CKM γ at LHCb

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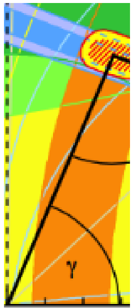
Neckarzimmern, Feb 2013



Plan

- Part I: introduction
- Part II: time integrated measurements
- Part III: time dependent measurements
- Part IV: γ combination

Part I:

Introduction

CKM matrix

Cabibbo
Kobayashi
Maskawa

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \cdot \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Flavor-Eigen-zustände

Massen-Eigen-zustände

$$V^\dagger V = 1$$

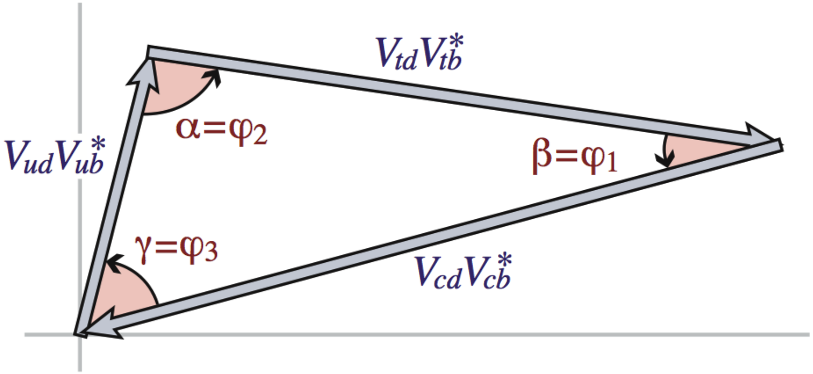
$$V_{\mathrm{CKM}} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & \lambda^3 A(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & \lambda^2 A \\ \lambda^3 A(1-\rho-i\eta) & -\lambda^2 A & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

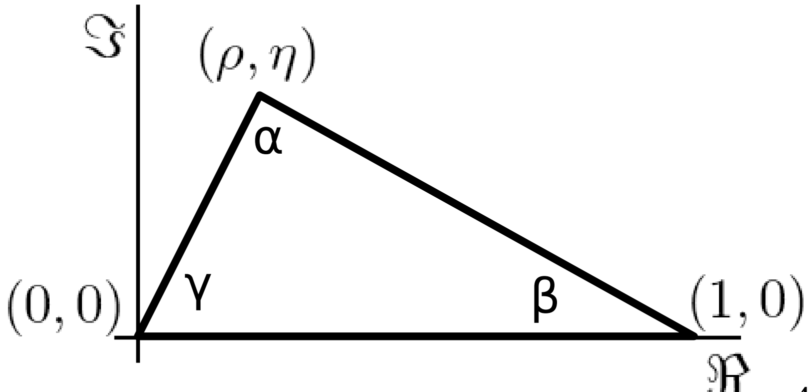
$$\alpha = \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right)$$

$$\beta = \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right)$$

$$\gamma = \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$$

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$





angle γ

- γ is the least well known angle of the unitarity triangle.

"combined γ measurements"		**"full triangle fit"**
$\gamma = (66^{+12}_{-12})^\circ$	CKMfitter ICHEP 2012	$\gamma = (67.7^{+4.1}_{-4.3})^\circ$
$\gamma = (75.5 \pm 10.5)^\circ$	UTfit pre-ICHEP 2012	$\gamma = (68.5 \pm 3.1)^\circ$

- Difficult to measure, as the decay rates are small (they contain V_{ub}...).
- γ can be determined entirely from **tree decays**.
  - this is a unique property among all CP violation parameters
  - examples:

$$\mathrm{BR}(B^- \to DK^-, D \to K_S\pi^+\pi^-) = 3.7\times10^{-4}\cdot 2.8\times10^{-2} = 10^{-5}$$

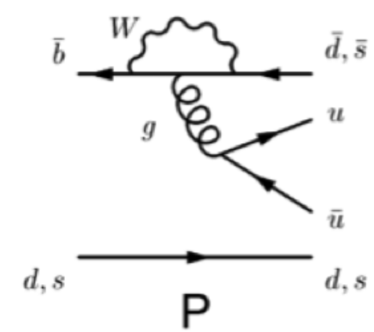
$$\mathrm{BR}(B^- \to DK^-, D \to \pi K) \approx 2\times10^{-7}$$

(!!) LHCb first observation with 100 events

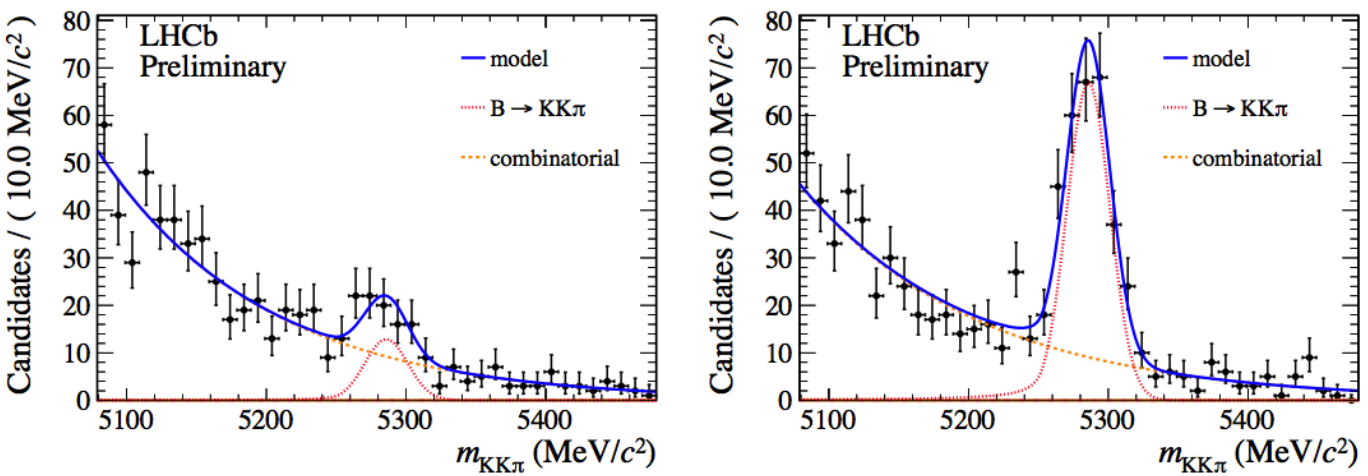
angle γ

- **Tree** decays:
  - negligible theoretical uncertainty: $\delta\gamma/\gamma = \mathcal{O}(10^{-6})$
  - provides an important Standard Model set point ("standard candle")
  - hadronic parameters can all be determined from the data

- γ from **loop** decays:
  - for example B → hh (or at a later point B → hhh)
  - one can (eventually) look for New Physics by comparing to tree decays



$B^- \to K^+K^-\pi^-$ $B^+ \to K^+K^-\pi^+$

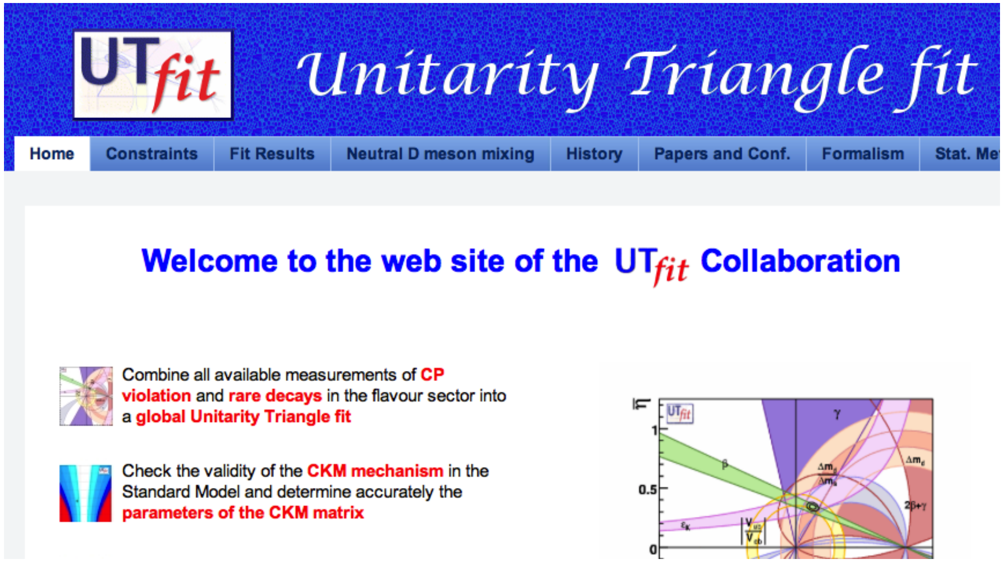


CP asymmetry in B → KKπ in selected kinematic range

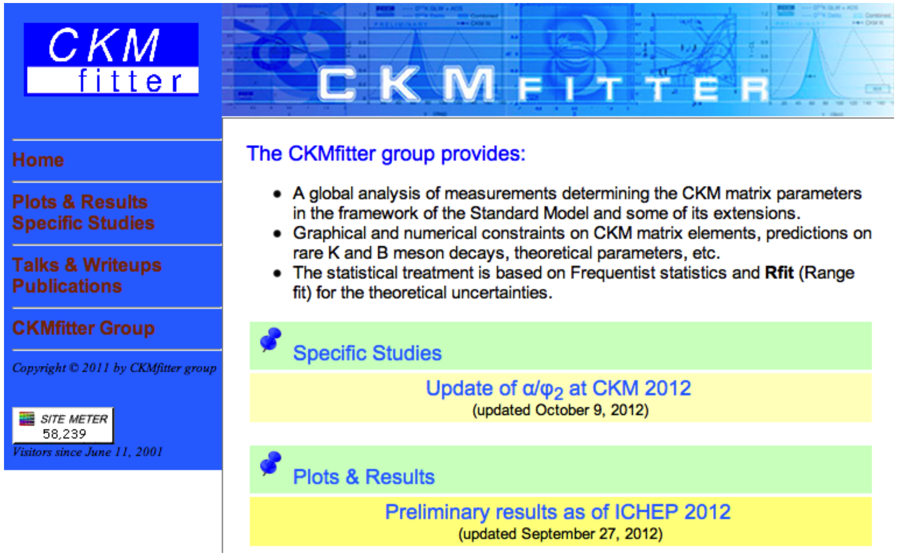
LHCb-CONF-2012-028

angle γ

- The γ-related equations contain features that make the statistical treatment very **challenging**.
- There are two established groups combining γ measurements: **UTfit** (Bayesian) and **CKMfitter** (frequentist). There are always lively discussions.
- **HFAG** humbly refrains.
- Both B factories (BaBar, Belle) and LHCb have performed their own γ combinations (all frequentist).

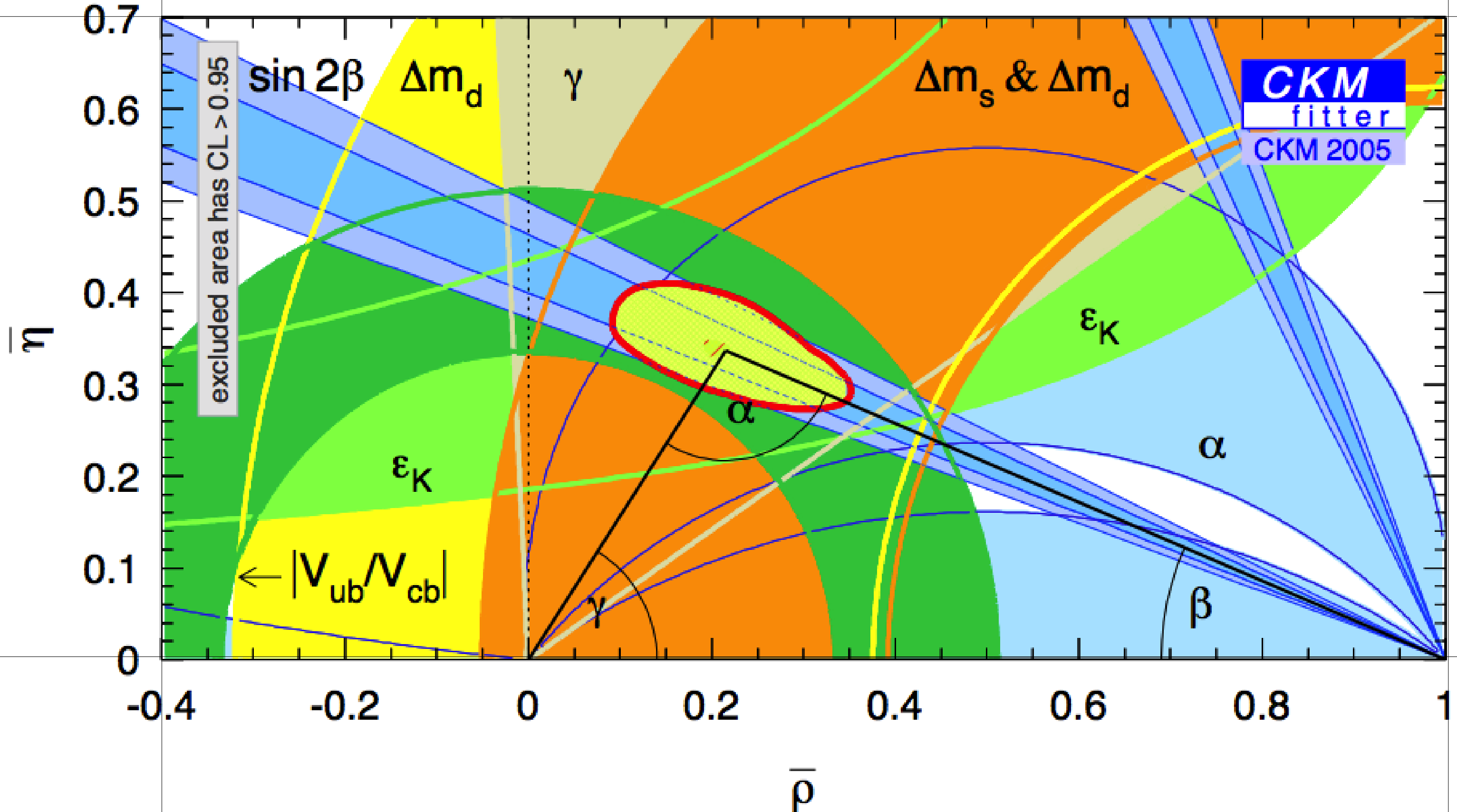


Bayesian



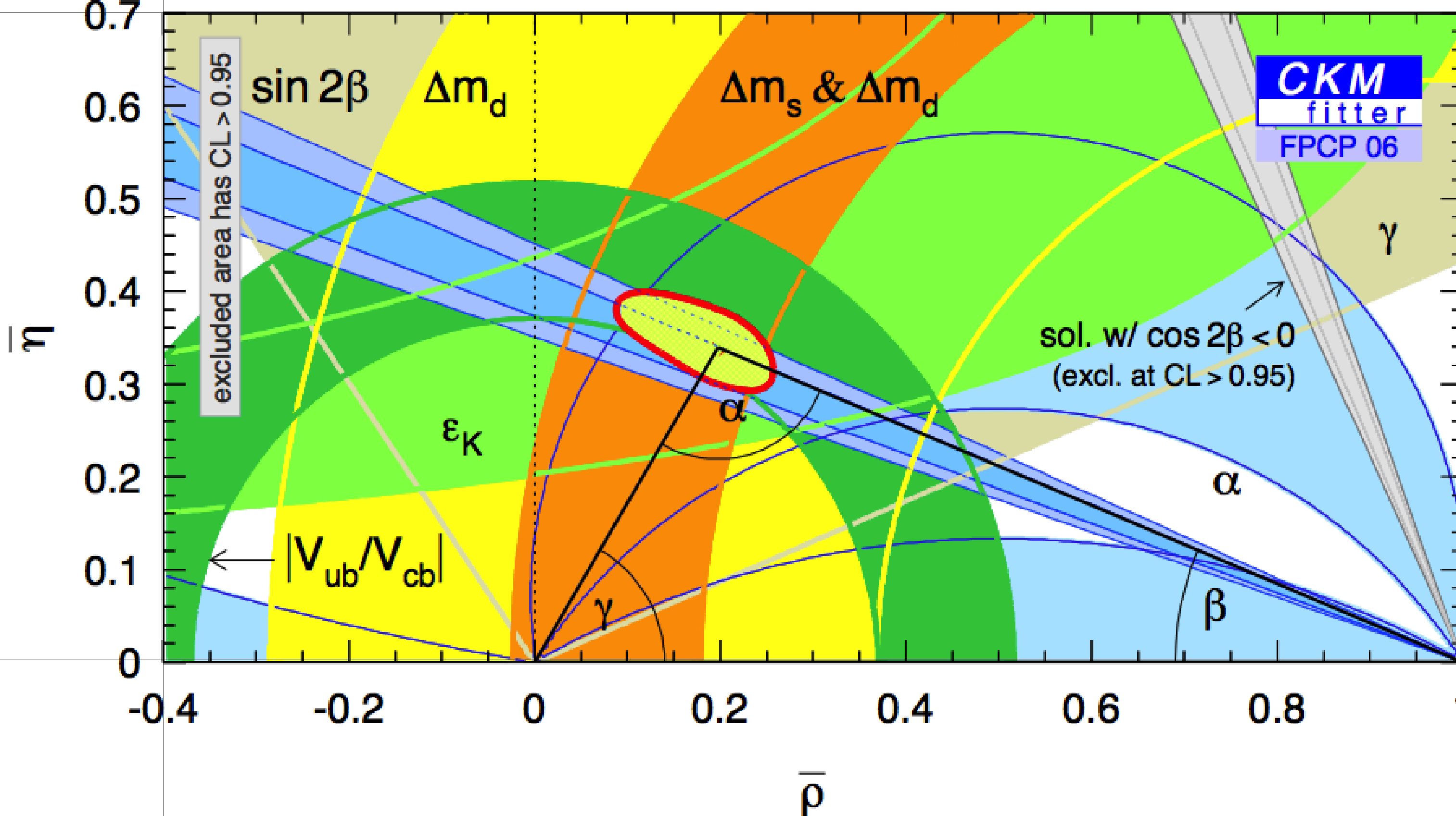
frequentist

2005



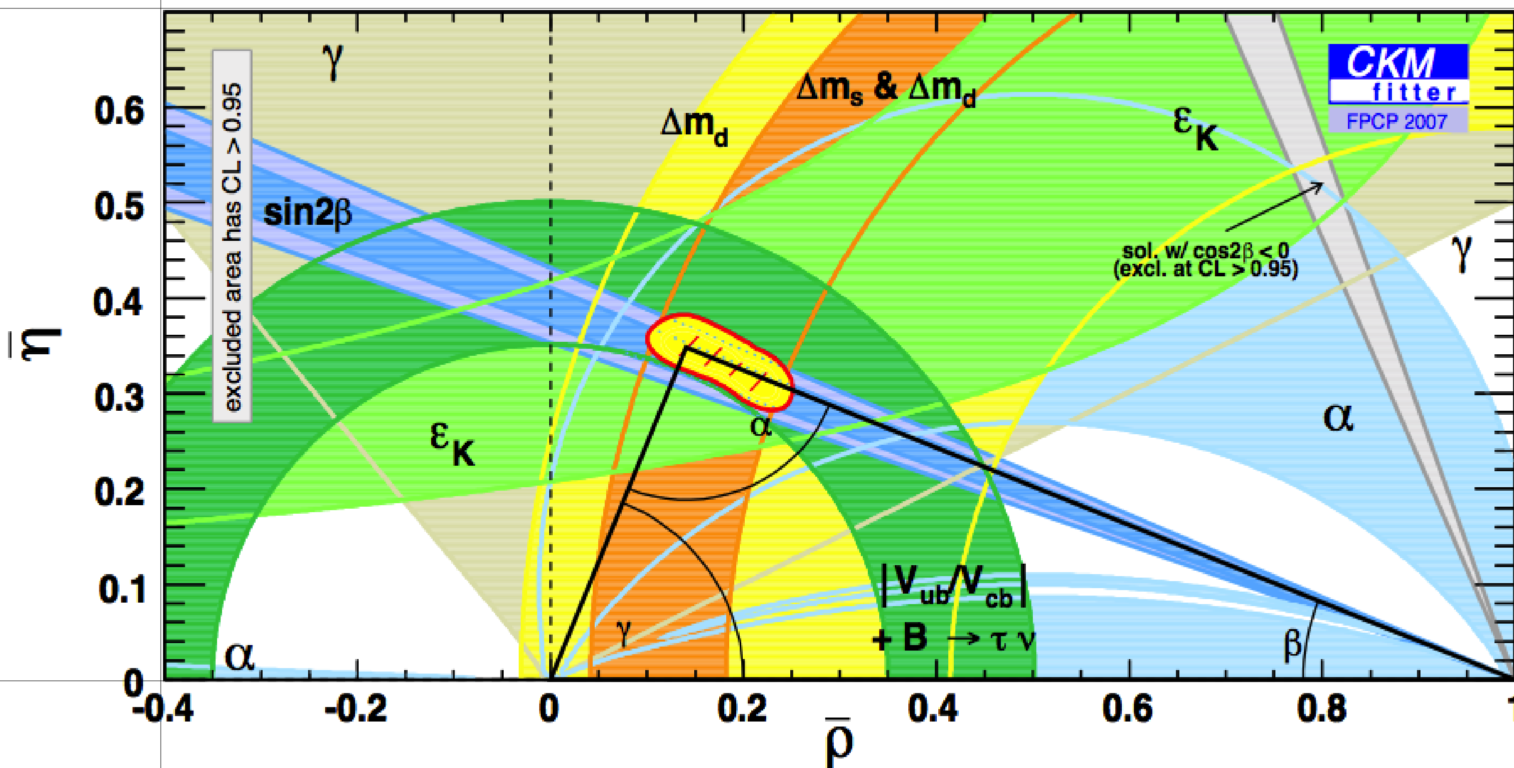
sin 2β
Δm_d
γ
Δm_s & Δm_d
CKM fitter
CKM 2005
excluded area has CL > 0.95
α
ε_K
α
ε_K
γ
β
←|V_ub/V_cb|
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
η̄
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
ρ̄

2006



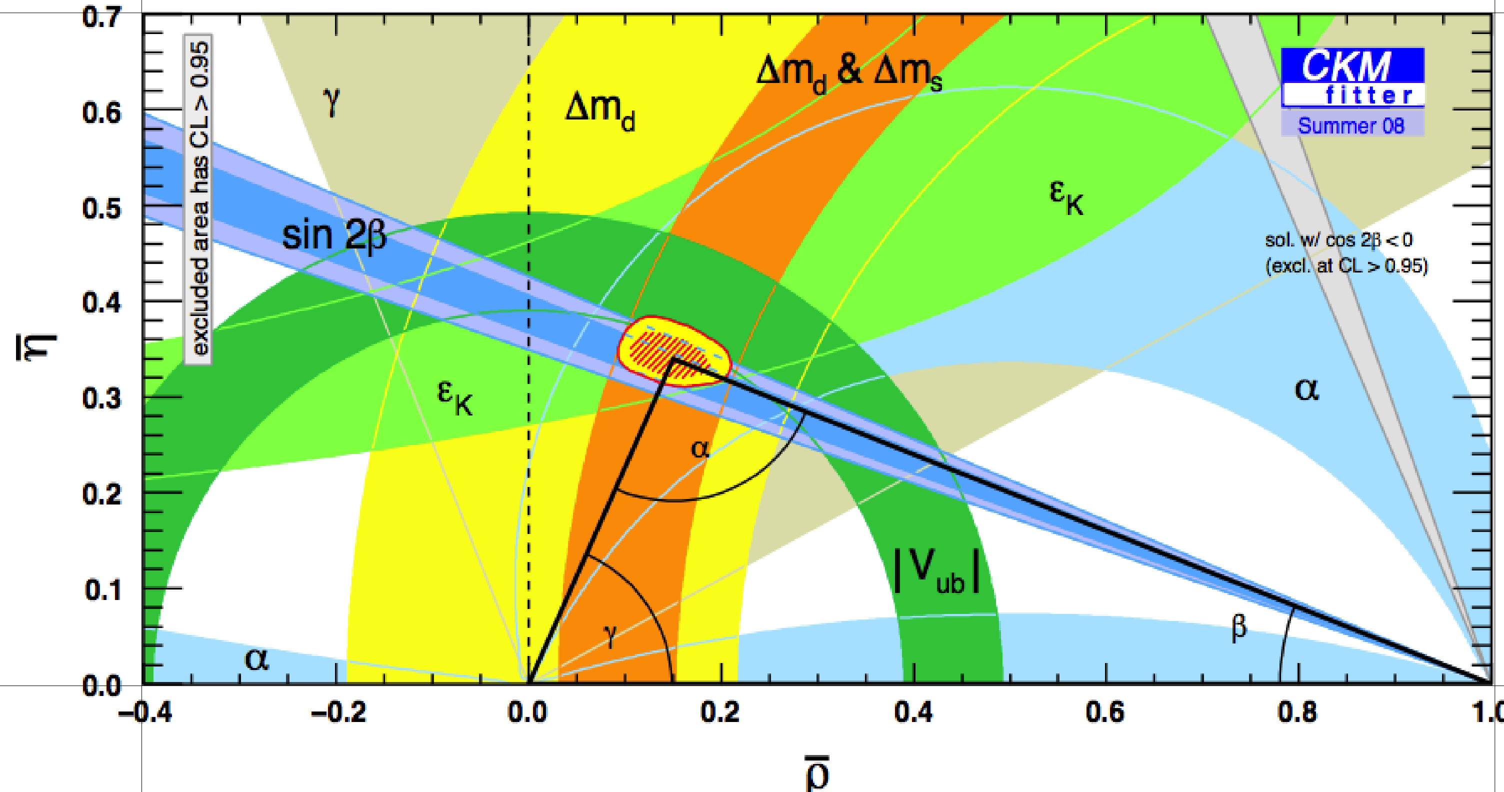
sin 2β
Δm_d
Δm_s & Δm_d
CKM fitter
FPCP 06
excluded area has CL > 0.95
γ
sol. w/ cos 2β < 0
(excl. at CL > 0.95)
α
ε_K
α
|V_ub/V_cb|
γ
β
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
η̄
ρ̄

2007



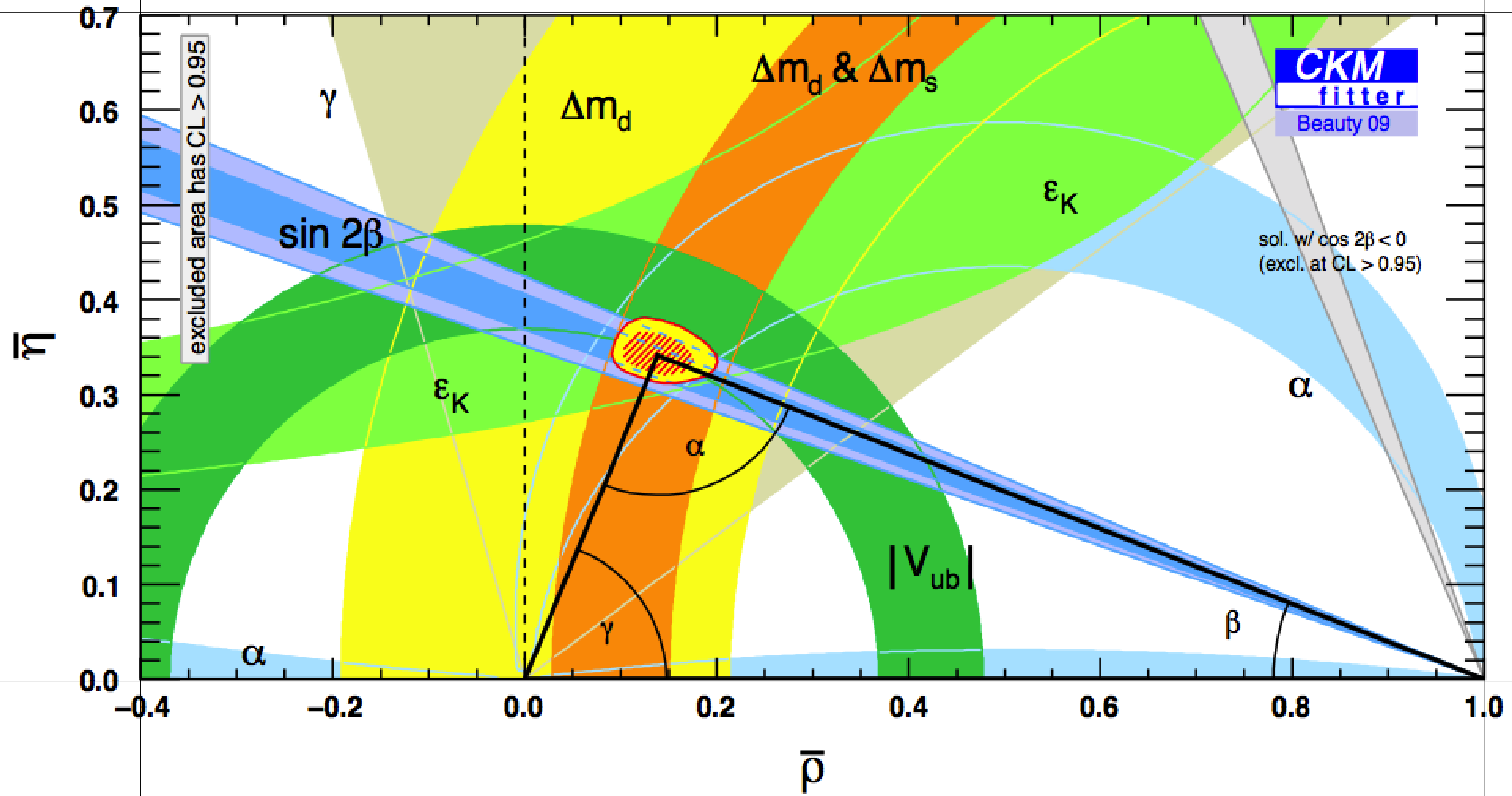
CKM fitter
FPCP 2007
excluded area has CL > 0.95
γ
sin2β
Δm$_d$
Δm$_s$ & Δm$_d$
ε$_K$
sol. w/ cos2β < 0
(excl. at CL > 0.95)
γ
α
ε$_K$
α
|V$_{ub}$/V$_{cb}$|
+ B → τ ν
γ
β
α
$\bar{\eta}$
$\bar{\rho}$
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1

