News on $B \to K^* \mu^+ \mu^-$

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B–Workshop Neckarzimmern
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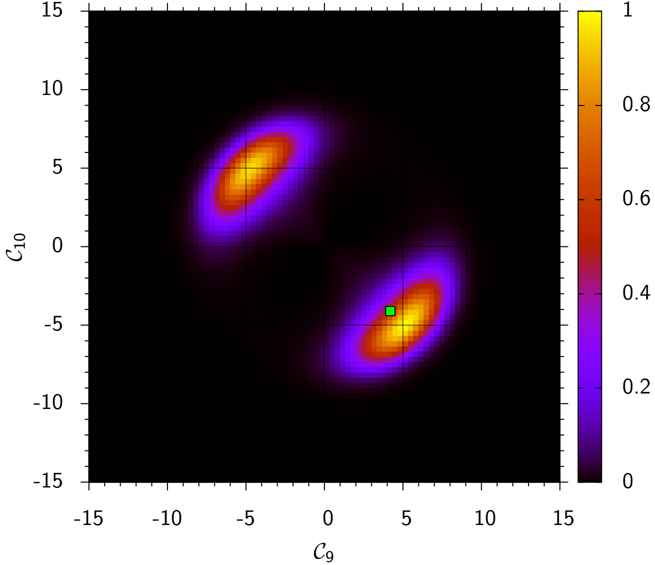
The Goal



15
10
5
0
-5
-10
-15
$\mathcal{C}_{10}$
-15
-10
-5
0
5
10
15
$\mathcal{C}_9$
1
0.8
0.6
0.4
0.2
0

Outline

Outline

From Scratch to a Partonic Matrix Element

From Quarks to Mesons

From Matrix Elements to Observables

News from Fits at Large Dimuon Invariant Mass

Motivation

Why is $B \to K^* \mu^+ \mu^-$ so damn interesting?

- Transition at quark level: $b \to s \mu^+ \mu^-$
- Flavor Changing Neutral Current (FCNC), forbidden at tree level in the Standard Model (SM)
- Needs loops, which allow heavy particles to enter virtually
- These heavy particles might stem from Beyond the SM (BSM)

Effective Theory

$$\mathcal{H}^{\text{eff}}(b \to s\mu^+\mu^-, b \to s\gamma) = -\frac{4\,G_{\mathrm{F}}}{\sqrt{2}} V_{tb} V_{ts}^* \sum_i \mathcal{C}_i(\mu)\mathcal{O}_i(\mu) + O\left(V_{ub} V_{us}^*\right)$$

- Similar to 4-Fermi interaction
- Calculate the complete theory (SM, SUSY) at a large energy scale (M_W)
- Match coefficient $\mathcal{C}_i(M_W)$ onto theory
- Calculate $\mathcal{C}_i(m_b)$ from $\mathcal{C}_i(M_W)$ via QCD running.

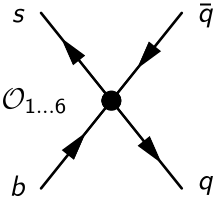
Operator Product Expansion

$$\sum_i \mathcal{C}_i(\mu)\mathcal{O}_i(\mu)$$

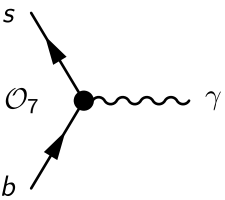
- Operators $\mathcal{O}_i$ have mass dimension 6 $\Rightarrow$ not renormalizable
- Coefficients $\mathcal{C}_i$ are called Wilson coefficients
- $\mathcal{C}_i$ are running couplings of new effective interations
- $\mathcal{C}_i$ are real numbers in this work (and in models with Minimal Flavor Violation)

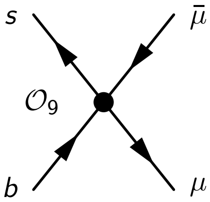
Operators

e.g. $\propto [\bar{s}\Gamma^{i}_{\mu} q]\,[\bar{q}\Gamma^{i,\mu} b]$

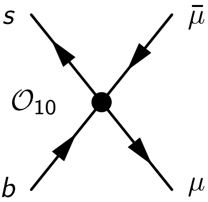


$\frac{e}{(4\pi)^2} m_b\,[\bar{s}\sigma^{\mu\nu} P_R b]\, F_{\mu\nu}$





$\frac{e^2}{(4\pi)^2}[\bar{s}\gamma_\mu P_L b]\,[\bar{\mu}\gamma^\mu \mu]$



$\frac{e^2}{(4\pi)^2}[\bar{s}\gamma_\mu P_L b]\,[\bar{\mu}\gamma^\mu \gamma_5 \mu]$

