Introduction to Neutrino Physics

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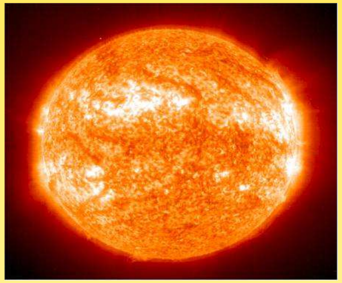
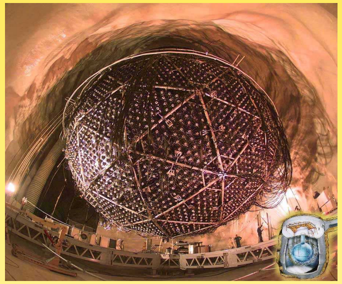
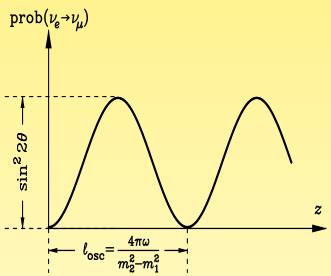
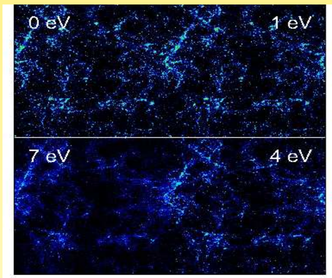
24/02/12

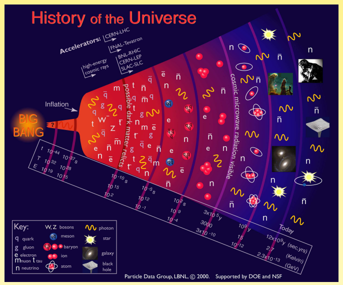
Max-Planck-Institut
für Kernphysik

MANITOP
Massive Neutrinos: Investigating their Theoretical Origin and Phenomenology





Literature

- ArXiv:
  - Bilenky, Giunti, Grimus: *Phenomenology of Neutrino Oscillations*, `hep-ph/9812360`
  - Akhmedov: *Neutrino Physics*, `hep-ph/0001264`
  - Grimus: *Neutrino Physics – Theory*, `hep-ph/0307149`
- Textbooks:
  - Fukugita, Yanagida: *Physics of Neutrinos and Applications to Astrophysics*
  - Kayser: *The Physics of Massive Neutrinos*
  - Giunti, Kim: *Fundamentals of Neutrino Physics and Astrophysics*
  - Schmitz: *Neutrinophysik*

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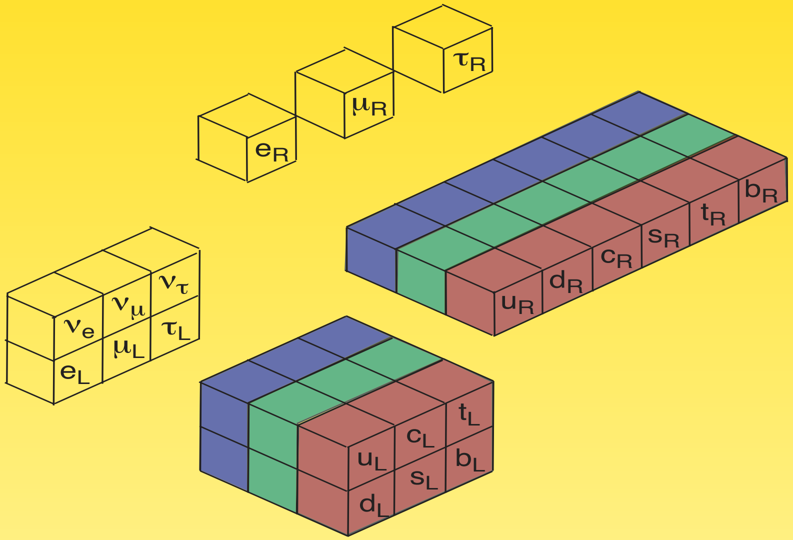
I Basics

I1) Introduction

Standard Model of Elementary Particle Physics: $SU(3)_C \times SU(2)_L \times U(1)_Y$

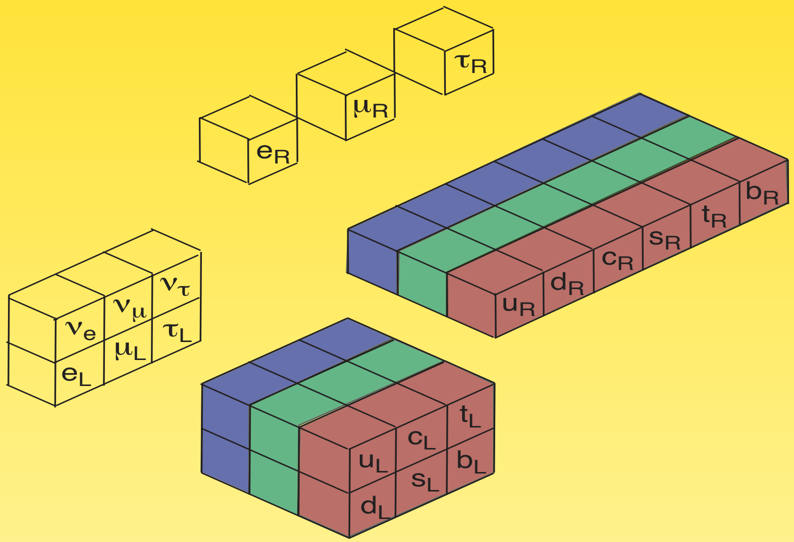


Species	$\#$	$\sum$
Quarks	10	10
Leptons	3	13
Charge	3	16
Higgs	2	18

18 free parameters...

+ Dark Matter
+ Gravitation
+ Dark Energy
+ Baryon Asymmetry

Standard Model of Elementary Particle Physics: $SU(3)_C \times SU(2)_L \times U(1)_Y$



Species	#	$\sum$
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+ Neutrino Mass m_ν

Standard Model* of Particle Physics

add neutrino mass matrix m_ν (and a new energy scale?)

Species	$\#$	$\sum$
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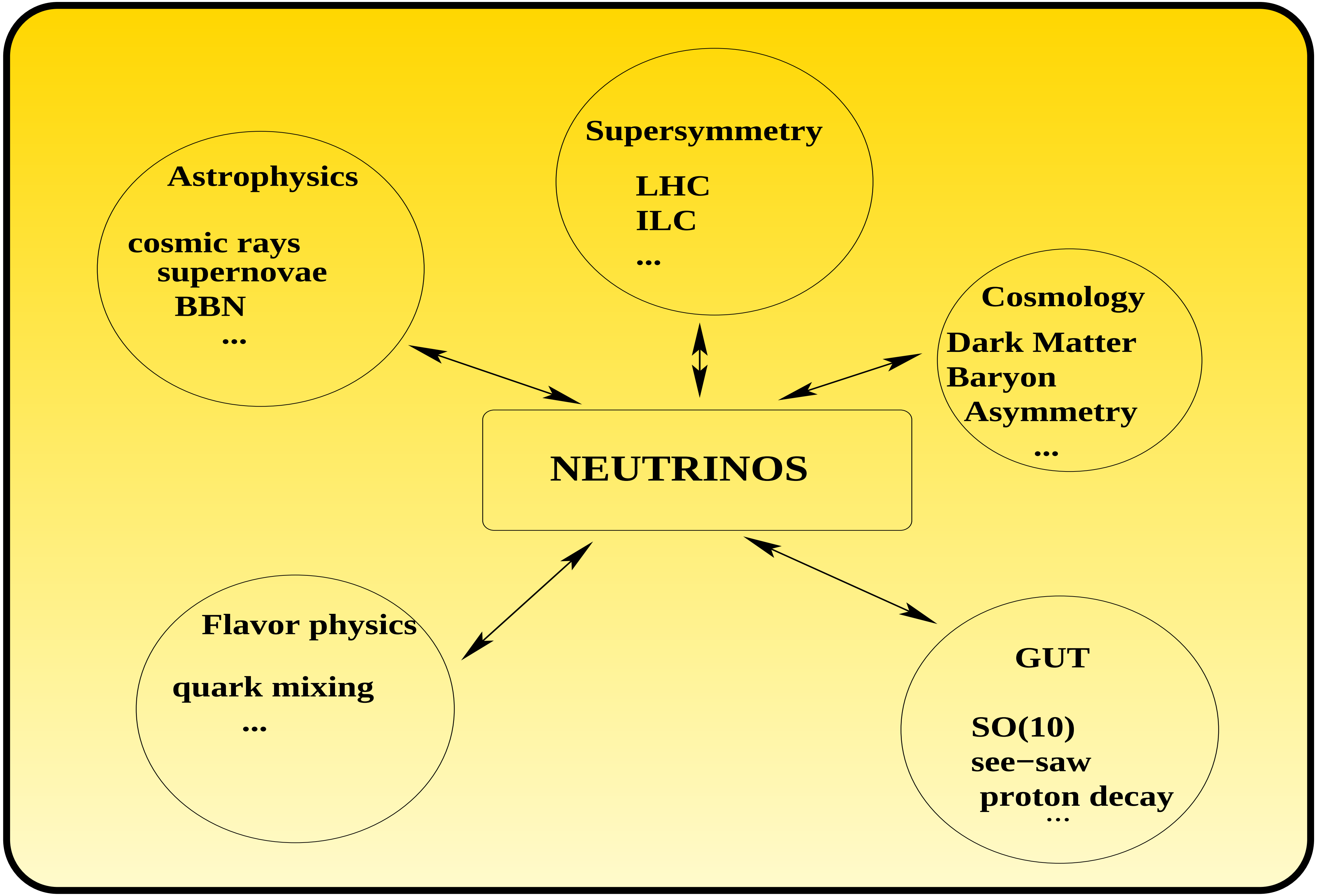
Standard Model* of Particle Physics

add neutrino mass matrix m_ν (and a new energy scale?)

Species	#	$\sum$		Species	#	$\sum$
Quarks	10	10		Quarks	10	10
Leptons	3	13	$\longrightarrow$	Leptons	12 (10)	22 (20)
Charge	3	16		Charge	3	25 (23)
Higgs	2	18		Higgs	2	27 (25)

Two roads towards more understanding: Higgs and Flavor


Supersymmetry
LHC
ILC
...
Astrophysics
cosmic rays
supernovae
BBN
...
Cosmology
Dark Matter
Baryon
Asymmetry
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Flavor physics
quark mixing
...
GUT
SO(10)
see−saw
proton decay
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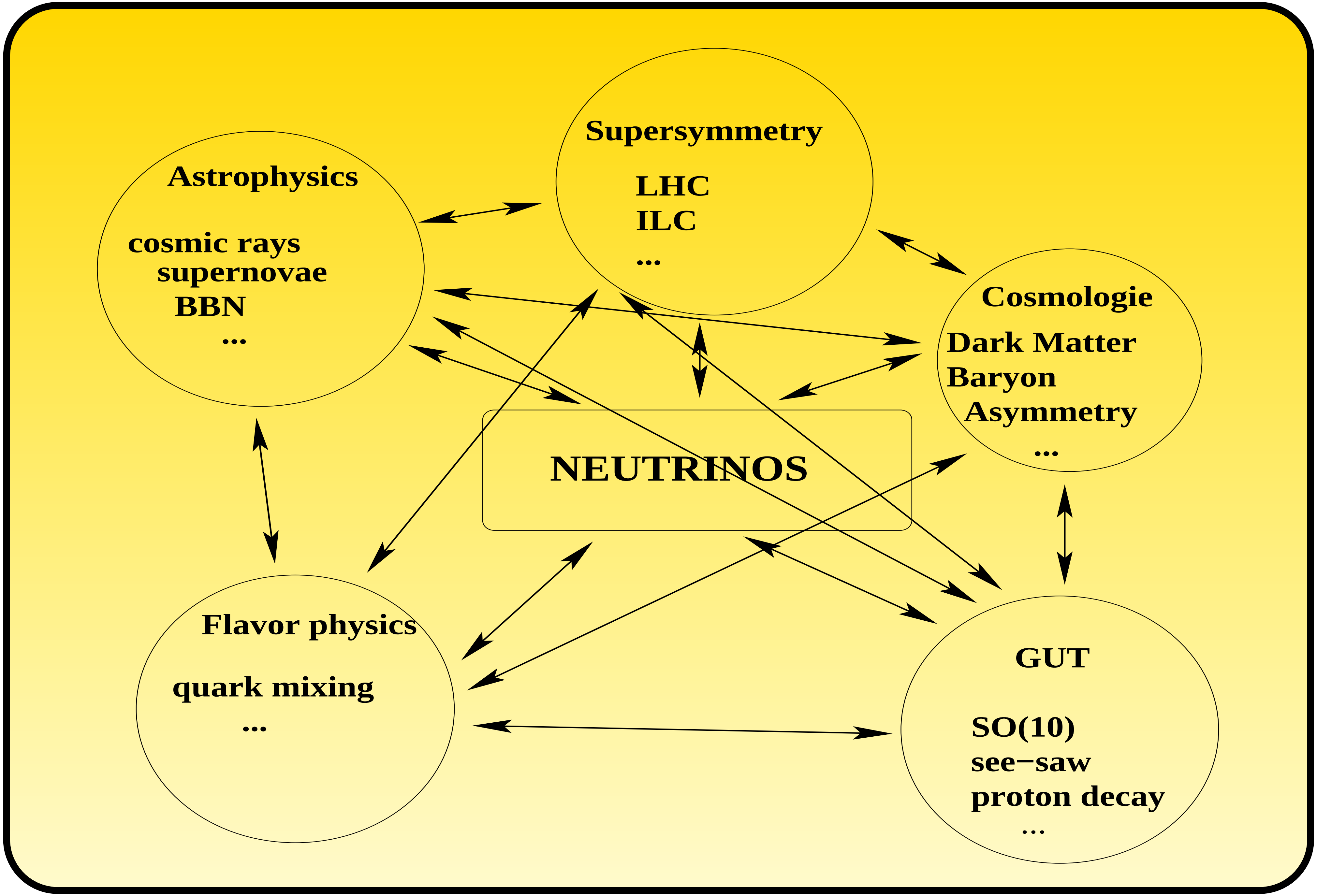
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General Remarks

- Neutrinos interact weakly: can probe things not testable by other means
  - solar interior
  - geo-neutrinos
  - cosmic rays
- Neutrinos have no mass in SM
  - probe scales $m_\nu \propto 1/\Lambda$
  - happens in GUTs
  - connected to new concepts, e.g. Lepton Number Violation

$\Rightarrow$ particle and source physics

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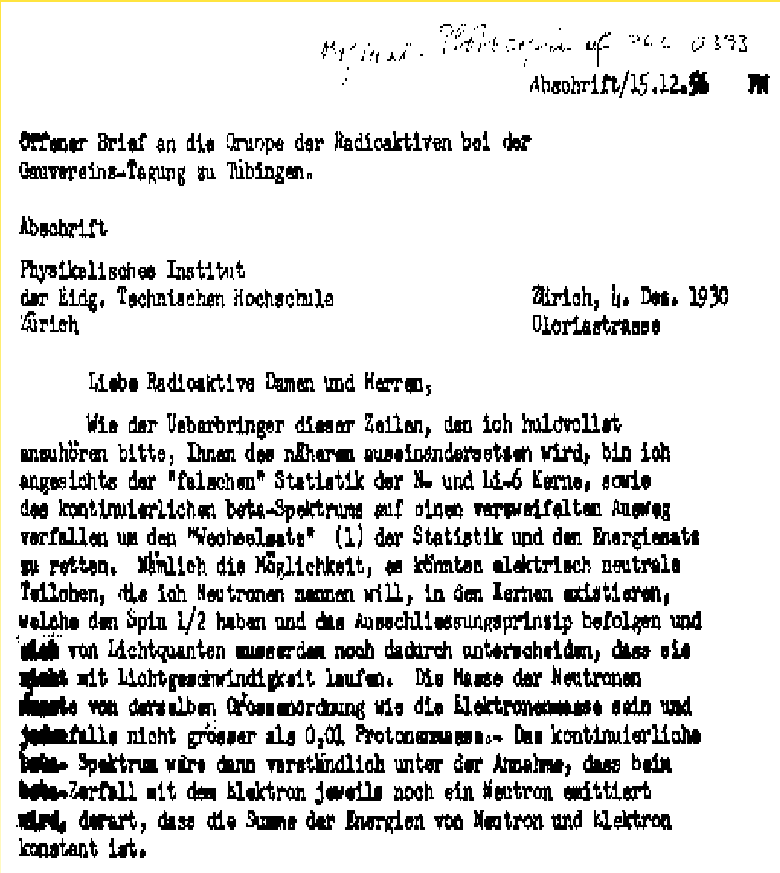
I Basics

I2) History

1926 problem in spectrum of β-decay

1930 Pauli postulates “neutron”



Abschrift/15.12.56 PM

Offener Brief an die Gruppe der Radioaktiven bei der Gauvereins-Tagung zu Tübingen.

Abschrift

Physikalisches Institut
der Eidg. Technischen Hochschule
Zürich

Zürich, 4. Dez. 1930
Gloriastrasse

Liebe Radioaktive Damen und Herren,

Wie der Ueberbringer dieser Zeilen, den ich huldvollst anzuhören bitte, Ihnen des näheren auseinandersetzen wird, bin ich angesichts der "falschen" Statistik der N- und Li-6 Kerne, sowie des kontinuierlichen beta-Spektrums auf einen verzweifelten Ausweg verfallen um den "Wechselsatz" (1) der Statistik und den Energiesatz zu retten. Nämlich die Möglichkeit, es könnten elektrisch neutrale Teilchen, die ich Neutronen nennen will, in den Kernen existieren, welche den Spin 1/2 haben und das Ausschliessungsprinzip befolgen und sich von Lichtquanten ausserdem noch dadurch unterscheiden, dass sie nicht mit Lichtgeschwindigkeit laufen. Die Masse der Neutronen müsste von derselben Grössenordnung wie die Elektronenmasse sein und jedenfalls nicht grösser als 0,01 Protonenmasse.- Das kontinuierliche beta- Spektrum wäre dann verständlich unter der Annahme, dass beim beta-Zerfall mit dem Elektron jeweils noch ein Neutron emittiert wird, derart, dass die Summe der Energien von Neutron und Elektron konstant ist.

1932 Fermi theory of β-decay

1956 discovery of $\bar{\nu}_e$ by Cowan and Reines (NP 1985)

1957 Pontecorvo suggests neutrino oscillations

1958 helicity $h(\nu_e) = -1$ by Goldhaber $\Rightarrow V - A$

1962 discovery of ν_μ by Lederman, Steinberger, Schwartz (NP 1988)

1970 first discovery of solar neutrinos by Ray Davis (NP 2002); solar neutrino problem

1987 discovery of neutrinos from SN 1987A (Koshiba, NP 2002)

1991 $N_\nu = 3$ from invisible Z width

1998 SuperKamiokande shows that atmospheric neutrinos oscillate

2000 discovery of ν_τ

2002 SNO solves solar neutrino problem

2010 the third mixing angle

Contents

I Basics

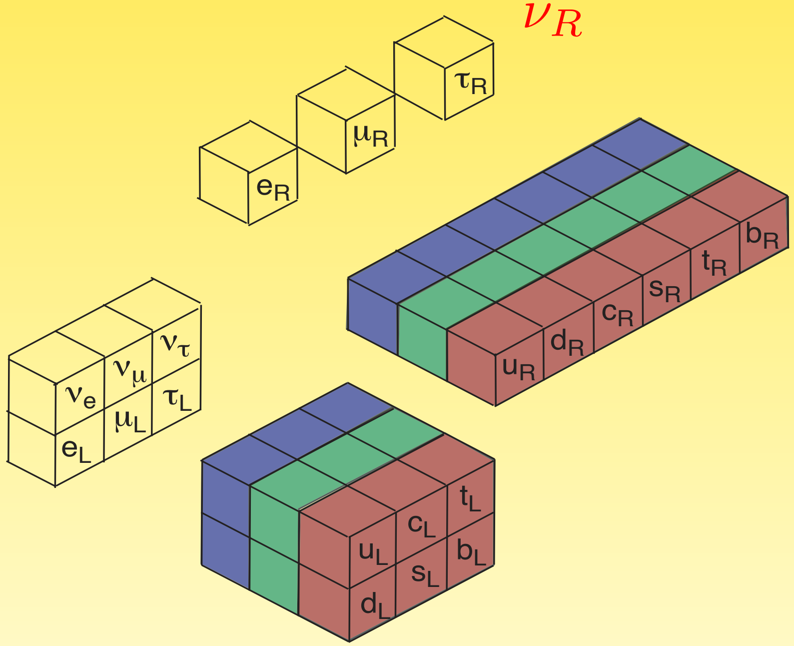
I3) Neutrinos and the Standard Model

$SU(3)_c \times SU(2)_L \times U(1)_Y \to SU(3)_c \times U(1)_{\rm em}$ with $Q = I_3 + \frac{1}{2}\,Y$

$$L_e = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \qquad \sim (1, 2, -1)$$

$$e_R \qquad \sim (1, 1, -2)$$

$$\nu_R \qquad \sim (1, 1, 0)$$ total SINGLET!!



Masses in the SM:

$$-\mathcal{L}_Y = g_e\,\overline{L}\,\Phi\,e_R + g_\nu\,\overline{L}\,\tilde{\Phi}\,\nu_R + h.c.$$

with

$$L = \left(\begin{array}{c}\nu_e\\ e\end{array}\right)_L \text{ and } \tilde{\Phi} = i\tau_2\,\Phi^* = i\tau_2\left(\begin{array}{c}\phi^+\\ \phi^0\end{array}\right)^* = \left(\begin{array}{c}\phi^0\\ -\phi^+\end{array}\right)^*$$

after EWSB: $\langle\Phi\rangle \to (0, v/\sqrt{2})^T$ and $\langle\tilde{\Phi}\rangle \to (v/\sqrt{2}, 0)^T$

$$-\mathcal{L}_Y = g_e\,\frac{v}{\sqrt{2}}\,\overline{e_L}\,e_R + g_\nu\,\frac{v}{\sqrt{2}}\,\overline{\nu_L}\,\nu_R + h.c. \equiv m_e\,\overline{e_L}\,e_R + m_\nu\,\overline{\nu_L}\,\nu_R + h.c.$$

$\Leftrightarrow$ in a renormalizable, lepton number conserving model with Higgs doublets the absence of ν_R means absence of m_ν

($\rightarrow$ ν_R don't even interact gravitationally)

Mass Matrices

3 generations of quarks

$$L'_1 = \begin{pmatrix} u' \\ d' \end{pmatrix}_L \,, \quad L'_2 = \begin{pmatrix} c' \\ s' \end{pmatrix}_L \,, \quad L'_3 = \begin{pmatrix} t' \\ b' \end{pmatrix}_L$$

$$u'_R\,,\ c'_R\,,\ t'_R \equiv u'_{i,R} \quad \text{and} \quad d'_R\,,\ s'_R\,,\ b'_R \equiv d'_{i,R}$$

gives mass term

$$\begin{aligned} -\mathcal{L}_Y &= \sum_{i,j} \overline{L'_i} \left[g_{ij}^{(d)}\, \Phi\, d'_{j,R} + g_{ij}^{(u)}\, \tilde{\Phi}\, u'_{j,R} \right] \\ &\xrightarrow{\text{EWSB}} \sum_{i,j} \frac{v}{\sqrt{2}}\, g_{ij}^{(d)}\, \overline{d'_{i,L}}\, d'_{j,R} + \frac{v}{\sqrt{2}}\, g_{ij}^{(u)}\, \overline{u'_{i,L}}\, u'_{j,R} \\ &= \overline{d'_L}\, M^{(d)}\, d'_R + \overline{u'_L}\, M^{(u)}\, u'_R \end{aligned}$$

arbitrary complex 3×3 matrices in "flavor (interaction, weak) basis"

Diagonalization

$$U_d^\dagger \, M^{(d)} \, V_d = D^{(d)} = \mathrm{diag}(m_d, m_s, m_b)$$
$$U_u^\dagger \, M^{(u)} \, V_u = D^{(u)} = \mathrm{diag}(m_u, m_c, m_t)$$

with unitary matrices $U_{u,d} U_{u,d}^\dagger = U_{u,d}^\dagger U_{u,d} = V_{u,d} V_{u,d}^\dagger = V_{u,d}^\dagger V_{u,d} = \mathbb{1}$

in Lagrangian:

$$-\mathcal{L}_Y = \overline{d'_L} \, M^{(d)} \, d'_R + \overline{u'_L} \, M^{(u)} \, u'_R$$
$$\underbrace{\overline{d'_L} \, U_d}_{\overline{d_L}} \; \underbrace{U_d^\dagger \, M^{(d)} \, V_d}_{D^{(d)}} \; \underbrace{V_d^\dagger \, d'_R}_{d_R} \; + \; \underbrace{\overline{u'_L} \, U_u}_{\overline{u_L}} \; \underbrace{U_u^\dagger \, M^{(u)} \, V_u}_{D^{(u)}} \; \underbrace{V_u^\dagger \, u'_R}_{u_R}$$

physical (mass, propagation) states $u_L = \begin{pmatrix} u \\ c \\ t \end{pmatrix}_L$

in interaction terms:

$$-\mathcal{L}_{\rm CC} = \frac{g}{\sqrt{2}}\, W_\mu^+\, \overline{u'_L}\, \gamma^\mu\, d'_L$$

$$\frac{g}{\sqrt{2}}\, W_\mu^+ \quad \underbrace{\overline{u'_L}\, U_u}_{\overline{u_L}} \quad \gamma^\mu \quad \underbrace{U_u^\dagger\, U_d}_{V} \quad \underbrace{U_d^\dagger\, d'_L}_{d_L}$$

Cabibbo-Kobayashi-Maskawa (CKM) matrix survives:

$$\boxed{V = U_u^\dagger\, U_d}$$

Structure in Wolfenstein-parametrization:

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\,\lambda^3\,(\rho - i\,\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\,\lambda^2 \\ A\,\lambda^3\,(1-\rho-i\,\eta) & -A\,\lambda^2 & 1 \end{pmatrix}$$

with $\lambda = \sin\theta_C = 0.2253 \pm 0.0007,\ A = 0.808^{+0.022}_{-0.015},$

$\overline{\rho} = (1-\frac{\lambda^2}{2})\,\rho = 0.132^{+0.022}_{-0.014},\ \overline{\eta} = 0.341 \pm 0.013$

Lesson to learn:

$$|V| = \begin{pmatrix} 0.97428 \pm 0.00015 & 0.2253 \pm 0.0007 & 0.00347^{+0.00016}_{-0.00012} \\ 0.2252 \pm 0.0007 & 0.97345^{+0.00015}_{-0.00016} & 0.0410^{+0.0011}_{-0.0007} \\ 0.00862^{+0.00026}_{-0.00020} & 0.0403^{+0.0011}_{-0.0007} & 0.999152^{+0.000030}_{-0.000045} \end{pmatrix}$$

small mixing in the quark sector

related to hierarchy of masses?

$$M = \begin{pmatrix} 0 & a \\ a & b \end{pmatrix} = U\, D\, U^T \;\; \text{with } U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

where $D = \mathrm{diag}(m_1, m_2)$

from 11-entry one gets

$$\tan\theta = \sqrt{\frac{m_1}{m_2}}$$

compare with $\sqrt{m_d/m_s} \simeq 0.22$ and $\tan\theta_C \simeq 0.23$

Number of parameters in V for N families:

complex $N \times N$	$2\,N^2$	$2\,N^2$
unitarity	$-N^2$	N^2
rephase u_i, d_i	$-(2\,N-1)$	$(N-1)^2$

a real matrix would have $\frac{1}{2}\,N\,(N-1)$ rotations around ij-axes

in total:

families	angles	phases
2	1	0
3	3	1
4	6	3
N	$\frac{1}{2}\,N\,(N-1)$	$\frac{1}{2}\,(N-2)\,(N-1)$

Lepton Masses

$$-\mathcal{L}_Y = \overline{e'_L}\, M^{(\ell)}\, e'_R$$

$$= \underbrace{\overline{e'_L}\, U_\ell}_{\overline{e_L}} \; \underbrace{U_\ell^\dagger\, M^{(\ell)}\, V_\ell}_{D^{(\ell)}} \; \underbrace{V_\ell^\dagger\, e'_R}_{e_R}$$

and in charged current term:

$$-\mathcal{L}_{\rm CC} = \frac{g}{\sqrt{2}}\, W_\mu^+\, \overline{e'_L}\, \gamma^\mu\, \nu'_L$$

$$\frac{g}{\sqrt{2}}\, W_\mu^+ \; \underbrace{\overline{e'_L}\, U_\ell}_{\overline{e_L}} \; \gamma^\mu \; \underbrace{U_\ell^\dagger\, U_\nu}_{U} \; \underbrace{U_\nu^\dagger\, \nu'_L}_{\nu_L}$$

Rotation of ν_L is arbitrary in absence of m_ν: choose $U_\nu = U_\ell$

$\Rightarrow$ Pontecorvo-Maki-Nakagawa-Saki (PMNS) matrix

$U = \mathbb{1}$ for massless neutrinos!!

$\Rightarrow$ individual lepton numbers L_e, L_μ, L_τ are conserved

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II Neutrino Oscillations

II1) The PMNS matrix

Neutrinos have mass, so:

$$-\mathcal{L}_{\rm CC} = \frac{g}{\sqrt{2}}\,\overline{\ell_L}\,\gamma^\mu\,U\,\nu_L\,W_\mu^- \quad \text{with } U = U_\ell^\dagger\,U_\nu$$

Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix

$$\boxed{\nu_\alpha = U_{\alpha i}^*\,\nu_i}$$

connects flavor states ν_α $(\alpha = e, \mu, \tau)$ to mass states ν_i $(i = 1, 2, 3)$

Number of parameters in U for N families:

complex $N \times N$	$2\,N^2$	$2\,N^2$
unitarity	$-N^2$	N^2
rephase ν_i, ℓ_i	$-(2\,N-1)$	$(N-1)^2$

a real matrix would have $\frac{1}{2}\,N\,(N-1)$ rotations around ij-axes

in total:

families	angles	phases
2	1	0
3	3	1
4	6	3
N	$\frac{1}{2}\,N\,(N-1)$	$\frac{1}{2}\,(N-2)\,(N-1)$

this assumes $\bar{\nu}\nu$ mass term, what if $\nu^T\nu$?

