance
2. (Compare with LHCb: \(E(b\bar{b})\) and \(p(b\bar{b})\) not \textit{a priori} known!)
3. Kinematic constraints from \(b\bar{b}\) production help to suppress background
4. Construct “invariant” masses of 2 combinations of \(\pi\)’s:
$e^+ + e^- \rightarrow \Upsilon(4S) \rightarrow (b\bar{b}): \text{at } \sqrt{s} = 10.5 \text{ GeV}$

1. Use kinematics of $(b\bar{b})$ production at resonance
2. (Compare with LHCb: $E(b\bar{b})$ and $p(b\bar{b})$ not \textit{a priori} known!)
3. Kinematic constraints from $b\bar{b}$ production help to suppress background
4. Construct “invariant” masses of 2 combinations of $\pi$’s:
Analysis Techniques

$e^+ + e^- \rightarrow \Upsilon(4S) \rightarrow (b\bar{b})$ : at $\sqrt{s} = 10.5$ GeV

1. Use kinematics of $(b\bar{b})$ production at resonance
2. (Compare with LHCb: $E(b\bar{b})$ and $p(b\bar{b})$ not a priori known!)
3. Kinematic constraints from $b\bar{b}$ production help to suppress background
4. Construct “invariant” masses of 2 combinations of $\pi$'s:
Analysis Techniques

$e^+ + e^- \rightarrow \Upsilon(4S) \rightarrow (b\bar{b}) : \text{at } \sqrt{s} = 10.5 \text{ GeV}$

1. Use kinematics of $(b\bar{b})$ production at resonance
2. (Compare with LHCb: $E(b\bar{b})$ and $p(b\bar{b})$ not $a$ $priori$ known!)
3. Kinematic constraints from $b\bar{b}$ production help to suppress background
4. Construct “invariant” masses of 2 combinations of $\pi$’s:
Analysis Techniques

\[ e^+ + e^- \rightarrow \Upsilon(4S) \rightarrow (b\bar{b}) : \text{at } \sqrt{s} = 10.5 \text{ GeV} \]

1. Use kinematics of \((b\bar{b})\) production at resonance
2. (Compare with LHCb: \(E(b\bar{b})\) and \(p(b\bar{b})\) not \textit{a priori} known!)
3. Kinematic constraints from \(b\bar{b}\) production help to suppress background
4. Construct “invariant” masses of 2 combinations of \(\pi\)’s:

![Plot of invariant mass distribution]
$e^+ + e^- \rightarrow \Upsilon(4S) \rightarrow (b\bar{b})$ : at $\sqrt{(s)} = 10.5$ GeV

1. Use kinematics of $(b\bar{b})$ production at resonance
2. (Compare with LHCb: $E(b\bar{b})$ and $p(b\bar{b})$ not \textit{a priori} known!)
3. Kinematic constraints from $b\bar{b}$ production help to suppress background
4. Construct “invariant” masses of 2 combinations of $\pi$’s:

\[ \text{e.g. } B^+ \rightarrow \pi^+_1(\pi^-_2 \pi^+_3) \]
Analysis Techniques

\[ e^+ + e^- \rightarrow \gamma(4S) \rightarrow (b\bar{b}) : \text{at } \sqrt{s} = 10.5 \text{ GeV} \]

1. Use kinematics of \((b\bar{b})\) production at resonance
2. (Compare with LHCb: \(E(b\bar{b})\) and \(p(b\bar{b})\) not \textit{a priori} known!)
3. Kinematic constraints from \(b\bar{b}\) production help to suppress background
4. Construct “invariant” masses of 2 combinations of \(\pi\)’s:

\[ s_{13} = m_{\pi_1^{\pm}\pi_2^{\mp}}^2 \]
\[ s_{23} = m_{\pi_2^{\pm}\pi_3^{\mp}}^2 \]

\( e.g. \ B^+ \rightarrow \pi_1^+(\pi_2^- \pi_3^+) \)
Introduction

Dalitz Plot Analysis

Example: $D^0 \rightarrow K^0_s \pi^+ \pi^-$
Dalitz Plot Analysis

Example: $D^0 \rightarrow K_s^0 \pi^+ \pi^-$

1. Plot invariant masses
2. Why observe structure in Dalitz plot?
3. What did we expect?

- $R_k$: mass distribution
- $T_k$: angular-dependent amplitude
- $R_k$ usual a Breit-Wigner
- $T_k^{(0)} = 1$, $T_k^{(1)} = -2\vec{p} \cdot \vec{q}$
- $T_k^{(2)} = \frac{4}{3} [3(\vec{p} \cdot \vec{q})^2 - (|\vec{p}| |\vec{q}|)^2$

