Phenomenology of Exclusive $b \to s\ell^+\ell^-$ Decays

Danny van Dyk

Universität Siegen

LHCb Workshop Neckarzimmern
March 6th 2014

Theor. Physik 1

DFG FOR 1873

Program

Phenomenology of $\bar{B} \to \bar{K}\ell^+\ell^-$

Phenomenology of $\bar{B} \to \bar{K}^*(\to \bar{K}\pi)\ell^+\ell^-$

Beyond Naive Factorization

- QCD Improved Factorization
- Low Recoil OPE

Global Analysis

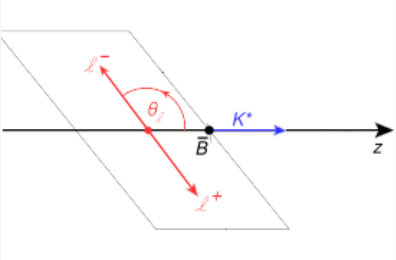
Phenomenology of $\Lambda_b \to \Lambda\ell^+\ell^-$

Section 1

Phenomenology of $\bar{B} \to \bar{K}\ell^+\ell^-$

Kinematics

$$\bar{B}(p) \to \bar{K}(k)\,\ell^+(q_1)\,\ell^-(q_2)$$



Momenta

$$q = q_1 + q_2$$
$$\bar{q} = q_1 - q_2$$
$$k$$

Phase Space

$$4m_\ell^2 \leq q^2 \leq (M_B - M_K)^2$$
$$-1 \leq \cos\theta_\ell \leq 1$$
$$k^2 = M_K^2$$

Helicity Angles ($\ell\ell$ rest frame)

one angle

$$k \cdot \bar{q} \propto \cos\theta_\ell$$

Effective Theory

Effective Hamiltonian

$$\mathcal{H}^{\text{eff}} = -\frac{4G_{\mathrm{F}}}{\sqrt{2}}\left[V_{tb}V_{ts}^* \sum_i \mathcal{C}_i \mathcal{O}_i + O\left(V_{ub}V_{us}^*\right)\right] + \text{h.c.}$$

Semileptonic Operators (SM-like: $9, 10$ chirality-flipped: $9', 10'$)

$$\mathcal{O}_{9(')} = \frac{\alpha_e}{4\pi}\left[\bar{s}\gamma_\mu P_{L(R)} b\right]\left[\bar{\ell}\gamma^\mu \ell\right] \qquad \mathcal{O}_{10(')} = \frac{\alpha_e}{4\pi}\left[\bar{s}\gamma_\mu P_{L(R)} b\right]\left[\bar{\ell}\gamma^\mu \gamma_5 \ell\right]$$

Penguins (photonic: $7(')$ gluonic: $8(')$ $\bar{q}q$: 3-6)

$$\mathcal{O}_{7(')} = \left[\bar{s}\sigma_{\mu\nu} P_{R(L)} b\right] F^{\mu\nu} \qquad \mathcal{O}_{8(')} = \left[\bar{s}\sigma_{\mu\nu} P_{R(L)} b\right] G^{\mu\nu}$$

- $\mathcal{O}_{7(')}$ dominant when dilepton system is almost lightlike
- current/current ($\mathcal{O}_{1,2}$) and QCD Penguins ($\mathcal{O}_{3\ldots6}$) $\rightarrow$ talk by Th. Feldmann

Hadronic Matrix Elements

Aim

$\mathcal{H}^{\text{eff}}$: effective $|\Delta B| = |\Delta S| = 1$ Hamiltonian

$$i\mathcal{M} = \langle \bar{K}\ell^+\ell^- | \mathcal{H}^{\text{eff}} | \bar{B} \rangle$$

Factorizable Contributions

leptonic and hadronic currents of operators $\mathcal{O}_7$, $\mathcal{O}_9$, $\mathcal{O}_{10}$ factorize up to α_e corr.

$$\begin{aligned}\langle \bar{K}\ell^+\ell^- | \mathcal{O}_{9(10)} | \bar{B} \rangle &= \frac{1}{2} \langle \bar{K} | \bar{s}\gamma_\mu (1-\gamma_5) b | \bar{B} \rangle \times \langle \ell^+\ell^- | \bar{\ell}\gamma^\mu(\gamma_5)\ell | 0 \rangle + O(\alpha_e) \\ &= \frac{1}{2} \langle \bar{K} | \bar{s}\gamma_\mu (1-\gamma_5) b | \bar{B} \rangle \times \left[\bar{u}_\ell \gamma^\mu (\gamma_5) v_\ell\right]\end{aligned}$$

Hadronic Matrix Elements

parametrized in terms of three *formfactors*

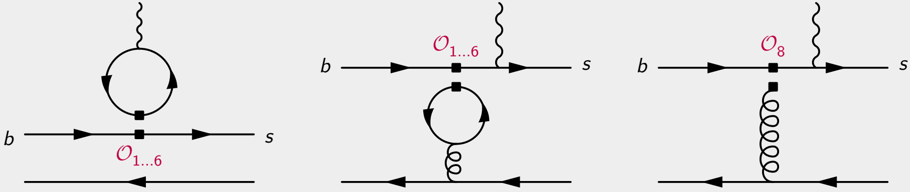
$$\langle \bar{K} | \bar{s}\gamma^\mu b | \bar{B} \rangle = f_+(q^2) \left[(p+k)^\mu + \left(1 - \frac{M_B^2 - M_K^2}{q^2}\right) q^\mu \right] + f_0(q^2) \frac{M_B^2 - M_\pi^2}{q^2} q^\mu$$

$$\langle \bar{K} | \bar{s} i \sigma^{\mu\nu} q_\nu b | \bar{B} = -\frac{f_T(q^2)}{M_B + M_K} \left[q^2 (p+k)^\mu - (M_B^2 - M_\pi^2) q^\mu \right]$$

Hadronic Matrix Elements (II)

Non-Factorizable Contributions

matrix elements of current-current operators ($\mathcal{O}_1$, $\mathcal{O}_2$), QCD penguins ($\mathcal{O}_3 \dots \mathcal{O}_6$) and chromomagnetic penguin $\mathcal{O}_8$ do not neccessarily factorize



Calculation

- several methods, applicability depends on q^2
- problem occurs in *all* exclusive decays, not just $\bar{B} \to \bar{K}$
- $\to$ we will return to this

Fully-Differential Decay Width

3-body Decay

[see PDG, section 45]

$$\mathrm{d}\Gamma = \frac{1}{(2\pi)^5}\frac{1}{16M^2}\overline{|\mathcal{M}|^2}|\mathbf{p_1^*}|\,|\mathbf{p_3}|\,\mathrm{d}m_{12}\mathrm{d}\Omega_1^*\,\mathrm{d}\Omega_3$$

identify $q^2 = m_{12}^2$, $M = M_B$, $\mathrm{d}\Omega_1^* = 2\pi \mathrm{d}\cos\theta_\ell$, $p_1 = q_2$, k

Ω_3 to be integrated out

Full Angular Distribution

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}q^2\,\mathrm{d}\cos\theta_\ell} = a(q^2) + b(q^2)\cos\theta_\ell + c(q^2)\cos^2\theta_\ell$$

