Introduction to OPE / Factorization / EFT

(and its applications in B-meson decays)

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Theoretische Physik 1

UNIVERSITÄT SIEGEN

DFG FOR 1873
quark flavour physics and effective field theories

Disclaimer:

The dynamics of strong and weak interactions in B-decays is very complex and has many faces . . .

. . . I will not be able to cover everything, . . .

. . . but I hope that some theoretical and phenomenological concepts become clearer.

. . . Some introductory remarks . . .

- Physical processes involve Different typical Energy/Length Scales:

$\Rightarrow$ Short-distance Dynamics vs. Long-distance Dynamics

- e.g. for *b*-decays:

New physics	:	$\delta x \sim 1/\Lambda_{\rm NP}$
Electroweak interactions	:	$\delta x \sim 1/M_W$
Short-distance QCD(QED) corrections	:	$\delta x \sim 1/M_W \to 1/m_b$
Hadronic effects	:	$\delta x < 1/m_b$

$\rightarrow$ Sequence of Effective Field Theories (EFT)

$\rightarrow$ Perturbative and Non-Perturbative Strong Interaction Effects
($\mapsto$ renormalization-group improved perturbation theory)

$\rightarrow$ Definition of Hadronic Input Parameters (Functions)

Central Notions / Theoretical Jargon

- **Factorization:**
  1. Separation of Scales in (RG-improved) Perturbation Theory
  2. Simplification of Exclusive Hadronic Matrix Elements

- **Operator-Product Expansion (OPE):**
  Short-distance expansion ($x \to 0$) of time-ordered operator products, corresponding to $|q^2| \to \infty$ in Fourier transform:

$$\int d^4x\, e^{iq\cdot x}\, T\left(\phi(x)\,\phi(0)\right) = \sum_i c_i(q^2)\, \mathcal{O}_i(0)$$

"Wilson Coefficients" $c_i(q^2)$

"Effective" Operators $\mathcal{O}_i(0)$

- **Effective (Quantum) Field Theories:**
  Effective Lagrangian / Hamiltonian:
  - Feynman rules reproduce the dynamics of low-energy modes.
  - High-energy (short-distance) information in coefficient (functions).

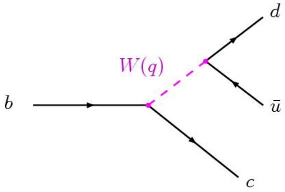
Outline

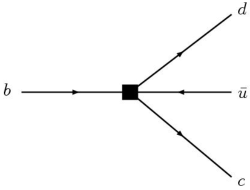
- Example: $b \to c d \bar{u}$ Decays
  - separation of scales in loop diagrams
  - current-current operators (chirality, colour)
  - matching and running of Wilson coefficients
- Another Example: $b \to s(d) q \bar{q}$ Decays
  - strong penguin operators
  - electroweak operators
- From $b \to c d \bar{u}$ to $B \to D\pi$
  - naive factorization
  - QCD factorization (BBNS)
  - factorizable and non-factorizable topologies
- $B \to \pi\pi$ and $B \to \pi K$
  - factorization theorem with spectator interaction
  - effective-theory description

Example: $b \to c d \bar{u}$ decays

$b \to c d\bar{u}$ decay at Born level

Full theory (SM) $\to$ Fermi model





$$\left(\frac{g}{2\sqrt{2}}\right)^2 J_\alpha^{(b\to c)} \frac{-g^{\alpha\beta} + \frac{q^\alpha q^\beta}{M_W^2}}{q^2 - M_W^2} \overline{J}_\beta^{(d\to u)} \quad \xrightarrow{|q| \ll M_W} \quad \frac{G_F}{\sqrt{2}} J_\alpha^{(b\to c)} g^{\alpha\beta} \overline{J}_\beta^{(d\to u)}$$

- Energy/Momentum transfer limited by mass of decaying *b*-quark.
- *b*-quark mass much smaller than *W*-boson mass.

$$\boxed{|q| \leq m_b \ll m_W}$$

Effective Theory:

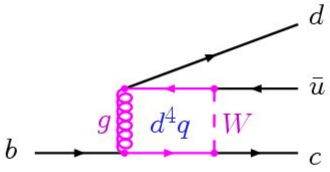
- Analogously to muon decay, transition described in terms of current-current interaction, with left-handed charged currents

$$J_\alpha^{(b\to c)} = V_{cb}\,[\bar c\,\gamma_\alpha(1-\gamma_5)\,b]\ , \qquad \bar J_\beta^{(d\to u)} = V_{ud}^*\,[\bar d\,\gamma_\beta(1-\gamma_5)\,u]$$

- Effective operators only contain light fields (!)
  ("light" quarks, leptons, gluons, photons).
- Effect of large scale M_W in effective Fermi coupling constant:

$$\boxed{\frac{g^2}{8M_W^2} \longrightarrow \frac{G_F}{\sqrt{2}} \simeq 1.16639\cdot 10^{-5}\,\mathrm{GeV}^{-2}}$$

Quantum-loop corrections to $b \to c d \bar{u}$ decay



- 4-momentum of the W-boson in the loop is an

  internal integration parameter d^4q,

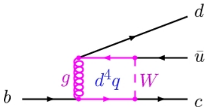
  each component taking values between $-\infty$ and $+\infty$.

