Workshop Neckarzimmern

Symmetries

Standard Model Langrangian

Higgs Coupling to Quarks and Mass Generation

CKM Matrix

Unitarity Triangles

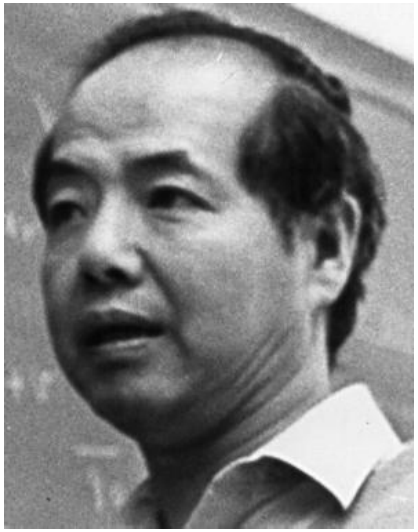
Mixing

Symmetries

T.D.Lee:
"The root to all symmetry principles lies in the assumption that it is impossible to observe certain basic quantities; the non-observables"

There are four main types of symmetry:

- Permutation symmetry:
  Bose-Einstein and Fermi-Dirac Statistics
- Continuous space-time symmetries:
  translation, rotation, acceleration,…
- Discrete symmetries:
  space inversion, time inversion, charge inversion
- Unitary symmetries: gauge invariances:
  U_1(charge), SU_2(isospin), SU_3(color),..



Noether Theorem: symmetry ⇔ conservation law

Non-observables	Symmetry Transformations	Conservation Laws or Selection Rules
Difference between identical particles	Permutation	B.-E. or F.-D. statistics
Absolute spatial position	Space translation $\vec{r} \to \vec{r} + \vec{\Delta}$	momentum
Absolute time	Time translation $t \to t + \tau$	energy
Absolute spatial direction	Rotation	angular momentum
Absolute right (or left)	$\vec{r} \to -\vec{r}$	parity
Absolute sign of electric charge	$e \to -e$	charge conjugation
Relative phase between states of different charge Q	$\psi \to e^{iQ\theta}\psi$	charge
Relative phase between states of different baryon number B	$\psi \to e^{iN\theta}\psi$	baryon number
Relative phase between states of different lepton number L	$\psi \to e^{iL\theta}\psi$	lepton number
Difference between different coherent mixture of p and n states	$\begin{pmatrix} p \\ n \end{pmatrix} \to U \begin{pmatrix} p \\ n \end{pmatrix}$	isospin

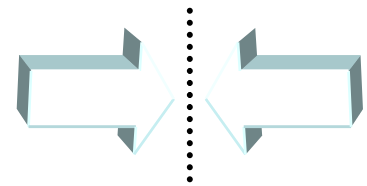
Lepton / baryon number conservation:

$$q(x) \to e^{i\omega/3} q(x) \quad \text{and} \quad \ell(x) \to \ell(x)$$

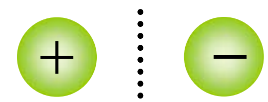
$$\ell(x) \to e^{i\lambda} \ell(x) \quad \text{and} \quad q(x) \to q(x)$$

Diskrete Symmetrien C, P, T

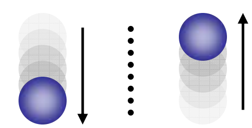
- Parity, *P*
  - Parity reflects a system through the origin. Converts right-handed coordinate systems to left-handed ones.
  - Vectors change sign but axial vectors remain unchanged
    - $\vec{x} \rightarrow -\vec{x}$, $\vec{p} \rightarrow -\vec{p}$, but $\vec{L} = \vec{x} \times \vec{p} \rightarrow \vec{L}$



- Charge Conjugation, *C*
  - Charge conjugation turns a particle into its anti-particle
    - $e^+ \rightarrow e^-$, $K^- \rightarrow K^+$



- Time Reversal, *T*
  - Changes, for example, the direction of motion of particles
    - $t \rightarrow -t$



Discrete symmetries ⇔ multiplicative quantum numbers

Transformation Properties

			P	C
	$(\vec{x},t)$	$\rightarrow$	$(-\vec{x},t)$	$(\vec{x},t)$
Scalar Field :	$\phi(\vec{x},t)$	$\rightarrow$	$\phi(-\vec{x},t)$	$\phi^\dagger(\vec{x},t)$
Pseudo Field :	$P(\vec{x},t)$	$\rightarrow$	$-P(-\vec{x},t)$	$P^\dagger(\vec{x},t)$
Dirac Field :	$\psi(\vec{x},t)$	$\rightarrow$	$\gamma_0\psi(-\vec{x},t)$	$i\gamma^2\gamma^0\overline{\psi}^T(\vec{x},t)$
Vector Field :	$V_\mu(\vec{x},t)$	$\rightarrow$	$V^\mu(-\vec{x},t)$	$-V_\mu^\dagger(\vec{x},t)$
Axial Field :	$A_\mu(\vec{x},t)$	$\rightarrow$	$-A^\mu(-\vec{x},t)$	$A_\mu^\dagger(\vec{x},t)$

Transformation properties of Dirac spinor bilinears:

			P	C	CP	T	CPT
$S:$	$\overline{\psi}_1\psi_2$	$\rightarrow$	$\overline{\psi}_1\psi_2$	$\overline{\psi}_2\psi_1$	$\overline{\psi}_2\psi_1$	$\overline{\psi}_1\psi_2$	$\overline{\psi}_2\psi_1$
$P:$	$\overline{\psi}_1\gamma_5\psi_2$	$\rightarrow$	$-\overline{\psi}_1\gamma_5\psi_2$	$\overline{\psi}_2\gamma_5\psi_1$	$-\overline{\psi}_2\gamma_5\psi_1$	$-\overline{\psi}_1\gamma_5\psi_2$	$\overline{\psi}_2\gamma_5\psi_1$
$V:$	$\overline{\psi}_1\gamma_\mu\psi_2$	$\rightarrow$	$\overline{\psi}_1\gamma^\mu\psi_2$	$-\overline{\psi}_2\gamma_\mu\psi_1$	$-\overline{\psi}_2\gamma^\mu\psi_1$	$-\overline{\psi}_1\gamma^\mu\psi_2$	$-\overline{\psi}_2\gamma_\mu\psi_1$
$A:$	$\overline{\psi}_1\gamma_\mu\gamma_5\psi_2$	$\rightarrow$	$-\overline{\psi}_1\gamma^\mu\gamma_5\psi_2$	$\overline{\psi}_2\gamma_\mu\gamma_5\psi_1$	$-\overline{\psi}_2\gamma^\mu\gamma_5\psi_1$	$-\overline{\psi}_1\gamma^\mu\gamma_5\psi_2$	$-\overline{\psi}_2\gamma_\mu\gamma_5\psi_1$
$T:$	$\overline{\psi}_1\sigma_{\mu\nu}\psi_2$	$\rightarrow$	$\overline{\psi}_1\sigma^{\mu\nu}\psi_2$	$-\overline{\psi}_2\sigma_{\mu\nu}\psi_1$	$-\overline{\psi}_1\sigma^{\mu\nu}\psi_2$	$-\overline{\psi}_2\sigma^{\mu\nu}\psi_1$	$\overline{\psi}_2\sigma_{\mu\nu}\psi_1$
						$c\rightarrow c^*$	$c\rightarrow c^*$

T Reversal

- Wigner found that T operator is **antiunitary:**

$$\langle \psi' | \phi' \rangle = \langle \psi T^{\dagger} | T\phi \rangle = \langle \psi | \phi \rangle^{*}$$

- This leaves the physical content of a system unchanged, since:

$$|\langle \psi' | \phi' \rangle| = |\langle \phi | \psi \rangle| = |\langle \psi | \phi \rangle|$$

- Antiunitary operators may be interpreted as the product of an unitary operator by an operator which complex-conjugates.

- As a consequence, T is **antilinear:**

$$T\left(C_1 |\psi\rangle + C_2 |\phi\rangle\right) = C_1^{*} T |\psi\rangle + C_2^{*} T |\phi\rangle$$

Parity Violation in Weak Interaction

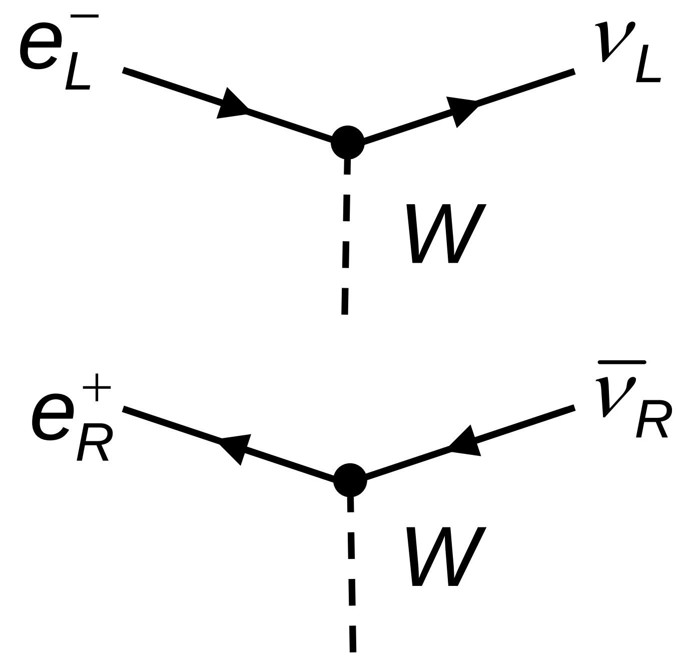
(Charged current)

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L$$

e_R^-, ~~ν_{eR}~~ $\quad$ μ_R^-, ~~$\nu_{\mu R}$~~ $\quad$ τ_R^-, ~~$\nu_{\tau R}$~~

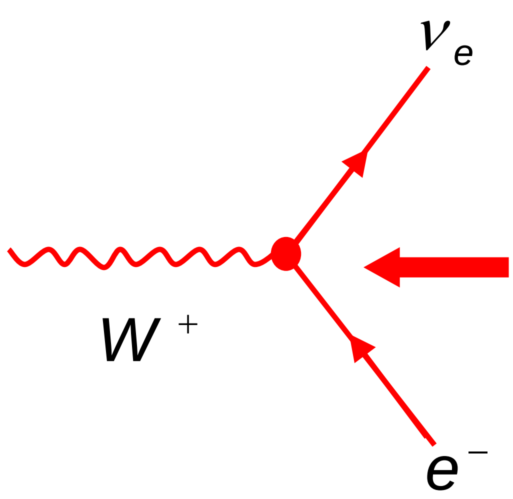
$$\begin{pmatrix} u \\ d \end{pmatrix}_L \quad \begin{pmatrix} c \\ s \end{pmatrix}_L \quad \begin{pmatrix} t \\ b \end{pmatrix}_L$$

$$u_R, d_R \quad c_R, s_R \quad t_R, b_R$$



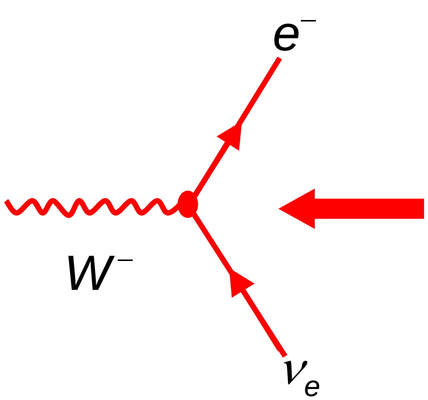
Maximal Violation of P and C

			P
$S:$	$\overline{\psi}_1\psi_2$	$\rightarrow$	$\overline{\psi}_1\psi_2$
$P:$	$\overline{\psi}_1\gamma_5\psi_2$	$\rightarrow$	$-\overline{\psi}_1\gamma_5\psi_2$
$V:$	$\overline{\psi}_1\gamma_\mu\psi_2$	$\rightarrow$	$\overline{\psi}_1\gamma^\mu\psi_2$
$A:$	$\overline{\psi}_1\gamma_\mu\gamma_5\psi_2$	$\rightarrow$	$-\overline{\psi}_1\gamma^\mu\gamma_5\psi_2$
$T:$	$\overline{\psi}_1\sigma_{\mu\nu}\psi_2$	$\rightarrow$	$\overline{\psi}_1\sigma^{\mu\nu}\psi_2$



Charge raising current:

