Heavy-Quark Expansion

(for exclusive B decays)

Thorsten Feldmann

Neckarzimmern, February 2010

Outline

1. Introduction
2. The effective Weak Hamiltonian
   - Example: $b \to c d \bar{u}$ decay
   - Other cases
3. Hadronic matrix elements for exclusive B-meson decays
   - $B \to D^{(*)}$ form factors $\to$ HQET
   - Non-leptonic $B \to D\pi$ decays $\to$ QCDF
   - Non-leptonic $B \to \pi\pi$ and $B \to \pi K$ decays $\to$ QCDF
   - $B \to K^*$ form factors $\to$ SCET
   - QCDF corrections to $B \to K^* \mu^+ \mu^-$

Overview: Phenomenological Aspects of B-Physics

Semi-leptonic decays:

Extraction of $|V_{cb}|$ and $|V_{ub}|$ from $b \to c\ell\nu$, $b \to u\ell\nu$.

- Exclusive decays: Needs $B \to D^{(*)}$ and $B \to \pi(\rho)$ form factors.
  (HQET symmetries, QCD sum rules, Lattice QCD, ...)
- [Inclusive decays: study moments of decay spectra ...]

CP Asymmetries in "Golden Channels":

E.g. time-dependent CP asymmetry in $B_d \to J/\psi K_S$:

- Dominated by B_d–$\bar{B}_d$ mixing phase;
  Pollution from $b \to sq\bar{q}$ penguins CKM-suppressed;

$\to$ Theoretically clean determination of CKM angle $\sin 2\beta$

- Open issue: *How small* are corrections from penguin pollution?
  (experimental data + flavour symmetry of QCD [Faller et al. 2009], ...)

Overview: Phenomenological Aspects of B-Physics

CP asymmetries in "penguin-polluted/dominated" channels:

e.g. $B \to \phi K_S$, ..., $B \to \pi K$, ...

- Theoretical estimate of *P*enguin/*T*ree ratio (QCD Factorization [Beneke et al.], ...)
- $+$ Experimental data $+$ Isospin/$SU(3)$ symmetry.

Radiative decays: $b \to s(d)\gamma$, $b \to s(d)\ell^+\ell^-$

Particularly sensitive to New Physics contributions:

- Exclusive decay modes, e.g. $B \to K^*\mu^+\mu^-$:
  - Hadronic transition form factors for vector/tensor currents. (SCET symmetries, Light-cone sum rules, Lattice-QCD, ...)
  - "Non-factorizable" effects from long-distance photon radiation. (QCD factorization [Beneke/TF/Seidel], ...)
  - Part of hadronic uncertainties drops out in decay asymmetries (forward-backward, isospin, CP, ...)
  - Angular analysis of $K^* \to K\pi$ helpful [Egede et al., Altmannshofer et al., ...]

...Some introductory remarks ...

- Physical processes involve
  different typical energy/length scales
- Separate short-distance and long-distance effects:

New physics	:	$\delta x \sim 1/\Lambda_{\mathrm{NP}}$
Electroweak interactions	:	$\delta x \sim 1/M_W$
Short-distance QCD(QED) corrections	:	$\delta x \sim 1/M_W \to 1/m_b$
Hadronic effects	:	$\delta x > 1/m_b$

$\longrightarrow$ Sequence of Effective Theories.

$\longrightarrow$ Short-distance coefficients (functions)
(calculable in RG-improved perturbation theory)

$\longrightarrow$ Non-perturbative hadronic input parameters (functions)

Central Notions to be explained

<u>Disclaimer:</u>

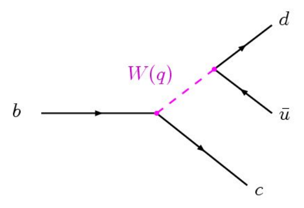
The dynamics of strong interactions in B-decays is very complex and has many faces. I will not be able to cover everything, but I hope that some theoretical and phenomenological concepts become clearer . . .

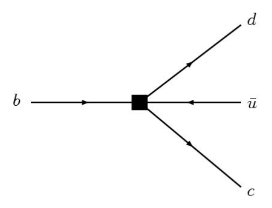
- *Separation of scales* in perturbation theory (Effective Weak Hamiltonian)
- *QCD Factorization* for exclusive hadronic matrix elements
- *Heavy quark effective theory* + Isgur-Wise form factors
- *Soft collinear effective theory* + Symmetry relations large recoil
- Limitations of the heavy quark expansion

Outline

$b \to c d \bar{u}$ decay at Born level

Full theory (SM) $\longrightarrow$ Fermi model





$$\left(\frac{g}{2\sqrt{2}}\right)^2 J_\alpha^{(b\to c)} \frac{-g^{\alpha\beta} + \frac{q^\alpha q^\beta}{M_W^2}}{q^2 - M_W^2} \bar{J}_\beta^{(d\to u)} \quad \xrightarrow{|q| \ll M_W} \quad \frac{G_F}{\sqrt{2}} J_\alpha^{(b\to c)} g^{\alpha\beta} \bar{J}_\beta^{(d\to u)}$$