Number of parameters in U for N families:

complex $N \times N$	$2\,N^2$	$2\,N^2$
unitarity	$-N^2$	N^2
rephase ℓ_α	$-N$	$N(N-1)$

a real matrix would have $\frac{1}{2}\,N\,(N-1)$ rotations around ij-axes

in total:

families	angles	phases	extra phases
2	1	1	1
3	3	3	2
4	6	6	3
N	$\frac{1}{2}\,N\,(N-1)$	$\frac{1}{2}\,N\,(N-1)$	$N-1$

Extra $N-1$ "Majorana phases" because of mass term $\nu^T\,\nu$

(absent for Dirac neutrinos)

Majorana Phases

- connected to Majorana nature, hence to Lepton Number Violation
- I can always write: $U = \tilde{U}\,P$, where all Majorana phases are in $P = \mathrm{diag}(1, e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3}, \ldots)$:
- 2 families:

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & e^{i\alpha} \end{pmatrix}$$

- 3 families: $U = R_{23}\,\tilde{R}_{13}\,R_{12}\,P$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}\,e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}\,e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} P$$

$$= \begin{pmatrix} c_{12}\,c_{13} & s_{12}\,c_{13} & s_{13}\,e^{i\delta} \\ -s_{12}\,c_{23} - c_{12}\,s_{23}\,s_{13}\,e^{-i\delta} & c_{12}\,c_{23} - s_{12}\,s_{23}\,s_{13}\,e^{-i\delta} & s_{23}\,c_{13} \\ s_{12}\,s_{23} - c_{12}\,c_{23}\,s_{13}\,e^{-i\delta} & -c_{12}\,s_{23} - s_{12}\,c_{23}\,s_{13}\,e^{-i\delta} & c_{23}\,c_{13} \end{pmatrix} P$$

with $P = \mathrm{diag}(1, e^{i\alpha}, e^{i\beta})$

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II Neutrino Oscillations

II2) Neutrino Oscillations in Vacuum and Matter

a) Neutrino Oscillations in Vacuum

Neutrino produced with charged lepton α is **flavor state**

$$|\nu(0)\rangle = |\nu_\alpha\rangle = U^*_{\alpha j}\,|\nu_j\rangle$$

evolves with time as

$$|\nu(t)\rangle = U^*_{\alpha j}\,e^{-i\,E_j\,t}\,|\nu_j\rangle$$

amplitude to find state $|\nu_\beta\rangle = U^*_{\beta i}\,|\nu_i\rangle$:

$$\mathcal{A}(\nu_\alpha \to \nu_\beta, t) = \langle\nu_\beta|\nu(t)\rangle = U_{\beta i}\,U^*_{\alpha j}\,e^{-i\,E_j\,t}\quad \underbrace{\langle\nu_i|\nu_j\rangle}_{\delta_{ij}}$$

$$= U^*_{\alpha i}\,U_{\beta i}\,e^{-i\,E_i\,t}$$

Probability:

$$P(\nu_\alpha \to \nu_\beta, t) \equiv P_{\alpha\beta} = |\mathcal{A}(\nu_\alpha \to \nu_\beta, t)|^2$$

$$= \sum_{ij} \underbrace{U_{\alpha i}^* \, U_{\beta i} \, U_{\beta j}^* \, U_{\alpha j}}_{\mathcal{J}_{ij}^{\alpha\beta}} \quad \underbrace{e^{-i\,(E_i - E_j)\,t}}_{e^{-i\Delta_{ij}}}$$

$$= \ldots =$$

$$P_{\alpha\beta} = \delta_{\alpha\beta} - 4 \sum_{j>i} \mathrm{Re}\{\mathcal{J}_{ij}^{\alpha\beta}\} \, \sin^2 \frac{\Delta_{ij}}{2} + 2 \sum_{j>i} \mathrm{Im}\{\mathcal{J}_{ij}^{\alpha\beta}\} \sin \Delta_{ij}$$

with phase

$$\tfrac{1}{2}\Delta_{ij} = \tfrac{1}{2}\,(E_i - E_j)\,t \simeq \tfrac{1}{2}\left(\sqrt{p_i^2 + m_i^2} - \sqrt{p_j^2 + m_j^2}\right) L$$

$$\simeq \tfrac{1}{2}\left(p_i\,(1 + \tfrac{m_i^2}{2p_i^2}) - p_j\,(1 + \tfrac{m_j^2}{2p_j^2})\right) L \simeq \tfrac{m_i^2 - m_j^2}{2E} L$$

$$\frac{1}{2}\Delta_{ij} = \frac{m_i^2 - m_j^2}{4\,E}\, L \simeq 1.27 \left(\frac{\Delta m_{ij}^2}{\mathrm{eV}^2}\right) \left(\frac{L}{\mathrm{km}}\right) \left(\frac{\mathrm{GeV}}{E}\right)$$

$$P_{\alpha\beta} = \delta_{\alpha\beta} - 4 \sum_{j>i} \mathrm{Re}\{\mathcal{J}_{ij}^{\alpha\beta}\} \sin^2 \frac{\Delta_{ij}}{2} + 2 \sum_{j>i} \mathrm{Im}\{\mathcal{J}_{ij}^{\alpha\beta}\} \sin \Delta_{ij}$$

- $\alpha = \beta$: survival probability
- $\alpha \neq \beta$: transition probability
- requires $U \neq \mathbb{1}$ and $\Delta m_{ij}^2 \neq 0$
- $\sum\limits_{\alpha} P_{\alpha\beta} = 1 \leftrightarrow$ conservation of probability
- $\mathcal{J}_{ij}^{\alpha\beta}$ invariant under $U_{\alpha j} \to e^{i\phi_\alpha} \, U_{\alpha j} \, e^{i\phi_j}$

  $\Rightarrow$ Majorana phases drop out!

CP Violation

In oscillation probabilities: $U \to U^*$ for anti-neutrinos

Define asymmetries:

$$\Delta_{\alpha\beta} = P(\nu_\alpha \to \nu_\beta) - P(\overline{\nu}_\alpha \to \overline{\nu}_\beta) = P(\nu_\alpha \to \nu_\beta) - P(\nu_\beta \to \nu_\alpha)$$
$$= 4 \sum_{j>i} \mathrm{Im}\{\mathcal{J}_{ij}^{\alpha\beta}\} \sin \Delta_{ij}$$

- 2 families: U is real and $\mathrm{Im}\{\mathcal{J}_{ij}^{\alpha\beta}\} = 0 \; \forall \alpha, \beta, i, j$
- 3 families:

$$\Delta_{e\mu} = -\Delta_{e\tau} = \Delta_{\mu\tau} = \left(\sin \frac{\Delta m_{21}^2}{2\,E} L + \sin \frac{\Delta m_{32}^2}{2\,E} L + \sin \frac{\Delta m_{13}^2}{2\,E} L \right) J_{\mathrm{CP}}$$

where $J_{\mathrm{CP}} = \mathrm{Im}\left\{U_{e1}\, U_{\mu 2}\, U_{e2}^*\, U_{\mu 1}^*\right\}$

$$= \tfrac{1}{8} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos\theta_{13} \sin\delta$$

vanishes for one $\Delta m_{ij}^2 = 0$ or one $\theta_{ij} = 0$ or $\delta = 0, \pi$

- CP violation in survival probabilities vanishes:

$$P(\nu_\alpha \to \nu_\alpha) - P(\overline{\nu}_\alpha \to \overline{\nu}_\alpha) \propto \sum_{j>i} \mathrm{Im}\{\mathcal{J}_{ij}^{\alpha\alpha}\} = \sum_{j>i} \mathrm{Im}\{U_{\alpha i}^* \, U_{\alpha i} \, U_{\alpha j}^* \, U_{\alpha j}\} = 0$$

- Recall that $U = U_\ell^\dagger \, U_\nu$

  If charged lepton masses diagonal, then m_ν is diagonalized by PMNS matrix:

$$m_\nu = U \, \mathrm{diag}(m_1, m_2, m_3) \, U^T$$

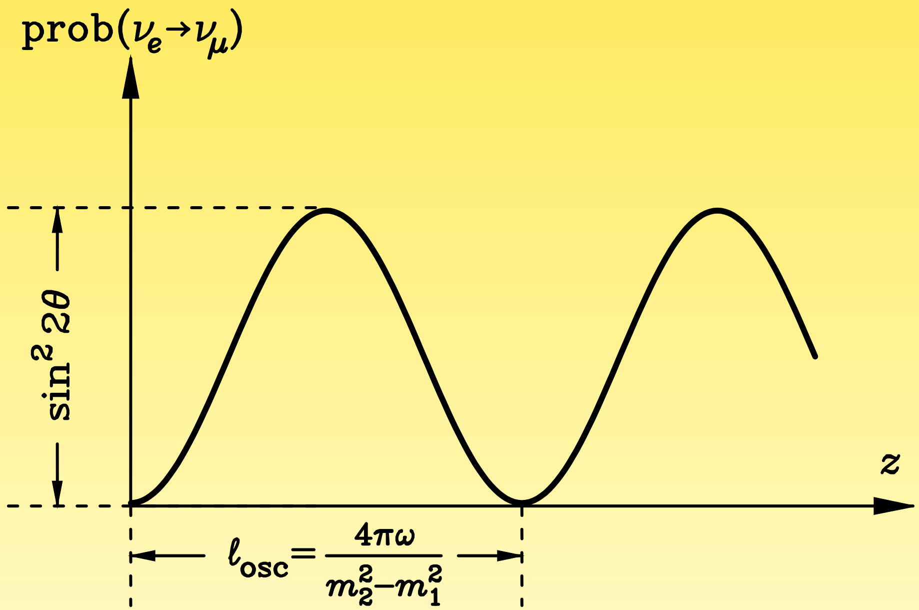
  Define $h = m_\nu \, m_\nu^\dagger$ and find that

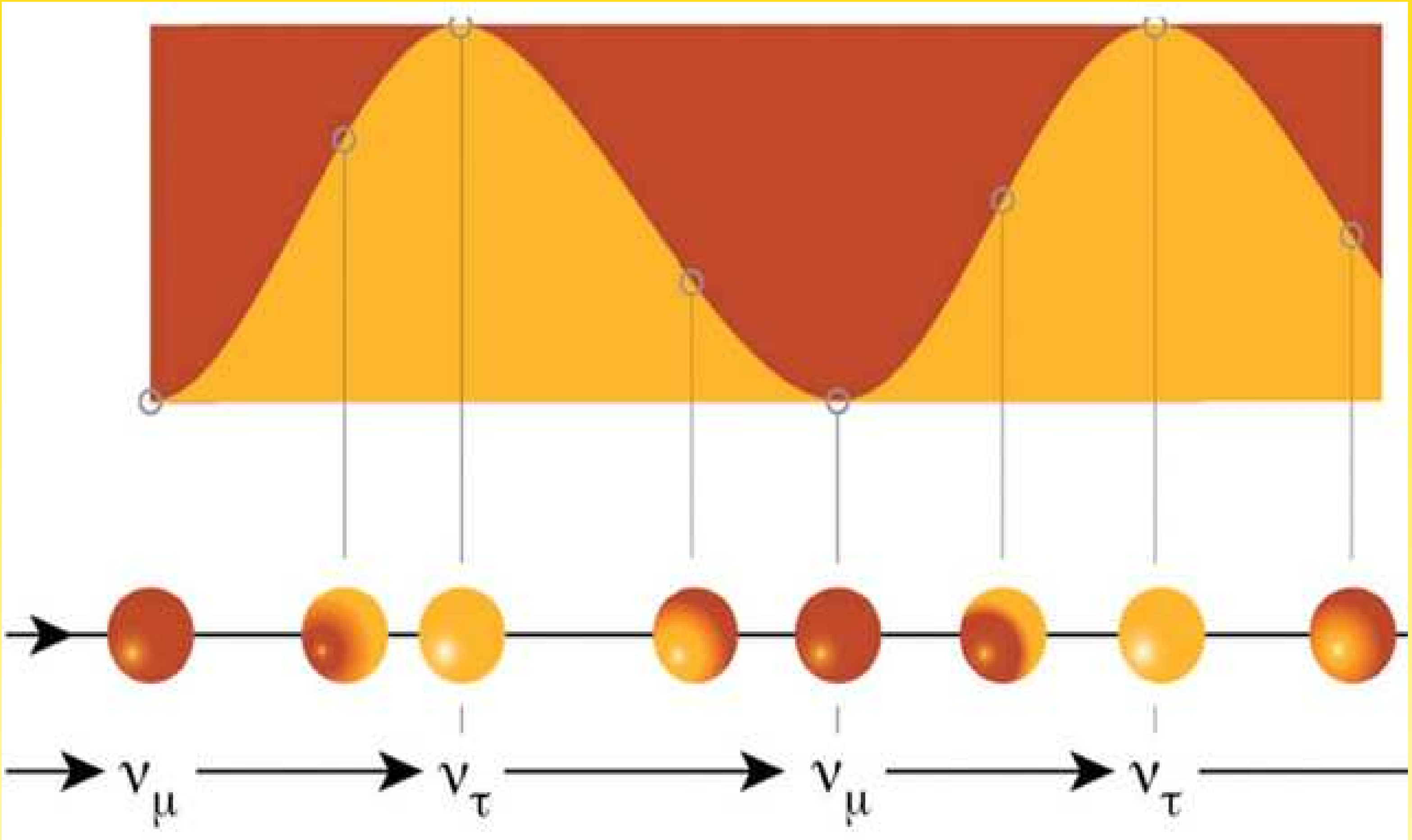
$$\mathrm{Im}\,\{h_{12} \, h_{23} \, h_{31}\} = \Delta m_{21}^2 \, \Delta m_{31}^2 \, \Delta m_{32}^2 \, J_{\mathrm{CP}}$$

Two Flavor Case

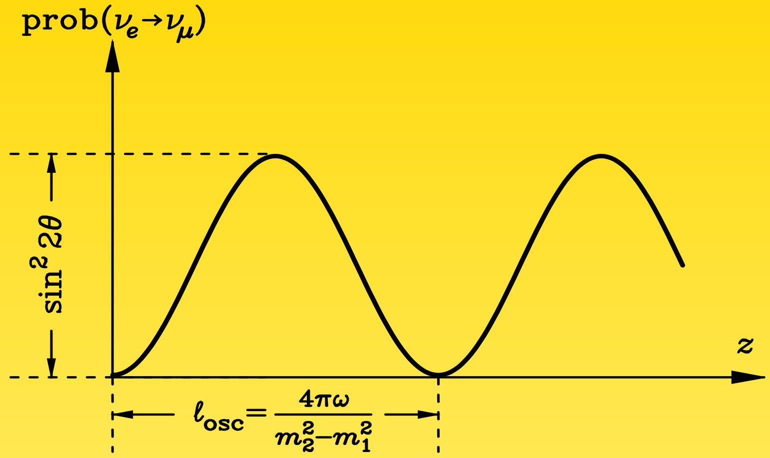
$$U = \begin{pmatrix} U_{\alpha 1} & U_{\alpha 2} \\ U_{\beta 1} & U_{\beta 2} \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \Rightarrow \mathcal{J}_{12}^{\alpha\alpha} = |U_{\alpha 1}|^2 |U_{\alpha 2}|^2 = \frac{1}{4}\sin^2 2\theta$$

and transition probability is $\boxed{P_{\alpha\beta} = \sin^2 2\theta \, \sin^2 \frac{\Delta m_{21}^2}{4\,E} L}$





ν_μ
ν_τ
ν_μ
ν_τ

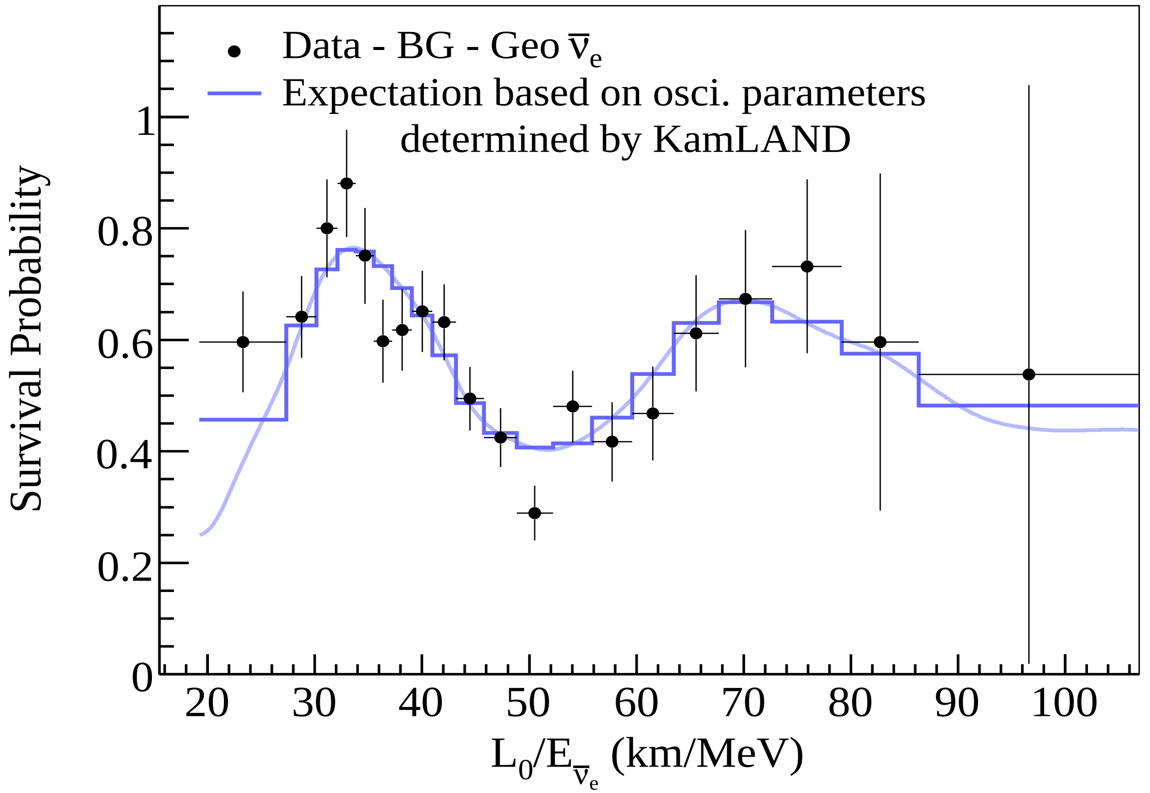


- amplitude $\sin^2 2\theta$
- maximal mixing for $\theta = \pi/4 \Rightarrow \nu_\alpha = \sqrt{\frac{1}{2}}\,(\nu_1 + \nu_2)$
- oscillation length $L_{\rm osc} = 4\pi\, E/\Delta m_{21}^2 = 2.48\,\frac{E}{\rm GeV}\,\frac{\rm eV^2}{\Delta m_{21}^2}$ km

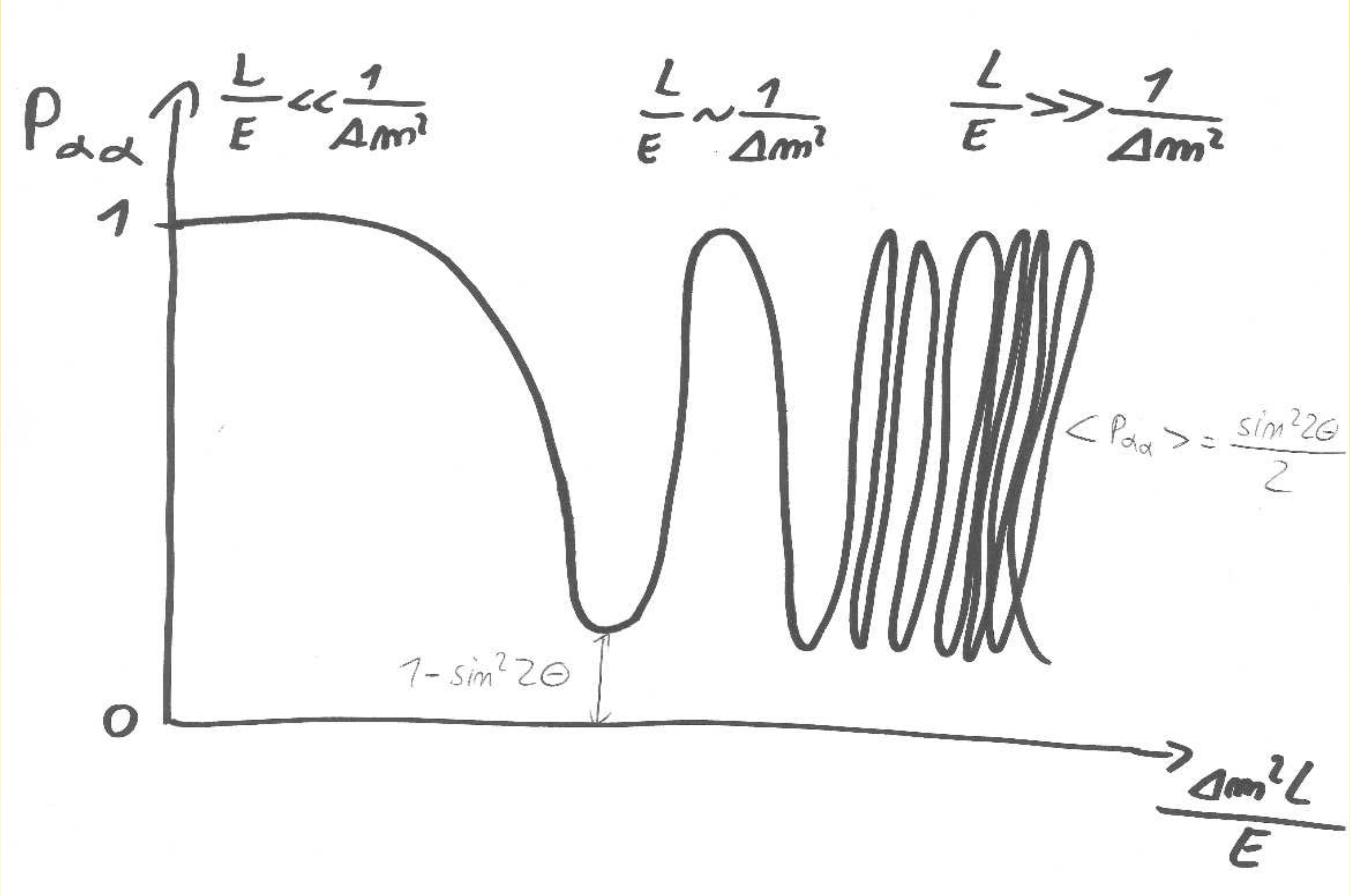
  $$\Rightarrow P_{\alpha\beta} = \sin^2 2\theta\, \sin^2 \pi \frac{L}{L_{\rm osc}}$$

  is distance between two maxima (minima)

  e.g.: $E =$ GeV and $\Delta m^2 = 10^{-3}$ eV^2: $L_{\rm osc} \simeq 10^3$ km



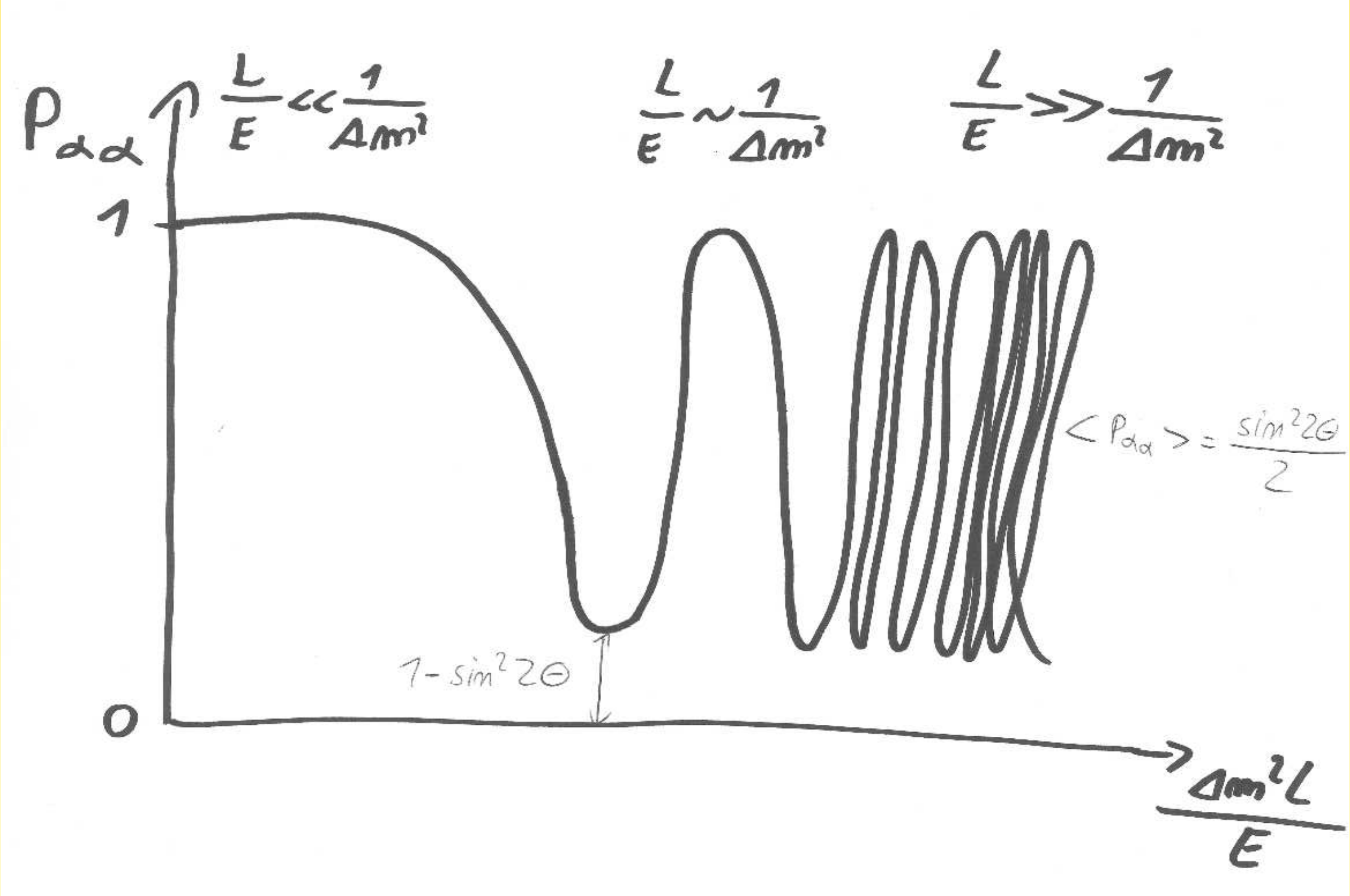
Data - BG - Geo $\bar{\nu}_e$
Expectation based on osci. parameters determined by KamLAND
Survival Probability
0
0.2
0.4
0.6
0.8
1
20
30
40
50
60
70
80
90
100
$L_0/E_{\bar{\nu}_e}$ (km/MeV)



$L \gg L_{\rm osc}$: fast oscillations $\langle \sin^2 \pi L/L_{\rm osc} \rangle = \frac{1}{2}$

and $P_{\alpha\alpha} = 1 - 2\,|U_{\alpha 1}|^2\,|U_{\alpha 2}|^2 = |U_{\alpha 1}|^4 + |U_{\alpha 2}|^4$

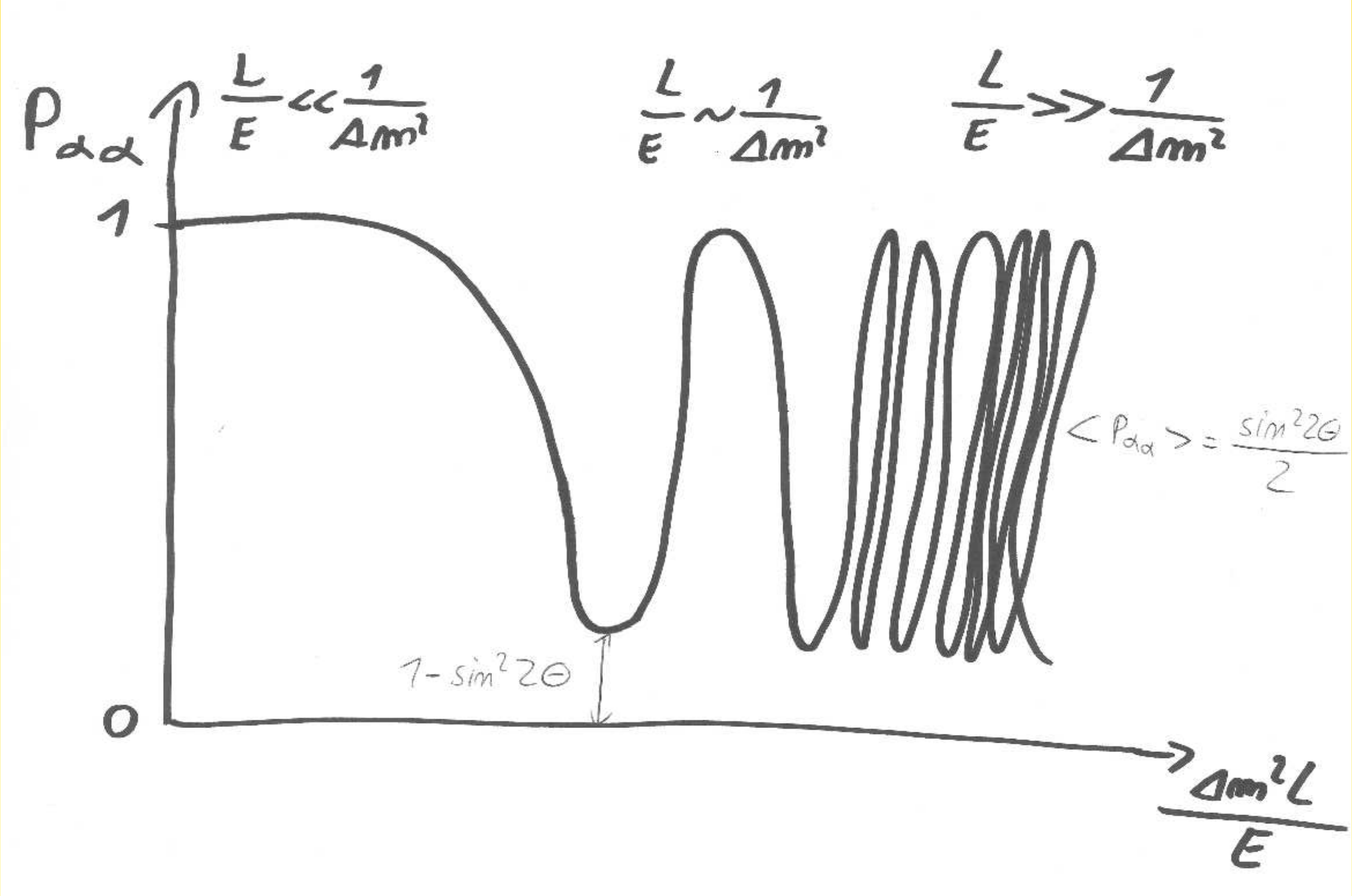
sensitivity to mixing



$L \gg L_{\rm osc}$: fast oscillations $\langle \sin^2 \pi L/L_{\rm osc} \rangle = \frac{1}{2}$

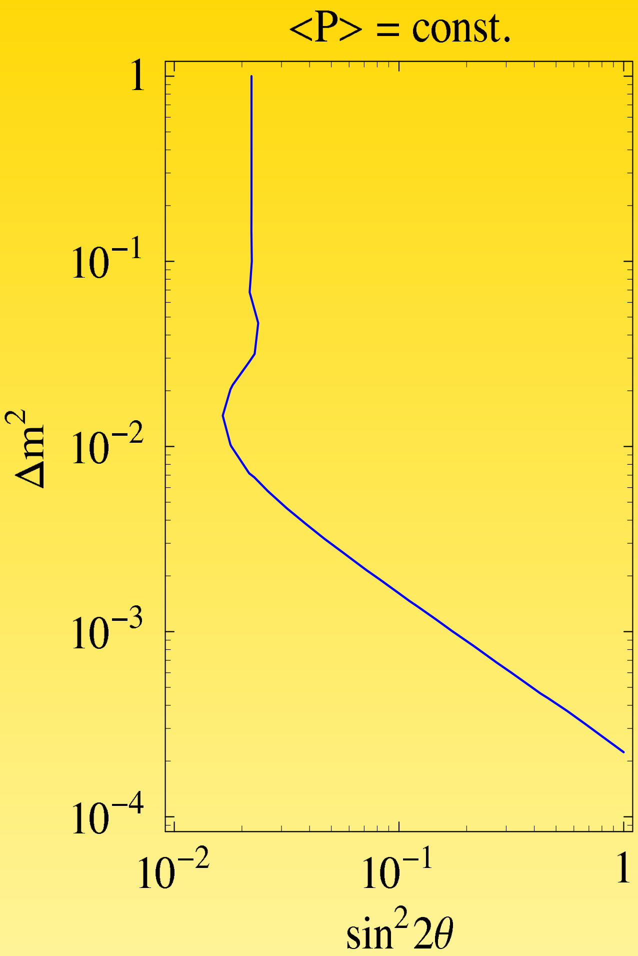
and $P_{\alpha\beta} = 2\,|U_{\alpha 1}|^2\,|U_{\alpha 2}|^2 = |U_{\alpha 1}|^2\,|U_{\beta 1}|^2 + |U_{\alpha 2}|^2\,|U_{\beta 2}|^2 = \frac{1}{2}\sin^2 2\theta$

sensitivity to mixing



$L \ll L_{\mathrm{osc}}$: hardly oscillations and $P_{\alpha\beta} = \sin^2 2\theta\,(\Delta m^2 L/(4E))^2$

sensitivity to product $\sin^2 2\theta\,\Delta m^2$



large Δm^2: sensitivity to mixing

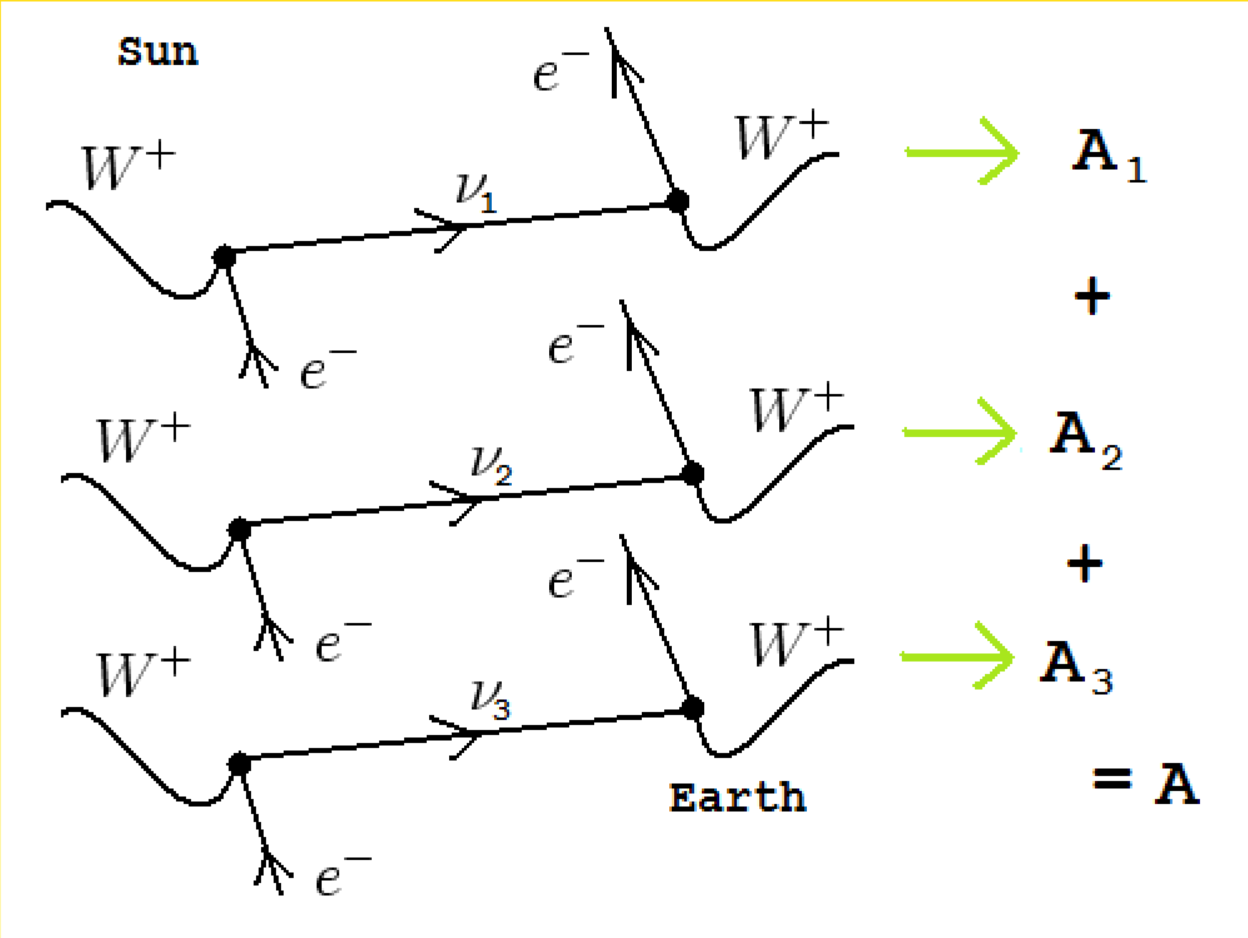
small Δm^2: sensitivity to $\sin^2 2\theta \, \Delta m^2$

maximal sensitivity when $\Delta m^2 L/E \simeq 2\pi$

Characteristics of typical oscillation experiments

Source	Flavor	E [GeV]	L [km]	$(\Delta m^2)_{\min}$ [eV^2]
Atmosphere	$\overset{(-)}{\nu}_e$, $\overset{(-)}{\nu}_\mu$	$10^{-1} \ldots 10^{2}$	$10 \ldots 10^{4}$	10^{-6}
Sun	ν_e	$10^{-3} \ldots 10^{-2}$	10^{8}	10^{-11}
Reactor SBL	$\bar{\nu}_e$	$10^{-4} \ldots 10^{-2}$	10^{-1}	10^{-3}
Reactor LBL	$\bar{\nu}_e$	$10^{-4} \ldots 10^{-2}$	10^{2}	10^{-5}
Accelerator LBL	$\overset{(-)}{\nu}_e$, $\overset{(-)}{\nu}_\mu$	$10^{-1} \ldots 1$	10^{2}	10^{-1}
Accelerator SBL	$\overset{(-)}{\nu}_e$, $\overset{(-)}{\nu}_\mu$	$10^{-1} \ldots 1$	1	1

Quantum Mechanics



Can't distinguish the individual m_i: coherent sum of amplitudes and *interference*

Quantum Mechanics

Textbook calculation is completely wrong!!