Resonances from
($\pi^- + \pi^+ = \rho^0, f_0(989)$)
($\pi^+ + K_s^0 = K^{*+}(892)$)

J. Blouw
Measuring the CKM angle $\gamma$
Dalitz Plot Analysis

Example: \( D^0 \rightarrow K_s^0 \pi^+ \pi^- \)

1. Plot invariant masses
2. Why observe structure in Dalitz plot?
3. What did we expect?

- \( R_k \): mass distribution
- \( T_k \): angular-dependent amplitude
- \( R_k \) usual a Breit-Wigner
- \( T_k^{(0)} = 1, \ T_k^{(1)} = -2\vec{p} \cdot \vec{q} \)
- \( T_k^{(2)} = \frac{4}{3} [3(\vec{p} \cdot \vec{q})^2 - (|\vec{p}| |\vec{q}|)^2] \)

1. Resonances from \((\pi^- + \pi^+ = \rho^0, f_0(989))\)
2. Resonances from \((\pi^+ + K_s^0 = K^{*+}(892))\)

J. Blouw
Measuring the CKM angle \( \gamma \)
Dalitz Plot Analysis

Example: $D^0 \rightarrow K_s^0 \pi^+ \pi^-$

1. Plot invariant masses
2. Why observe structure in Dalitz plot?
3. What did we expect?

- $R_k$: mass distribution
- $T_k$: angular-dependent amplitude
- $R_k$ usual a Breit-Wigner

1. Resonances from $(\pi^- + \pi^+ = \rho^0, f_0(989))$
2. Resonances from $(\pi^+ + K_s^0 = K^{*+}(892))$

\[ T_k^{(0)} = 1, \quad T_k^{(1)} = -2 \vec{p} \cdot \vec{q} \]

\[ T_k^{(2)} = \frac{4}{3} [3(\vec{p} \cdot \vec{q})^2 - (|\vec{p}| |\vec{q}|)^2] \]
Dalitz Plot Analysis

Example: $D^0 \rightarrow K^0_s \pi^+ \pi^-$

1. Plot invariant masses
2. Why observe structure in Dalitz plot?
3. What did we expect?

Resonances...

1. Resonances from $(\pi^- + \pi^+ = \rho^0, f_0(989))$
2. Resonances from $(\pi^+ + K^0_s = K^{*+}(892))$

$R_k$: mass distribution

$T_k$: angular-dependent amplitude

$R_k$ usual a Breit-Wigner

$T_k^{(0)} = 1$, $T_k^{(1)} = -2 \vec{p} \cdot \vec{q}$

$T_k^{(2)} = \frac{4}{3} [3(\vec{p} \cdot \vec{q})^2 - (|\vec{p}| |\vec{q}|)^2]$
**Dalitz Plot Analysis**

**Example:** $D^0 \rightarrow K_s^0 \pi^+ \pi^-$

1. Plot invariant masses
2. Why observe structure in Dalitz plot?
3. What did we expect?

**Resonances...**

1. Resonances from $(\pi^- + \pi^+ = \rho^0, f_0(989))$
2. Resonances from $(\pi^+ + K_s^0 = K^{*+}(892))$

---

**$R_k$:** mass distribution

**$T_k$:** angular-dependent amplitude

$R_k$ usual a Breit-Wigner

$T_k^{(0)} = 1$, $T_k^{(1)} = -2\vec{p} \cdot \vec{q}$

$T_k^{(2)} = \frac{4}{3}[3(\vec{p} \cdot \vec{q})^2 - (|\vec{p}| |\vec{q}|)^2]$
Example: $D^0 \rightarrow K^0_s \pi^+ \pi^-$

1. Plot invariant masses
2. Why observe structure in Dalitz plot?
3. What did we expect?

Resonances...