$\Rightarrow$ We cannot simply expand in $|q|/M_W$!

$\Rightarrow$ Need a method to separate the cases $|q| \geq M_W$ and $|q| \ll M_W$.

IR and UV regions in the Effective Theory

full theory $=$ IR region $\left(\begin{array}{c}|q| \ll M_W \\ M_W \to \infty\end{array}\right)$ $+$ UV region $\left(\begin{array}{c}|q| \gtrsim M_W \\ m_{b,c} \to 0\end{array}\right)$



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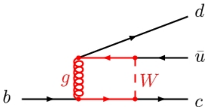

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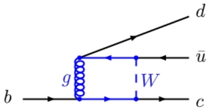

$$I(\alpha_s;\, \frac{m_b}{M_W},\, \frac{m_c}{m_b})/G_F \;\simeq\; I_{IR}(\alpha_s;\, \frac{m_b}{\mu},\, \frac{m_c}{m_b}) \;+\; I_{UV}(\alpha_s;\, \frac{\mu}{m_W})$$

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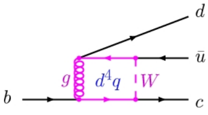
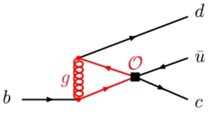


$\simeq$



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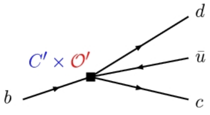

$$I(\alpha_s;\, \tfrac{m_b}{M_W},\, \tfrac{m_c}{m_b})/G_F \;\simeq\; \langle \mathcal{O} \rangle^{\rm loop}(\alpha_s;\, \tfrac{m_b}{\mu},\, \tfrac{m_c}{m_b}) \;+\; C'(\alpha_s;\, \tfrac{\mu}{m_W}) \times \langle \mathcal{O}' \rangle^{\rm tree}$$

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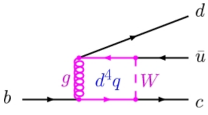

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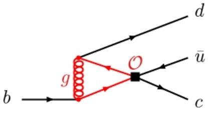

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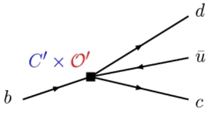

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$\updownarrow$

1-loop matrix element of operator $\mathcal{O}$ in Eff. Th.

- independent of M_W
- UV divergent $\to \mu$

$\updownarrow$

1-loop coefficient for new operator $\mathcal{O}'$ in EFT

- independent of $m_{b,c}$
- IR divergent $\to \mu$

Effective Operators for $b \to c\bar{u}$

- short-distance QCD corrections preserve chirality;
- quark-gluon vertices induce second colour structure.

$$H_{\rm eff} = \frac{G_F}{\sqrt{2}} \, V_{cb} V_{ud}^* \sum_{i=1,2} C_i(\mu) \, \mathcal{O}_i + \text{h.c.} \qquad (b \to c d\bar{u})$$

- **Current-Current Operators:** ($b \to c d\bar{u}$, analogously for $b \to q q' \bar{q}''$ decays)

$$\mathcal{O}_1 = (\bar{d}_L^a \gamma_\alpha u_L^b)\,(\bar{c}_L^b \gamma^\alpha b_L^a)$$
$$\mathcal{O}_2 = (\bar{d}_L^a \gamma_\alpha u_L^a)\,(\bar{c}_L^b \gamma^\alpha b_L^b)$$

- The Wilson Coefficients $C_i(\mu)$ contain all information about

  Short-Distance Physics $\equiv$ Dynamics above a Scale μ

Wilson Coefficients in Perturbation Theory

- 1-loop result:

$$C_i(\mu) = \left\{ \begin{array}{c} 0 \\ 1 \end{array} \right\} + \frac{\alpha_s(\mu)}{4\pi} \left(\ln \frac{\mu^2}{M_W^2} + \frac{11}{6} \right) \left\{ \begin{array}{c} 3 \\ -1 \end{array} \right\} + \mathcal{O}(\alpha_s^2)$$

Question : How do we choose the renormalization scale μ ?

Wilson Coefficients in Perturbation Theory

- 1-loop result:

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Question : How do we choose the renormalization scale μ ?

Answer :

"Matching"

- For $\mu \sim M_W$ the logarithmic term is small, and $\frac{\alpha_s(M_W)}{\pi} \ll 1$

$\rightarrow$ $C_i(M_W)$ can be calculated in Fixed-order Perturbation Theory

- In this context, M_W is called the Matching Scale.

Anomalous Dimensions

- In order to compare with experiment / hadronic models, the matrix elements of EFT operators are needed at low-energy scale $\mu \sim m_b$
  - Only the combination

$$\sum_i C_i(\mu) \, \langle \mathcal{O}_i \rangle(\mu)$$

    is μ-independent (in perturbation theory).

  $\Rightarrow$ Need Wilson coefficients at low scale !

- Scale dependence can be calculated in perturbation theory:
  - Loop diagrams in EFT are UV divergent
    $\Rightarrow$ anomalous dimensions (matrix):

$$\frac{\partial}{\partial \ln \mu} C_i(\mu) \equiv \gamma_{ji}(\mu) \, C_j(\mu) = \left(\frac{\alpha_s(\mu)}{4\pi} \, \gamma_{ji}^{(1)} + \ldots \right) C_j(\mu)$$

  - $\gamma = \gamma(\alpha_s)$ has a perturbative expansion.