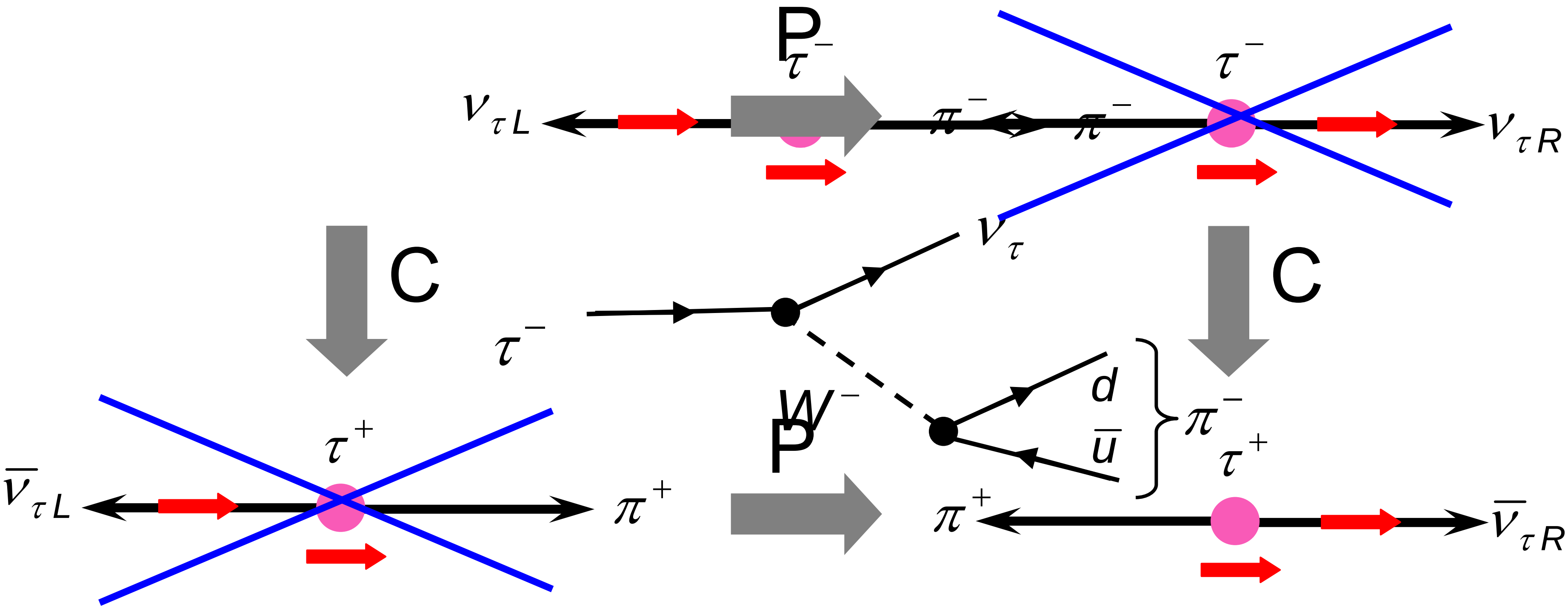
$$J_\mu^+ = \overline{\psi}_\nu\gamma_\mu\frac{1-\gamma^5}{2}\psi_e == \overline{\nu}_L\gamma_\mu e_L$$



Charge lowering current:

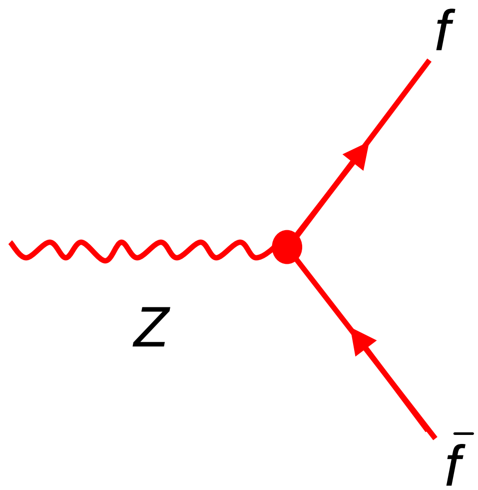
$$J_\mu^- = \overline{\psi}_e\gamma_\mu\frac{1-\gamma^5}{2}\psi_\nu = \overline{e}_L\gamma_\mu\nu_L$$

C and P Violation



Auf ersten Blick scheint schwache Wechselwirkung invariant unter CP.

Partity Violation in Neutral Current ?



J_μ ? $\overline{\psi}_f \gamma_\mu (g_V - g_A \gamma^5) \psi_f$

$$\text{LH}: \; g_L \cdot \frac{g}{\cos\theta_w} \quad \text{mit} \quad g_L = T_3^f - Q_f \sin^2\theta_w$$

$$\text{RH}: \; g_R \cdot \frac{g}{\cos\theta_w} \quad \text{mit} \quad g_R = \quad - Q_f \sin^2\theta_w$$

Beispiel:

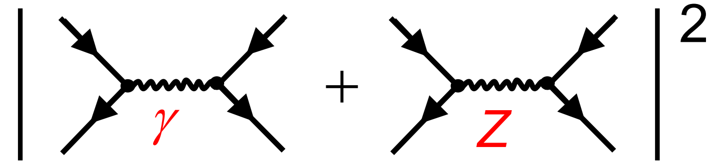
$\nu: \quad g_L = +\frac{1}{2} \quad g_R = 0$

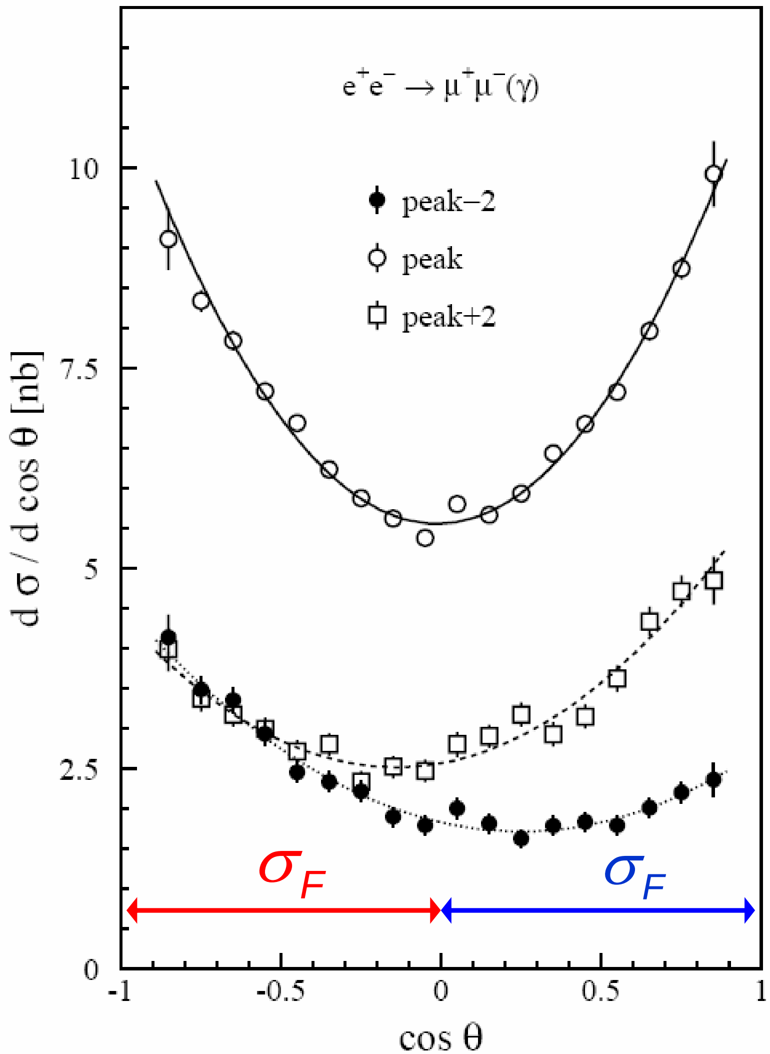
$e: \quad g_L = -0.27 \quad g_R = +0.23$

Instead of g_l and g_R the vector and axial vector couplings are often used:

$$g_V = g_L + g_R \qquad g_A = g_L - g_R$$

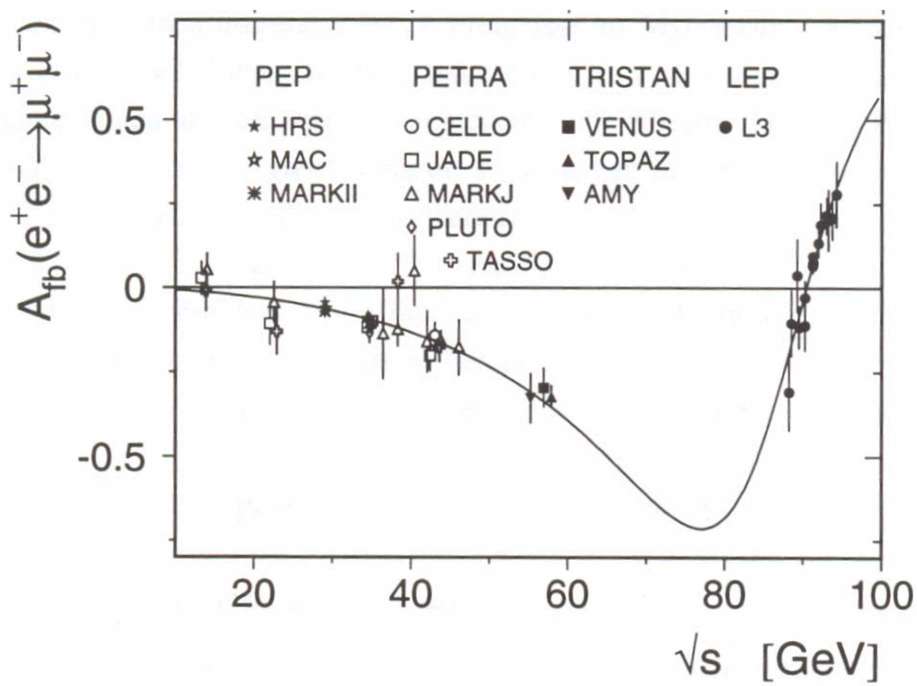
$$e^+ e^- \rightarrow Z \rightarrow \mu^+ \mu^-$$

$|M|^2 =$ 



$$\frac{d\sigma}{d\cos\theta} \sim (1+\cos^2\theta) + \frac{8}{3} A_{FB} \cos\theta$$

with $$A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}$$



CPT Symmetrie

Local Field theories always respect:

- Lorentz Invariance
- Symmetry under CPT operation (an electron = a positron travelling back in time)

=> Consequence: mass of particle = mass of anti-particle: (Lüders, Pauli, Schwinger)

$$M(p) = \langle p|H|p\rangle = \langle p|(CPT)^{\dagger}(CPT)H(CPT)^{-1}(CPT)|p\rangle^{*}$$

$$= \langle \overline{p}|(CPT)H(CPT)^{-1}|\overline{p}\rangle^{*} = \langle \overline{p}|H|\overline{p}\rangle^{*} = M(\overline{p})$$

(anti-unitarity)

=> Similarly the total decay-rate of a particle is equal to that of the anti-particle

- Question 1:

The mass difference between K_L and K_S: Δm = 3.5 x 10^{-6} eV => CPT violation?

- Question 2:

How come the lifetime of K_S = 0.089 ns while the lifetime of the K_L = 51.7 ns?

- Question 3:

BaBar measures decay rate B-> J/ψ K_S and Bbar-> J/ψ K_S. Clearly not the same: how can it be?

Dirac Equation for free Spin ½ Fermion

$$\gamma^{\mu}\partial_{\mu}\psi - m\psi = 0$$

Solutions ψ are four-component spinors.