- Energy/Momentum transfer limited by mass of decaying *b*-quark.
- *b*-quark mass much smaller than *W*-boson mass.

$$\boxed{|q| \leq m_b \ll m_W}$$

Effective Theory:

- Analogously to muon decay, transition described in terms of current-current interaction, with left-handed charged currents

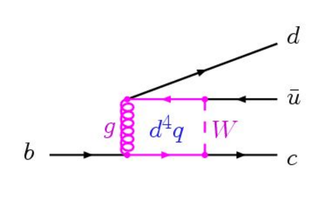
$$J_\alpha^{(b\to c)} = V_{cb}\,[\bar{c}\,\gamma_\alpha(1-\gamma_5)\,b]\;, \qquad \overline{J}_\beta^{(d\to u)} = V_{ud}^*\left[\bar{d}\,\gamma_\beta(1-\gamma_5)\,u\right]$$

- Effective operators only contain light fields
  ("light" quarks, electron, neutrinos, gluons, photons). $\surd$

- Effect of large scale M_W in effective Fermi coupling constant:

$$\boxed{\frac{g^2}{8M_W^2} \longrightarrow \frac{G_F}{\sqrt{2}} \simeq 1.16639 \cdot 10^{-5}\,\mathrm{GeV}^{-2}}$$

Quantum-loop corrections to $b \to c d \bar{u}$ decay



- Momentum q of the W-boson is an internal loop parameter that is integrated over and can take values between $-\infty$ and $+\infty$.

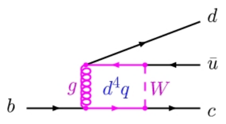
$\Rightarrow$ We cannot simply expand in $|q|/M_W$!

$\Rightarrow$ Need a method to separate the cases $|q| \geq M_W$ and $|q| \ll M_W$.

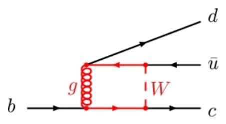
$\rightarrow$ Factorization / Effective Hamiltonian

IR and UV regions in the Effective Theory

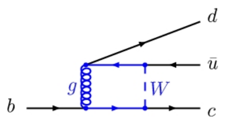
full theory = IR region $(M_W \to \infty)$ + UV region $(m_{b,c} \to 0)$



=



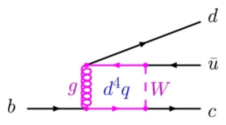
+



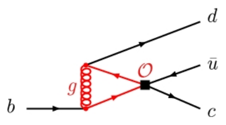
$$I(\alpha_s;\ \frac{m_b}{M_W},\ \frac{m_c}{m_b})/G_F \simeq I_{IR}(\alpha_s;\ \frac{m_b}{\mu},\ \frac{m_c}{m_b}) + I_{UV}(\alpha_s;\ \frac{\mu}{m_W})$$

IR and UV regions in the Effective Theory

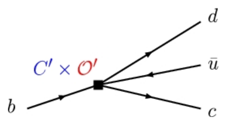
full theory $=$ IR region $(M_W \to \infty)$ $+$ UV region $(m_{b,c} \to 0)$



$\simeq$



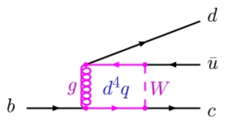
$+$



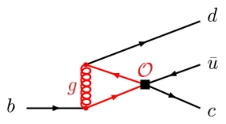
$$I(\alpha_s;\ \frac{m_b}{M_W},\ \frac{m_c}{m_b})/G_F \simeq \langle\mathcal{O}\rangle^{\rm loop}(\alpha_s;\ \frac{m_b}{\mu},\ \frac{m_c}{m_b}) + C'(\alpha_s;\ \frac{\mu}{m_W}) \times \langle\mathcal{O}'\rangle^{\rm tree}$$

IR and UV regions in the Effective Theory

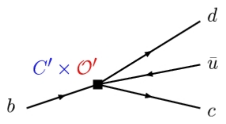
full theory $=$ IR region $(M_W \to \infty)$ $+$ UV region $(m_{b,c} \to 0)$



$\simeq$



$+$



$$I(\alpha_s;\, \frac{m_b}{M_w},\, \frac{m_c}{m_b})/G_F \simeq \langle \mathcal{O} \rangle^{\rm loop}(\alpha_s;\, \frac{m_b}{\mu},\, \frac{m_c}{m_b}) + C'(\alpha_s;\, \frac{\mu}{m_W}) \times \langle \mathcal{O}' \rangle^{\rm tree}$$

$\updownarrow$

1-loop matrix element of operator $\mathcal{O}$ in Eff. Th.