2008



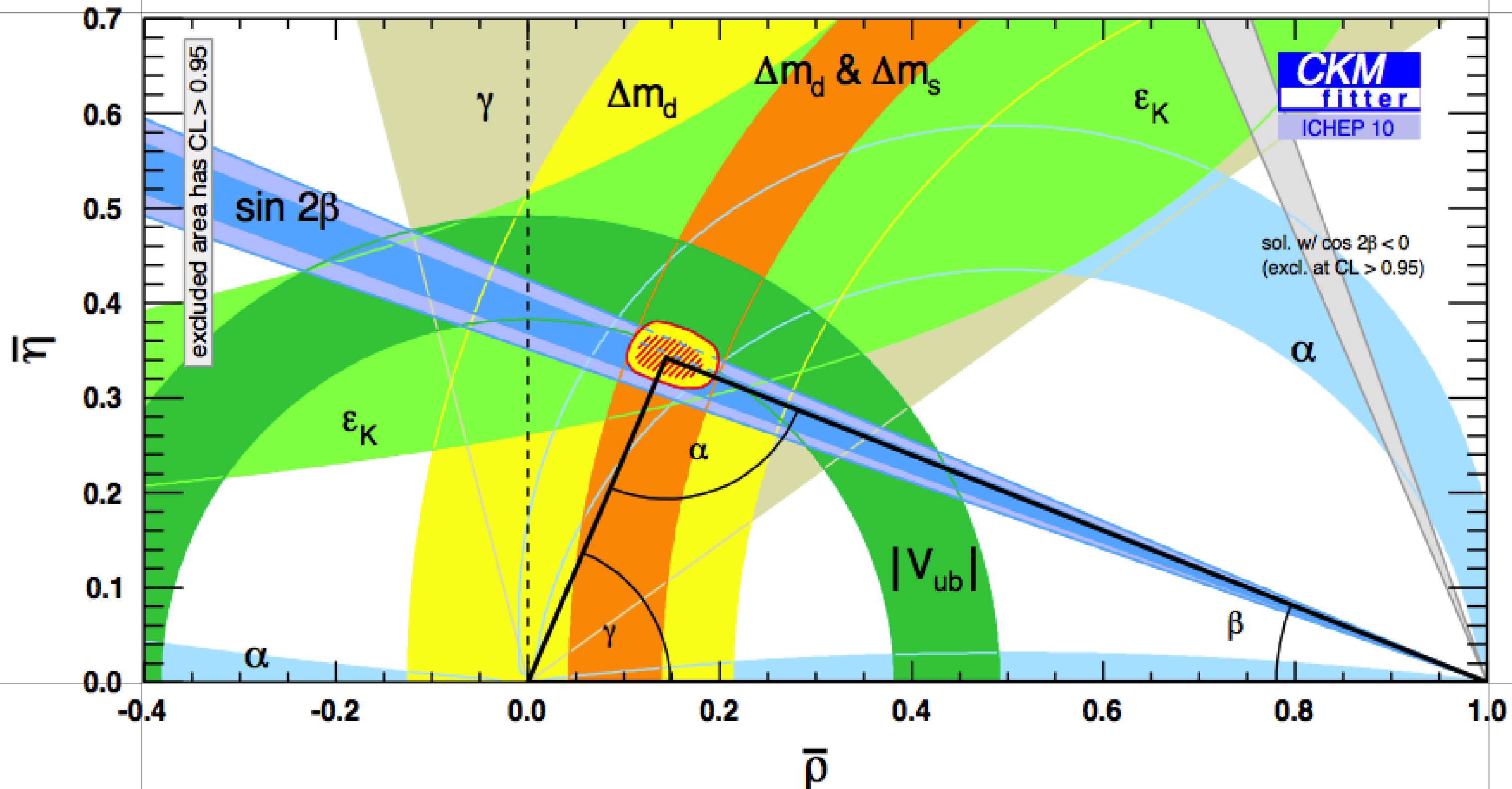
CKM fitter
Summer 08
excluded area has CL > 0.95
γ
Δm_d
Δm_d & Δm_s
ε_K
sin 2β
sol. w/ cos 2β < 0
(excl. at CL > 0.95)
α
|V_ub|
β
η̄
ρ̄
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
−0.4
−0.2
0.0
0.2
0.4
0.6
0.8
1.0

2009



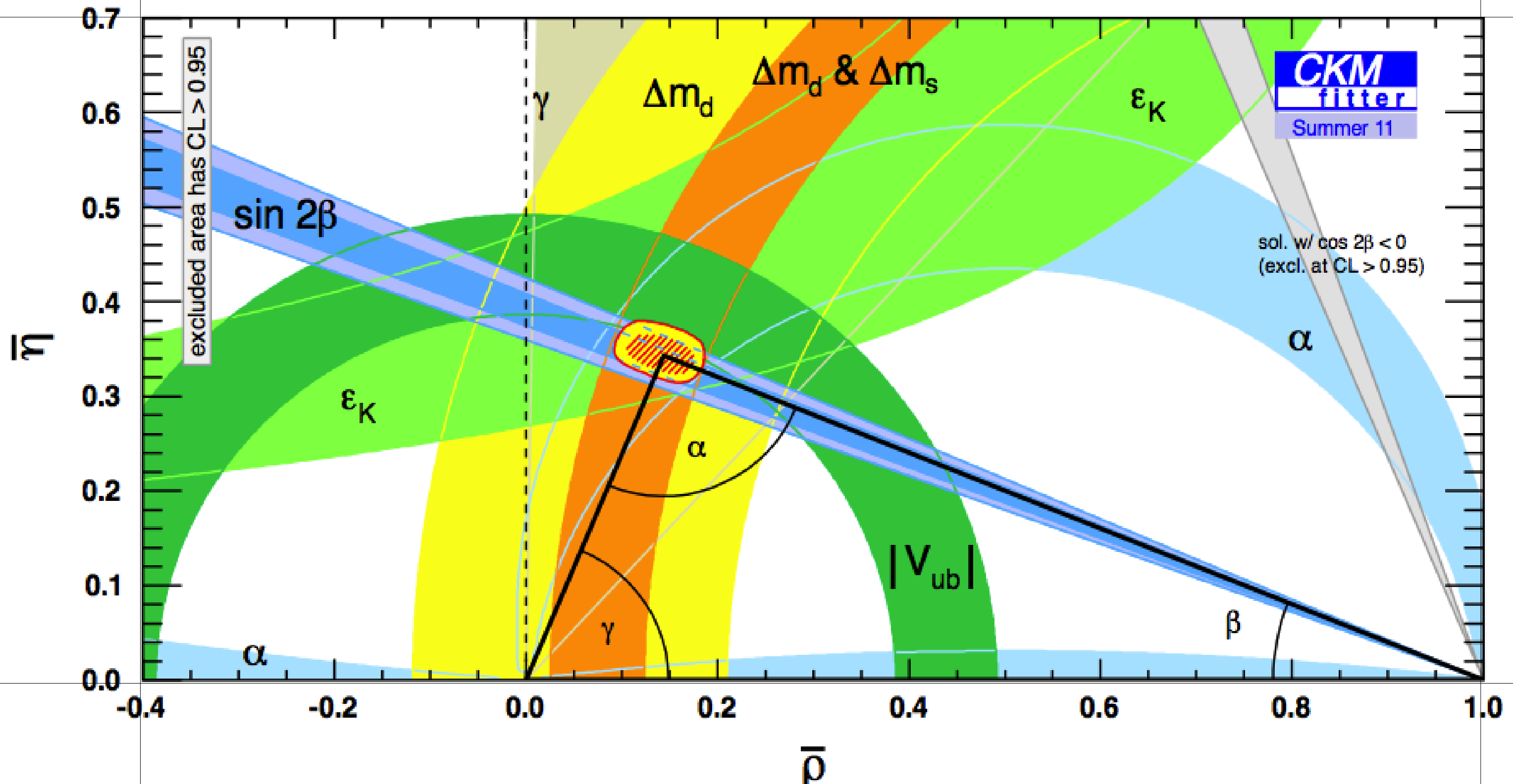
excluded area has CL > 0.95
γ
Δm_d
Δm_d & Δm_s
CKM fitter
Beauty 09
sin 2β
ε_K
sol. w/ cos 2β < 0
(excl. at CL > 0.95)
α
|V_ub|
β
γ
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
−0.4
−0.2
0.0
0.2
0.4
0.6
0.8
1.0
η̄
ρ̄

2010



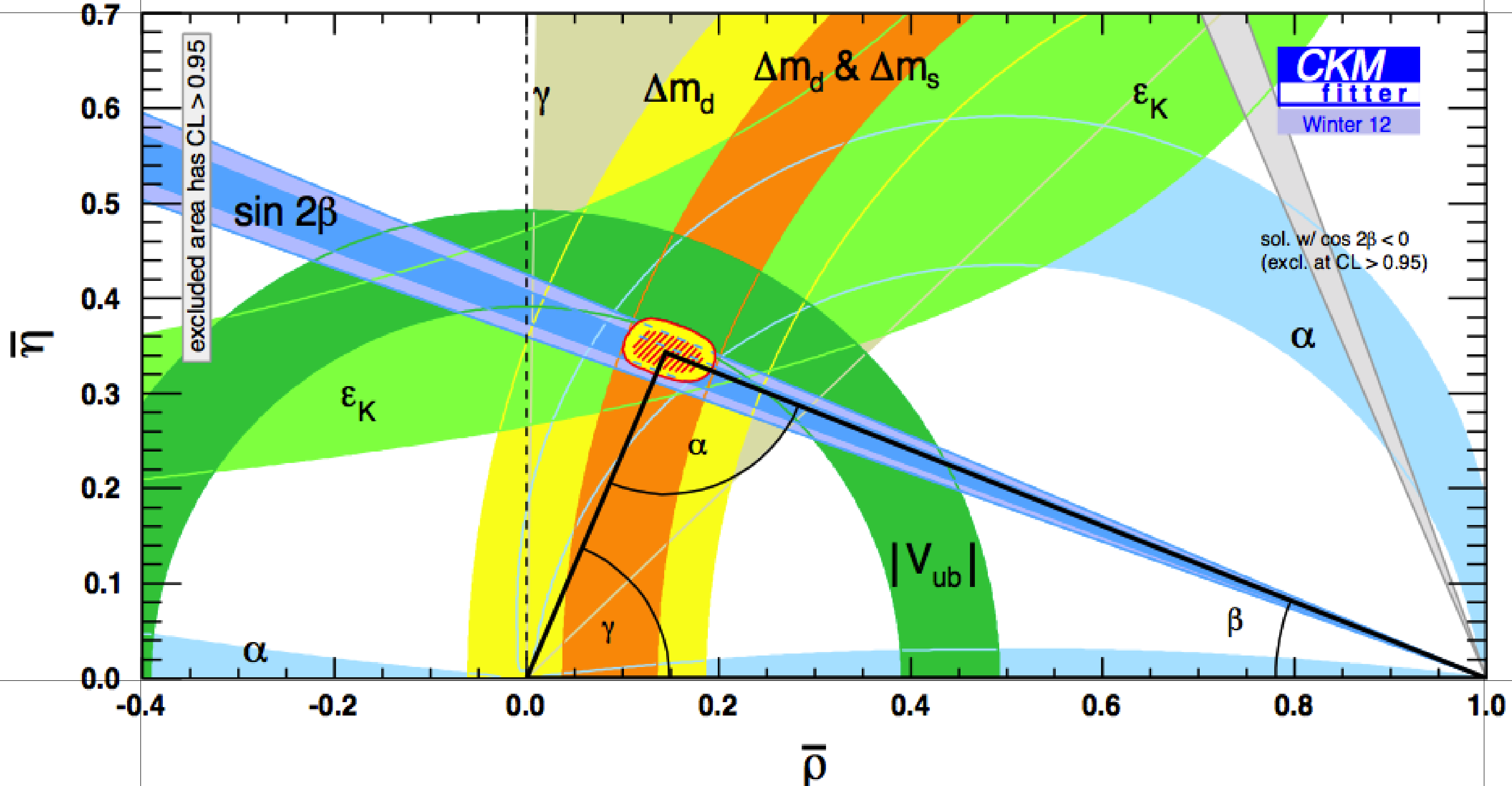
excluded area has CL > 0.95
γ
Δm_d
Δm_d & Δm_s
ε_K
CKM fitter
ICHEP 10
sin 2β
sol. w/ cos 2β < 0
(excl. at CL > 0.95)
α
ε_K
α
|V_ub|
γ
β
α
η̄
ρ̄
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
-0.4
-0.2
0.0
0.2
0.4
0.6
0.8
1.0

2011



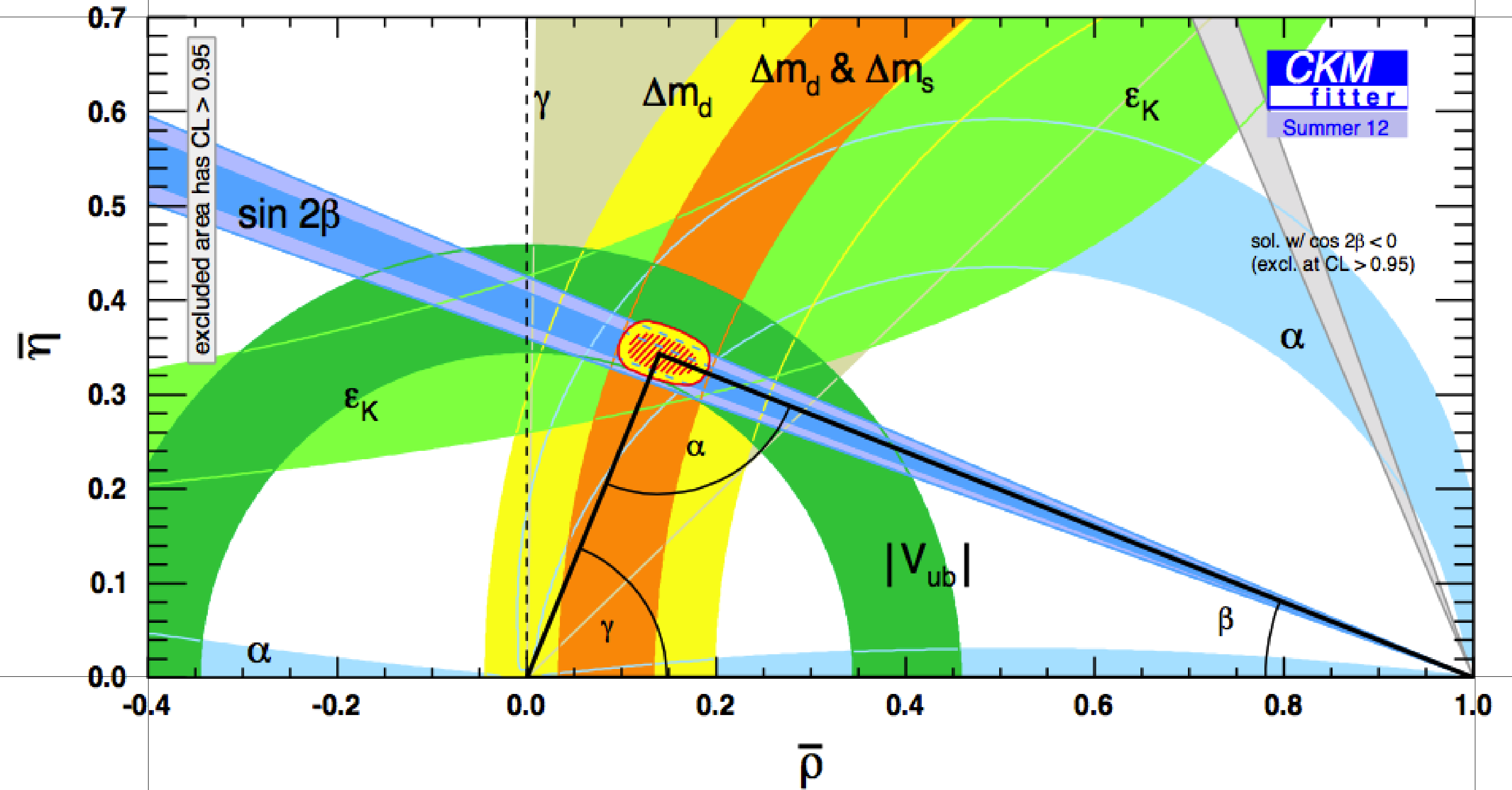
excluded area has CL > 0.95
sin 2β
γ
Δm_d
Δm_d & Δm_s
ε_K
CKM fitter
Summer 11
sol. w/ cos 2β < 0
(excl. at CL > 0.95)
α
ε_K
α
|V_ub|
γ
β
α
η̄
ρ̄
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
-0.4
-0.2
0.0
0.2
0.4
0.6
0.8
1.0

2012



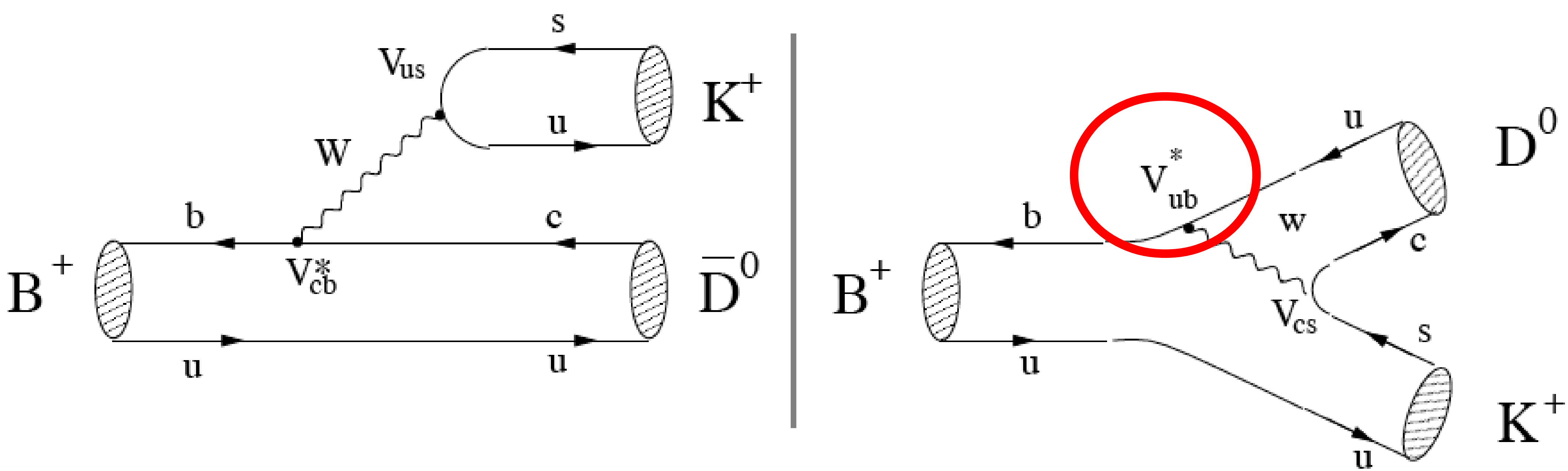
excluded area has CL > 0.95
sin 2β
γ
Δm_d
Δm_d & Δm_s
ε_K
CKM fitter
Winter 12
sol. w/ cos 2β < 0
(excl. at CL > 0.95)
α
ε_K
α
γ
|V_ub|
β
α
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
-0.4
-0.2
0.0
0.2
0.4
0.6
0.8
1.0
η̄
ρ̄

2012



excluded area has CL > 0.95
γ
Δm_d
Δm_d & Δm_s
ε_K
CKM fitter
Summer 12
sin 2β
sol. w/ cos 2β < 0
(excl. at CL > 0.95)
α
ε_K
α
γ
|V_ub|
β
α
η̄
ρ̄

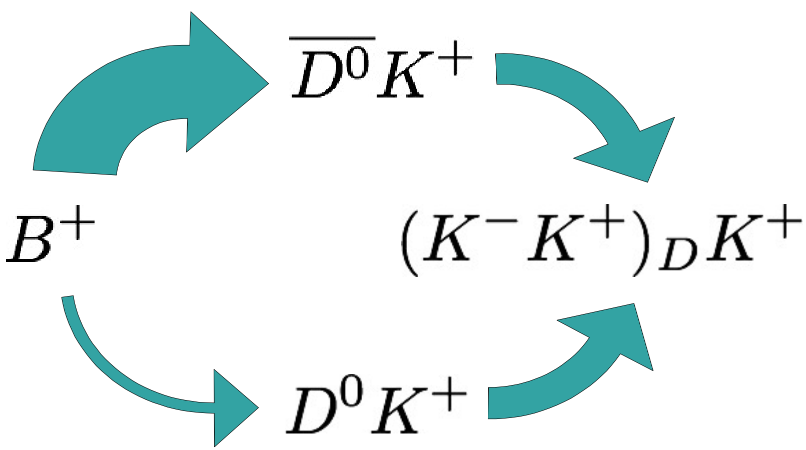
$B \to DK$



- This was, and still is, the most important channel to measure γ.
- We need to reconstruct the D / D-bar in a final state accessible to both to achieve interference.
- Choice of final state labels the "method": GLW, ADS, GGSZ
- Can also use excited D's and K's
- Or any final state with the same K quantum numbers, e.g. $B \to DK\pi\pi$

$B \to DK$

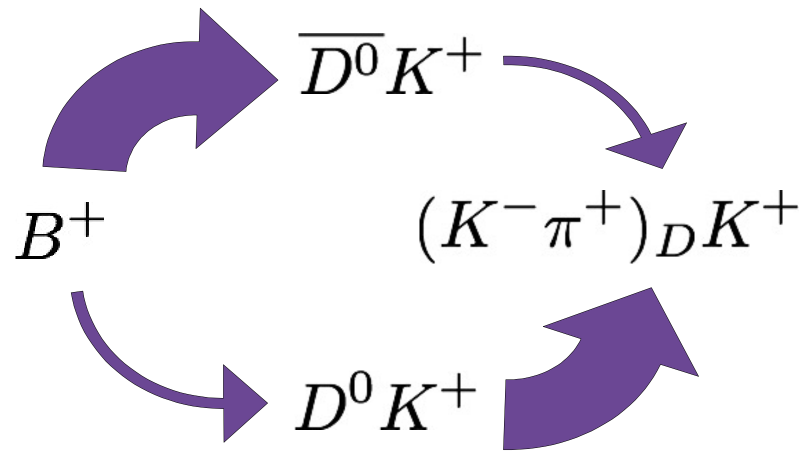
"GLW"



Phys.Lett. B253 (1991) 483
Phys.Lett. B265 (1991) 172

Gronau, London, Wyler

"ADS", "suppressed"



Phys.Rev.Lett 78 (1997) 3257
Phys.Rev. D63 (2001) 036005

Atwood, Dunietz, Soni

$B \to DK$

- **GLW**
  - Use CP eigenstates such as **$D \to K^+K^-$**
  - Therefore $r_D = 1$ and $\delta_D = 0, \pi$ (for CP+, CP-)
  - Normalize rates to the Cabibbo-allowed $D \to K^+\pi^-$

measure yields

- **ADS**
  - Use doubly Cabibbo-suppressed states such as **$D \to K^-\pi^+$**
  - Enhanced interference (allowed → suppressed / suppressed → allowed) but bad statistics.
  - needs external input on hadronic parameters r_D and δ_D

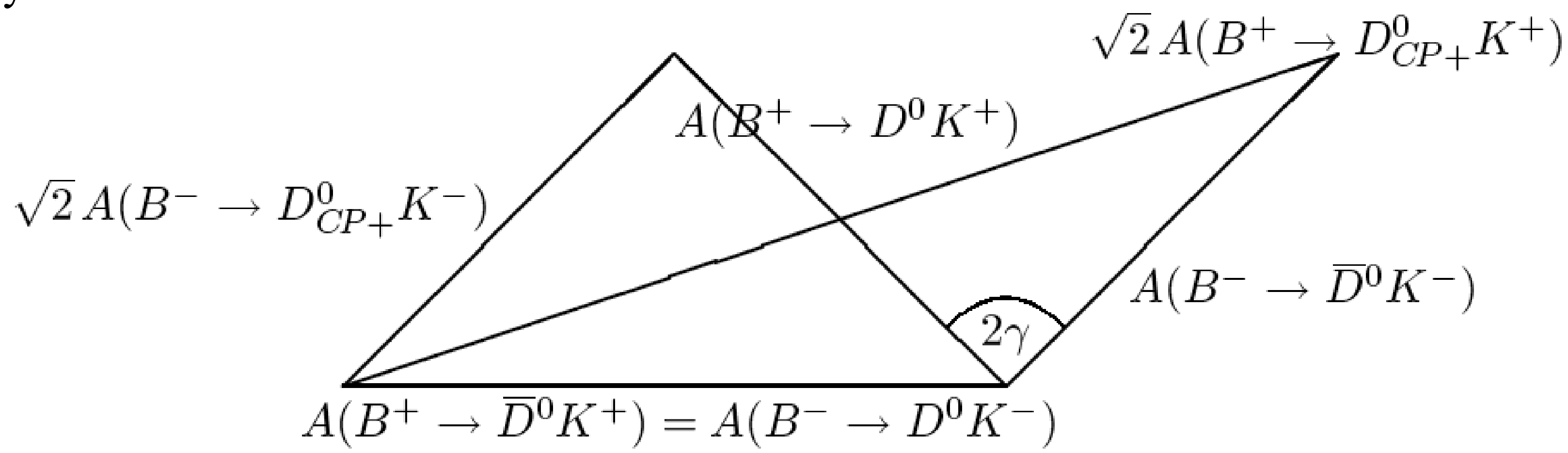
measure yields

- **GGSZ** ("Dalitz")
  - Use 3-body self-conjugate modes such as **$D \to K_S\pi^+\pi^-$**
  - hadronic D parameters vary across Dalitz plot
  - Model dependent or independent

fit Dalitz plot

Gronau, London, Wyler

originally:



modified:

$$R_{CP\pm} = \frac{\Gamma(B^- \to D^0_{CP\pm}K^-) + \Gamma(B^+ \to D^0_{CP\pm}K^+)}{\left[\Gamma(B^- \to D^0 K^-) + \Gamma(B^+ \to \overline{D}^0 K^+)\right]/2}$$

$$A_{CP\pm} = \frac{\Gamma(B^- \to D^0_{CP\pm}K^-) - \Gamma(B^+ \to D^0_{CP\pm}K^+)}{\Gamma(B^- \to D^0_{CP\pm}K^-) + \Gamma(B^+ \to D^0_{CP\pm}K^+)}$$

$$R_{CP\pm} = 1 + r^2 \pm 2r\cos\delta_s\cos\gamma,$$

$$A_{CP\pm} = \frac{\pm 2r\sin\delta_s\sin\gamma}{R_{CP\pm}}.$$

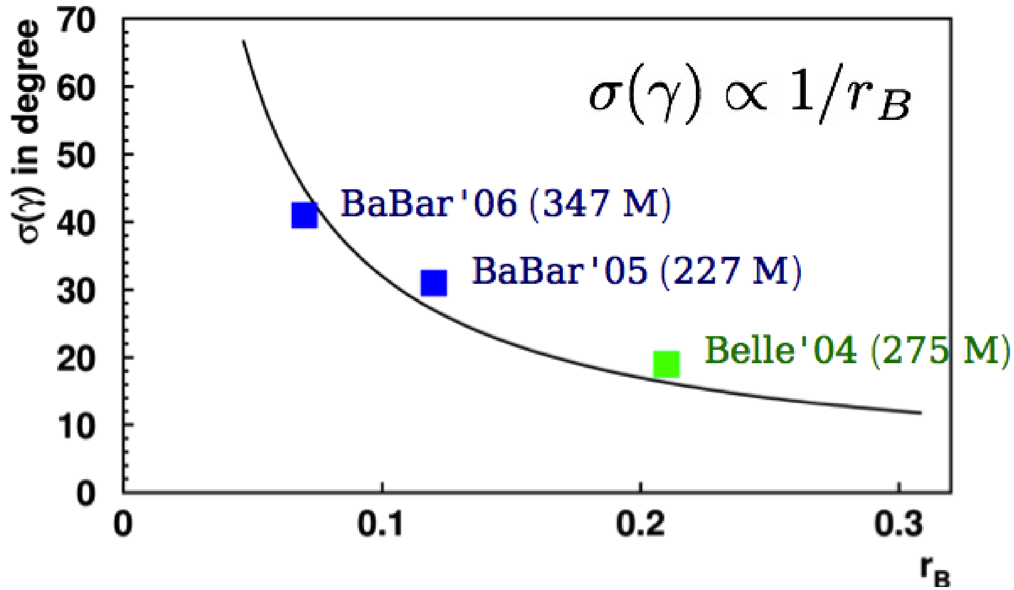
$A_{CP\pm} = 0$ means no direct CP violation – but still can measure γ!