They describes the fundamental spin ½ particles:

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix}$$

With adjoint spinor $\overline{\psi} = \psi^{+}\gamma^{0}$

$$\overline{\psi}(i\partial_{\mu}\gamma^{\mu} + m) = 0$$

Extremely compressed description

$$j = 1...4: \quad \sum_{k=1}^{4}\left(\sum_{\mu} i \cdot (\gamma^{\mu})_{jk} \frac{\partial}{\partial x^{\mu}} - m\delta_{jk}\right)\psi_k$$

Gamma Matrices

$$\gamma^0 = \beta$$

$$\gamma^i = \beta\alpha_i\ ,\ \mathrm{i} = 1,2,3$$

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^2\gamma^3$$

$$\alpha_i = \begin{pmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{pmatrix} \text{ and } \beta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\gamma^i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix},\quad \mathrm{i} = 1...3$$

$$\gamma^5 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Rules

$$\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu},\quad \gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 0 \text{ for } \mu \neq \nu$$

$$(\gamma^\mu)^+ = \gamma^0\gamma^\mu\gamma^0,\quad (\gamma^0)^+ = \gamma^0,\quad (\gamma^k)^+ = -\gamma^k$$

$$\gamma^0\gamma^0 = 1,\quad \gamma^k\gamma^k = -1,\quad k = 1..3$$

$$\gamma^5\gamma^\mu + \gamma^\mu\gamma^5 = 0\ ,\quad (\gamma^5)^+ = \gamma^5$$

Continuity Equation

$$\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi + (\partial_{\mu}\bar{\psi})\gamma^{\mu}\psi = 0$$

$$\Rightarrow \quad \underbrace{\partial_{\mu}(\bar{\psi}\gamma^{\mu}\psi)}_{} = 0$$

Fermion current

$$j^{\mu} = (\bar{\psi}\gamma^{\mu}\psi)$$

$$\begin{cases} \text{here}: \quad \rho = j^{0} > 0 \\ j^{0} = \bar{\psi}\gamma^{0}\psi = \psi^{+}\gamma^{0}\gamma^{0}\psi \\ \quad = \psi^{+}\psi > 0 \end{cases}$$

Langrangian

Local field theories work with Lagrangian densities:

$$L\left(\vec{x},t\right) = L\left(\phi_j\left(\vec{x},t\right), \partial^\mu \phi_j\left(\vec{x},t\right)\right)$$

with $\phi_j\left(\vec{x},t\right),\ j=1,2,\ldots,N$ the fields taken at $\vec{x},t$

The fundamental quantity, when discussing symmetries is the Action:

$$A = \int d^4x\, L\left(\vec{x},t\right)$$

If the action is (is not) invariant under a symmetry operation then the symmetry in question is a good (broken) one

=> Unitarity of the interaction requires the Lagrangian to be Hermitian

$\left(S = \exp iA_{\text{int}}\right)$ (S matrix theory)

Toy Theory

Consider a spin-1/2 (Dirac) particle ("nucleon") interacting with a spin-0 (Scalar) object ("meson")

$$(i\gamma^{\mu}\partial_{\mu} - m)\psi = 0$$

$$L(\vec{x},t) = i\bar{\psi}(\vec{x},t)\gamma^{\mu}\partial_{\mu}\psi(\vec{x},t) - m\bar{\psi}(\vec{x},t)\psi(\vec{x},t)$$

$$(i\partial^{\mu}\partial_{\mu} + M^{2})\phi = 0$$

Interaction: Scalar and pseudo scalar couplings

Local Gauge Invariance

= Lagrangian must be invariant under local gauge transformations

Theory of massless Fermions: $\mathcal{L} = i\bar{\psi}\left(\gamma^{\mu}\partial_{\mu}\right)\psi$

"global" U(1) gauge transformation: $\psi(x) \rightarrow \psi'(x) = e^{i\alpha}\psi(x)$

"local" U(1) gauge transformation: $\psi(x) \rightarrow \psi'(x) = e^{i\alpha(x)}\psi(x)$

Is the Lagrangian invariant?

$$\psi(x) \rightarrow e^{i\alpha(x)}\psi(x) \quad ; \quad \bar{\psi}(x) \rightarrow e^{-i\alpha(x)}\bar{\psi}(x)$$

$$\partial_{\mu}\psi(x) \rightarrow e^{i\alpha(x)}\partial_{\mu}\psi(x) \quad + \quad ie^{i\alpha(x)}\psi(x)\partial_{\mu}\alpha(x)$$

Then:

$$i\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi \rightarrow i\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi \quad - \quad \bar{\psi}\gamma^{\mu}\psi\partial_{\mu}\alpha(x)$$

Not invariant!

Gauge Field

=> Introduce the covariant derivative:

$$D_\mu \equiv \partial_\mu - ieA_\mu$$

and <u>demand</u> that A_μ transforms as:

$$A_\mu \to A_\mu' = A_\mu + \frac{1}{e}\partial_\mu \alpha(x)$$

$$\mathcal{L} \to \mathcal{L}' = \mathcal{L}$$

<u>Conclusion:</u>

- Introduce charged fermion field (electron)
- Demand invariance under local gauge transformations (U(1))
- The price to pay is that a gauge field A_μ and the IA with the field must be introduced at the same time.

Attention: Mass terms are not gauge invariant!

Standard Modell Lagrangian

$$L = L(f,G) + L(f,H) + L(G,H) + L(G) - L(H)$$

f = fermions (quarks, leptons)

G = gauge bosons (W, B)

H = Higgs doublet

$$\begin{pmatrix} q_{jL} \\ q'_{jL} \end{pmatrix}, \quad q_{jR}, \quad q'_{jR} \quad \text{mit } j = 1,2,3$$

$$q_R = \frac{1+\gamma_5}{2} q \qquad q_L = \frac{1-\gamma_5}{2} q$$

$$L(f,G) = \sum_{j=1}^{N} \left\{ \overline{(q,q')_{jL}}\; i\gamma^\mu [\partial_\mu I - ig_2 \frac{\vec{\sigma}}{2}.\vec{W}_\mu - ig_1(\frac{1}{6}) B_\mu I\,] \begin{pmatrix} q_{jL} \\ q'_{jL} \end{pmatrix} \right.$$

$$\left. + \overline{q_{jR}}\; i\gamma^\mu [\partial_\mu - ig_1(\frac{2}{3})\; B_\mu] q_{jR} + \overline{q'_{jR}}\; i\gamma^\mu [\partial_\mu - ig_1(\frac{-1}{3})\; B_\mu] q'_{jR} \right\}$$

Langrangian formally violates P and C, but conserves CP

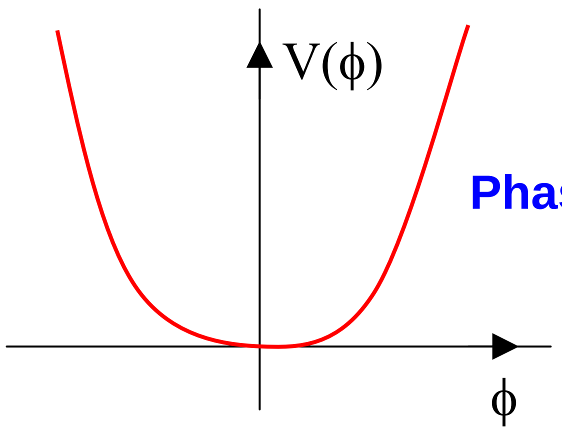
L(H) and Symmetrie Breaking

$$\mathcal{L}_{Higgs} = D_\mu \phi^\dagger D^\mu \phi - V_{Higgs} \qquad V_{Higgs} = \frac{1}{2}\mu^2 \left(\phi^\dagger \phi\right) + |\lambda| \left(\phi^\dagger \phi\right)^2$$

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_0 + i\phi_3 \end{pmatrix}$$

$\mu^2 > 0:$

$< \varphi >= 0$

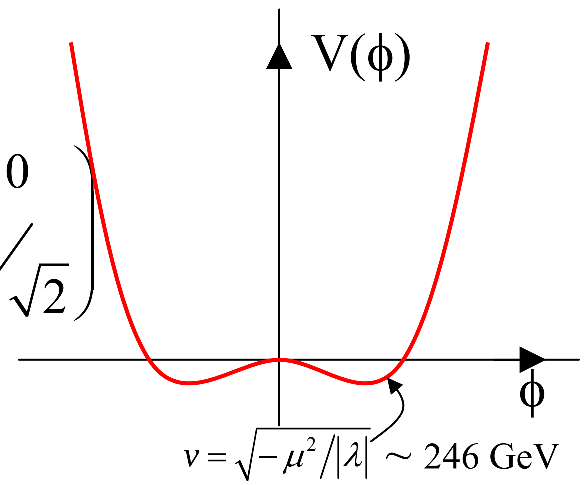


Phase change

SSB

$\mu^2 < 0:$

$< \varphi >= \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$



Spontaneous Symmetry Breaking:
The Higgs field adopts a non-zero vacuum expectation value

→Note $\mathcal{L}_{Higgs}$ = CP conserving

L(f,H) and Symmetrie Breaking

$$L(f,H) = \sum_{j,k=1}^{N} \left\{ Y_{jk} \overline{(q\ ,q')_{jL}} \begin{pmatrix} \phi^{(0)*} \\ -\phi^{(-)} \end{pmatrix} q_{kR} + Y'_{jk} \overline{(q,q')_{jL}} \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix} q'_{kR} + h.c. \right\}$$

eichinvariant

Spontaneous Symmetry Breaking:

$\phi_0 \rightarrow \phi_0 + \upsilon$ $\phi_i\ ,i = 1,2,3$

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \phi_0 + \upsilon \end{pmatrix}$$

„Eaten“ up by $W^{\pm}$ and Z bosons becoming massive

$$L(f,H) \overset{SSB}{\longrightarrow} - \sum_{j,k=1}^{N} \{ m_{jk}\ \overline{q_{jL}}\ q_{kR} + m'_{jk} \overline{q'_{jL}}\, q'_{kR} \ + h.c.\}(1 + \frac{1}{\upsilon}\phi_0)$$

u-type $m_{jk} = -\frac{\upsilon}{\sqrt{2}} Y_{jk}$ $m'_{jk} = -\frac{\upsilon}{\sqrt{2}} Y'_{jk}$ Quark mass matrices

Non physical quark fields

Physical Quark Fields

convention

➡ Diagonalize mass matrices:

$$U_L m U_R^{+} = D \equiv Diag.(m_u, m_c, m_t)$$

(matrix theory: possible w/ help of 2 unitary matices)

$$U_L' m' U_R'^{+} = D' \equiv Diag.(m_d, m_s, m_b)$$

$$U_L U_L^{+} = 1$$

Substituting into L(f,H) one obtains for u-type quarks:

$$\bar{q}_{jL} m_{jk} q_{kR} = \bar{q}_L m \, q_R = \bar{q}_L U_L^{+} U_L m U_R^{+} U_R q_R$$

$$= \overline{U_L q_L} \, D U_R q_R = \overline{U_L q_L} \begin{pmatrix} m_u & & \\ & m_c & \\ & & m_t \end{pmatrix} U_R q_R$$

$$q_L^{phys} = U_L q_L$$

analog for d-type quarks

$$q_L'^{phys} = U_L' q_L'$$

Similar relations also for right-handed quarks

Mass terms

Substitution into L(f,H):

$$L^{phys}(f,H) = -(1+\frac{\phi_0}{v})\Big[m_u\bar{u}\frac{(1+\gamma_5)}{2}u + m_c\bar{c}\frac{(1+\gamma_5)}{2}c + \ldots$$

$$+m_d\bar{d}\frac{(1+\gamma_5)}{2}d + m_s\bar{s}\frac{(1+\gamma_5)}{2}s + \ldots\Big] + h.c.$$

$$= -(1+\frac{\phi_0}{v})[m_u\bar{u}u + m_c\bar{c}c + \ldots + m_d\bar{d}d + m_s\bar{s}s + \ldots].$$

Remark: $\bar{q}_L = q_L^+\gamma^0 = q^+\frac{1}{2}(1-\gamma^5)\gamma^0 = \bar{q}\frac{1}{2}(1-\gamma^5)$

Note: $L^{phys}(f,H)$ conserves C and P separately, and therefore also CP.

Charged Current Interaction

$$X_C \equiv [W_\mu^1 - iW_\mu^2]\, \overline{q_L}\, \gamma^\mu q'_L \; + h.c. = [W_\mu^1 - iW_\mu^2]\, \overline{q_L^{phys}}\, \gamma^\mu U_L U_L'^\dagger q_L'^{phys} + h.c.$$

$$= [W_\mu^1 - iW_\mu^2]\, \overline{q_L^{phys}}\, \gamma^\mu V q_L'^{phys} \; + h.c.$$

$$\equiv [W_\mu^1 - iW_\mu^2]\, J_C^\mu \; + h.c. \qquad ($$

with $V \equiv U_L U_L'^{+}$ and $J_C^\mu \equiv \overline{(u,c,t)}_L\, \gamma^\mu \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix}$

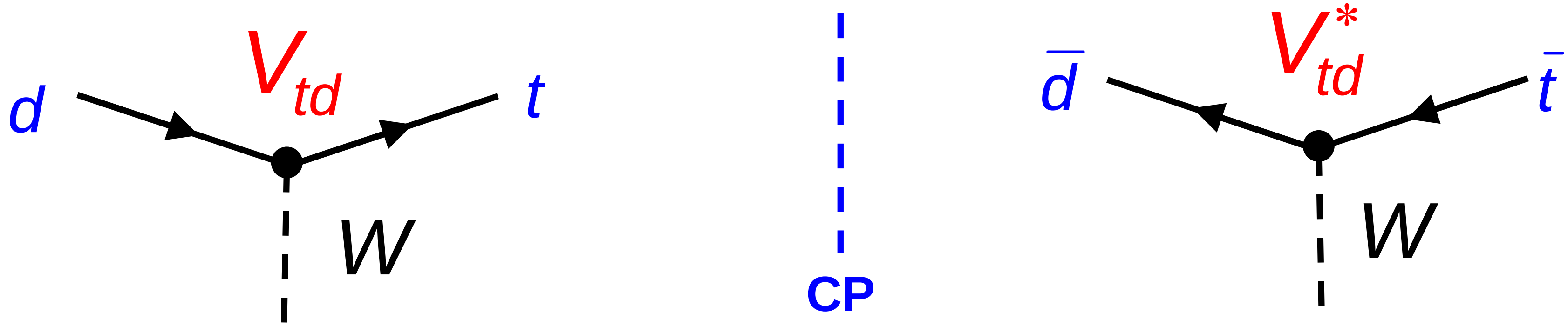
V is unitary

CP Violation and Mixing Matrix

X_C is maximally P and C violating and CP conservation requires V to be real

$$X_C = (W_\mu^1 - iW_\mu^2)\overline{q_j}\gamma^\mu V_{jk}(1-\gamma_5)q'_k + (W_\mu^1 + iW_\mu^2)\overline{q'_k}\gamma^\mu V^*_{jk}(1-\gamma_5)q_j$$

$$\xrightarrow{CP} (W_\mu^1 + iW_\mu^2)\overline{q'_k}\gamma^\mu V_{jk}(1-\gamma_5)q_j + (W_\mu^1 - iW_\mu^2)\overline{q_j}\gamma^\mu V^*_{jk}(1-\gamma_5)q'_k$$