- independent of M_W
- UV divergent $\to \mu$

$\updownarrow$

1-loop coefficient for new operator $\mathcal{O}'$ in ET

- independent of $m_{b,c}$
- IR divergent $\to \mu$

Effective Operators for $b \to c\bar{u}$

- short-distance QCD corrections preserve chirality;
- quark-gluon vertices induce second colour structure.

$$H_{\rm eff} = \frac{4G_F}{\sqrt{2}}\, V_{cb} V_{ud}^* \sum_{i=1,2} C_i(\mu)\, \mathcal{O}_i + \text{h.c.} \qquad (b \to c d\bar{u})$$

- Current-current operators: $(b \to cd\bar{u},$ analogously for $b \to qq'\bar{q}''$ decays)

$$\begin{aligned} \mathcal{O}_1 &= (\bar{d}_L^i \gamma_\alpha u_L^j)(\bar{c}_L^j \gamma^\alpha b_L^i) \\ \mathcal{O}_2 &= (\bar{d}_L^i \gamma_\alpha u_L^i)(\bar{c}_L^j \gamma^\alpha b_L^j) \end{aligned}$$

- The so-called Wilson coefficients $C_i(\mu)$ contain all information about short-distance physics above the scale μ

Wilson Coefficients in Perturbation Theory

- 1-loop result:

$$C_i(\mu) = \left\{ \begin{array}{c} 0 \\ 1 \end{array} \right\} + \frac{\alpha_s(\mu)}{4\pi} \left(\ln \frac{\mu^2}{M_W^2} + \frac{11}{6} \right) \left\{ \begin{array}{c} 3 \\ -1 \end{array} \right\} + \mathcal{O}(\alpha_s^2)$$

Question : How do we choose the renormalization scale μ ?

Wilson Coefficients in Perturbation Theory

- 1-loop result:

$$C_i(\mu) = \left\{ \begin{array}{c} 0 \\ 1 \end{array} \right\} + \frac{\alpha_s(\mu)}{4\pi} \left(\ln \frac{\mu^2}{M_W^2} + \frac{11}{6} \right) \left\{ \begin{array}{c} 3 \\ -1 \end{array} \right\} + \mathcal{O}(\alpha_s^2)$$

Question : How do we choose the renormalization scale μ ?

"Matching"

Answer : For $\mu \sim M_W$ the logarithmic term is small, and $C_i(M_W)$ can be calculated in fixed-order perturbation theory, since $\frac{\alpha_s(M_W)}{\pi} \ll 1$.

Here M_W is called the matching scale.

Anomalous Dimensions

- In order to compare with experiment / hadronic models, the matrix elements of ET operators are needed at low-energy scale $\mu \sim m_b$
  - Only the combination
$$\sum_i C_i(\mu)\,\langle \mathcal{O}_i \rangle(\mu)$$
is μ-independent (in perturbation theory).

  $\Rightarrow$ Need Wilson coefficients at low scale !

- Scale dependence can be calculated in perturbation theory:
  - Loop diagrams in ET are UV divergent $\Rightarrow$ anomalous dimensions (matrix):
$$\frac{\partial}{\partial \ln \mu}\, C_i(\mu) \equiv \gamma_{ji}(\mu)\, C_j(\mu) \;=\; \left(\frac{\alpha_s(\mu)}{4\pi}\, \gamma_{ji}^{(1)} + \ldots \right) C_j(\mu)$$
  - $\gamma = \gamma(\alpha_s)$ has a perturbative expansion.

RG Evolution ("running")

In our case:

$$\gamma^{(1)} = \begin{pmatrix} -2 & 6 \\ 6 & -2 \end{pmatrix} \qquad \begin{cases} \text{Eigenvectors: } C_\pm = \frac{1}{\sqrt{2}}(C_2 \pm C_1) \\ \text{Eigenvalues: } \gamma_\pm^{(1)} = +4, \ -8 \end{cases}$$

- Formal solution of differential equation: (separation of variables)

$$\ln \frac{C_\pm(\mu)}{C_\pm(M)} = \int_{\ln M}^{\ln \mu} d\ln\mu' \, \gamma_\pm(\mu') = \int_{\alpha_s(M)}^{\alpha_s(\mu)} \frac{d\alpha_s}{2\beta(\alpha_s)} \gamma_\pm(\alpha_s)$$

- Perturbative expansion of anomalous dimension and β-function:

$$\gamma = \frac{\alpha_s}{4\pi} \gamma^{(1)} + \ldots , \qquad 2\beta \equiv \frac{d\alpha_s}{d\ln\mu} = -\frac{2\beta_0}{4\pi} \alpha_s^2 + \ldots$$

$$C_\pm(\mu) \simeq C_\pm(M_W) \cdot \left(\frac{\alpha_s(\mu)}{\alpha_s(M_W)}\right)^{-\gamma_\pm^{(1)}/2\beta_0} \qquad \text{(LeadingLogApprox)}$$

Numerical values for $C_{1,2}$ in the SM

[Buchalla/Buras/Lautenbacher 96]

operator:	$\mathcal{O}_1 = (\bar{d}_L^i \gamma_\mu u_L^i)(\bar{c}_L^j \gamma^\mu b_L^j)$	$\mathcal{O}_2 = (\bar{d}_L^i \gamma_\mu T u_L^i)(\bar{c}_L^j \gamma^\mu b_L^j)$
$C_i(m_b)$:	-0.514 (LL) -0.303 (NLL)	1.026 (LL) 1.008 (NLL)

(modulo parametric uncertainties from M_W, m_b, $\alpha_s(M_Z)$ and QED corr.)