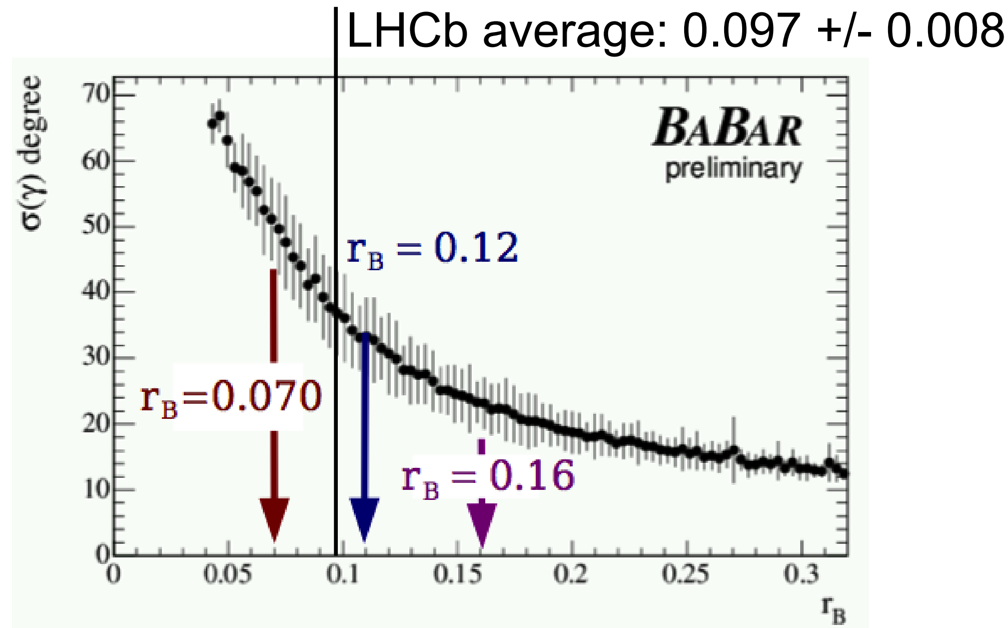
Eight-fold ambiguity in the angle γ.

amplitude ratio r_B

The B amplitude ratio drives the sensitivity:

$$r_B \equiv |A(b \to u)/A(b \to c)| \qquad r_B = \frac{A(B^+ \to D^0 K^+)}{A(B^+ \to \overline{D^0} K^+)}$$





Plots taken from slides by Karim Tabelsi:

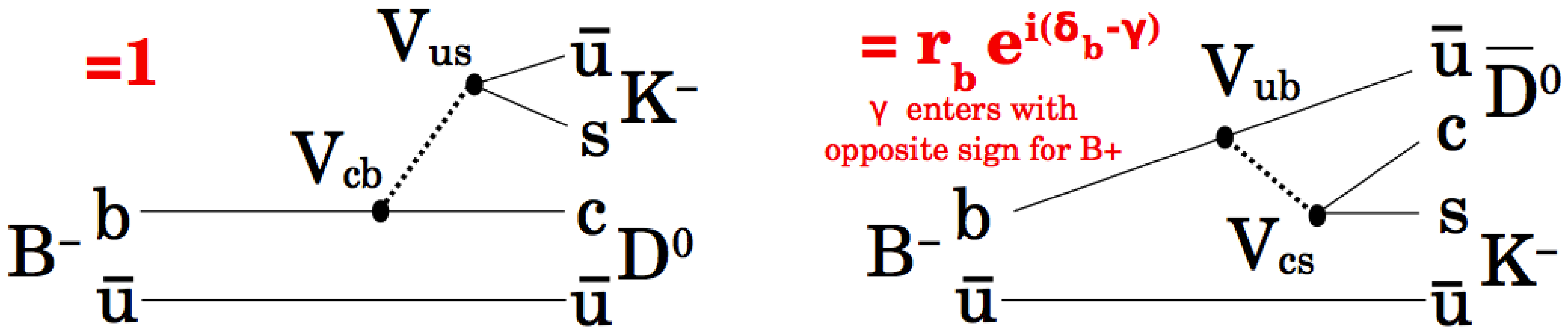
http://beauty2009.physi.uni-heidelberg.de/Programme/talks/tuesday-session4/karim_ckmfitter.pdf

http://agenda.infn.it/getFile.py/access?contribId=115&sessionId=29&resId=0&materialId=slides&confId=1066

Part II:

time integrated observables

Derivation of usual observables



Define B decay amplitudes:

$$A(B^- \to D^0 K^-) = A_c e^{i\delta_c} ,$$

$$A(B^- \to \overline{D}^0 K^-) = A_u e^{i(\delta_u - \gamma)}$$

CP conjugated amplitudes:

$$A(B^+ \to \overline{D}^0 K^+) = A_c e^{i\delta_c} ,$$

$$A(B^+ \to D^0 K^+) = A_u e^{i(\delta_u + \gamma)}$$

strong phase:
constant under CP

weak phase:
sign under CP

A's: real, positive

Derivation of usual observables

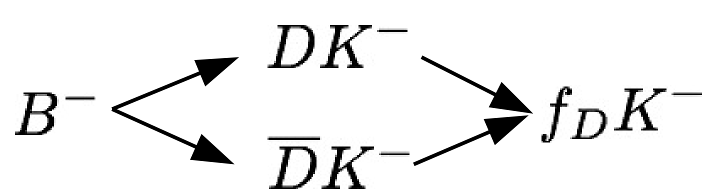
Define D decay amplitudes:

$$A(D^0 \to f) = A_f e^{i\delta_f}$$
$$A(D^0 \to \bar{f}) = A_{\bar{f}} e^{i\delta_{\bar{f}}}$$

CP conjugated amplitudes (neglecting CP violation in D decay):

$$A(\overline{D}^0 \to \bar{f}) = A_f e^{i\delta_f}$$
$$A(\overline{D}^0 \to f) = A_{\bar{f}} e^{i\delta_{\bar{f}}}$$
$$(A_f \equiv \bar{A}_{\bar{f}},\ A_{\bar{f}} \equiv \bar{A}_f)$$

Form combined amplitude:



$$A(B^- \to D[\to f]K^-) = A_c A_f e^{i(\delta_c + \delta_f)} + A_u A_{\bar{f}} e^{i(\delta_u + \delta_{\bar{f}} - \gamma)}$$

Take squared modulus to get rate:

master equation

$$\Gamma(B^- \to D[\to f]K^-) = A_c^2 A_f^2 + A_u^2 A_{\bar{f}}^2 + 2 A_c A_f A_u A_{\bar{f}} \Re(e^{i(\delta_B + \delta_D - \gamma)})$$
$$= A_c^2 A_{\bar{f}}^2 \left(r_D^2 + r_B^2 + 2 r_B r_D \cos(\delta_B + \delta_D - \gamma)\right)$$

Derivation of usual observables

Have defined amplitude ratios:

$$r_B = A_u/A_c$$
$$r_D = A_f/A_{\bar{f}}$$

Have defined strong phase differences:

$$\delta_B = \delta_u - \delta_c$$
$$\delta_D = \delta_{\bar{f}} - \delta_f$$

Sum with other B charge: $\cos(a-b) + \cos(a+b) = 2\cos(a)\cos(b)$

$$\Gamma(B^- \to D[\to f]h^-) + \Gamma(B^+ \to D[\to f]h^+)$$
$$= 2A_c^2 A_{\bar{f}}^2 \left(r_D^2 + r_B^2 + 2r_B r_D \cos(\delta_B + \delta_D)\cos\gamma\right)$$

Analogously form the difference, divide: $\cos(a-b) - \cos(a+b) = 2\sin(a)\sin(b)$

$$A_{CP} = \frac{\Gamma(B^- \to D[\to f]h^-) - \Gamma(B^+ \to D[\to f]h^+)}{\Gamma(B^- \to D[\to f]h^-) + \Gamma(B^+ \to D[\to f]h^+)}$$
$$= \frac{2r_B r_D \sin(\delta_B + \delta_D)\sin\gamma}{r_D^2 + r_B^2 + 2r_B r_D \cos(\delta_B + \delta_D)\cos\gamma},$$

observable:
CP asymmetry

Derivation of usual observables

We can also form the ratio to rates from **another D decay.** Example: D → KK and D → Kπ

$$R = \frac{\Gamma(B^- \to D[\to f]h^-) + \Gamma(B^+ \to D[\to f]h^+)}{\Gamma(B^- \to D[\to g]h^-) + \Gamma(B^+ \to D[\to g]h^+)}$$

$$= \frac{A_{\bar{f}}^2}{A_{\bar{g}}^2} \cdot \frac{r_{Df}^2 + r_B^2 + 2r_B r_{Df}\cos(\delta_B + \delta_{Df})\cos\gamma}{r_{Dg}^2 + r_B^2 + 2r_B r_{Dg}\cos(\delta_B + \delta_{Dg})\cos\gamma}$$

observable: charge-averaged ratio

Residual term:

$$\frac{A_{\bar{f}}^2}{A_{\bar{g}}^2} = \frac{\mathcal{B}(D \to \bar{f})}{\mathcal{B}(D \to \bar{g})}$$

Sometimes it is more convenient to work with the inverse ratio rD = rD':

$$\Gamma(B^- \to D[\to f]h^-) = A_c^2 A_f^2 \left(1 + r_B^2 r_D'^2 + 2r_B r_D' \cos(\delta_B + \delta_D - \gamma)\right)$$

$$R = \frac{A_{\bar{f}}^2}{A_g^2} \cdot \frac{r_{Df}^2 + r_B^2 + 2r_B r_{Df}\cos(\delta_B + \delta_{Df})\cos\gamma}{1 + r_B^2 r_{Dg}'^2 + 2r_B r_{Dg}'\cos(\delta_B + \delta_{Dg})\cos\gamma}$$

Derivation of usual observables

In case of GLW, we know specific hadronic values:

rD = 1 (CP eigenstates!) and
δD = 0 (CP+ eigenvalue 1) and
δD = π (CP- eigenvalue -1)

Also, rB ~ 0.1 and rD(Kπ) ~ 0.06, thus the denominator is sometimes assumed to equal 1:

$$R = \underbrace{\frac{\mathcal{B}(D \to K^+K^-)}{\mathcal{B}(D^0 \to K^-\pi^+)}}_{= 0.1021 \pm 0.0024} \cdot (1 + r_B^2 \pm 2 r_B \cos\delta_B \cos\gamma)$$

Or, when using the “CP notation”:

$$D_{CP\pm} = \frac{D^0 \pm \overline{D}^0}{\sqrt{2}} \qquad \frac{\mathcal{B}(D \to f_{CP})}{\mathcal{B}(D \to f_{\text{flavor}})} = \frac{1}{2}$$

$$R_{CP\pm} = 2 \cdot \frac{\Gamma(B^- \to D[\to CP\pm]K^-) + \Gamma(B^+ \to D[\to CP\pm]K^+)}{\Gamma(B^- \to D[\to \text{flav}]K^-) + \Gamma(B^+ \to D[\to \text{flav}]K^+)}$$
$$= 1 + r_B^2 \pm 2 r_B \cos\delta_B \cos\gamma \ .$$

Derivation of usual observables

One can also form the ratio to a **different B decay**. (Example: B → Dπ)

Then, we need additional hadronic B decay parameters.

$$r_B \to r_{DK},\ r_{D\pi}$$
$$\delta_B \to \delta_{DK},\ \delta_{D\pi}$$

$$R_{K/\pi} = \frac{\Gamma(B^- \to D[\to f]K^-) + \Gamma(B^+ \to D[\to f]K^+)}{\Gamma(B^- \to D[\to f]\pi^-) + \Gamma(B^+ \to D[\to f]\pi^+)}$$
$$= \frac{A_K^2}{A_\pi^2} \cdot \frac{r_D^2 + r_{DK}^2 + 2 r_{DK} r_D \cos(\delta_{DK} + \delta_D) \cos\gamma}{r_D^2 + r_{D\pi}^2 + 2 r_{D\pi} r_D \cos(\delta_{D\pi} + \delta_D) \cos\gamma}$$

One can go further and form a **double ratio** to measure Rcp. Then, the residual ratio of branching ratios cancels:

$$R_{CP+} = \frac{R_{K/\pi}^{KK}}{R_{K/\pi}^{K\pi}}$$

GLW and ADS observables

GLW observables

$$R_{CP\pm} = \frac{2[\Gamma(B^- \to D_{CP\pm}K^-) + \Gamma(B^+ \to D_{CP\pm}K^+)]}{\Gamma(B^- \to D^0K^-) + \Gamma(B^+ \to \bar{D}^0K^+)} ,$$

$$A_{CP\pm} = \frac{\Gamma(B^- \to D_{CP\pm}K^-) - \Gamma(B^+ \to D_{CP\pm}K^+)}{\Gamma(B^- \to D_{CP\pm}K^-) + \Gamma(B^+ \to D_{CP\pm}K^+)} .$$

$$R_{CP\pm} = 1 + r_B^2 \pm 2r_B \cos\delta_B \cos\gamma$$

$$A_{CP\pm} = \pm 2r_B \sin\delta_B \sin\gamma / R_{CP\pm} .$$

4 observables, 3 parameters.

But there is an inherent 8 fold ambiguity.

GLW and ADS observables

- traditional ADS observables

$$R_{\mathrm{ADS}} = \frac{\Gamma(B^- \to D[\to \pi^- K^+]K^-) + \Gamma(B^+ \to D[\to \pi^+ K^-]K^+)}{\Gamma(B^- \to D[\to K^- \pi^+]K^-) + \Gamma(B^+ \to D[\to K^+ \pi^-]K^+)}$$

$$A_{\mathrm{ADS}} = \frac{\Gamma(B^- \to D[\to \pi^- K^+]K^-) - \Gamma(B^+ \to D[\to \pi^+ K^-]K^+)}{\Gamma(B^- \to D[\to \pi^- K^+]K^-) + \Gamma(B^+ \to D[\to \pi^+ K^-]K^+)}$$

correlated observables

$$R_{\mathrm{ADS}} = r_B^2 + r_{K\pi}^2 + 2 r_B\, r_{K\pi} \cos\gamma \cos(\delta_B + \delta_{K\pi})$$

$$A_{\mathrm{ADS}} = 2 r_B\, r_{K\pi} \sin\gamma \sin(\delta_B + \delta_{K\pi}) / R_{\mathrm{ADS}}$$

- ADS observables

$$R_{\pm} \equiv \frac{\Gamma(B^{\pm} \to [\pi^{\pm} K^{\mp}]_D K^{\pm})}{\Gamma(B^{\pm} \to [K^{\pm} \pi^{\mp}]_D K^{\pm})}$$

$$= \frac{1}{N}\left(r_B^2 + r_{K\pi}^2 + 2 r_B r_{K\pi} \cos(\delta_B + \delta_{K\pi} \pm \gamma)\right)$$

N ~ 1

2 observables, 5 parameters (2 are “external” input on D system)

Gives good precision on rB.

GLW and ADS observables

"LHCb-style" observables

- We use single ratios and their full truth relations.
- We also use the asymmetry of the ADS favored modes:

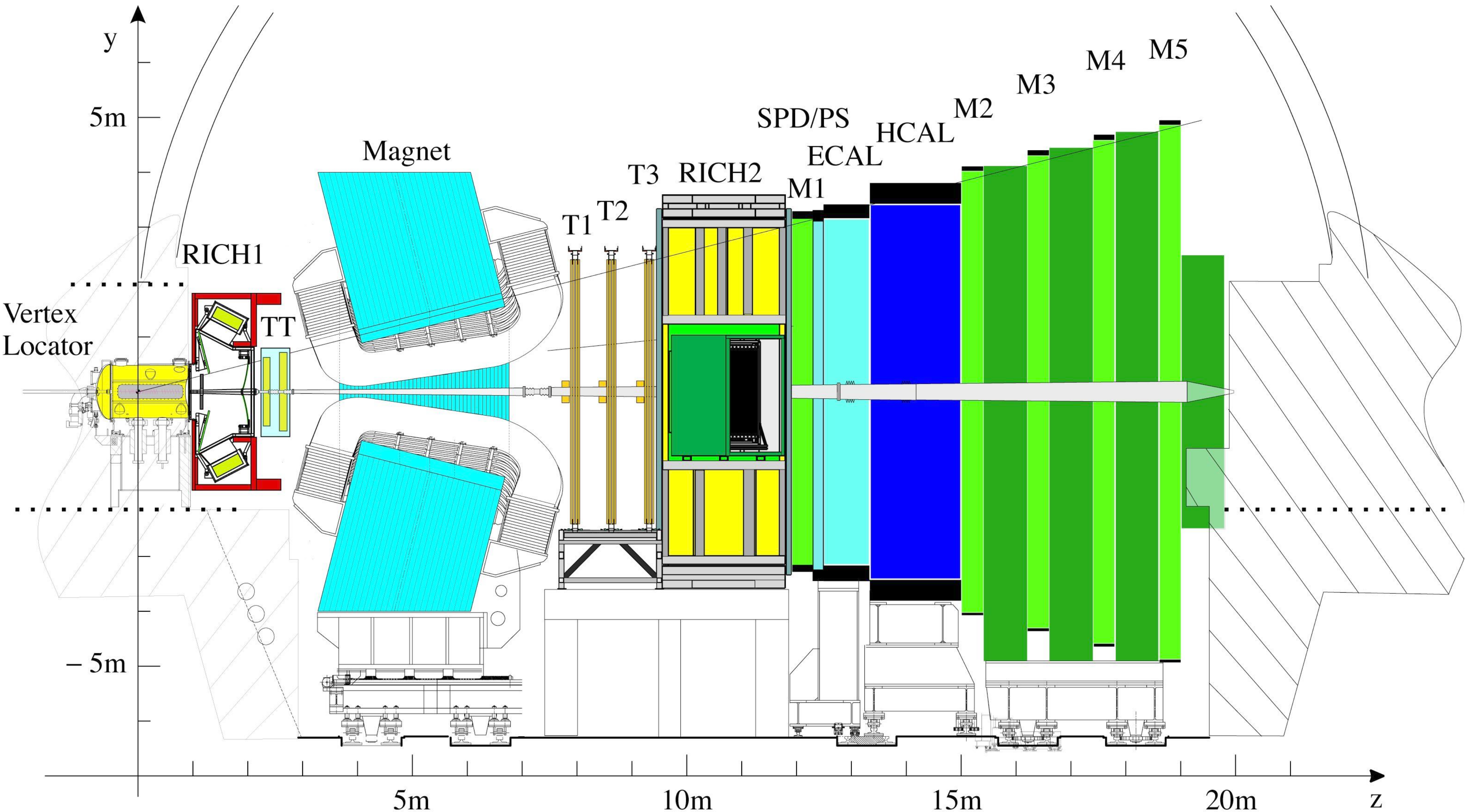
$$A_{\text{fav}} = \frac{\Gamma(B^- \to D[\to K^-\pi^+]K^-) - \Gamma(B^+ \to D[\to K^+\pi^-]K^+)}{\Gamma(B^- \to D[\to K^-\pi^+]K^-) + \Gamma(B^+ \to D[\to K^+\pi^-]K^+)}$$
$$= \frac{2r_B\, r_{K\pi} \sin\gamma \sin(\delta_B - \delta_{K\pi})}{1 + r_{K\pi}^2 r_B^2 + 2r_B\, r_{K\pi} \cos\gamma \cos(\delta_B - \delta_{K\pi})} \ .$$

- We are very bad at reconstructing neutral particles, so we miss the GLW CP- states.
- We also suffer in D → $K_S\pi\pi$ (will talk about it in a minute!), so the best precision will come from GLW/ADS like measurements.
- But counting parameters, this only works when using many final states.
- At LHCb, we chose a more **"factory like" approach**.

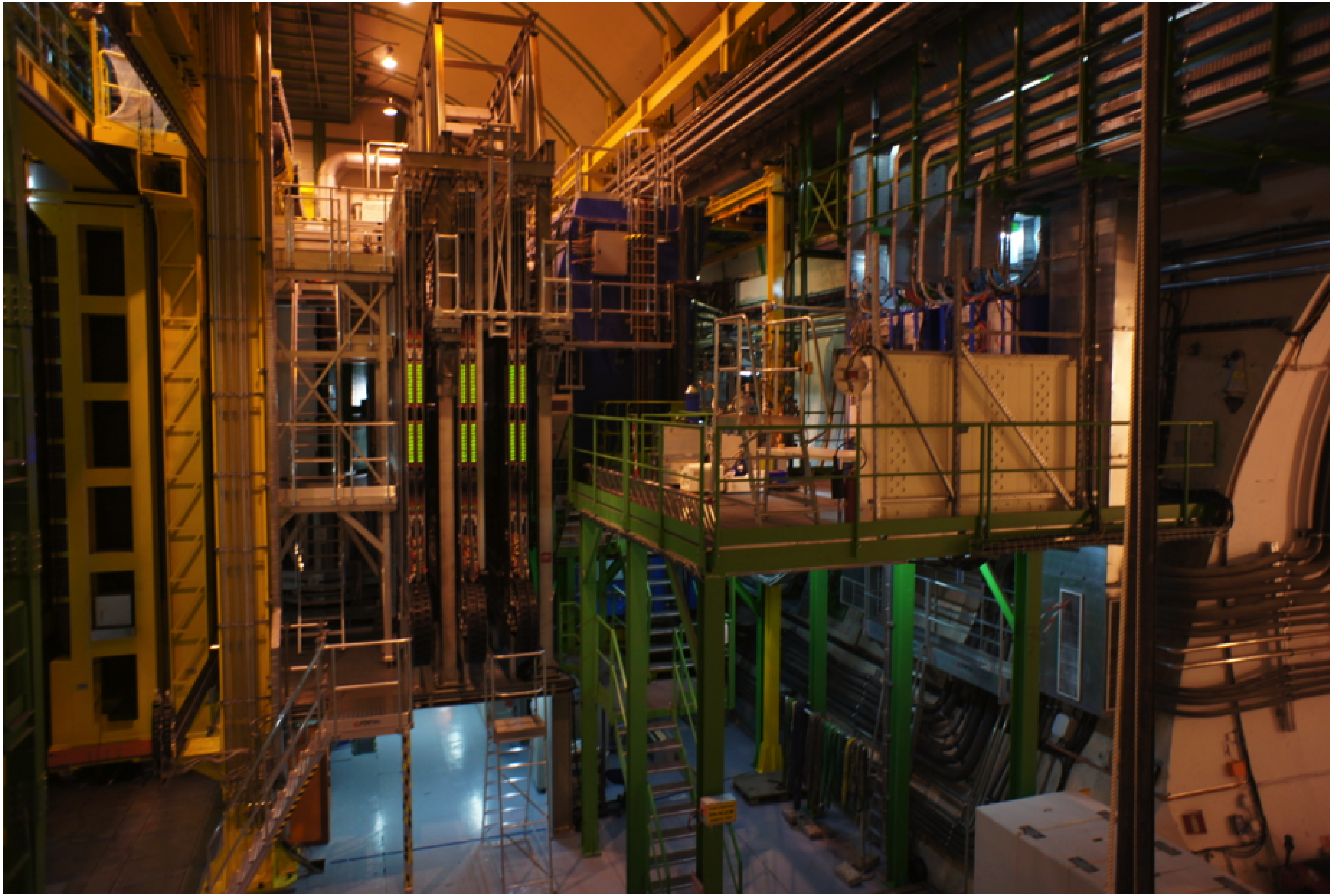
$R_{K/\pi}^{K\pi}$	=	$0.0774 \pm$
$R_{K/\pi}^{KK}$	=	$0.0773 \pm$
$R_{K/\pi}^{\pi\pi}$	=	$0.0803 \pm$
$A_{\pi}^{K\pi}$	=	$-0.0001 \pm$
$A_{K}^{K\pi}$	=	$0.0044 \pm$
A_{K}^{KK}	=	0.148 ± 0
$A_{K}^{\pi\pi}$	=	0.135 ± 0
A_{π}^{KK}	=	-0.020 ± 0
$A_{\pi}^{\pi\pi}$	=	-0.001 ± 0
R_K^-	=	$0.0073 \pm$
R_K^+	=	$0.0232 \pm$
R_π^-	=	$0.00469 \pm$
R_π^+	=	$0.00352 \pm$

all part of same analysis!

LHCb

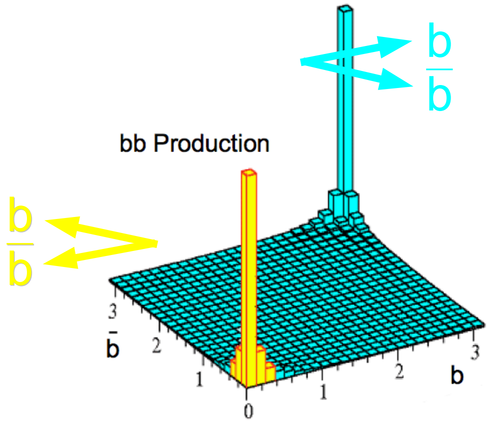


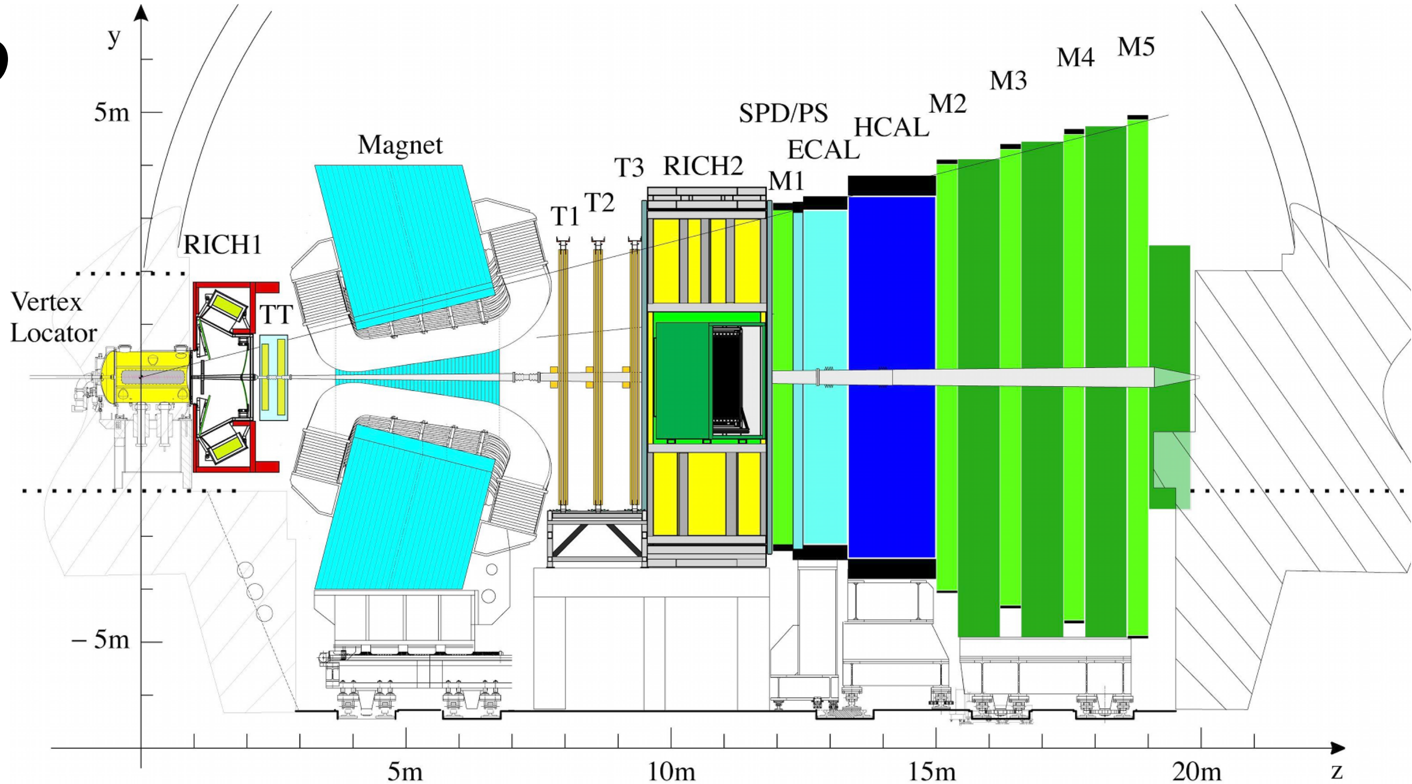
y
5m
– 5m
Magnet
RICH1
Vertex
Locator
TT
T1
T2
T3
RICH2
M1
SPD/PS
ECAL
HCAL
M2
M3
M4
M5
5m
10m
15m
20m
z



LHCb

- one arm forward spectrometer
- b pair production correlated
- covers **1.9 < η < 4.9**
- tracking stations before and after magnet
- particle identification by two RICH detectors