CP Violation $\Rightarrow$ **Phase $\neq$ 0, π**

Parameters of the CKM Matrix

		N=3
Unitary $N \times N$ Matrix:	$\rightarrow$ N^2 Parameters:	9
	$N(N-1)/2$ Euler angles (rotation angles)	3
	Remaining parameters are phases:	6
	2N-1 are unmeasurable phase diff	5
	$(N-1)^2$ observable parameters	4

Unobservable Phases

Phases of left-handed fields in J_C are unobservable: possible redefinition

$$u_L \to e^{i\phi(u)} u_L \qquad c_L \to e^{i\phi(c)} c_L \qquad t_L \to e^{i\phi(t)} t_L$$

$$d_L \to e^{i\phi(d)} d_L \qquad s_L \to e^{i\phi(s)} s_L \qquad b_L \to e^{i\phi(b)} b_L$$

↑ Real numbers

Under phase transformation:

$$V \to \begin{pmatrix} e^{-i\phi(u)} & 0 & 0 \\ 0 & e^{-i\phi(c)} & 0 \\ 0 & 0 & e^{-i\phi(t)} \end{pmatrix} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} e^{i\phi(d)} & 0 & 0 \\ 0 & e^{i\phi(s)} & 0 \\ 0 & 0 & e^{i\phi(b)} \end{pmatrix}$$

$$V\alpha j \to \exp[i(\phi(j) - \phi(\alpha))] V\alpha j$$

$L^{phys}(f,G)$ invariant

$L(f,H)$ affected …. rephasing q_R

CP violation with 3 families

CKM Phase $\delta \neq 0, \pi \Leftrightarrow$ CP Violation ⇦ Not fully correct !

Assume that s and b quark have the same mass: extra symmetry

= rotations in s/b quark space

New s-quark: $s^{new} \sim V_{us} s + V_{ub} b$

➡ u-quark only couples two d and s^{new} but not to b^{new}

New mixing matrix:

(contructed from 1st row using unitarity)

$$\begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta\cos\phi & \cos\theta\cos\phi & \sin\phi \\ \sin\theta\sin\phi & -\cos\theta\sin\phi & \cos\phi \end{pmatrix}$$

⇨ **NO CP Violation**

Necessary (but not sufficient) conditions for CP Violation

$$m_u \neq m_c \quad m_u \neq m_t \quad m_c \neq m_t$$

$$m_d \neq m_s \quad m_d \neq m_b \quad m_s \neq m_b$$

CKM Parametrization

PDG choice

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

where $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$

CKM Parametrization

Lincon Wolfenstein 1983:

λ, A, ρ, η

→ hierarchy expressed by orders of λ **= sinθ_c≈ 0.22**

$$V_{CKM} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O\left(\lambda^4\right)$$

CKM Parametrization

PDG choice + Wolfenstein inspired parametrization

$s_{12} = \lambda \quad s_{23} = A\lambda^2$

$s_{13} \sin\delta_{13} = A\lambda^3\eta$

$s_{13} \cos\delta_{13} = A\lambda^3\rho$

$$\mathbf{V} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{1-A^2\lambda^4} & A\lambda^2 \\ 0 & -A\lambda^2 & \sqrt{1-A^2\lambda^4} \end{pmatrix} \cdot$$

$$\cdot \begin{pmatrix} \sqrt{1-A^2\lambda^6(\rho^2+\eta^2)} & 0 & A\lambda^3(\rho - \mathrm{i}\eta) \\ 0 & 1 & 0 \\ -A\lambda^3(\rho+\mathrm{i}\eta) & 0 & \sqrt{1-A^2\lambda^6(\rho^2+\eta^2)} \end{pmatrix} \cdot$$

$$\cdot \begin{pmatrix} \sqrt{1-\lambda^2} & \lambda & 0 \\ -\lambda & \sqrt{1-\lambda^2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1-\frac{\lambda^2}{2}-\frac{\lambda^4}{8} & \lambda & A\lambda^3(\rho-\mathrm{i}\eta) \\ -\lambda - A^2\lambda^5(\rho+\mathrm{i}\eta-\frac{1}{2}) & 1-\frac{\lambda^2}{2}-(\frac{1}{8}+\frac{A}{2})\lambda^4 & A\lambda^2 \\ A\lambda^3[1-(\rho+\mathrm{i}\eta)(1-\frac{\lambda^2}{2})] & -A\lambda^2 - A\lambda^4(\rho+\mathrm{i}\eta-\frac{1}{2}) & 1-\frac{1}{2}A^2\lambda^4 \end{pmatrix}$$

$$+\mathcal{O}(\lambda^6)$$

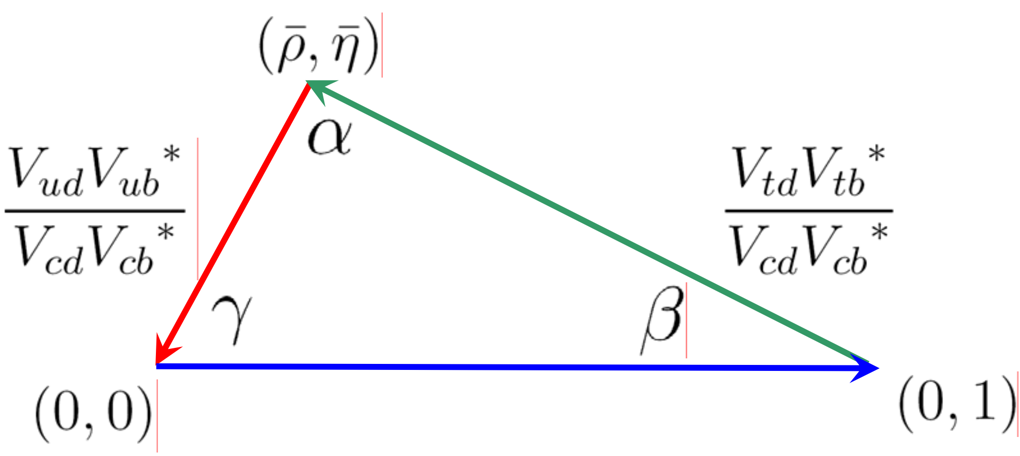
in O(λ^3) equal to Wolfenstein parametrization.

Unitarity Triangle „bd“ in O(λ^3)

$$\begin{pmatrix} V_{ud}^* & V_{cd}^* & V_{td}^* \\ V_{us}^* & V_{cs}^* & V_{ts}^* \\ V_{ub}^* & V_{cb}^* & V_{tb}^* \end{pmatrix} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\Rightarrow V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0$$

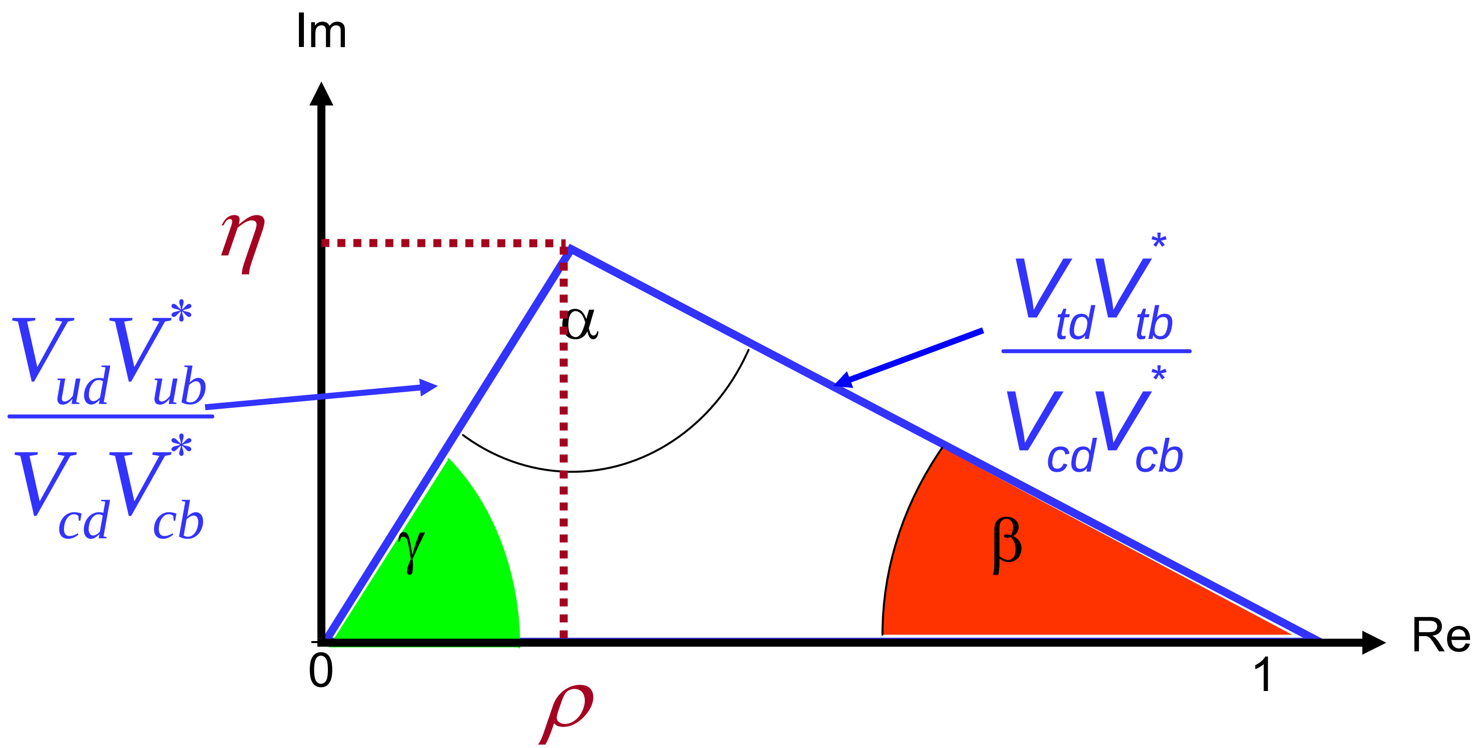
Unitaritätsdreieck



Angles

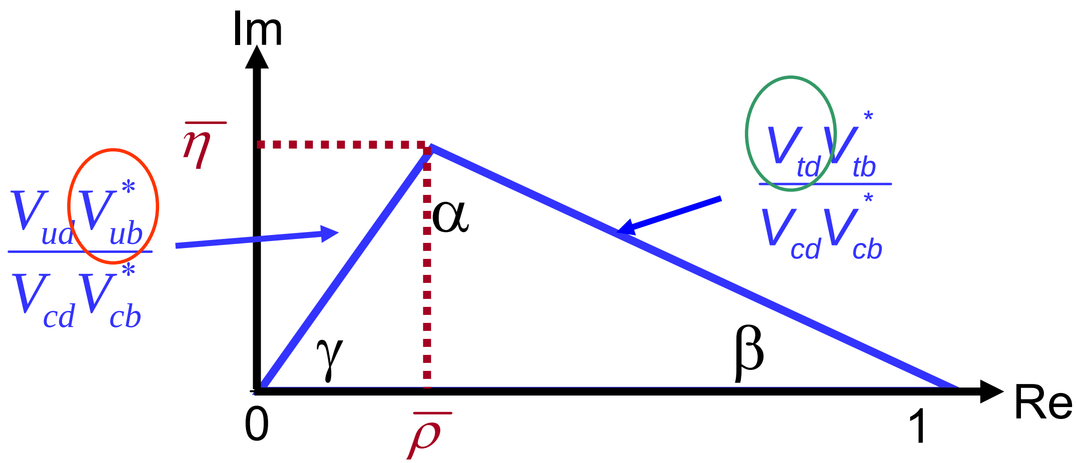
In complex plane: Angle α–β between two vectors $A = ae^{i\alpha}$ and $B = be^{i\beta}$

$$e^{i(\alpha-\beta)} = AB^* / |AB|$$



$$\gamma \equiv \arg\left[-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right] \qquad \alpha \equiv \arg\left[-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right] \qquad \beta \equiv \arg\left[-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right]$$

CKM Phases and Angles



$$V_{CKM} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

CKM Phasen

b→u

$$\begin{pmatrix} 1 & 1 & e^{-i\gamma} \\ 1 & 1 & 1 \\ e^{-i\beta} & 1 & 1 \end{pmatrix}$$

t→d

All triangles ...