$a(q^2)\ldots c(q^2)$: angular observables, depend on Wilson coefficients

Observables

Normalized Angular Distribution

$$\frac{1}{\mathrm{d}\Gamma/\mathrm{d}q^2}\frac{\mathrm{d}\Gamma}{\mathrm{d}q^2\,\mathrm{d}\cos\theta_\ell} = \frac{3}{4}\Big(1 - F_H\Big)\left(1-\cos^2\theta_\ell\right) + \frac{1}{2}F_H + A_{\mathrm{FB}}\cos\theta_\ell$$

Forward-Backward Asymmetry

counting: how many ℓ^- in positive z direction vs in negative z direction

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}q^2}A_{\mathrm{FB}} = \int \mathrm{d}\cos\theta_\ell\,\mathrm{sign}\cos\theta_\ell\frac{\mathrm{d}\Gamma}{\mathrm{d}q^2\,\mathrm{d}\cos\theta_\ell}$$

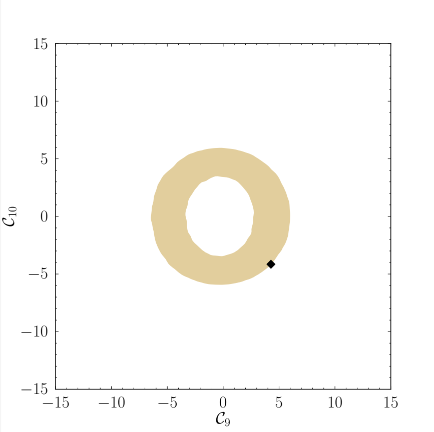
Flat Term

appears as constant (flat) term in the angular distribution as well as $\propto \sin^2\theta_\ell$

Sensitivity

if considering only $\mathcal{C}_{7,9,10,7',9',10'}$

$$\frac{d\Gamma}{dq^2} \propto |(\mathcal{C}_9 + \mathcal{C}_{9'}) + (\mathcal{C}_7 + \mathcal{C}_{7'})|^2 + |\mathcal{C}_{10} + \mathcal{C}_{10'}|^2 \qquad A_{\text{FB}} \simeq 0 \simeq F_H$$



Fit to $\mathcal{C}_7$, $\mathcal{C}_9$, $\mathcal{C}_{10}$

- only $\bar{B} \to \bar{K}\ell^+\ell^-$ and $\bar{B} \to \bar{K}^*\gamma$
- based on data up to winter 2011/2012

[Beaujean/Bobeth/DvD/Wacker '12]

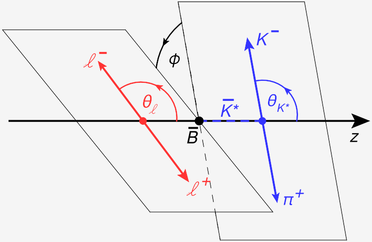
◆: SM value

Section 2

Phenomenology of $\bar{B} \to \bar{K}^*(\to \bar{K}\pi)\ell^+\ell^-$

Kinematics

$$\bar{B}(p) \to \bar{K}(k_1)\,\pi(k_2)\,\ell^+(q_1)\,\ell^-(q_2)$$



Momenta

$$q = q_1 + q_2$$
$$\bar{q} = q_1 - q_2$$
$$k = k_1 + k_2$$
$$\bar{k} = k_1 - k_2$$

Phase Space

$$4m_\ell^2 \le q^2 \le (M_B - M_K)^2$$
$$-1 \le \cos\theta_\ell \le 1$$
$$-1 \le \cos\theta_{K^*} \le 1$$
$$(M_K + M_\pi)^2 \le k^2 \le (M_B - \sqrt{q^2})^2$$
$$0 \le \phi \le 2\pi$$

Helicity Angles ($\ell\ell$ and $\bar{K}\pi$ rest frames)

2 helicity angles

$$k \cdot \bar{q} \propto \cos\theta_\ell$$
$$q \cdot \bar{k} \propto \cos\theta_{K^*}$$
$$\bar{k} \cdot \bar{q} \sim \cos\phi$$

one azimuthal angle ϕ

Partial Wave Expansion and Hadronic Matrix Elements

Expand in $\cos\theta_{K^*}$

- use Legendre polynomials (eigenfunction of angular momentum operator $\hat{J}$)
- $J = 0$ is S wave: broad resonance κ or $K_0^*(800)$
- $J = 1$ is P wave: $K^*(892)$

S Wave Form Factors [Meißner/Wang '13]

using Light Cone Sum Rules

$$\langle (\bar{K}\pi)_{J=0} | \bar{s}\Gamma b | \bar{B} \rangle$$

not so well known as other hadronic matrix elements

Hadronic Matrix Elements

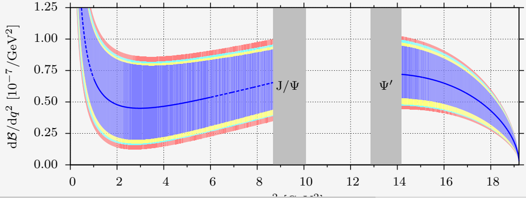
P Wave Form Factors

- hadronic matrix elements $\langle \bar{K}^* | \bar{s} \Gamma b | \bar{B} \rangle$ parametrized through 7 form factors:

$$\langle \bar{K}^* | \bar{s}\gamma^\mu b | \bar{B} \rangle \sim V \quad \langle \bar{K}^* | \bar{s}\gamma^\mu \gamma_5 b | \bar{B} \rangle \sim A_{0,1,2} \quad \langle \bar{K}^* | \bar{s}\sigma^{\mu\nu} b | \bar{B} \rangle \sim T_{1,2,3}$$

- form factors largest source of theory uncertainty
  amplitude $\sim 10\% - 15\% \Rightarrow$ observables: $\sim 20\% - 50\%$
  - from Light Cone Sum Rules [Ball/Zwicky '04, Khodjamirian et al. '11]
  - from Lattice QCD [Horgan/Liu/Meinel/Wingate '13]
  - extract ratios from low recoil data
    [Hambrock/Hiller/Schacht/Zwicky '13, Beaujean/Bobeth/DvD '13]

blue band:
form factor uncertainty



Angular Distribution [Krüger/Matias '05, Altmannshofer et al. '08]

Differential Decay Rate for pure P-wave state

$$
\begin{aligned}
\frac{d^4\Gamma}{dq^2\, d\cos\theta_\ell\, d\cos\theta_{K^*}\, d\phi} \sim\; & J_{1s}\sin^2\theta_{K^*} + J_{1c}\cos^2\theta_{K^*} + J_{1i}\cos\theta_{K^*} \\
& + \left(J_{2s}\sin^2\theta_{K^*} + J_{2c}\cos^2\theta_{K^*} + J_{2i}\cos\theta_{K^*}\right)\cos 2\theta_\ell \\
& + \left(J_3\cos 2\phi + J_9\sin 2\phi\right)\sin^2\theta_{K^*}\sin^2\theta_\ell \\
& + \left(J_4\sin 2\theta_{K^*} + J_{4i}\cos\theta_{K^*}\right)\sin 2\theta_\ell\cos\phi \\
& + \left(J_5\sin 2\theta_{K^*} + J_{5i}\cos\theta_{K^*}\right)\sin\theta_\ell\cos\phi \\
& + \left(J_{6s}\sin^2\theta_{K^*} + J_{6c}\cos^2\theta_{K^*}\right)\cos\theta_\ell \\
& + \left(J_7\sin 2\theta_{K^*} + J_{7i}\cos\theta_{K^*}\right)\sin\theta_\ell\sin\phi \\
& + \left(J_8\sin 2\theta_{K^*} + J_{8i}\cos\theta_{K^*}\right)\sin 2\theta_\ell\sin\phi\,,
\end{aligned}
$$