1. Resonances from $(\pi^- + \pi^+ = \rho^0, f_0(989))$
2. Resonances from $(\pi^+ + K^0_s = K^{*+}(892))$

$R_k$: mass distribution

$T_k$: angular-dependent amplitude

$R_k$ usual a Breit-Wigner

$T_k^{(0)} = 1$, $T_k^{(1)} = -2 \vec{p} \cdot \vec{q}$

$T_k^{(2)} = \frac{4}{3} [3(\vec{p} \cdot \vec{q})^2 - (|\vec{p}| |\vec{q}|)^2]$
Dalitz Plot Analysis

Example: $D^0 \to K_s^0 \pi^+ \pi^-$

1. Plot invariant masses
2. Why observe structure in Dalitz plot?
3. What did we expect?

Decay rate:

$$\frac{d\Gamma}{ds_{13}ds_{23}} = |\mathcal{M}|^2 \propto \left| \sum_k c_k e^{i\theta_k} \mathcal{D}_k(s_{12}, s_{23}) \right|^2$$

$$\mathcal{D}_k = R_k \times T_k$$

1. $R_k$: mass distribution
2. $T_k$: angular-dependent amplitude
3. $R_k$ usual a Breit-Wigner
4. $T_k^{(0)} = 1, T_k^{(1)} = -2 \vec{p} \cdot \vec{q}$
5. $T_k^{(2)} = \frac{4}{3} [3(\vec{p} \cdot \vec{q})^2 - (|\vec{p}| |\vec{q}|)^2]$

Resonances...

1. Resonances from $(\pi^- + \pi^+ = \rho^0, f_0(989))$
2. Resonances from $(\pi^+ + K_s^0 = K^*(892))$
**Dalitz Plot Analysis**

**Example:** $D^0 \rightarrow K^0_s \pi^+ \pi^-$

1. Plot invariant masses
2. Why observe structure in Dalitz plot?
3. What did we expect? Decay rate:

$$\frac{d\Gamma}{ds_{13} ds_{23}} = |\mathcal{M}|^2 \propto \left| \sum_k c_k e^{i\theta_k} D_k(s_{12}, s_{23}) \right|^2$$

$$D_k = R_k \times T_k$$

1. $R_k$: mass distribution
2. $T_k$: angular-dependent amplitude
3. $R_k$ usual a Breit-Wigner
4. $T_k^{(0)} = 1$, $T_k^{(1)} = -\vec{\rho} \cdot \vec{q}$
5. $T_k^{(2)} = \frac{4}{3} [3(\vec{\rho} \cdot \vec{q})^2 - (|\vec{\rho}| \cdot |\vec{q}|)^2]

**Resonances...**

1. Resonances from $(\pi^- + \pi^+ = \rho^0, f_0(989))$
2. Resonances from $(\pi^+ + K^0_s = K^{*+}(892))$
Dalitz Plot Analysis

Example: $D^0 \rightarrow K_s^0 \pi^+ \pi^-$

1. Plot invariant masses
2. Why observe structure in Dalitz plot?
3. What did we expect?

Decay rate:

$$\frac{d\Gamma}{ds_{13} ds_{23}} = |M|^2 \propto \left| \sum_k c_k e^{i\theta_k} D_k(s_{12}, s_{23}) \right|^2$$

$$D_k = R_k \times T_k$$

1. $R_k$: mass distribution
2. $T_k$: angular-dependent amplitude
3. $R_k$ usual a Breit-Wigner
4. $T_k^{(0)} = 1$, $T_k^{(1)} = -2\vec{p} \cdot \vec{q}$
5. $T_k^{(2)} = \frac{4}{3}[3(\vec{p} \cdot \vec{q})^2 - (|\vec{p}| |\vec{q}|)^2]$
Dalitz Plot Analysis

Example: $D^0 \rightarrow K^0_S \pi^+ \pi^-$

1. Plot invariant masses
2. Why observe structure in Dalitz plot?
3. What did we expect?

Decay rate:

$$\frac{d\Gamma}{ds_{13} ds_{23}} = |\mathcal{M}|^2 \propto \left| \sum_k c_k e^{i\theta_k} D_k(s_{12}, s_{23}) \right|^2$$