RG Improvement ("running")

In our case:

$$\gamma^{(1)} = \begin{pmatrix} -2 & 6 \\ 6 & -2 \end{pmatrix} \qquad \begin{cases} \text{Eigenvectors: } C_\pm = \frac{1}{\sqrt{2}}(C_2 \pm C_1) \\ \text{Eigenvalues: } \gamma_\pm^{(1)} = +4, \ -8 \end{cases}$$

- Formal solution of differential equation: (separation of variables)

$$\ln \frac{C_\pm(\mu)}{C_\pm(M)} = \int_{\ln M}^{\ln \mu} d\ln\mu' \, \gamma_\pm(\mu') = \int_{\alpha_s(M)}^{\alpha_s(\mu)} \frac{d\alpha_s}{2\beta(\alpha_s)} \, \gamma_\pm(\alpha_s)$$

- Perturbative expansion of anomalous dimension and β-function:

$$\gamma = \frac{\alpha_s}{4\pi} \gamma^{(1)} + \ldots, \qquad 2\beta \equiv \frac{d\alpha_s}{d\ln\mu} = -\frac{2\beta_0}{4\pi} \alpha_s^2 + \ldots$$

$$C_\pm(\mu) \simeq C_\pm(M_W) \cdot \left(\frac{\alpha_s(\mu)}{\alpha_s(M_W)}\right)^{-\gamma_\pm^{(1)}/2\beta_0} \qquad \text{(LeadingLogApprox)}$$

Numerical values for $C_{1,2}$ in the SM

[Buchalla/Buras/Lautenbacher 96]

operator:	$\mathcal{O}_1$	$\mathcal{O}_2$
$C_i(m_b)$:	-0.514 (LL) -0.303 (NLL)	1.026 (LL) 1.008 (NLL)

(modulo parametric uncertainties from M_W, m_b, $\alpha_s(M_Z)$ and QED corr.)

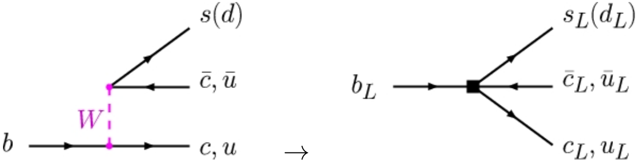
(potential) New Physics modifications:

- new left-handed interactions (incl. new phases)

$$C_{1,2}(M_W) \to C_{1,2}(M_W) + \delta_{\rm NP}(M_W, M_{\rm NP})$$

- new chiral structures $\Rightarrow$ extend operator basis (LR,RR currents)

Next Example: $b \to s(d)\, q\bar{q}$ decays



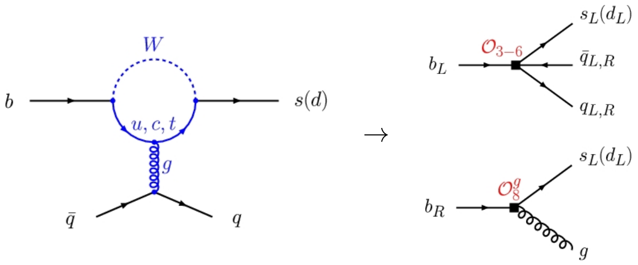
- Now, there are two possible flavour structures:

$$V_{ub} V^*_{us(d)}\, (\bar{u}_L \gamma_\mu b_L)(\bar{s}(d)_L \gamma^\mu u_L) \equiv \lambda_u\, \mathcal{O}_2^{(u)}\,,$$
$$V_{cb} V^*_{cs(d)}\, (\bar{c}_L \gamma_\mu b_L)(\bar{s}(d)_L \gamma^\mu c_L) \equiv \lambda_c\, \mathcal{O}_2^{(c)}\,,$$

- Again, α_s corrections induce independent colour structures $\mathcal{O}_1^{(u,c)}$.

$b \to s(d)\, q\bar{q}$ decays – strong penguin operators

- New feature: **Penguin Diagrams** $\to$ additional operator structures



smaller Wilson coefficients
(suppressed by α_s / loop factor)

- Strong penguin operators: $\mathcal{O}_{3-6}$
- Chromomagnetic operator: $\mathcal{O}_8^g$

Question : CKM factor of Penguin Pperators? (for $m_{u,c} \ll m_t$)
Answer :

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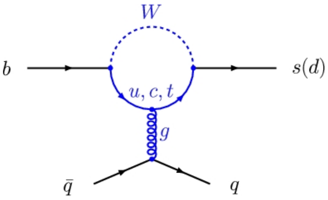

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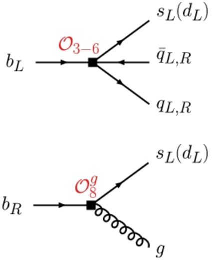

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Answer : $-\lambda_t = (\lambda_u + \lambda_c) = -V_{tb} V^*_{ts(d)}$