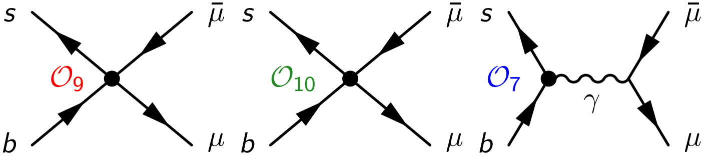
Wilson Coefficients in the SM

- Calculated from α_s, m_t, M_W and θ_W
- At $\mu = m_b \simeq 4.8$ GeV

$\mathcal{C}_1$	-0.257	$\mathcal{C}_6$	+0.001	
$\mathcal{C}_2$	+1.009	$\mathcal{C}_7$	-0.298	
$\mathcal{C}_3$	-0.005	$\mathcal{C}_8$	-0.164	
$\mathcal{C}_4$	-0.078	$\mathcal{C}_9$	+4.211	$\leftarrow$
$\mathcal{C}_5$	0	$\mathcal{C}_{10}$	-4.103	$\leftarrow$

Our First Matrix Element

For $b \to s\mu^+\mu^-$



$$
\begin{aligned}
i\mathcal{M} = \frac{G_\mathrm{F}}{\sqrt{2}} V_{tb} V_{ts}^* \frac{\alpha_e}{\pi} \times \Big\{ & \mathcal{C}_9 \; \left[\bar{s}\gamma^\mu P_L b\right] \left[\bar{\mu}\gamma_\mu \mu\right] \\
& + \mathcal{C}_{10} \; \left[\bar{s}\gamma^\mu P_L b\right] \left[\bar{\mu}\gamma_\mu \gamma_5 \mu\right] \\
& - \mathcal{C}_7 \; \frac{2 m_b q_\nu}{q^2} \left[\bar{s}\sigma^{\mu\nu} P_R b\right] \left[\bar{\mu}\gamma_\mu \mu\right] \Big\}
\end{aligned}
$$

Outline

Naive Factorization

- Factorize leptonic and hadronic parts

$$\langle \mu^+ \mu^- K^* | \mathcal{H}^{\text{eff}} | B \rangle \mapsto \langle \mu^+ | \Gamma | \mu^- \rangle \times \langle K^* | \bar{s} \Gamma b | B \rangle$$

- Take the hadronic matrix element:

$$[\bar{s} \Gamma b] \mapsto \langle K^*(k, \eta) | \bar{s} \Gamma b | B(p) \rangle$$

- Factor out the Lorentz structure, e.g.

$$\langle K^*(k, \eta) | \bar{s} \gamma_\mu b | B(p) \rangle \equiv \frac{2V(q^2)}{m_B + m_V} \varepsilon_{\mu\rho\sigma\tau} \eta^{*,\rho} p^\sigma k^\tau, \quad q \equiv p - k$$

- Form factor $V(q^2)$ covers all hadronic contributions

Hadronic Form Factors

Total of 7 form factors (reduceable by symmetries) in $B \to K^* \mu^+ \mu^-$:

$$\langle K^*(k,\eta)|\bar{s}\gamma_\mu b|B(p)\rangle = \frac{2V(q^2)}{m_B + m_V}\varepsilon_{\mu\rho\sigma\tau}\eta^{*\rho}p^\sigma k^\tau$$

$$\begin{aligned}\langle K^*(k,\eta)|\bar{s}\gamma_\mu\gamma_5 b|B(p)\rangle = i\eta^{*\rho}\Big[&2m_V A_0(q^2)\frac{q_\mu q_\rho}{q^2} + (m_B + m_V)A_1(q^2)\left(g_{\mu\rho} - \frac{q_\mu q_\rho}{q^2}\right)\\ &- A_2(q^2)\frac{q_\rho}{m_B + m_V}\left((p+k)_\mu - \frac{m_B^2 - m_V^2}{q^2}(p-k)_\mu\right)\Big]\end{aligned}$$