(potential) New Physics modifications:

- new left-handed interactions (incl. new phases)

$$C_{1,2}(M_W) \to C_{1,2}(M_W) + \delta_{\rm NP}(M_W, M_{\rm NP})$$

- new chiral structures ⇒ extend operator basis (LR,RR currents)

Outline

Other Cases

$b \to s(d)\, q\bar{q}$ decays

- Current–current operators (as above):
  - Now, *two possible flavour structures*

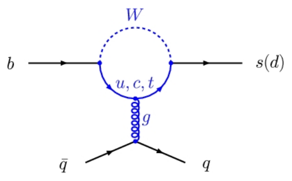
$$V_{ub} V^*_{us(d)}\; (\bar{u}_L \gamma_\mu b_L)(\bar{s}(d)_L \gamma^\mu u_L) \equiv \lambda_u\, \mathcal{O}_2^{(u)}\,,$$
$$V_{cb} V^*_{cs(d)}\; (\bar{c}_L \gamma_\mu b_L)(\bar{s}(d)_L \gamma^\mu c_L) \equiv \lambda_c\, \mathcal{O}_2^{(c)}\,,$$

  - Again, α_s corrections induce 2 independent colour structures $\mathcal{O}_{1,2}^{(u,c)}$.
- penguin operators (see next page)

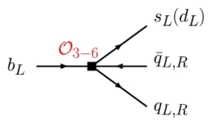
Other Cases

$b \to s(d)\, q\bar{q}$ decays

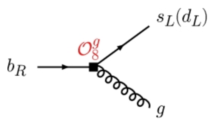
- New feature: penguin diagrams



$\to$



,



  - Strong penguin operators: $\mathcal{O}_{3-6}$, Chromomagnetic operator: $\mathcal{O}_8^g$
  - Wilson coefficients numerically suppressed by α_s / loop factor.

- Penguin and box diagrams with additional γ/Z exchange:
  - Electroweak penguin operators $\mathcal{O}_{7-10}$, Electromagnetic operator $\mathcal{O}_7^\gamma$
    (relevant for isospin-sensitive effects in non-leptonic decays, sensitive to NP)

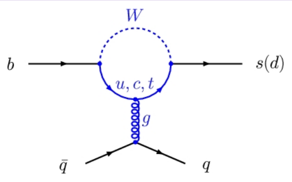
Question : What is the CKM factor of the penguin operators? (for $m_{u,c} \ll m_t$)

Answer :

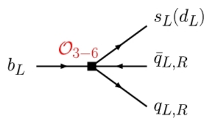
Other Cases

$b \to s(d)\, q\bar{q}$ decays

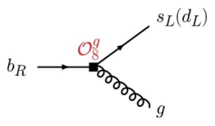
- New feature: penguin diagrams



$\to$



,



  - Strong penguin operators: $\mathcal{O}_{3-6}$, Chromomagnetic operator: $\mathcal{O}_8^g$
  - Wilson coefficients numerically suppressed by α_s / loop factor.

- Penguin and box diagrams with additional γ/Z exchange:
  - Electroweak penguin operators $\mathcal{O}_{7-10}$, Electromagnetic operator $\mathcal{O}_7^\gamma$
    (relevant for isospin-sensitive effects in non-leptonic decays, sensitive to NP)

Question : What is the CKM factor of the penguin operators? (for $m_{u,c} \ll m_t$)
Answer : $\lambda_t = -(\lambda_u + \lambda_c) = V_{tb} V_{ts(d)}^*$

Radiative decays $b \to s(d)\gamma$, $b \to s(d)\ell^+\ell^-$

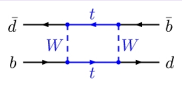
- Operator basis from $b \to s(d)q\bar{q}$,
  (with long-distance $q\bar{q} \to \gamma$ or $q\bar{q} \to \gamma^* \to \ell^+\ell^-$)
- New semileptonic operators (from short-distance box diagrams):

$$\begin{aligned} \mathcal{O}_{9V} &= (\bar{s}_L\, \gamma_\mu\, b_L)\,(\bar{\ell}\,\gamma^\mu\,\ell)\,, \\ \mathcal{O}_{10A} &= (\bar{s}_L\, \gamma_\mu\, b_L)\,(\bar{\ell}\,\gamma^\mu\gamma_5\,\ell) \end{aligned}$$

  - Particularly sensitive to New Physics!