- $E_i - E_j$ is not Lorentz invariant
- massive particles with different p_i and same E violates energy and/or momentum conservation
- definite p: in space this is e^{ipx}, thus no localization

Quantum Mechanics

consider E_j and $p_j = \sqrt{E_j^2 - m_j^2}$:

$$p_j \simeq E + m_j^2 \left.\frac{\partial p_j}{\partial m_j^2}\right|_{m_j=0} \equiv E - \xi \frac{m_j^2}{2E}, \quad \text{with } \xi = -2E \left.\frac{\partial p_j}{\partial m_j^2}\right|_{m_j=0}$$

$$E_j \simeq p_j + m_j^2 \left.\frac{\partial E_j}{\partial m_j^2}\right|_{m_j=0} = p_j + \frac{m_j^2}{2p_j} = E + \frac{m_j^2}{2E}(1-\xi)$$

in pion decay $\pi \to \mu\nu$:

$$E_j = \frac{m_\pi^2}{2}\left(1 - \frac{m_\mu^2}{m_\pi^2}\right) + \frac{m_j^2}{2m_\pi^2}$$

thus,

$$\xi = \frac{1}{2}\left(1 + \frac{m_\mu^2}{m_\pi^2}\right) \simeq 0.8 \ \text{ in } E_i - E_j \simeq (1-\xi)\frac{\Delta m_{ij}^2}{2E}$$

wave packet with size $\sigma_x (\gtrsim 1/\sigma_p)$ and group velocity $v_i = \partial E_i / \partial p_i = p_i / E_i$:

$$\psi_i \propto \exp\left\{ -i(E_i\, t - p_i\, x) - \frac{(x - v_i\, t)^2}{4\sigma_x^2} \right\}$$

1) wave packet separation should be smaller than σ_x!

$$L\, \Delta v < \sigma_x \Rightarrow \frac{L}{L_{\rm osc}} < \frac{p}{\sigma_p}$$

(loss of *coherence*: interference impossible)

2) m_ν^2 should NOT be known too precisely!

if known too well: $\Delta m^2 \gg \delta m_\nu^2 = \frac{\partial m_\nu^2}{\partial p_\nu}\, \delta p_\nu \Rightarrow \delta x_\nu \gg \frac{2\, p_\nu}{\Delta m^2} = \frac{L_{\rm osc}}{2\pi}$

(I know which state ν_i is exchanged, *localization*)

In both cases: $P_{\alpha\alpha} = |U_{\alpha 1}|^4 + |U_{\alpha 2}|^4$ (same as for $L \gg L_{\rm osc}$)

Quantum Mechanics

total amplitude for $\alpha \to \beta$ should be given by

$$A \propto \sum_j \int \frac{d^3p}{2E_j} \mathcal{A}^*_{\beta j}\, \mathcal{A}_{\alpha j} \exp\{-i(E_j t - px)\}$$

with production and detection amplitudes

$$\mathcal{A}_{\alpha j}\, \mathcal{A}^*_{\beta j} \propto \exp\left\{-\frac{(p-\tilde{p}_j)^2}{4\sigma_p^2}\right\}$$

we expand around $\tilde{p}_j$:

$$E_j(p) \simeq E_j(\tilde{p}_j) + \left.\frac{\partial E_j(p)}{\partial p}\right|_{p=\tilde{p}_j} (p-\tilde{p}_j) = \tilde{E}_j + v_j\,(p-\tilde{p}_j)$$

and perform the integral over p:

$$A \propto \sum_j \exp\left\{-i(\tilde{E}_j t - \tilde{p}_j x) - \frac{(x-v_j t)^2}{4\sigma_x^2}\right\}$$

the probability is the integral of $|A|^2$ over t:

$$P = \int dt\, |A|^2 \propto \exp\left\{-i\left[(\tilde{E}_j - \tilde{E}_k)\frac{v_j + v_k}{v_j^2 + v_k^2} - (\tilde{p}_j - \tilde{p}_k)\right] x\right\}$$
$$\times \exp\left\{-\frac{(v_j - v_k)^2 x^2}{4\sigma_x^2 (v_j^2 + v_k^2)} - \frac{(\tilde{E}_j - \tilde{E}_k)^2}{4\sigma_p^2 (v_j^2 + v_k^2)}\right\}$$

now express average momenta, energy and velocity as

$$\tilde{p}_j \simeq E - \xi \frac{m_j^2}{2E}$$

$$\tilde{E}_j \simeq E + (1 - \xi)\frac{m_j^2}{2E}\,, \qquad v_j = \frac{\tilde{p}_j}{\tilde{E}_j} \simeq 1 - \frac{m_j^2}{2E^2}$$

this we insert in first exponential of P:

$$\left[(\tilde{E}_j - \tilde{E}_k)\frac{v_j + v_k}{v_j^2 + v_k^2} - (\tilde{p}_j - \tilde{p}_k)\right] = \frac{\Delta m_{jk}^2 L}{2E}$$

the second exponential (damping term) can also be rewritten and the final probability is

$$P \propto \exp\left\{ -i\frac{\Delta m_{ij}^2}{2E}L - \left(\frac{L}{L_{jk}^{\rm coh}}\right)^2 - 2\pi^2(1-\xi)^2 \left(\frac{\sigma_x}{L_{jk}^{\rm osc}}\right)^2 \right\}$$

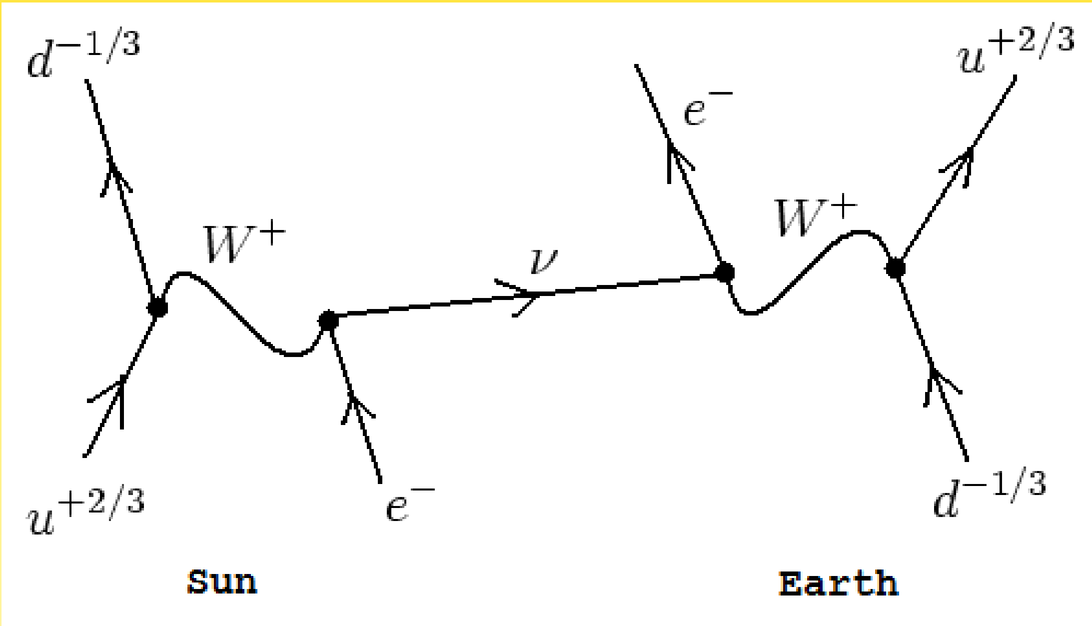
with

$$L_{jk}^{\rm coh} = \frac{4\sqrt{2}E^2}{|\Delta m_{jk}^2|}\sigma_x \text{ and } L_{jk}^{\rm osc} = \frac{4\pi E}{|\Delta m_{jk}^2|}$$

expressing the two conditions (coherence and localization) for oscillation discussed before

Quantum Mechanics

derivation of formula also works in QFT, when everything is a big Feynman diagram:



(Lorentz invariance, energy and momentum conservation at every vertex, etc.)

b) Neutrino Oscillations in Matter

- ν can witness coherent ($\sigma \propto G_F$) elastic scattering with e^-, p, n in matter
- creates mean potential $V = \mathcal{O}(G_F\, n_e) = \mathcal{O}(\Delta m^2/E)$
- Formalism easy when Hamiltonian approach is used:

$$(\gamma_\mu p^\mu - M)\,\Psi = 0 \Rightarrow (p^2 - M^2)\,\Psi = 0$$

$$\text{with } \Psi = \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix},\; M^2 = \begin{pmatrix} m_1^2 & 0 \\ 0 & m_2^2 \end{pmatrix}$$

$$\text{use } p^2 = E^2 + \partial_x^2 = (E + i\partial_x)\,(E - i\partial_x)$$

$$= (E + i\partial_x)\,(E + p) \simeq 2\,E\,(E + i\partial_x)$$

gives Hamiltonian (global phase E has no effect)

$$i\partial_x\,\Psi = \left[-E + \frac{M^2}{2\,E}\right]\,\Psi \Rightarrow \boxed{i\partial_x\,\Psi = \mathcal{H}\,\Psi = \frac{M^2}{2\,E}\,\Psi}$$

in flavor basis $\Psi_{\rm fl} = U\,\Psi$

$$\mathcal{H}_{\rm fl} = U\,\mathcal{H}\,U^\dagger = \frac{\Delta m^2}{4\,E}\begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix}$$

diagonalizing this Hamiltonian gives mixing angle θ and eigenvalues $\pm\frac{\Delta m^2}{4\,E}$

Potential due to matter effects from CC term:

$$\mathcal{H}_{\rm CC} = \frac{G_F}{\sqrt{2}}\,[\overline{e}\,\gamma_\mu(1-\gamma_5)\,e]\,[\overline{\nu_e}\,\gamma_\mu(1-\gamma_5)\,\nu_e]$$

integrate over e such that $\overline{\nu_e}\,V\,\nu_e$ survives

$\langle\overline{e}\,\gamma_\mu\gamma_5\,e\rangle = 0$ unpolarized matter

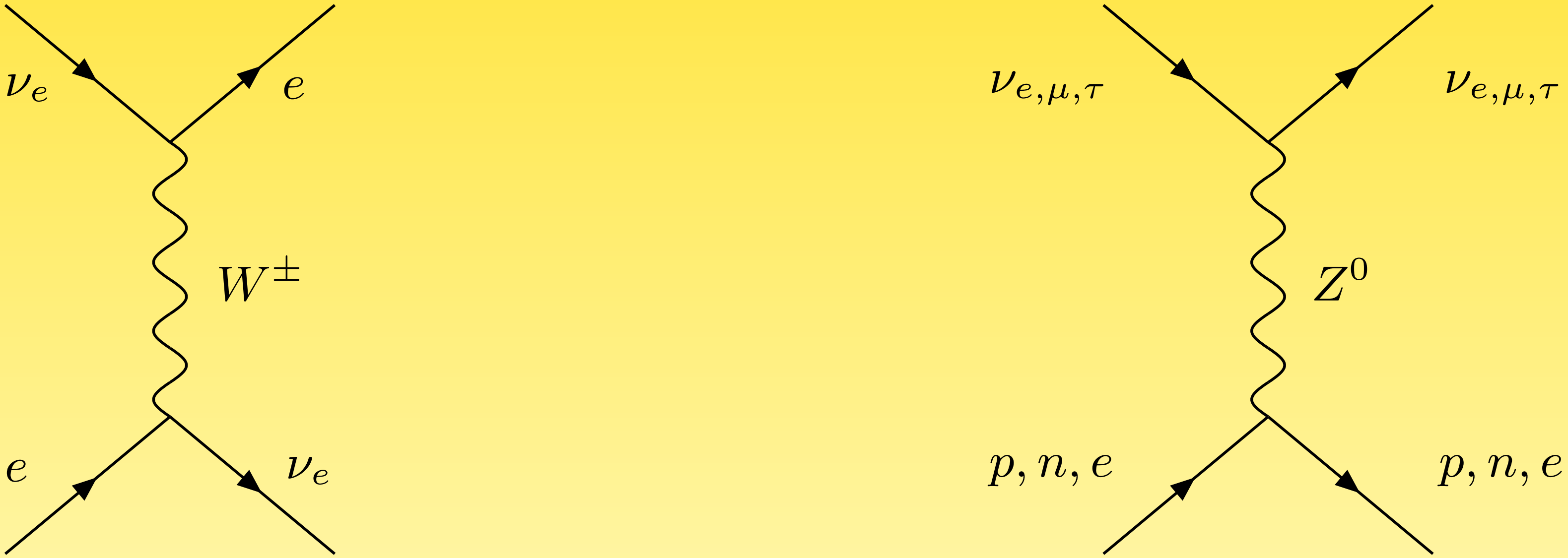
$\langle\overline{e}\,\gamma_i\,e\rangle = 0$ zero momentum of matter

$\langle\overline{e}\,\gamma_0\,e\rangle = n_e$

it follows

$$V_{ee} = \sqrt{2}\, G_F\, n_e$$

if matter electrically neutral: $V_{\rm NC}(e) = V_{\rm NC}(p)$



Electron neutrinos have CC + NC, muon and tau neutrinos only NC

$$\mathcal{H}_{\text{fl}} = \frac{\Delta m^2}{4\,E} \begin{pmatrix} -\cos 2\theta + 2\,\dfrac{A}{\Delta m^2} & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix}$$

where $A = 2\sqrt{2}\,G_F\,n_e\,E \simeq \begin{cases} 10^{-5}\ \text{eV}^2\ \frac{E}{\text{MeV}} & \text{Sun} \\ 10^{-7}\ \text{eV}^2\ \frac{E}{\text{MeV}} & \text{Earth} \end{cases}$

diagonalize to find mass^2 and θ in matter

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_m & \sin\theta_m \\ -\sin\theta_m & \cos\theta_m \end{pmatrix} \begin{pmatrix} \nu_1^m \\ \nu_2^m \end{pmatrix}$$

and

$$P_{e\mu} = \sin^2 2\theta_m\ \sin^2 \frac{(\Delta m^2)^m}{4\,E}\,L$$

$$L_{\rm osc}^m = \frac{4\pi\, E}{(\Delta m^2)^m} = \frac{4\pi\, E}{\sqrt{(\cos 2\theta - A/\Delta m^2)^2 + \sin^2 2\theta}}$$

$$\sin^2 2\theta_m = \frac{\sin^2 2\theta}{(\cos 2\theta - A/\Delta m^2)^2 + \sin^2 2\theta}$$

- Resonance at $\frac{\Delta m^2}{2E} \cos 2\theta = \sqrt{2}\, G_F\, n_e$ or $L_{\rm osc} = \cos 2\theta\, L_{\rm magic}$

with $L_{\rm magic} = \frac{\sqrt{2}\pi}{G_F\, n_e}$ "magic baseline" $\simeq$ 7500 km in Earth

- at resonance: $L_{\rm osc}^m = L_{\rm osc}/\sin 2\theta$
- for matter dominance: $L_{\rm osc}^m = L_{\rm magic}$
- note: depends on octant of θ and sign of Δm^2

MSW effect

$$\sin^2 2\theta_m = \frac{\sin^2 2\theta}{(\cos 2\theta - A/\Delta m^2)^2 + \sin^2 2\theta} \ , \quad A = 2\sqrt{2}\, G_F\, n_e\, E$$

propagation through medium (Sun) with varying density $n_e(x)$

but: adiabatic variation

1) $A/\Delta m^2 \gg 1$: $\Rightarrow \nu_e \simeq \nu_2^m$ and $\nu_\mu \simeq -\nu_1^m$

2) hits resonance $\nu_2^m = \sqrt{\frac{1}{2}}\,(\nu_e + \nu_\mu)$

3) exits sun, $\theta_m = \theta$:

$$\nu_1^m = \nu_e \, \cos\theta - \nu_\mu \, \sin\theta$$
$$\nu_2^m = \nu_\mu \, \cos\theta + \nu_e \, \sin\theta$$

gives mean probability: $P_{e\mu} = \cos^2\theta \Rightarrow$ if θ is small, complete conversion!

`Mikheev, Smirnov (1978); Wolfenstein (1985)`

- Condition for adiabacity:

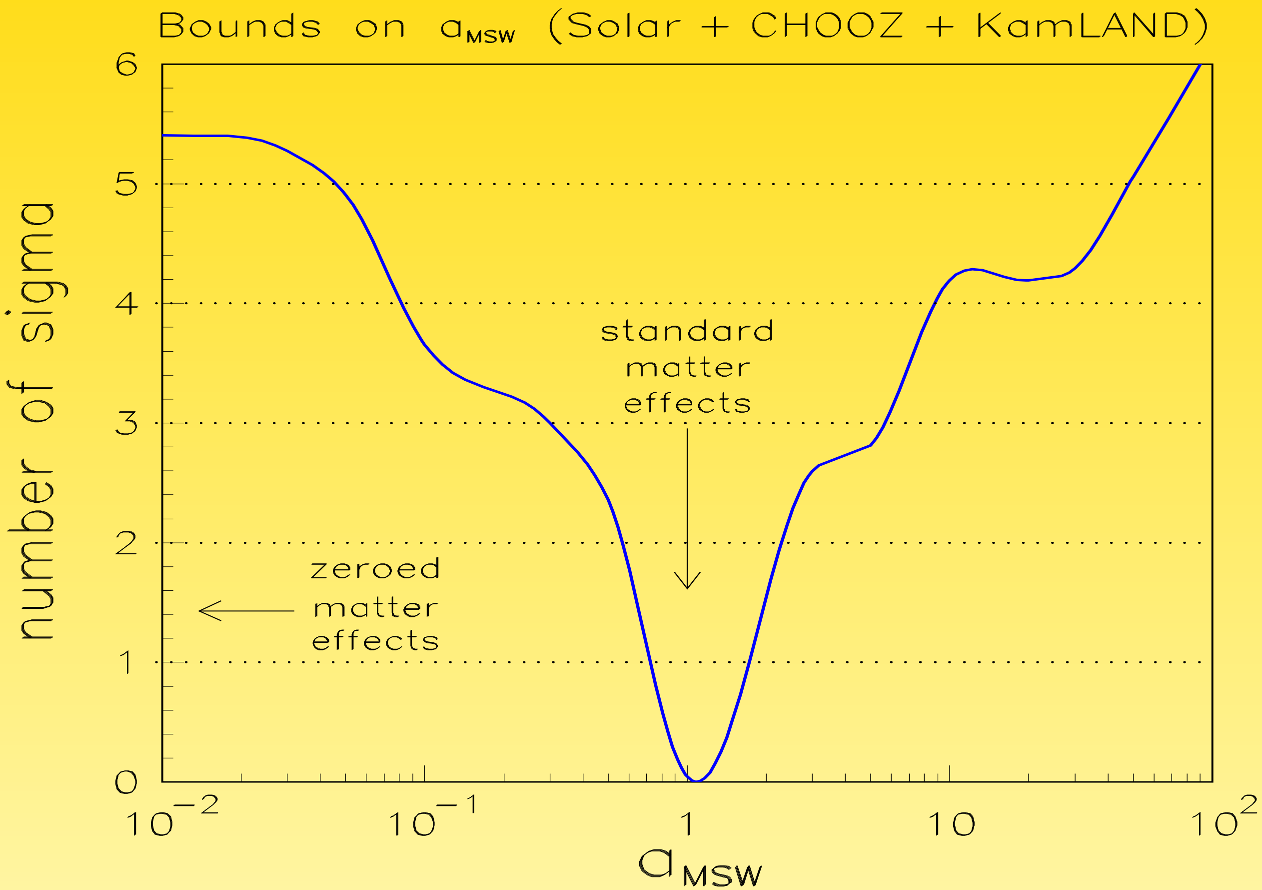
$$\gamma \equiv \frac{\Delta m^2}{2E} \frac{\sin 2\theta}{\cos 2\theta} \left(\frac{1}{n_e} \frac{dn_e}{dr} \right)^{-1} \gg 1$$

at resonance: n_e basically constant over many $L_{\rm osc}^m$

- Condition for resonance

$$A > \Delta m^2 \cos 2\theta$$

matter effects are indeed occurring in Sun, though with large mixing θ



fit to solar neutrino data with $V = a_{\rm MSW}\,\sqrt{2}\,G_F\,n_e$

Fogli, Lisi, Marrone, Palazzo

Contents

II Neutrino Oscillations

II3) Results and their interpretation – what have we learned?

- Main results as by-products:
  - check solar fusion in Sun $\longrightarrow$ solar neutrino problem
  - look for nucleon decay $\longrightarrow$ atmospheric neutrino oscillations
- almost all current data described by 2-flavor formalism
- future goal: confirm genuine 3-flavor effects:
  - third mixing angle
  - mass ordering
  - CP violation
- have entered precision era

Solar Neutrinos

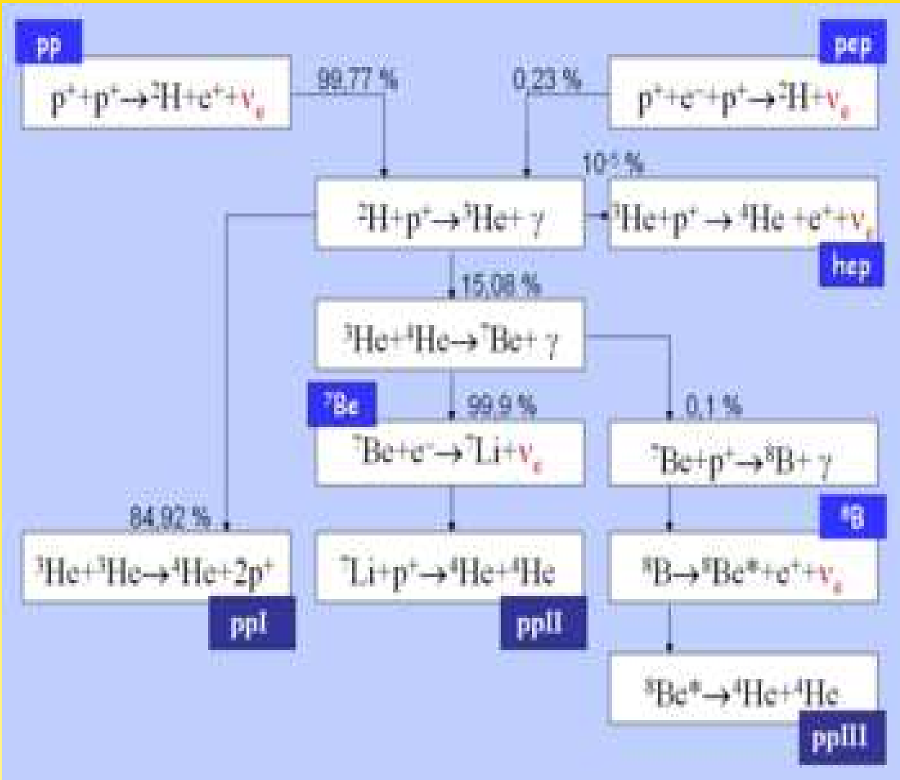
98% of energy production in fusion of net reaction

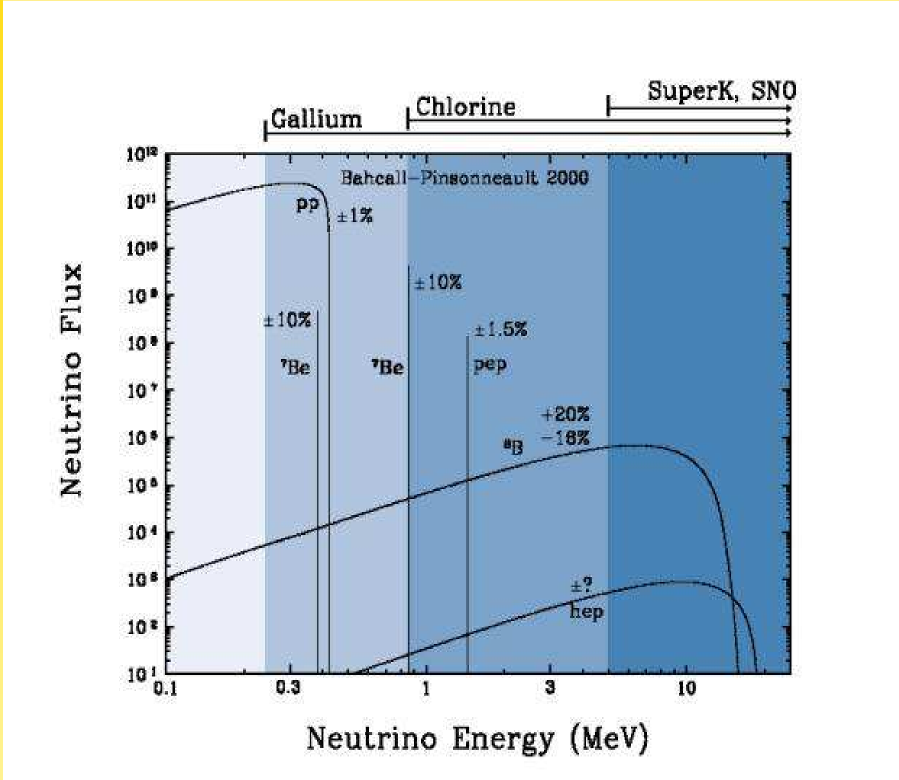
$$4\,p + 2\,e^- \rightarrow {}^4\mathrm{He}^{++} + 2\,\nu_e + 26.73\ \mathrm{MeV}$$

26 MeV of the energy go in photons, i.e., 13 MeV per ν_e;

get neutrino flux from solar constant

$$S = 8.5 \times 10^{11}\ \mathrm{MeV\,cm^{-2}\,s^{-1}} \Rightarrow \Phi_\nu = \frac{S}{13\ \mathrm{MeV}} = 6.5 \times 10^{10}\ \mathrm{cm^{-2}\,s^{-1}}$$





Solar Standard Model (SSM) predicts 5 sources of neutrinos from pp-chain

Bahcall *et al.*

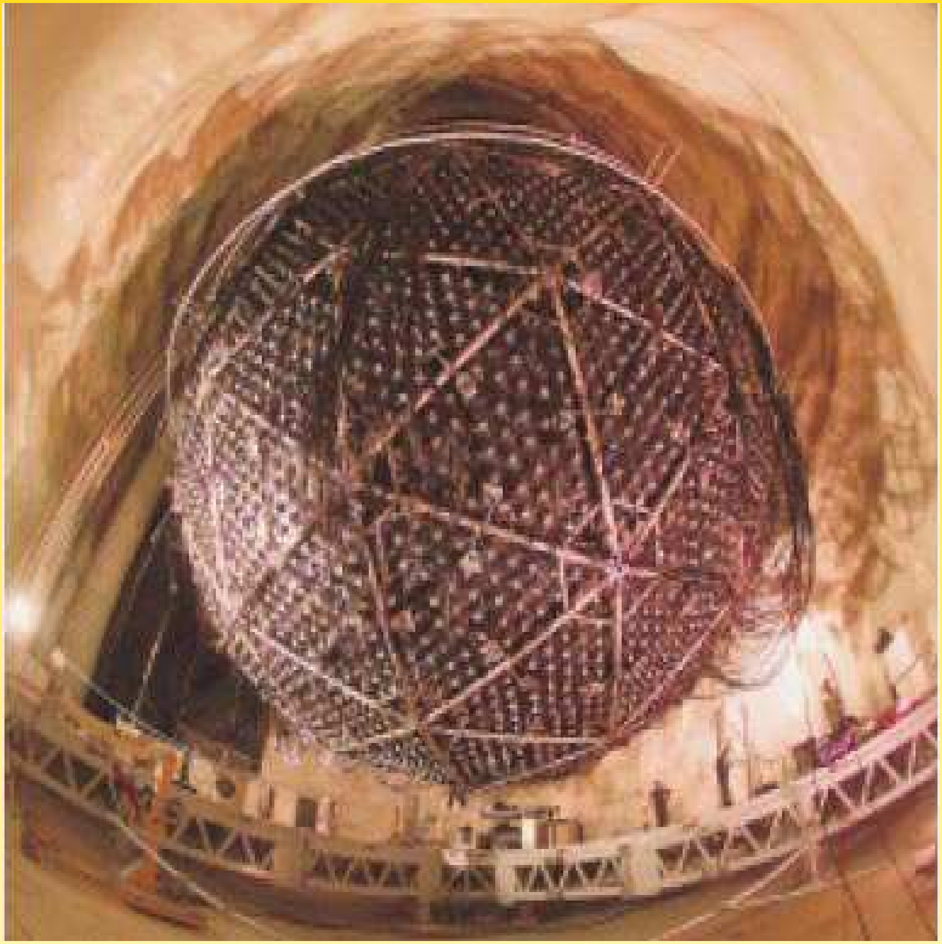
Different experiments sensitive to different energy, hence different neutrinos

