

Measuring the CKM angle γ

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Introduction

- The CKM Triangle
- Why γ ?
- Possible ways to measure γ
- Rare decays



Particle Decays

Corollary

CKM matrix describes quark-flavour changing

Example

- 1 decays described by quark-flavour transformations
- 2 3 up-type quarks, 3 down-type quarks
- 3 Unitarity requirement imposes constraints on elements



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$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$



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Unitarity yields ($V_{\text{CKM}} \cdot V_{\text{CKM}}^* = \mathbb{1}$)

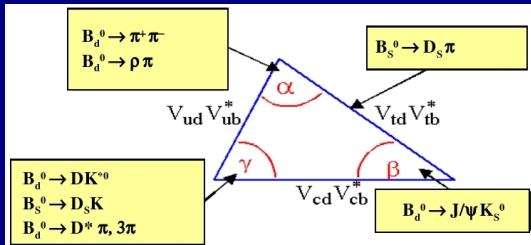


Unitarity of the CKM matrix

Conclusion

The CKM Triangle:

$$V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0$$

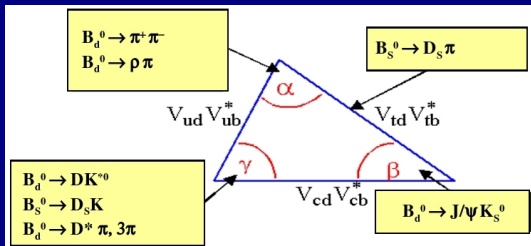


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More triangle exist, *e.g.* for B_s physics:

$$V_{ud} V_{us}^* + V_{cd} V_{cs}^* + V_{td} V_{ts}^* = 0$$



Angles of the Triangle(s)

The 3 Angles

$$1 \quad \alpha = \text{Arg} \left[-\frac{V_{td} V_{tb}^*}{V_{ud} V_{ub}^*} \right]$$

$$2 \quad \beta = \text{Arg} \left[-\frac{V_{cd} V_{cb}^*}{V_{td} V_{tb}^*} \right]$$

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More Angles

$$\beta_s = \text{Arg} \left[-\frac{V_{ts} V_{tb}^*}{V_{cs} V_{cb}^*} \right]$$

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Direct Measurement of γ

$$\mathbf{B}_u^\pm \rightarrow \pi^\pm (\pi^\pm + \pi^\mp)$$

- 1 Small branching fraction
 - 2 Measure γ through mixing of resonances w/o CP phase
 - 3 Direct CP-violating asymmetries
- $\mathcal{B} = (16.2 \pm 1.2 \pm 0.9) \cdot 10^{-6}$
 - $(\pi^+ + \pi^-)$ - resonance: $\rho^0, f_0, \chi_{c0} \dots$
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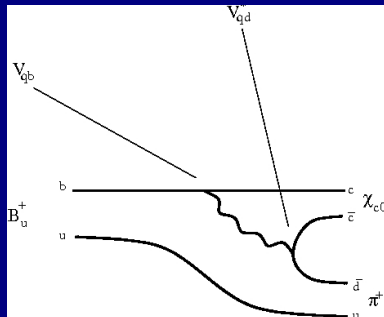
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 - access to cp-violation in interference
- interference between (ρ^0, f_0) and (χ_{c0}) : sensitivity to CP.



Feynmann Diagram



- $M(\chi_{c0}) = 3414.76 \pm 0.35 \text{ GeV}$
- χ_{c0} is a $c\bar{c}$ -state



A Little Complication

- Apply common known Dalitz-plot analysis technique.
 - need fit function for each type of resonance:
- 1 a_i : unkown (!!) parameter, amplitude fraction
 - 2 θ_i : unkown parameter, phase(. . .)
 - 3 F_j : amplitudes from resonances included in fit
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- 1 B^+ yields $\theta_i = \delta_i + \phi_j$
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→ need another decay to disentangle sum of phases

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$b\bar{b}$ production at BaBar**PEP-II at SLAC in Kalifornia****Electron-Positron-Collisions:** $e^- : 9.0 \text{ GeV} \quad e^+ : 3.1 \text{ GeV}$

- ➔ two storage rings
- ➔ Boost
- ➔ center of mass energy: 10.58 GeV
 - corresponds to $\Upsilon(4S)$ resonance