$$\begin{aligned}\langle K^*(k,\eta)|\bar{s}i\sigma_{\mu\nu}q^\nu b|B(p)\rangle &= -2T_1(q^2)\varepsilon_{\mu\rho\sigma\tau}\eta^{*\rho}p^\sigma k^\tau\\ \langle K^*(k,\eta)|\bar{s}i\sigma_{\mu\nu}\gamma_5 q^\nu b|B(p)\rangle &= iT_2(q^2)\left(\eta^*_\mu(m_B^2 - m_V^2) - (\eta^*\cdot q)(p+k)_\mu\right)\\ &\quad + iT_3(q^2)\left(\eta^*\cdot q\right)\left(q_\mu - \frac{q^2}{m_B^2 - m_V^2}(p+k)_\mu\right)\end{aligned}$$

OPE in $1/\sqrt{q^2}$

- So far: Operator Product Expansion (OPE) in $1/M_W^2 \sim G_\mathrm{F}$
- Now additionally: OPE in $1/m_b$ and $1/\sqrt{q^2}$
- $q^2 =$ dilepton invariant mass, $4m_\mu^2 \leq q^2 \leq (M_B - M_{K^*})^2$

- Uses Heavy Quark Effective Theory (HQET)
- Result: Systematic approach, model independent up to $\Lambda/m_b, \Lambda/\sqrt{q^2}, m_c^2/m_b^2, m_c^2/q^2$ (B. Grinstein, D. Pirjol '04)
- Valid for large $q^2 \sim O(m_b^2)$

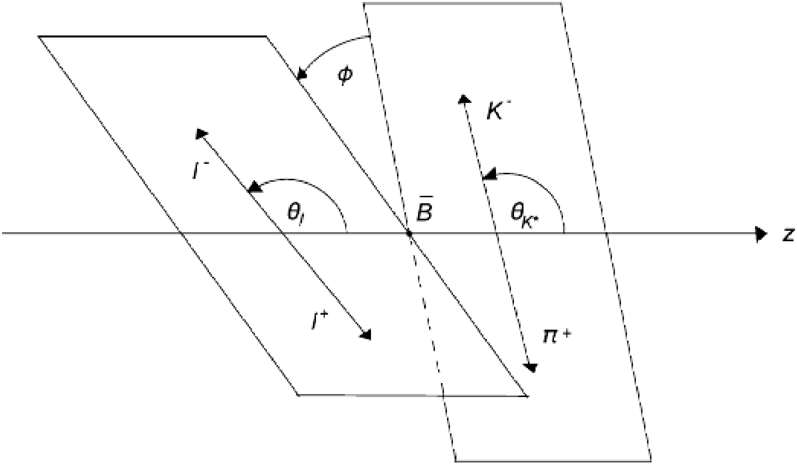
Meson Matrix Elements

$$
\begin{aligned}
i\mathcal{M}(B \to K^*\mu^+\mu^-) = {} & \frac{G_\mathrm{F}}{\sqrt{2}} V_{tb} V_{ts}^* \frac{\alpha_e}{\pi} \\
& \times \Big\{ \mathcal{C}_9^\mathrm{eff} \; \langle K^*(k,\eta)|\bar{s}\gamma^\mu P_L b|B(p)\rangle \, [\bar{\mu}\gamma_\mu\mu] \\
& + \mathcal{C}_{10} \; \langle K^*(k,\eta)|\bar{s}\gamma^\mu P_L b|B(p)\rangle \, [\bar{\mu}\gamma_\mu\gamma_5\mu] \\
& - \mathcal{C}_7^\mathrm{eff} \; \frac{2 m_b q_\nu}{q^2} \langle K^*(k,\eta)|\bar{s}\sigma^{\mu\nu} P_R b|B(p)\rangle \, [\bar{\mu}\gamma_\mu\mu] \Big\}
\end{aligned}
$$

Outline

Kinematics

$$B \to K^*(\to K\pi)\, q(\to \mu^+\mu^-)$$



To $B \to K^*(\to K\pi)\,\mu^+\mu^-$

- Depends on 4 kinematic variables: $q^2, \theta_l, \theta_{K^*}, \phi$
- Differential decay width is 4-differential:

$$\frac{\mathrm{d}^4\Gamma}{\mathrm{d}q^2\,\mathrm{d}\cos\theta_l\,\mathrm{d}\cos\theta_{K^*}\,\mathrm{d}\phi} = \frac{3}{8\pi} J(q^2, \theta_l, \theta_{K^*}, \phi)$$

- $J(q^2, \theta_l, \theta_{K^*}, \phi)$ can be expanded in the angles

$$J(q^2, \theta_l, \theta_{K^*}, \phi) = \sum J_i(q^2) f_i(\theta_l, \theta_{K^*}, \phi)$$

- Total of 9 angular coefficients: $J_1, \ldots, J_9$
- J_i can be expressed through transversity amplitudes $A_{0,\perp,\parallel}$

Transversity Amplitudes

- What are $A_{0,\perp,\parallel}$?