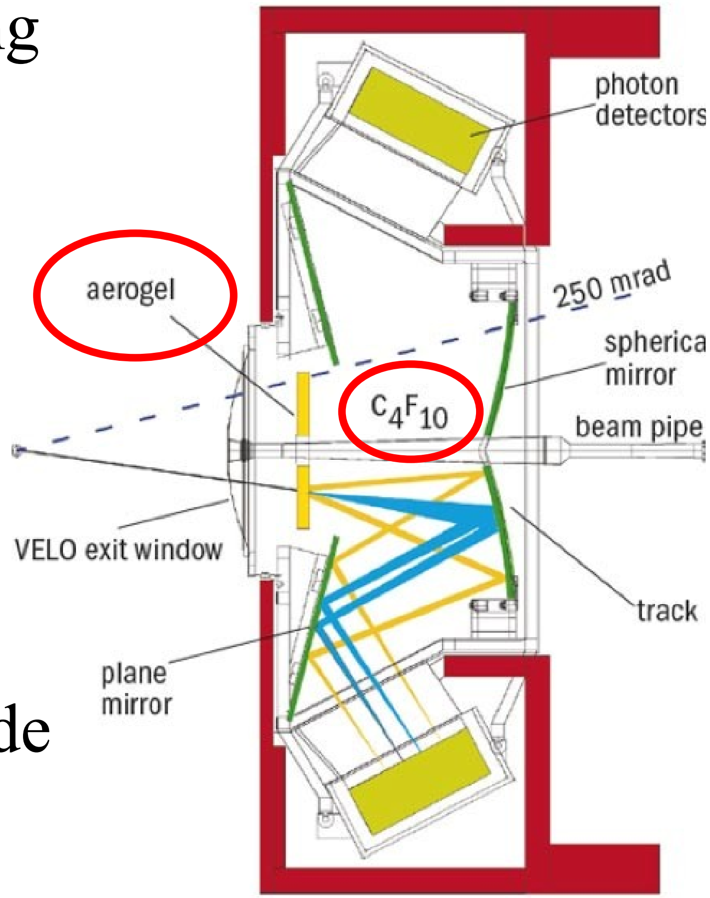


PID system

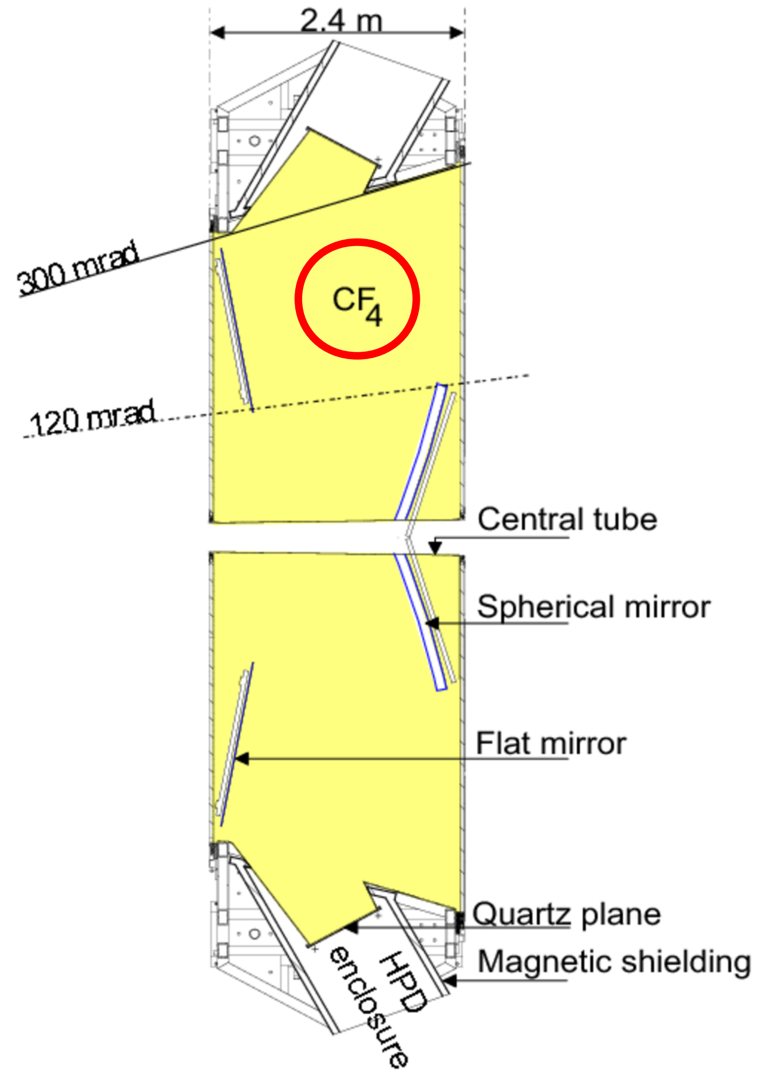
- Ring Imaging Cherenkov Detectors

$$\cos\theta = \frac{1}{\beta n}$$

- 3 radiators covering wide momentum range

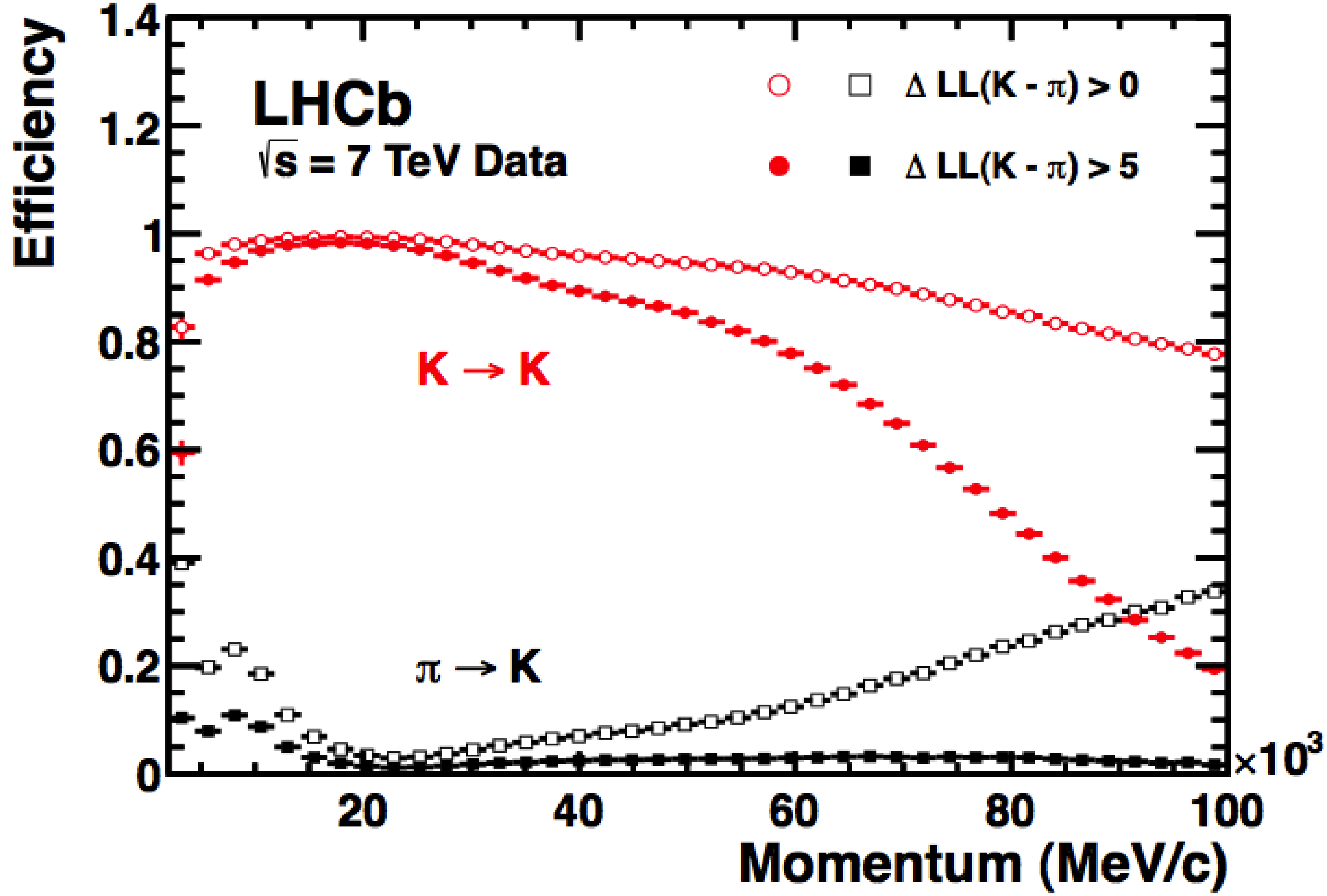


RICH 1
(side view)

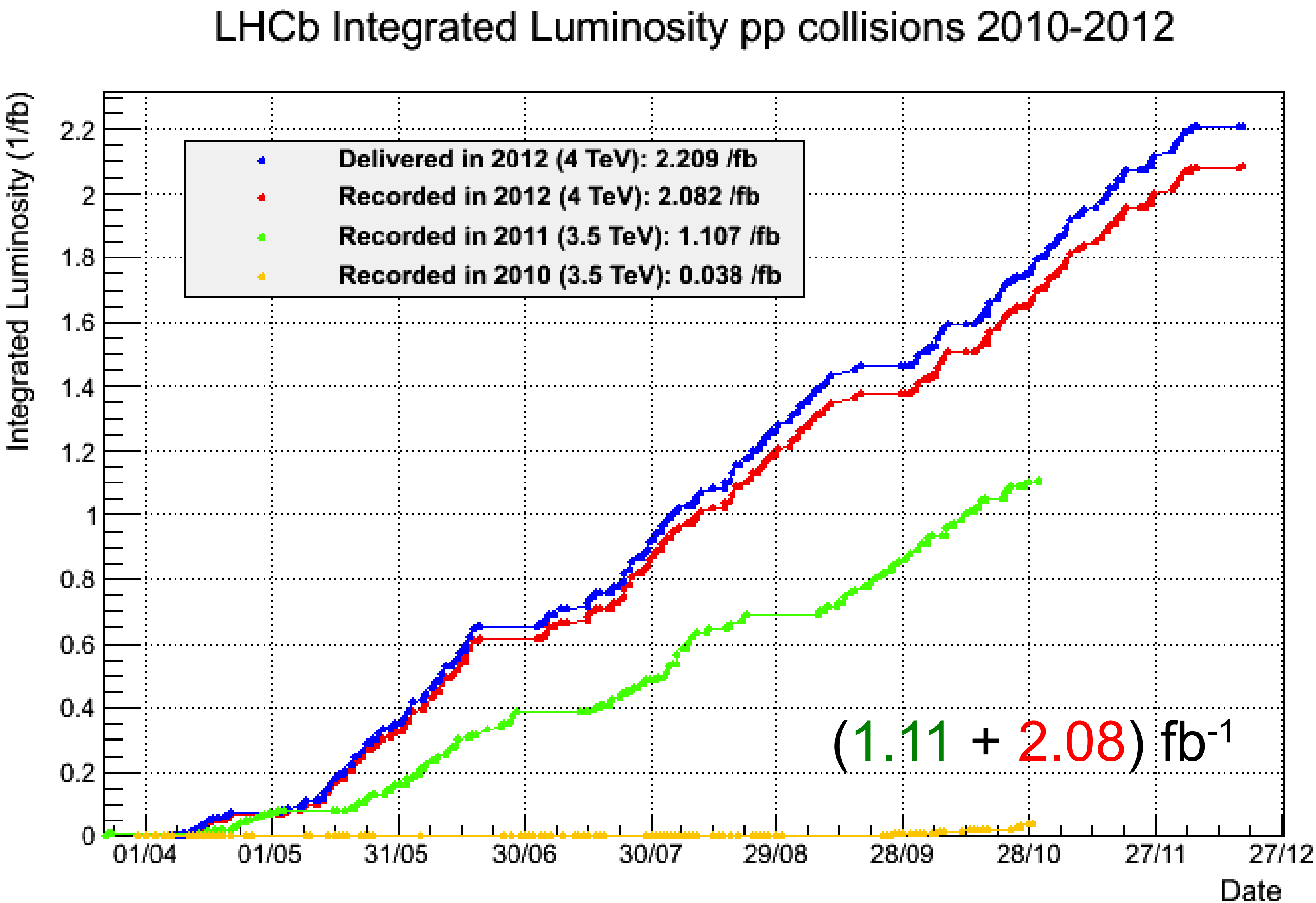


RICH 2
(top view)

LHCb – Kaon/pion separation



Luminosity Plot



B → D(hh)K: Observables

- Define observables as yield ratios (some systematics cancel).

- Charge **asymmetries**:

$$A_h^f = \frac{\Gamma(B^- \to [f]_D h^-) - \Gamma(B^+ \to [f]_D h^+)}{\Gamma(B^- \to [f]_D h^-) + \Gamma(B^+ \to [f]_D h^+)}$$

final states $[f]_D$:
KK, ππ
Kπ, πK

- Charge averaged **Kaon/pion** ratio:

$$R_{K/\pi}^f = \frac{\Gamma(B^\pm \to [f]_D K^\pm)}{\Gamma(B^\pm \to [f]_D \pi^\pm)}$$

- **Suppressed/favored** decay ratio:

13 observables

$$R_h^\pm = \frac{\Gamma(B^\pm \to [\pi^\pm K^\mp]_D h^\pm)}{\Gamma(B^\pm \to [K^\pm \pi^\mp]_D h^\pm)}$$
$$= r_B^2 + r_D^2 + 2 r_B r_D \cos(\pm\gamma + \underbrace{\delta_B + \delta_D}_{\text{strong phase diff.}})$$

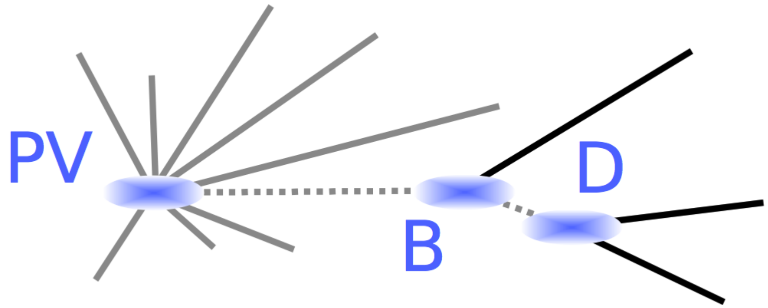
$B \to D(hh)K$: Analysis method

- Most backgrounds are combinatorial

  multivariate analysis (BDT) with 20 variables

- Charmless backgrounds

  exploit large forward boost of the D



cut on D flight distance

- Simultaneous fit on 16 slices

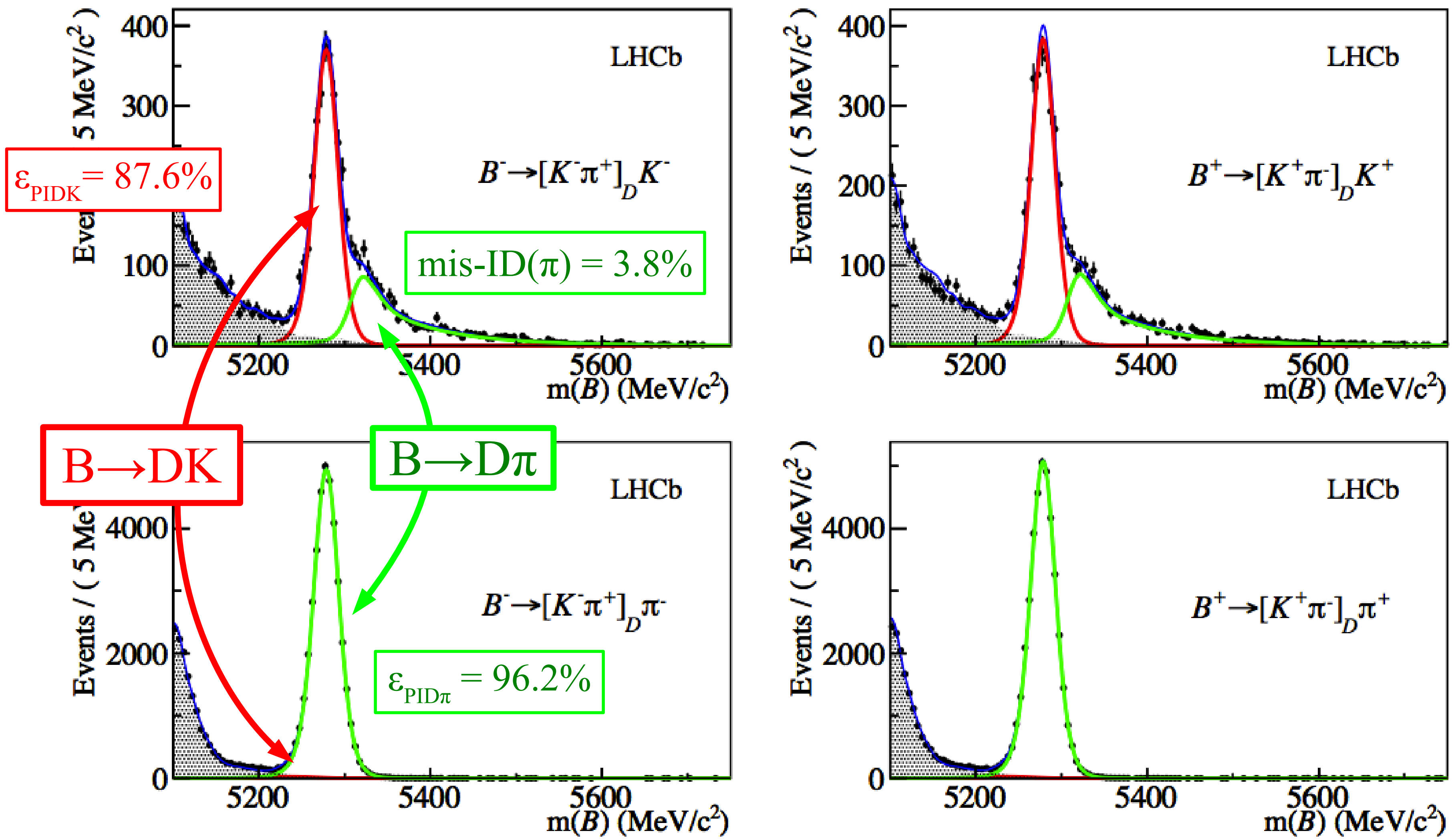
  2 (charges) x 4 (D modes) x 2 (K/π)

- Dominant systematics

  intrinsic charge asymmetries (A_{CP})

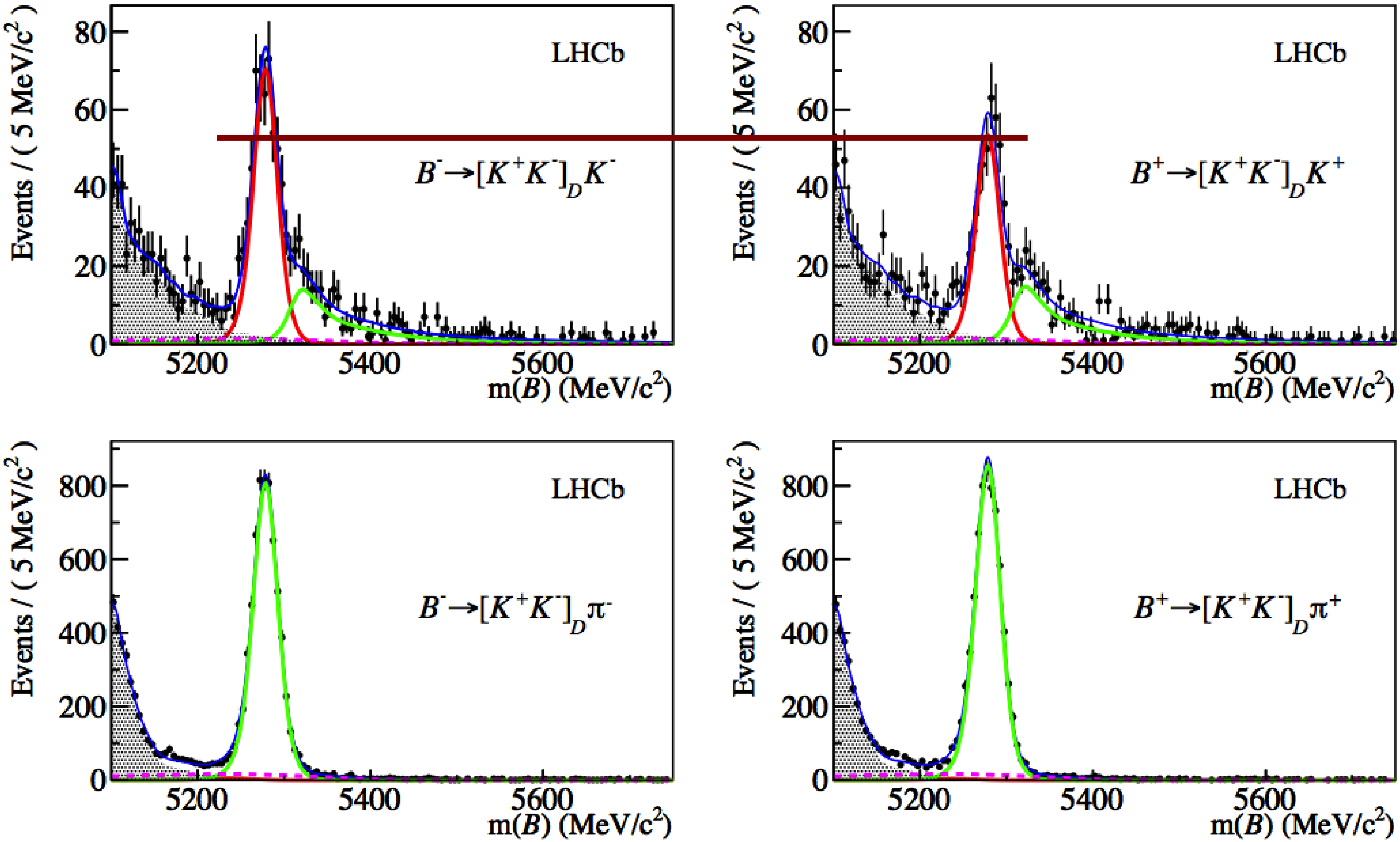
  particle ID ($R_{K\pi}$)

B → D(hh)K: favored ADS mode



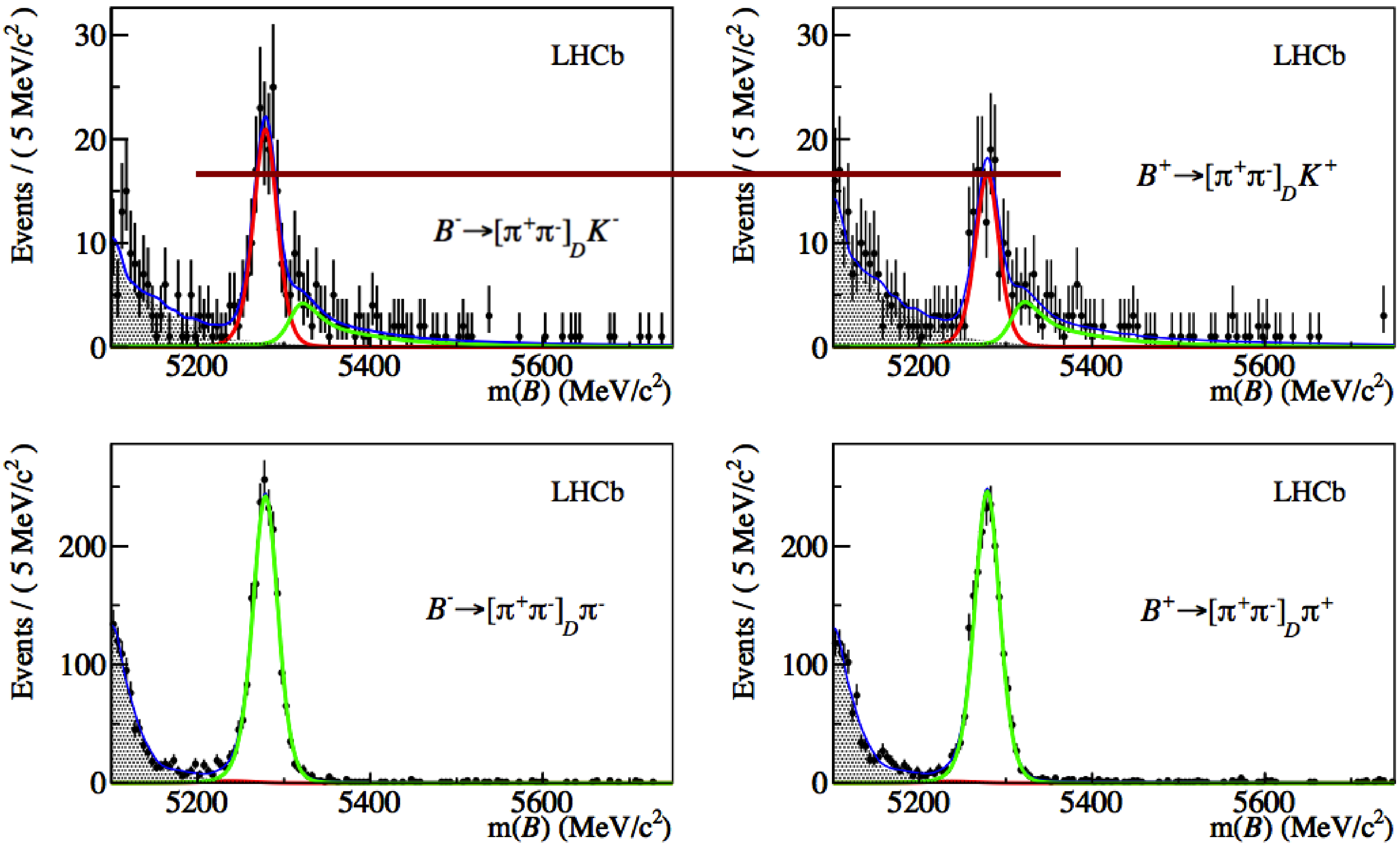
ARXIV:1203.3662

B → D(hh)K: GLW CP+ mode



ARXIV:1203.3662

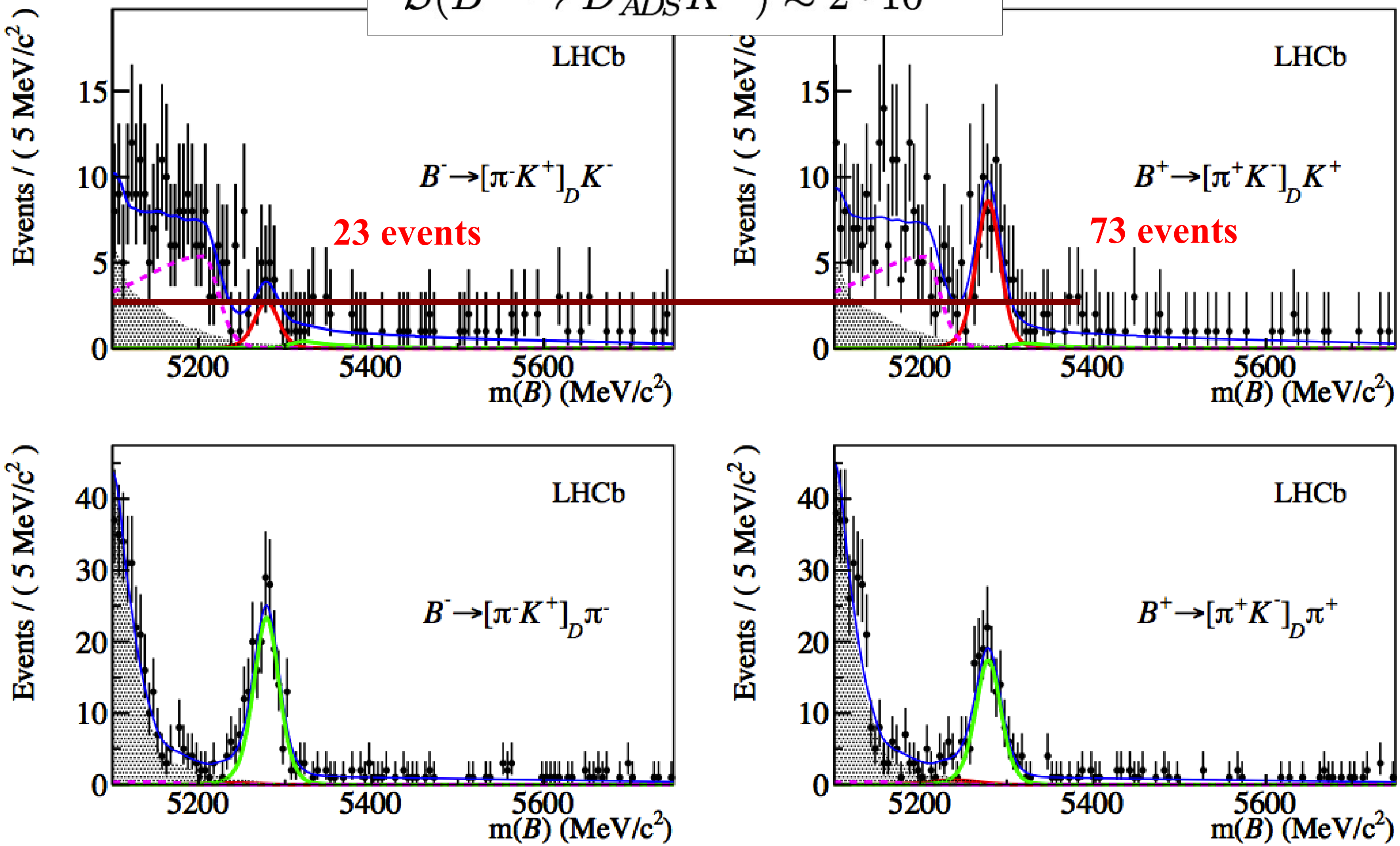
B → D(hh)K: GLW CP+ mode (II)



ARXIV:1203.3662

B → D(hh)K: suppressed ADS mode

$$\mathcal{B}(B^{\pm} \to D_{ADS} K^{\pm}) \approx 2 \cdot 10^{-7}$$



ARXIV:1203.3662

B → D(hh)K: Results

$$
\begin{aligned}
R^{K\pi}_{K/\pi} &= 0.0774 \pm 0.0012 \pm 0.0018 \\
R^{KK}_{K/\pi} &= 0.0773 \pm 0.0030 \pm 0.0018 \\
R^{\pi\pi}_{K/\pi} &= 0.0803 \pm 0.0056 \pm 0.0017 \\
A^{K\pi}_{\pi} &= -0.0001 \pm 0.0036 \pm 0.0095 \\
A^{K\pi}_{K} &= 0.0044 \pm 0.0144 \pm 0.0174 \\
A^{KK}_{K} &= 0.1480 \pm 0.0369 \pm 0.0097 \\
A^{\pi\pi}_{K} &= 0.1351 \pm 0.0661 \pm 0.0095 \\
A^{KK}_{\pi} &= -0.0199 \pm 0.0091 \pm 0.0116 \\
A^{\pi\pi}_{\pi} &= -0.0009 \pm 0.0165 \pm 0.0099 \\
R^{-}_{K} &= 0.0073 \pm 0.0023 \pm 0.0004 \\
R^{+}_{K} &= 0.0232 \pm 0.0034 \pm 0.0007 \\
R^{-}_{\pi} &= 0.00469 \pm 0.00038 \pm 0.00008 \\
R^{+}_{\pi} &= 0.00352 \pm 0.00033 \pm 0.00007
\end{aligned}
$$

ARXIV:1203.3662

B → D(hh)K: Results

$$R^{K\pi}_{K/\pi} = 0.0774 \pm 0.0012 \pm 0.0018$$
$$R^{KK}_{K/\pi} = 0.0773 \pm 0.0030 \pm 0.0018$$
$$R^{\pi\pi}_{K/\pi} = 0.0803 \pm 0.0056 \pm 0.0017$$
$$A^{K\pi}_{\pi} = -0.0001 \pm 0.0036 \pm 0.0095$$
$$A^{K\pi}_{K} = 0.0044 \pm 0.0144 \pm$$
$$A^{KK}_{K} = 0.1480 \pm 0.0369 \pm 0.0097$$
$$A^{\pi\pi}_{K} = 0.1351 \pm 0.0661 \pm 0.0095$$
$$A^{KK}_{\pi} = -0.0199 \pm 0.0091 \pm 0.0116$$
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$$R^{-}_{\pi} = 0.00469 \pm 0.00038 \pm 0.00008$$
$$R^{+}_{\pi} = 0.00352 \pm 0.00033 \pm 0.00007$$

$$R_{CP+} \approx < R^{\pi\pi}_{K/\pi}, R^{KK}_{K/\pi} > / R^{K\pi}_{K/\pi}$$
$$= 1.007 \pm 0.038 \pm 0.012$$

The GLW charge-averaged ratio, D(CP) over D(flavor).