**B-Factory**

$$e^+ e^- \rightarrow B \bar{B}:$$

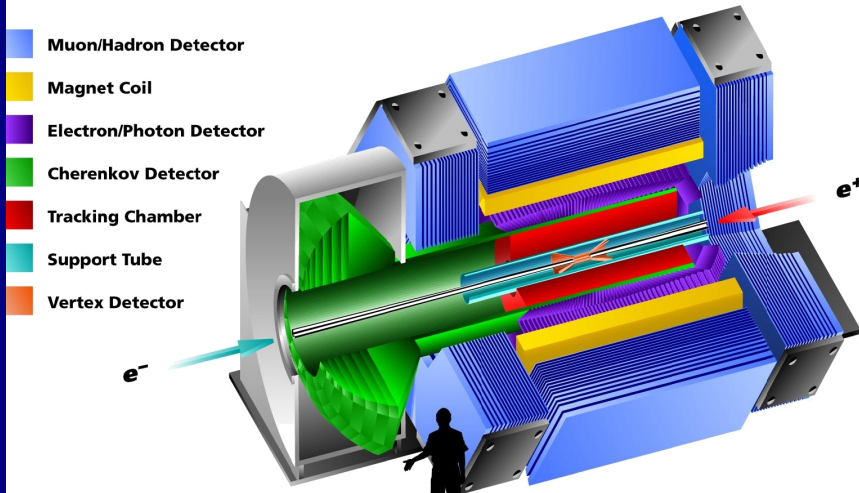
$$\sigma = 1.05 \text{ nb}$$

... but also Tau-Factory

$$e^+ e^- \rightarrow \tau \tau:$$

$$\sigma = 0.89 \text{ nb}$$



$b\bar{b}$ production at BaBar**BABAR Detector**

How to Analyse?

- reconstruct B^\pm
 - which π belongs to which?
 - through which resonance did the π 's go?
 - Example 1: BaBar analysis:
 - 1 3 identical particles in final state
 - 2 use Dalitz-plot analysis
 - 3 $\Delta E = E_B^* - \sqrt{s}/2,$
 - 4 $m_{ES} = \sqrt{(s/2 + p_i \cdot p_B)^2 / E_i^2 - p_B^2}$
 - 5 2 independent variables



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(Babar: Amplitude Analysis of the Decay $B^\pm \rightarrow \pi^\pm \pi^\pm \pi^\mp$, hep-ex/0507025)

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Analysis Techniques

$$e^+ + e^- \rightarrow \Upsilon(4S) \rightarrow (b\bar{b}) : \text{at } \sqrt{s} = 10.5 \text{ GeV}$$

- 1 Use kinematics of $(b\bar{b})$ production at resonance
- 2 (Compare with LHCb: $E(b\bar{b})$ and $p(b\bar{b})$ not *a priori* known!)
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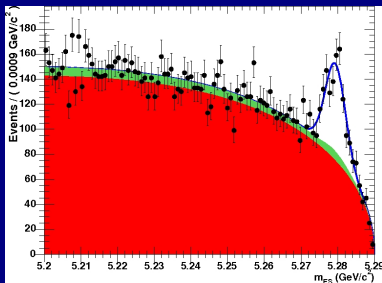
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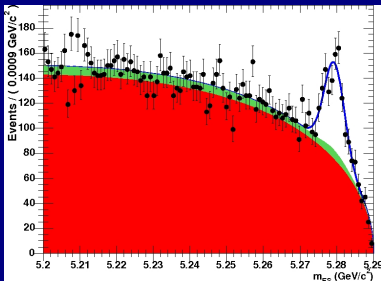
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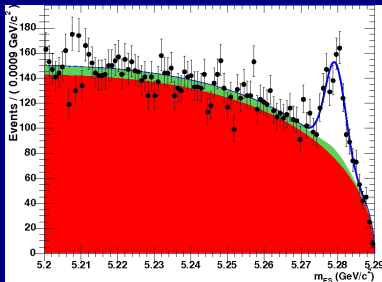
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$$e^+ + e^- \rightarrow \Upsilon(4S) \rightarrow (b\bar{b}) : \text{at } \sqrt{s} = 10.5 \text{ GeV}$$

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- 2 (Compare with LHCb: $E(b\bar{b})$ and $p(b\bar{b})$ not *a priori* known!)
- 3 kinematic constraints from $b\bar{b}$ production help to suppress background
- 4 construct “invariant” masses of 2 combinations of π 's:



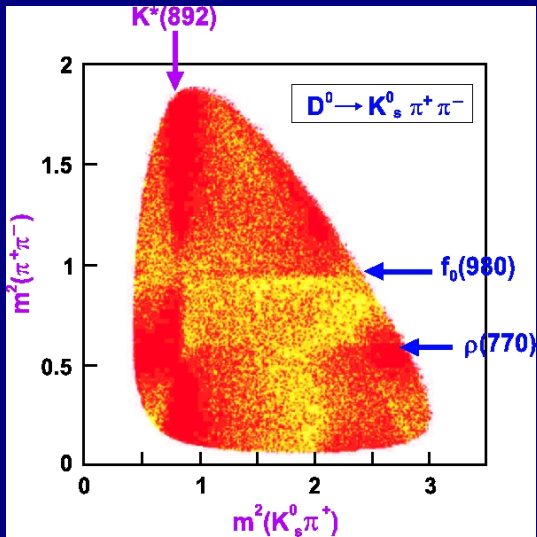
$$\text{e.g. } B^+ \rightarrow \pi_1^+ (\pi_2^- \pi_3^+)$$