$J_i \equiv J_i(q^2, k^2)$: 12 angular observables

Angular Distribution [Krüger/Matias '05, Altmannshofer et al. '08, Blake/Egede/Shires '12]

Differential Decay Rate for mixed P- and S-wave state

$$\begin{aligned}
\frac{d^4\Gamma}{dq^2 d\cos\theta_\ell d\cos\theta_{K^*} d\phi} \sim\ & J_{1s}\sin^2\theta_{K^*} + J_{1c}\cos^2\theta_{K^*} + J_{1i}\cos\theta_{K^*} \\
& + (J_{2s}\sin^2\theta_{K^*} + J_{2c}\cos^2\theta_{K^*} + J_{2i}\cos\theta_{K^*})\cos 2\theta_\ell \\
& + (J_3\cos 2\phi + J_9\sin 2\phi)\sin^2\theta_{K^*}\sin^2\theta_\ell \\
& + (J_4\sin 2\theta_{K^*} + J_{4i}\cos\theta_{K^*})\sin 2\theta_\ell\cos\phi \\
& + (J_5\sin 2\theta_{K^*} + J_{5i}\cos\theta_{K^*})\sin\theta_\ell\cos\phi \\
& + (J_{6s}\sin^2\theta_{K^*} + J_{6c}\cos^2\theta_{K^*})\cos\theta_\ell \\
& + (J_7\sin 2\theta_{K^*} + J_{7i}\cos\theta_{K^*})\sin\theta_\ell\sin\phi \\
& + (J_8\sin 2\theta_{K^*} + J_{8i}\cos\theta_{K^*})\sin 2\theta_\ell\sin\phi\,,
\end{aligned}$$

$J_i \equiv J_i(q^2, k^2)$: 18 angular observables

Transversity Amplitudes

Angular Observables

J_i functionals of $A_S, A_a, A_{ab},\ a, b = t, 0, \|, \perp$ e.g.

$$J_3(q^2) = \frac{3\beta_\ell}{4}\left[|A_\perp|^2 - |A_\||^2\right]$$

β_ℓ: lepton velocity in dilepton rest frame $m_\ell^2/q^2 \to 0 \Rightarrow \beta_\ell \to 1$

Transversity Amplitudes

contractions of hadronix matrix elements with polarization vectors

$$\begin{aligned}
A_\perp &\propto \langle K^*(k,\eta^*)|\bar{s}\gamma^\mu P_L b|\bar{B}\rangle \varepsilon^*_\mu(+) + \left[\varepsilon(+) \to \varepsilon(-)\right] && \sim V \\
A_\| &\propto \langle K^*(k,\eta^*)|\bar{s}\gamma^\mu P_L b|\bar{B}\rangle \varepsilon^*_\mu(+) - \left[\varepsilon(+) \to \varepsilon(-)\right] && \sim A_1 \\
A_0 &= \langle K^*(k,\eta^*)|\bar{s}\gamma^\mu P_L b|\bar{B}\rangle \varepsilon^*_\mu(0) && \sim A_{12}
\end{aligned}$$

ε: artifical polarization vector η: polarization vector of K^* meson

Observables

Decay Width

$$\frac{d\Gamma}{dq^2} = 2J_{1s} + J_{1c} - \frac{2}{3}J_{2s} - \frac{1}{3}J_{2c}$$

Leptonic Forward-Backward Asymmetry

as in $\bar{B} \to \bar{K}\ell^+\ell^-$, integrated over $\cos\theta_{K^*}$ and ϕ

$$\frac{d\Gamma}{dq^2}A_{\text{FB}} = \int d\cos\theta_\ell \,\text{sign}\cos\theta_\ell \frac{d\Gamma}{dq^2\, d\cos\theta_\ell} = J_{6s} + \frac{1}{2}J_{6c} \quad \xrightarrow{\text{SM}} J_{6s}$$

some form factor uncertainties cancel

Longitudinal/Transverse Polarization

$$\frac{1}{d\Gamma/dq^2}\frac{d\Gamma}{dq^2\, d\cos\theta_{K^*}} = \frac{3}{4}F_T \sin^2\theta_{K^*} + \frac{3}{2}F_L\cos^2\theta_{K^*}$$

considerable theory uncertainty due to form factors

$$\frac{d\Gamma}{dq^2}F_L = J_{1c} - \frac{1}{3}J_{2c} \qquad \frac{d\Gamma}{dq^2}F_L = 2J_{1s} - \frac{2}{3}J_{2s}$$

Sensitivity to Wilson Coefficients if considering only $\mathcal{C}_{7,9,10,7',9',10'}$

Transversity Amplitudes

$$A_\perp^{L(R)} \propto \left((\mathcal{C}_9 - \mathcal{C}_{9'}) + \frac{M_B^2}{q^2} (\mathcal{C}_7 - \mathcal{C}_{7'}) \mp (\mathcal{C}_{10} - \mathcal{C}_{10'}) \right)$$

$$A_\parallel^{L(R)} \propto \left((\mathcal{C}_9 + \mathcal{C}_{9'}) + \frac{M_B^2}{q^2} (\mathcal{C}_7 + \mathcal{C}_{7'}) \mp (\mathcal{C}_{10} + \mathcal{C}_{10'}) \right)$$

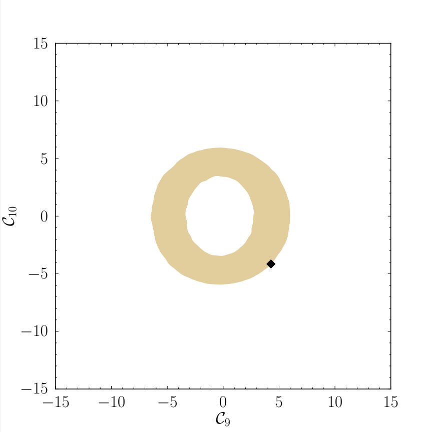
$$A_0^{L(R)} \propto \left((\mathcal{C}_9 + \mathcal{C}_{9'}) + \qquad (\mathcal{C}_7 + \mathcal{C}_{7'}) \mp (\mathcal{C}_{10} + \mathcal{C}_{10'}) \right)$$

complementary sensitivity to $\bar{B} \to \bar{K}\ell^+\ell^-$!