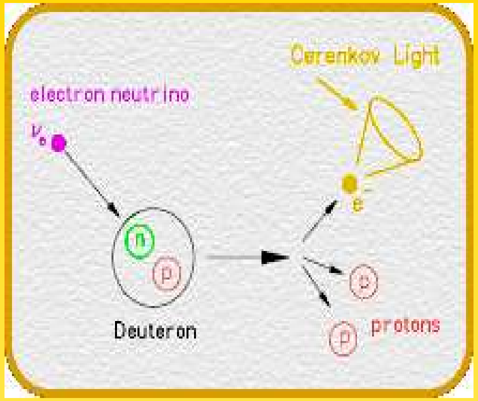
- Homestake: $\nu_e + {}^{37}\mathrm{Cl} \rightarrow {}^{37}\mathrm{Ar} + e^-$
- Gallex, GNO, SAGE: $\nu_e + {}^{71}\mathrm{Ga} \rightarrow {}^{71}\mathrm{Ge} + e^-$
- (Super)Kamiokande: $\nu_e + e^- \rightarrow \nu_e + e^-$

All find less neutrinos than predicted by SSM, deficit is energy dependent:

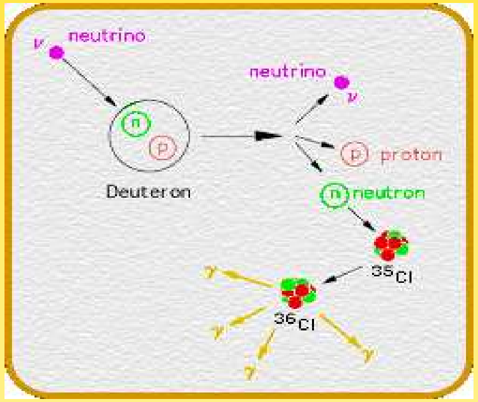
"solar neutrino problem"

Breakthrough came with SNO experiment, using heavy water

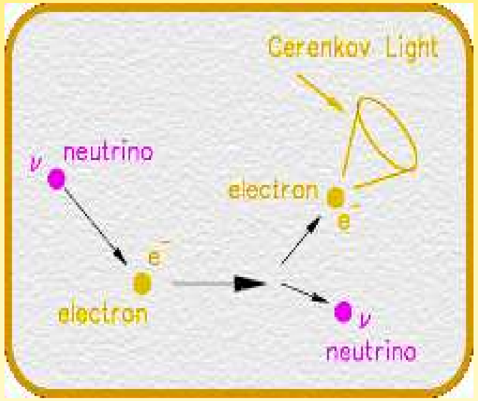




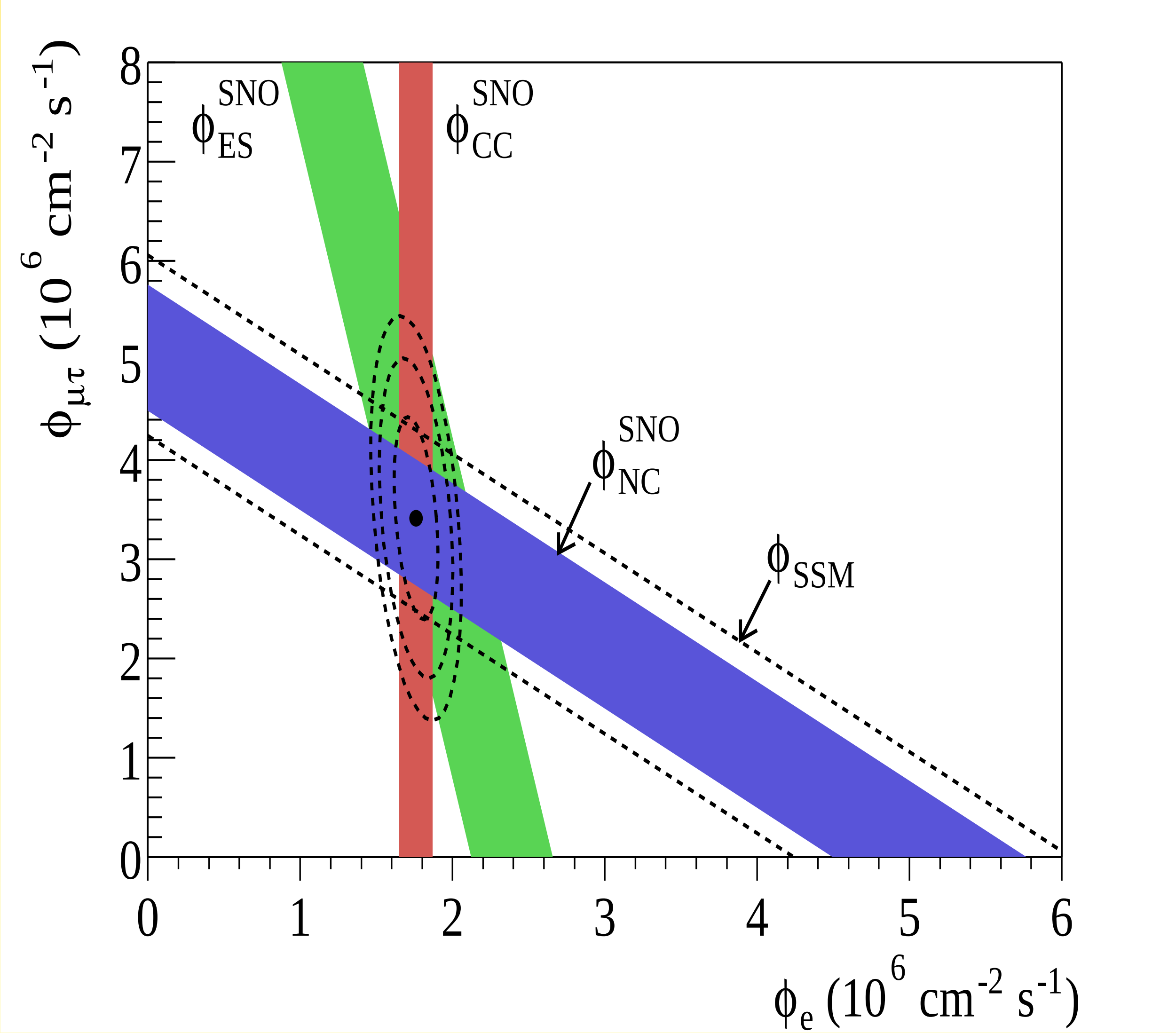
charged current: $\Phi(\nu_e)$



neutral current: $\Phi(\nu_e) + \Phi(\nu_{\mu\tau})$



elastic scattering: $\Phi(\nu_e) + 0.15\,\Phi(\nu_{\mu\tau})$



ϕ_{ES}^{SNO}
ϕ_{CC}^{SNO}
ϕ_{NC}^{SNO}
ϕ_{SSM}
$\phi_{\mu\tau}$ (10^6 cm^{-2} s^{-1})
ϕ_e (10^6 cm^{-2} s^{-1})
0 1 2 3 4 5 6
0 1 2 3 4 5 6 7 8

Results of fits give

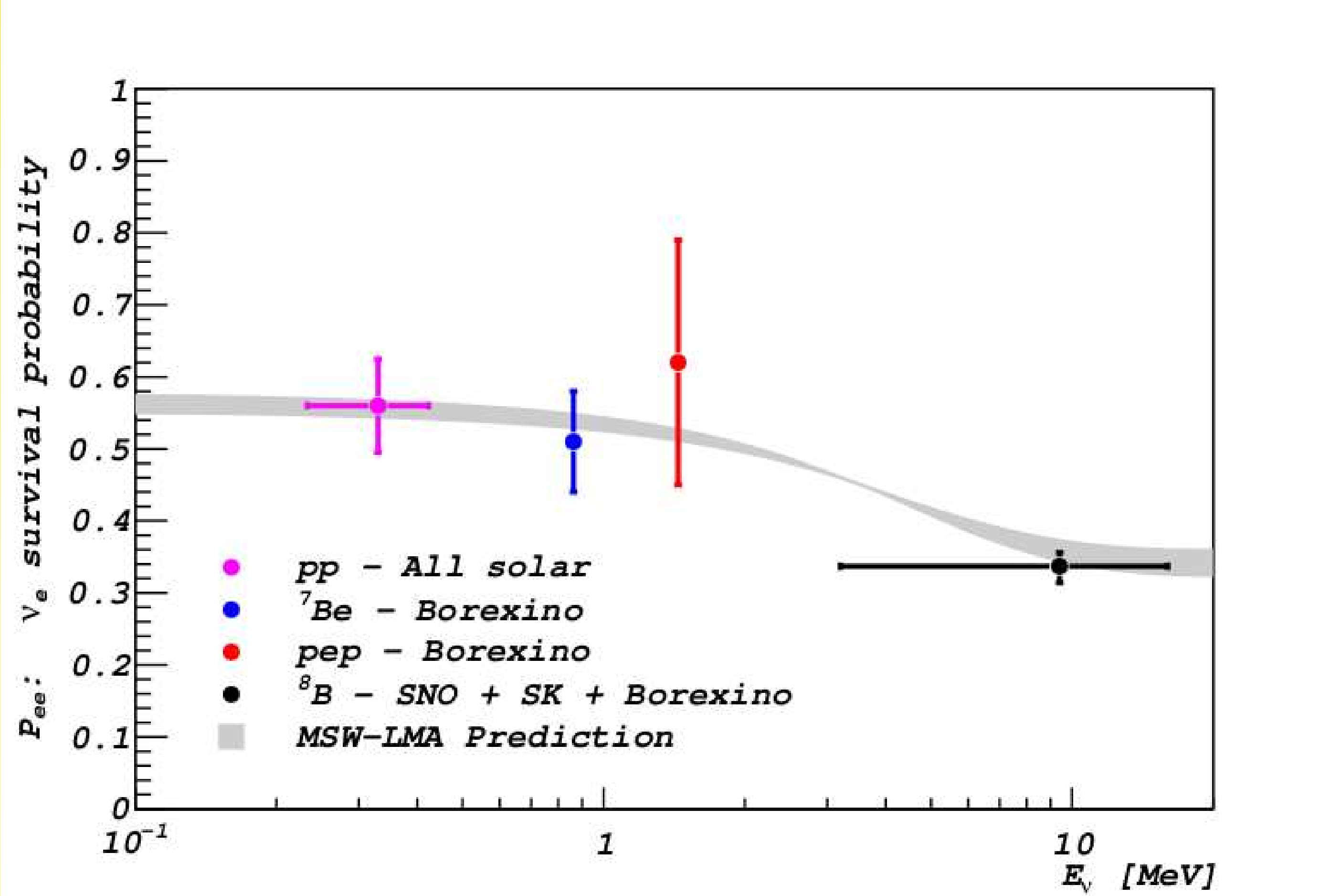
$$\sin^2\theta_{12} \simeq 0.33$$

$$\Delta m^2_{21} \equiv \Delta m^2_\odot \simeq 8\times 10^{-5}\ \mathrm{eV}^2$$

only works with matter effects and resonance in Sun

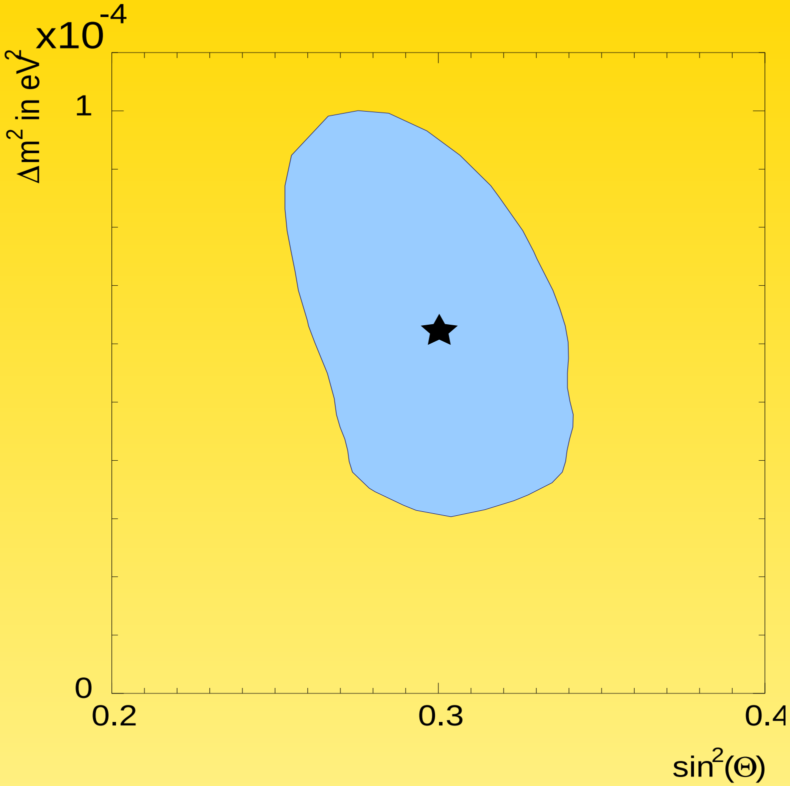
$$\Rightarrow \Delta m^2_\odot\ \cos 2\theta_{12} = (m_2^2 - m_1^2)\,(\cos^2\theta_{12} - \sin^2\theta_{12}) > 0$$

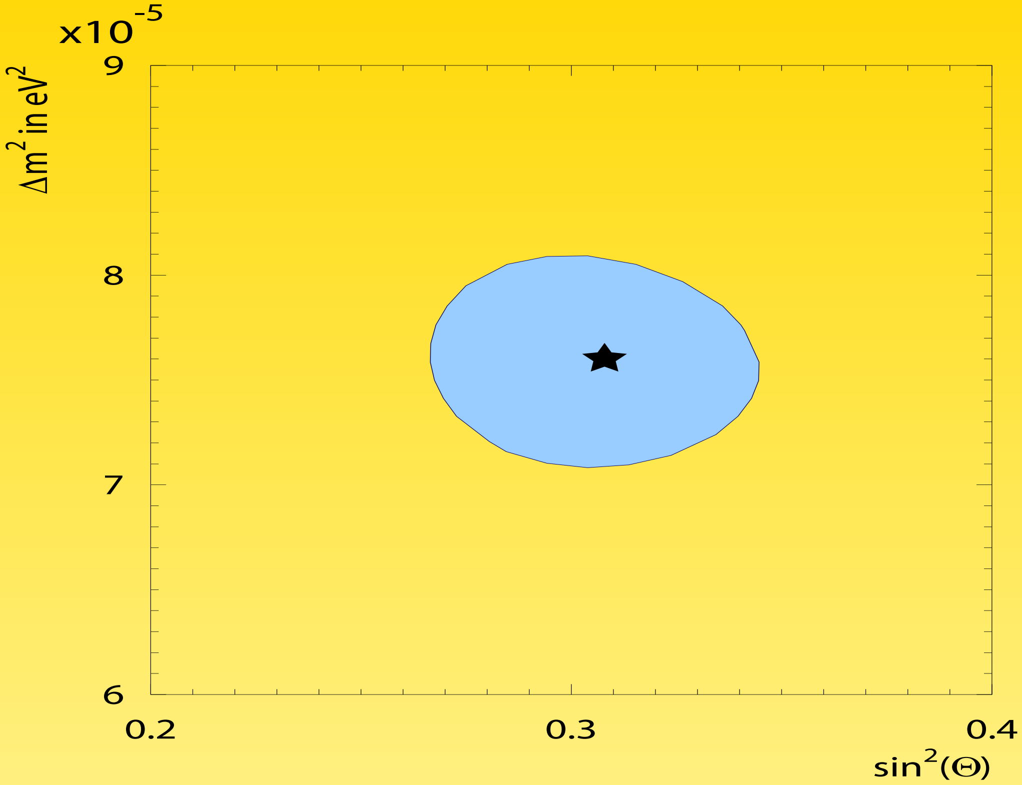
choosing $\cos 2\theta_{12} > 0$ fixes $\Delta m^2_\odot > 0$



low E: $P_{ee} = 1 - \frac{1}{2}\sin^2 2\theta_{12} \simeq \frac{5}{9}$

large E: $P_{ee} = \sin^2\theta_{12} = \frac{1}{3}$



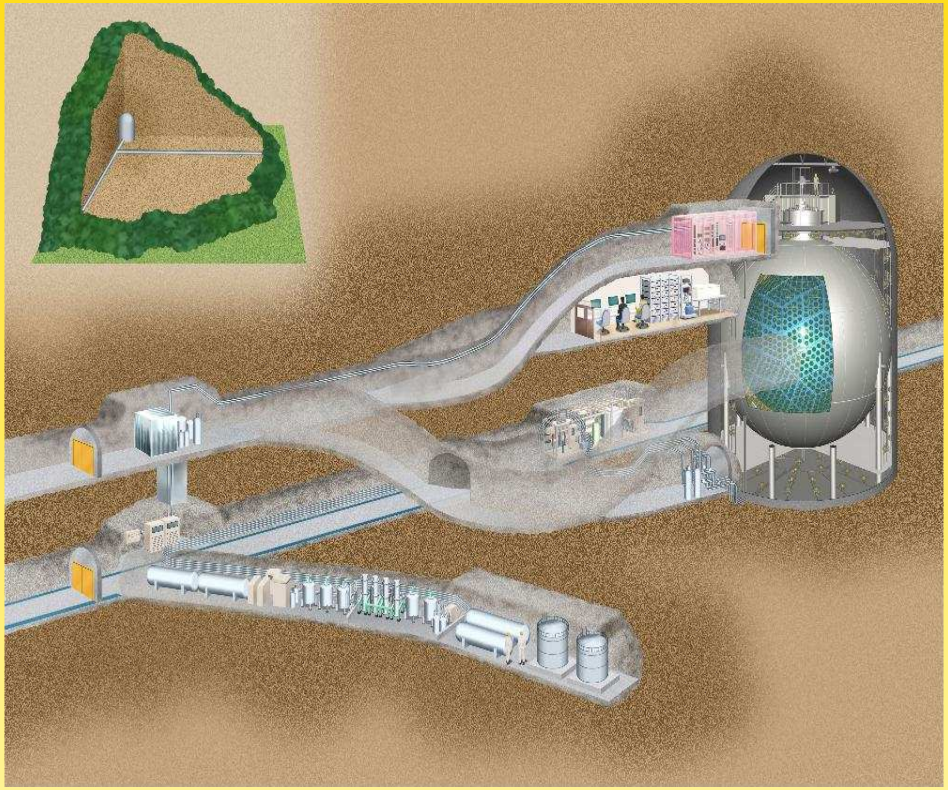


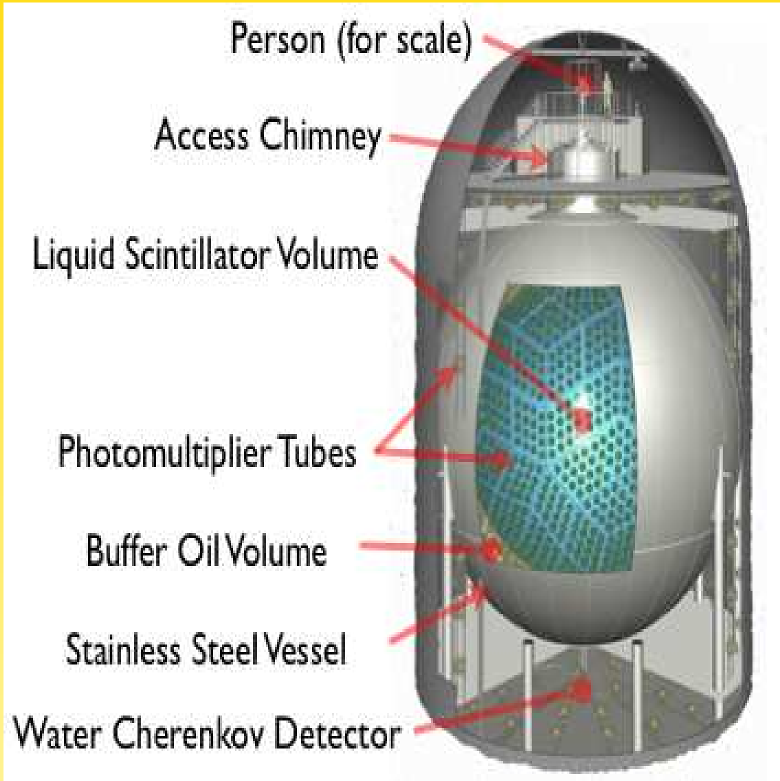
KamLAND: reactor neutrinos

$n \to p + e^- + \overline{\nu_e}$ with $E \simeq \text{few MeV}$

If $L \simeq 100$ km:

$$\frac{\Delta m^2_\odot}{E} L \sim 1 \Rightarrow \text{ solar } \nu \text{ parameters!!}$$

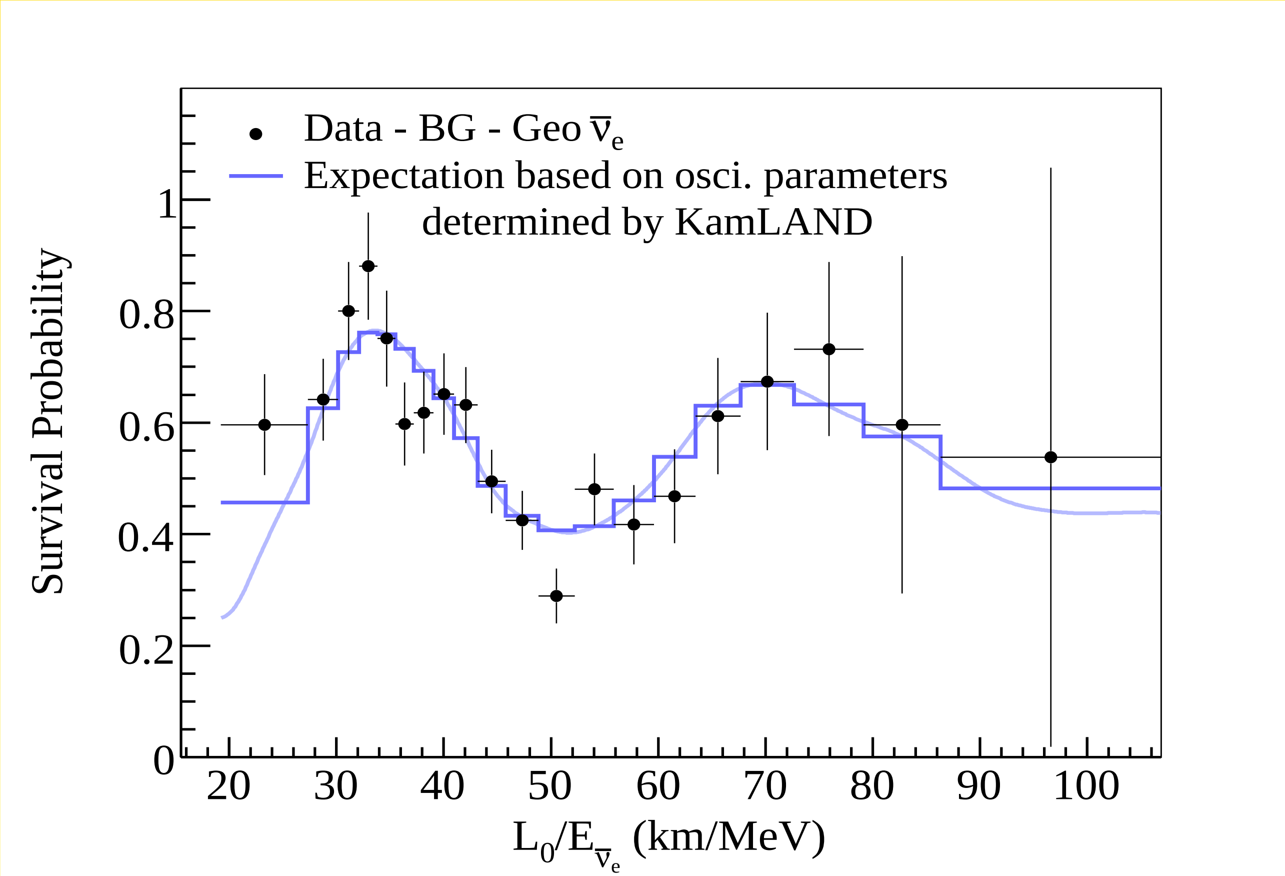




$\overline{\nu_e} + p \to n + e^+$ with $E_\nu \simeq E_{\text{prompt}} + E_n^{\text{recoil}} + 0.8$ MeV

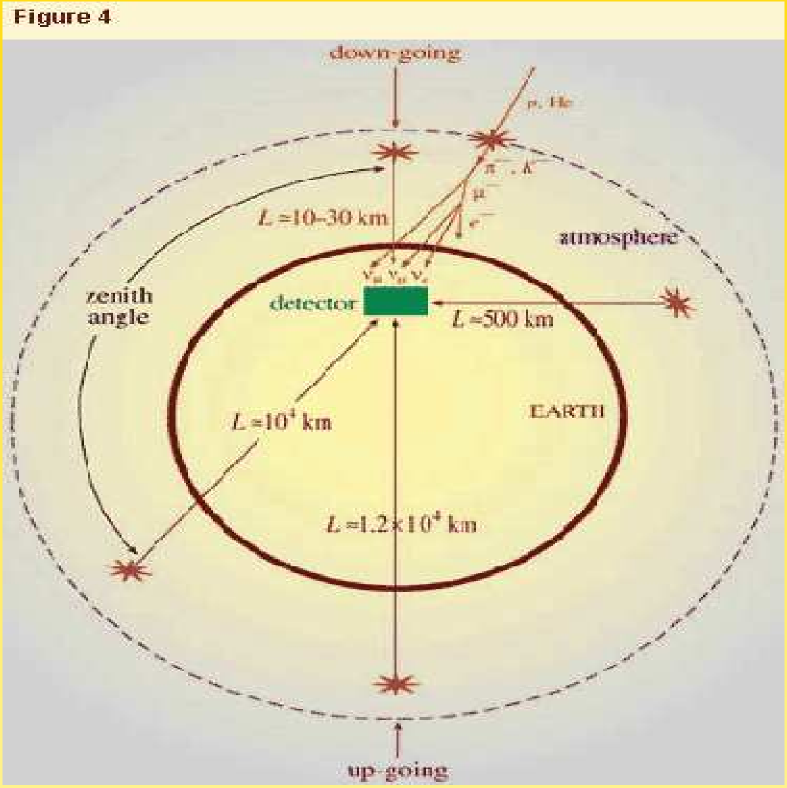
200 μs later: $n + p \to d + \gamma$

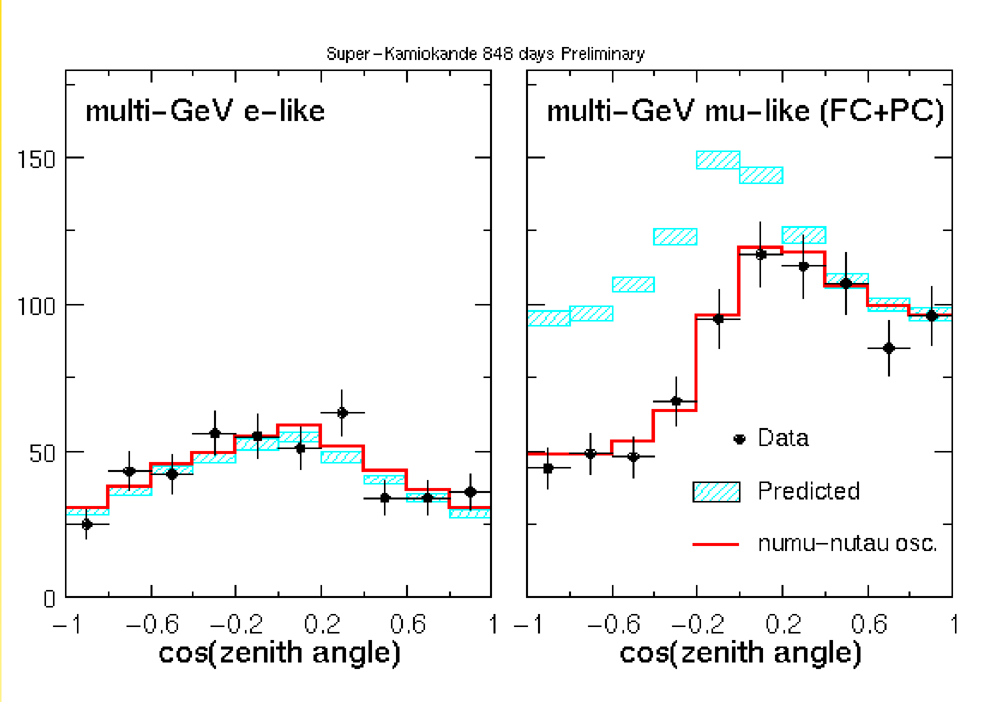
Neutrinos do oscillate



KamLAND

Atmospheric Neutrinos



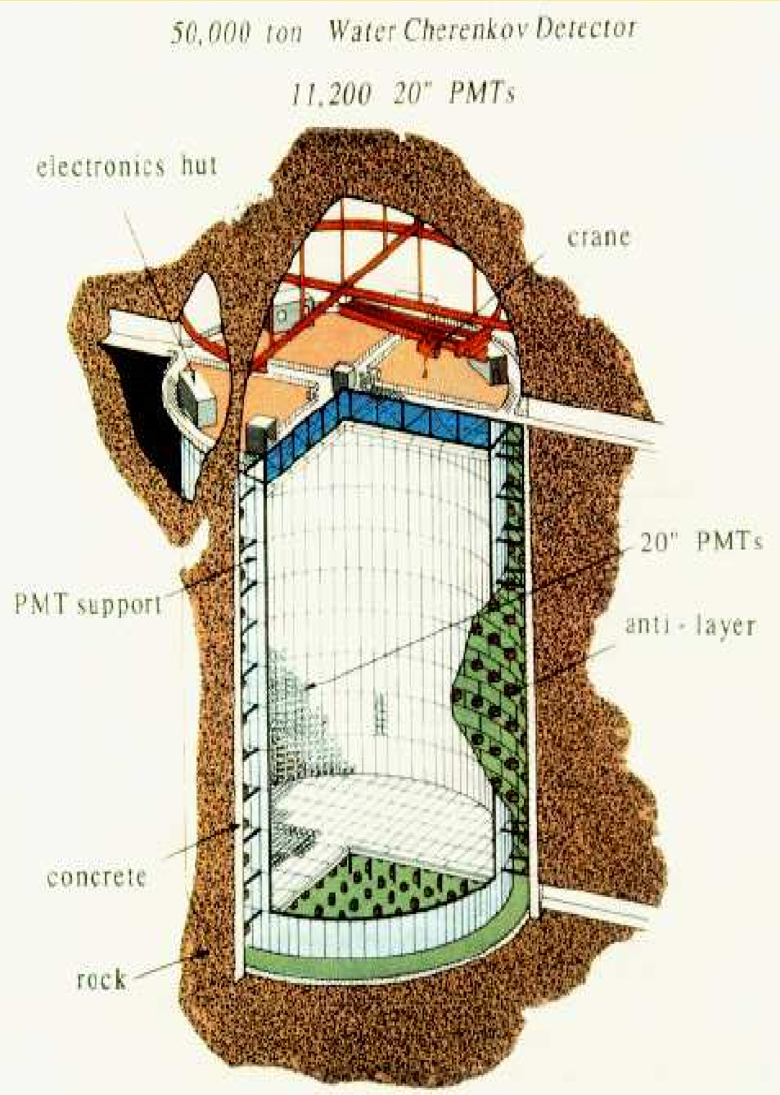


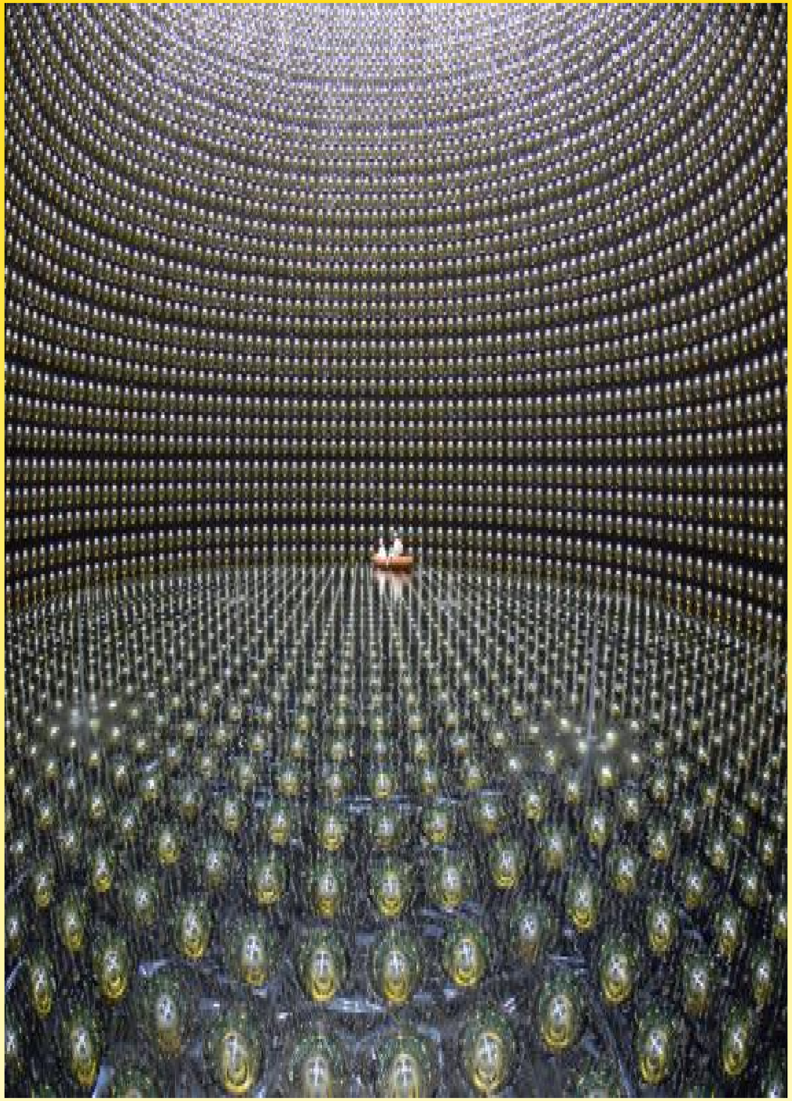
zenith angle $\cos\theta = 1$ $L \simeq 500$ km