B–$\bar{B}$ Mixing

- $\Delta B = 2$ operators require two $W^\pm$ exchanges $\to$ box diagrams, e.g.:



$\to$

$$H_{\rm eff}^{\Delta B=2} = \frac{G_F^2}{16\pi^2}\, M_W^2\, (V_{tb}^* V_{td})^2\; C(x_t,\mu)\; (\bar{b}_L\,\gamma^\mu\, d_L)(\bar{b}_L\,\gamma_\mu\, d_L) + \text{h.c.}$$

Summary: Effective Theory for b–Quark Decays

"Full theory" $\leftrightarrow$ all modes propagate Parameters: $M_{W,Z}, M_H, m_t, m_q, g, g', \alpha_s \ldots$	$\uparrow \mu > M_W$
$C_i(M_W) = C_i\big\|_{\rm tree} + \delta_i^{(1)} \frac{\alpha_s(M_W)}{4\pi} + \ldots$	matching: $\mu \sim M_W$
"Eff. theory" $\leftrightarrow$ Low-energy modes propagate. High-energy modes are "integrated out". Parameters: $m_b, m_c, G_F, \alpha_s, C_i(\mu) \ldots$	$\downarrow \mu < M_W$
$\frac{\partial}{\partial \ln \mu} C_i(\mu) = \gamma_{ji}(\mu)\, C_j(\mu)$	anomalous dimensions
Expectation values of operators $\langle O_i \rangle$ at $\mu = m_b$. All dependence on M_W absorbed into $C_i(m_b)$	resummation of logs

Outline

1. Introduction
2. The effective Weak Hamiltonian
   - Example: $b \to c d \bar{u}$ decay
   - Other cases
3. Hadronic matrix elements for exclusive B-meson decays
   - $B \to D^{(*)}$ form factors $\to$ HQET
   - Non-leptonic $B \to D\pi$ decays $\to$ QCDF
   - Non-leptonic $B \to \pi\pi$ and $B \to \pi K$ decays $\to$ QCDF
   - $B \to K^*$ form factors $\to$ SCET
   - QCDF corrections to $B \to K^* \mu^+ \mu^-$

$B \to D^{(*)}\ell\nu$

- In experiment, we cannot see the quark transition directly.

- Rather, we observe exclusive hadronic transitions, described by hadronic matrix elements, like e.g.

$$\langle D^* \ell\nu | \mathcal{H}_{\rm eff}^{b\to c\ell\nu} | B\rangle = \frac{G_F}{\sqrt{2}} \underbrace{\langle \ell\nu | \bar\ell \gamma_\mu (1-\gamma_5)\nu | 0\rangle}_{\text{leptonic}} \underbrace{\langle D^* | \bar c \gamma^\mu (1-\gamma_5) b | B\rangle}_{\text{hadronic}}$$

$\to$ Here: $B \to D^*$ transition form factor

- Contains non-perturbative dynamics about hadronic binding effects.
- Depends on two scales: $m_b \simeq 3 m_c \gg \Lambda_{\rm QCD}$

Can one disentangle the dependence on the heavy quark masses ?

Simplifications for $m_{b,c} \to \infty$?

Heavy Quark Symmetries

- heavy quark in heavy meson $\sim$ heavy nucleus in hydrogen atom
  (with light spectator system playing the role of the electron)
  - heavy quark acts as "quasi-static source of colour"
  - "wave function' almost independent of "isotope" (i.e. heavy flavour)
  - "wave function" almost independent of relative spin orientation

Field-theoretical formalism $(Q = b, c)$

$$Q(x) \equiv e^{-i m_Q\, v\cdot x} \left\{ h_v(x) + H_v(x) \right\} ,$$

with
$$h_v = e^{i m_Q\, v\cdot x}\, \frac{1+\not{v}}{2}\, Q(x) , \quad H_v = e^{i m_Q\, v\cdot x}\, \frac{1-\not{v}}{2}\, Q(x) .$$

- v^μ : 4-velocity of heavy quark, conserved for $m_Q \to \infty$.
- $\frac{1\pm\not{v}}{2}$: Projectors onto large/small spinor components
- corresponds to non-relativ. limit of Dirac equation (for $v^\mu = (1,0,0,0)$)

Question : Which scale controls the residual x–dependence of h_v and H_v ?

Answer :

Heavy Quark Symmetries

- heavy quark in heavy meson $\sim$ heavy nucleus in hydrogen atom
  (with light spectator system playing the role of the electron)
  - heavy quark acts as "quasi-static source of colour"
  - "wave function' almost independent of "isotope" (i.e. heavy flavour)
  - "wave function" almost independent of relative spin orientation

Field-theoretical formalism $(Q = b, c)$

$$Q(x) \equiv e^{-im_Q\, v\cdot x}\, \{h_v(x) + H_v(x)\}\ ,$$

with $$h_v = e^{im_Q\, v\cdot x}\, \frac{1+\not{v}}{2}\, Q(x)\,, \quad H_v = e^{im_Q\, v\cdot x}\, \frac{1-\not{v}}{2}\, Q(x)\,.$$

- v^μ : 4-velocity of heavy quark, conserved for $m_Q \to \infty$.
- $\frac{1\pm\not{v}}{2}$: Projectors onto large/small spinor components
- corresponds to non-relativ. limit of Dirac equation (for $v^\mu = (1,0,0,0)$)