Observables

$$A_{\mathrm{FB}} \propto \mathrm{Re}\left(A_\perp A_\parallel^*\right) \longrightarrow \mathrm{Re}\left((\mathcal{C}_9 + \frac{M_B^2}{q^2}\mathcal{C}_7)\, \mathcal{C}_{10}^* \right)$$

Sensitivity

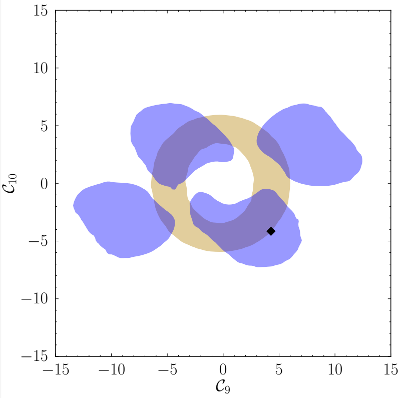


♦: SM value [Beaujean/Bobeth/DvD/Wacker '12]

Fits to $\mathcal{C}_7$, $\mathcal{C}_9$, $\mathcal{C}_{10}$

- based on $\bar{B} \to \bar{K}^*\gamma$ and
  - $\bar{B} \to \bar{K}\ell^+\ell^-$

Sensitivity

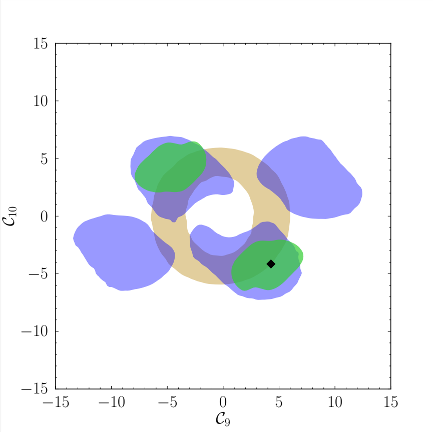


◆: SM value [Beaujean/Bobeth/DvD/Wacker '12]

Fits to $\mathcal{C}_7$, $\mathcal{C}_9$, $\mathcal{C}_{10}$

- based on $\bar{B} \to \bar{K}^*\gamma$ and
  - $\bar{B} \to \bar{K}\ell^+\ell^-$
  - $\bar{B} \to \bar{K}^*\ell^+\ell^-$ small q^2

Sensitivity



♦: SM value [Beaujean/Bobeth/DvD/Wacker '12]

Fits to $\mathcal{C}_7$, $\mathcal{C}_9$, $\mathcal{C}_{10}$

- based on $\bar{B} \to \bar{K}^*\gamma$ and
  - $\bar{B} \to \bar{K}\ell^+\ell^-$
  - $\bar{B} \to \bar{K}^*\ell^+\ell^-$ small q^2
  - $\bar{B} \to \bar{K}^*\ell^+\ell^-$ large q^2

Section 3

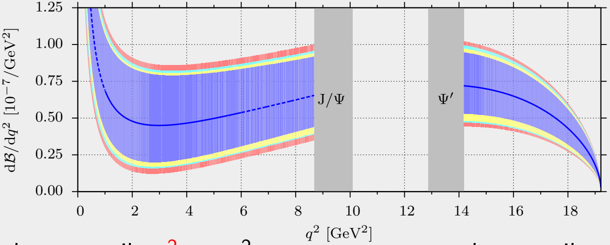
Beyond Naive Factorization

Charm Resonances

$$\bar{B} \to \bar{K}^{(*)}\psi(n)(\to \ell^+\ell^-)$$

Narrow Resonances: J/ψ and $\psi(2s)$

- experiments veto q^2-region of narrow charmonia J/ψ and $\psi(2s)$
- however: resonance affects observables outside the veto!



large recoil: $q^2 \ll m_b^2$ low recoil: $q^2 \simeq m_b^2$

- treat region below J/ψ (aka *large recoil*) differently than above $\psi(2s)$

Subsection 1

QCD Improved Factorization

QCD Improved Factorization

Preliminaries

when $q^2 \ll m_b^2 \Rightarrow E_{K^{(*)}} \sim m_b$

- final state $K^{(*)}$ almost light-like (collinear)
- decompose four-momenta in collinear (n_+) and anticollinear (n_-) direction

$$k^\mu = \frac{n_- k}{2} n_+^\mu + k_\perp^\mu + \frac{n_+ k}{2} n_-^\mu$$

$n_- k \sim m_b$, $n_+ k \sim \Lambda_{\mathrm{QCD}} \rightarrow$ Soft Collinear Effective Theory

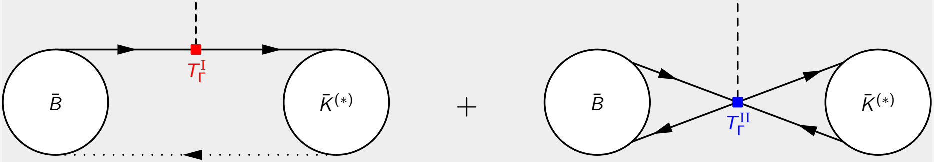
Ingredients (I)

Form Factor Symmetry Relations

- reduce 3 $B \to K$ form factors down to 1 (ξ_P)
- reduce 7 $B \to K^*$ form factors down to 2 ($\xi_\perp$, $\xi_\parallel$)
- ξs are called *soft form factors*

Symmetry-breaking Corrections

- hadronic matrix elements that cannot be expressed through ξs
- new *universal* hadronic parameters enter (e.g. $\lambda_{B,+}^{-1}$ from $B^- \to \gamma\ell^-\bar{\nu}_\ell$)



Ingredients (II)

Loop Corrections

- $q\bar{q}$ loops (up to two loop) perturbatively calculated [Greub et al, Seidel '04]
- calculate hadronic matrix elements [Beneke/Feldmann/Seidel '01 & '04]
  - $C_7 \times \xi_{\perp(\|)} \to \mathcal{T}_{\perp(\|)}(q^2)$, polarization dep.

Reduction of Hadronic Uncertainties: A_{FB} Zero Crossing

s_0: $A_{\mathrm{FB}}(q^2 = s_0) = 0$

$$s_0 = +3.4 \pm 0.6 \quad \xrightarrow[\text{QCDF}]{} \quad s_0^{\text{QCDF}} = 4.0 \pm 0.3$$

Results

Factorization

- transversity amplitudes factorize up to power supressed terms

$$A_\perp^{L,R} \sim X_\perp^{L,R} \times \xi_\perp \qquad A_\parallel^{L,R} \sim X_\parallel^{L,R} \times \xi_\perp \qquad A_0^{L,R} \sim X_0^{L,R} \times \xi_\parallel$$

$\xi_{\perp,\parallel}$: soft form factors $\qquad X_i^{L,R}$: combinations of Wilson coefficients

[Beneke/Feldmann/Seidel '01 & '04, Bobeth/Hiller/Piranishvili '08, Altmannshofer et al. '08]

Optimized Observables

enhanced sensitivity to right-handed currents, reduced form factor dependence

[Krüger/Matias '05, Egede et al. '08 & '10]

$$A_T^{(2)} = \frac{|A_\perp|^2 - |A_\parallel|^2}{|A_\perp|^2 + |A_\parallel|^2} \sim J_3 \qquad A_T^{(3)} = \frac{|A_0^L A_\parallel^{L*} + A_0^{R*} A_\parallel^R|}{\sqrt{|A_0|^2 |A_\parallel|^2}} \sim J_4, J_7$$

$$A_T^{(4)} = \frac{|A_0^L A_\perp^{L*} - A_0^{R*} A_\perp^R|}{\sqrt{|A_0|^2 |A_\perp|^2}} \sim J_5, J_8 \qquad A_T^{(5)} = \frac{|A_\perp^L A_\parallel^{R*} + A_\perp^{R*} A_\parallel^L|}{|A_\perp|^2 + |A_\parallel|^2}$$