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{i=1,2} C_i(\mu) \left(\lambda_u \, \mathcal{O}_i^{(u)} + \lambda_c \, \mathcal{O}_i^{(c)} \right) - \frac{G_F}{\sqrt{2}} \lambda_t \sum_{i=3}^{6} C_i(\mu) \, \mathcal{O}_i - \frac{G_F}{\sqrt{2}} \lambda_t \, C_8^g(\mu) \, \mathcal{O}_8^g$$

$$\mathcal{O}_3 = (\bar{s}_L^a \gamma_\mu b_L^a) \sum_{q \neq t} (\bar{q}_L^b \gamma^\mu q_L^b) \, , \qquad \mathcal{O}_4 = (\bar{s}_L^a \gamma_\mu b_L^b) \sum_{q \neq t} (\bar{q}_L^b \gamma^\mu q_L^a) \, ,$$

$$\mathcal{O}_5 = (\bar{s}_L^a \gamma_\mu b_L^a) \sum_{q \neq t} (\bar{q}_R^b \gamma^\mu q_R^b) \, , \qquad \mathcal{O}_6 = (\bar{s}_L^a \gamma_\mu b_L^b) \sum_{q \neq t} (\bar{q}_R^b \gamma^\mu q_R^a) \, ,$$

$$\mathcal{O}_8^g = \frac{g_s}{8\pi^2} \, m_b \, (\bar{s}_L \, \sigma^{\mu\nu} \, T^A \, b_R) \, G_{\mu\nu}^A \, .$$

- gluon couples to left- and right-handed currents.
- chromomagnetic operator requires one chirality flip !

(m_s is set to zero)

Matching and running for strong penguin operators

- Matching coefficients depend on top mass,

$$C_i = C_i(\mu, x_t)\,, \qquad x_t = m_t^2/M_W^2$$

- Matching of chromomagnetic operator is scheme-dependent. Usually, one considers scheme-independent linear combination:

$$C_8^{g,\,\mathrm{eff}} = C_8^g + \sum_{i=1}^{6} z_i\, C_i$$

- Again, operators mix under RG running (→ anomalous-dimension matrix)

Electroweak Corrections

- Penguin and box diagrams with additional γ/Z exchange:

  $\to$ Electroweak Penguin Operators $\mathcal{O}_{7-10}$

$$\mathcal{O}_7 = \frac{2}{3}\,(\bar s_L^a \gamma_\mu b_L^a) \sum_{q\neq t} e_q\,(\bar q_L^b \gamma^\mu q_L^b)\,, \quad \mathcal{O}_8 = \frac{2}{3}\,(\bar s_L^a \gamma_\mu b_L^b) \sum_{q\neq t} e_q\,(\bar q_L^b \gamma^\mu q_L^a)\,,$$

$$\mathcal{O}_9 = \frac{2}{3}\,(\bar s_L^a \gamma_\mu b_L^a) \sum_{q\neq t} e_q\,(\bar q_R^b \gamma^\mu q_R^b)\,, \quad \mathcal{O}_{10} = \frac{2}{3}\,(\bar s_L^a \gamma_\mu b_L^b) \sum_{q\neq t} e_q\,(\bar q_R^b \gamma^\mu q_R^a)\,.$$

  depend on electromagnetic charge of final state quarks !

  $\to$ Electromagnetic operators $\mathcal{O}_7^\gamma$
  main contribution to $b \to s(d)\gamma$ decays.

  $\to$ Semileptonic operators $\mathcal{O}_{9V}$, $\mathcal{O}_{10A}$
  main contribution to $b \to s\ell^+\ell^-$ decays.

  [$\to$ more in Danny's lecture]

  $\to$ electroweak corrections to matching coefficients

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  - $\rightarrow$ Semileptonic operators $\mathcal{O}_{9V}$, $\mathcal{O}_{10A}$

$$\begin{aligned} \mathcal{O}_{9V} &= (\bar{s}_L \gamma_\mu b_L)(\bar{\ell} \gamma^\mu \ell), \\ \mathcal{O}_{10A} &= (\bar{s}_L \gamma_\mu b_L)(\bar{\ell} \gamma^\mu \gamma_5 \ell) \end{aligned}$$

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Summary: Effective Theory for b-quark decays

"Full theory" $\leftrightarrow$ all modes propagate
Parameters: $M_{W,Z}, M_H, m_t, m_q, g, g', \alpha_s \ldots$ | $\uparrow \mu > M_W$

$C_i(M_W) = C_i\Big|_{\rm tree} + \delta_i^{(1)} \frac{\alpha_s(M_W)}{4\pi} + \ldots$ | matching: $\mu \sim M_W$

"Eff. theory" $\leftrightarrow$ low-energy modes propagate.
High-energy modes are "integrated out".
Parameters: $m_b, m_c, G_F, \alpha_s, C_i(\mu) \ldots$ | $\downarrow \mu < M_W$

$\frac{\partial}{\partial \ln \mu} C_i(\mu) = \gamma_{ji}(\mu)\, C_j(\mu)$ | anomalous dimensions

Expectation values of operators $\langle O_i \rangle$ at $\mu = m_b$.
All dependence on M_W absorbed into $C_i(m_b)$ | resummation of logs