1. Project out helicity amplitudes

$$\mathcal{M} = \eta^{*\mu}_{K^*}(\lambda) L^\nu \mathcal{M}_{\mu\nu} \quad \xrightarrow{L\to\eta_q} \quad H_\lambda \equiv \eta^{*\mu}_{K^*}(\lambda)\eta^{*\nu}_q(\lambda)\mathcal{M}_{\mu\nu}$$

L^ν: Lepton current

2. Build linear combinations of helicity amplitudes $H_{0,\pm}$

$$A_{\perp,\parallel} = (H_+ \mp H_-)/\sqrt{2} \qquad A_0 = H_0$$

(Signs may change depending upon convention)

Differential Decay Width / Longitudinal Fraction

Differential Decay Width $\mathrm{d}\Gamma/\mathrm{d}q^2$:

(For massless leptons)

$$\mathrm{d}\Gamma/\mathrm{d}q^2 = J_1 - \frac{J_2}{3} = |A_\perp|^2 + |A_\parallel|^2 + |A_0|^2$$

Longitudinal Fraction F_L:

Longitudinal K^* versus total K^*

$$F_L = \frac{|A_0|^2}{\mathrm{d}\Gamma/\mathrm{d}q^2}$$

Forward-Backward Asymmetry

- Definition

$$A_{FB} = \frac{1}{\mathrm{d}\Gamma/\mathrm{d}q^2}\left(\int_0^{+1} \mathrm{d}\cos\theta_l \frac{\mathrm{d}^2\Gamma}{\mathrm{d}q^2\mathrm{d}\cos\theta_l} - \int_0^{-1} \mathrm{d}\cos\theta_l \frac{\mathrm{d}^2\Gamma}{\mathrm{d}q^2\mathrm{d}\cos\theta_l}\right)$$

- Can be written as

$$A_{FB} = \frac{J_6}{\mathrm{d}\Gamma/\mathrm{d}q^2}, \quad J_6 \sim \mathrm{Re}\left\{A_\parallel A_\perp\right\}$$

$$A_{FB} \propto \frac{(\mathcal{C}_9 + \kappa\frac{m_b^2}{q^2}\mathcal{C}_7)\,\mathcal{C}_{10}^*}{(\mathcal{C}_9 + \kappa\frac{m_b^2}{q^2}\mathcal{C}_7)^2 + \mathcal{C}_{10}^2}$$

- At $q^2 \sim m_b^2$: Tests if $\mathcal{C}_9\,\mathcal{C}_{10}^*$ is SM-like

Data (as of February 2010)

- Measured are: $\mathrm{d}\Gamma/\mathrm{d}q^2$, A_{FB} and F_L
- Measured by: BaBar, Belle and CDF
- Few to very few bins with few events per bin
- Theorists want: Measurement of all J_i