Words of Caution

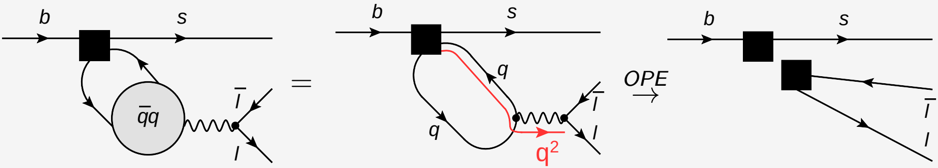
- $c\bar{c}$ loops should be under control for $q^2 \ll 4m_c^2$
- however: QCDF only includes some corrections, not all of them
- more inclusively: Light Cone Sum Rules (with its own caveats)
- $1/m_b$ corrections are of interest, could explain putative $B \to K^*\ell^+\ell^-$-Anomaly

Subsection 2

Low Recoil OPE

Local OPE [Grinstein/Pirjol '04, Beylich/Buchalla/Feldmann '11]

$$i \int \mathrm{d}^4 x e^{iqx} \langle \bar{K}^* | T\{\mathcal{O}_i(0), j_\mu^{\mathrm{e.m.}}(x)\} | \bar{B} \rangle = \sum_{j,k} \mathcal{C}_{i,j,k}(q^2/m_b^2, \mu) \langle \mathcal{O}_j^{(k)} \rangle_\mu$$



Operators

- $k = 3$ form factors, α_s corrections known, absorbed into effective Wilson coefficients $\mathcal{C}_{7,9} \to \mathcal{C}_{7,9}^{\mathrm{eff}}$
- $k = 4$ absent
- $k = 5$ $\Lambda^2/m_b^2 \sim 2\%$ corrections, first new had. matrix elements explicitly: $< 1\%$ for $q^2 = 15\mathrm{GeV}^2$ [Beylich/Buchalla/Feldmann]
- $k = 6$ first isospin breaking correction, Λ^3/m_b^3 suppressed

Low Recoil

SM basis + chirality flipped [Bobeth/Hiller/DvD '10 & '12]

- transversity amplitudes factorize

$$A^{L,R}_{\perp,\parallel,0} \sim C^{L,R}_{\pm} \times f_{\perp,\parallel,0} + O\left(\frac{\alpha_s \Lambda}{m_b}, \frac{\mathcal{C}_7 \Lambda}{\mathcal{C}_9 m_b}\right)$$

f_i: helicity form factors $\quad$ $C^{L,R}_{\pm}$: combinations of Wilson coeff.

$$C^{L(R)}_{+} = (\mathcal{C}_9 + \mathcal{C}_{9'}) + \frac{M_B^2}{q^2}(\mathcal{C}_7 + \mathcal{C}_{7'}) \mp (\mathcal{C}_{10} + \mathcal{C}_{10'})$$

$$C^{L(R)}_{-} = (\mathcal{C}_9 - \mathcal{C}_{9'}) + \frac{M_B^2}{q^2}(\mathcal{C}_7 - \mathcal{C}_{7'}) \mp (\mathcal{C}_{10} - \mathcal{C}_{10'})$$

- 4 combinations of Wilson coefficients enter observables:

$$\rho_1^{\pm} \sim |C^R_{\pm}|^2 + |C^L_{\pm}|^2$$

$$\mathrm{Re}\,(\rho_2) \sim \mathrm{Re}\left(C^R_+ C^{R*}_- - C^L_- C^{L*}_+\right) \quad \text{and } \mathrm{Re}\,(\,\cdot\,) \leftrightarrow \mathrm{Im}\,(\,\cdot\,)$$

Optimized Observables at Low Recoil

"Form Factor Free" Observables

- optimized for low recoil: $H_T^{(1,2,3,4,5)}$ [Bobeth/Hiller/DvD '10 & '12]
- $H_T^{(1)}$: probes low-recoil framework before new physics
- $H_T^{(2,3,4,5)}$: access to combination of Wilson coefficients

$$\rho_2/\sqrt{\rho_1^+\rho_1^-} \quad \underset{\text{SM basis}}{\longrightarrow} \quad \frac{\mathcal{C}_9\mathcal{C}_{10}}{|\mathcal{C}_9|^2+|\mathcal{C}_{10}|^2}$$

up to $O\left(\frac{\alpha_s\Lambda}{m_b}, \frac{\mathcal{C}_7\Lambda}{\mathcal{C}_9 m_b}\right)$ corrections, complementary to large recoil

"Short-Distance Free" Observables

- form factor ratios, relevant for comparison with lattice
- SM: all ratios f_i/f_j available, chirality-flipped: only $f_0/f_\parallel$

Words of Caution

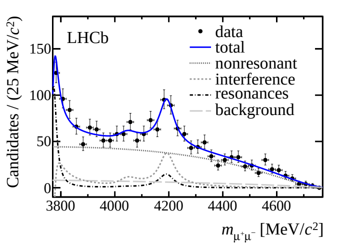
- OPE at low recoil makes use of quark-hadron duality
  $X(q^2)$ be some observable, then

$$\int \mathrm{d}q^2 X_{\mathrm{OPE}}(q^2) \simeq \int \mathrm{d}q^2 X_{\mathrm{hadronic}}(q^2)$$

  but

$$X_{\mathrm{OPE}}(q^2) \neq X_{\mathrm{hadronic}}(q^2)$$

- expectation and data [Beylich/Buchalla/Feldmann '11]: good agreement



- LHCb 2013: Surprise, we see a broad resonance!
- Theorists: Not surprised!
- Theorists: Also, $X_{\mathrm{OPE}} \neq X_{\text{non-resonant}}$!
- LHCb 2013: ...

Section 4

Global Analysis

Model-Independent Framework

Definition of model-independent for the purpose of this work:

Basis of Operators $\mathcal{O}_i$

- include as many $\mathcal{O}_i$ beyond SM as needed/as few as possible
- balancing act, test statistically if choice of basis describes data well!

Wilson Coefficients $\mathcal{C}_i$

- treat $\mathcal{C}_i$ as uncorrelated, generalized couplings
- constrain their values from data
- model builders: confront new physics models with constraints

Fit Scenarios: SM(ν-only)

SM-like Coefficients

- fix $\mathcal{C}_{7,9,10}$ to SM values (NNLL)

Chirality-flipped Coefficients

- fix $\mathcal{C}_{7'} = m_s/m_b\,\mathcal{C}_7$, fix $\mathcal{C}_{9',10'} = 0$

Nuisance Parameters

- fit nuisance parameters
- informative priors
  - form factors: light-cone sum rules [Khodjamirian/Mannel/Pivovarov/Wang '10]
  - power corrections: power-counting assumptions
  - CKM: tree-level fit [UTfit]
  - quark masses [PDG]

Fit Scenarios: SM Basis

SM-like Coefficients

- fit $\mathcal{C}_{7,9,10}$

Chirality-flipped Coefficients

- fix $\mathcal{C}_{7'} = m_s/m_b\,\mathcal{C}_7$, fix $\mathcal{C}_{9',10'} = 0$

Nuisance Parameters

- fit nuisance parameters
- informative priors
  - form factors: light-cone sum rules [Khodjamirian/Mannel/Pivovarov/Wang '10]
  - power corrections: power-counting assumptions
  - CKM: tree-level fit [UTfit]
  - quark masses [PDG]