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

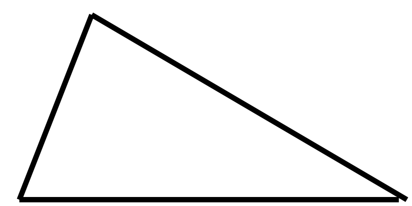
„bd“ $V_{ud}V_{ub}^{*} + V_{cd}V_{cb}^{*} + V_{td}V_{tb}^{*} = 0$

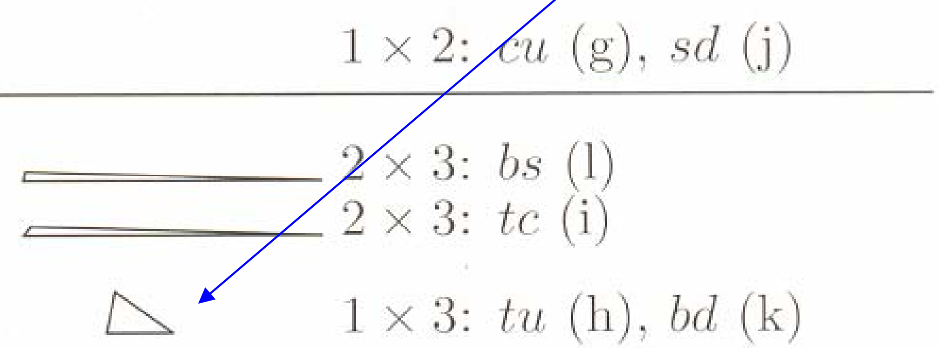
$A\lambda^3(\rho + i\eta) - A\lambda^3 + A\lambda^3(1-\rho-i\eta) = 0$

„tu“ $V_{ud}^{*}V_{td} + V_{us}^{*}V_{ts} + V_{ub}^{*}V_{tb} = 0$

$A\lambda^3(1-\rho-i\eta) - A\lambda^3 + A\lambda^3(\rho + i\eta) = 0$

$O(\lambda^3)$

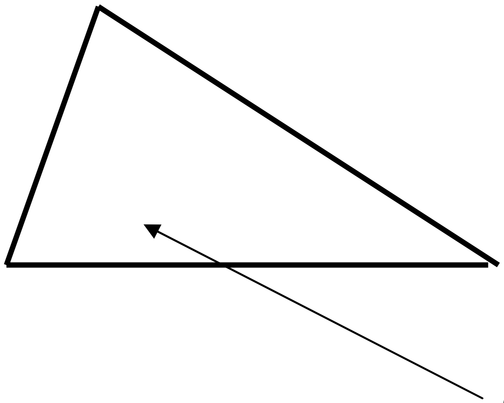




„The Unitarity Triangle“

Jarlskog Invariant

$$J = \pm \mathrm{Im} V_{ij} V_{kl} V_{il}^{*} V_{kj}^{*} \approx A^2 \lambda^6 \eta \approx 3 \cdot 10^{-5}$$



$$area = J/2$$

(all triangles)

$$\begin{pmatrix} V_{ud} & V_{us}^{*} & V_{ub} \\ V_{cd}^{*} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

CP Violation:

$$J \cdot (m_t - m_c)(m_t - m_u)(m_u - m_c)$$
$$\times (m_b - m_s)(m_b - m_d)(m_s - m_d) \neq 0$$

Triangles in Higher Order

„bd" $V_{ud}V_{ub}^{*} + V_{cd}V_{cb}^{*} + V_{td}V_{tb}^{*} = 0$

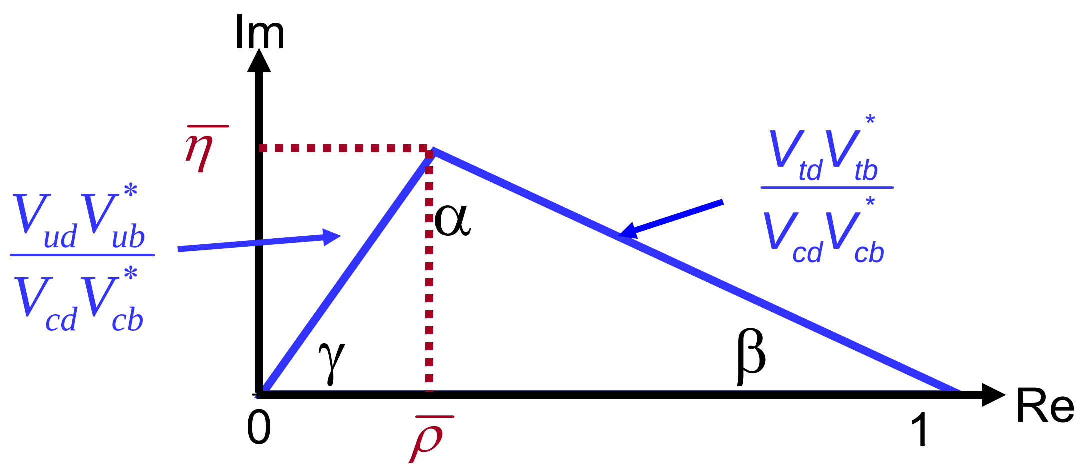
$$A\lambda^3(\rho + i\eta) - A\lambda^3 + A\lambda^3(1-\rho-i\eta) = 0 \qquad O(\lambda^3)$$

$$-\frac{1}{2}A\lambda^5(\rho+i\eta) \qquad +\frac{1}{2}A\lambda^5(\rho+i\eta) \qquad O(\lambda^5)$$

$$+O(\lambda^7)$$

$$\bar{\eta} = \eta\cdot(1-\frac{\lambda^2}{2}) \qquad \bar{\rho} = \rho\cdot(1-\frac{\lambda^2}{2})$$

$$A\lambda^3(\bar{\rho}+i\bar{\eta}) - A\lambda^3 + A\lambda^3(1-\bar{\rho}-i\bar{\eta}) = 0 + O(\lambda^7)$$

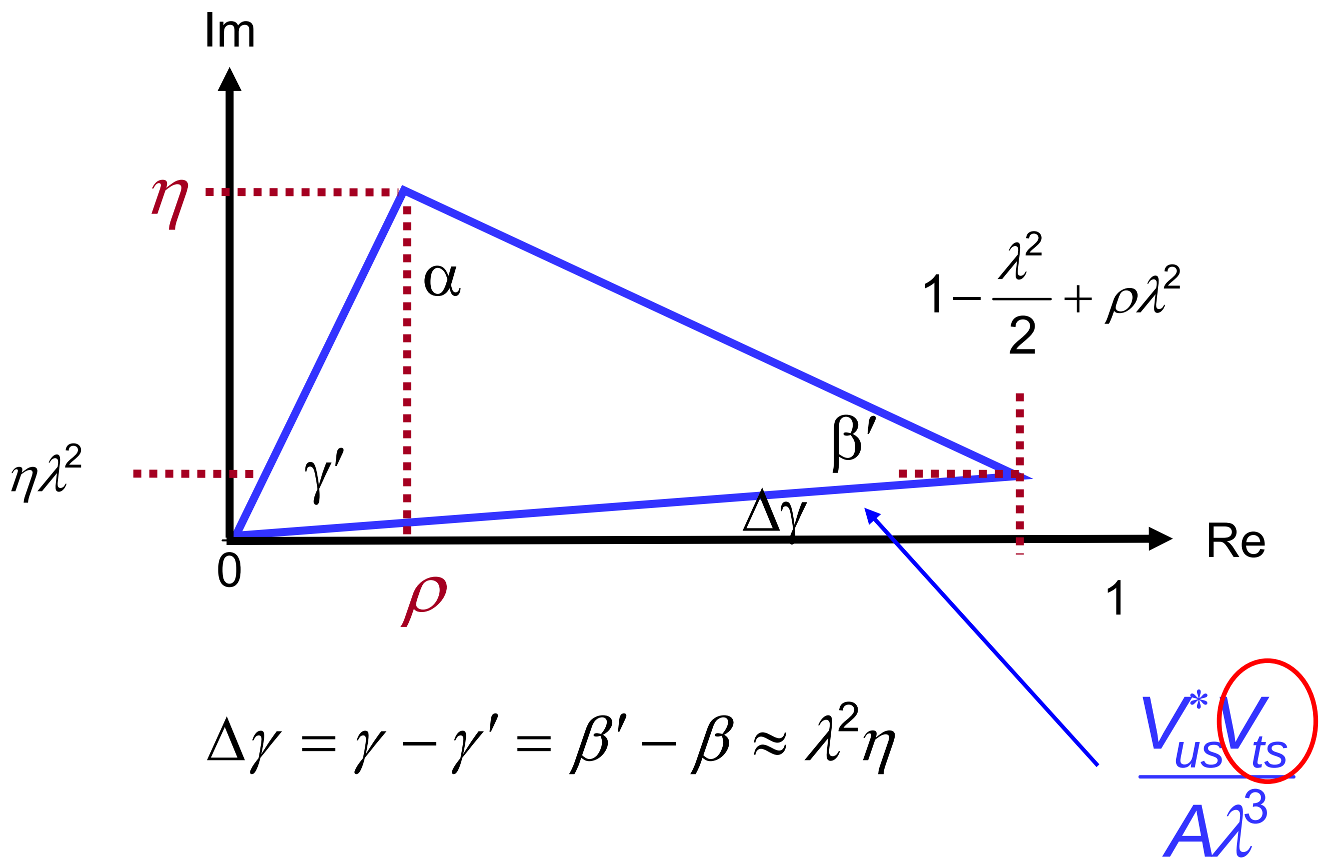


„tu“ $V_{ud}^{*} V_{td} + V_{us}^{*} V_{ts} + V_{ub}^{*} V_{tb} = 0$

$$A\lambda^3(1-\rho-i\eta) \quad - A\lambda^3 \quad + A\lambda^3(\rho+i\eta) = 0 \qquad O(\lambda^3)$$

$$-A\lambda^5\left(\frac{1}{2}-(\rho+i\eta)\right) \quad + A\lambda^5\left(\frac{1}{2}-(\rho+i\eta)\right) \qquad O(\lambda^5)$$

$$+O(\lambda^7)$$



$$\Delta\gamma = \gamma - \gamma' = \beta' - \beta \approx \lambda^2\eta$$

Standard Modell

1. CP is explicitly broken
2. There is a single source of CP violation: $\delta_{\mathbf{CKM}}$
3. CP violation appears only in charged current interactions of quarks
4. CP violation is closely related to flavor changing interactions

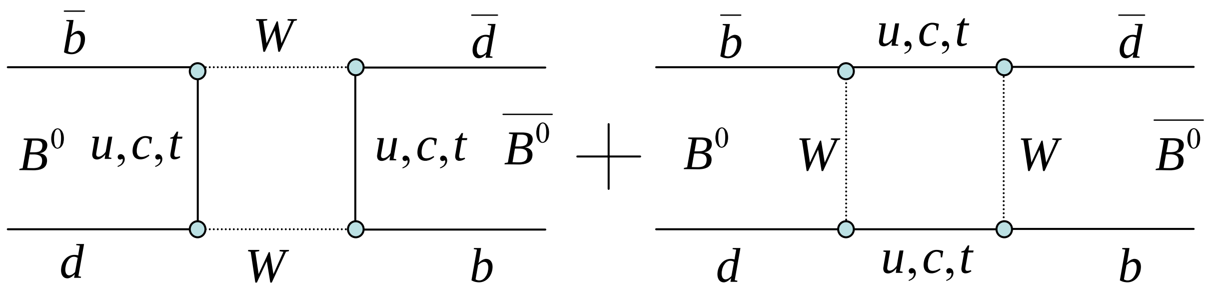
Mixing of neutral mesons

Neutral mesons: $\left|P^0\right\rangle$: $K^0 = \left|d\bar{s}\right\rangle$ $D^0 = \left|\bar{u}c\right\rangle$ $B_d^0 = \left|d\bar{b}\right\rangle$ $B_s^0 = \left|s\bar{b}\right\rangle$

$\left|\overline{P^0}\right\rangle$: $\overline{K^0} = \left|\bar{d}s\right\rangle$ $\overline{D^0} = \left|\bar{u}c\right\rangle$ $\overline{B_d^0} = \left|d\bar{b}\right\rangle$ $\overline{B_s^0} = \left|s\bar{b}\right\rangle$

Standard Model predicts oscillations of neutral Mesons:

$$B_d^0 \Leftrightarrow \overline{B_d^0}$$



Transition can be described by matrix element: $\left\langle \overline{B_d^0} \middle| H_W \middle| B_d^0 \right\rangle$