ARXIV:1203.3662

B → D(hh)K: Results

$R^{K\pi}_{K/\pi}$	$=$	$0.0774 \pm 0.0012 \pm 0.0018$
$R^{KK}_{K/\pi}$	$=$	$0.0773 \pm 0.0030 \pm 0.0018$
$R^{\pi\pi}_{K/\pi}$	$=$	$0.0803 \pm 0.0056 \pm 0.0017$
$A^{K\pi}_{\pi}$	$=$	$-0.0001 \pm 0.0036 \pm 0.0095$
$A^{K\pi}_{K}$	$=$	$0.0044 \pm 0.0144 \pm 0.0174$
A^{KK}_{K}	$=$	$0.1480 \pm 0.0369 \pm 0.0097$
$A^{\pi\pi}_{K}$	$=$	$0.1351 \pm 0.0661 \pm 0.0095$
A^{KK}_{π}	$=$	$-0.0199 \pm 0.0091 \pm 0.0116$
$A^{\pi\pi}_{\pi}$	$=$	$-0.0009 \pm 0.0165 \pm 0.$[illegible]
R^{-}_{K}	$=$	$0.0073 \pm 0.0023 \pm 0.$[illegible]
R^{+}_{K}	$=$	$0.0232 \pm 0.0034 \pm 0.0007$
R^{-}_{π}	$=$	$0.00469 \pm 0.00038 \pm 0.00008$
R^{+}_{π}	$=$	$0.00352 \pm 0.00033 \pm 0.00007$

The GLW
charge asymmetry

$$A_{CP+} \approx < A^{\pi\pi}_{K/\pi}, A^{KK}_{K/\pi} >$$

$$= 0.145 \pm 0.032 \pm 0.010$$

4.5σ

ARXIV:1203.3662

B → D(hh)K: Results

$$R^{K\pi}_{K/\pi} = 0.0774 \pm 0.0012 \pm 0.0018$$

$$R^{KK}_{K/\pi} = 0.0773 \pm 0.$$

$$R^{\pi\pi}_{K/\pi} = 0.0803 \pm 0.$$

$$A^{K\pi}_{\pi} = -0.0001 \pm 0.$$

$$A^{K\pi}_{K} = 0.0044 \pm 0.$$

$$A^{KK}_{K} = 0.1480 \pm 0.$$

$$A^{\pi\pi}_{K} = 0.1351 \pm 0.0661 \pm 0.0095$$

$$A^{KK}_{\pi} = -0.0199 \pm 0.0091 \pm 0.0116$$

$$A^{\pi\pi}_{\pi} = -0.0009 \pm 0.0165 \pm 0.0099$$

$$R^{-}_{K} = 0.0073 \pm 0.0023 \pm 0.0004$$

$$R^{+}_{K} = 0.0232 \pm 0.0034 \pm 0.0007$$

$$R^{-}_{\pi} = 0.00469 \pm 0.00038 \pm 0.00008$$

$$R^{+}_{\pi} = 0.00352 \pm 0.00033 \pm 0.00007$$

The ADS observables

$$R_{ADS(K)} = 0.0152 \pm 0.0020 \pm 0.0004$$

10σ

$$A_{ADS(K)} = -0.520 \pm 0.150 \pm 0.021$$

4σ

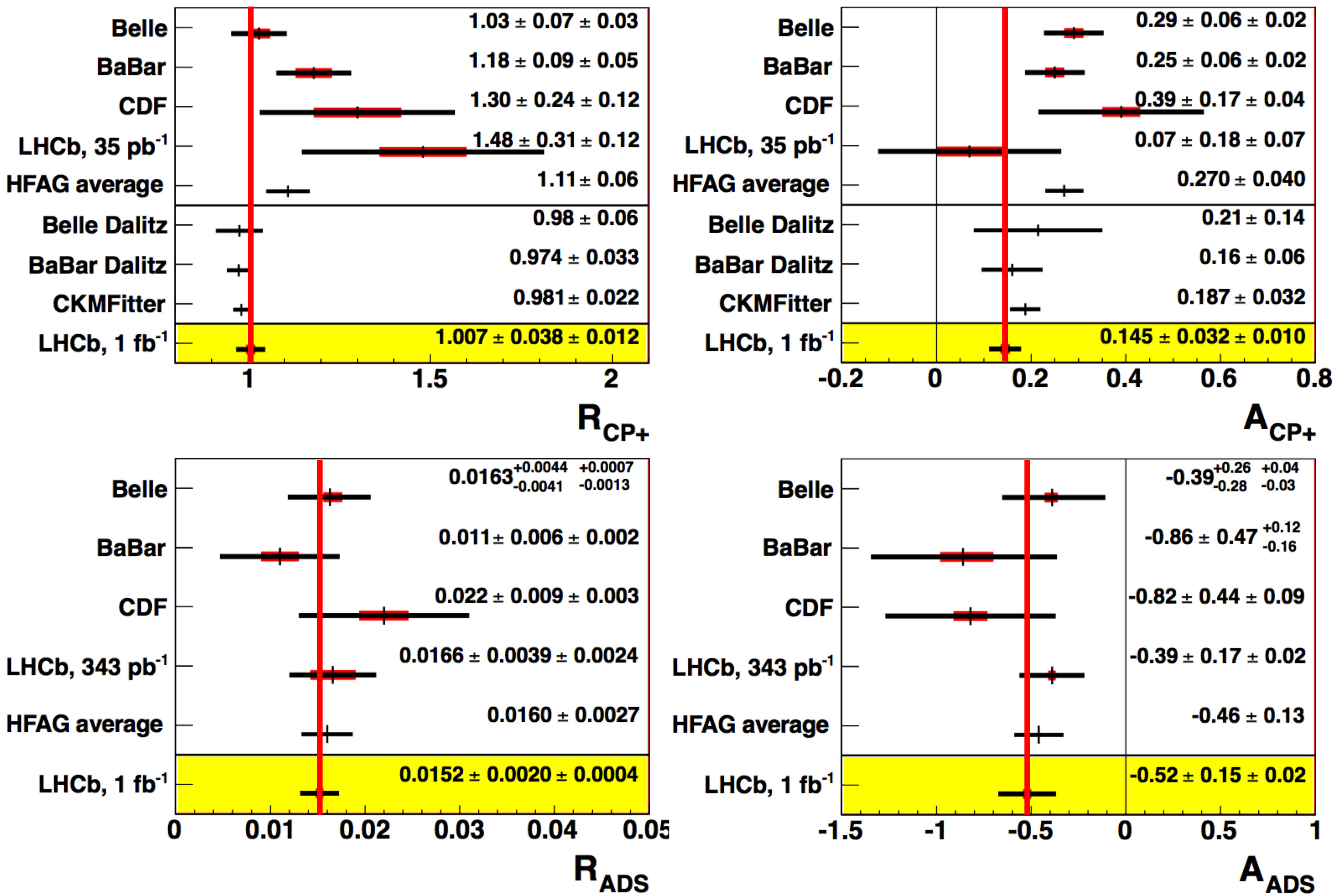
$$R_{ADS(\pi)} = 0.00410 \pm 0.00025 \pm 0.00005$$

$$A_{ADS(\pi)} = 0.143 \pm 0.062 \pm 0.011$$

2.4σ

ARXIV:1203.3662

$B \to D(hh)K$: Results



multi-body D decays

multi-body D decays

- Interference can only occur at same points in phase space, i.e. the requirement "same final state" is not enough.
- The magnitudes of the D decay amplitudes and the strong phase difference become **functions of the phase space**.
- Introduce effective quantities averaged over phase space!

$$r_{K3\pi}^2 = \frac{\int \bar{A}_D(\vec{m})^2 d\vec{m}}{\int A_D(\vec{m})^2 d\vec{m}}$$

phase space point

$$\kappa_{K3\pi} e^{i\delta_{K3\pi}} = \frac{\int A_D(\vec{m})\bar{A}_D(\vec{m}) e^{i\delta(\vec{m})} d\vec{m}}{\sqrt{\int \bar{A}_D(\vec{m})^2 d\vec{m} \times \int A_D(\vec{m})^2 d\vec{m}}}$$

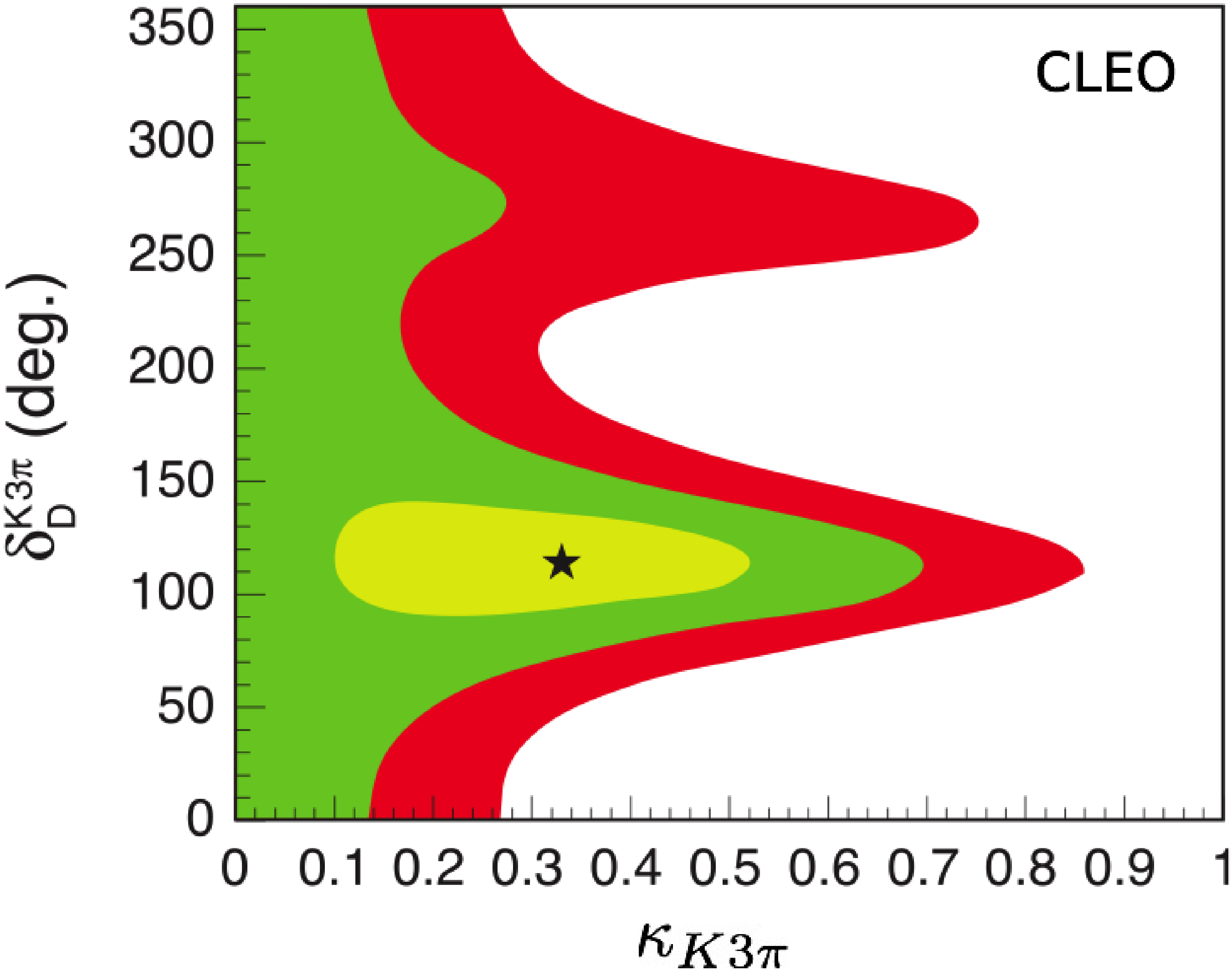
$$R_\pm = r_B^2 + r_{K3\pi}^2 + 2\underbrace{\kappa_{K3\pi}}r_B r_{K3\pi}\cos(\pm\gamma + \delta_B + \underbrace{\delta_{K3\pi}})$$

the "coherence factor", external input

a new (eff.) strong phase diff.

multi-body D decays

The Ghana Plot



four-body ADS

"LHCb-style" observables:

similar as before; only now each interference term has the extra coherence factor

$$R^{K3\pi}_{K/\pi} = R_{\rm cab}\frac{1 + r_B^2 r_{K3\pi}^2 + 2\kappa_{K3\pi} r_B r_{K3\pi}\cos(\delta_B - \delta_{K3\pi})\cos\gamma}{1 + r_B^{\pi 2} r_{K3\pi}^2 + 2\kappa_{K3\pi} r_B^\pi r_{K3\pi}\cos(\delta_B^\pi - \delta_{K3\pi})\cos\gamma},$$

$$A^{K3\pi}_{\pi} = \frac{2\kappa_{K3\pi} r_B^\pi r_{K3\pi}\sin(\delta_B^\pi - \delta_{K3\pi})\sin(\gamma)}{1 + r_B^{\pi 2} r_{K3\pi}^2 + 2\kappa_{K3\pi} r_B^\pi r_{K3\pi}\cos(\delta_B^\pi - \delta_{K3\pi})\cos\gamma},$$

$$A^{K3\pi}_{K} = \frac{2\kappa_{K3\pi} r_B r_{K3\pi}\sin(\delta_B - \delta_{K3\pi})\sin\gamma}{1 + r_B^2 r_{K3\pi}^2 + 2\kappa_{K3\pi} r_B r_{K3\pi}\cos(\delta_B - \delta_{K3\pi})\cos\gamma},$$

$$R^{K3\pi}_{\pi-} = \frac{r_B^{\pi 2} + r_{K3\pi}^2 + 2\kappa_{K3\pi} r_B^\pi r_{K3\pi}\cos(\delta_B^\pi + \delta_{K3\pi} - \gamma)}{1 + r_B^{\pi 2} r_{K3\pi}^2 + 2\kappa_{K3\pi} r_B^\pi r_{K3\pi}\cos(\delta_B^\pi - \delta_{K3\pi} - \gamma)},$$

$$R^{K3\pi}_{\pi+} = \frac{r_B^{\pi 2} + r_{K3\pi}^2 + 2\kappa_{K3\pi} r_B^\pi r_{K3\pi}\cos(\delta_B^\pi + \delta_{K3\pi} + \gamma)}{1 + r_B^{\pi 2} r_{K3\pi}^2 + 2\kappa_{K3\pi} r_B^\pi r_{K3\pi}\cos(\delta_B^\pi - \delta_{K3\pi} + \gamma)},$$

$$R^{K3\pi}_{K-} = \frac{r_B^2 + r_{K3\pi}^2 + 2\kappa_{K3\pi} r_B r_{K3\pi}\cos(\delta_B + \delta_{K3\pi} - \gamma)}{1 + r_B^2 r_{K3\pi}^2 + 2\kappa_{K3\pi} r_B r_{K3\pi}\cos(\delta_B - \delta_{K3\pi} - \gamma)},$$

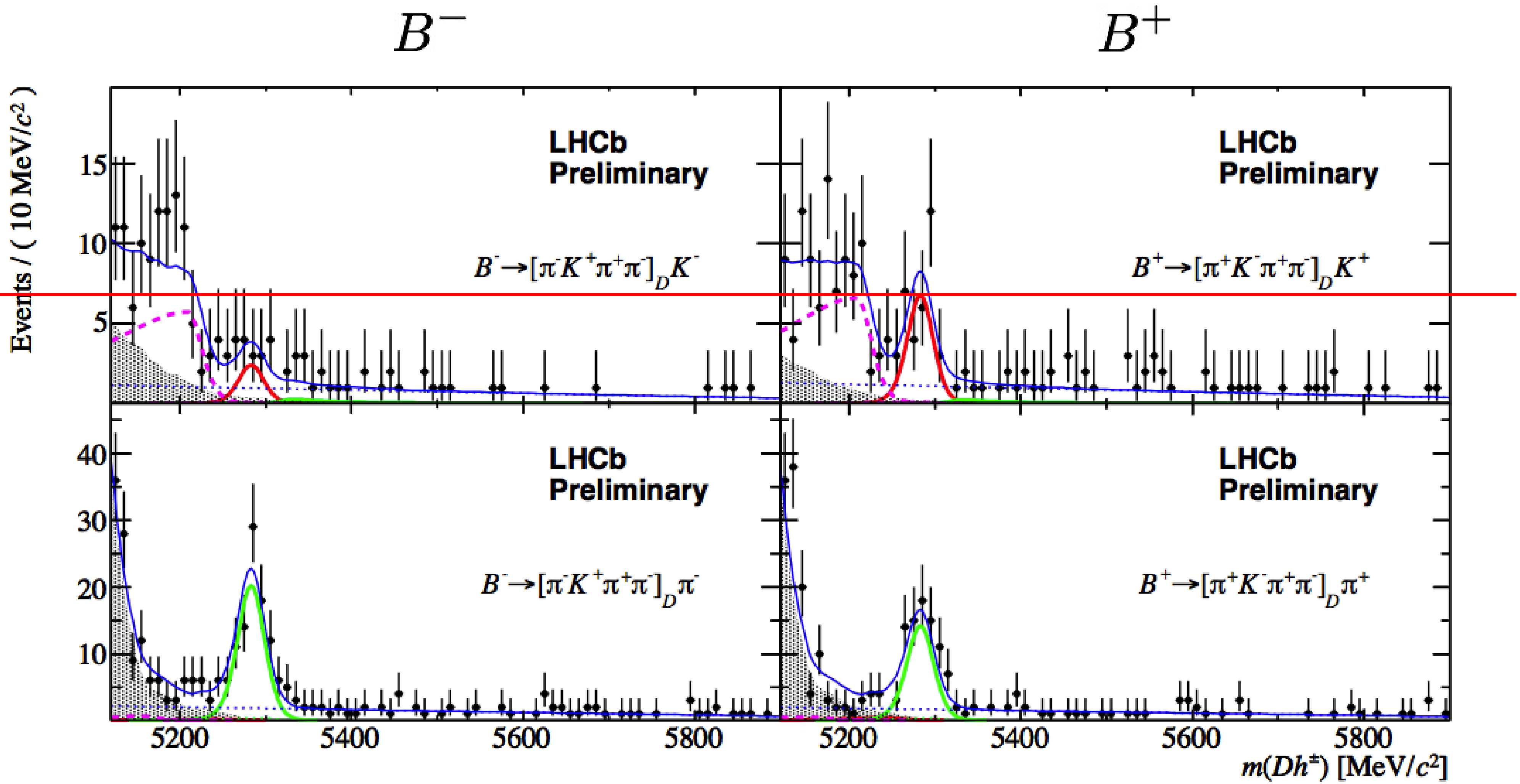
$$R^{K3\pi}_{K+} = \frac{r_B^2 + r_{K3\pi}^2 + 2\kappa_{K3\pi} r_B r_{K3\pi}\cos(\delta_B + \delta_{K3\pi} + \gamma)}{1 + r_B^2 r_{K3\pi}^2 + 2\kappa_{K3\pi} r_B r_{K3\pi}\cos(\delta_B - \delta_{K3\pi} + \gamma)}.$$

$$D^0 \to K\pi\pi\pi$$

LHCb-CONF-2012-030

four-body ADS

First observations of these decay modes!



four-body ADS

$$R_{K/\pi}^{K3\pi} \equiv \frac{\Gamma(B^- \to [K^-\pi^+\pi^-\pi^+]_D K^-) + \Gamma(B^+ \to [K^+\pi^-\pi^+\pi^-]_D K^+)}{\Gamma(B^- \to [K^-\pi^+\pi^-\pi^+]_D \pi^-) + \Gamma(B^+ \to [K^+\pi^-\pi^+\pi^-]_D \pi^+)},$$

$$A_h^{K3\pi} \equiv \frac{\Gamma(B^- \to [K^-\pi^+\pi^+\pi^-]_D h^-) - \Gamma(B^+ \to [K^+\pi^-\pi^+\pi^-]_D h^+)}{\Gamma(B^- \to [K^-\pi^+\pi^+\pi^-]_D h^-) + \Gamma(B^+ \to [K^+\pi^-\pi^+\pi^-]_D h^+)},$$

$$R_h^{K3\pi,\pm} \equiv \frac{\Gamma(B^\pm \to [\pi^\pm K^\mp \pi^+\pi^-]_D h^\pm)}{\Gamma(B^\pm \to [K^\pm \pi^\mp \pi^+\pi^-]_D h^\pm)}.$$

$$\begin{aligned}
R_{K/\pi}^{K3\pi} &= 0.0771 \pm 0.0017 \pm 0.0026 \\
A_K^{K3\pi} &= -0.029 \pm 0.020 \pm 0.018 \\
A_\pi^{K3\pi} &= -0.006 \pm 0.005 \pm 0.010 \\
R_K^{K3\pi,-} &= 0.0072 \,{}^{+0.0036}_{-0.0032} \pm 0.0008 \\
R_K^{K3\pi,+} &= 0.0175 \,{}^{+0.0043}_{-0.0039} \pm 0.0010 \\
R_\pi^{K3\pi,-} &= 0.00417 \,{}^{+0.00054}_{-0.00050} \pm 0.00011 \\
R_\pi^{K3\pi,+} &= 0.00321 \,{}^{+0.00048}_{-0.00045} \pm 0.00011
\end{aligned}$$

Giri, Grossman, Soffer, Zupan, hep-ph/0303187

GGSZ

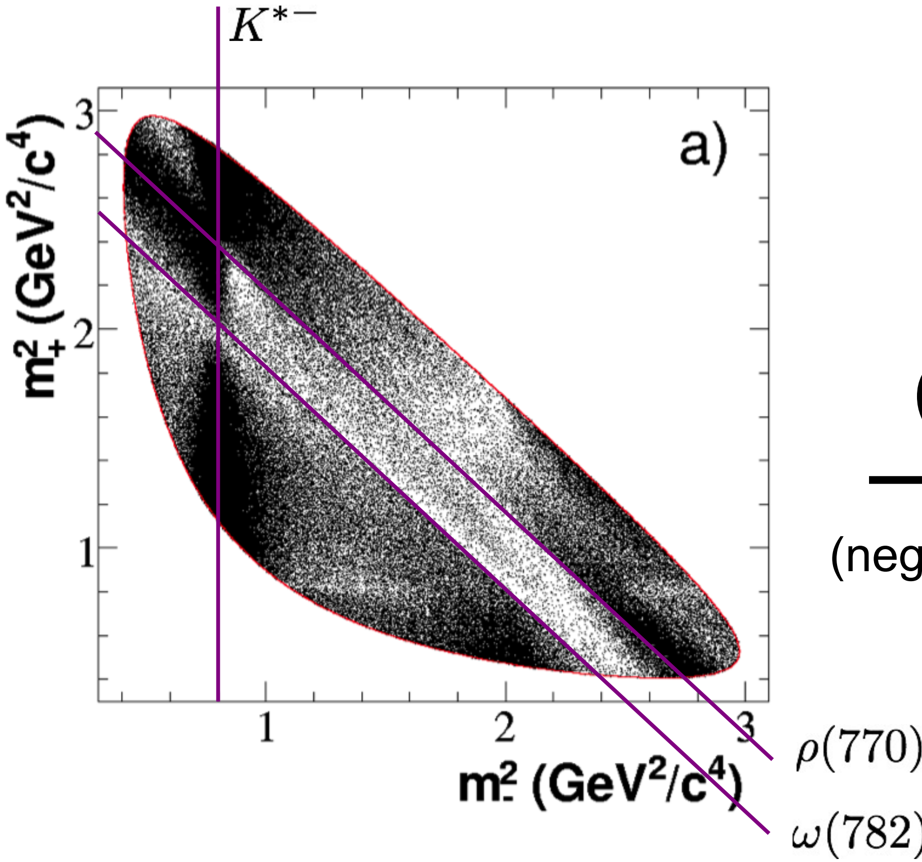
- **Idea**: take advantage of the D strong phase variation over D phase space rather than averaging it away
- then no external input on D hadronic parameters is needed, no coherence factor
- Fit the Dalitz plot: "Dalitz method". Most precise at B-factories.
- GGSZ use self-conjugate three-body final states

$$D^0 \to K^0_S \pi^- \pi^+ \qquad D^0 \to K^0_S K^- K^+$$

- Need to input the D decay amplitude. Chose CP eigenstates, neglect CP violation:

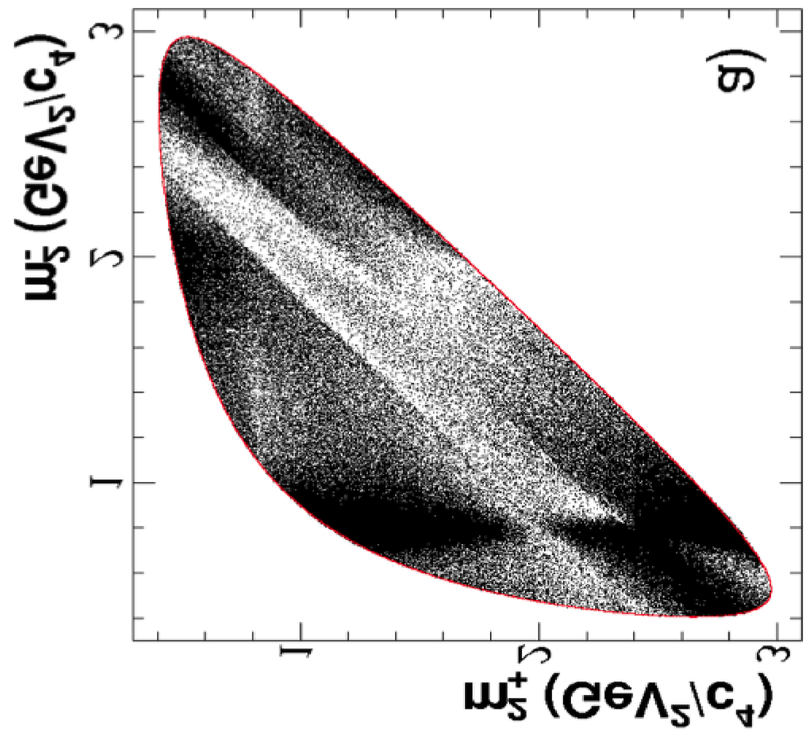
$$A(D^0 \to f) = A_f e^{i\delta_f} = f(m_-^2, m_+^2)$$
$$A(\overline{D^0} \to f) = A_{\bar{f}} e^{i\delta_{\bar{f}}} = f(m_+^2, m_-^2)$$

Dalitz Plot



CP

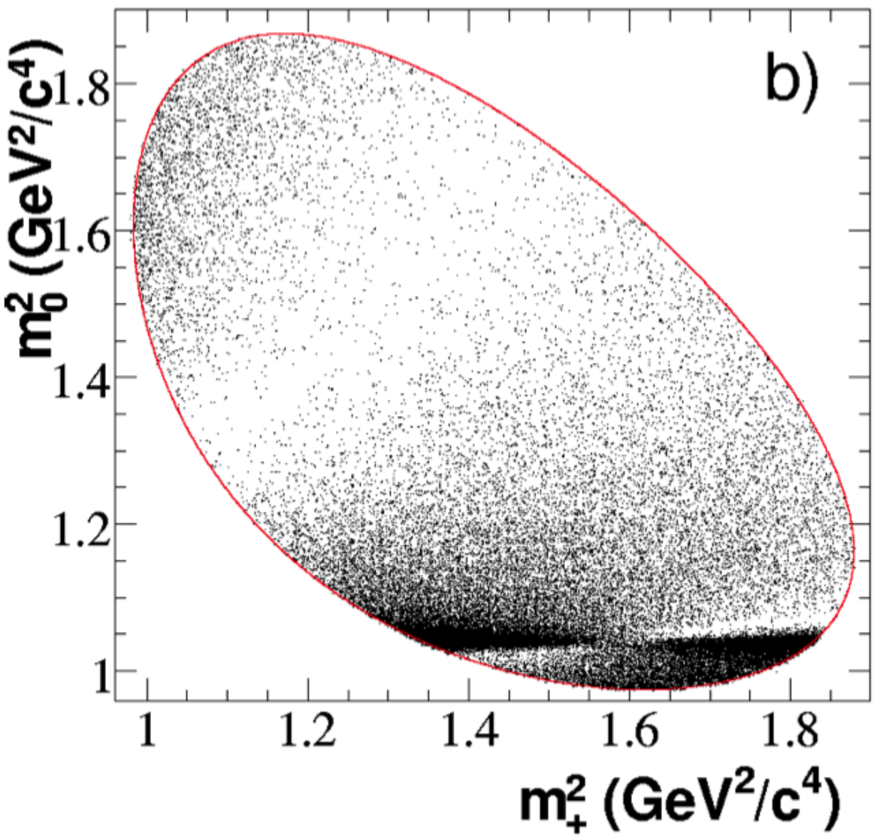
(neglecting CPV)



$$D^0 \to K_S^0 \pi^- \pi^+$$

$$\overline{D^0} \to K_S^0 \pi^- \pi^+$$

Dalitz Plot



$$D^0 \to K_S^0 K^- K^+$$

GGSZ

Remember the master equation:

$$\Gamma(B^- \to D[\to f]K^-) = A_c^2 A_f^2 + A_u^2 A_{\bar{f}}^2 + 2A_c A_f A_u A_{\bar{f}} \Re(e^{i(\delta_B+\delta_D-\gamma)})$$
$$= A_c^2 A_{\bar{f}}^2 \left(r_D^2 + r_B^2 + 2r_B r_D \cos(\delta_B + \delta_D - \gamma)\right)$$

Allow for non-constant D amplitudes and strong phases:

$$\Gamma(B^\mp \to D[\to K_S^0 \pi^- \pi^+]K^\mp) \propto |f(m_\mp^2, m_\pm^2)|^2 + r_B^2 |f(m_\pm^2, m_\mp^2)|^2$$
$$+ 2r_B |f(m_\mp^2, m_\pm^2)||f(m_\pm^2, m_\mp^2)| \cos(\delta_B + \delta_D(m_\mp^2, m_\pm^2) \mp \gamma)$$

We can now plug in an amplitude **model** (Breit-Wigners, …): **model dependent analysis**

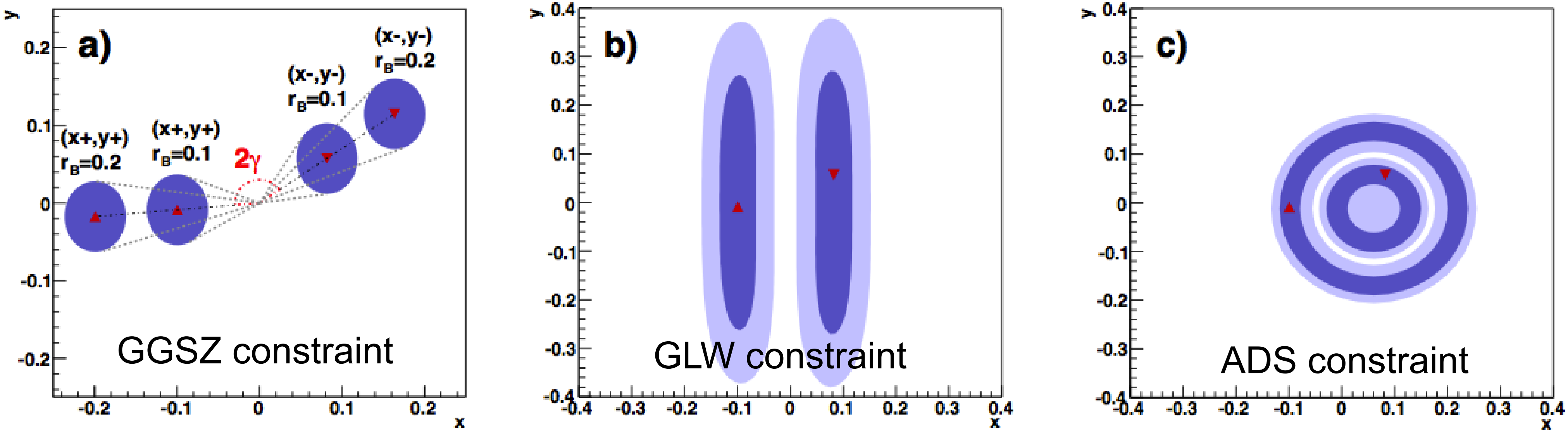
But the fit for rB, δB, and γ was found to be biased, when rB not far enough from zero!