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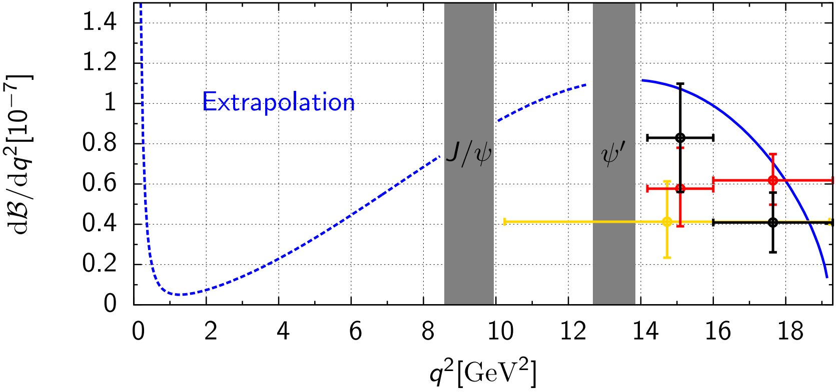
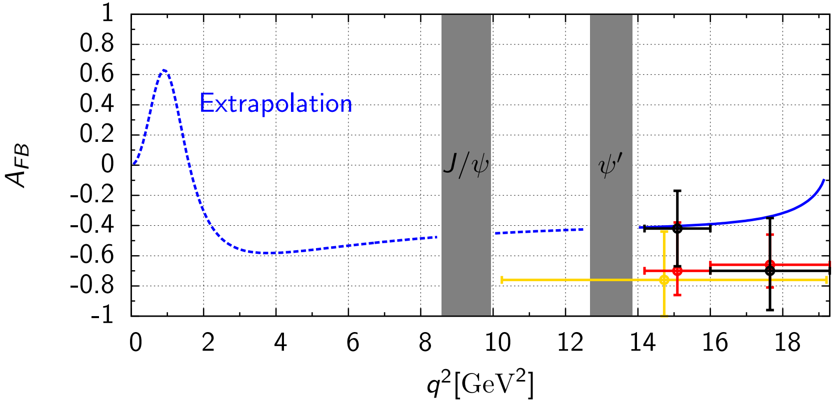
$\mathrm{d}\mathcal{B}/\mathrm{d}q^2(B \to K^* l^+ l^-), l = e, \mu$

Preliminary Result for $\mathrm{d}\mathcal{B}/\mathrm{d}q^2$ at Low Recoil, BaBar'06, Belle'09, CDF'09



$A_{FB}(B \to K^* l^+ l^-), l = e, \mu$

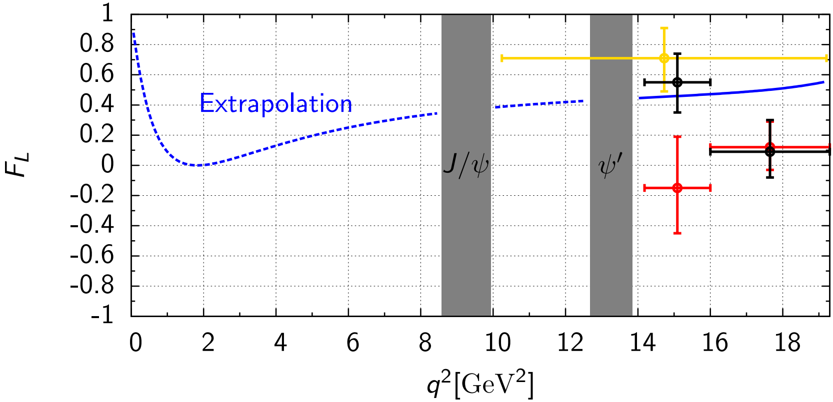
Preliminary Result for A_{FB} at Low Recoil, BaBar'08, Belle'09, CDF'09



$\mathcal{C}_9 \mathcal{C}_{10}^*$ is SM-like

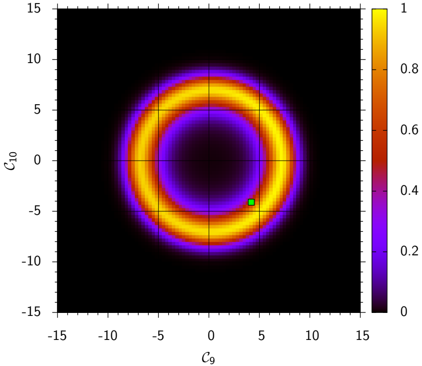
$F_L(B \to K^* l^+ l^-), l = e, \mu$

Preliminary Result for F_L at Low Recoil, BaBar'08, Belle'09, CDF'09

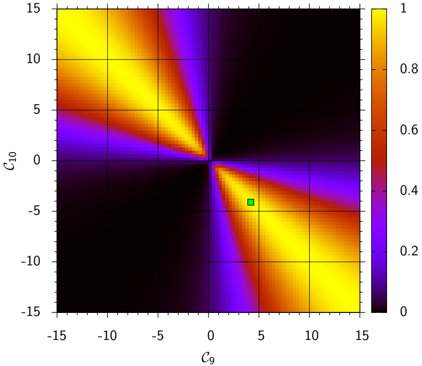


The Result

Preliminary Result: Scan for $\mathcal{C}_{9,10}$, plotted is $\mathcal{L}$ normalized to max.