Phenomenological description of mixing

Schrödinger equation for unstable meson:

$$i\frac{d}{dt}|\psi\rangle = H|\psi\rangle = \left(m - \frac{i}{2}\Gamma\right)|\psi\rangle \quad \Rightarrow \quad \begin{cases} |\psi(t)\rangle = |\psi_0\rangle \cdot e^{-imt} \cdot e^{-\frac{1}{2}\Gamma t} \\ \left\||\psi(t)\rangle\right\|^2 = \left\||\psi_0\rangle\right\|^2 \cdot e^{-\Gamma t} \end{cases}$$

For neutral mesons $(K^0, \overline{K}^0), (D^0, \overline{D}^0), (B^0, \overline{B}^0), (B_s^0, \overline{B}_s^0)$ consider 2 compon.

$$i\frac{d}{dt}\begin{pmatrix} P^0 \\ \overline{P}^0 \end{pmatrix} = \mathbf{H}\begin{pmatrix} P^0 \\ \overline{P}^0 \end{pmatrix} = \begin{pmatrix} H_{11} & H_{21} \\ H_{12} & H_{22} \end{pmatrix}\begin{pmatrix} P^0 \\ \overline{P}^0 \end{pmatrix} = \left(\mathbf{M} - \frac{i}{2}\Gamma\right)\begin{pmatrix} P^0 \\ \overline{P}^0 \end{pmatrix} = \begin{pmatrix} m_{11} - \frac{i}{2}\Gamma_{11} & m_{12}^* - \frac{i}{2}\Gamma_{12}^* \\ m_{12} - \frac{i}{2}\Gamma_{12} & m_{22} - \frac{i}{2}\Gamma_{22} \end{pmatrix}\begin{pmatrix} P^0 \\ \overline{P}^0 \end{pmatrix}$$

$\langle B^0|\mathbf{H}_W|\overline{B^0}\rangle$

CP $\Rightarrow H_{12} = H_{21}$

CPT $\Rightarrow$ $H_{11} = H_{22} = H$, $m_{11} = m_{22} = m$, $\Gamma_{11} = \Gamma_{22} = \Gamma$

M and **Γ** hermitian:

$m_{21} = m_{12}^*$

$\Gamma_{21} = \Gamma_{12}^*$

Phase $e^{i\phi_s}$

Mass eigenstates (by diagonalizing matrix)

Heavy and light mass eigenstate:

$$|P_L\rangle = p|P^0\rangle + q|\overline{P^0}\rangle \quad \text{with } m_L, \Gamma_L$$

$$|P_H\rangle = p|P^0\rangle - q|\overline{P^0}\rangle \quad \text{with } m_H, \Gamma_H$$

$|p|^2 + |q|^2 = 1$ complex coefficients

$$|P^0\rangle = \frac{1}{2p}(|P_L\rangle + |P_H\rangle)$$

$$|\overline{P}^0\rangle = \frac{1}{2q}(|P_L\rangle - |P_H\rangle)$$

Parameters of the mass states

$$m_{H,L} = m \pm \mathrm{Re}\sqrt{H_{12}H_{21}}$$

$$\Gamma_{H,L} = \Gamma \mp 2\mathrm{Im}\sqrt{H_{12}H_{21}}$$

$$\Delta m = m_H - m_L = 2\mathrm{Re}\sqrt{H_{12}H_{21}}$$

$$\Delta\Gamma = \Gamma_H - \Gamma_L = -4\,\mathrm{Im}\sqrt{H_{12}H_{21}}$$

$$x \equiv \frac{\Delta m}{\Gamma} \quad \text{und} \quad y \equiv \frac{\Delta\Gamma}{2\Gamma}$$

Neutral B mesons

$$\Delta m_d = 0.502 \pm 0.007\ \mathrm{ps}^{-1}\ \text{(PDG)}$$

$$\Delta m_s > 14.4\ \mathrm{ps}^{-1}\ \text{(PDG)}$$

$$\Delta\Gamma / \Gamma < 0.07\ (90\%\ CL)$$

$$\Delta\Gamma_s / \Gamma_s = 0.09 \pm 0.64\ \text{(Moriond07)}$$

Time evolution

Generic particle (before P)

$$\left|B_{H,L},t\right\rangle = b_{H,L}(t)\left|B_{H,L}\right\rangle \quad \text{mit} \quad b_{H,L}(t) = e^{-\Gamma_{H,L}t}e^{-im_{H,L}t}$$

$$\left|\psi_{B^0}(t)\right\rangle = \frac{\left|B_L,t\right\rangle + \left|B_{H,t}\right\rangle}{2p} = \frac{1}{2p}\left(b_L(t)\cdot\left(p\left|B^0\right\rangle + q\left|\overline{B^0}\right\rangle\right) + b_H(t)\cdot\left(p\left|B^0\right\rangle - q\left|\overline{B^0}\right\rangle\right)\right)$$

$$= f_+(t)\cdot\left|B^0\right\rangle - \frac{q}{p}f_-(t)\cdot\left|\overline{B^0}\right\rangle$$

$$f_\pm(t) = \frac{1}{2}\cdot\left[e^{-im_H t}e^{-\Gamma_H t/2} \pm e^{-im_L t}e^{-\Gamma_L t/2}\right]$$

$$\left|\psi_{\overline{B}^0}(t)\right\rangle = f_+(t)\cdot\left|\overline{B}^0\right\rangle + \frac{p}{q}f_-(t)\cdot\left|B^0\right\rangle$$

B^0

$$P\left(B^0 \to B^0, t\right) = \left|f_+(t)\right|^2$$

$$P\left(B^0 \to \overline{B}^0, t\right) = \left|\frac{q}{p}\right|^2\left|f_-(t)\right|^2$$

$\overline{B}^0$

$$P\left(\overline{B}^0 \to \overline{B}^0, t\right) = \left|f_+(t)\right|^2$$

$$P\left(\overline{B}^0 \to B^0, t\right) = \left|\frac{p}{q}\right|^2\left|f_-(t)\right|^2$$

Mixing of neutral mesons

Time evolution of rates:

$$\underbrace{P(B^0 \to B^0) = P(\overline{B^0} \to \overline{B^0})}_{\text{CPT}} = \frac{1}{4}\left[e^{-\Gamma_L t} + e^{-\Gamma_H t} + 2e^{-(\Gamma_L + \Gamma_H)t/2} \cos \Delta m t\right]$$

$$P(B^0 \to \overline{B^0}) = \frac{1}{4}\left|\frac{q}{p}\right|^2 \left[e^{-\Gamma_L t} + e^{-\Gamma_H t} - 2e^{-(\Gamma_L + \Gamma_H)t/2} \cos \Delta m t\right]$$

$$P(\overline{B^0} \to B^0) = \frac{1}{4}\left|\frac{p}{q}\right|^2 \left[e^{-\Gamma_L t} + e^{-\Gamma_H t} - 2e^{-(\Gamma_L + \Gamma_H)t/2} \cos \Delta m t\right]$$

CP, T- violation in mixing: $P(B^0 \to \overline{B^0}) \neq P(\overline{B^0} \to B^0) \Rightarrow \left|\frac{q}{p}\right| \neq 1$

Mixing Mechanism

$$\mathbf{H} = \begin{pmatrix} m - \frac{i}{2}\Gamma & -\frac{\Delta m - \frac{i}{2}\Delta\Gamma}{2\eta_m} \\ -\eta_m \frac{\Delta m - \frac{i}{2}\Delta\Gamma}{2} & m - \frac{i}{2}\Gamma \end{pmatrix} = \begin{pmatrix} m - \frac{i}{2}\Gamma & -\frac{\Gamma}{2}\frac{1}{\eta_m}(x - iy) \\ -\frac{\Gamma}{2}\eta_m (x - iy) & m - \frac{i}{2}\Gamma \end{pmatrix}$$

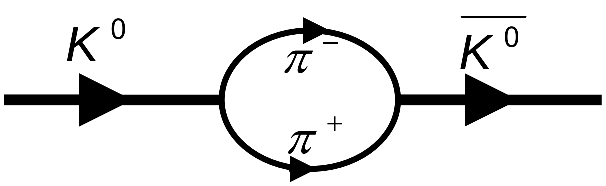
dispersive (Δm) absorbtive ($\Delta\Gamma$)

If CP conserved: $\eta_m = e^{-i\phi_\eta}$

<u>Two mixing mechanisms:</u>

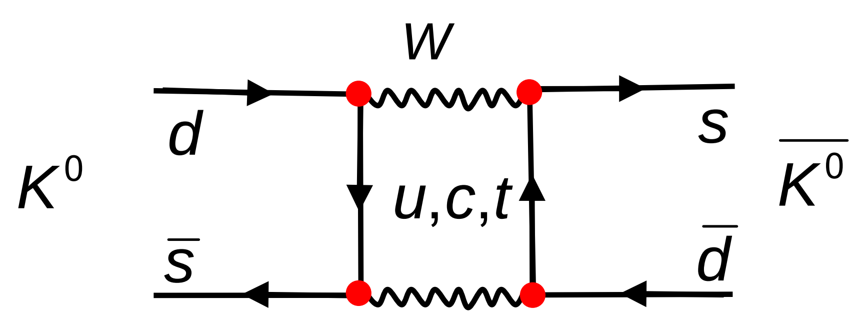
- Mixing through decays → $y = \frac{\Delta\Gamma}{2\Gamma} \approx O(1)$
- Mixing through oscillation → $x = \frac{\Delta m}{\Gamma} \approx O(1)$

$(K^0, \overline{K}^0), (D^0, \overline{D}^0), (B^0, \overline{B}^0), (B_s^0, \overline{B}_s^0)$ show different oscillation behavior



„long distant, on-shell states“

For K^0 important, for B^0 negligible



„short distant, virtual states“

Table 1. Parameters of the four neutral oscillating meson pairs [9].

	$K^0/\overline{K}^0$	$D^0/\overline{D}^0$	$B^0/\overline{B}^0$	$B_s/\overline{B}_s$
τ [ps]	89.4 ± 0.1; 51700 ± 400	$0.413 \pm .003$	1.548 ± 0.021	1.49 ± 0.06
Γ [s^{-1}]	$5.61 \cdot 10^9$	$2.4 \cdot 10^{12}$	$(6.41 \pm 0.16) \cdot 10^{11}$	$(6.7 \pm 0.3) \cdot 10^{11}$
$y = \frac{\Delta\Gamma}{2\Gamma}$	-0.9966	$\|y\| < 0.06$	$\|y\| \lesssim 0.01^*$	$-(0.01 \ldots 0.10)^*$
Δm [s^{-1}]	$(5.300 \pm 0.012) \cdot 10^9$	$< 7 \cdot 10^{10}$	$(4.89 \pm 0.09) \cdot 10^{11}$	$> 15 \cdot 10^{12}$
Δm [eV]	$(3.49 \pm 0.01) \cdot 10^{-6}$	$< 5 \cdot 10^{-6}$	$(3.2 \pm 0.1) \cdot 10^{-4}$	$> 1.0 \cdot 10^{-2}$
$x = \frac{\Delta m}{\Gamma}$	0.945 ± 0.002	< 0.03	0.76 ± 0.02	$21 \ldots 40^*$
δ_ϵ	$(3.27 \pm 0.12) \cdot 10^{-3}$		$\sim -10^{-3}\,*$	$\|\delta_\epsilon\| < 10^{-3}\,*$
$\|\eta_m\|^2$	0.99348 ± 0.00024	$\approx 1\,*$	$1 \ldots 1.002^*$	$\approx 1\,*$

* Standard Model expectation [39]

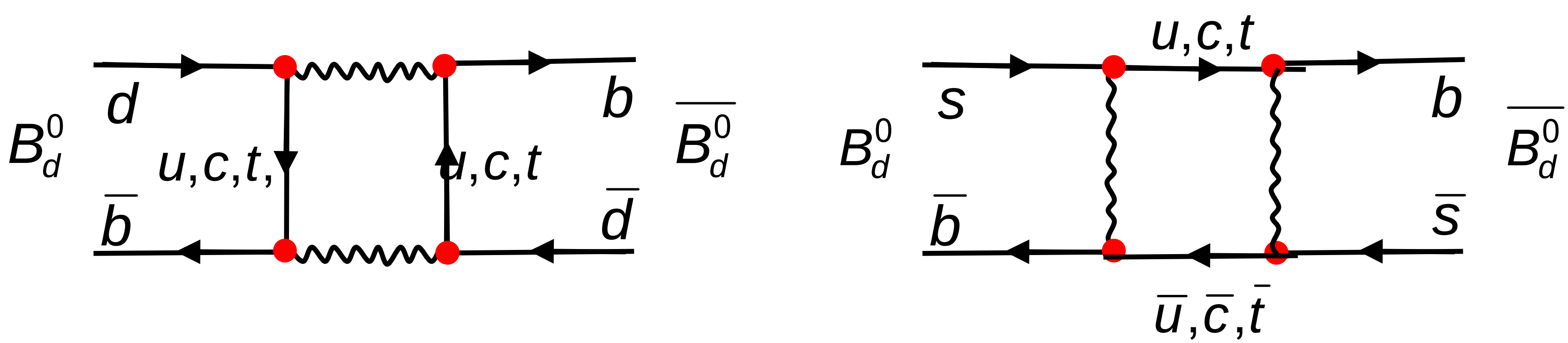
$B^0_{(d,s)}$ Mixing

Mixing mechanisms:

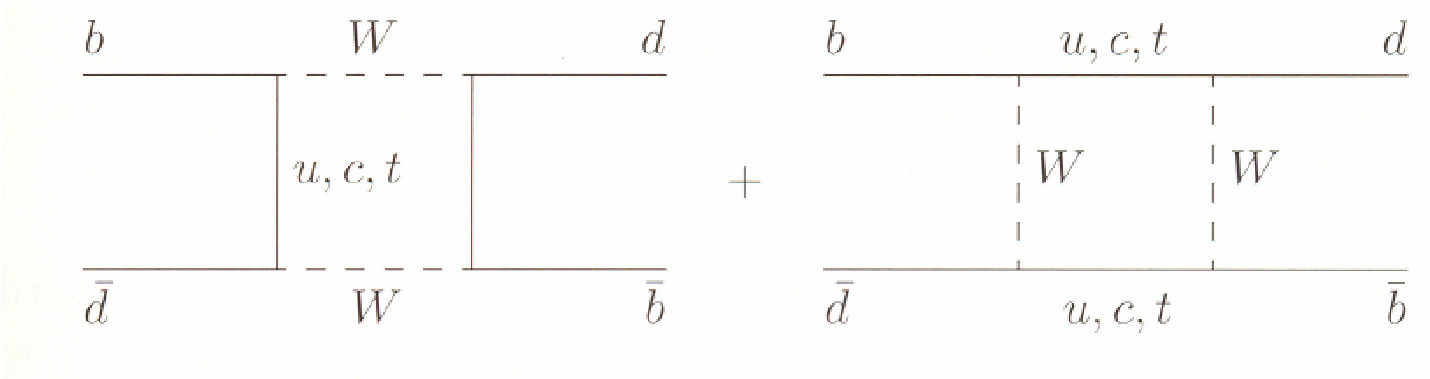
- **Mixing through decay:** many possible hadronic decays $\rightarrow \Gamma$ is large
  in addition decays like $B \rightarrow \pi\pi$ are suppressed

$$\Rightarrow y = \frac{\Delta\Gamma}{2\Gamma} \quad \text{is small} = \begin{cases} \approx 0 \text{ for } B^0_d \\ \approx O(0.1) \text{ for } B^0_s \end{cases} \quad \Rightarrow \text{don't expect mixing via decay}$$

- **Mixing through oscillation** $\rightarrow$ Significant contribution only from top loop



Matrix Element



$$
\begin{aligned}
\mathcal{M} \sim\ & V_{tb}V_{td}^{*}\Pi_{t} \cdot V_{tb}V_{td}^{*}\Pi_{t} \\
&+ V_{cb}V_{cd}^{*}\Pi_{c} \cdot V_{tb}V_{td}^{*}\Pi_{t} \\
&+ V_{ub}V_{ud}^{*}\Pi_{u} \cdot V_{tb}V_{td}^{*}\Pi_{t} \\
&+ V_{tb}V_{td}^{*}\Pi_{t} \cdot V_{cb}V_{cd}^{*}\Pi_{c} \\
&+ V_{tb}V_{td}^{*}\Pi_{t} \cdot V_{ub}V_{ud}^{*}\Pi_{u} \\
&+ V_{cb}V_{cd}^{*}\Pi_{c} \cdot V_{cb}V_{cd}^{*}\Pi_{c} \\
&+ V_{ub}V_{ud}^{*}\Pi_{c} \cdot V_{ub}V_{ud}^{*}\Pi_{u} \\
&+ V_{cb}V_{cd}^{*}\Pi_{c} \cdot V_{ub}V_{ud}^{*}\Pi_{u} \\
&+ V_{ub}V_{ud}^{*}\Pi_{u} \cdot V_{cb}V_{cd}^{*}\Pi_{c}
\end{aligned}
$$

Π = *Propagator*

Light Quark Contribution

For $m_u = m_c \approx 0$: $\quad \Pi_u = \Pi_c = \Pi_0$ (similar to GIM)

$$\begin{aligned}
\mathcal{M} \sim\ & V_{tb}V_{td}^* \Pi_t \cdot V_{tb}V_{td}^* \Pi_t \\
& + (V_{cb}V_{cd}^* + V_{ub}V_{ud}^*)\Pi_0 \cdot V_{tb}V_{td}^* \Pi_t \\
& + V_{tb}V_{td}^* \Pi_t \cdot (V_{cb}V_{cd}^* + V_{ub}V_{ud}^*)\Pi_0 \\
& + (V_{cb}V_{cd}^* + V_{ub}V_{ud}^*)\Pi_0 \cdot (V_{cb}V_{cd}^* + V_{ub}V_{ud}^*)\Pi_0
\end{aligned}$$

Using unitarity: $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$

$$\begin{aligned}
\mathcal{M} &\sim (V_{td}V_{tb}^*)^2 [\Pi_t\Pi_t - \Pi_0\Pi_t - \Pi_t\Pi_0 + \Pi_0\Pi_0] \\
&= (V_{td}V_{tb}^*)^2 \underbrace{[\Pi_t - \Pi_0][\Pi_t - \Pi_0]}
\end{aligned}$$

Corresponds to single matrix element w/ inner fermion „t – u “

$$\Delta m = 2|m_{12}|$$

$$H_{12} = \left\langle B^0 \left| \mathbf{H} \right| \overline{B}^0 \right\rangle$$

$$\approx m_{12} = -\frac{G_F^2}{12\pi^2} e^{-i\phi_{CPB}} V_{tb}^2 V_{td}^{*2} m_W^2 m_B [f_B^2 B_B] \cdot \underbrace{[S(m_t^2 / m_W^2)\eta_{QCD}]}_{\text{Electroweak and QCD corrections}}$$

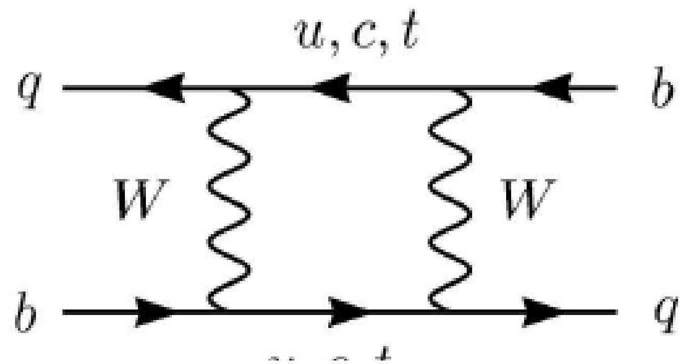
CP Phase introduced while eval. hadronic part of matrix element

Electroweak and QCD corrections

Hadronic part of matrix element:

$$\langle B^0 | J_\mu J^\mu | \overline{B}^0 \rangle = \sum_X \langle B^0 | J_\mu | X \rangle \langle X | J^\mu | \overline{B}^0 \rangle$$
$$= B_B \cdot \langle B^0 | J_\mu | 0 \rangle \langle 0 | J^\mu | \overline{B}^0 \rangle = B_B f_B^2 p_\mu p^\mu$$

$M_{12}^{\rm SM} \; =$



- $\sqrt{\hat{B}_{B_s}} f_{B_s} = (0.281 \pm 0.021)\,{\rm GeV}$ (HPQCD 06, unquenched, $N_f = 2+1$ staggered light quarks)

- $\sqrt{\hat{B}_{B_s}} f_{B_s} = (0.245 \pm 0.021^{+0.003}_{-0.002})\,{\rm GeV}$ (JLQCD 03, unquenched, $N_f = 2$ Wilson light quarks)

- $S_0($
- $\hat{\eta}^B$ = 0.552: NLO QCD correction (Buras/Jamin/Weiss '90)
- $\hat{B}_{B_s} f_{B_s}^2 \propto \langle B_s^0 | (\bar{s}b)_{V-A} (\bar{s}b)_{V-A} | \bar{B}_s^0 \rangle$: hadronic matrix element, from lattice
- $V_{ts}^* V_{tb}$: from tree-level processes

Patricia Ball, Moriond 2007

$$\frac{\Delta m_s}{\Delta m_d} = \frac{|V_{ts}|^2}{|V_{td}|^2} \xi^2 \frac{m_{B_s}}{m_{B_d}} \quad \text{with} \quad \xi = \frac{f_{B_s}\sqrt{B_{B_s}}}{f_{B_d}\sqrt{B_{B_d}}}$$

~ 26

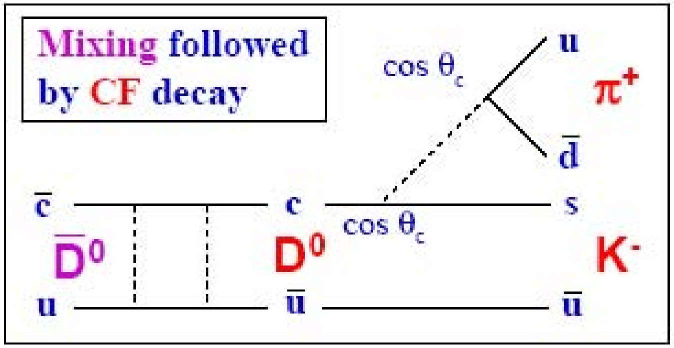
$$\xi = 1.24 \pm 0.07$$

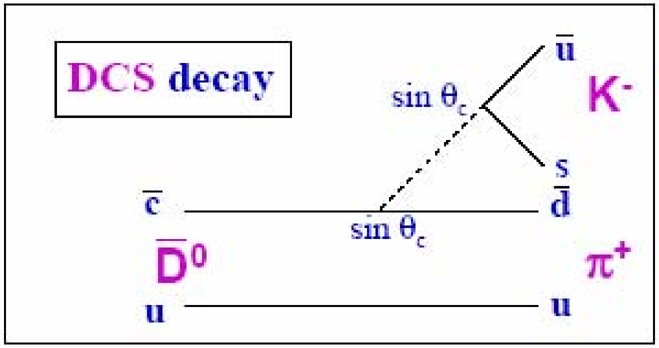
$$\Delta m_d = 0.506 \pm 0.006 \pm 0.004 \text{ ps}^{-1}$$

$$\Delta m_s = 17.77 \pm 0.10 (\text{stat.}) \pm 0.07 (\text{syst.}) \text{ ps}^{-1}$$

X 35

D^0 Mixing Formalism





- Right-sign (RS) CF decay
- Wrong-sign (WS) decays
  - mixing, DCS diagrams

- Mixing implies that the weak eigenstates are not pure flavor states →

$$|D_{1,2}\rangle = p|D^0\rangle \pm q|\overline{D}^0\rangle, \quad |p|^2 + |q|^2 = 1$$

- Charm mixing values typically quoted using scaled parameters *x*, *y* →

$$x = \frac{\Delta M}{\Gamma}, \quad y = \frac{\Delta\Gamma}{2\Gamma} \qquad \Gamma = \tfrac{1}{2}(\Gamma_2 + \Gamma_1) \quad \Delta M = M_2 - M_1 \quad \Delta\Gamma = \Gamma_2 - \Gamma_1$$

Time Dependence of Mixed Final States

- For |x|, |y| << 1, time-dependence of a hadronic final state with mixing and DCS (R_D) amplitudes →

$$\frac{\Gamma_{WS}(t)}{\Gamma_{RS}(t)} = R_D + y'\sqrt{R_D}\,\Gamma t + \frac{x'^2 + y'^2}{4}(\Gamma t)^2$$

in the limit of no CP violation, and where

$$x' = x\cos\delta_{K\pi} + y\sin\delta_{K\pi}\,, \quad y' = y\cos\delta_{K\pi} - x\sin\delta_{K\pi}$$

with $\delta_{K\pi}$ being the relative strong phase between DCS and CF amplitudes

- Time-integrated mixing rate →

$$R_M = \frac{x^2 + y^2}{2} = \frac{x'^2 + y'^2}{2}$$

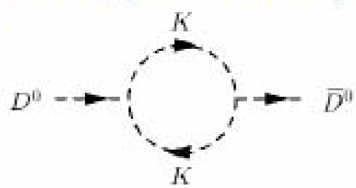
- If CP is not conserved, the time distribution for D^0 and $\overline{D}^0$ can differ

$$\frac{\Gamma^{\pm}_{WS}(t)}{e^{-\Gamma t}} = R^{\pm}_D + y'^{\pm}\sqrt{R^{\pm}_D}(\Gamma t) + \frac{x'^{\pm 2} + y'^{\pm 2}}{4}(\Gamma t)^2$$