Reparametrize in terms of unbiased **Cartesian Coordinates**:

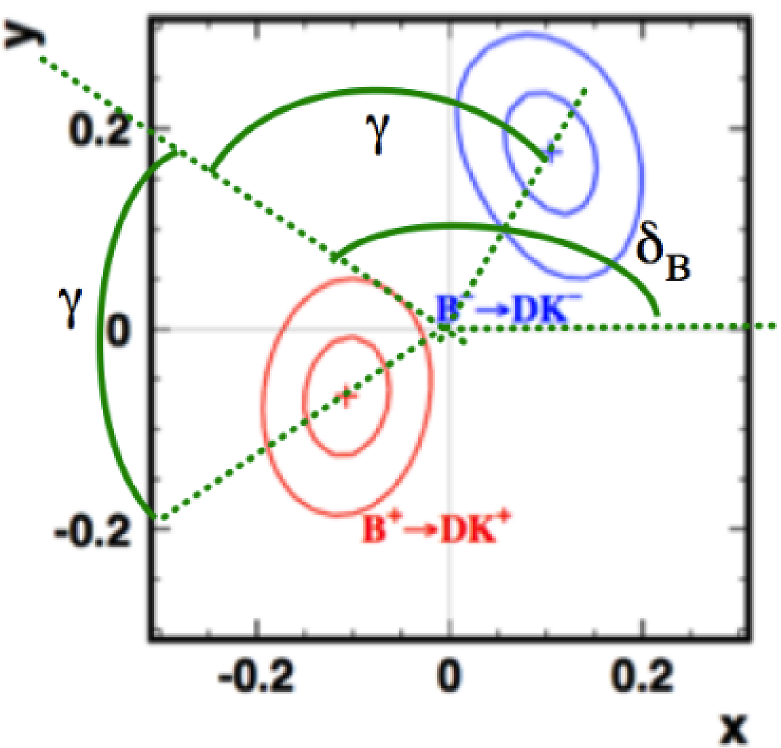
$$x_\pm = r_B \cos(\delta_B \pm \gamma) \qquad y_\pm = r_B \sin(\delta_B \pm \gamma)$$

$$\Gamma(B^\mp \to D^0[\to K_S^0 \pi^+ \pi^-]K^\mp) \propto |f_\mp|^2 + (x_\mp^2 + y_\mp^2)|f_\pm|^2 + 2\left[x_\mp \mathrm{Re}[f_\mp f_\pm^*] + y_\mp \mathrm{Im}[f_\mp f_\pm^*]\right]$$

GGSZ Cartesian Coordinates



Matteo Rama at FPCP2009

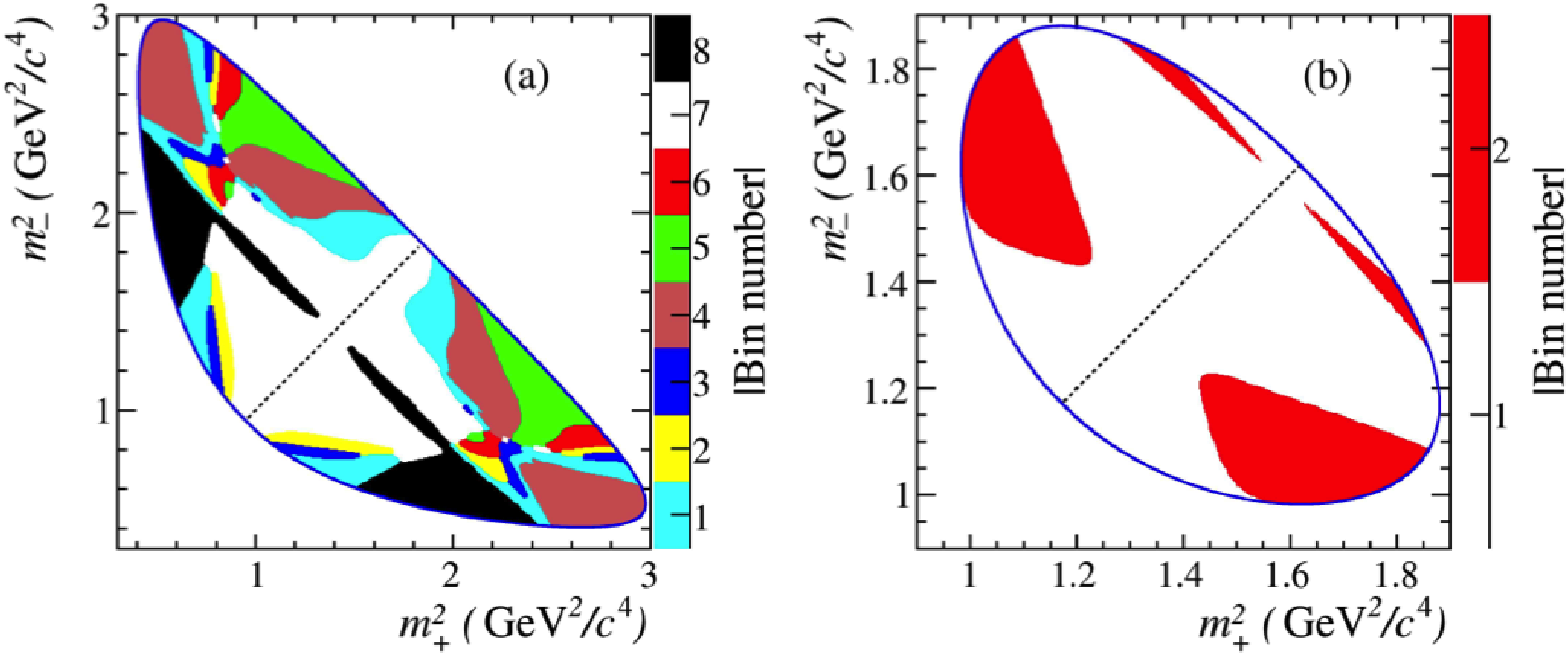


Express GLW observables in terms of cart. coordinates:

$$x_\pm = \frac{R_{CP+}(1 \mp A_{CP+}) - R_{CP-}(1 \mp A_{CP-})}{4}$$

$$r^2 = x_\pm^2 + y_\pm^2 = \frac{R_{CP+} + R_{CP-} - 2}{2}.$$

model independent GGSZ



One can avoid the model dependence by performing analysis in bins of Dalitz plot.

Then, one uses external information about the effective strong phase and magnitude in the bins.

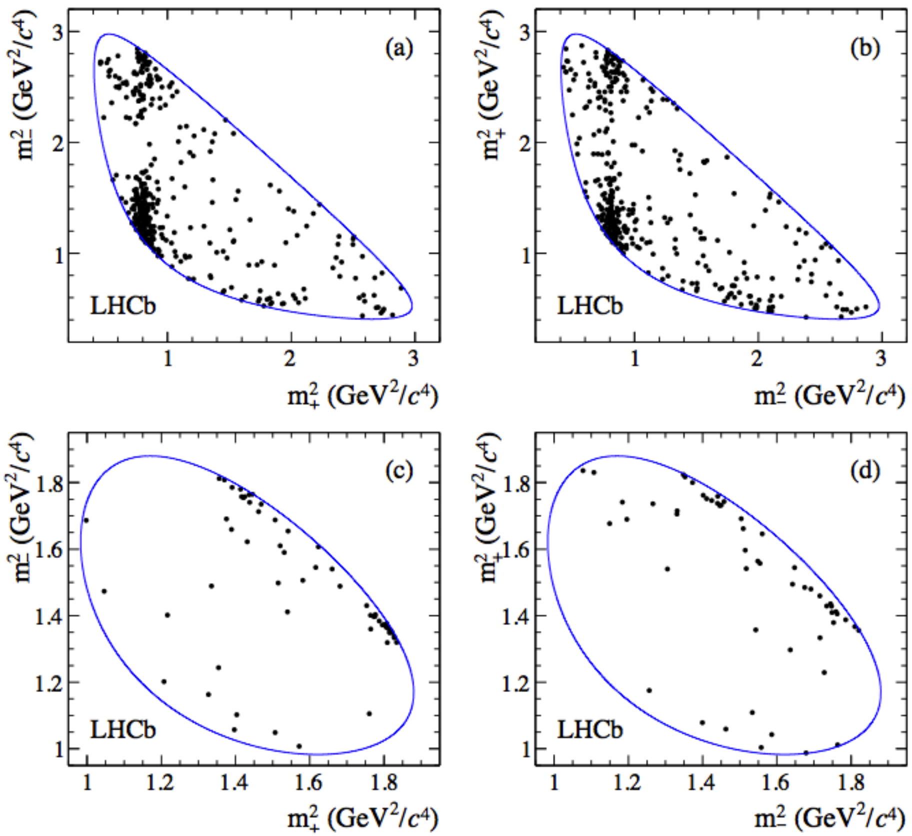
$$N_{\pm i}^{+} = h_{B^+}\left[K_{\mp i} + (x_+^2 + y_+^2)K_{\pm i} + 2\sqrt{K_i K_{-i}}(x_+ c_{\pm i} \mp y_+ s_{\pm i})\right]$$
$$N_{\pm i}^{-} = h_{B^-}\left[K_{\pm i} + (x_-^2 + y_-^2)K_{\mp i} + 2\sqrt{K_i K_{-i}}(x_- c_{\pm i} \pm y_- s_{\pm i})\right]$$

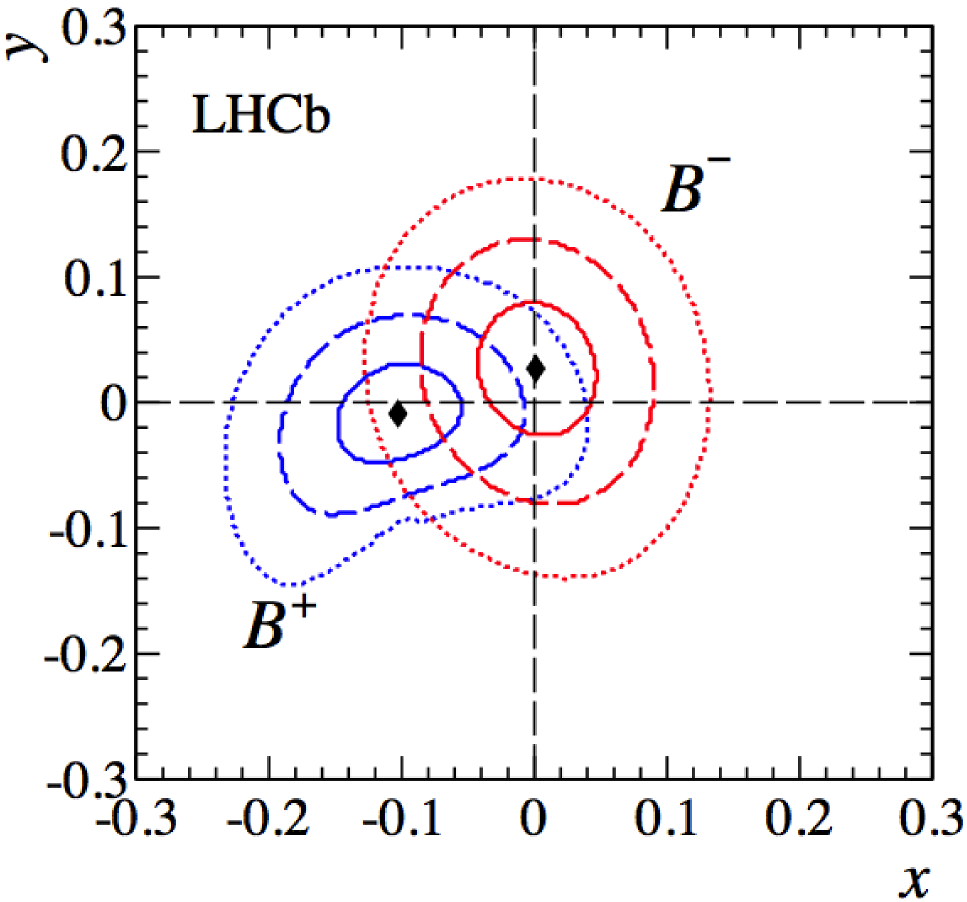
$$c_i = \frac{\int_{\mathcal{D}_i}(|A||\overline{A}|\cos\delta_D)\, d\mathcal{D}}{\sqrt{\int_{\mathcal{D}_i}|A|^2\, d\mathcal{D}}\sqrt{\int_{\mathcal{D}_i}|\overline{A}|^2\, d\mathcal{D}}},$$

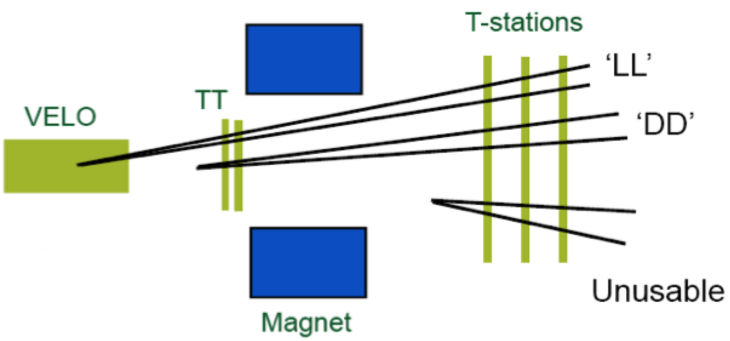
Ki = yields in bins

(s_i analogously for sin)

model independent GGSZ

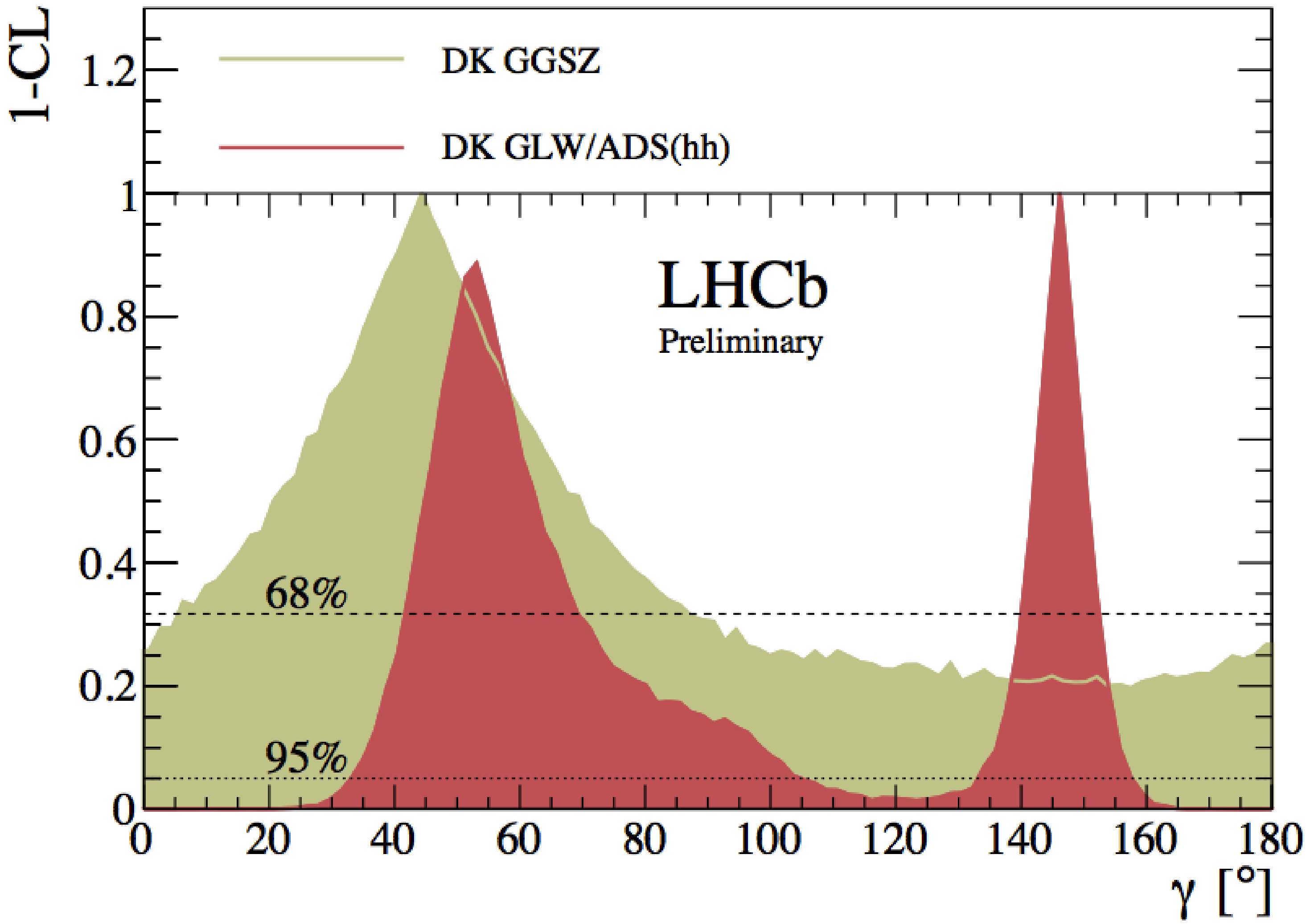






$$x_- = (\quad 0.0 \pm 4.3 \pm 1.5 \pm 0.6) \times 10^{-2}$$
$$y_- = (\quad 2.7 \pm 5.2 \pm 0.8 \pm 2.3) \times 10^{-2}$$
$$x_+ = (-10.3 \pm 4.5 \pm 1.8 \pm 1.4) \times 10^{-2}$$
$$y_+ = (\ -0.9 \pm 3.7 \pm 0.8 \pm 3.0) \times 10^{-2}$$

γ from GGSZ and GLW/ADS(hh)

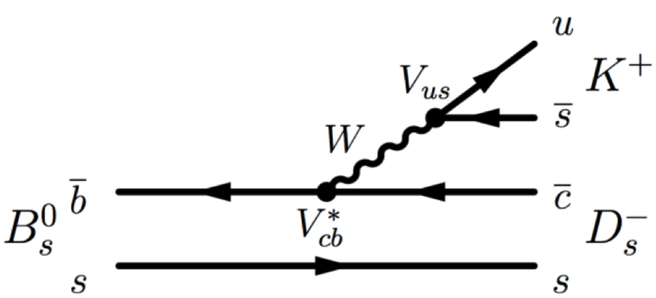


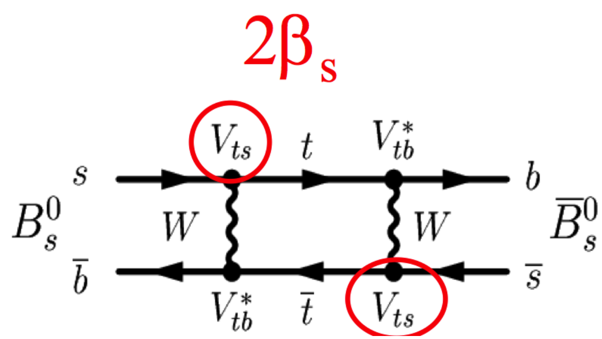
Part III:

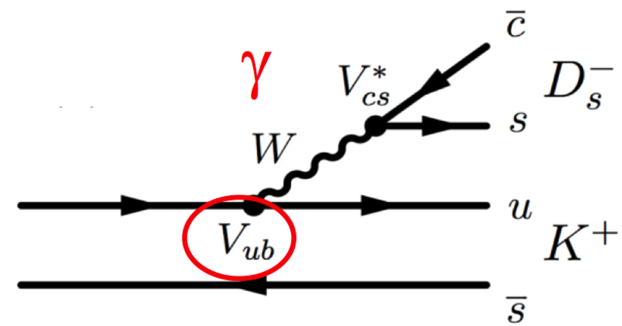
time dependent measurements

method

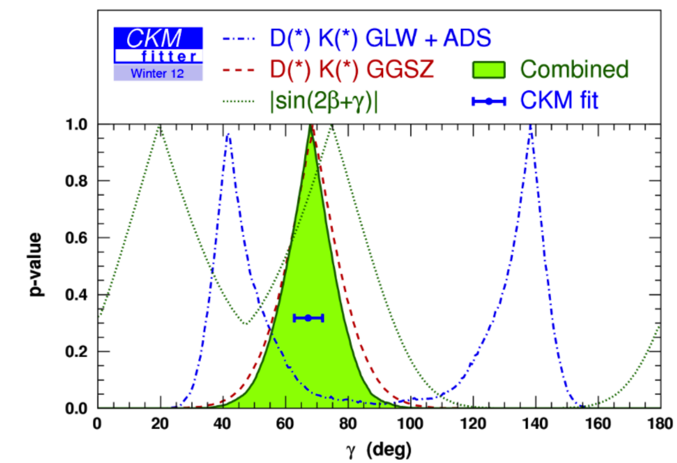
- It is also possible to use tree decays of neutral B mesons [1]!
- Using charged final states, interference is achieved through mixing.





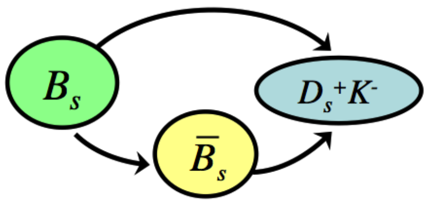


- B-factories performed such measurements with $B^0 \rightarrow D^+ \pi^-$, constraining $\sin(2\beta+\gamma)$
- Much better sensitivity expected in $B_s \rightarrow D_s K$: large amplitude ratio, finite decay width difference: $\Delta\Gamma = 0.091 \pm 0.011\ \text{ps}^{-1}$ (HFAG fall 2012)

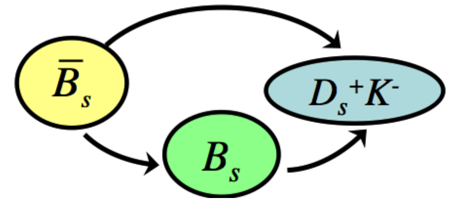


[1] R. Fleischer. New strategies to obtain insights into CP violation through $B_{(s)} \rightarrow D^{\pm}_{(s)} K^{\mp}$, $D^{*\pm}_{(s)} K^{\mp}$, ... and $B_{(d)} \rightarrow D^{\pm}\pi^{\mp}$, $D^{*\pm}\pi^{\mp}$, ... decays. *Nucl.Phys.*, B671:459–482, 2003.

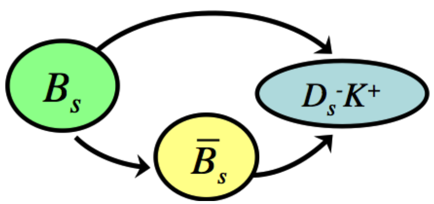
four decay rates



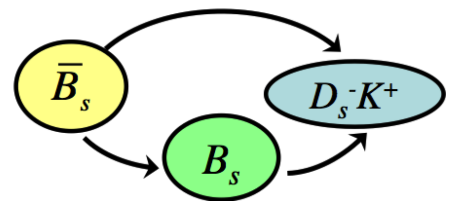
$$B_s^0 \to D_s^+ K^-$$



$$\overline{B_s^0} \to D_s^+ K^-$$



$$B_s^0 \to D_s^- K^+$$



$$\overline{B_s^0} \to D_s^- K^+$$

each has their own time dependence

strictly, only two are needed for γ

four decay rates

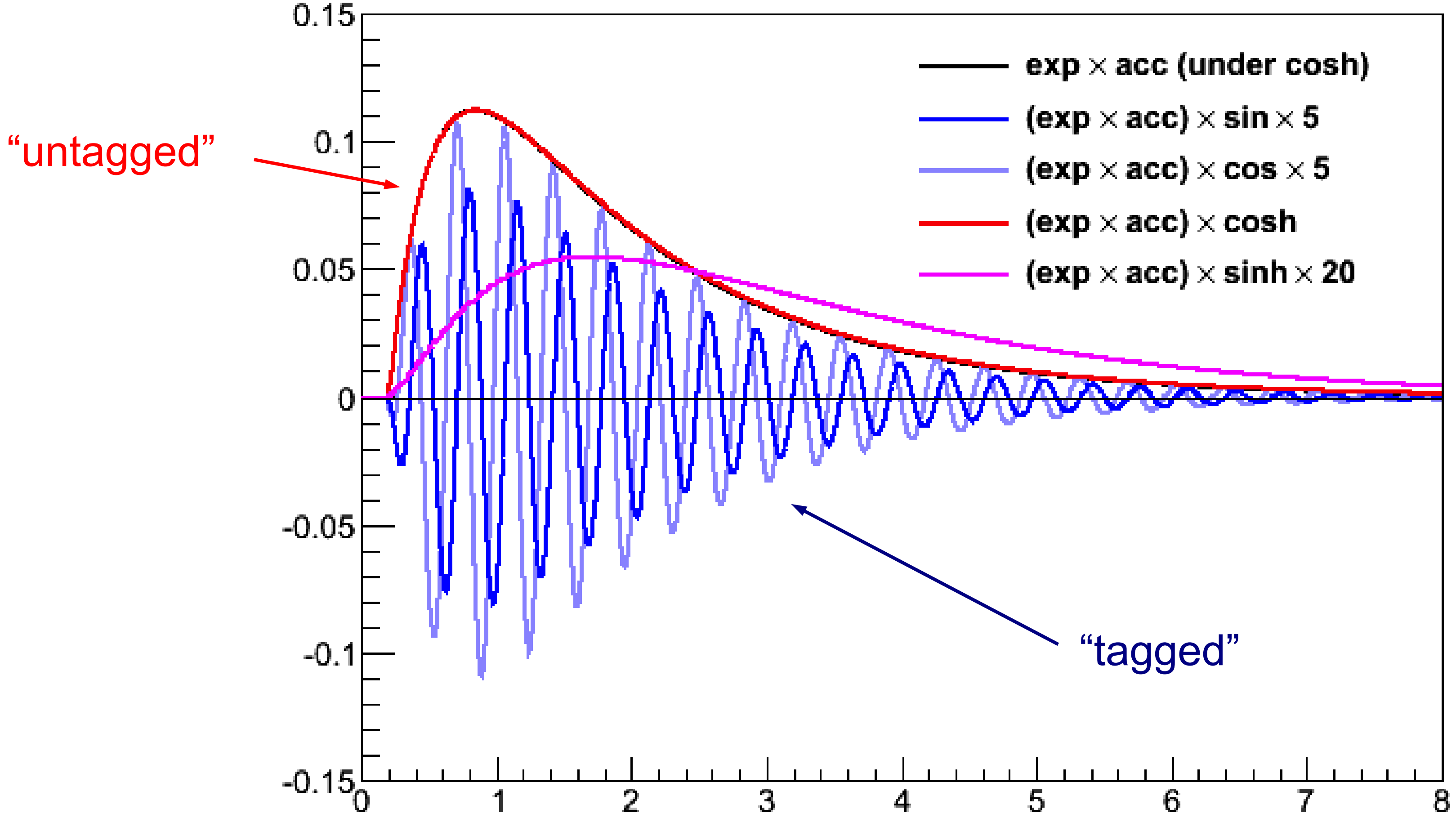
$$\frac{d\Gamma_{B_s^0\to f}(t)}{dt\, e^{-\Gamma_s t}} = \frac{1}{2}|A_f|^2(1+|\lambda_f|^2) \left[\cosh\left(\frac{\Delta\Gamma_s t}{2}\right) + D_f \sinh\left(\frac{\Delta\Gamma_s t}{2}\right) + C_f \cos(\Delta m_s t) - S_f \sin(\Delta m_s t)\right] \tag{1}$$

$$\frac{d\Gamma_{\bar{B}_s^0\to f}(t)}{dt\, e^{-\Gamma_s t}} = \frac{1}{2}|A_f|^2\left|\frac{p}{q}\right|^2(1+|\lambda_f|^2) \left[\cosh\left(\frac{\Delta\Gamma_s t}{2}\right) + D_f \sinh\left(\frac{\Delta\Gamma_s t}{2}\right) - C_f \cos(\Delta m_s t) + S_f \sin(\Delta m_s t)\right] \tag{2}$$

$$\frac{d\Gamma_{\bar{B}_s^0\to \bar{f}}(t)}{dt\, e^{-\Gamma_s t}} = \frac{1}{2}|\bar{A}_{\bar{f}}|^2(1+|\bar{\lambda}_{\bar{f}}|^2) \left[\cosh\left(\frac{\Delta\Gamma_s t}{2}\right) + D_{\bar{f}} \sinh\left(\frac{\Delta\Gamma_s t}{2}\right) + C_{\bar{f}} \cos(\Delta m_s t) - S_{\bar{f}} \sin(\Delta m_s t)\right] \tag{3}$$

$$\frac{d\Gamma_{B_s^0\to \bar{f}}(t)}{dt\, e^{-\Gamma_s t}} = \frac{1}{2}|\bar{A}_{\bar{f}}|^2\left|\frac{q}{p}\right|^2(1+|\bar{\lambda}_{\bar{f}}|^2) \left[\cosh\left(\frac{\Delta\Gamma_s t}{2}\right) + D_{\bar{f}} \sinh\left(\frac{\Delta\Gamma_s t}{2}\right) - C_{\bar{f}} \cos(\Delta m_s t) + S_{\bar{f}} \sin(\Delta m_s t)\right] \tag{4}$$

terms of decay rates



parameters

$$\lambda_f = \left(\frac{q}{p}\right)\frac{\bar{A}_f}{A_f} = \left(\frac{V_{tb}^* V_{ts}}{V_{tb} V_{ts}^*}\right)\left(\frac{V_{ub} V_{cs}^*}{V_{cb}^* V_{us}}\right)\left|\frac{A_2}{A_1}\right| e^{i\Delta_{T2/T1}}$$
$$= |\lambda_f| e^{i(\Delta_{T2/T1} - (\gamma + \phi_M))} \quad ,$$

$$\bar{\lambda}_{\bar{f}} = \left(\frac{p}{q}\right)\frac{A_{\bar{f}}}{\bar{A}_{\bar{f}}} = \left(\frac{V_{tb} V_{ts}^*}{V_{tb}^* V_{ts}}\right)\left(\frac{V_{ub}^* V_{cs}}{V_{cb} V_{us}^*}\right)\left|\frac{A_2}{A_1}\right| e^{i\Delta_{T2/T1}}$$
$$= |\lambda_f| e^{i(\Delta_{T2/T1} + (\gamma + \phi_M))} \quad .$$

Here, Δ represents the strong phase difference between the interfering amplitudes.