Question : Which scale controls the residual x–dependence of h_v and H_v ?
Answer : $\Lambda_{\rm QCD}$ (as m_Q-dependence already in phase-factor)

Derivation of the HQET (heavy quark effective theory) Lagrangian

- Express Dirac-Lagrangian in QCD in terms of h_v and H_v:

$$\begin{aligned}\mathcal{L} &= \bar{Q}\,(i\not{D} - m_Q)\,Q \\ &= \bar{h}_v\,(v\cdot D)\,h_v - \bar{H}_v\,(v\cdot D + 2m_Q)\,H_v + \bar{h}_v\,i\not{D}\,H_v + \bar{H}_v\,i\not{D}\,h_v\end{aligned}$$

- Solve classical equations of motion for H_v:

$$\begin{aligned}& -(i\,v\cdot D + 2m_Q)\,H_v \stackrel{!}{=} i\not{D}\,h_v \\ \Leftrightarrow \qquad & H_v = (i\,v\cdot D + 2m_Q)^{-1}\,i\not{D}\,h_v = \mathcal{O}(1/m_Q)\,h_v\end{aligned}$$

- HQET Lagrangian in the infinite mass limit:

$$\mathcal{L}_{\rm HQET} = \bar{h}_v\,(i\,v\cdot D)\,h_v + \ldots$$

  - independent of m_Q $\Rightarrow$ heavy-quark flavour symmetry
  - trivial coupling of h_v spinor components $\Rightarrow$ heavy-quark spin symmetry

Spectroscopic Implications

Spin Symmetry:

- Heavy Mesons H with different relative spin orientation have mass splittings of order $1/m_Q \sim 1/m_H$

$$
\begin{aligned}
(m_{H^*} - m_H)(m_{H^*} + m_H) &\stackrel{!}{\simeq} \text{const.} \\
(m_{B^*}^2 - m_B^2) &\simeq 0.49\ \mathrm{GeV}^2\,, \\
(m_{D^*}^2 - m_D^2) &\simeq 0.55\ \mathrm{GeV}^2 \qquad \surd
\end{aligned}
$$

Flavour Symmetry:

- Mass splittings for heavy mesons with different spectator system, e.g.

$$
\begin{aligned}
(m_{H_s} - m_H) &\stackrel{!}{\simeq} \text{const.} \\
(m_{B_s} - m_B) &\simeq 100\ \mathrm{MeV}\,, \\
(m_{D_s} - m_D) &\simeq 100\ \mathrm{MeV} \qquad \surd
\end{aligned}
$$

The Isgur-Wise function in HQET

- Consider any $b \to c$ current with some Dirac structure Γ

$$\bar{c}(x)\,\Gamma\,b(x) \quad \simeq \quad e^{-im_b v\cdot x + im_c v'\cdot x}\;\bar{h}_{v'}^{(c)}(x)\,\frac{1+\not{v}'}{2}\,\Gamma\,\frac{1+\not{v}}{2}\,h_v^{(b)}(x)$$

- Hadronic matrix element written as a Dirac trace ← (spin symmetry)

$$\langle D(v')|\bar{h}_{v'}^{(c)}\Gamma h_v^{(b)}|B(v)\rangle \quad \propto \quad \mathrm{tr}\left[\Xi(v,v')\,\gamma_5\,\frac{1+\not{v}'}{2}\,\Gamma\,\frac{1+\not{v}}{2}\,(-\gamma_5)\right]$$

$$\langle D^*(v',\epsilon)|\bar{h}_{v'}^{(c)}\Gamma h_v^{(b)}|B(v)\rangle \quad \propto \quad \mathrm{tr}\left[\Xi(v,v')\,\not{\epsilon}^*\,\frac{1+\not{v}'}{2}\,\Gamma\,\frac{1+\not{v}}{2}(-\gamma_5)\right]$$

with some generic Dirac matrix $\Xi(v,v')$.

- Because $\not{v}^2 = \not{v}'^2 = 1$, the most general expression is

$$\Xi(v,v') \quad = \quad -\,\xi(v\cdot v')\,\mathbf{1}$$

Normalization of the Isgur-Wise function

- For $v = v'$ and $\Gamma = \gamma_\mu$, we encounter the *Noether Current* corresponding to the heavy quark flavour symmetry

$$\mathcal{L} = \sum_{Q=b,c} \bar{h}_v^{(Q)} \, iv \cdot D \, h_v^{(Q)} \qquad \Rightarrow \qquad \partial_\mu \left(\bar{h}_v^{(Q)} \, v^\mu \, h_v^{(Q)} \right) = 0$$

Conserved charge $\Rightarrow \xi(v \cdot v' = 1) = 1$.