$$S_{13} = m_{\pi_1^+ \pi_2^-}^2$$

$$S_{23} = m_{\pi_2^- \pi_3^+}^2$$



Dalitz Plot Analysis



Example: $D^0 \rightarrow K_S^0 \pi^+ \pi^-$

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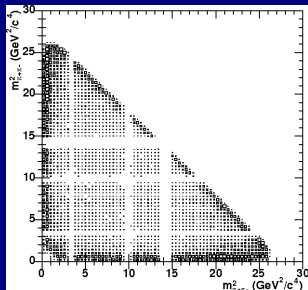
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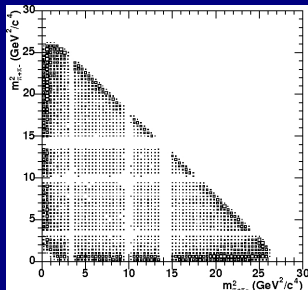
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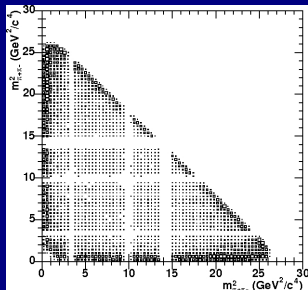
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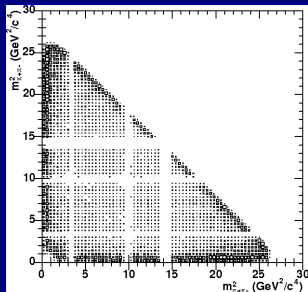
$$\frac{d\Gamma}{ds_{13} ds_{23}} = |\mathcal{M}|^2 \propto \left| \sum_k c_k e^{i\theta_k} \mathcal{D}_k(s_{12}, s_{23}) \right|^2$$

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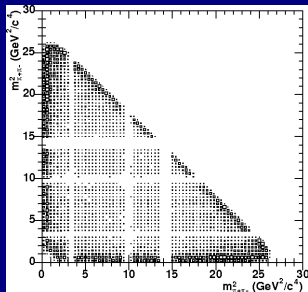
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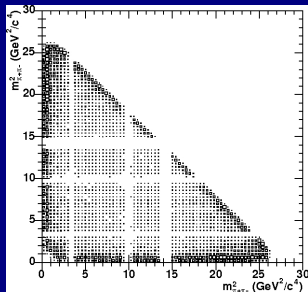
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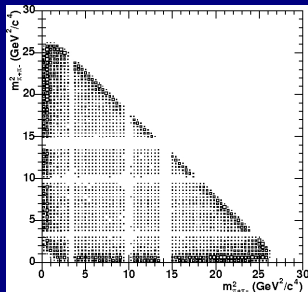
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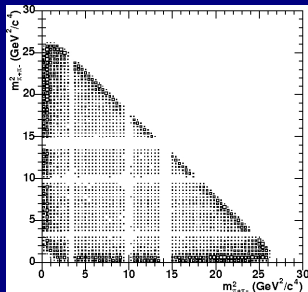
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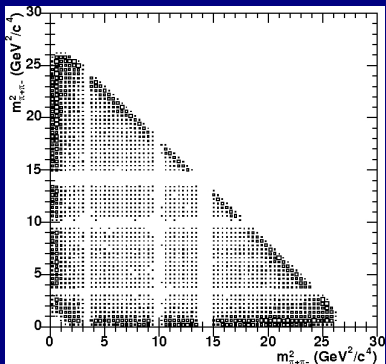
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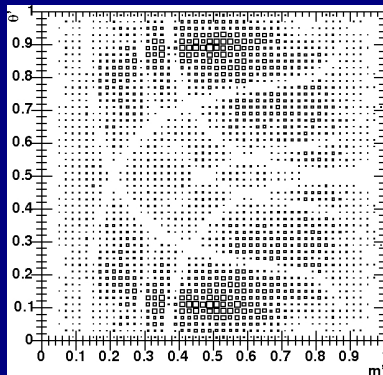
$$\text{Dalitz: } B^\pm \rightarrow \pi^\mp \pi^\pm \pi^\pm$$

Usual Dalitz plot



Note: resonances occur at boundaries...