Weak phase: γ and the Bs mixing phase

$$C_f = \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2} \quad , \quad S_f = \frac{2\mathcal{I}m(\lambda_f)}{1 + |\lambda_f|^2} \quad , \quad D_f = \frac{2\mathcal{R}e(\lambda_f)}{1 + |\lambda_f|^2}$$
$$C_{\bar{f}} = \frac{1 - |\bar{\lambda}_{\bar{f}}|^2}{1 + |\bar{\lambda}_{\bar{f}}|^2} \quad , \quad S_{\bar{f}} = \frac{2\mathcal{I}m(\bar{\lambda}_{\bar{f}})}{1 + |\bar{\lambda}_{\bar{f}}|^2} \quad , \quad D_{\bar{f}} = \frac{2\mathcal{R}e(\bar{\lambda}_{\bar{f}})}{1 + |\bar{\lambda}_{\bar{f}}|^2}$$

parameters

Assuming CP conservation in mixing, $|p/q| = 1$

and CP conservation in the direct decays, $|\bar{A}_{\bar{f}}| = |A_f| \quad , \quad |\bar{A}_f| = |A_{\bar{f}}|$

we find $|\lambda_f| = |\bar{\lambda}_{\bar{f}}|$

and thus $C_f = C_{\bar{f}}$

renaming

$r_{D_sK} = |\lambda_f|$

$\phi_M = -2\beta_s$

Five observables

$$C = \frac{1 - r_{D_sK}^2}{1 + r_{D_sK}^2} ,$$

$$D_f = \frac{2r_{D_sK}\cos(\Delta - (\gamma - 2\beta_s))}{1 + r_{D_sK}^2} , \quad D_{\bar{f}} = \frac{2r_{D_sK}\cos(\Delta + (\gamma - 2\beta_s))}{1 + r_{D_sK}^2} ,$$

$$S_f = \frac{2r_{D_sK}\sin(\Delta - (\gamma - 2\beta_s))}{1 + r_{D_sK}^2} , \quad S_{\bar{f}} = \frac{2r_{D_sK}\sin(\Delta + (\gamma - 2\beta_s))}{1 + r_{D_sK}^2} .$$

$B_s \rightarrow D_s K$

- Sensitivity to γ expected to be large.

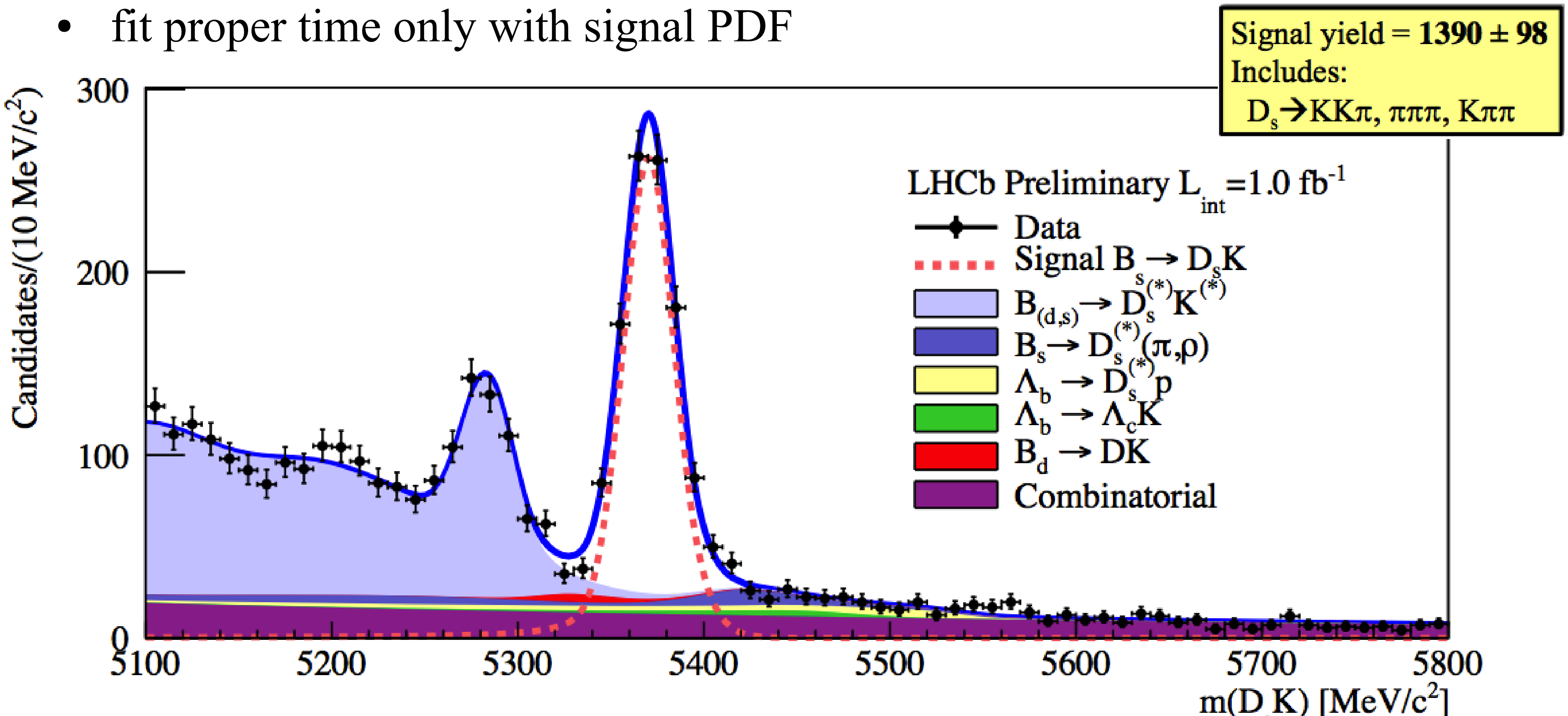
- Eventually, there could be some sensitivity to CPT violation:

  Kundu, Nandi, Patra, Soni, ARXIV:1209.6063

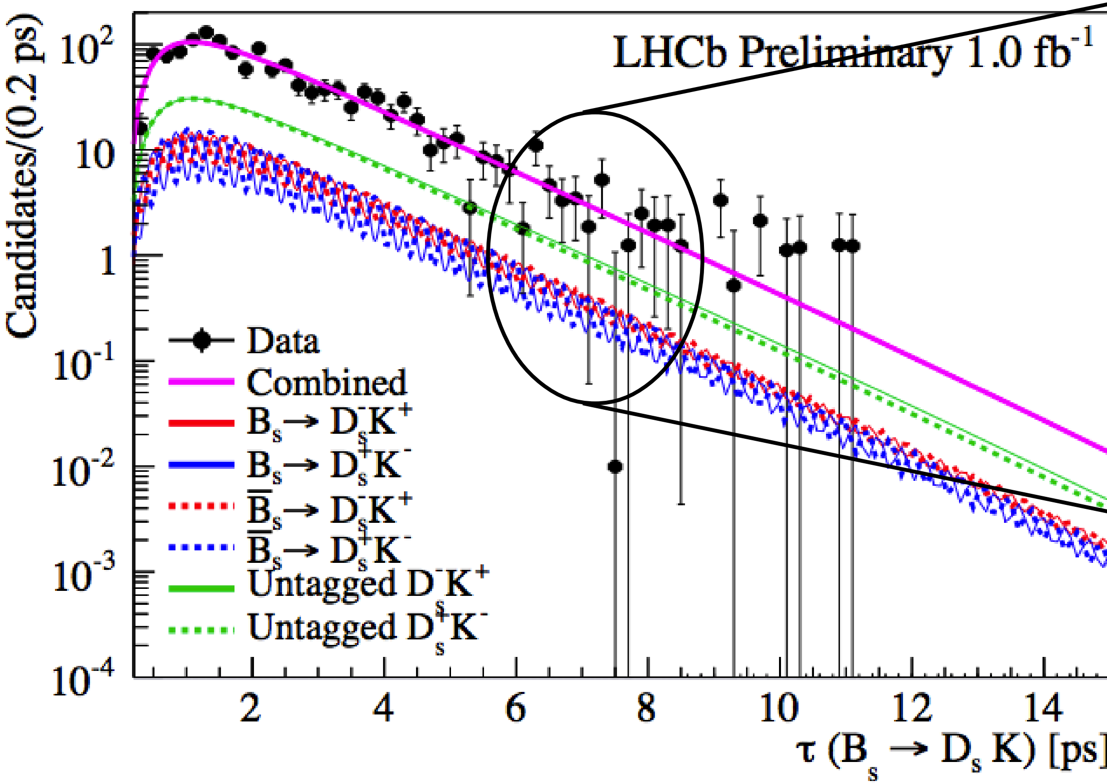
- Uses the best of LHCb:
  - excellent time resolution needed to resolve fast B_s oscillations
  - relies on flavor tagging to tell B_s from B_s-bar
  - fully hadronic decay, need excellent PID
  - relies on hadronic trigger that reconstructs secondary vertices online
  - time acceptance is important

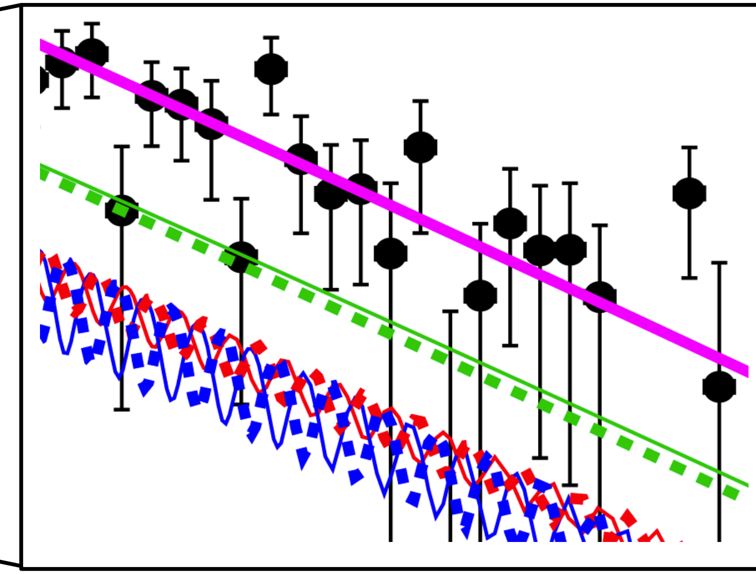
mass plot

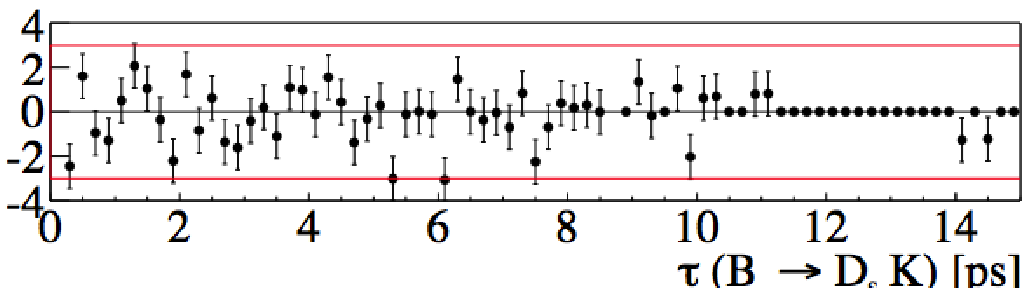
- selection based on boosted decision tree
- rich physics backgrounds – all could have different time structure!
- employ an "sFit" technique ("cFit" available):
    - use B_s mass as discriminating variable to compute per-event weights
    - fit proper time only with signal PDF



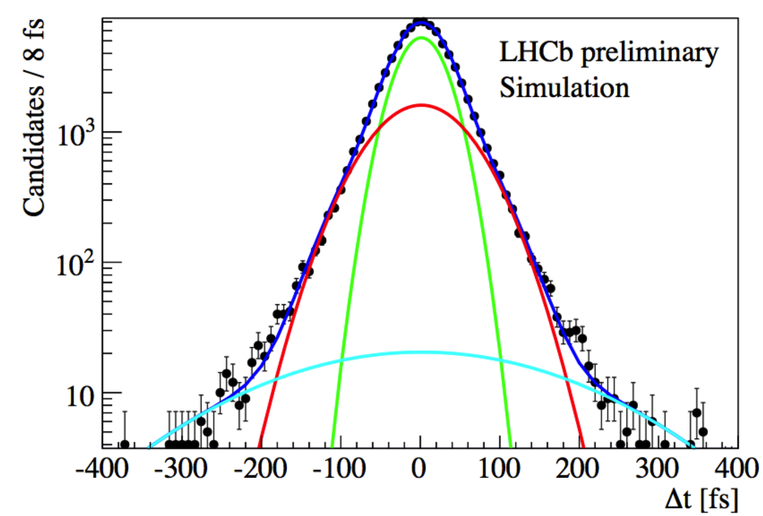
time dependent DsK







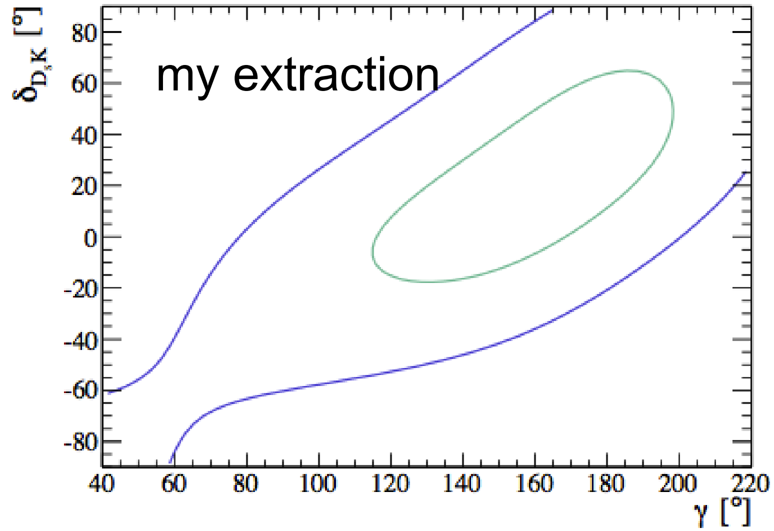
time resolution ~50fs

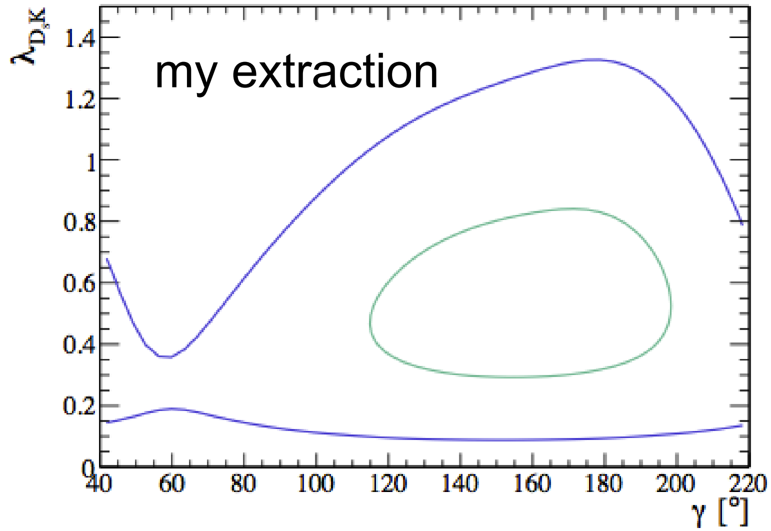


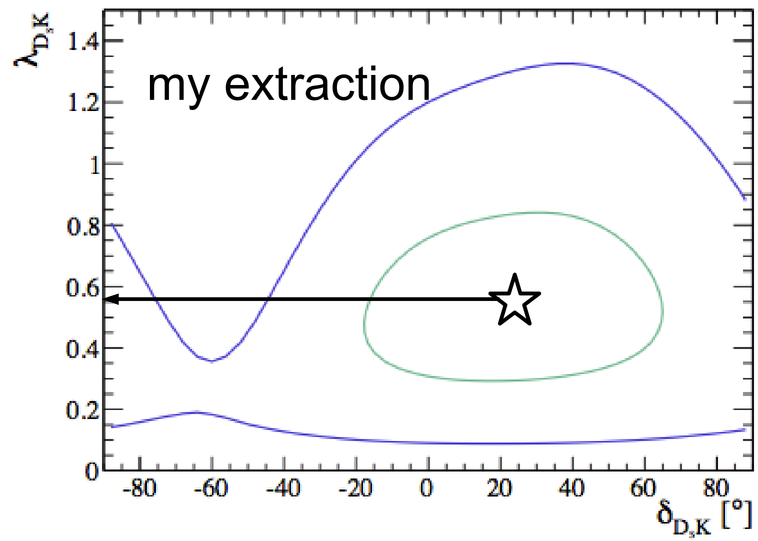
systematic uncertainties

	C	S_f	$S_{\bar{f}}$	D_f	$D_{\bar{f}}$
Toy corrected central value	1.01	-1.25	0.08	-1.33	-0.81
Statistical uncertainty	0.50	0.56	0.68	0.60	0.56
Systematic uncertainties (σ_{stat})					
Decay-time bias	0.03	0.05	0.05	0.00	0.00
Decay-time resolution	0.11	0.08	0.09	0.00	0.00
Tagging calibration	0.23	0.17	0.16	0.00	0.00
Backgrounds	0.15	0.07	0.07	0.07	0.07
Fixed parameters	0.15	0.22	0.20	0.40	0.42
Asymmetries	0.12	0.01	0.04	0.00	0.02
Momentum/length scale	0.00	0.00	0.00	0.00	0.00
k-factors	0.27	0.27	0.27	0.08	0.08
Bias correction	0.03	0.03	0.03	0.03	0.03
Total systematic (σ_{stat})	0.46	0.50	0.35	0.43	0.46

time dependent DsK







WARNING:
syst. correlations were neglected, but extraction depends on them

Part IV:

gamma combination

Combination

- We now have measured a total of **24 γ-related observables**.

- What does it mean for γ?
- What about the other parameters (rB, ...)?
- There is no easy answer (yet) to "what is the error on γ?".

A few choices to make:

- What about the Dπ system? We're the first including it.
- What about external input on the hadronic D systems? We use CLEO information at a deep level.
- What about CP violation in charm decays?
- Frequentist / Bayesian?

Parameters

Table 2: Overview of prominent parameters of the input analyses. The symbol h stands for $h = K, \pi$. The colour code is: **green** are parameters that enter more than one analysis; **purple** are parameters describing hadronic D systems; **orange** are parameters describing (mostly not-well-known) hadronic B systems; **blue** are B mixing parameters. The second part of the table is not included into the combination yet.

Analysis	N_{obs}	Parameters
$B^+ \to Dh^+$, $D \to hh$, GLW/ADS	13	γ, r_B, δ_B, r_B^π, δ_B^π, $R_{K/\pi}$, $r_{K\pi}$, $\delta_{K\pi}$, $A_{CP}^{D\to KK}$, $A_{CP}^{D\to\pi\pi}$
$B^+ \to DK^+$, $D \to K_s^0 h^+ h^-$, GGSZ	4	γ, r_B, δ_B
$B^+ \to Dh^+$, $D \to K\pi\pi\pi$, ADS	7	γ, r_B, δ_B, r_B^π, δ_B^π, $R_{K/\pi}$, $r_{K3\pi}$, $\delta_{K3\pi}$, $\kappa_{K3\pi}$
Cleo $D^0 \to K\pi$, $D^0 \to K\pi\pi\pi$	9	x_D, y_D, $\delta_{K\pi}$, $\delta_{K3\pi}$, $\kappa_{K3\pi}$, $r_{K\pi}$, $r_{K3\pi}$, $\mathcal{B}(K\pi)$, $\mathcal{B}(K\pi\pi\pi)$
CP violation in the charm system	2	$A_{CP}^{D\to KK}$, $A_{CP}^{D\to\pi\pi}$
charm mixing	3	x_D, y_D, $\delta_{K\pi}$, $r_{K\pi}$
$B^0 \to DK^{0*}$, $D \to hh$, $K^* \to K\pi$, GLW	2	γ, $r_B^{K^{0*}}$, $\delta_B^{K^{0*}}$, $\kappa_B^{K^{0*}}$
$B^+ \to DK^+\pi^+\pi^-$, $D \to K\pi$, ADS	2	γ, $r_B^{DK\pi\pi}$, $\delta_B^{DK\pi\pi}$, $\kappa_B^{DK\pi\pi}$, $r_{K\pi}$, $\delta_{K\pi}$
$B^0 \to D^+\pi^-$ time dependent	5	γ, $\lambda_{D^+\pi^-}$, $\delta_{D^+\pi^-}$, Δm_d, $\sin 2\beta$
$B_s^0 \to D_s^+ K^-$ time dependent	5	γ, λ_{D_sK}, δ_{D_sK}, Δm_s, Γ_s, $\Delta\Gamma_s$, ϕ_s

combination strategy

- Form combined likelihood (using the GLW example):
  - Observables $\vec{y} = (A_{CP+}, R_{CP+})^T$
  - Physics parameters $\vec{x} = (\gamma, r_B, \delta_B)^T$
  - Truth relations $\vec{y}_t = \vec{f}(\vec{x})$

$$R_{CP\pm} = 1 + r_B^2 \pm 2r_B \cos\delta_B \cos\gamma\,,$$
$$A_{CP\pm} = \pm 2r_B \sin\delta_B \sin\gamma / R_{CP\pm}.$$

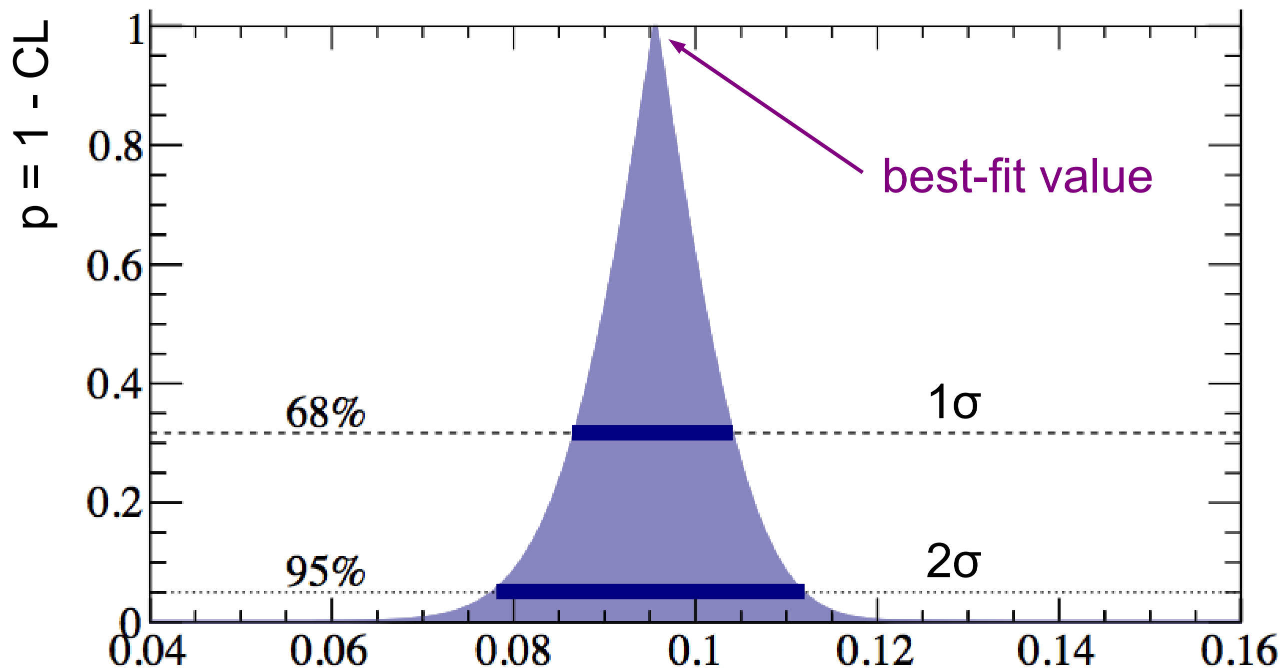
  - Construct likelihood and χ^2

$$\mathcal{L}(\vec{y}) = \frac{1}{N} \exp\left(-\frac{1}{2}(\vec{y} - \vec{y}_{\mathrm{t}})^T V_{\mathrm{cov}}^{-1} (\vec{y} - \vec{y}_{\mathrm{t}})\right)$$
$$\chi^2(\vec{y}) = -2\ln\mathcal{L}(\vec{y})\ .$$

- We use a frequentist method to compute confidence intervals.

statistical treatment

- The “usual” approach is, to find the global minimum of the χ^2 and to compute the profile likelihood.
- Then, one can turn the $\Delta\chi^2$ into a confidence level, assuming it is distributed “like that of a Gaussian”: $1 - CL = Prob(\Delta\chi^2, ndof=1)$
- This is equivalent to “Minos”, where one “goes up by 0.5”
- Using the profile likelihood in this way is neither frequentist nor Bayesian.



Not sufficient here!

statistical treatment

- The combined likelihood has a very rich structure:
  - many nuisance parameters
  - many trigonometrical functions, thus many local minima
  - dimensionality of the likelihood depends on the value of the nuisance parameters, potentially affecting the coverage

$$R_{CP\pm} = 1 + r_B^2 \pm 2 r_B \cos\delta_B \cos\gamma$$
$$A_{CP\pm} = \pm 2 r_B \sin\delta_B \sin\gamma / R_{CP\pm}\,.$$

direct product of rB and angular terms

- We use a Feldman-Cousins based frequentist method.
  - likelihood ratio ordering
- We compute the actual distribution of the test statistic ($\Delta\chi^2$) using toy Monte Carlo.

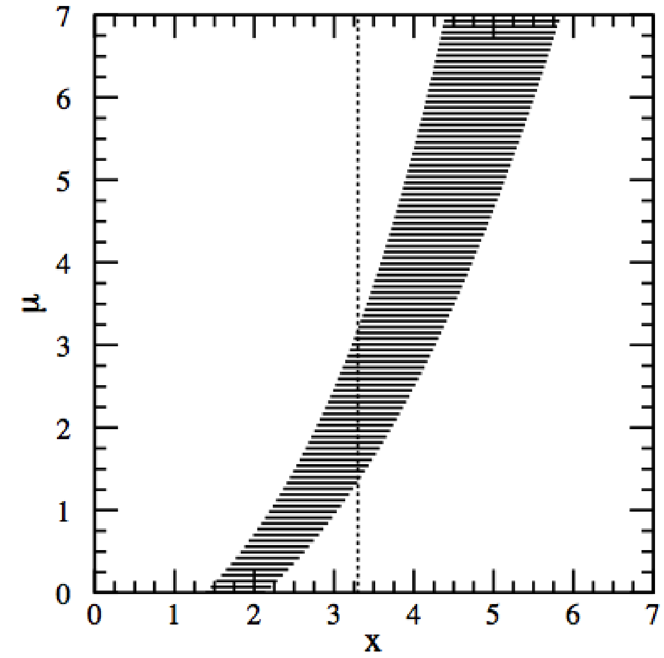


Figure: Confidence belt (plot from FC paper physics/9711021).