Fit Scenarios: SM+SM$'$ Basis

SM-like Coefficients

- fit $\mathcal{C}_{7,9,10}$

Chirality-flipped Coefficients

- fit $\mathcal{C}_{7',9',10'}$

Nuisance Parameters

- fit nuisance parameters
- informative priors
  - form factors: light-cone sum rules [Khodjamirian/Mannel/Pivovarov/Wang '10]
  - power corrections: power-counting assumptions
  - CKM: tree-level fit [UTfit]
  - quark masses [PDG]

Sensitivity to Fit Parameters

Overview

	$\mathcal{C}_{7(')}$	$\mathcal{C}_{9(')}$	$\mathcal{C}_{10(')}$	hadronic parameters
$B_s \to \mu^+\mu^-$	–	–	✓	f_{B_s}
$B \to X_s\gamma$	✓	–	–	2 HQE matrix elements
$B \to X_s\ell^+\ell^-$	✓	✓	✓	2 HQE matrix elements
$B \to K^*\gamma$	✓	–	–	1 FF parameter
$B \to K^*\ell^+\ell^-$	✓	✓	✓	6 FF parameters
$B \to K\ell^+\ell^-$	✓	✓	✓	2 FF parameters

Form Factors

- interplay between $B \to X_s\{\gamma, \ell^+\ell^-\}$ and $B \to K^*\{\gamma, \ell^+\ell^-\}$
- some $B \to K^*\ell^+\ell^-$ obs. form-factor insensitive by construction
- some $B \to K^*\ell^+\ell^-$ obs. dominantly sensitive to form factor ratios

$B \to K^* \ell^+ \ell^-$ $q^2 \in [1,6]\,\mathbf{GeV}^2$, $q^2 \geq M^2_{\psi'}$

- $\mathcal{B}$, A_{FB}, F_{L}, A_{T}^2
- **new**: $A_{\mathrm{T}}^{\mathrm{re}}$, P'_4, P'_5, P'_6
- ATLAS, BaBar, Belle, CDF, CMS, LHCb

$B \to K \ell^+ \ell^-$ $q^2 \in [1,6]\,\mathbf{GeV}^2$, $q^2 \geq M^2_{\psi'}$

- $\mathcal{B}$
- BaBar, Belle, CDF, LHCb

$B_s \to \mu^+ \mu^-$

- $\int \mathrm{d}\tau\, \mathcal{B}(\tau)$
- CMS, LHCb

$B \to K^* \gamma$

- $\mathcal{B}$, $S_{K^*\gamma}$, $C_{K^*\gamma}$
- BaBar, Belle, CLEO

$B \to X_s \gamma$ $E^{\gamma}_{\mathbf{min}} = 1.8\,\mathbf{GeV}$

- $\mathcal{B}$
- BaBar, Belle, CLEO

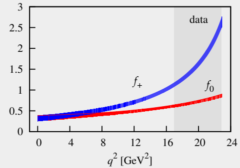
$B \to X_s \ell^+ \ell^-$ $q^2 \in [1,6]\,\mathbf{GeV}^2$

- $\mathcal{B}$
- BaBar, Belle

Further Theory Constraints

Form Factors from Lattice QCD (LQCD) [HPQCD arxiv:1306.2384]

- $B \to K$ form factors available from LQCD
  - data only at high q^2: $17 - 23\,\mathrm{GeV}^2$
  - no data points given
- reproduce 3 data points from z-parametrization
  - $q^2 \in \{17, 20, 23\}\,\mathrm{GeV}^2$
  - use as constraint, incl. covariance matrix



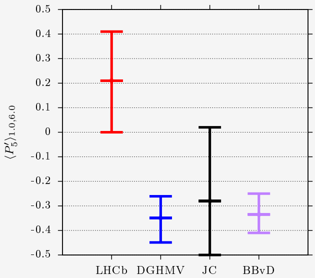
$B \to K^*$ Form Factor (FF) Relation at $q^2 = 0$

- FF $V, A_1 \propto \xi_\perp + \ldots$ [Charles et al. hep-ph/9901378]
  - no α_s corrections [Burdmann/Hiller hep-ph/0011266, Beneke/Feldmann hep-ph/0008255]
  - Large Energy Limit: $V(0) = (1.33 \pm 0.4) \times A_1(0)$
- LCSR constraint: $\xi_\parallel(0) = 0.10^{+0.03}_{-0.02}$, to avoid $\xi_\parallel(q^2) \propto A_0(q^2) < 0$.
- see also FF fits by [Hambrock/Hiller/Schacht/Zwicky 1308.4379]

The $B \to K^* \ell^+ \ell^-$ "Anomaly"

- **LHCb** measurement [1308.1707]
  - deviation from SM prediction in form factor-free obs. $\langle P_5' \rangle_{[1,6]}$
  - LHCb uses one SM prediction (**DGHMV**)

    [Descotes-Genon/Hurth/Matias/Virto 1303.5794]



- however: further SM prediction exist, much larger uncertainty (**JC**)

  [Jäger/Camalich 1212.2263]

- our take on SM prediction $\langle P_5' \rangle_{[1,6]} = -0.34^{+0.09}_{-0.08}$ (**BBvD**)

  see also backups for $P'_{4,6}$ and $[2, 4.3]$ bins

difference: treatment of unknown power corrections
(form factor corrections, $\bar{c}c$ resonances)

Results SM(ν-only)

Pull Values at Best-Fit Point

- largest pulls

-3.4σ $\langle F_L \rangle_{[1,6]}$, BaBar 2012

-2.6σ $\langle F_L \rangle_{[1,6]}$, ATLAS 2013

-2.4σ $\langle P'_4 \rangle_{[14.18,16]}$, LHCb 2013

$+2.5\sigma$ $\langle \mathcal{B} \rangle_{[16,19.21]}$, Belle 2009

$+2.2\sigma$ $\langle A_{\mathrm{FB}} \rangle_{[16,19]}$, ATLAS 2013

$+2.1\sigma$ $\langle P'_5 \rangle_{[1,6]}$, LHCb 2013

p Values

- p value 0.10
- taking out ATLAS, BaBar $\langle F_L \rangle_{[1,6]}$: p value increases to 0.38

Summary

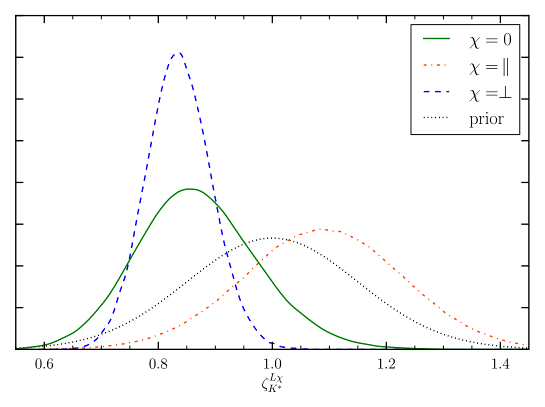
- decent to good fit, no New Physics signal
- we find power corrections on top of QCDF results at large recoil