Rescale co-ordinates:

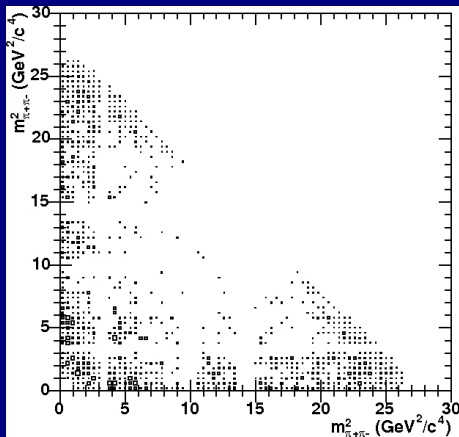


Empty space from charm vetoes



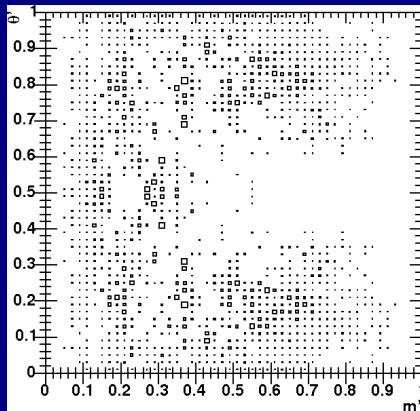
Dalitz from $B\bar{B}$ background

$B\bar{B}$ background: usual co-ordinates



Holes are from charm vetoes

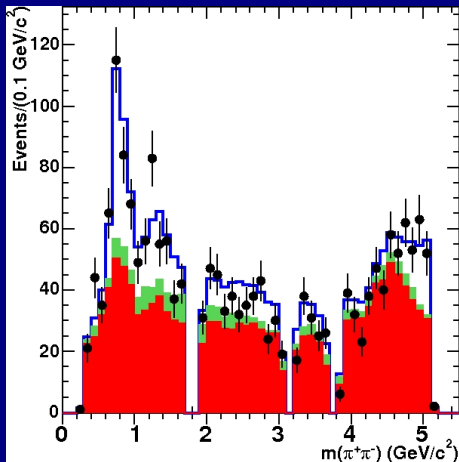
re-scaled co-ordinates:



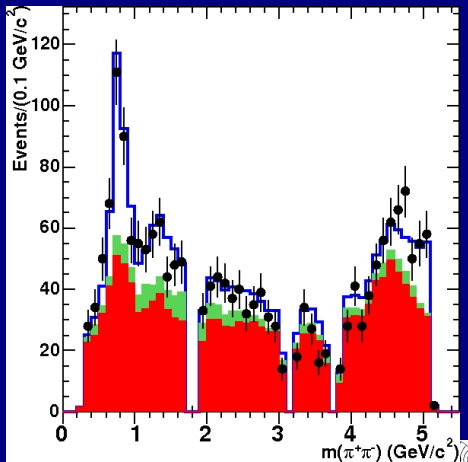
Results

Mass projections

Projection from B^+ onto $m(\pi^+\pi^-)$



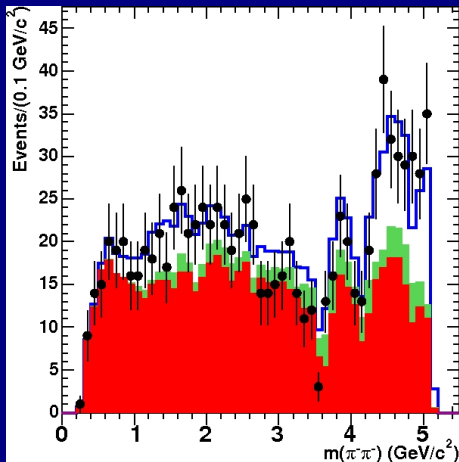
Projection from B^- onto $m(\pi^-\pi^-)$



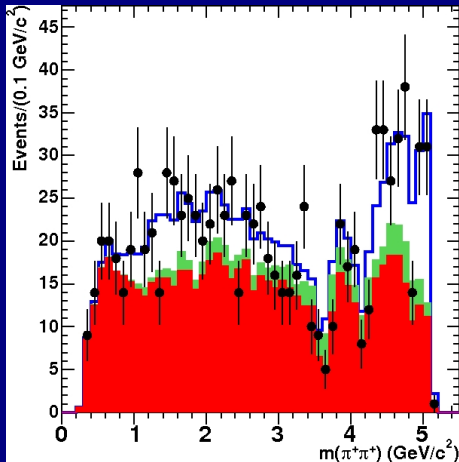
Results

Mass projections

Project from B^- onto $m(\pi^- \pi^-)$



Projection from B^+ onto $m(\pi^+ \pi^+)$



Results & Conclusions

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- 2 $\rho^0(770)\pi^\pm (\rho^0 \rightarrow \pi^+\pi^-) : (8.8 \pm 1.0 \pm 0.6) \times 10^{-6}$
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Conclusion

No χ_{c0} signal(!) \Rightarrow CP-violation measurement not possible!!!