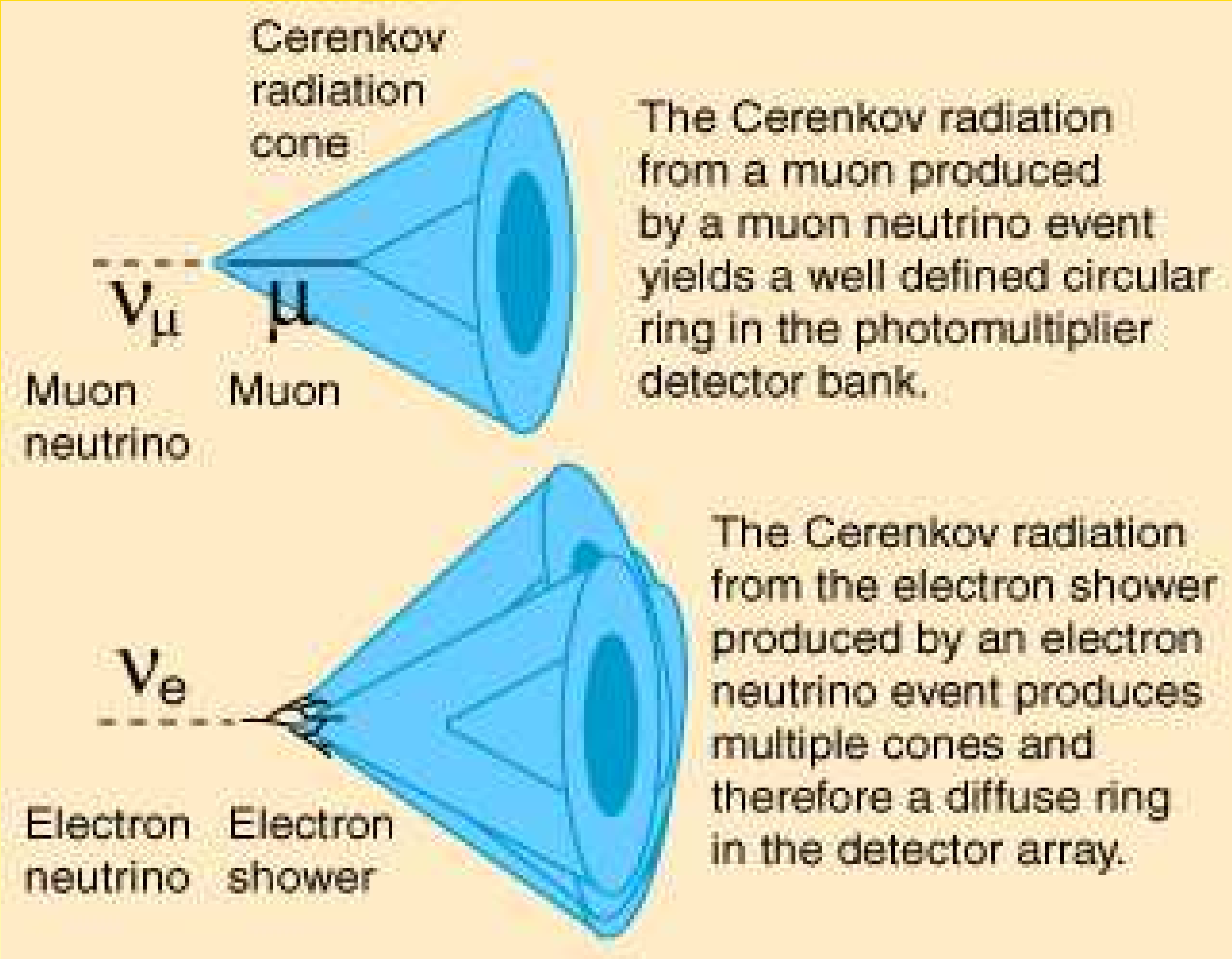
zenith angle $\cos\theta = 0$ $L \simeq 10$ km down-going

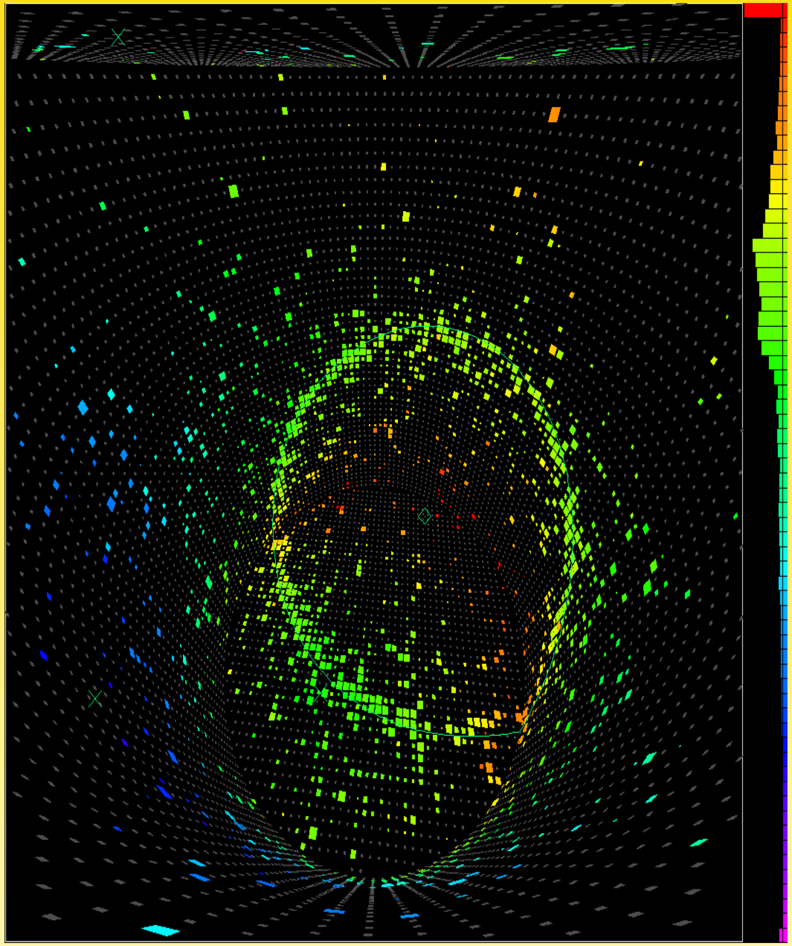
zenith angle $\cos\theta = -1$ $L \simeq 10^4$ km up-going

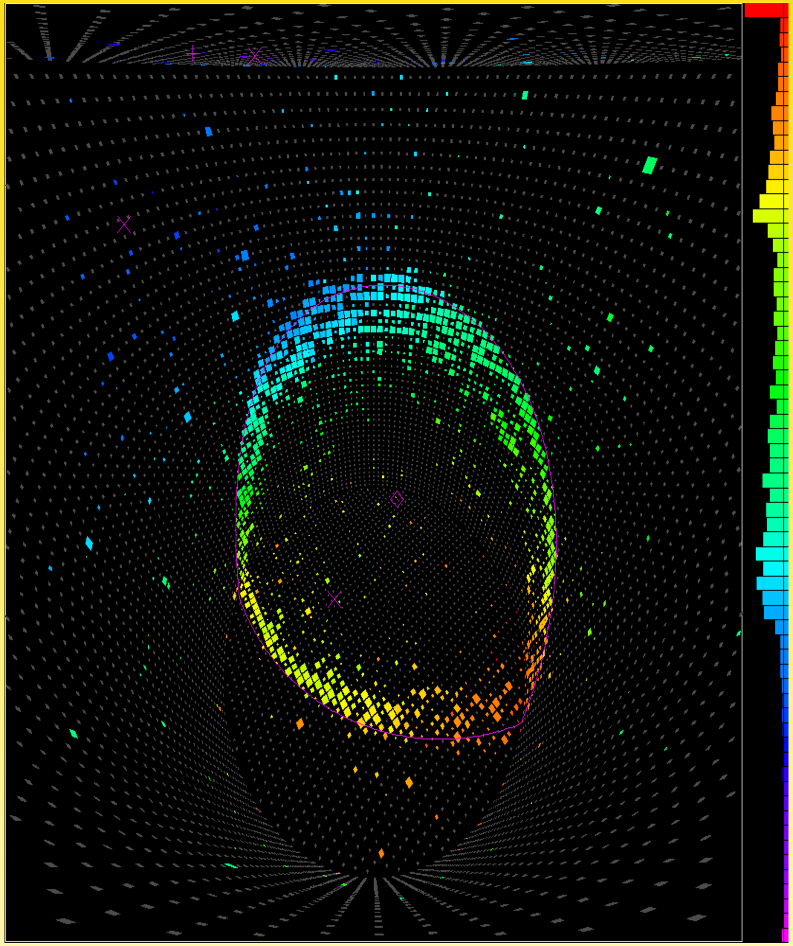
SuperKamiokande



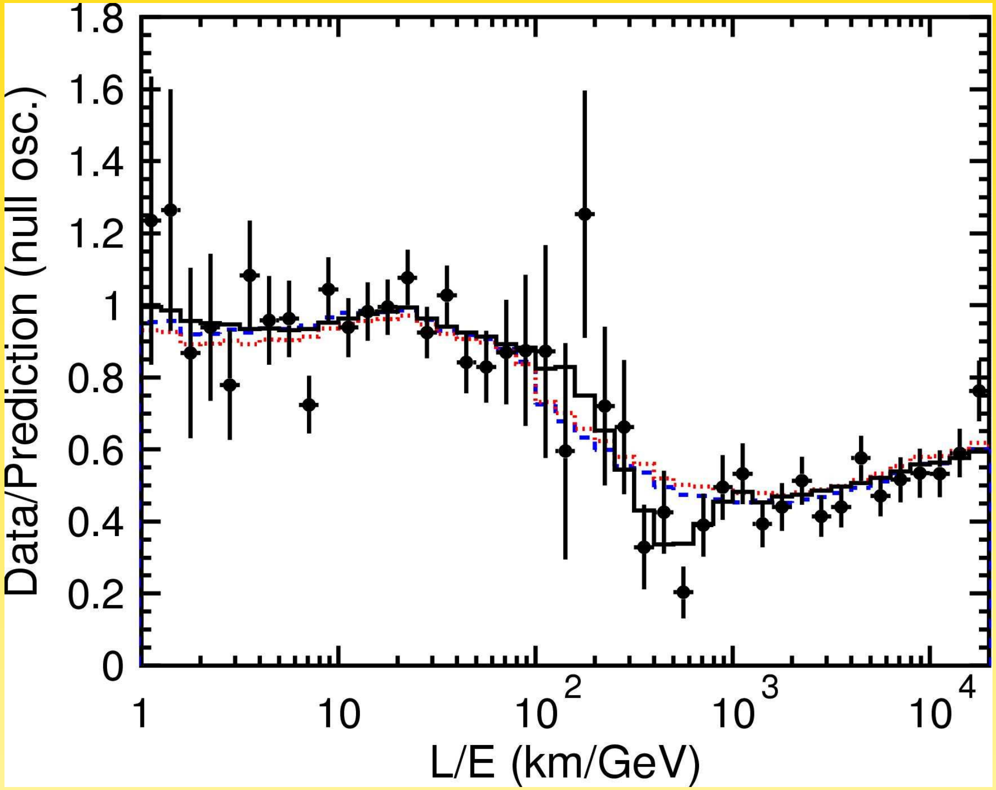



Cerenkov radiation cone
ν_μ
μ
Muon neutrino
Muon
The Cerenkov radiation from a muon produced by a muon neutrino event yields a well defined circular ring in the photomultiplier detector bank.
ν_e
Electron neutrino
Electron shower
The Cerenkov radiation from the electron shower produced by an electron neutrino event produces multiple cones and therefore a diffuse ring in the detector array.





Atmospheric Neutrinos



Dip at $L/E \simeq 500\ \mathrm{km/GeV} \Rightarrow$ Oscillatory Behavior!!

(No ν_τ observed yet)

Testing Atmospheric Neutrinos with Accelerators: K2K, MINOS, T2K, OPERA, NoνA

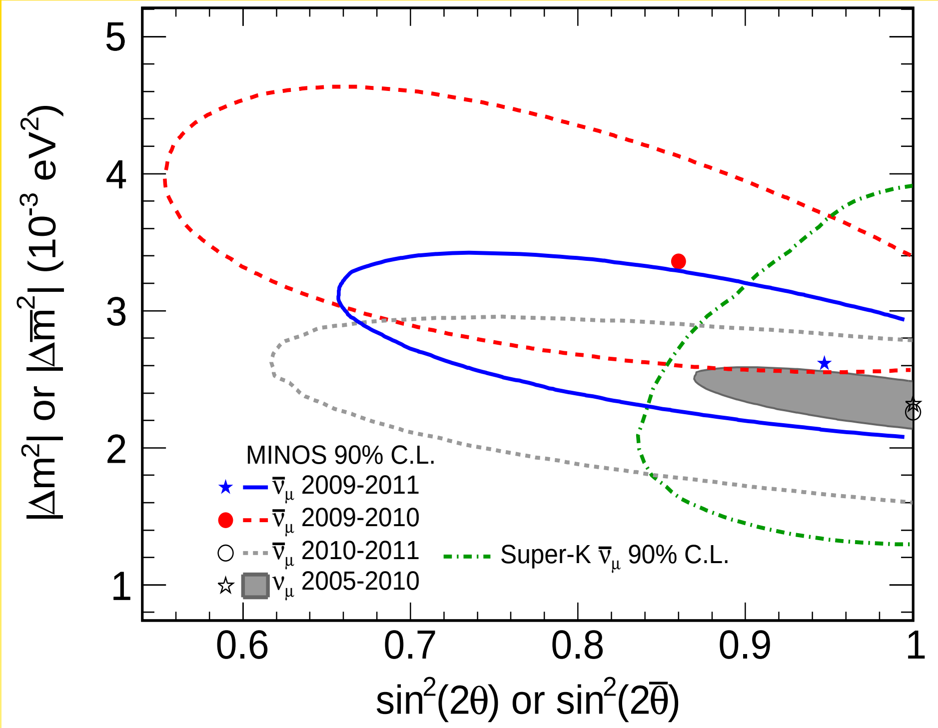
Proton beam

$$p + X \to \pi^{\pm}, \; K^{\pm} \to \pi^{\pm} \to \overset{(-)}{\nu}_{\mu} \quad \text{with } E \simeq \mathrm{GeV}$$

If $L \simeq 100$ km:

$$\frac{\Delta m^2_{\rm A}}{E} \, L \sim 1 \Rightarrow \text{ atmospheric } \nu \text{ parameters!!}$$

$$P(\nu_\mu \to \nu_\mu) = 1 - \sin^2 2\theta_{23} \, \sin^2 \frac{\Delta m^2_{31}}{4E} L$$



Results of fits give

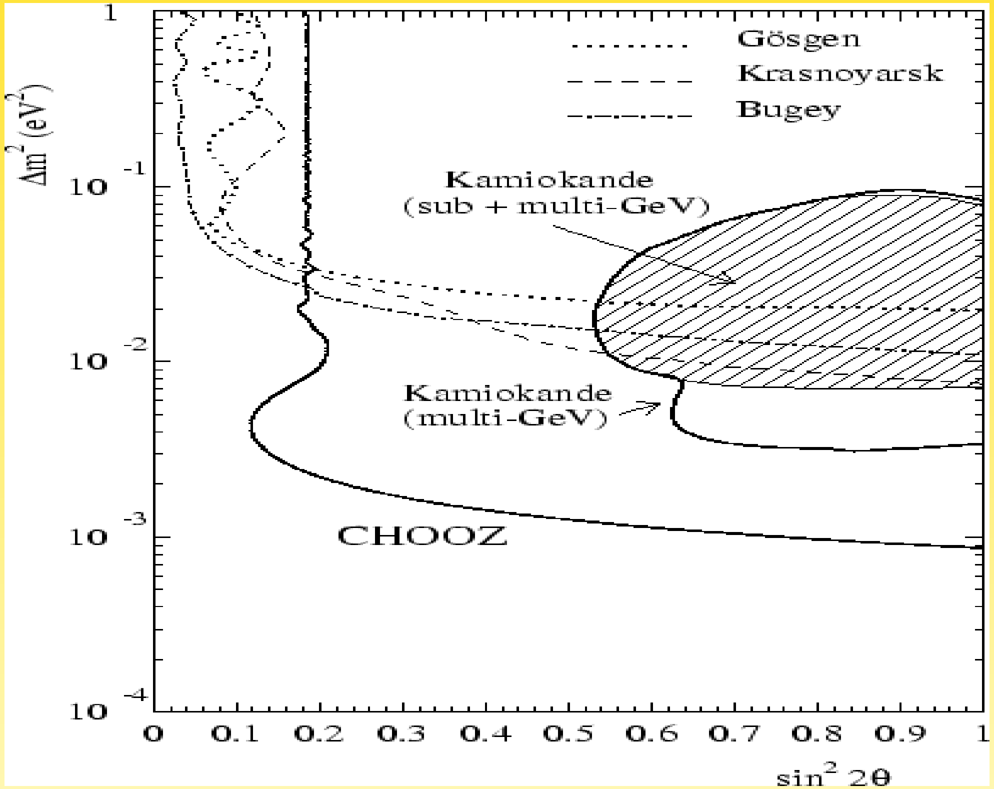
$\sin^2\theta_{23} \simeq 0.50$ maximal mixing?!

$$|\Delta m^2_{31}| \equiv \Delta m^2_{\rm A} \simeq 2.5 \times 10^{-3}\ {\rm eV}^2 \simeq 30\,\Delta m^2_{\odot}$$

The third mixing: Short-Baseline Reactor Neutrinos

$E_\nu \simeq \mathrm{MeV}$ and $L \simeq 0.1$ km:

$\frac{\Delta m^2_{\rm A}}{E} L \sim 1 \Rightarrow$ atmospheric ν parameters!!



$$P_{ee} = 1 - \sin^2 2\theta_{13} \sin^2 \frac{\Delta m^2_{\rm A}}{4E} L$$

3 families: $U = R_{23}\, \tilde{R}_{13}\, R_{12}\, P$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}\, e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}\, e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} P$$

$$= \begin{pmatrix} c_{12}\, c_{13} & s_{12}\, c_{13} & s_{13}\, e^{i\delta} \\ -s_{12}\, c_{23} - c_{12}\, s_{23}\, s_{13}\, e^{-i\delta} & c_{12}\, c_{23} - s_{12}\, s_{23}\, s_{13}\, e^{-i\delta} & s_{23}\, c_{13} \\ s_{12}\, s_{23} - c_{12}\, c_{23}\, s_{13}\, e^{-i\delta} & -c_{12}\, s_{23} - s_{12}\, c_{23}\, s_{13}\, e^{-i\delta} & c_{23}\, c_{13} \end{pmatrix} P$$

with $P = \mathrm{diag}(1, e^{i\alpha}, e^{i\beta})$

Interpretation in 3 Neutrino Framework

assume $\Delta m^2_{21} \ll \Delta m^2_{31} \simeq \Delta m^2_{32}$ and small θ_{13}:

- atmospheric and accelerator neutrinos: $\Delta m^2_{21} L/E \ll 1$

$$P(\nu_\mu \to \nu_\tau) \simeq \sin^2 2\theta_{23} \, \sin^2 \frac{\Delta m^2_{31}}{4\,E} \, L$$

- solar and KamLAND neutrinos: $\Delta m^2_{31} L/E \gg 1$

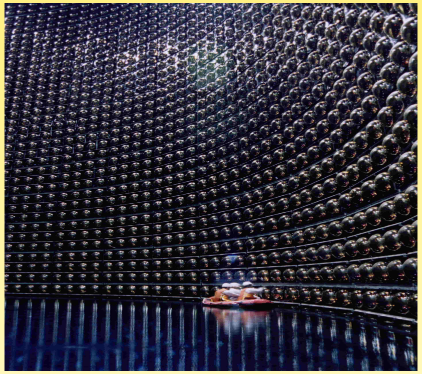
$$P(\nu_e \to \nu_e) \simeq 1 - \sin^2 2\theta_{12} \, \sin^2 \frac{\Delta m^2_{21}}{4\,E} \, L$$

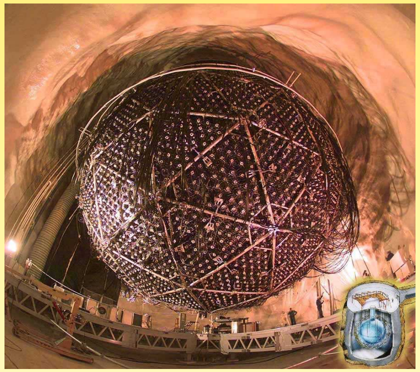
- short baseline reactor neutrinos: $\Delta m^2_{21} L/E \ll 1$

$$P(\nu_e \to \nu_e) \simeq 1 - \sin^2 2\theta_{13} \, \sin^2 \frac{\Delta m^2_{31}}{4\,E} \, L$$

$$U = \begin{pmatrix} c_{12}\, c_{13} & s_{12}\, c_{13} & s_{13}\, e^{i\delta} \\ -s_{12}\, c_{23} - c_{12}\, s_{23}\, s_{13}\, e^{-i\delta} & c_{12}\, c_{23} - s_{12}\, s_{23}\, s_{13}\, e^{-i\delta} & s_{23}\, c_{13} \\ s_{12}\, s_{23} - c_{12}\, c_{23}\, s_{13}\, e^{-i\delta} & -c_{12}\, s_{23} - s_{12}\, c_{23}\, s_{13}\, e^{-i\delta} & c_{23}\, c_{13} \end{pmatrix} =$$

$$\underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\text{atmospheric and LBL accelerator}} \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13}\, e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}\, e^{i\delta} & 0 & c_{13} \end{pmatrix}}_{\text{SBL reactor}} \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{solar and LBL reactor}}$$





$$U = \begin{pmatrix} c_{12}\, c_{13} & s_{12}\, c_{13} & s_{13}\, e^{i\delta} \\ -s_{12}\, c_{23} - c_{12}\, s_{23}\, s_{13}\, e^{-i\delta} & c_{12}\, c_{23} - s_{12}\, s_{23}\, s_{13}\, e^{-i\delta} & s_{23}\, c_{13} \\ s_{12}\, s_{23} - c_{12}\, c_{23}\, s_{13}\, e^{-i\delta} & -c_{12}\, s_{23} - s_{12}\, c_{23}\, s_{13}\, e^{-i\delta} & c_{23}\, c_{13} \end{pmatrix} =$$

$$\underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\substack{\text{atmospheric and} \\ \text{LBL accelerator}}} \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13}\, e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}\, e^{i\delta} & 0 & c_{13} \end{pmatrix}}_{\text{SBL reactor}} \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\substack{\text{solar and} \\ \text{LBL reactor}}}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{\frac{1}{2}} & -\sqrt{\frac{1}{2}} \\ 0 & \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} \end{pmatrix} \qquad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{3}} & \sqrt{\frac{2}{3}} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

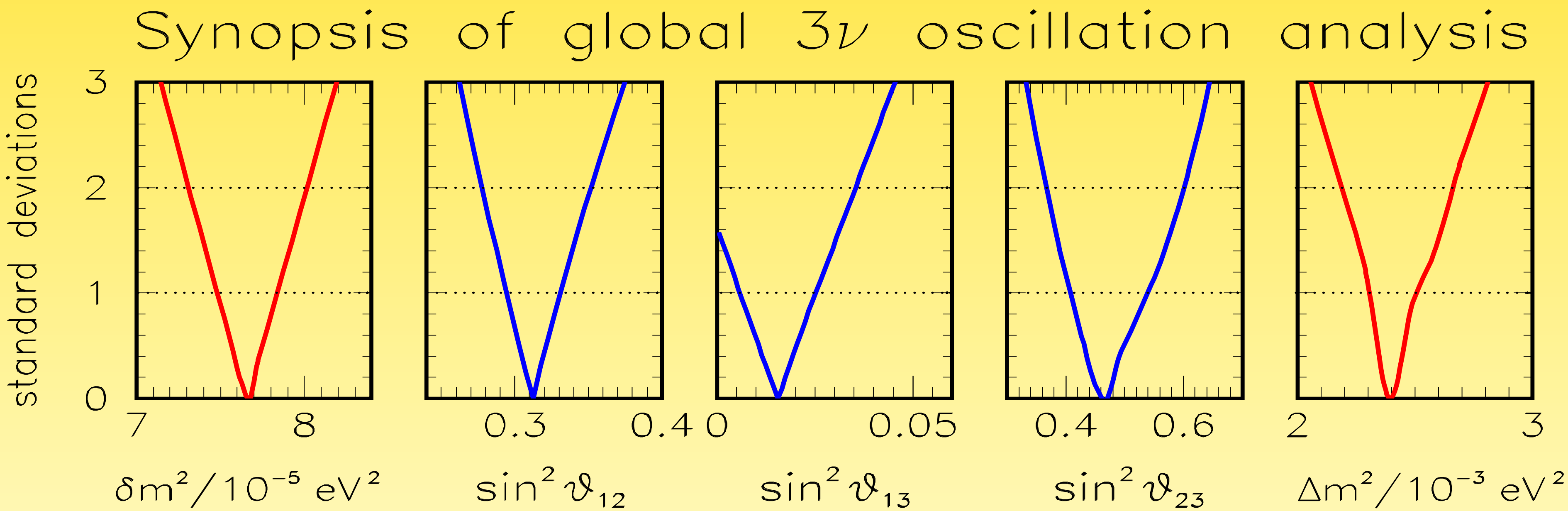
$$(\sin^2\theta_{23} = \tfrac{1}{2}) \qquad (\sin^2\theta_{13} = 0) \qquad (\sin^2\theta_{12} = \tfrac{1}{3})$$

$$\Delta m^2_{\mathrm{A}} \qquad \Delta m^2_{\mathrm{A}} \qquad \Delta m^2_{\odot}$$

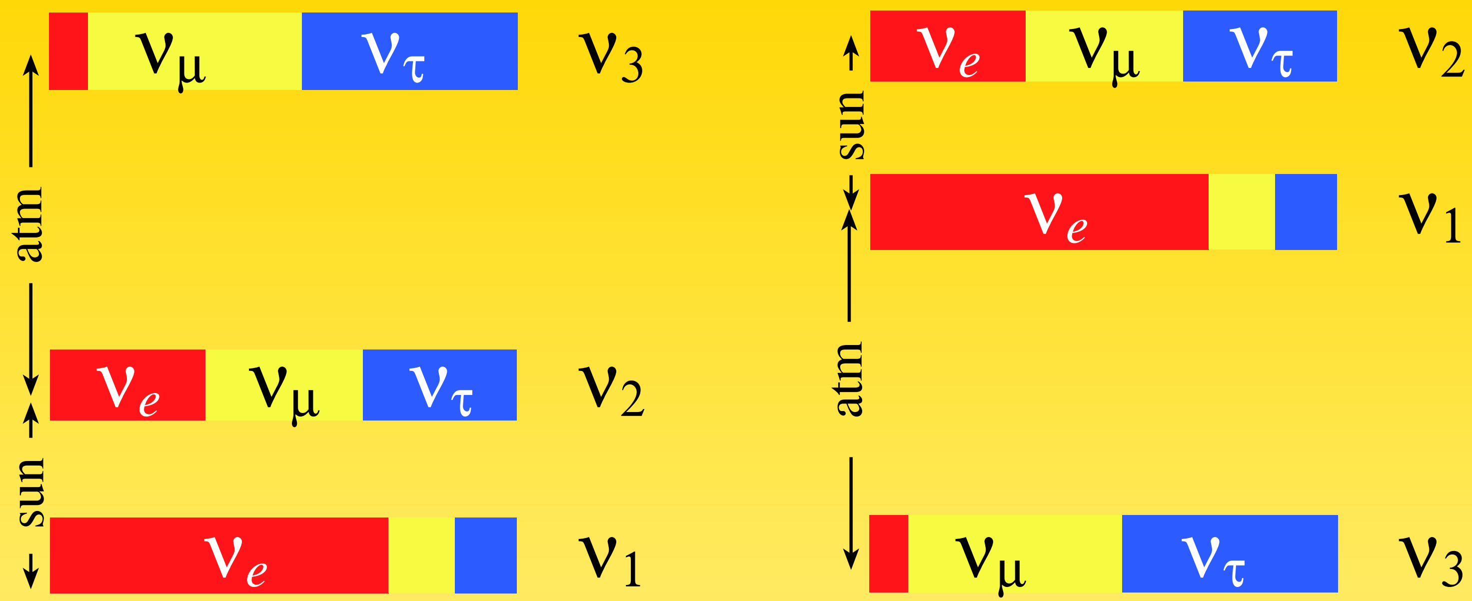
Tri-bimaximal Mixing

$$U_{\rm TBM} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix}$$

Harrison, Perkins, Scott (2002)



Fogli *et al.*



$$|U|^2 \simeq \begin{pmatrix} 0.67 & 0.33 & 0 \\ 0.17 & 0.33 & 0.50 \\ 0.17 & 0.33 & 0.50 \end{pmatrix}$$

- normal ordering: $\Delta m^2_{31} > 0$
- inverted ordering: $\Delta m^2_{31} < 0$


45°
40°
35°
30°
25°
20°
15°
10°
θ_{23}
θ_{12}

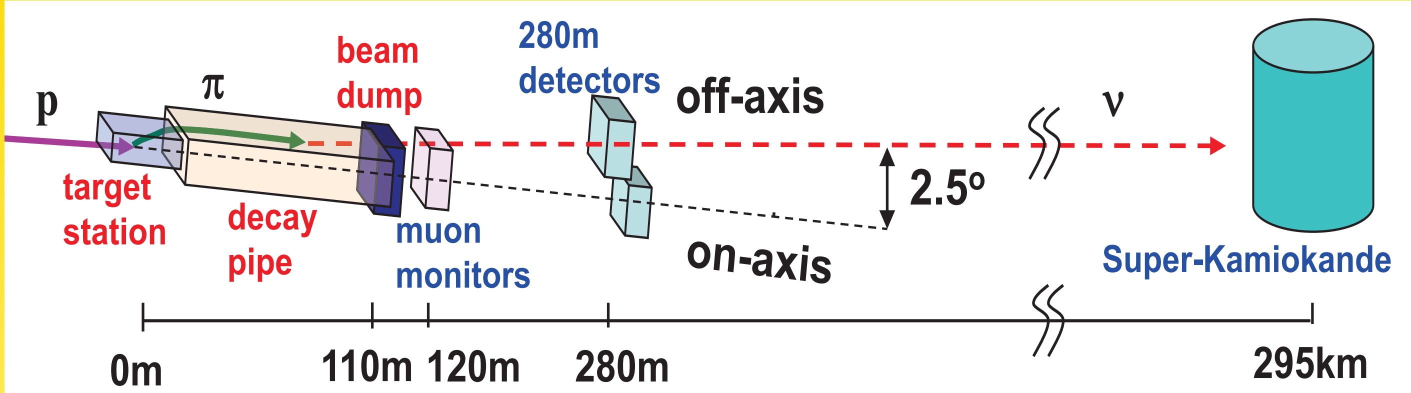
Non-zero $|U_{e3}|$?