Parametrization of Power Corrections @ Large Recoil

- six parameters $\zeta_\chi^{L(R)}$ for the $[1, 6]$ bin

$$A_\chi^{L(R)}(q^2) \mapsto \zeta_\chi^{L(R)} A_\chi^{L(R)}(q^2)\,, \quad \chi = \perp, \parallel, 0$$

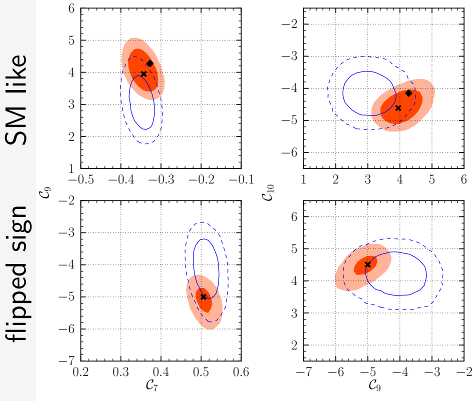
- on top of QCDF correction to transversity amplitudes



- tension diluted by parameters $\zeta_\chi^{L(R)}$
- shift by $\simeq -20\%$ for $\zeta_{\perp,\parallel}^L$
- shift by $\simeq +10\%$ for ζ_0^L
- few percents for ζ_χ^R

improved understanding of power corrections desirable

Results (SM Basis)



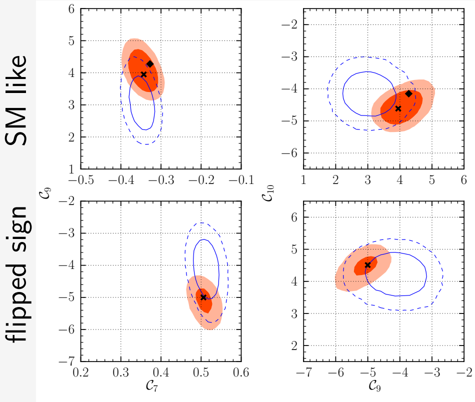
♦: Standard Model, ×: best-fit point

(light-) red: 68% CL (95% CL) for full dataset

blue solid (blue dashed): 68% CL (95% CL) for selection

post HEP'13 (selection)

- with $B \to X_s\{\gamma, \ell^+\ell^-\}$
- $B_s \to \mu^+\mu^-$ from LHCb and CMS
- same data as [Descotes-Genon/Matias/Virto 1307.5683] exclusive decays limited:
  - only $B \to K^*\ell^+\ell^-$!
  - only LHCb data!
  - only $q^2 \in [1, 6]\mathrm{GeV}^2$
- best-fit point similar to [Descotes-Genon et al. 1307.5683]
  - less tension, only $\lesssim 2\sigma$
  - $\mathcal{C}_9 - \mathcal{C}_9^{\mathrm{SM}} \simeq -1.3 \pm 0.5$



♦: Standard Model, ×: best-fit point

(light-) red: 68% CL (95% CL) for full dataset

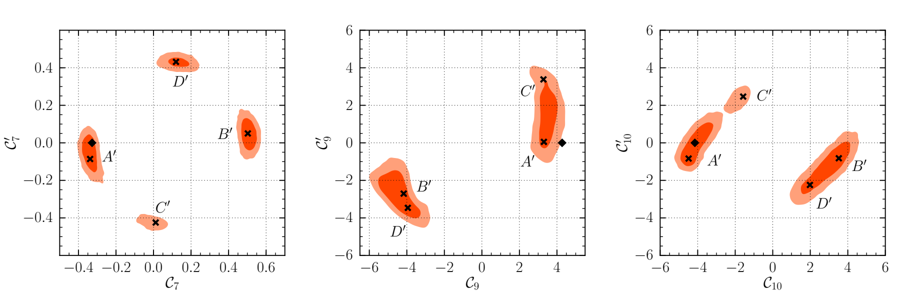
blue solid (blue dashed): 68% CL (95% CL) for selection

post HEP'13 (full)

- SM-like uncertainty reduced by ~ 2 compared to 2012
- SM at the border of 1σ
- flipped-sign barely allowed at 1σ (26% of evidence)
- cannot confirm NP findings
  - in $(\mathcal{C}_7, \mathcal{C}_9)$

    [Descotes-Genon et al. 1307.5683]
- $\zeta_\chi^{L(R)}$ as in SM(ν-only)
- p value: 0.08 (@SM-like sol.)

Results (SM+SM′ Basis)



♦: Standard Model, ×: best-fit points, (light-) red: 68% CL (95% CL) for full dataset

- four solutions A' through D'
  - $A' =$ SM like, 39% of ev.
  - $B' =$ flipped signs, 41% of ev.
  - C', D' suppressed: 5% and 15% of evidence
- for A' (SM-like sol.)
  - p value 0.09
  - $\mathcal{C}_9 - \mathcal{C}_9^{\mathrm{SM}} = -0.8^{+0.2}_{-0.5}$
  - 2σ deviation from SM
  - $\zeta_\chi^{L(R)}$ decrease wrt. SM(ν-only) and SM basis

(Statistical) Model Comparison

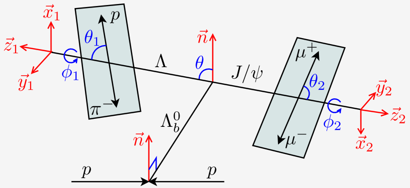
- model comparison using Bayes factor and model priors
- compare scenarios only at SM-like solution $A(')$
- adjust priors to contain only $A(')$
- results
  - SM(ν-only) wins over SM basis: odds of 100:1
  - SM(ν-only) wins over SM+SM$'$ basis: odds of 22:1
  - SM+SM$'$ basis wins over SM basis: odds of 4.5:1

Section 5

Phenomenology of $\Lambda_b \to \Lambda \ell^+ \ell^-$

Kinematics (unpolarized Λ_b)

$$\Lambda_b(p, s_b) \to \Lambda(k, s_\Lambda)(\to N(k_1)\,\pi(k_2))\,\ell^+(q_1)\,\ell^-(q_2)$$



$\theta_2 = \theta_\ell,\ \phi_2 = 0$ $\qquad$ $\theta = 0$

$\theta_1 = \theta_\Lambda,\ \phi_1 = \phi$ $\qquad$ [LHCb '13]

Momenta

$$q = q_1 + q_2$$
$$\bar{q} = q_1 - q_2$$
$$k = k_1 + k_2$$
$$\bar{k} = k_1 - k_2$$

Phase Space

$$4m_\ell^2 \le q^2 \le (M_{\Lambda_b}^2 - M_\Lambda)^2$$
$$-1 \le \cos\theta_\ell \le 1$$
$$k^2 = M_\Lambda^2$$
$$-1 \le \cos\theta_\Lambda \le 1$$
$$0 \le \phi < 2\pi$$

Helicity Angles ($\ell\ell$ and $N\pi$ rest frames)

two hel. angles, similar to $B \to K^*\ell^+\ell^-$

$$k \cdot \bar{q} \propto \cos\theta_\ell$$
$$\bar{k} \cdot q \propto \cos\theta_\Lambda$$
$$\bar{k} \cdot \bar{q} \sim \cos\phi$$

one azimuthal angle ϕ

$\Lambda \to N\pi$

Branching Fractions [PDG '13]