Plugin method

Scan for one specific physics parameter, x:

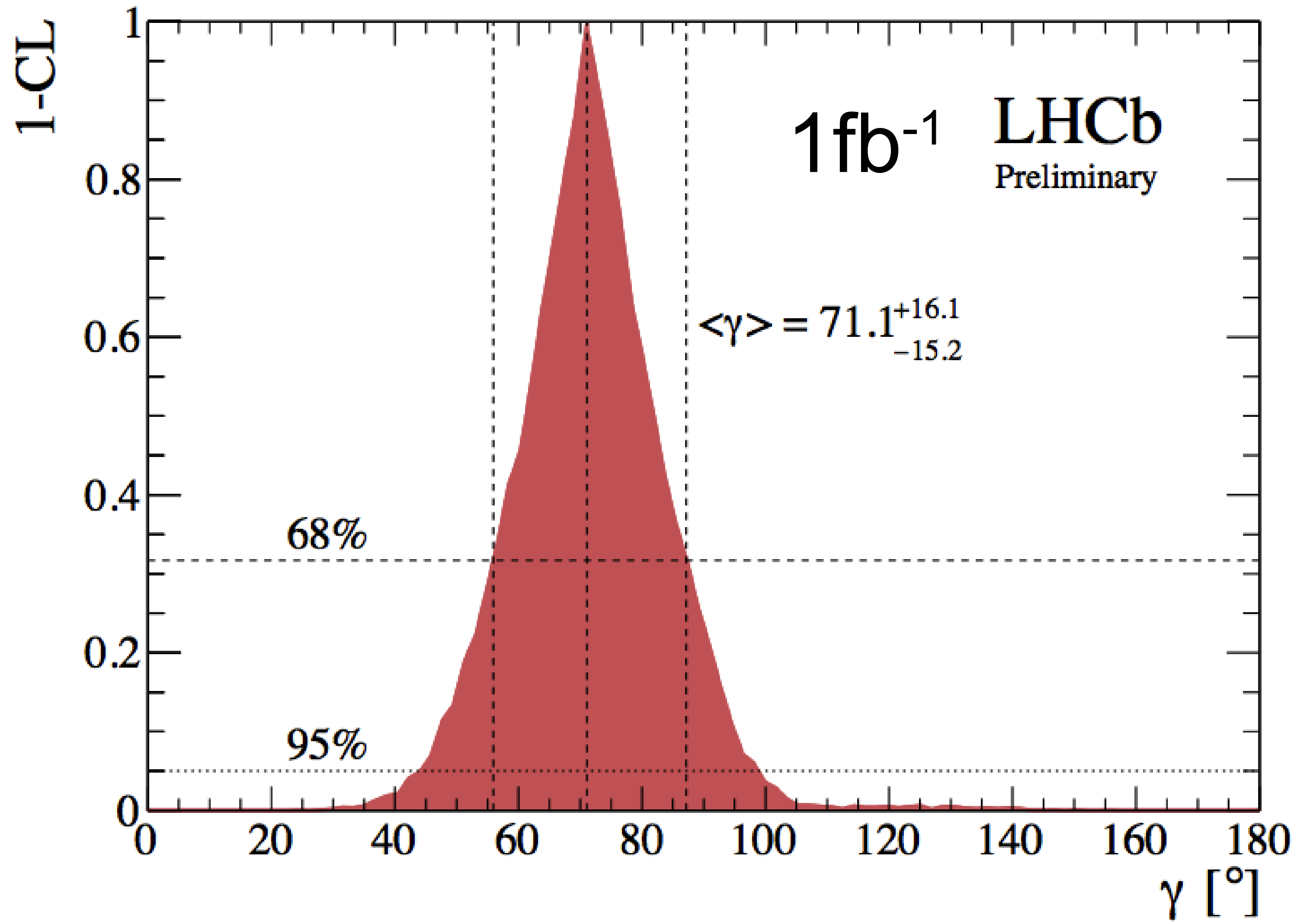
1. Find global minimum $\chi^2_{\rm min}$ and the most probable values for $\vec{x}$.
2. Fix x to x_0 and minimize with respect to the non-fixed parameters, i.e. obtain $\vec{x}'$, and $(\chi^2_{\rm min})'$. Calculate $\Delta\chi^2 = \chi^2_{\rm min} - (\chi^2_{\rm min})'$.
3. Generate a Toy MC result for $\vec{y}$, $\vec{y}_{\rm toy}$, by interpreting the likelihood as a PDF of $\vec{y}$.
4. Repeat the first two steps on the toy result, i.e. calculate $\Delta\chi^2_{\rm toy}$.
5. Calculate $(1 - {\rm CL})$ as the fraction

$$1 - {\rm CL} = \frac{N(\Delta\chi^2_{\rm toy} > \Delta\chi^2)}{N_{\rm toy}}. \qquad (5)$$

What the values to use for the parameters?
The best-fit values! (Plug them in here.)

Doesn't guarantee coverage (but tends to be close).

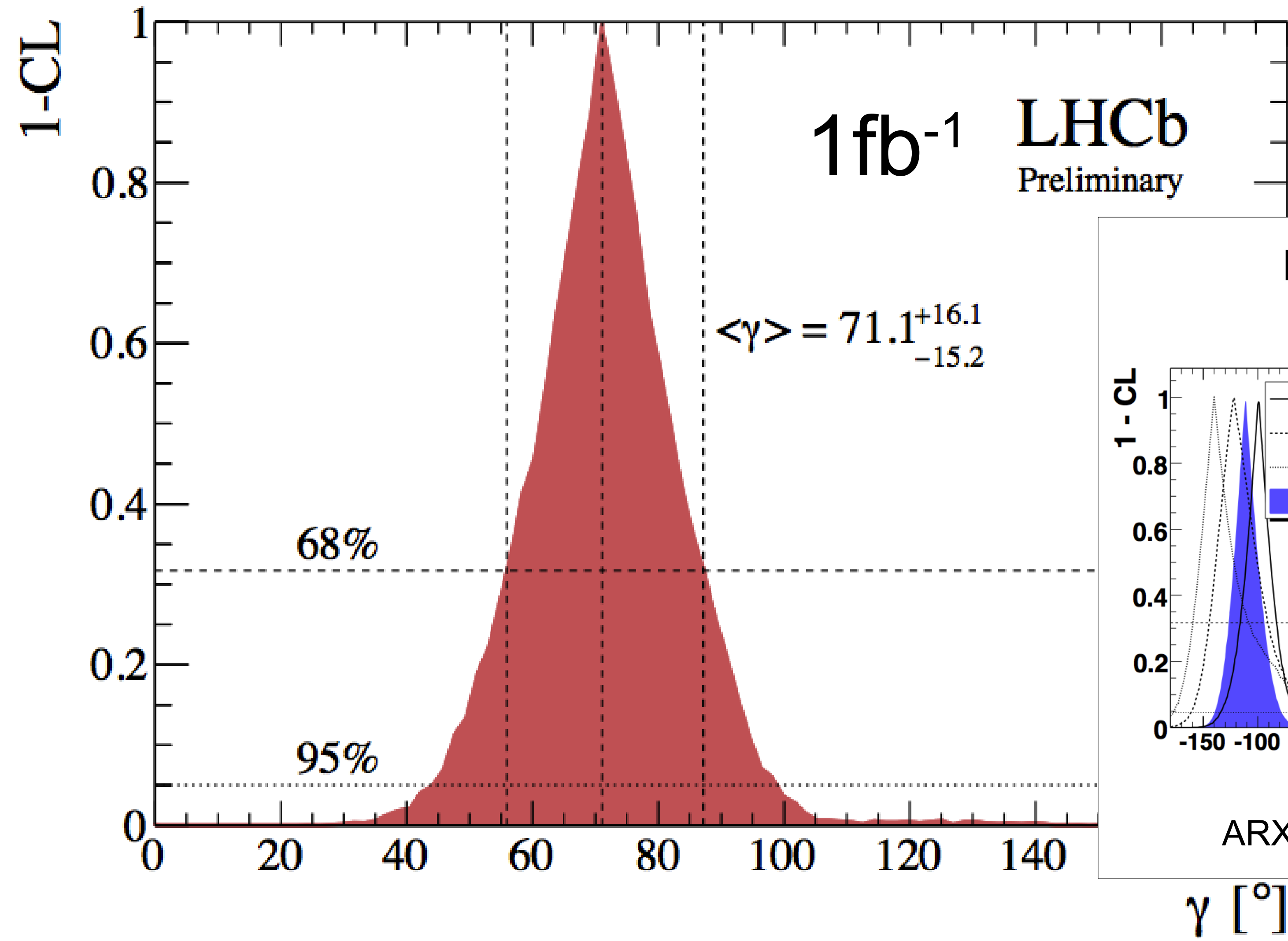
$B \to DK$ only

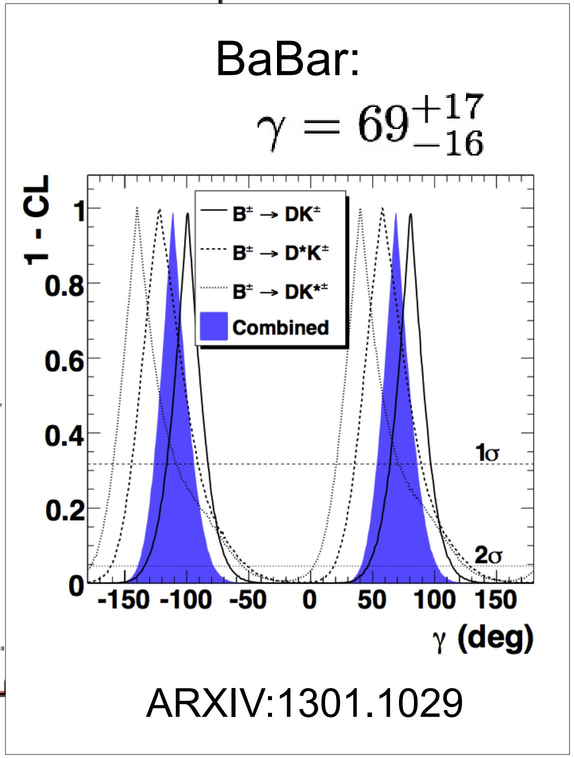


This is about as precise as the B-factory legacy combinations!

LHCb-CONF-2012-032

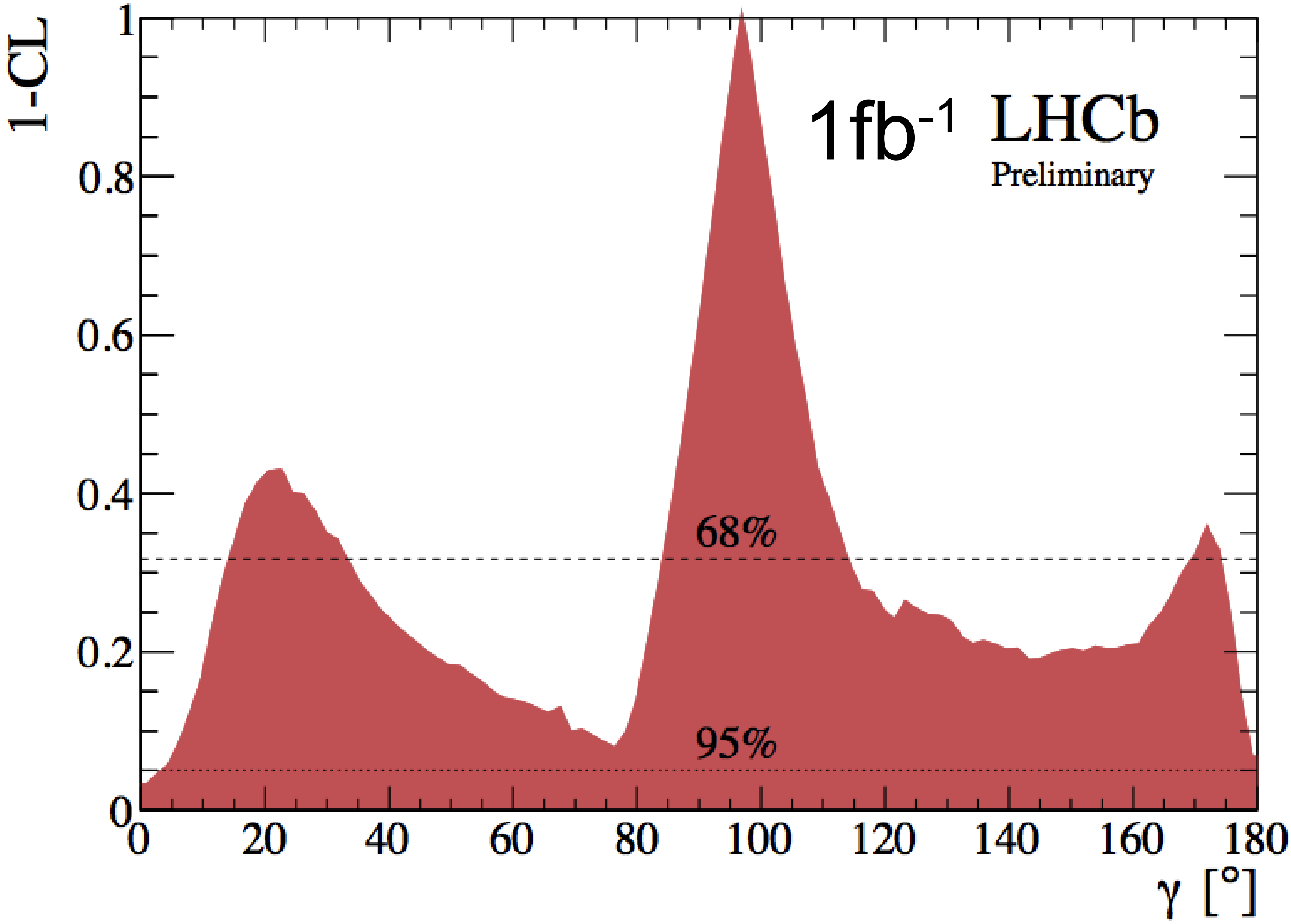
$B \to DK$ only





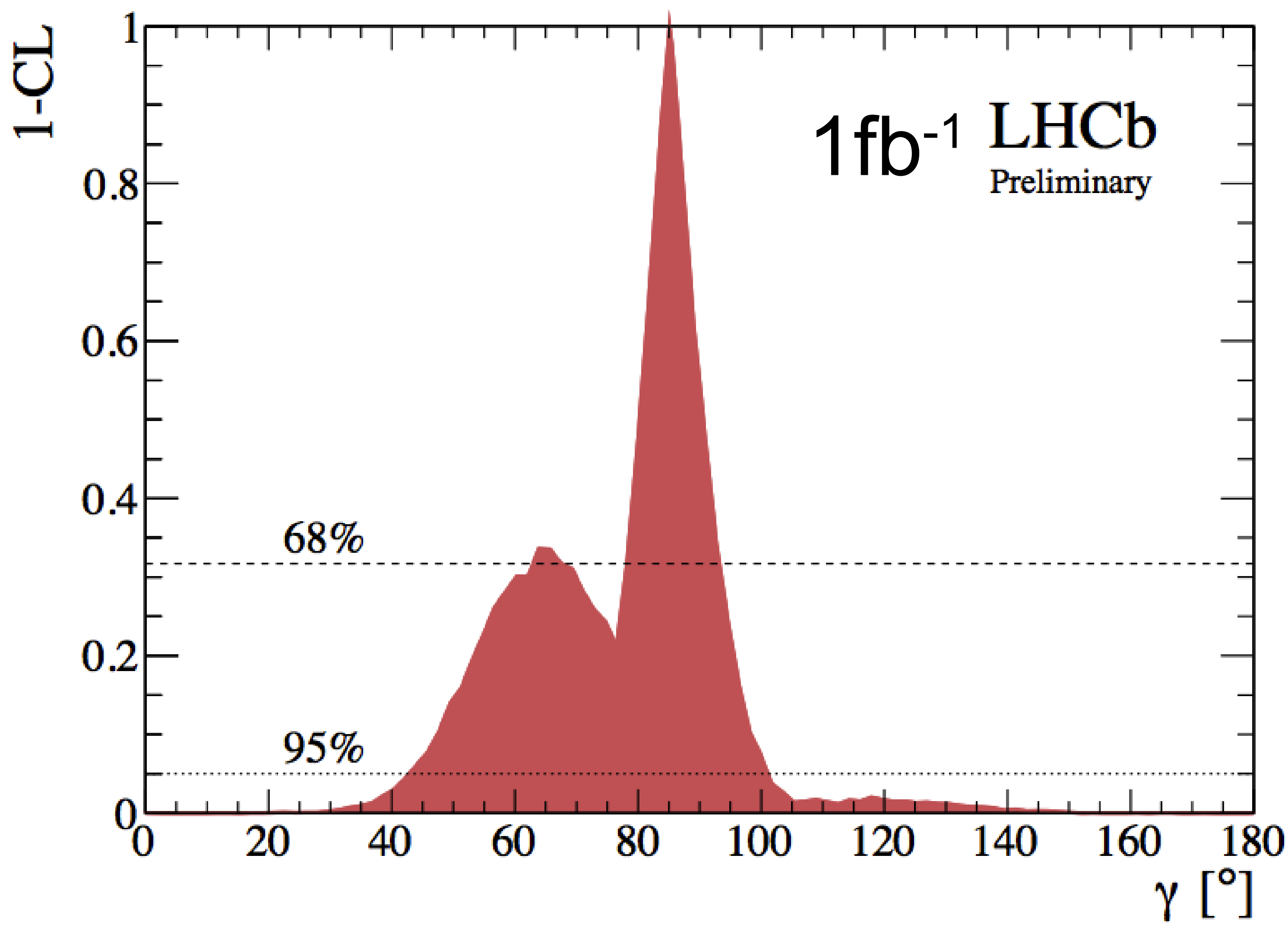
This is about as precise as the B-factory legacy combinations!

B → Dπ only



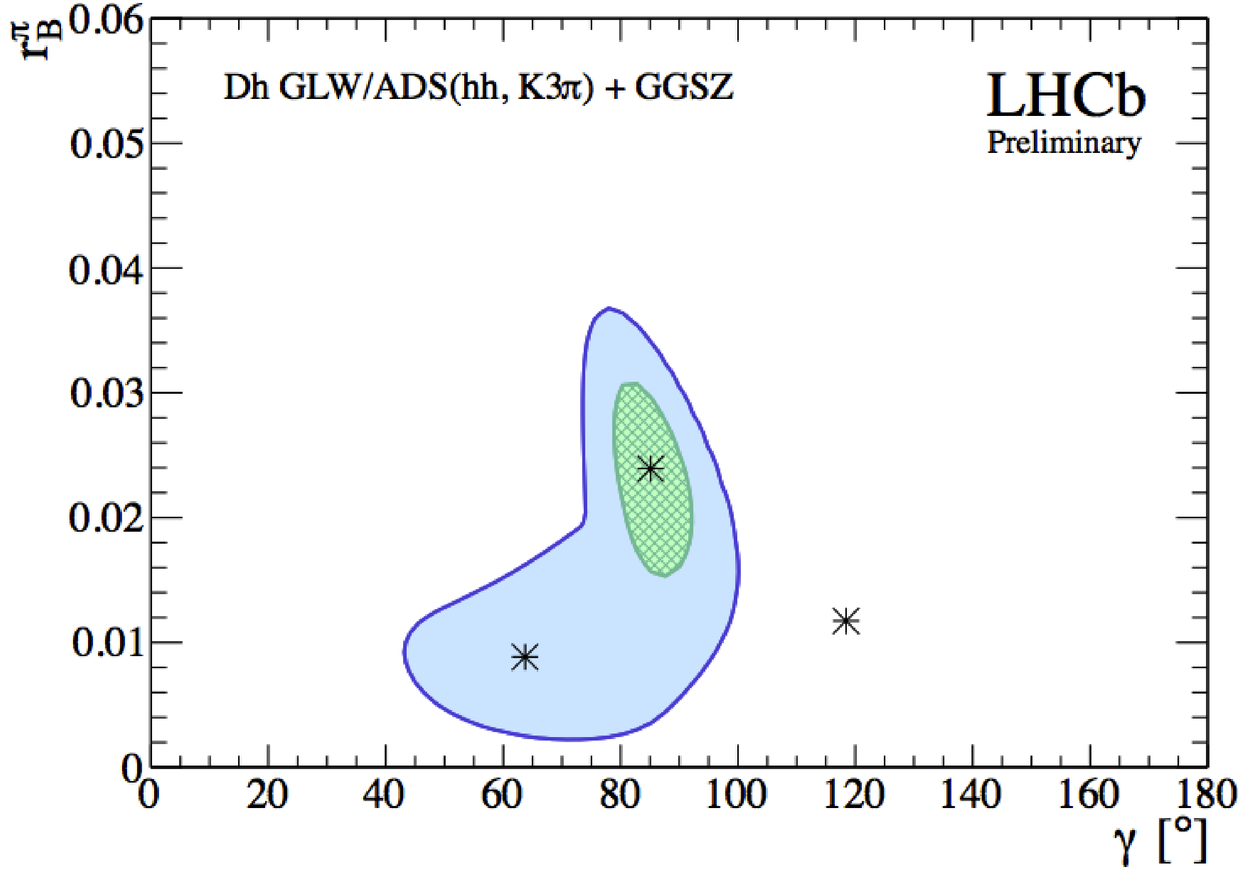
This is much more precise than expected.
But (almost) no constraint at 2σ.

full DK & Dπ



The sharp Dπ minimum affects the full combination!

full DK & Dπ



Profile likelihood for illustration

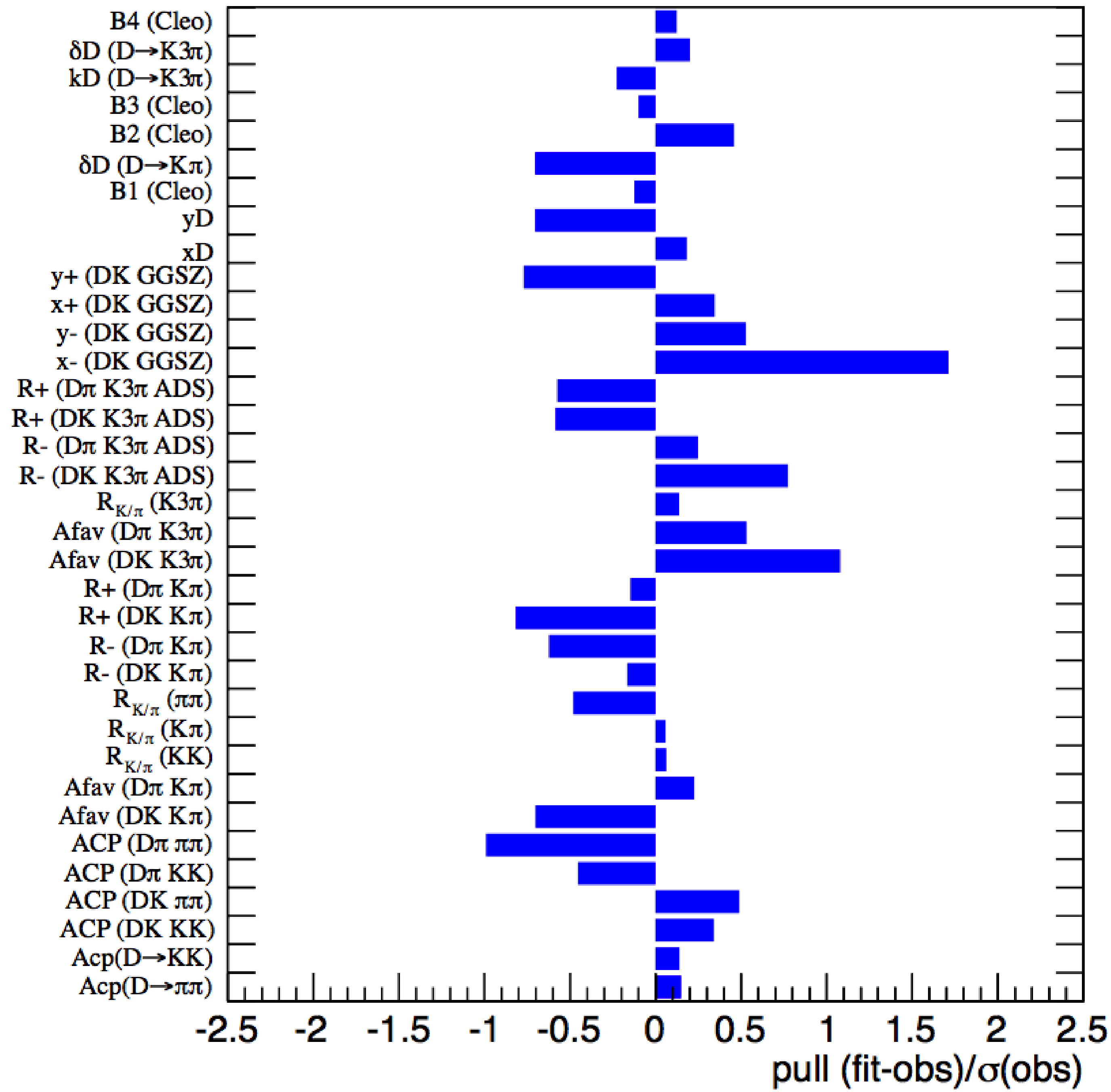
Large value of rBπ is favored.
This is about 10x larger than the expectation.
But at 2σ the expectation is more or less covered.

Conclusion

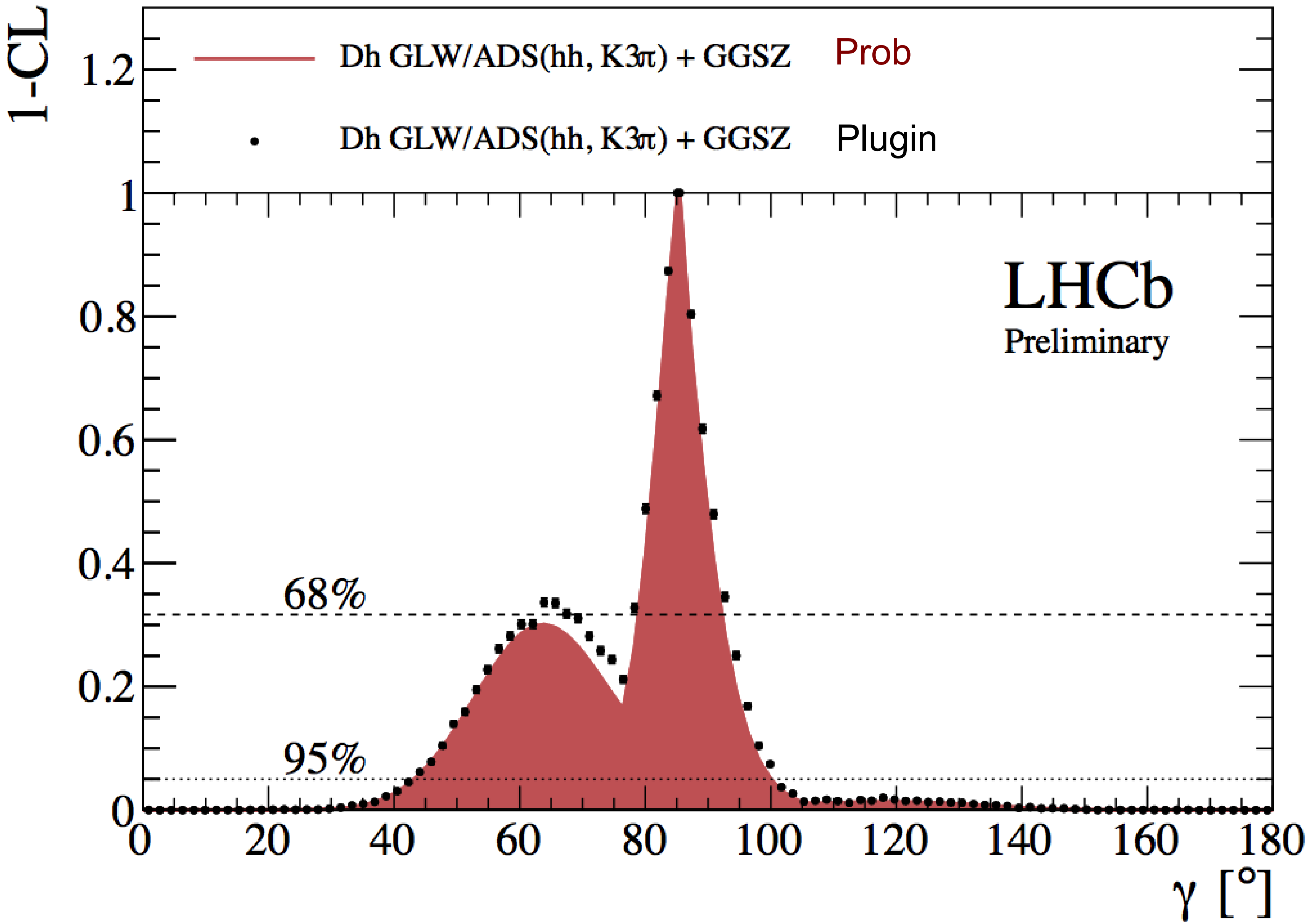
$$\gamma = (71.1\,^{+16.6}_{-15.7})^\circ$$

DK only, LHCb at CKM2012 preliminary

Backup



χ^2/(nObs-nPar) = 12.14/(35-17), P(χ^2,18)=0.840


1-CL
1.2
1
0.8
0.6
0.4
0.2
0
Dh GLW/ADS(hh, K3π) + GGSZ
Prob
Dh GLW/ADS(hh, K3π) + GGSZ
Plugin
LHCb
Preliminary
68%
95%
0
20
40
60
80
100
120
140
160
180
γ [°]