$D_k = R_k \times T_k$

1. $R_k$: mass distribution
2. $T_k$: angular-dependent amplitude
3. $R_k$ usual a Breit-Wigner
4. $T_k^{(0)} = 1$, $T_k^{(1)} = -2\vec{p} \cdot \vec{q}$
5. $T_k^{(2)} = \frac{4}{3} [3(\vec{p} \cdot \vec{q})^2 - (|\vec{p}| |\vec{q}|)^2$
Dalitz Plot Analysis

Example: $D^0 \rightarrow K_s^0 \pi^+ \pi^-$

1. Plot invariant masses
2. Why observe structure in Dalitz plot?
3. What did we expect? Decay rate:

$$\frac{d\Gamma}{ds_{13} ds_{23}} = |\mathcal{M}|^2 \propto \left| \sum_k c_k e^{i\theta_k} D_k(s_{12}, s_{23}) \right|^2$$

$D_k = R_k \times T_k$

1. $R_k$: mass distribution
2. $T_k$: angular-dependent amplitude
3. $R_k$ usual a Breit-Wigner
4. $T_k^{(0)} = 1$, $T_k^{(1)} = -2\bar{\rho} \cdot \bar{q}$
5. $T_k^{(2)} = \frac{4}{3} [3(\bar{\rho} \cdot \bar{q})^2 - (|\bar{\rho}| |\bar{q}|)^2]$

Resonances...

1. Resonances from $(\pi^- + \pi^+ = \rho^0, f_0(989))$
2. Resonances from $(\pi^+ + K_s^0 = K^{*+}(892))$

J. Blouw

Measuring the CKM angle $\gamma$
Introduction

**Dalitz:** $B^\pm \rightarrow \pi^\mp \pi^\pm \pi^\pm$

*Usual Dalitz plot*

Note: resonances occur at boundaries...

*Rescale co-ordinates:*

Empty space from charm vetoes

---

J. Blouw

Measuring the CKM angle $\gamma$
Dalitz from $B\bar{B}$ background

$B\bar{B}$ background: usual co-ordinates

re-scaled co-ordinates:

Holes are from charm vetoes
Results
Mass projections

Projection from $B^+$ onto $m(\pi^+\pi^-)$

Projection from $B^-$ onto $m(\pi^-\pi^-)$
Results
Mass projections

Project from $B^-$ onto $m(\pi^-\pi^-)$

Projection from $B^+$ onto $m(\pi^+\pi^+)$
Results & Conclusions

1. $B^+ \rightarrow \pi^\pm \pi^\pm \pi^\mp : (16.2 \pm 1.2 \pm 0.9) \times 10^{-6}$
2. $\rho^0(770)\pi^\pm (\rho^0 \rightarrow \pi^+ \pi^-) : (8.8 \pm 1.0 \pm 0.6) \times 10^{-6}$
3. $\rho^0(1450)\pi^\pm (\rho^0 \rightarrow \pi^+ \pi^-) : (1.0 \pm 0.6 \pm 0.4) \times 10^{-6}$
4. $f_0(980)\pi^\pm (f_0 \rightarrow \pi^+ \pi^-) : (1.2 \pm 0.6 \pm 0.5) \times 10^{-6}$
5. $\chi_{c0}\pi^\pm (\chi_{c0} \rightarrow \pi^+ \pi^-) : < 0.3$ at 90% CL
Results & Conclusions

1. $B^+ \rightarrow \pi^\pm \pi^\pm \pi^\mp : (16.2 \pm 1.2 \pm 0.9) \times 10^{-6}$
2. $\rho^0(770) \pi^\pm (\rho^0 \rightarrow \pi^+ \pi^-) : (8.8 \pm 1.0 \pm 0.6) \times 10^{-6}$
3. $\rho^0(1450) \pi^\pm (\rho^0 \rightarrow \pi^+ \pi^-) : (1.0 \pm 0.6 \pm 0.4) \times 10^{-6}$
4. $f_0(980) \pi^\pm (f_0 \rightarrow \pi^+ \pi^-) : (1.2 \pm 0.6 \pm 0.5) \times 10^{-6}$
5. $\chi_{c0} \pi^\pm (\chi_{c0} \rightarrow \pi^+ \pi^-) : < 0.3$ at 90% CL
Results & Conclusions