From $b \to c d \bar{u}$ to $\bar{B}^0 \to D^+ \pi^-$

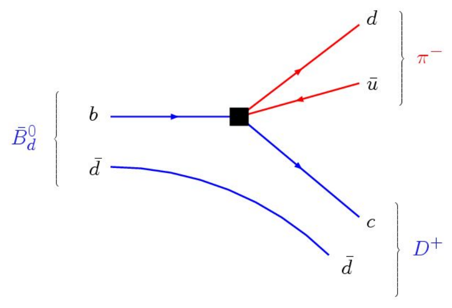
- In experiment, we cannot see the quark transition directly.
- Rather, we observe exclusive hadronic transitions, described by hadronic matrix elements, like e.g.

$$\langle D^+\pi^-|\mathcal{H}_{\rm eff}^{b\to cd\bar{u}}|\bar{B}_d^0\rangle = V_{cb}V_{ud}^* \frac{G_F}{\sqrt{2}} \sum_{i=1,2} C_i(\mu)\, r_i(\mu)$$

$$r_i(\mu) = \langle D^+\pi^-|\mathcal{O}_i|\bar{B}_d^0\rangle \Big|_\mu$$

- The hadronic matrix elements r_i contain QCD (and also QED) dynamics below the scale $\mu \sim m_b$.

"Naive" Factorization of hadronic matrix elements



$$r_i = \underbrace{\langle D^+ | J_i^{(b\to c)} | \bar{B}_d^0 \rangle}\ \underbrace{\langle \pi^- | J_i^{(d\to u)} | 0 \rangle}$$

- Quantum fluctuations above $\mu \sim m_b$ already in Wilson coefficients

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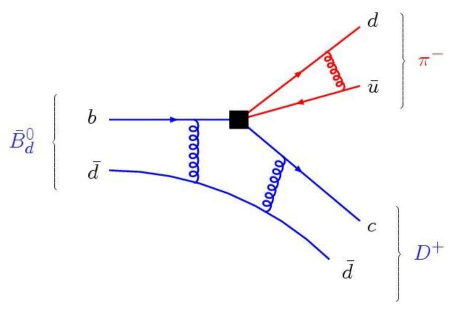
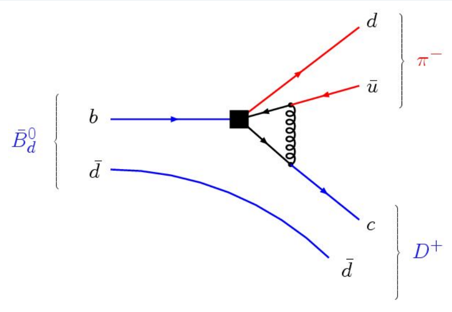


$$r_i = \underbrace{\langle D^+ | J_i^{(b\to c)} | \bar{B}_d^0 \rangle}_{\text{form factor}} \underbrace{\langle \pi^- | J_i^{(d\to u)} | 0 \rangle}_{\text{decay constant}}$$

- Part of (low-energy) gluon effects encoded in simple/universal had. quantities

"Naive" Factorization of hadronic matrix elements



$$r_i \quad = \quad \underbrace{\langle D^+|J_i^{(b\to c)}|\bar{B}_d^0\rangle}\ \underbrace{\langle \pi^-|J_i^{(d\to u)}|0\rangle}$$

Question : Why is naive factorization not exact ?

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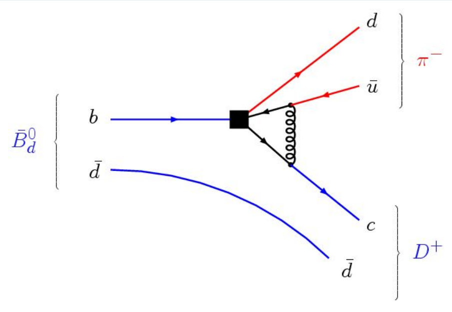

$$r_i(\mu) = \underbrace{\langle D^+ | J_i^{(b\to c)} | \bar{B}_d^0 \rangle}\ \underbrace{\langle \pi^- | J_i^{(d\to u)} | 0 \rangle} + \text{corrections}(\mu)$$

Answer : Gluon cross-talk between π^- and $B \to D$

- light quarks in π^- have large energy (in B rest frame)
- gluons from the $B \to D$ transition see "small colour-dipole"

$\Rightarrow$ corrections to naive factorization dominated by gluon exchange at short distances $\delta x \sim 1/m_b$

New feature: Light-cone distribution amplitudes $\phi_\pi(u)$

- Short-distance corrections to naive factorization given as convolution

$$r_i(\mu) \simeq \sum_j F_j^{(B\to D)} \int_0^1 du \left(1 + \frac{\alpha_s C_F}{4\pi}\, t_{ij}(u,\mu) + \ldots\right) f_\pi\, \phi_\pi(u,\mu)$$

- $\phi_\pi(u)$: distribution of momentum fraction u of a quark in the pion.
- $t_{ij}(u,\mu)$: perturbative coefficient function (depends on u)
- $F_j^{(B\to D)}$: form factors known from $B \to D\ell\nu$

QCD factorization

[Beneke/Buchalla/Neubert/Sachrajda 2000]

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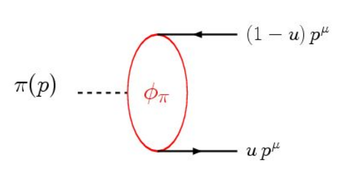
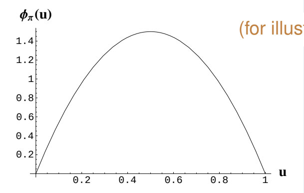
Light-cone distribution amplitude for the pion