From $\mathrm{d}\mathcal{B}/\mathrm{d}q^2$, Belle+CDF

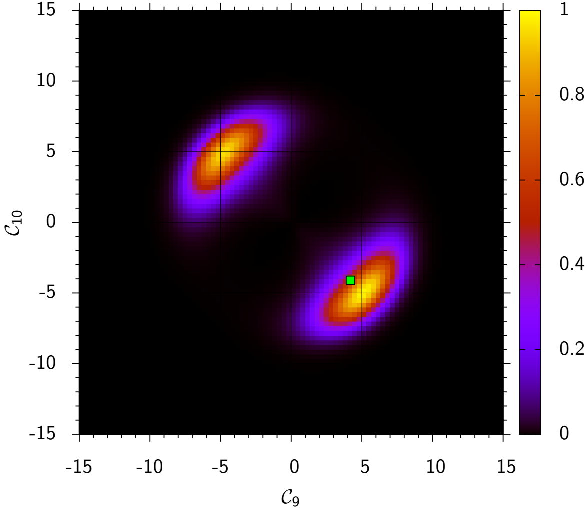


From A_{FB}, Belle+CDF

□: SM values

The Result

Preliminary Result: Scan for $\mathcal{C}_{9,10}$, plotted is $\mathcal{L}$ normalized to max.



Conclusion

- Large q^2 can be explored
- We calculated J_i, $A_{0,\perp,\parallel}$ for large q^2
- $\mathrm{sign}\,(\mathcal{C}_9\,\mathcal{C}_{10}^*)$ could be determined; is SM-like
- More data and orthogonal data needed ...
- Enhanced precisions needed ...
- ... to test the SM
- ... to see where it fails

Literature

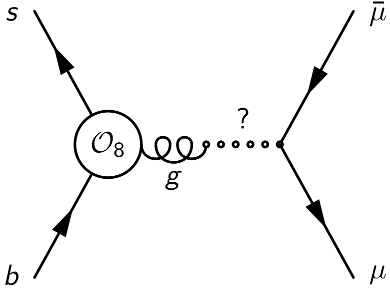
Literature on this decay includes (amongst others)

- Overview (*W. Altmannshofer et al*) `arxiv:0811.1214 [hep-ph]`
- Overview (*A. Ali et al*) `arxiv:hep-ph/9910221`
- CP Asymmetries (*C. Bobeth et al*) `arxiv:0805.2525 [hep-ph]`
- $1/\sqrt{q^2}$ Expansion (*B. Grinstein, D. Pirjol*) `arxiv:hep-ph/0404250`

Outline

Backup Slides

Why no $\mathcal{O}_8$?



- $\mu\bar{\mu}$ is colorless
- g carries color!

QCD Factorization (QCDF) + Soft Collinear Effective Theory (SCET)

- Replace $\mathcal{C}_7\, T_i(q^2) \mapsto \mathcal{T}_i(q^2), \mathcal{C}_9^{\text{eff}}(q^2) \mapsto \mathcal{C}_9$
- All $q\bar{q}$-loop contributions can now be resorbed into $\mathcal{T}_i(q^2)$
- Applicable at $q^2 \leq 6 - 7\,\mathrm{GeV}^2$
- At leading order only 2 form factors $\xi_{\perp,\parallel}$

Form Factor Values

Small q^2 (Large Recoil)

- 2 independent form factors in QCD Factorization (QCDF) + Soft Collinear Effective Theory(SCET)
- Obtained from Light Cone Sum Rules (LCSR) and consequent fits
- Uncertainties of $\sim 10 - 20\%$ (theory and input)
- In agreement with measurements for $q^2 = 0$

Large q^2 (Low Recoil)

- 4 independent form factors at LO
- Obtained from Lattice QCD calculations

Non-factorizable contributions

- Operators $\mathcal{O}_{1\dots6}$ are not factorizable
- Example

$$[\bar{s}\Gamma_\mu b][\bar{q}\Gamma^\mu q]$$

- $\mathcal{O}_{7,9,10}$ are completely factorizable!
- In $B \to K^{(*)}\mu^+\mu^-$ most non-factorizable contributions only appear at NLO in α_s.
- Resorb non-factorizable contributions into *effective* Wilson coefficients $\mathcal{C}_{7,9}^{\text{eff}}$