1. $B^+ \to \pi^\pm \pi^\pm \pi^\mp : (16.2 \pm 1.2 \pm 0.9) \times 10^{-6}$
2. $\rho^0(770)\pi^\pm (\rho^0 \to \pi^+ \pi^-) : (8.8 \pm 1.0 \pm 0.6) \times 10^{-6}$
3. $\rho^0(1450)\pi^\pm (\rho^0 \to \pi^+ \pi^-) : (1.0 \pm 0.6 \pm 0.4) \times 10^{-6}$
4. $f_0(980)\pi^\pm (f_0 \to \pi^+ \pi^-) : (1.2 \pm 0.6 \pm 0.5) \times 10^{-6}$
5. $\chi_{c0}\pi^\pm (\chi_{c0} \to \pi^+ \pi^-) : < 0.3$ at 90% CL
Results & Conclusions

1. $B^+ \rightarrow \pi^\pm \pi^\pm \pi^\mp : (16.2 \pm 1.2 \pm 0.9) \times 10^{-6}$
2. $\rho^0(770)\pi^\pm (\rho^0 \rightarrow \pi^+ \pi^-) : (8.8 \pm 1.0 \pm 0.6) \times 10^{-6}$
3. $\rho^0(1450)\pi^\pm (\rho^0 \rightarrow \pi^+ \pi^-) : (1.0 \pm 0.6 \pm 0.4) \times 10^{-6}$
4. $f_0(980)\pi^\pm (f_0 \rightarrow \pi^+ \pi^-) : (1.2 \pm 0.6 \pm 0.5) \times 10^{-6}$
5. $\chi_{c0}\pi^\pm (\chi_{c0} \rightarrow \pi^+ \pi^-) : < 0.3$ at 90% CL
Results & Conclusions

1. $B^+ \rightarrow \pi^\pm \pi^\pm \pi^\mp : (16.2 \pm 1.2 \pm 0.9) \times 10^{-6}$
2. $\rho^0(770)\pi^\pm (\rho^0 \rightarrow \pi^+ \pi^-) : (8.8 \pm 1.0 \pm 0.6) \times 10^{-6}$
3. $\rho^0(1450)\pi^\pm (\rho^0 \rightarrow \pi^+ \pi^-) : (1.0 \pm 0.6 \pm 0.4) \times 10^{-6}$
4. $f_0(980)\pi^\pm (f_0 \rightarrow \pi^+ \pi^-) : (1.2 \pm 0.6 \pm 0.5) \times 10^{-6}$
5. $\chi_{c0} \pi^\pm (\chi_{c0} \rightarrow \pi^+ \pi^-) : < 0.3$ at 90% CL
**Results & Conclusions**

1. $B^+ \to \pi^\pm \pi^\pm \pi^\mp : (16.2 \pm 1.2 \pm 0.9) \times 10^{-6}$
2. $\rho^0(770)\pi^\pm (\rho^0 \to \pi^+ \pi^-) : (8.8 \pm 1.0 \pm 0.6) \times 10^{-6}$
3. $\rho^0(1450)\pi^\pm (\rho^0 \to \pi^+ \pi^-) : (1.0 \pm 0.6 \pm 0.4) \times 10^{-6}$
4. $f_0(980)\pi^\pm (f_0 \to \pi^+ \pi^-) : (1.2 \pm 0.6 \pm 0.5) \times 10^{-6}$
5. $\chi_{c0}\pi^\pm (\chi_{c0} \to \pi^+ \pi^-) : < 0.3$ at 90% CL

**Conclusion**

*No $\chi_{c0}$ signal(!)*
Results & Conclusions

1. $B^+ \to \pi^+\pi^+\pi^- : (16.2 \pm 1.2 \pm 0.9) \times 10^{-6}$
2. $\rho^0(770)\pi^\pm(\rho^0 \to \pi^+\pi^-) : (8.8 \pm 1.0 \pm 0.6) \times 10^{-6}$
3. $\rho^0(1450)\pi^\pm(\rho^0 \to \pi^+\pi^-) : (1.0 \pm 0.6 \pm 0.4) \times 10^{-6}$
4. $f_0(980)\pi^\pm(f_0 \to \pi^+\pi^-) : (1.2 \pm 0.6 \pm 0.5) \times 10^{-6}$
5. $\chi_{c0}\pi^\pm(\chi_{c0} \to \pi^+\pi^-) : < 0.3$ at 90% CL

Conclusion

No $\chi_{c0}$ signal(!) $\Rightarrow$ CP-violation measurement not possible!!!