(for illustration only)

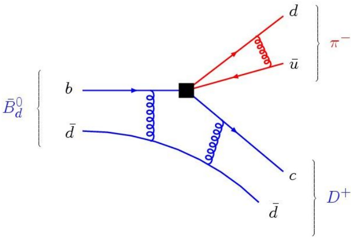
- Exclusive analogue of parton distribution function:
  - PDF: probability density (all Fock states)
  - LCDA: probability amplitude (one Fock state, here: $q\bar{q}$)
- Phenomenologically relevant

$$\langle u^{-1}\rangle_\pi = \int_0^1 \frac{du}{u}\,\phi_\pi(u) \simeq 3.3 \pm 0.3$$

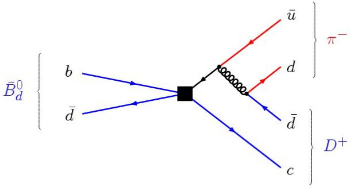
[from sum rules, lattice, exp.]

Complication: Annihilation in $\bar{B}_d \to D^+\pi^-$

Second topology for hadronic matrix element possible:



"Tree" (class-I)

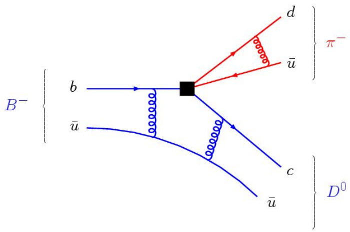


"Annihilation" (class-III)

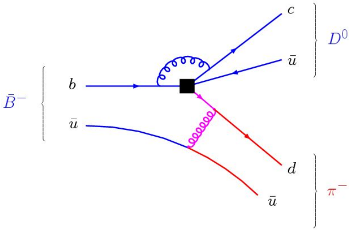
- annihilation is formally power-suppressed by $\Lambda_{\rm had}/m_b$
- more difficult to estimate (colour-dipole argument does not apply!)

Still more complicated: $B^- \to D^0\pi^-$

Second topology with spectator quark going into light meson:



"Tree" (class-I)

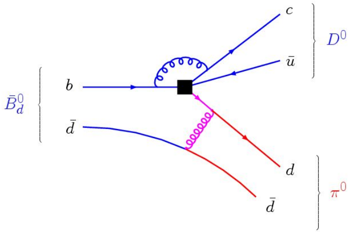


"Tree" (class-II)

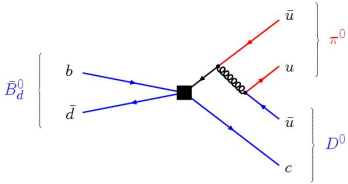
- class-II amplitude does not factorize into simpler objects
  (again, colour-transparency argument does not apply)
- again, it is power-suppressed compared to class-I topology

Non-factorizable: $\bar{B}^0 \to D^0 \pi^0$

In this channel, class-I topology is absent:



"Tree" (class-II)



"Annihilation" (class-III)

- The whole decay amplitude is power-suppressed!
- Naive factorization is not even a first-order approximation!

Isospin analysis for $B \to D\pi$

- Employ isospin symmetry between (u, d) of strong interactions.
- Final-state with π $(I = 1)$ and D $(I = 1/2)$ described by only two isospin amplitudes:

$$\begin{aligned} \mathcal{A}(\bar{B}_d \to D^+\pi^-) &= \sqrt{\frac{1}{3}}\,\mathcal{A}_{3/2} + \sqrt{\frac{2}{3}}\,\mathcal{A}_{1/2}\,, \\ \sqrt{2}\,\mathcal{A}(\bar{B}_d \to D^0\pi^0) &= \sqrt{\frac{4}{3}}\,\mathcal{A}_{3/2} - \sqrt{\frac{2}{3}}\,\mathcal{A}_{1/2}\,, \\ \mathcal{A}(B^- \to D^0\pi^-) &= \sqrt{3}\,\mathcal{A}_{3/2}\,, \end{aligned}$$

- QCDF: $\mathcal{A}_{1/2}/\mathcal{A}_{3/2} = \sqrt{2} +$ corrections, relative strong phase $\Delta\theta$ small

Isospin amplitudes from experimental data [Fleischer et al., arXiv:1012.2784]

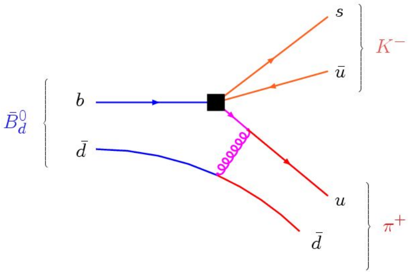
$$\left|\frac{\mathcal{A}_{1/2}}{\sqrt{2}\,\mathcal{A}_{3/2}}\right| = 0.676 \pm 0.038\,, \qquad \cos\Delta\theta = 0.930^{+0.024}_{-0.022}$$

(similar for $B \to D^*\pi$)

$\to$ Corrections to QCDF sizeable — Strong phases remain small

$B \to \pi\pi$ and $B \to \pi K$

Naive factorization:



- Both final-state mesons are light and energetic.
- Colour-transparency argument applies for class-I and class-II topologies.
- $B \to \pi(K)$ form factors at large recoil fairly well known (QCD sum rules)