Charm Mixing Predictions

Standard Model

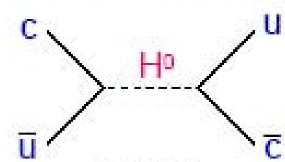
- Box diagram SM charm mixing rate naively expected to be very low (R_M~10^{-10}) (Datta & Kumbhakar)
  - Z.Phys. C27, 515 (1985)
  - **CKM suppression → $|V_{ub}V^*_{cb}|^2$**
  - **GIM suppression → $(m^2_s - m^2_d)/m^2_W$**
  - Di-penguin mixing, R_M~10^{-10}
    - **Phys. Rev. D 56, 1685 (1997)**
- Enhanced rate SM predictions generally due to long-distance ***y*** contributions:



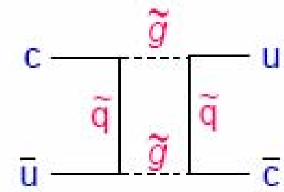
- Recent SM predictions can accommodate high mixing rate (Falk *et al.*)
  - ***x*,*y*** ≈ $\sin^2 \theta_C$ x [SU(3) breaking]^2 ~1%
    - ***y***: Phys.Rev. D 65, 054034 (2002)
    - ***x***: Phys.Rev. D 69, 114021 (2004)

New Physics

- Possible enhancements to mixing due to new particles and interactions in new physics models
- Most new physics predictions for ***x***
  - Extended Higgs, tree-level FCNC
  - Fourth generation down-type quarks
  - Supersymmetry: gluinos, squarks
  - Lepto-quarks



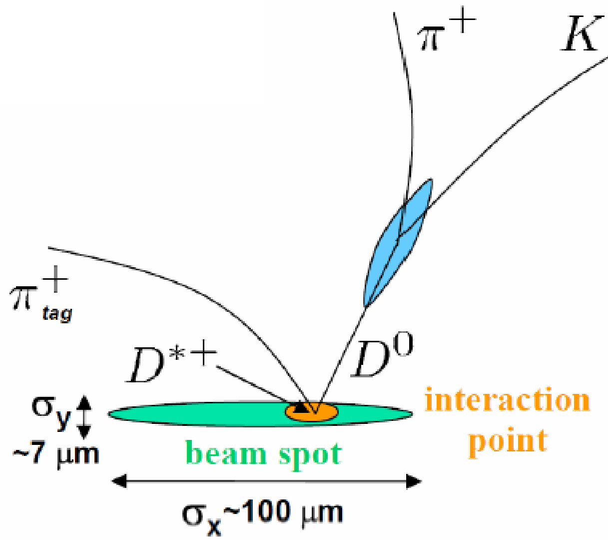
FCNC

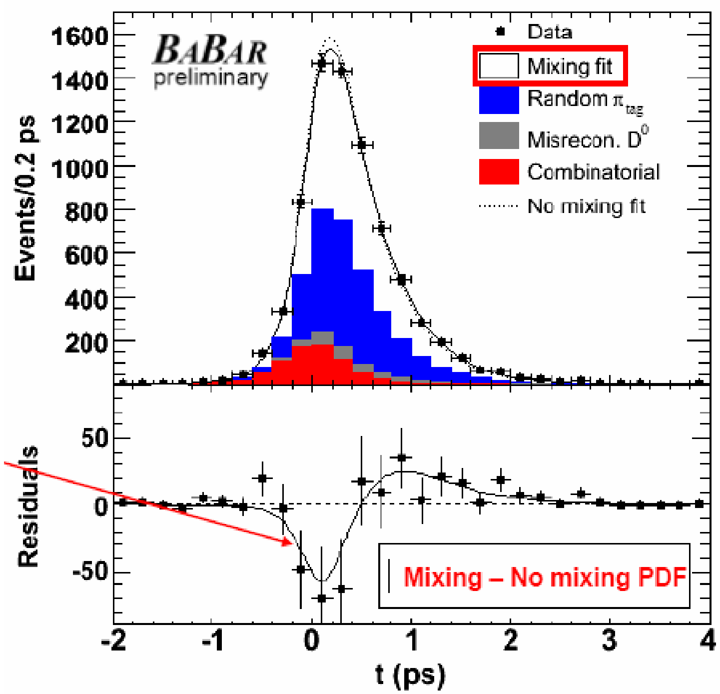


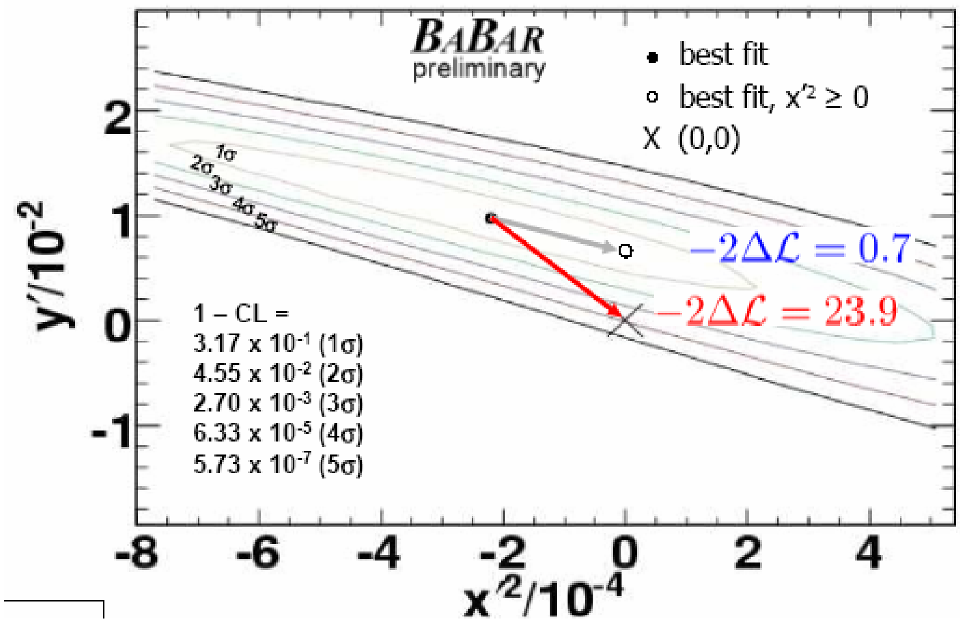
supersymmetry

- Large possible SM contributions to mixing require observation of either a CP-violating signal or | ***x*** | >> | ***y*** | to establish presence of NP
  - Ann.Rev.Nucl.Part.Sci 53 431-499 (2003)

$$D^{*+} \rightarrow D^0(K^-\pi^+)\pi^+$$





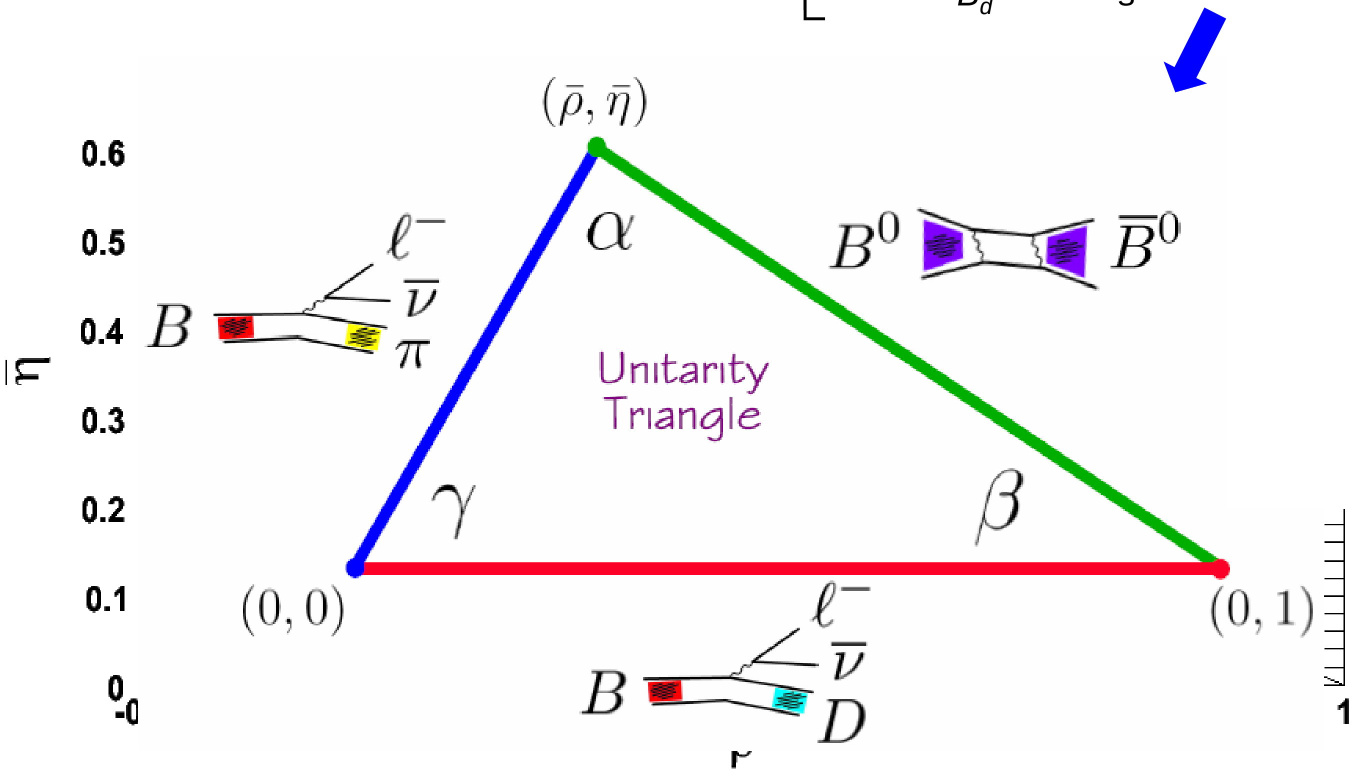


R_D: $(3.03 \pm 0.16 \pm 0.06) \times 10^{-3}$

x'^2: $(-0.22 \pm 0.30 \pm 0.20) \times 10^{-3}$

y': $(9.7 \pm 4.4 \pm 2.9) \times 10^{-3}$

$$R_t = \left[\xi^2 \frac{m_{B_s}}{m_{B_d}} \frac{\Delta m_d}{\Delta m_s} \frac{1-\lambda^2(1-2\overline{\rho})}{\lambda^2} \right]^{\frac{1}{2}}$$



Status aufgrund CP erhaltender Messgrößen

Couplings in electroweak Theorie

	I	II	III	T	T_3	Q	Y	
Leptonen	$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L$	$\begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L$	$\begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L$	$\frac{1}{2}$ $\frac{1}{2}$	$+\frac{1}{2}$ $-\frac{1}{2}$	0 -1	-1 -1	LH Dubletts
	e_R^-	μ_R^-	τ_R^-	0	0	-1	-2	RH Singletts
Quarks	$\begin{pmatrix} u \\ d' \end{pmatrix}_L$	$\begin{pmatrix} c \\ s' \end{pmatrix}_L$	$\begin{pmatrix} t \\ b' \end{pmatrix}_L$	$\frac{1}{2}$ $\frac{1}{2}$	$+\frac{1}{2}$ $-\frac{1}{2}$	$\frac{2}{3}$ $-\frac{1}{3}$	$\frac{1}{3}$ $\frac{1}{3}$	LH Dubletts
	u_R d_R	c_R s_R	t_R b_R	0 0	0 0	$\frac{2}{3}$ $-\frac{1}{3}$	$\frac{4}{3}$ $-\frac{2}{3}$	RH Singletts

1. Photonkopplung an Fermionen (an LH und RH Fermionen gleich) $\sim eQ_f$

2. W-Kopplung an Fermionen (nur an LH Fermionen)

$$\sim T_3 \cdot g = T_3 \cdot \frac{e}{\sin\theta_W}$$

3. Z-Kopplung an Fermionen (an LH und RH Fermionen verschieden)

$$\mathrm{LH}: \; g_L \cdot \frac{g}{\cos\theta_W} \quad \text{mit} \quad g_L = T_3^f - Q_f \sin^2\theta_W$$

$$\mathrm{RH}: \; g_R \cdot \frac{g}{\cos\theta_W} \quad \text{mit} \quad g_R = \quad - Q_f \sin^2\theta_W$$

Beispiel:

$\nu: \quad g_L = +\frac{1}{2} \quad g_R = 0$

$e: \quad g_L = -0.27 \quad g_R = +0.23$

Häufig werden statt g_l und g_R auch die Vektor und Axialvektorkopplungen verwendet:

$$g_V = g_L + g_R \qquad g_A = g_L - g_R$$