2010: `Fogli` *et al.* find a 1.6σ effect

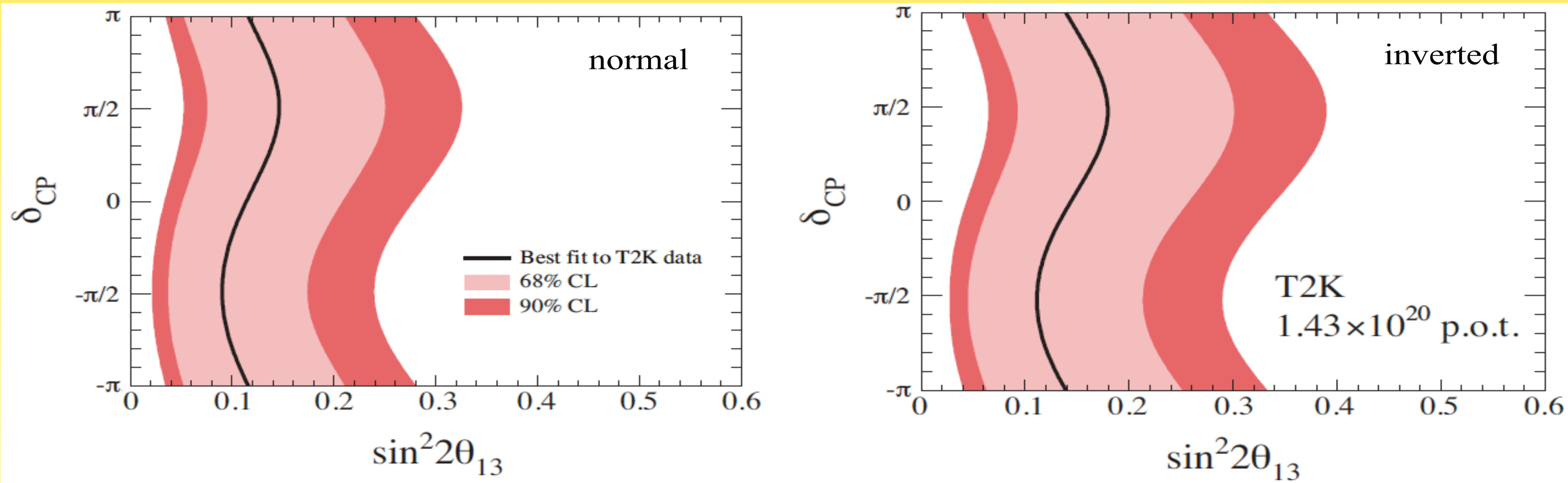
at $1\sigma : |U_{e3}|^2 = 0.016 \pm 0.010$

- SuperKamiokande atmospheric neutrinos (excess of sub-GeV e-like events, caused by sub-leading $\Delta m^2_\odot$)
- KamLAND favors slightly higher $\sin^2\theta_{12}$ than solar data
  ($P_{ee}^{\odot} = (1 - 2\,|U_{e3}|^2)\,\sin^2\theta_{12}$
  vs. $P_{ee}^{\rm KL} = (1 - 2\,|U_{e3}|^2)(1 - \sin^2 2\theta_{12}\,\sin^2\Delta_{21}/2)$)
- 2011: T2K, Double Chooz!

T2K: 2.5σ



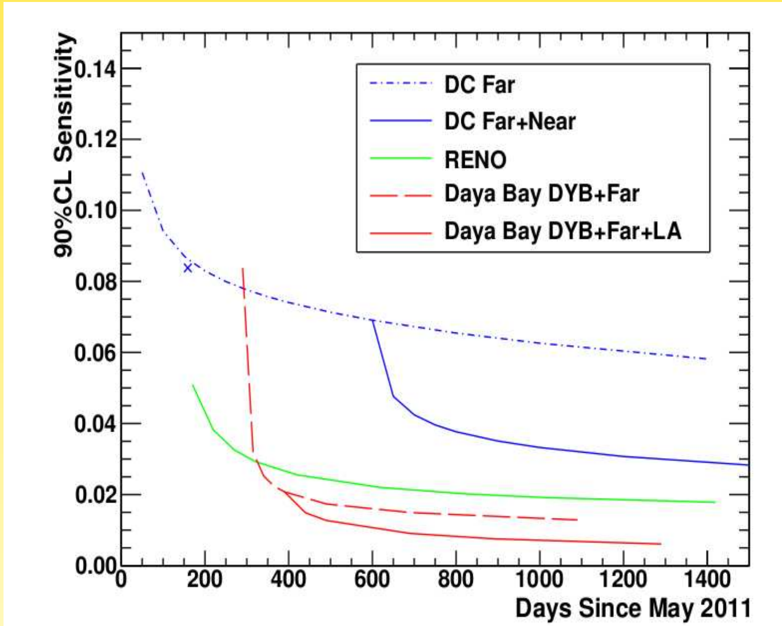
$$P(\nu_\mu \to \nu_e) \simeq \sin^2\theta_{23} \sin^2 2\theta_{13} \sin^2 \frac{\Delta m^2_A}{4E} L$$



More data

- MINOS: 1.7σ
- Double Chooz: $0.017 < \sin^2 2\theta_{13} < 0.16$ at 90 % C.L.

$$P_{ee} = 1 - \sin^2 2\theta_{13} \, \sin^2 \frac{\Delta m^2_{\rm A}}{4E} L$$



all this accumulates to larger than 3σ significance!

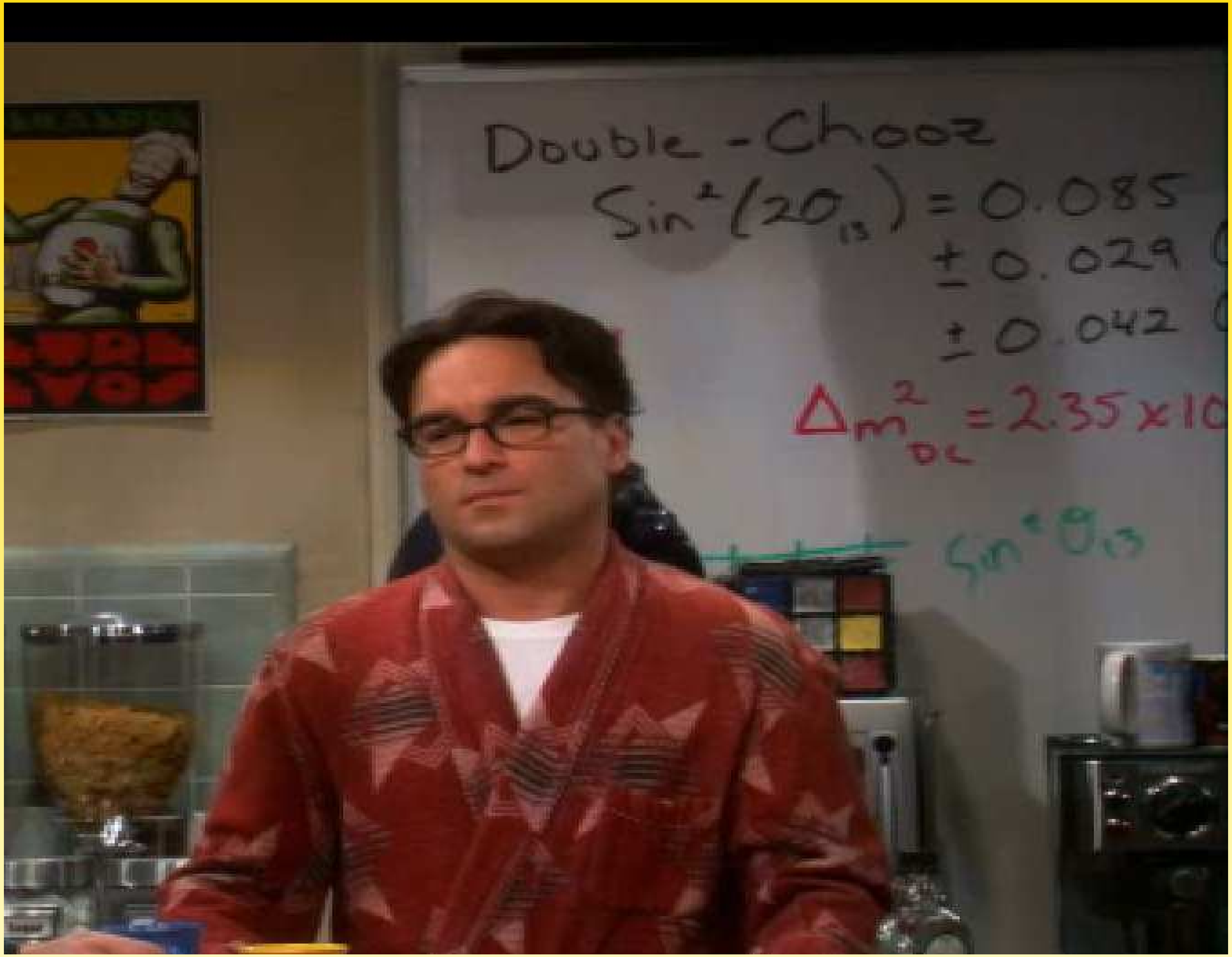
$$|U_{e3}| = 0.146^{+0.084}_{-0.119}$$

and PMNS matrix is more like

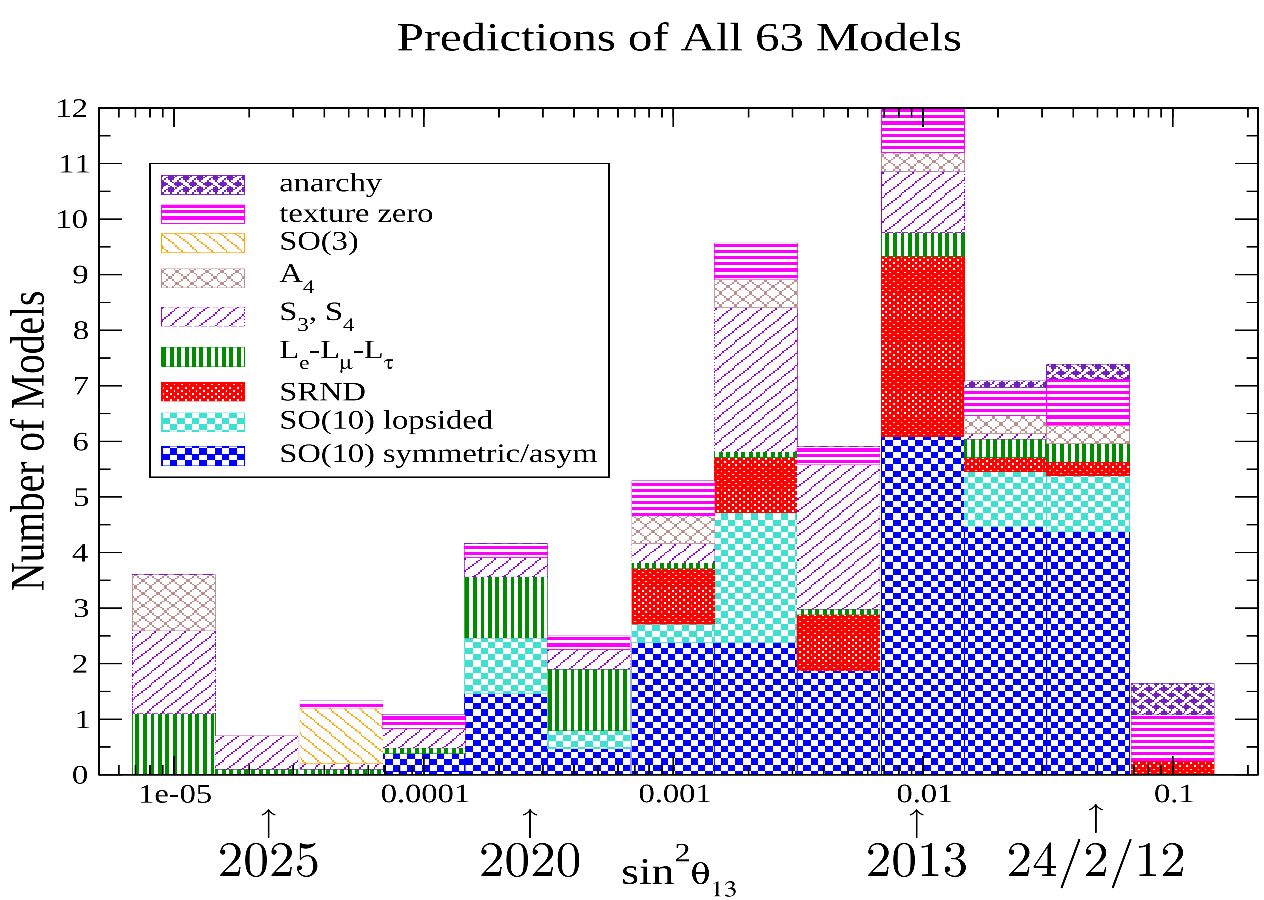
$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{\frac{1}{2}} & -\sqrt{\frac{1}{2}} \\ 0 & \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} \end{pmatrix} \begin{pmatrix} 1 & 0 & \epsilon\, e^{-i\delta} \\ 0 & 1 & 0 \\ -\epsilon\, e^{i\delta} & 0 & 1 \end{pmatrix} \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{3}} & \sqrt{\frac{2}{3}} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$\Rightarrow$ interesting phenomenological and theoretical implications...

Non-zero θ_{13}



What's that good for?



Albright, Chen

CKM vs. PMNS

$$|V_{\mathrm{CKM}}| \simeq \begin{pmatrix} 0.97419 & 0.2257 & 0.00359 \\ 0.2256 & 0.97334 & 0.0415 \\ 0.00874 & 0.0407 & 0.999133 \end{pmatrix}$$

$$|U_{\mathrm{PMNS}}| \simeq \begin{pmatrix} 0.82 & 0.58 & 0 \\ 0.64 & 0.58 & 0.71 \\ 0.64 & 0.58 & 0.71 \end{pmatrix}$$

Contents

II Neutrino Oscillations

II4) Prospects – what do we want to know?

9 physical parameters in m_ν

- θ_{12} and $m_2^2 - m_1^2$ (or $\theta_\odot$ and $\Delta m_\odot^2$)
- θ_{23} and $|m_3^2 - m_2^2|$ (or θ_A and Δm_A^2)
- θ_{13} (or $|U_{e3}|$)
- m_1, m_2, m_3
- sgn$(m_3^2 - m_2^2)$
- Dirac phase δ
- Majorana phases α and β (or α_1 and α_2, or ϕ_1 and ϕ_2, or...)

The future: open issues for neutrinos oscillations

Look for *three flavor effects:*

- precision measurements
  - how maximal is θ_{23} ? how small/large is U_{e3} ?
- sign of Δm^2_{32} ?

$$\tan 2\theta_m = f(\mathrm{sgn}(\Delta m^2))$$

- is there CP violation?
- Problems:
  - two *small* parameters: $\Delta m^2_\odot / \Delta m^2_\mathrm{A} \simeq 1/30$ and $|U_{e3}| \lesssim 0.2$
  - 8-fold degeneracy for fixed L/E and $\nu_e \to \nu_\mu$ channels

Degeneracies

Expand *3 flavor* oscillation probabilities in terms of $R = \Delta m^2_\odot / \Delta m^2_{\rm A}$ and $|U_{e3}|$:

$$P(\nu_e \to \nu_\mu) \simeq \sin^2 2\theta_{13} \, \sin^2 \theta_{23} \, \frac{\sin^2 (1-\hat{A})\Delta}{(1-\hat{A})^2} + R^2 \, \sin^2 2\theta_{12} \, \cos^2 \theta_{23} \frac{\sin^2 \hat{A}\Delta}{\hat{A}^2}$$

$$+\sin\delta \sin 2\theta_{13} \, R \, \sin 2\theta_{12} \, \cos\theta_{13} \, \sin 2\theta_{23} \sin\Delta \, \frac{\sin \hat{A}\Delta \, \sin (1-\hat{A})\Delta}{\hat{A}(1-\hat{A})}$$

$$+\cos\delta \sin 2\theta_{13} \, R \, \sin 2\theta_{12} \, \cos\theta_{13} \, \sin 2\theta_{23} \, \cos\Delta \frac{\sin \hat{A}\Delta \, \sin (1-\hat{A})\Delta}{\hat{A}(1-\hat{A})}$$

with $\hat{A} = 2\sqrt{2}\, G_F \, n_e \, E / \Delta m^2_{\rm A}$ and $\Delta = \frac{\Delta m^2_{\rm A}}{4\,E}\, L$

- $\theta_{23} \leftrightarrow \pi/2 - \theta_{23}$ degeneracy
- θ_{13}-δ degeneracy
- δ-sgn($\Delta m^2_{\rm A}$) degeneracy

Solutions: more channels, different L/E, high precision,...

Degeneracies

Expand *3 flavor* oscillation probabilities in terms of $R = \Delta m^2_\odot / \Delta m^2_{\rm A}$ and $|U_{e3}|$:

$$P(\nu_e \to \nu_\mu) \simeq \sin^2 2\theta_{13}\ \sin^2\theta_{23}\ \ \frac{\sin^2(1-\hat{A})\Delta}{(1-\hat{A})^2} + R^2\ \sin^2 2\theta_{12}\ \cos^2\theta_{23} \frac{\sin^2 \hat{A}\Delta}{\hat{A}^2}$$

$$+\sin\delta \sin 2\theta_{13}\ R\ \sin 2\theta_{12}\ \cos\theta_{13}\ \sin 2\theta_{23} \sin\Delta\ \frac{\sin\hat{A}\Delta\ \sin(1-\hat{A})\Delta}{\hat{A}(1-\hat{A})}$$

$$+\cos\delta \sin 2\theta_{13}\ R\ \sin 2\theta_{12}\ \cos\theta_{13}\ \sin 2\theta_{23}\ \cos\Delta \frac{\sin\hat{A}\Delta\ \sin(1-\hat{A})\Delta}{\hat{A}(1-\hat{A})}$$

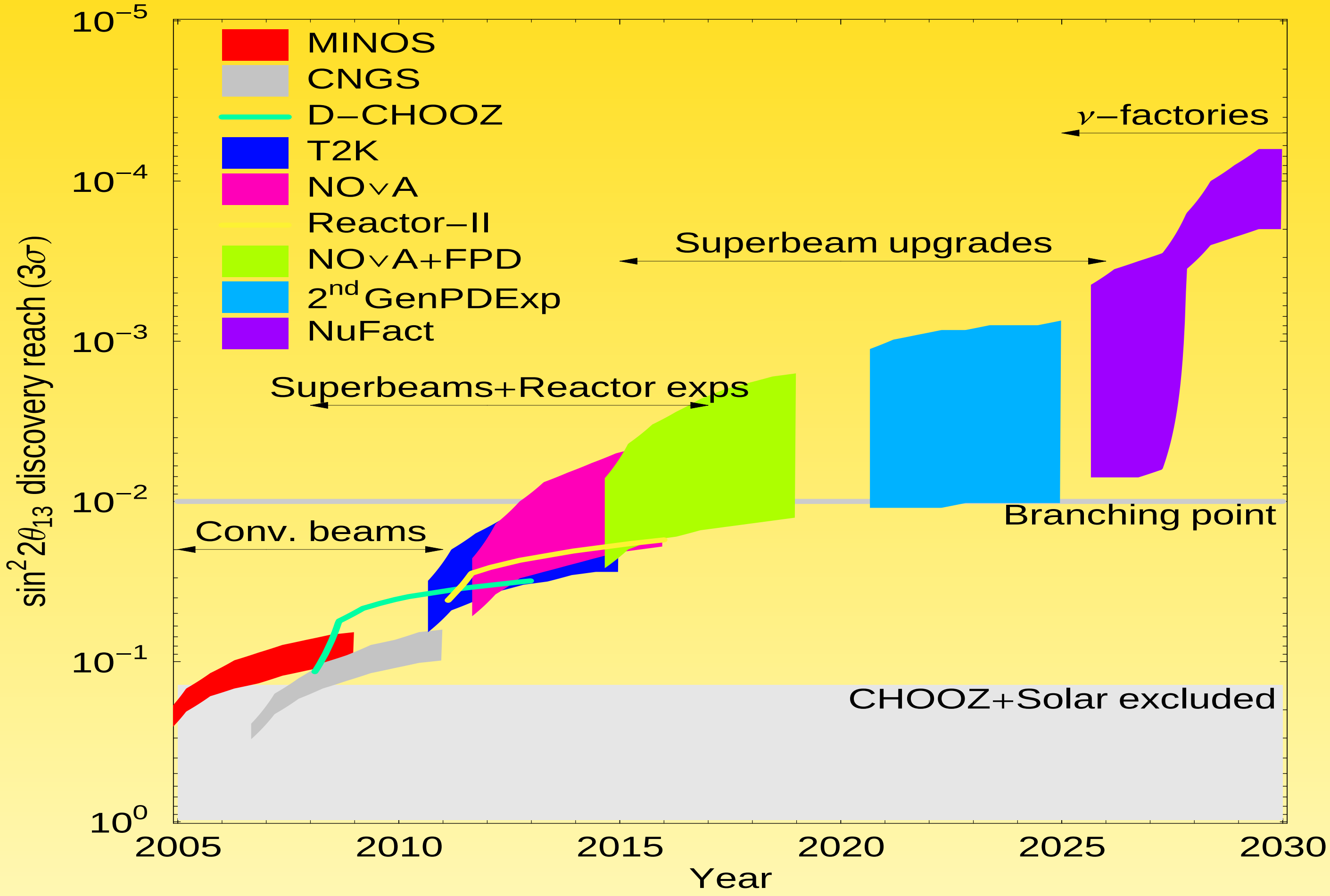
with $\hat{A} = 2\sqrt{2}\, G_F\, n_e\, E/\Delta m^2_{\rm A}$ and $\Delta = \dfrac{\Delta m^2_{\rm A}}{4\,E}\, L$

If $\hat{A}\,\Delta = \pi$:

$$P(\nu_e \to \nu_\mu) \simeq \sin^2 2\theta_{13}\ \sin^2\theta_{23}\ \ \frac{\sin^2(1-\hat{A})\Delta}{(1-\hat{A})^2}$$

This is the "magic baseline" of $L = \frac{\sqrt{2}\pi}{G_F\, n_e} \simeq 7500$ km

Typical time scale

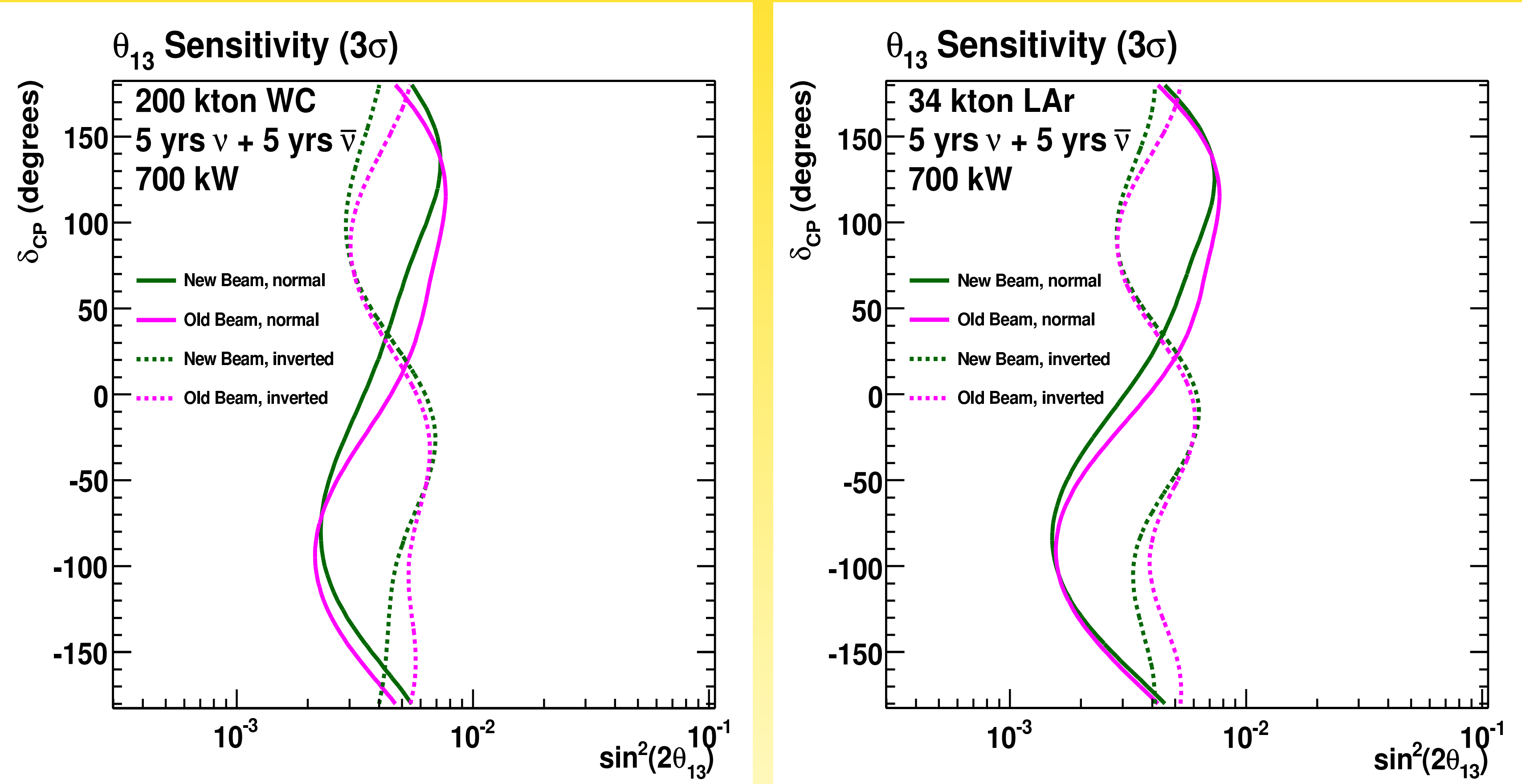


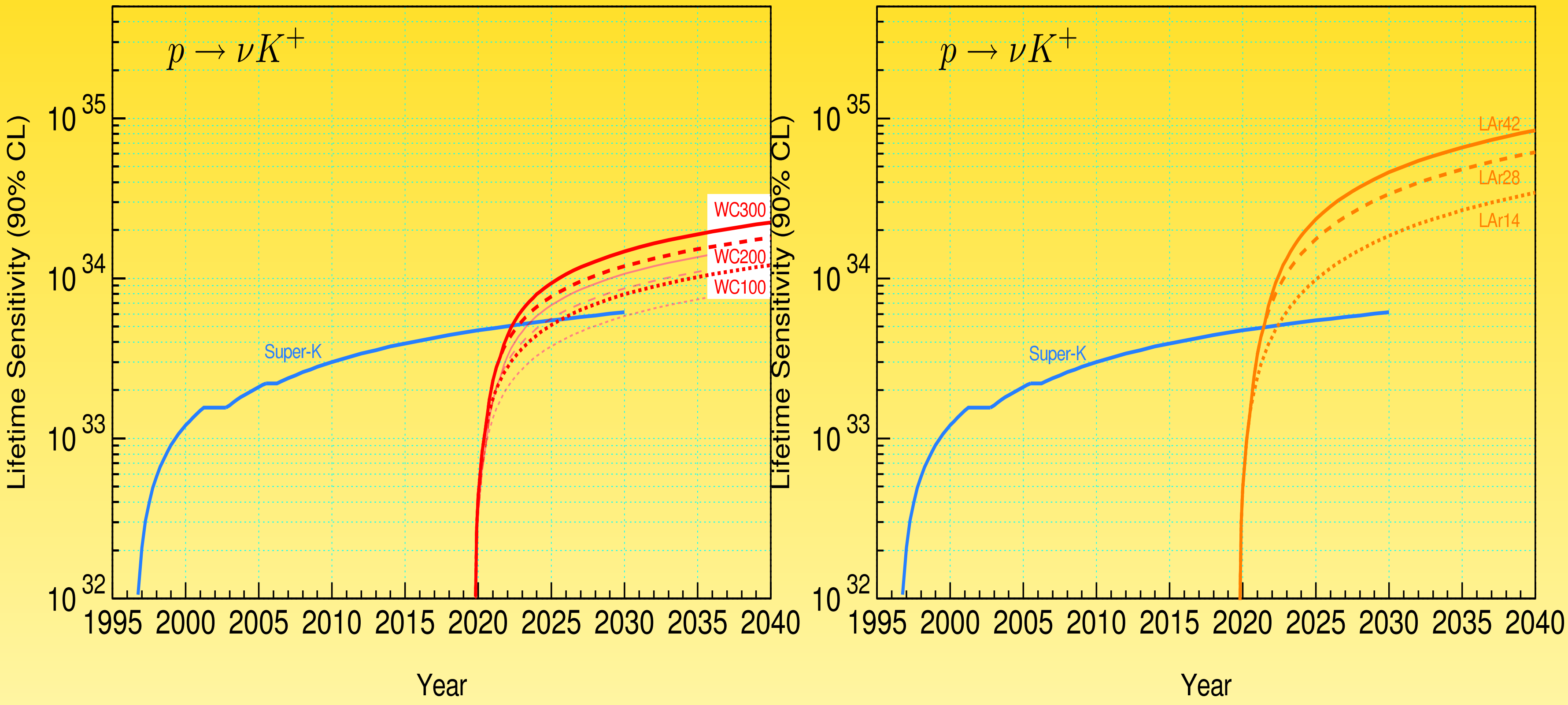
Future experiments

- what detector?
  - Water Cerenkov?
  - liquid scintillator?
  - liquid argon?
- Neutrino Physics
  - oscillations (hierarchy, CP, precision)
  - non-standard physics (NSIs, unitarity violation, steriles, extra forces,...)
- other physics
  - SN (burst and relic)
  - geo-neutrinos
  - p-decay

Example LBNE

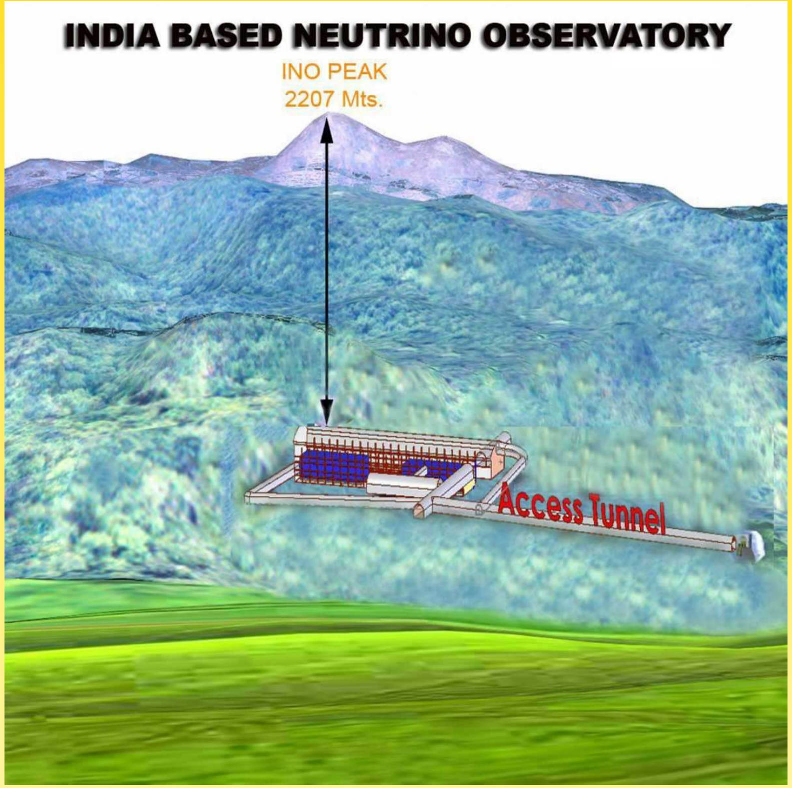
FNAL $\to$ Homestake, $L = 1300$ km

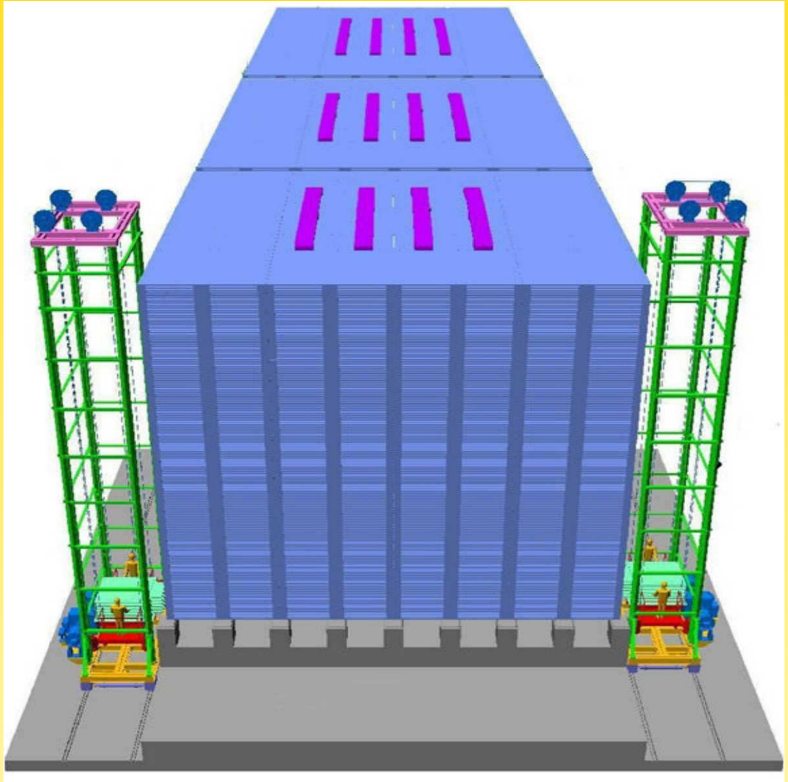




$p \to \nu K^+$
Lifetime Sensitivity (90% CL)
Year
Super-K
WC300
WC200
WC100
$p \to \nu K^+$
Lifetime Sensitivity (90% CL)
Year
Super-K
LAr42
LAr28
LAr14