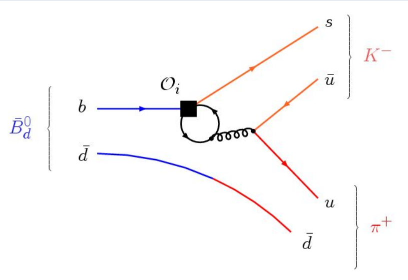
QCDF for $B \to \pi\pi$ and $B \to \pi K$ decays (BBNS 1999)

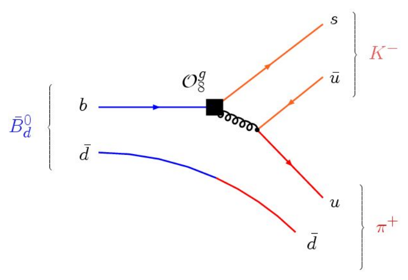
Factorization formula has to be extended:

- Vertex corrections are treated as in $B \to D\pi$
  - Include penguin (and electroweak) operators from $H_{\rm eff}$.
  - Take into account new (long-distance) penguin diagrams! (→ Fig.)

- Additional perturbative interactions involving spectator in B-meson (→ Fig.)
  - Sensitive to the distribution of the spectator momentum ω $\longrightarrow$ light-cone distribution amplitude $\phi_B(\omega)$

Additional diagrams for QCDF corrections in $B \to \pi K$ (example)





$\longrightarrow$ additional contributions to the hard coefficient functions $t_{ij}(u,\mu)$

$$r_i(\mu)\Big|_{\rm hard} \simeq \sum_j F_j^{(B\to\pi)}(m_K^2) \int_0^1 du \left(1 + \frac{\alpha_s}{4\pi}\, t_{ij}(u,\mu) + \ldots\right) f_K\, \phi_K(u,\mu)$$

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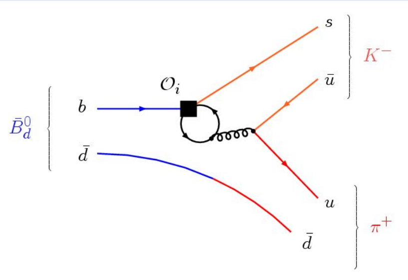

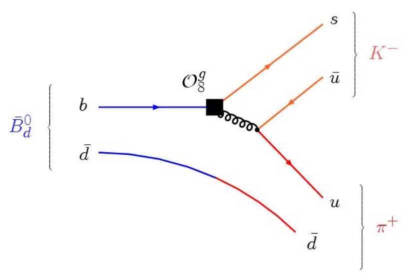

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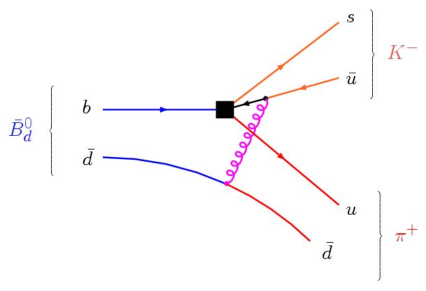
Spectator corrections in QCDF

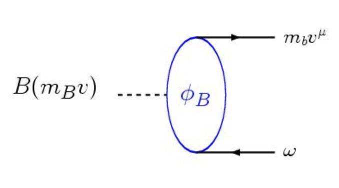


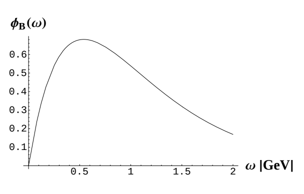
$\longrightarrow$ additive correction to naive factorization

$$\Delta r_i(\mu)\Big|_{\text{spect.}} = \int du\, dv\, d\omega \left(\frac{\alpha_s}{4\pi} h_i(u, v, \omega, \mu) + \dots \right) \times f_K\, \phi_K(u, \mu)\, f_\pi\, \phi_\pi(v, \mu)\, f_B\, \phi_B(\omega, \mu)$$

Distribution amplitudes for all three mesons involved!

New ingredient: LCDA for the *B*-meson





- Phenomenologically relevant:

$$\langle \omega^{-1} \rangle_B = \int_0^\infty \frac{d\omega}{\omega} \phi_B(\omega) \approx 2\ \mathrm{GeV}^{-1} \qquad \text{(at } \mu = \sqrt{m_b \Lambda} \simeq 1.5\ \text{GeV)}$$

(from QCD sum rules [Braun/Ivanov/Korchemsky])

(from HQET parameters [Lee/Neubert])

- Large logarithms $\ln m_b/\Lambda_{\rm had}$ can be resummed using SCET $\surd$

Complications for QCDF in $B \to \pi\pi, \pi K$ etc.

- Annihilation topologies are numerically important. BBNS use conservative model estimates.
- Some power-corrections are numerically enhanced by ”chiral factor”

$$\frac{\mu_\pi}{f_\pi} = \frac{m_\pi^2}{2f_\pi\, m_q}$$

- Many decay topologies interfere with each other.
- Many hadronic parameters to vary.

$\to$ Depending on specific mode, hadronic uncertainties sometimes quite large.