- $\Gamma[\Lambda \to p\pi^-]/\Gamma[\Lambda] = 0.639 \pm 0.005$
- $\Gamma[\Lambda \to n\pi^0]/\Gamma[\Lambda] = 0.358 \pm 0.005$

$\hookrightarrow$ $\Gamma[\Lambda \to N\pi]/\Gamma[\Lambda] = 0.997 \pm 0.008$

$s \to u\bar{u}d$ Decay

- secondary decay $\Lambda \to N\pi$ is *weak* decay

$\hookrightarrow$ parity violation

- one parameter

$$\alpha = 0.642 \pm 0.013$$

$\Lambda_b \to \Lambda$ Hadronic Matrix Elements / Background

Form Factors (for massless leptons)

$$\langle \Lambda(k, s_\Lambda) | \bar{s} \gamma^\mu b | \Lambda_b(p, s_b) \rangle \sim f_\perp^V, f_0^V$$
$$\langle \Lambda(k, s_\Lambda) | \bar{s} \gamma^\mu \gamma_5 b | \Lambda_b(p, s_b) \rangle \sim f_\perp^A, f_0^A$$
$$\langle \Lambda(k, s_\Lambda) | \bar{s} i \sigma^{\mu\nu} q_\nu b | \Lambda_b(p, s_b) \rangle \sim f_\perp^T, f_0^T$$
$$\langle \Lambda(k, s_\Lambda) | \bar{s} i \sigma^{\mu\nu} q_\nu \gamma_5 b | \Lambda_b(p, s_b) \rangle \sim f_\perp^{T5}, f_0^{T5}$$

Non-resonant Background

$$\Lambda_b \to N\pi\ell^+\ell^-$$

- supressed by small CKM matrix element: $\propto V_{tb} V_{td}^* \ll V_{tb} V_{ts}^*$
- Λ: very narrow peak

Form Factor Relations

Relations

- leading in $1/m_b$ expansion

$$f_\perp^{T(T5)} \sim \kappa f_\perp^{V(A)} \qquad\qquad f_0^{T(T5)} \sim \kappa \frac{m_b^2}{q^2} f_0^{V(A)}$$

  universality of transversity amplitudes, compare $\bar{B} \to \bar{K}^*$ FFs
- next-to-leading order in $1/m_b$

$$f_0^{T(T5)} \sim \kappa f_0^{V(A)} + \sum a_n \chi_n(q^2)$$

  $\chi_n(q^2)$: subleading Isgur-Wise functions

Impact on Amplitudes

$$A_0 \propto \mathcal{C}_9 + \frac{m_b^2}{q^2}\mathcal{C}_7 \qquad \delta A = O\left(\frac{\mathcal{C}_7 \Lambda_{\mathrm{QCD}}}{\mathcal{C}_9 m_b}\right) = O\left(1\%\right)$$

Transversity Amplitudes at Low Recoil

Universal Short-Distance Structure

up to corrections $\mathcal{C}_7/\mathcal{C}_9 \times \Lambda/m_b$

$$A_{\perp_1}^{L(R)} = -2NC_+^{L(R)} f_\perp^V \sqrt{s_-} \qquad A_{\parallel_1}^{L(R)} = +2NC_-^{L(R)} f_\perp^A \sqrt{s_+}$$

$$A_{\perp_0}^{L(R)} = +\sqrt{2}NC_+^{L(R)} f_0^V \frac{M_{\Lambda_b} + M_\Lambda}{\sqrt{q^2}} \sqrt{s_-} \qquad A_{\parallel_0}^{L(R)} = -\sqrt{2}NC_-^{L(R)} f_0^A \frac{M_{\Lambda_b} - M_\Lambda}{\sqrt{q^2}} \sqrt{s_+}$$

$s_\pm(q^2)$: kinematic functions

$$C_+^{L(R)} = \left((\mathcal{C}_9 + \mathcal{C}_{9'}) + \frac{m_b^2}{q^2} (\mathcal{C}_7 + \mathcal{C}_{7'}) \mp (\mathcal{C}_{10} + \mathcal{C}_{10'}) \right)$$

$$C_-^{L(R)} = \left((\mathcal{C}_9 - \mathcal{C}_{9'}) + \frac{m_b^2}{q^2} (\mathcal{C}_7 - \mathcal{C}_{7'}) \mp (\mathcal{C}_{10} - \mathcal{C}_{10'}) \right)$$

Angular Distribution

General Case

$$\frac{8\pi}{3}\frac{\mathrm{d}^4\Gamma}{\mathrm{d}q^2\,\mathrm{d}\cos\theta_\ell\,\mathrm{d}\cos\theta_\Lambda\,\mathrm{d}\phi} = K(q^2, \cos\theta_\ell, \sin\theta_\ell, \phi)$$

$$\begin{aligned}
K = {} & \left(K_{1ss}\sin^2\theta_\ell + K_{1cc}\cos^2\theta_\ell + K_{1sc}\sin\theta_\ell\cos\theta_\ell + K_{1s}\sin\theta_\ell + K_{1c}\cos\theta_\ell\right) \\
& + \left(K_{2ss}\sin^2\theta_\ell + K_{2cc}\cos^2\theta_\ell + K_{2sc}\sin\theta_\ell\cos\theta_\ell + K_{2s}\sin\theta_\ell + K_{2c}\cos\theta_\ell\right)\cos\theta_\Lambda \\
& + \left(K_{3ss}\sin^2\theta_\ell + K_{3cc}\cos^2\theta_\ell + K_{3sc}\sin\theta_\ell\cos\theta_\ell + K_{3s}\sin\theta_\ell + K_{3c}\cos\theta_\ell\right)\sin\theta_\Lambda\cos\phi \\
& + \left(K_{4ss}\sin^2\theta_\ell + K_{4cc}\cos^2\theta_\ell + K_{4sc}\sin\theta_\ell\cos\theta_\ell + K_{4s}\sin\theta_\ell + K_{4c}\cos\theta_\ell\right)\sin\theta_\Lambda\sin\phi
\end{aligned}$$

With SM-like and Chirality-flipped Operators

$$K_{1sc} = K_{1s} = K_{2sc} = K_{2s} = K_{3ss} = K_{3cc} = K_{3c} = K_{4ss} = K_{4sc} = K_{4c} = 0$$

Observables

Differential Decay Width

$$\frac{d\Gamma}{dq^2} = K_{1cc} + 2K_{1ss}$$

Forward-Backward Asymmetries

$$\frac{d\Gamma}{dq^2} A_{FB}^{\ell} = \frac{3}{2} K_{1c} \qquad \frac{d\Gamma}{dq^2} A_{FB}^{\Lambda} = \frac{1}{2}\left(K_{2cc} + 2K_{2ss}\right) \qquad \frac{d\Gamma}{dq^2} A_{FB}^{\ell\Lambda} = \frac{1}{4} K_{2c}$$

Chirality-flipped Operators

two new combinations of Wilson coefficients: $\rho_3^{\pm}$, $\rho 4$
complementary to $\rho_1^{\pm}$, ρ_2!

SM basis (no chirality-flipped operators)

$$A_{FB}^{\ell} \sim \frac{\rho_2}{\rho_1} \qquad A_{FB}^{\Lambda} \sim \alpha \qquad A_{FB}^{\ell\Lambda} \sim \alpha \frac{\rho_2}{\rho_1}$$