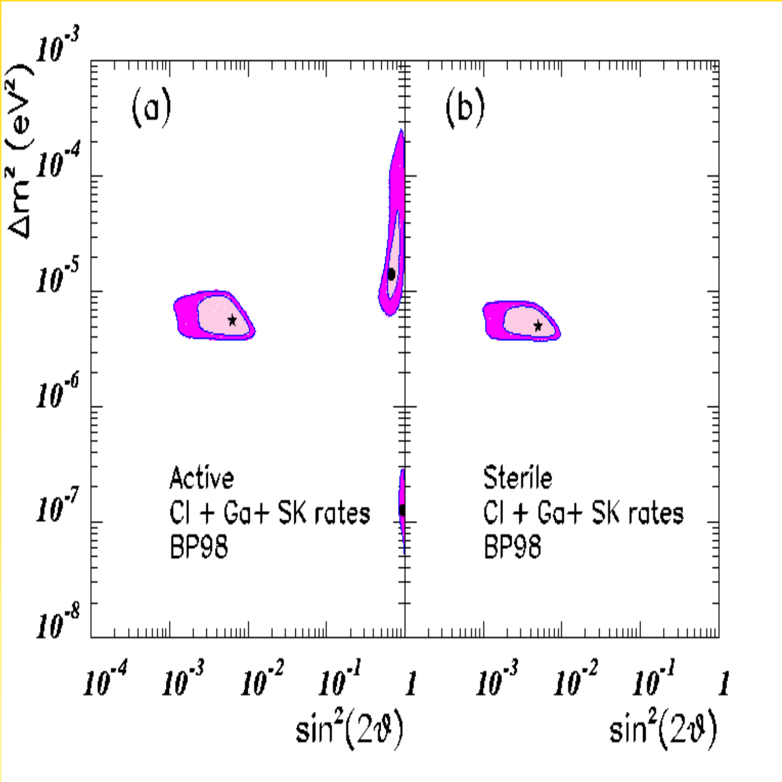
Example ICAL at INO



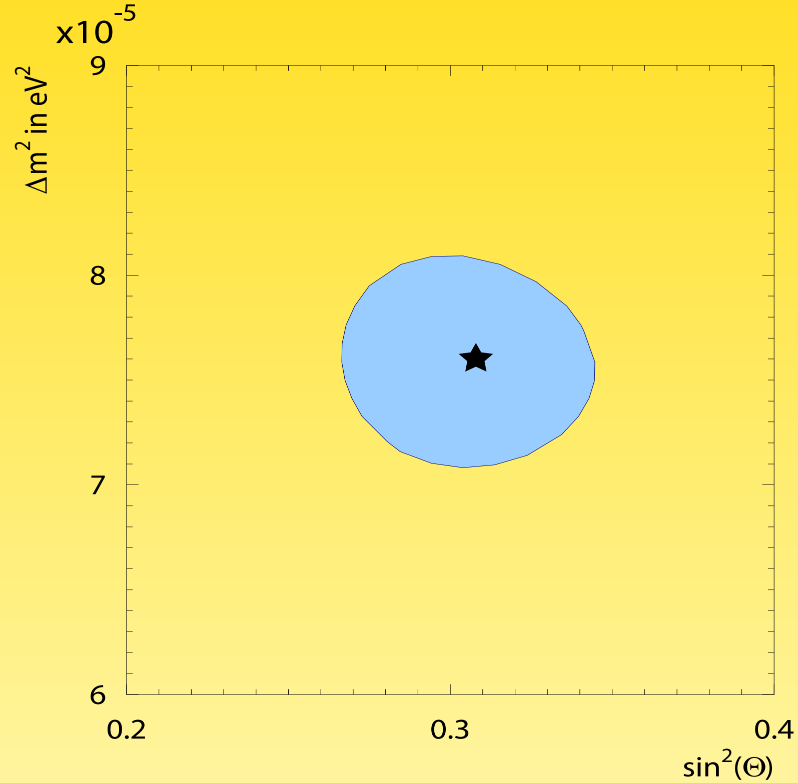


7300 km from CERN, 6600 km from JHF at Tokai

Precision era!



1998



today

Anomalies?

- light sterile neutrinos?
  -
- different Δm^2 for neutrinos and anti-neutrinos
  -
- faster than light neutrinos?
  -

Anomalies?

- light sterile neutrinos?
  - still there
- different Δm^2 for neutrinos and anti-neutrinos
  -
- faster than light neutrinos?
  -

Anomalies?

- light sterile neutrinos?
  - still there
- different Δm^2 for neutrinos and anti-neutrinos
  - went away
- faster than light neutrinos?
  -

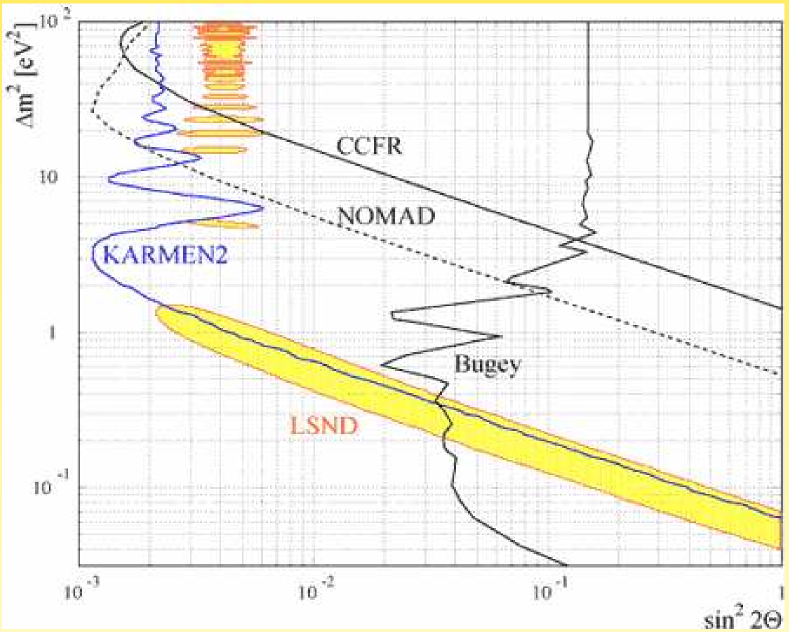
Anomalies?

- light sterile neutrinos?
  - still there
- different Δm^2 for neutrinos and anti-neutrinos
  - went away
- faster than light neutrinos?
  - obviously bullshit

Light sterile neutrinos?

it's all the fault of LSND

- 800 MeV proton beam on water target, detector is liquid scintillator, π^+
- $E \simeq 35$ MeV, $L \simeq 30$ m $\Rightarrow \Delta m^2 \simeq 1$ eV^2
- prompt signal $\bar{\nu}_e + p \to e^+ + n$, delayed signal $n + p \to d + \gamma$
- $P(\bar{\nu}_\mu \to \bar{\nu}_e) \simeq 2.6 \times 10^{-3}$, about 4σ

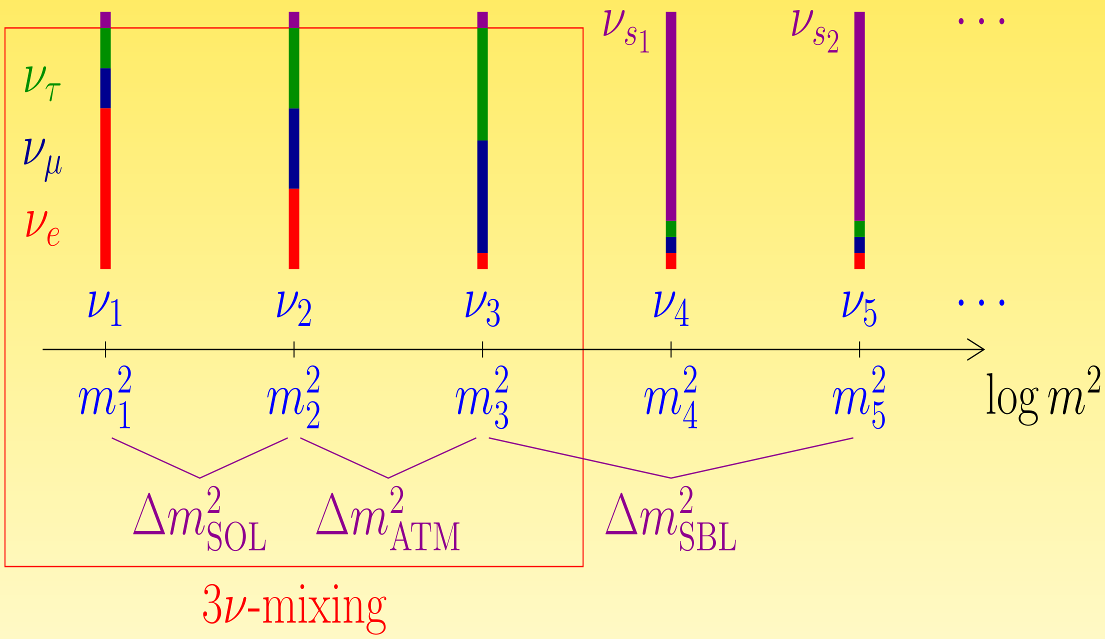


Light sterile neutrinos?

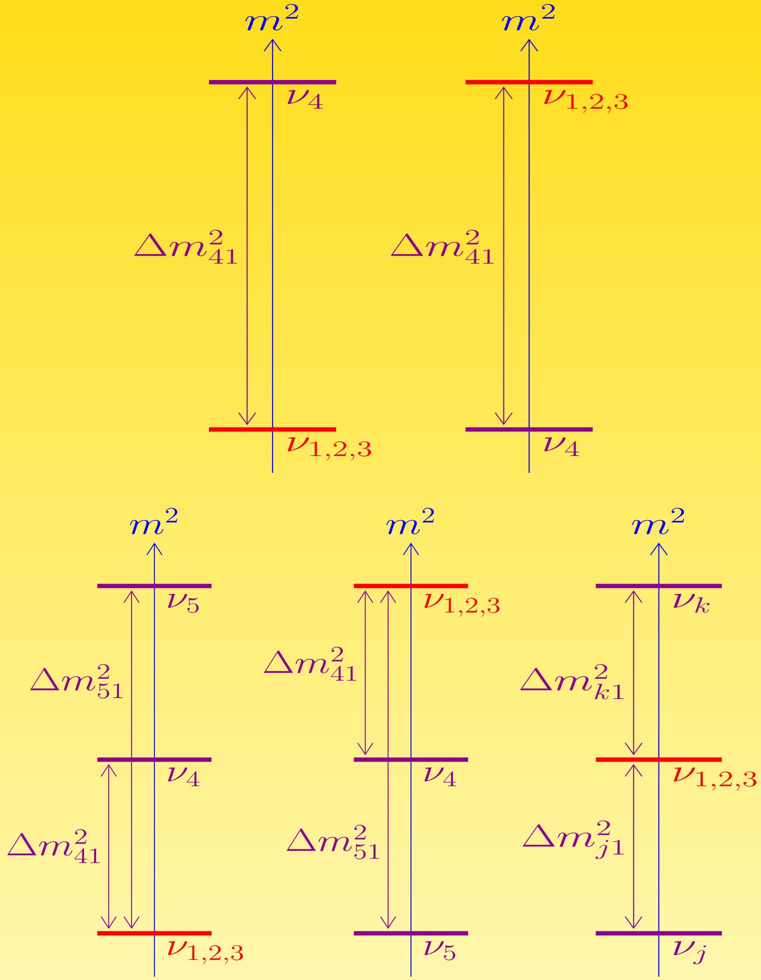
- Z-width says $N_\nu = 3$, and hence there are only two independent Δm^2
- $\Rightarrow$ fourth *sterile* neutrino, does not couple to W or Z
- mixing matrix is now $4 \times 4 \Rightarrow$ 6 angles, 3+3 phases

$$U = R_{34}\,\tilde{R}_{24}\,\tilde{R}_{14}\,R_{23}\,\tilde{R}_{13}\,R_{12}\,P$$

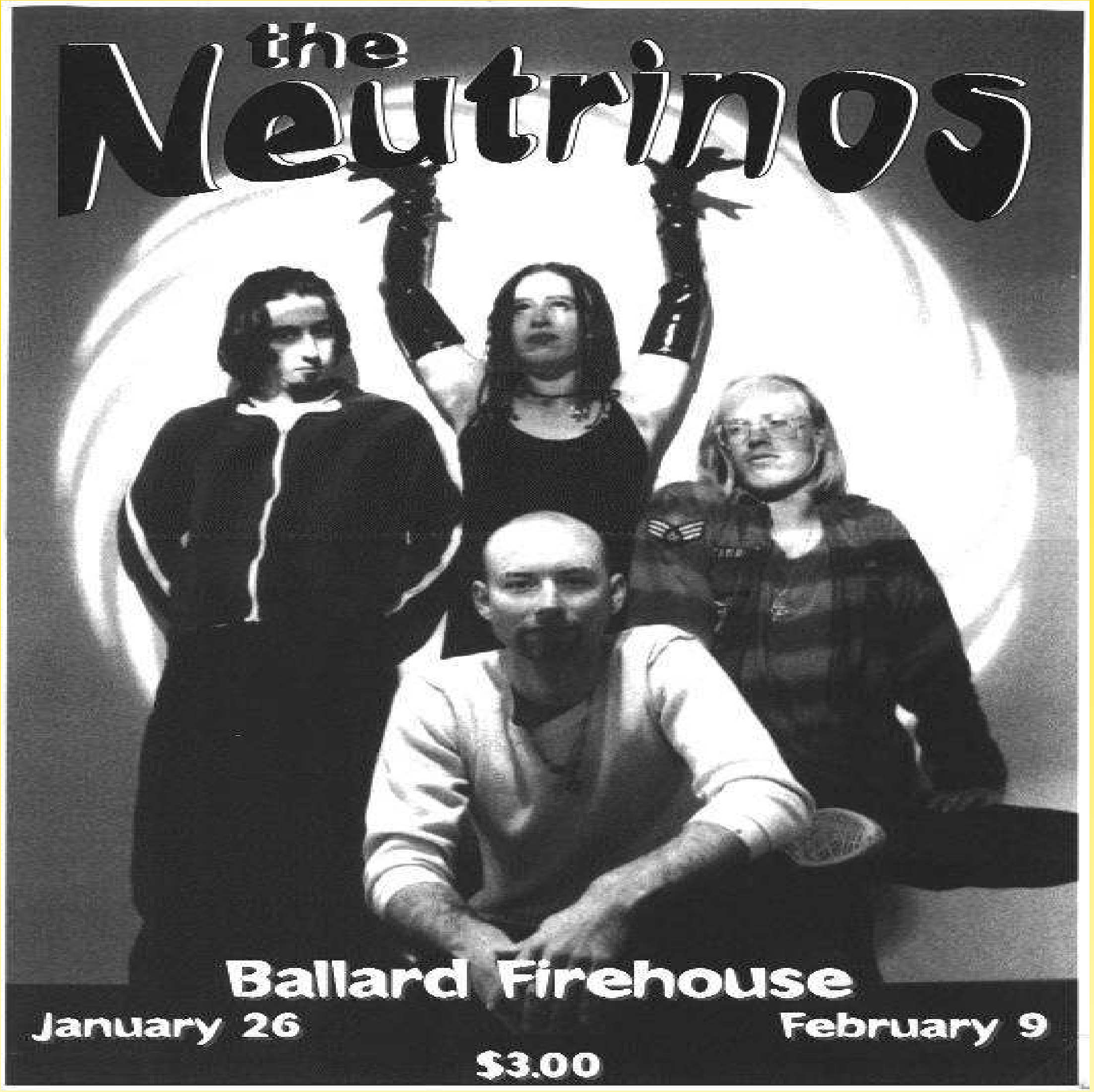
- influences cosmology, supernovae, neutrino mass measurements,...



Mass Orderings

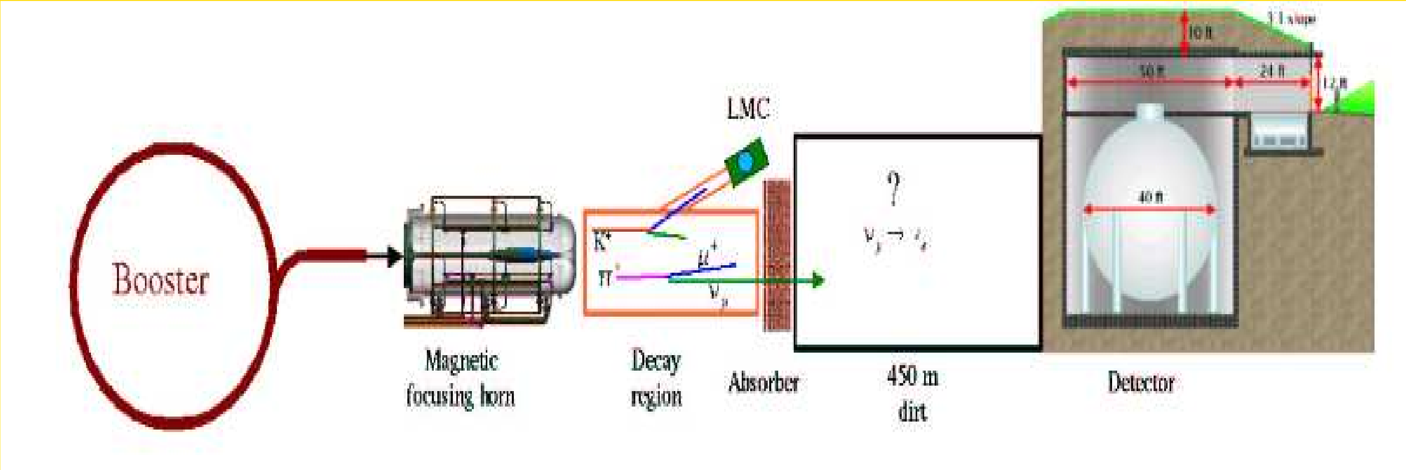


3 active neutrinos can be normally or inversely ordered



Which one is sterile?

MiniBooNE



- was supposed to test LSND
- same L/E
- can run in π^+ and π^- mode
- results:
  - ν-mode: inconsistent with LSND, unexplained low energy excess
  - $\bar{\nu}$ mode: somewhat consistent with LSND

	$\Delta m^2_{41}[\text{eV}^2]$	$\lvert U_{e4}\rvert$	$\lvert U_{\mu 4}\rvert$	$\Delta m^2_{51}[\text{eV}^2]$	$\lvert U_{e5}\rvert$	$\lvert U_{\mu 5}\rvert$
3+2/2+3	0.47	0.128	0.165	0.87	0.138	0.148
1+3+1	0.47	0.129	0.154	0.87	0.142	0.163

or $\Delta m^2_{41} = 1.78$ eV^2 and $|U_{e4}|^2 = 0.151$

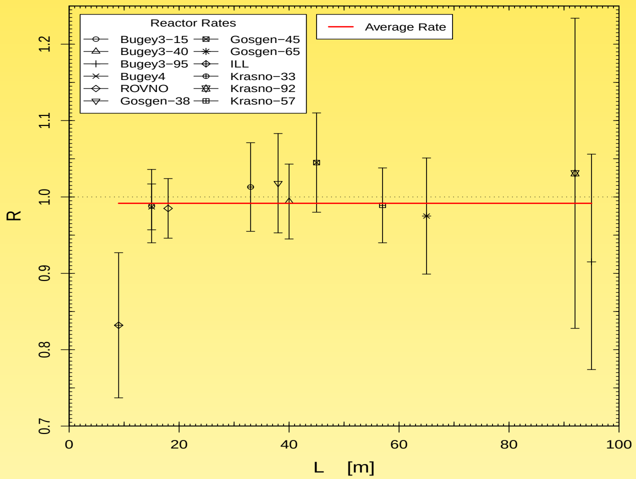
Kopp, Maltoni, Schwetz, 1103.4570

Other hints

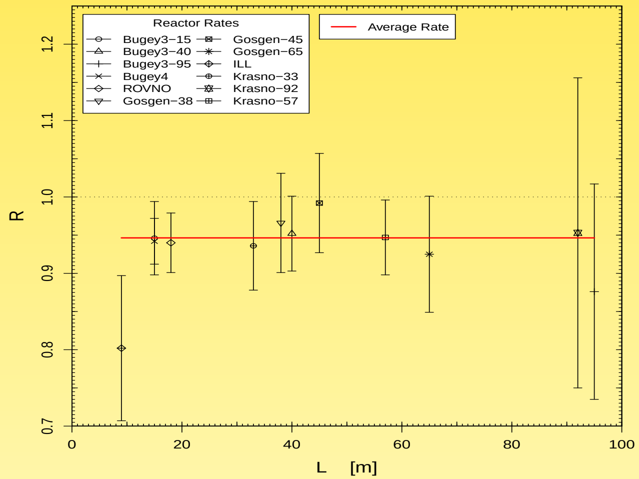
- cosmology
- BBN
- r-process nucleosynthesis in Supernovae
- reactor anomaly (Mention *et al.*, PRD **83**)

Reactor anomaly

- fission yield per isotope
- β decay branching ratios (allowed, forbidden)
- β shape (corrections: QED, weak magnetism, Coulomb)
- extraction from electron spectra



$R_{\rm old} = 0.992 \pm 0.024$



$R_{\rm new} = 0.946 \pm 0.024$

Contents

III Neutrino Mass

III1) Dirac vs. Majorana masses

Observation shows that neutrinos possess non-vanishing rest mass

Upper limits on masses imply $m_\nu \lesssim$ eV

How can we introduce neutrino mass terms?

a) Dirac masses

add $\nu_R \sim (1,1,0)$:

$$\mathcal{L}_D = g_\nu \, \overline{L} \, \tilde{\Phi} \, \nu_R \xrightarrow{\text{EWSB}} \frac{v}{\sqrt{2}} g_\nu \, \overline{\nu_L} \, \nu_R = m_\nu \, \overline{\nu_L} \, \nu_R$$

But $m_\nu \lesssim$ eV implies $g_\nu \lesssim 10^{-12} \lll g_e$

highly unsatisfactory fine-tuning...

actually, $m_e = 10^{-6}\, m_t$, so WTF?

point is that

$$\begin{pmatrix} u \\ d \end{pmatrix} \quad \text{with} \quad m_u \simeq m_d$$

has to be contrasted with

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix} \quad \text{with} \quad m_\nu \simeq 10^{-6}\, m_e$$

b) Majorana masses

need charge conjugation:

electron e^-: $[\gamma_\mu\,(i\partial^\mu + e\,A^\mu) - m]\,\psi = 0 \quad (1)$

positron e^+: $[\gamma_\mu\,(i\partial^\mu - e\,A^\mu) - m]\,\psi^c = 0 \quad (2)$

Try $\psi^c = S\,\psi^*$, evaluate $(S^*)^{-1}\,(2)^*$ and compare with (1):

$$S = i\gamma_2$$

and thus

$$\boxed{\psi^c = i\gamma_2\,\psi^* = i\gamma_2\,\gamma_0\overline{\psi}^T \equiv C\,\overline{\psi}^T}$$

flips all charge-like quantum numbers

Properties of C:

$$C^\dagger = C^T = C^{-1} = -C$$

$$C\,\gamma_\mu\,C^{-1} = -\gamma_\mu^T$$

$$C\,\gamma_5\,C^{-1} = \gamma_5^T$$

$$C\,\gamma_\mu\gamma_5\,C^{-1} = (\gamma_\mu\gamma_5)^T$$

properties of charged conjugate spinors:

$$(\psi^c)^c = \psi$$

$$\overline{\psi^c} = \psi^T\,C$$

$$\overline{\psi_1}\,\psi_2^c = \overline{\psi_2^c}\,\psi_1$$

$$\boxed{(\psi_L)^c = (\psi^c)_R}$$

$$\boxed{(\psi_R)^c = (\psi^c)_L}$$

C flips chirality: LH becomes RH

$$\mathcal{L} = m_\nu \, \overline{\nu_L} \, \nu_R + h.c. = m_\nu \left(\overline{\nu_L} \, \nu_R + \overline{\nu_R} \, \nu_L \right)$$

both chiralities a must for mass term!

Potential consequences for mass terms $\overline{\psi}\psi$:

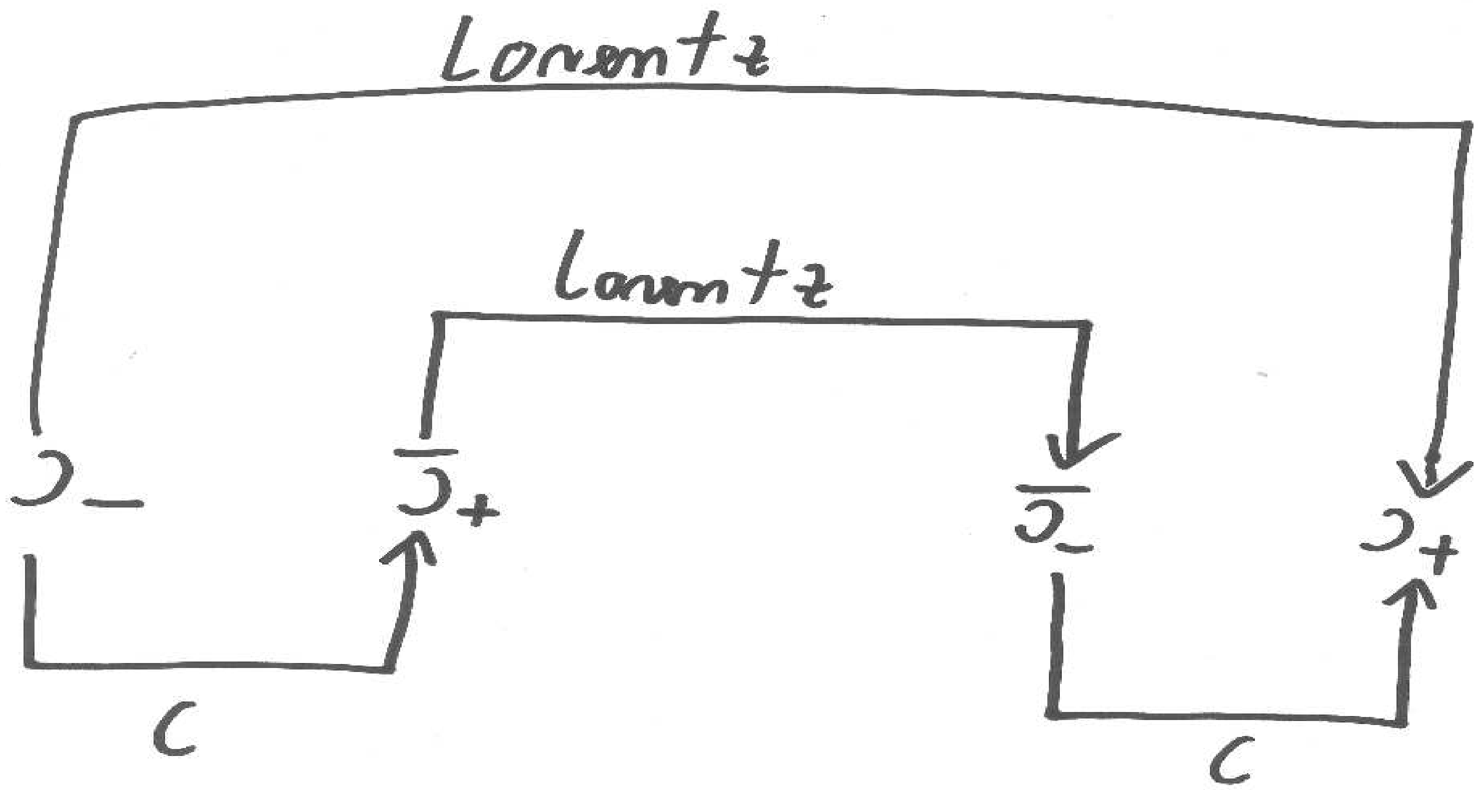
(i) ψ_L independent of ψ_R: **Dirac particle**

(ii) $\psi_L = (\psi_R)^c$: **Majorana particle**

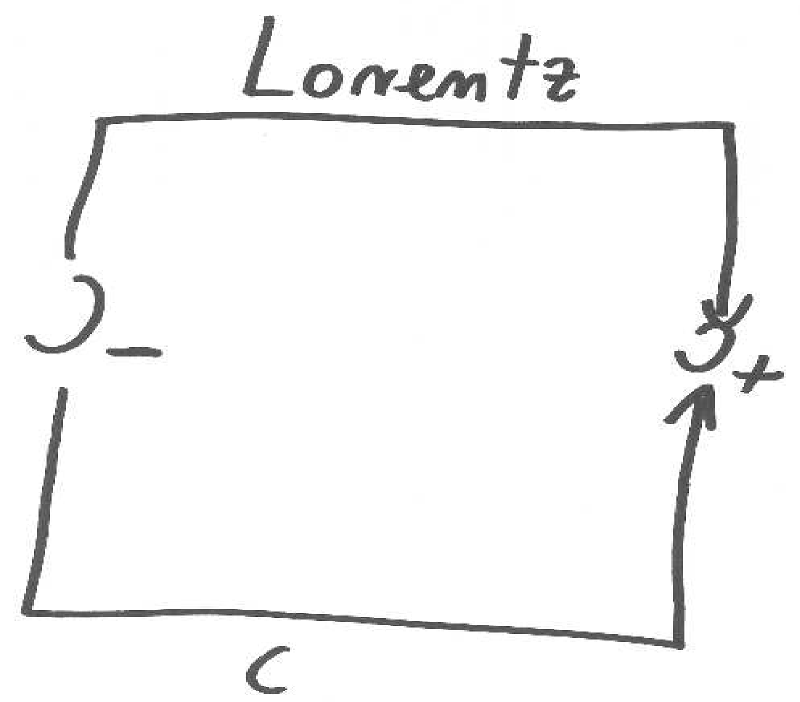
$$\Rightarrow \psi^c = (\psi_L + \psi_R)^c = (\psi_L)^c + (\psi_R)^c = \psi_R + \psi_L = \psi : \boxed{\psi^c = \psi}$$

$\Rightarrow$ Majorana fermion is identical to its antiparticle, a truly neutral particle

$\Rightarrow$ all additive quantum numbers (Q, L, B,...) are zero in terms of helicity states $\pm$: 4 d.o.f. for **Dirac particles:**



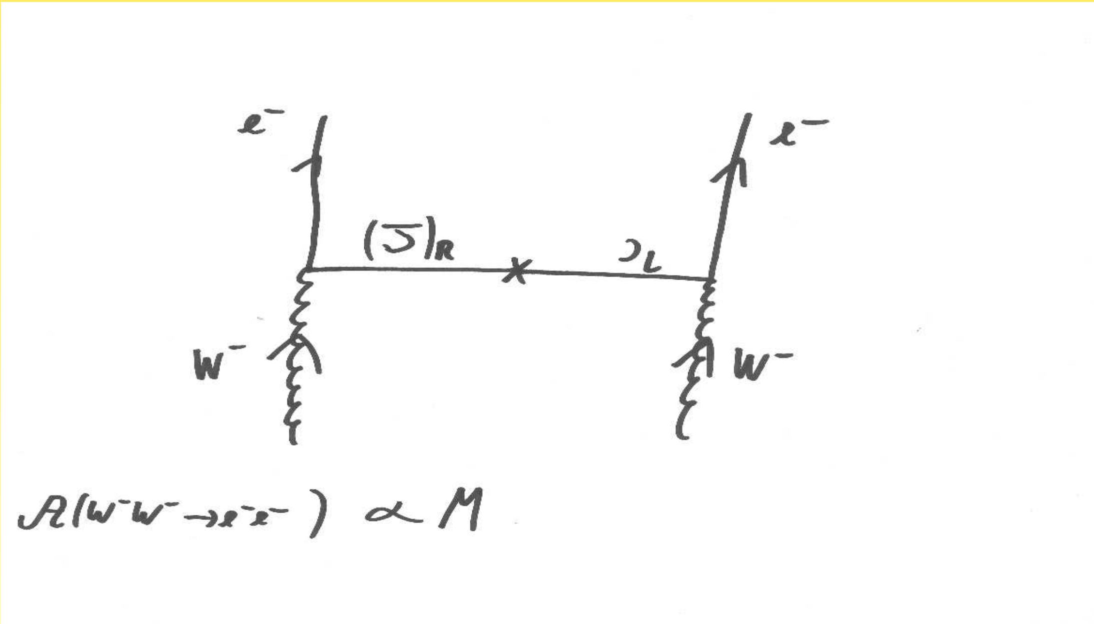
in terms of helicity states $\pm$: 2 d.o.f. for **Majorana particles:**



Mass term for Majorana particles:

$$\mathcal{L}_M = \frac{1}{2}\,\overline{\psi}\,M\,\psi = \frac{1}{2}\,\overline{\psi_L + (\psi_L)^c}\,M\,(\psi_L + (\psi_L)^c) = \tfrac{1}{2}\,\overline{\psi_L}\,M\,\psi_L^c + h.c.$$

- Majorana mass term (bare!)
- $\mathcal{L}_M \propto \psi\,\psi^T \Rightarrow$ NOT invariant under $\psi \to e^{i\alpha}\,\psi$
  $\Rightarrow$ breaks Lepton Number by 2



We should observe Lepton Number violation, right?