Summary

Weak b-quark decays described by Effective Hamiltonian:

- **Current-current** and **Penguin** and Box operators $\mathcal{O}_i$.
- **Wilson Coefficients** encode short-distance dynamics in SM or NP.
- QCD effects between M_W and m_b via **Renormalization-Group**.

Exclusive Amplitudes for non-leptonic decays:

- **Hadronic Matrix Elements** of $\mathcal{O}_i$ contain QCD dynamics below m_b.
- **“Naive” Factorization** in terms of form factors and decay constant.
- **QCD (improved) Factorization**: include complicated gluon cross talk.
- Factorizable and non-factorizable **“Flavour Topologies”**
- **Factorization Theorems**: soft and collinear modes in **HQET** / **SCET**.

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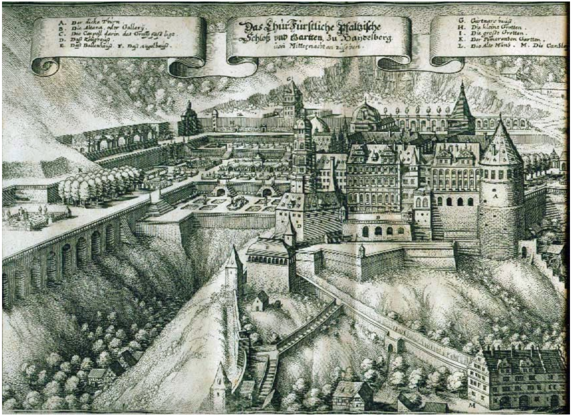
Summary

" When looking for New Physics, . . .
. . . do not forget about the complexity of the Old Physics ! "



. . . more in Danny's lecture . . .

Backup Slides

Characterization of relevant field modes below $\mu \sim m_b$: (in B-meson rest frame)

The heavy b-quark:

- Heavy quark approximately behaves as **Static Source of Colour**

$$p_b^\mu = m_b v^\mu + k^\mu , \qquad \text{with } |k^\mu| \ll m_b$$

k^μ: **soft** (residual) momentum. $v^\mu = (1, \vec{0})$: B-meson **velocity**.

- The b-quark propagator is approximated as

$$\frac{i}{\not{p}_b - m_b + i\epsilon} = \frac{i\,(\not{p}_b + m_b)}{p_b^2 - m_b^2 + i\epsilon} \simeq \frac{i m_b\,(\not{v} + 1)}{2 m_b\, v \cdot k + i\epsilon} = \frac{i}{v \cdot k + i\epsilon}\,\frac{1 + \not{v}}{2}$$

This corresponds to a kinetic term for an **effective b-quark field** h_v

$$\mathcal{L}_{\rm kin} = \bar{h}_v\,(i\, v \cdot D)\, h_v\,, \qquad \text{with} \quad (\not{v} - 1)\, h_v = 0$$

→ **Heavy Quark Effective Theory** (HQET)

Characterization of relevant field modes below $\mu \sim m_b$: (in B-meson rest frame)

Fast (massless) light quarks in energetic pions and kaons:

- Quarks move approximately **collinear** to their parent mesons.

$$p^\mu_{\rm coll} = p_+ \frac{n^\mu_-}{2} + p^\mu_\perp + p_- \frac{\bar{n}^\mu_+}{2}, \qquad \text{with } p_- \ll |p_\perp| \ll p_+$$

$p^\mu_\perp$: **small transverse momentum.** $n^\mu_\pm = (1, 0_\perp, \pm 1)$: **light-like.**

- Collinear quark propagator is approximated as

$$\frac{i \not{p}_{\rm coll}}{p^2_{\rm coll} + i\epsilon} \simeq \frac{i\, p_+ \not{n}_-}{p_+ p_- + p^2_\perp + i\epsilon} = \frac{i}{p_- + p^2_\perp / p_+ + i\epsilon} \frac{\not{n}_-}{2}$$

This corresponds to a kinetic term for an **effective collinear field** ξ_c

$$\mathcal{L}_{\rm kin} = \bar{\xi}_c \left(i\, n_- \cdot D + i\not{D}_\perp \frac{1}{i n_+ D} i\not{D}_\perp \right) \frac{\not{n}_+}{2}\, \xi_c\,, \qquad \text{with} \quad \not{n}_-\, \xi_c = 0$$

→ **Soft Collinear Effective Theory** (SCET)

Effective-Theory Description for $B \to \pi\pi, \pi K$

Characterization of relevant field modes below $\mu \sim m_b$: (in B-meson rest frame)

Soft-collinear interactions:

- Invariant mass of a gluon coupled to soft-collinear quark current:

$$(k_{\rm soft} - p_{\rm coll})^2 \simeq -p_+ \, (n_- \cdot k) \sim \mathcal{O}(E \, \Lambda_{\rm had})$$

$\to$ **hard-collinear modes** (relevant for spectator interactions)

Subtlety: Soft-collinear vertices have to be **multipole-expanded** according to the different sizes for the typical wave-lengths involved.

Heavy-to-light currents:

- A generic heavy-to-light current (with arbitrary Dirac matrix Γ) matches onto:

$$\bar{q}(0) \, \Gamma \, Q(0) \longrightarrow \bar{\xi}_c(0) \, \Gamma \, h_v(0) + \ldots$$

$\to$ **Soft Collinear Effective Theory** (SCET)