- $v = v'$ corresponds to $q^2 = q^2_{\max} = (m_B - m_D)^2$
- Take into account radiative corrections ($\alpha_s(m_Q)$)
- Estimate power corrections
  - $\mathcal{O}(1/m_Q)$ for $B \to D\ell\nu$
  - $\mathcal{O}(1/m_Q^2)$ for $B \to D^*\ell\nu$ — Luke's Theorem

$\Rightarrow$ Relatively precise extraction of

$$|V_{cb}| = \frac{\left[\mathcal{F}(q^2 \to q^2_{\max}) \, |V_{cb}|\right]_{\rm exp.}}{\left[\mathcal{F}(v \cdot v' = 1)\right]_{\rm theor.}}$$

Outline

From $b \to c d \bar{u}$ to $\bar{B}^0 \to D^+ \pi^-$

- Now, exclusive transitions are described by hadronic matrix elements,

$$\langle D^+ \pi^- | \mathcal{H}_{\rm eff}^{b \to c d \bar{u}} | \bar{B}_d^0 \rangle = V_{cb} V_{ud}^* \frac{G_F}{\sqrt{2}} \sum_i C_i(\mu \sim m_b)\, r_i(\mu \sim m_b)$$

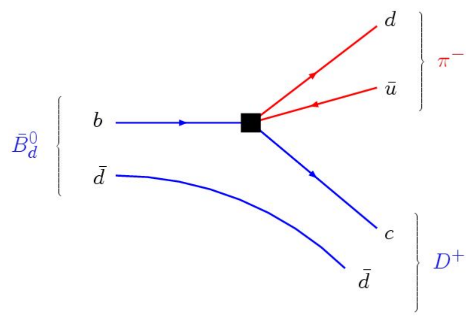
$$r_i(\mu) = \langle D^+ \pi^- | \mathcal{O}_i | \bar{B}_d^0 \rangle \Big|_\mu$$

  - The $r_i(\mu \sim m_b)$ contain QCD (and also QED) dynamics on scales below m_b
  - μ-dependence has to drop out when multiplied by Wilson coefficients.

Can one disentangle the dependence on the heavy quark masses ?

Simplifications for $m_{b,c} \to \infty$?

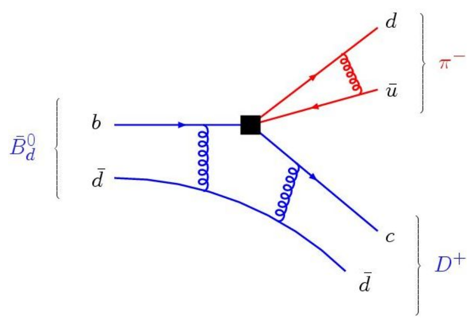
"Naive" Factorization of hadronic matrix elements



$$r_i \quad = \quad \underbrace{\langle D^+ | J_i^{(b\to c)} | \bar{B}_d^0 \rangle}\ \underbrace{\langle \pi^- | J_i^{(d\to u)} | 0 \rangle}$$

- Quantum fluctuations above $\mu \sim m_b$ already in Wilson coefficients

"Naive" Factorization of hadronic matrix elements

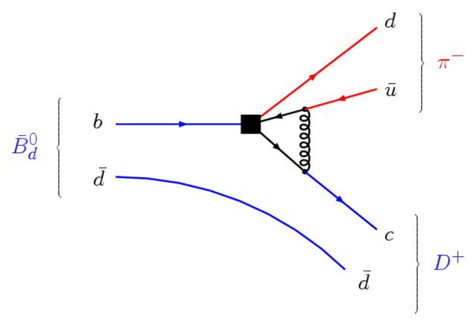


$$r_i = \underbrace{\langle D^+ | J_i^{(b\to c)} | \bar{B}_d^0 \rangle}_{\text{form factor}} \underbrace{\langle \pi^- | J_i^{(d\to u)} | 0 \rangle}_{\text{decay constant}}$$

⇈ Include hadronic binding effects from low-energy gluons

Question : Why is naive factorization not correct ?

"Naive" Factorization of hadronic matrix elements



$$r_i(\mu) \quad = \quad \underbrace{\langle D^+|J_i^{(b\to c)}|\bar{B}_d^0\rangle}_{\text{form factor}} \; \underbrace{\langle \pi^-|J_i^{(d\to u)}|0\rangle}_{\text{decay constant}} + \text{corrections}(\mu)$$

Answer : Gluon cross-talk between π^- and $B \to D$

QCD factorization [Beneke/Buchalla/Neubert/Sachrajda 2000]

- light quarks in π^- have large energy (in B rest frame)
- gluons from the $B \to D$ transition see "small colour-dipole"

$\Rightarrow$ corrections to naive factorization dominated by gluon exchange at short distances $\delta x \sim 1/m_b$

New feature: Light-cone distribution amplitudes $\phi_\pi(u)$

- Short-distance corrections to naive factorization given as convolution

$$r_i(\mu) \simeq \sum_j F_j^{(B\to D)} \int_0^1 du \left(1 + \frac{\alpha_s C_F}{4\pi}\, t_{ij}(u,\mu) + \ldots\right) f_\pi\, \phi_\pi(u,\mu)$$