Sun: $4\,p + 2\,e^- \rightarrow {}^4\mathrm{He}^{++} + 2\,\nu_e$

$\longrightarrow$ we observe $\nu_e + n \rightarrow p + e^-$, but we don't observe $\overline{\nu_e} + p \rightarrow n + e^+$

$\longrightarrow$ produced neutrino is left-handed due to $V - A$

$\longrightarrow$ should be right-handed to produce e^+

$\longrightarrow$ since chirality is not a good quantum number, it can produce a small right-handed component:

$$u_\downarrow = \sqrt{\frac{E+m}{2m}} \begin{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \\ \frac{\vec{\sigma}\vec{p}}{\sqrt{E+m}} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \end{pmatrix} \Rightarrow P_R\, u_\downarrow \propto \sqrt{\frac{m}{E}}$$

"spin flip" negligibly small since $m \lesssim$ eV and $E \simeq$ MeV

Another useful property: recall $\nu = \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$

appears in Majorana mass matrix

$$\overline{\nu}\, M\, \nu^c = \overline{\nu_\alpha}\, M_{\alpha\beta}\, (\nu^c)_\beta = \overline{\nu_\alpha}\, M_{\alpha\beta}\, C\, \overline{\nu_\beta}^T = -\overline{\nu_\beta}\, C^T\, M_{\alpha\beta}\, \overline{\nu_\alpha}^T$$
$$= \overline{\nu_\beta}\, M_{\alpha\beta}\, C\, \overline{\nu_\alpha}^T = \overline{\nu_\beta}\, M_{\alpha\beta}\, (\nu^c)_\alpha = \overline{\nu_\alpha}\, M_{\beta\alpha}\, (\nu^c)_\beta$$
$$= \overline{\nu^c}\, M^T\, \nu$$

Majorana neutrino mass matrices are symmetric!

$$\Rightarrow U_\nu^\dagger\, M\, U_\nu^* = D^\nu = \mathrm{diag}(m_1, m_2, m_3)$$

Contents

III Neutrino Mass

III2) Realization of Majorana masses beyond the SM

a) Higher dimensional operators

Renormalizability: only dimension 4 terms in $\mathcal{L}$

SM has several problems $\longrightarrow$ there is a theory beyond SM, whose low energy limit is the SM $\longrightarrow$ higher dimensional operators:

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \mathcal{L}_5 + \mathcal{L}_6 + \ldots = \mathcal{L}_{\rm SM} + \frac{1}{\Lambda}\,\mathcal{O}_5 + \frac{1}{\Lambda^2}\,\mathcal{O}_6 + \ldots$$

gauge and Lorentz invariant, only SM fields:

$$\frac{1}{\Lambda}\,\mathcal{O}_5 = \frac{c}{\Lambda}\,\overline{L^c}\,\tilde{\Phi}^*\,\tilde{\Phi}^\dagger\,L \stackrel{\rm EWSB}{\longrightarrow} \frac{c\,v^2}{2\,\Lambda}\,\overline{(\nu_L)^c}\,\nu_L \equiv m_\nu\,\overline{(\nu_L)^c}\,\nu_L$$

it follows $\boxed{\Lambda \gtrsim c\left(\frac{0.1\ {\rm eV}}{m_\nu}\right)10^{14}\ \text{GeV}}$ Weinberg 1979

General remarks: $SU(2)_L \times U(1)_Y$ with $2 \otimes 2 = 3 \oplus 1$:

$$\overline{L}\,\tilde{\Phi} \sim (2,-1) \otimes (2,1) = (3,0) \oplus (1,0)$$

To make a singlet, couple $(1,0)$ or $(3,0)$, because $3 \otimes 3 = 5 \oplus 3 \oplus 1$

Alternatively:

$$\overline{L}\,L^c \sim (2,-1) \otimes (2,-1) = (3,-2) \oplus (1,-2)$$

To make a singlet, couple to $(1,2)$ or $(3,2)$. However, singlet combination is $\overline{\nu}\,\ell^c - \overline{\ell}\,\nu^c$, which cannot generate neutrino mass term

$\Longrightarrow$	$(1,0)$	or	$(3,2)$	or	$(3,0)$
	type I		type II		type III

b) Fermion singlets (type I)

introduce $N_R \sim (1,0)$ and couple to $g_\nu \overline{L}\,\tilde{\Phi} \sim (1,0)$

Hence, $g_\nu \overline{L}\,\tilde{\Phi}\,N_R$ is also singlet and becomes $g_\nu\, v/\sqrt{2}\,\overline{\nu_L}\,N_R \equiv m_D\,\overline{\nu_L}\,N_R$

in addition: Majorana mass term for N_R

$$
\begin{aligned}
\mathcal{L} &= \overline{\nu_L}\,m_D\,N_R + \tfrac{1}{2}\,\overline{N_R^c}\,M_R\,N_R + h.c. \\
&= \tfrac{1}{2}\,(\overline{\nu_L}, \overline{N_R^c}) \begin{pmatrix} 0 & m_D \\ m_D & M_R \end{pmatrix} \begin{pmatrix} \nu_L^c \\ N_R \end{pmatrix} + h.c. \\
&= \tfrac{1}{2}\,\overline{\Psi}\,\mathcal{M}_\nu\,\Psi^c + h.c.
\end{aligned}
$$

Diagonalization with $\mathcal{U}^\dagger\,\mathcal{M}_\nu\,\mathcal{U}^* = D = \mathrm{diag}(m_\nu, M)$

and

$$
\mathcal{U} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}
$$

$$\mathcal{L} = \frac{1}{2}\,(\overline{\nu_L}, \overline{N_R^c}) \begin{pmatrix} 0 & m_D \\ m_D & M_R \end{pmatrix} \begin{pmatrix} \nu_L^c \\ N_R \end{pmatrix}$$

$$= \frac{1}{2} \quad \underbrace{(\overline{\nu_L}, \overline{N_R^c})\,\mathcal{U}}_{(\overline{\nu}, \overline{N^c})} \quad \underbrace{\mathcal{U}^\dagger \begin{pmatrix} 0 & m_D \\ m_D & M_R \end{pmatrix} \mathcal{U}^*}_{\mathrm{diag}(m_\nu, M)} \quad \underbrace{\mathcal{U}^T \begin{pmatrix} \nu_L^c \\ N_R \end{pmatrix}}_{(\nu^c, N)^T}$$

with

general formula:	if $m_D \ll M_R$:
$\tan 2\theta = \frac{2\, m_D}{M_R - 0}$	$\ll 1$
$m_\nu = \frac{1}{2}\left[(0 + M_R) - \sqrt{(0 - M_R)^2 + 4\, m_D^2}\right]$	$\simeq -m_D^2/M_R$
$M = \frac{1}{2}\left[(0 + M_R) + \sqrt{(0 - M_R)^2 + 4\, m_D^2}\right]$	$\simeq M_R$

Note: m_D associated with EWSB, part of SM, bounded by $v/\sqrt{2} = 174$ GeV

M_R is SM singlet, does whatever it wants: $\Rightarrow M_R \gg m_D$

Hence, $\theta \simeq m_D/M_R \ll 1$

$\nu = \nu_L \cos\theta - N_R^c \sin\theta \simeq \nu_L$ with mass $m_\nu \simeq -m_D^2/M_R$

$N = N_R \cos\theta + \nu_L^c \sin\theta \simeq N_R$ with mass $M \simeq M_R$

in effective mass terms

$$\mathcal{L} = \frac{1}{2}\, m_\nu\, \overline{\nu}\, \nu^c + \frac{1}{2}\, M\, \overline{N^c}\, N \simeq \frac{1}{2}\, m_\nu\, \overline{\nu_L}\, \nu_L^c + \frac{1}{2}\, M_R\, \overline{N_R^c}\, N_R$$

compare with Weinberg operator:

$$\Lambda = -\frac{c\, v^2}{m_D^2}\, M_R$$

also: integrate N_R away with Euler-Lagrange equation matrix case: block diagonalization

$$\mathcal{L} = \frac{1}{2}\,(\overline{\nu_L}, \overline{N_R^c}) \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix} \begin{pmatrix} \nu_L^c \\ N_R \end{pmatrix}$$

$$= \frac{1}{2} \underbrace{(\overline{\nu_L}, \overline{N_R^c})\,\mathcal{U}}_{(\overline{\nu}, \overline{N^c})} \underbrace{\mathcal{U}^\dagger \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix} \mathcal{U}^*}_{\mathrm{diag}(m_\nu, M)} \underbrace{\mathcal{U}^T \begin{pmatrix} \nu_L^c \\ N_R \end{pmatrix}}_{(\nu^c, N)^T}$$

with 6×6 diagonal matrix

$$\mathcal{U} = \begin{pmatrix} 1 & -\rho \\ \rho^\dagger & 1 \end{pmatrix}, \quad \mathcal{U}^\dagger = \begin{pmatrix} 1 & \rho \\ -\rho^\dagger & 1 \end{pmatrix}, \quad \mathcal{U}^* = \begin{pmatrix} 1 & -\rho^* \\ \rho^T & 1 \end{pmatrix}$$

write down individual components:

write down individual components:

$$
\begin{aligned}
m_\nu &= \rho\, m_D^T + m_D\, \rho + \rho\, M_R\, \rho^T \\
0 &= m_D - \rho\, m_D^T\, \rho^* + \rho\, M_R \\
M &= -\rho^\dagger\, m_D - m_D^T\, \rho^* + M_R
\end{aligned}
$$

now, ρ (aka θ from before) will be of order m_D/M_R:

$$
\begin{aligned}
m_\nu &= \rho\, m_D^T + m_D\, \rho + \rho\, M_R\, \rho^T \\
0 &\simeq m_D + \rho\, M_R \Rightarrow \rho = -m_D\, M_R^{-1} \\
M &\simeq M_R
\end{aligned}
$$

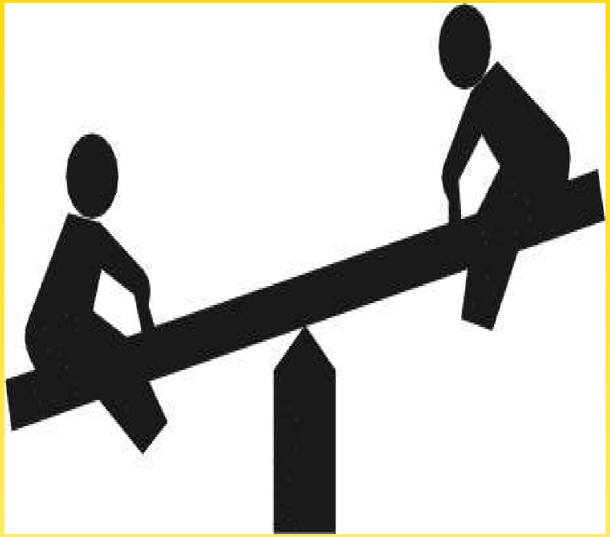
insert ρ in m_ν to find:

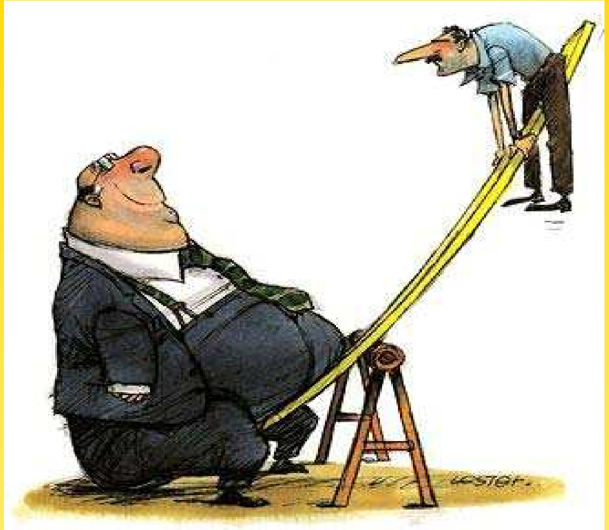
$$
m_\nu = -m_D\, M_R^{-1}\, m_D^T
$$

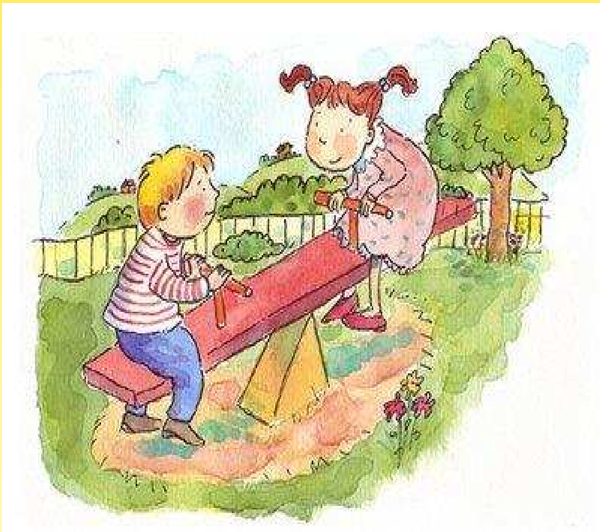
$$m_\nu = -\frac{m_D^2}{M_R}$$

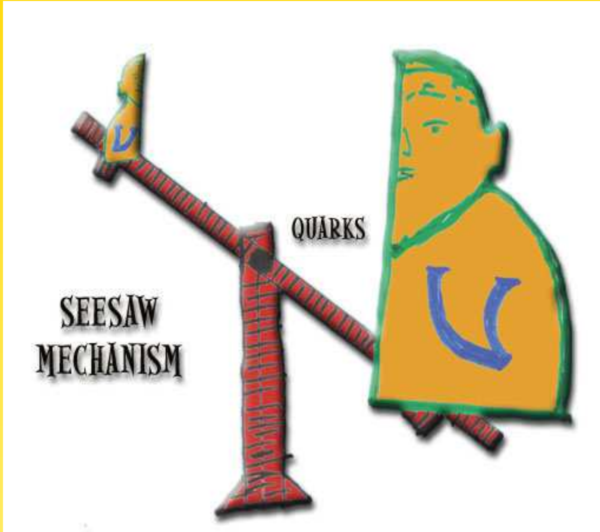
(type I) See-Saw Mechanism

Minkowski; Yanagida; Glashow; Gell-Mann, Ramond, Slansky; Mohapatra, Senjanović (77-80)








QUARKS
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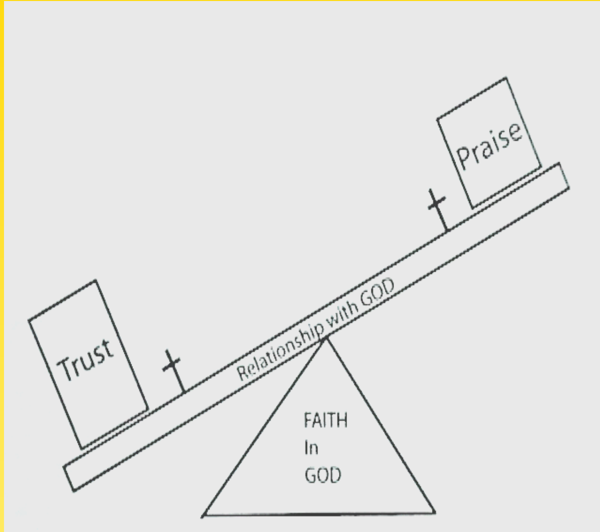
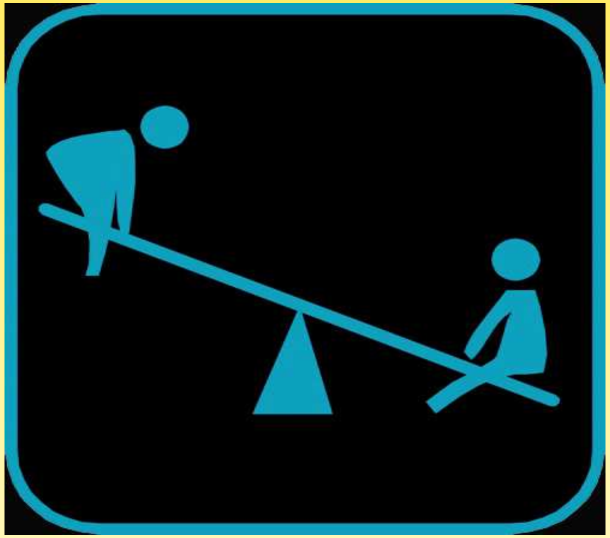
MECHANISM


SEE-SAW DON COVAY
DON COVAY SEE-SAW


THE SEESAW
Waiting


Praise
Trust
Relationship with GOD
FAITH
In
GOD



SEE SAW

See-Saw Scale

See-saw formula:

$$m_\nu = m_D^2/M_R \simeq v^2/M_R$$

with $m_\nu \simeq \sqrt{\Delta m_{\mathrm{A}}^2}$ it follows

$$\boxed{M_R \simeq 10^{15}\ \mathrm{GeV}}$$

c) Higgs triplet (type II)

$$\mathcal{L} \propto \overline{L}\, L^c \to \overline{\nu}\, \nu^*$$

has isospin $I_3 = +1$ and transforms as $\sim (3, -2)$

$\Rightarrow$ introduce Higgs triplet $\sim (3, +2)$ with $(I_3 = Q - Y/2)$:

$$\Delta = \begin{pmatrix} H^+ & \sqrt{2}\, H^{++} \\ \sqrt{2}\, H^0 & -H^+ \end{pmatrix} \to \begin{pmatrix} 0 & 0 \\ v_T & 0 \end{pmatrix}$$

with $SU(2)$ transformation property $\Delta \to U\, \Delta\, U^\dagger$:

$$\mathcal{L} = g_\nu\, \overline{L}\, i\tau_2\, \Delta\, L^c \xrightarrow{\text{vev}} g_\nu\, v_T\, \overline{\nu_L}\, \nu_L^c \equiv m_\nu\, \overline{\nu_L}\, \nu_L^c$$

Constraints on v_T

- $m_\nu = g_\nu \, v_T \lesssim$ eV $\Rightarrow v_T \lesssim \mathrm{eV}/g_\nu$
- ρ-parameter

$$\left(\frac{M_W}{M_Z \, \cos\theta_W}\right)^2 = \rho = \frac{\sum I_i \, (I_i + 1 - \frac{1}{4} \, Y_i^2) \, v_i^2}{\frac{1}{2} \, \sum v_i^2 \, Y_i^2}$$

$$= \begin{cases} 1 & I = \frac{1}{2} \text{ and } Y = 1 \\ \dfrac{v^2 + 2 \, v_T^2}{v^2 + 4 \, v_T^2} & I = 1 \text{ and } Y = 2 \end{cases}$$

$$\Rightarrow v_T \lesssim 8 \text{ GeV}$$

$v_T \ll v$ because

$$V = -M_\Delta^2 \,\mathrm{Tr}\left(\Delta\,\Delta^\dagger\right) + \mu\,\Phi^\dagger\, i\tau_2\,\Delta\,\Phi$$

with $\frac{\partial V}{\partial \Delta} = 0$ one has $\boxed{v_T = \frac{\mu\, v^2}{M_\Delta^2}}$

coupling of SM Higgs with triplet drives minimum v_T away from zero

v_T can be suppressed by M_Δ and/or μ

compare with Weinberg operator:

$$\boxed{\Lambda = \frac{c\, M_\Delta^2}{g_\nu\, \mu}}$$

Type II (or Triplet) See-Saw Mechanism

Magg, Wetterich; Mohapatra, Senjanovic; Lazarides, Shafi, Wetterich; Schechter, Valle (80-82)

d) Fermion triplets (type III)

The term

$$\mathcal{L} \propto \overline{L}\,\Sigma^c\,\tilde{\Phi}$$

is a singlet if $\Sigma \sim (3,0)$: "hyperchargeless triplets"

$$\Sigma = \begin{pmatrix} \Sigma^0/\sqrt{2} & \Sigma^+ \\ \Sigma^- & -\Sigma^0/\sqrt{2} \end{pmatrix}$$

additional terms in Lagrangian

$$\overline{L}\,\sqrt{2}\,Y_\Sigma\,\Sigma^c\,\tilde{\Phi} + \frac{1}{2}\mathrm{Tr}\left\{\overline{\Sigma}\,M_\Sigma\,\Sigma^c\right\}$$

give a "Dirac mass term" $m_D^\Sigma = v\,Y_\Sigma$ and Majorana mass term M_Σ for neutral component of Σ

overall mass term for neutrinos

$$m_\nu = -\frac{(m_D^\Sigma)^2}{M_\Sigma}$$

same structure as type I see-saw

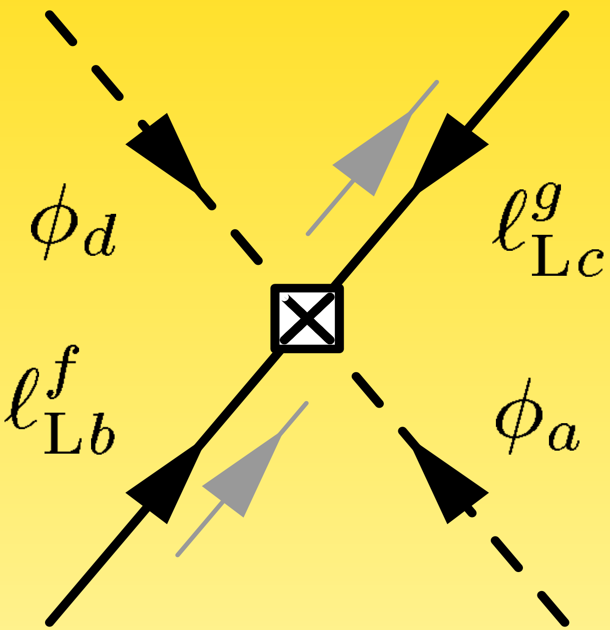
Type III See-Saw Mechanism

Foot, Lew, He, Joshi (1989)

compare with Weinberg operator:

$$\Lambda = -\frac{c\,v^2}{(m_D^\Sigma)^2}\,M_\Sigma$$

Weinberg operator is $LL\Phi\Phi$

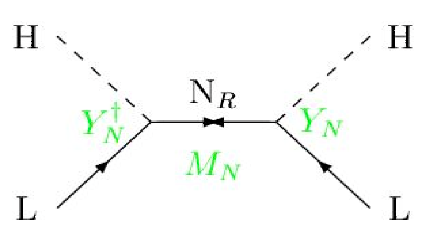


Seesaw Mechanisms are realizations of this effective operator by integrating out heavy physics:

The 3 basic seesaw models

→ i.e. tree level ways to generate the dim 5 operator

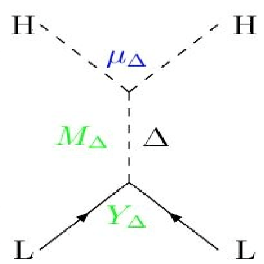
Right-handed singlet: (type-I seesaw)



$$m_\nu = Y_N^T \frac{1}{M_N} Y_N v^2$$

Minkowski; Gellman, Ramon, Slansky; Yanagida;Glashow; Mohapatra, Senjanovic

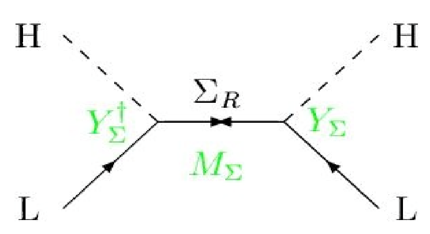
Scalar triplet: (type-II seesaw)



$$m_\nu = Y_\Delta \frac{\mu_\Delta}{M_\Delta^2} v^2$$

Magg, Wetterich; Lazarides, Shafi; Mohapatra, Senjanovic; Schechter, Valle

Fermion triplet: (type-III seesaw)



$$m_\nu = Y_\Sigma^T \frac{1}{M_\Sigma} Y_\Sigma v^2$$

Foot, Lew, He, Joshi; Ma; Ma, Roy;T.H., Lin, Notari, Papucci, Strumia; Bajc, Nemevsek, Senjanovic; Dorsner, Fileviez-Perez;....

slide by T. Hambye

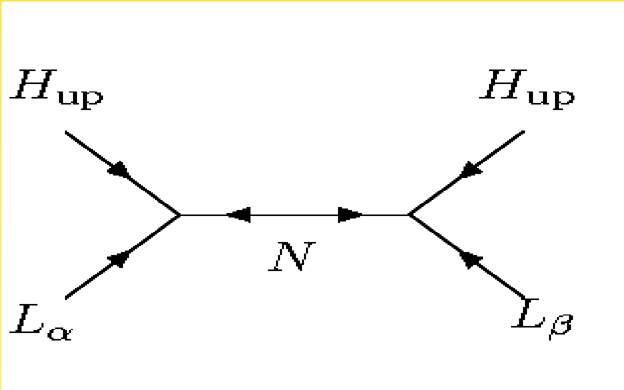
Remarks

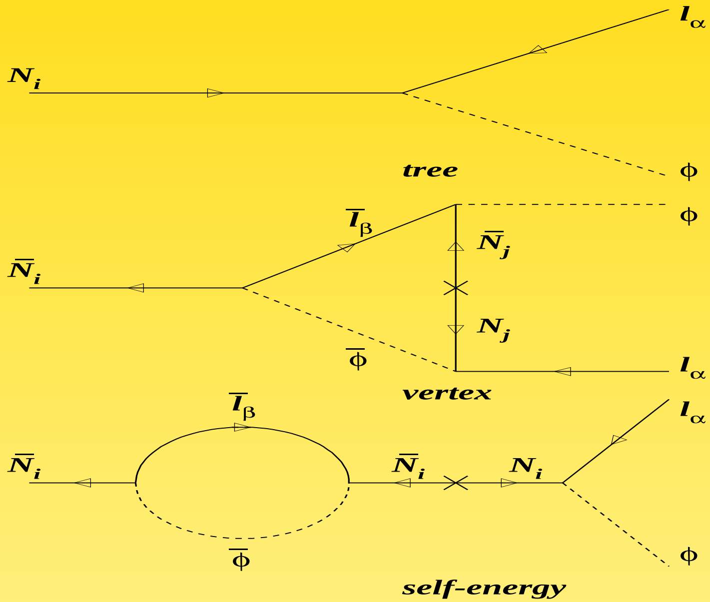
- Higgs and fermion triplets have SM charges $\Rightarrow$ coupling to gauge bosons in kinetic terms:
  - RG effects
  - production at colliders
  - FCNC
  - $\ldots$
- naturalness in GUTs: type I $\simeq$ type II $\gg$ type III
- note: one, two or three of the see-saw terms may be present in m_ν

Remarks

- Higgs and fermion triplets have SM charges $\Rightarrow$ coupling to gauge bosons in kinetic terms:
  - RG effects
  - production at colliders
  - FCNC
  - ...
- naturalness in GUTs: type I $\simeq$ type II $\gg$ type III
- note: none of the see-saw terms may be present in m_ν

Leptogenesis

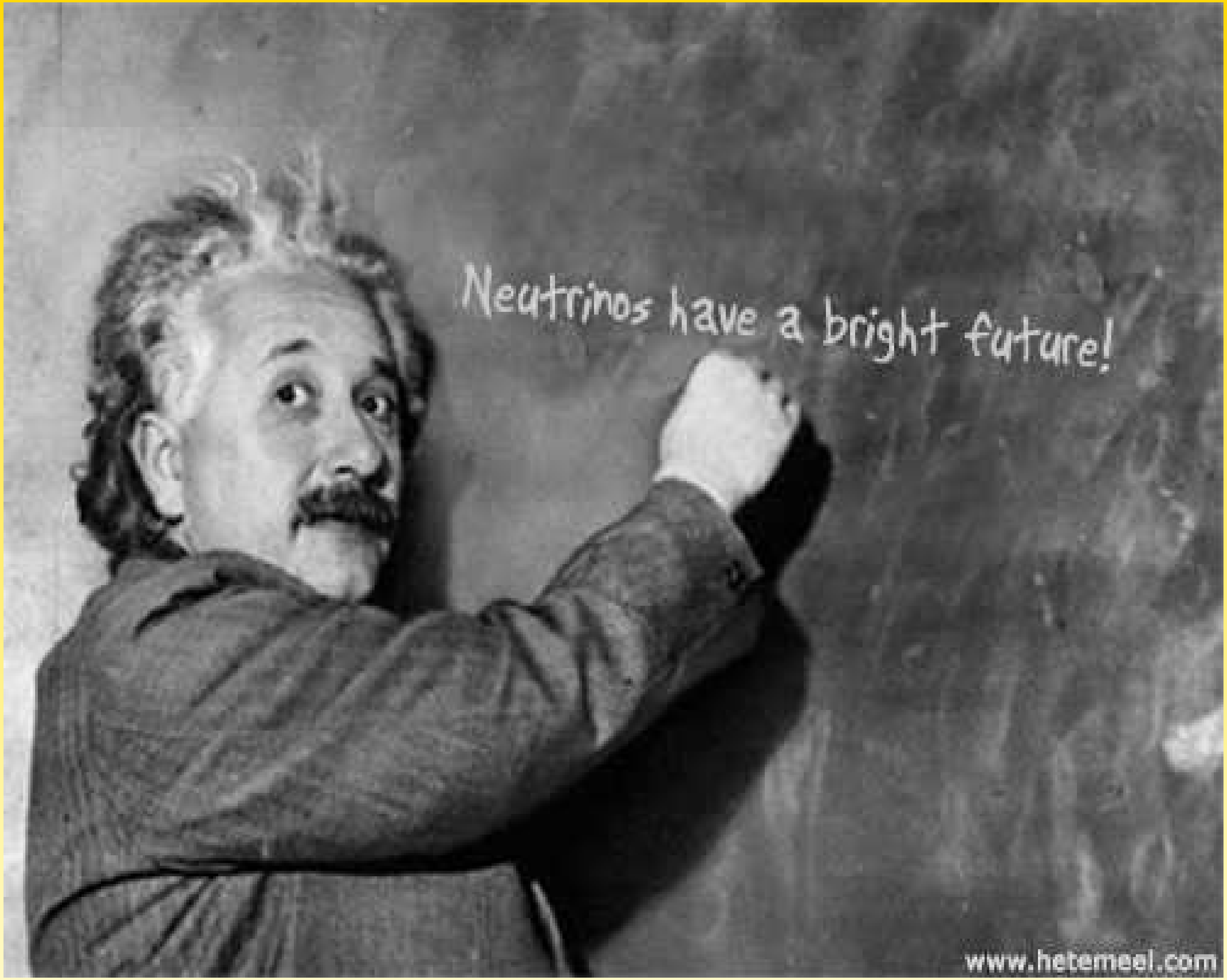




Take advantage of (L, $H_{\rm up}$, N_R) vertex in early Universe!

$$Y_B \propto \varepsilon_i^\alpha = \frac{\Gamma(N_i \to \Phi\, \bar{L}^\alpha) - \Gamma(N_i \to \Phi^\dagger\, L^\alpha)}{\Gamma(N_i \to \Phi\, \bar{L}) + \Gamma(N_i \to \Phi^\dagger\, L)}$$

Fukugita and Yanagida (1986)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