- $\phi_\pi(u)$: distribution of momentum fraction u of a quark in the pion.
- $t_{ij}(u,\mu)$: perturbative coefficient function (depends on u)

QCD factorization

[Beneke/Buchalla/Neubert/Sachrajda 2000]

- light quarks in π^- have large energy (in B rest frame)
- gluons from the $B \to D$ transition see "small colour-dipole"

$\Rightarrow$ corrections to naive factorization dominated by gluon exchange at short distances $\delta x \sim 1/m_b$

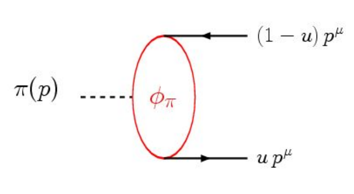
New feature: Light-cone distribution amplitudes $\phi_\pi(u)$

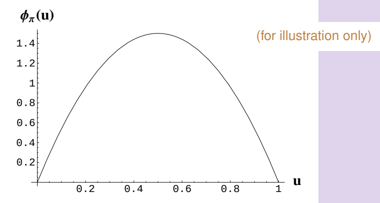
- Short-distance corrections to naive factorization given as convolution

$$r_i(\mu) \simeq \sum_j F_j^{(B\to D)} \int_0^1 du \left(1 + \frac{\alpha_s C_F}{4\pi}\, t_{ij}(u,\mu) + \ldots\right) f_\pi\, \phi_\pi(u,\mu)$$

- $\phi_\pi(u)$: distribution of momentum fraction u of a quark in the pion.
- $t_{ij}(u,\mu)$: perturbative coefficient function (depends on u)

Light-cone distribution amplitude for the pion



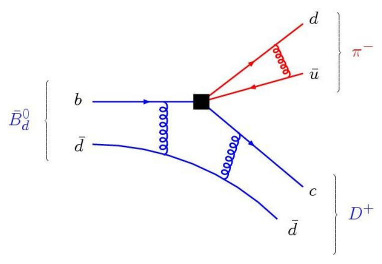


- Exclusive analogue of parton distribution function:
  - PDF: probability density (all Fock states)
  - LCDA: probability amplitude (one Fock state, e.g. $q\bar{q}$)
- Phenomenologically relevant $\langle u^{-1} \rangle_\pi \simeq 3.3 \pm 0.3$

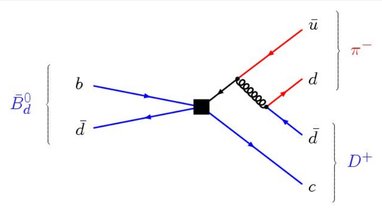
[from sum rules, lattice, exp.]

Complication: Annihilation in $\bar{B}_d \to D^+\pi^-$

Second topology for hadronic matrix element possible:



"Tree" (class-I)

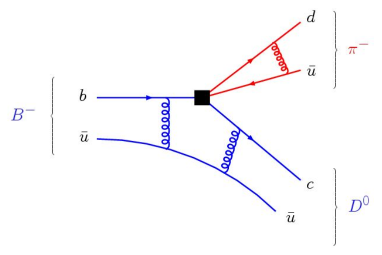


"Annihilation" (class-III)

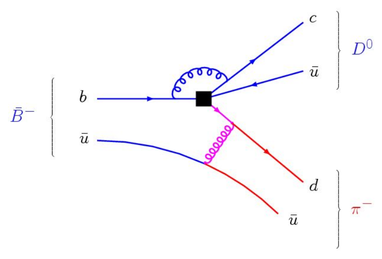
- annihilation is power-suppressed by Λ/m_b
- difficult to estimate (final-state interactions?)

Still more complicated: $B^- \to D^0\pi^-$

Second topology with spectator quark going into light meson:



"Tree" (class-I)

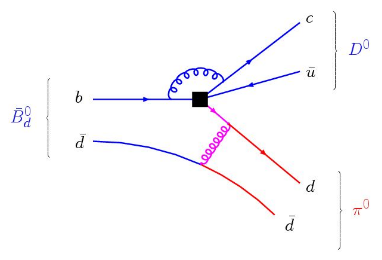


"Tree" (class-II)

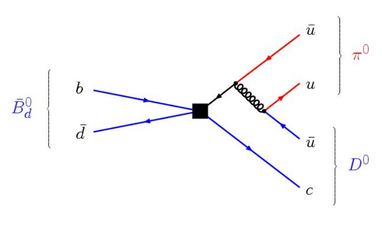
- class-II amplitude does not factorize into simpler objects
  (colour-transparency argument does not apply)
- again, it is power-suppressed compared to class-I topology

Non-factorizable: $\bar{B}^0 \to D^0 \pi^0$

In this channel, class-I topology is absent:



"Tree" (class-II)



"Annihilation" (class-III)

- The whole decay amplitude is power-suppressed!
- Naive factorization is not even an approximation!

Isospin analysis for $B \to D\pi$

- Employ isospin symmetry between (u, d) of strong interactions.
- Final-state with pion ($I = 1$) and D-meson ($I = 1/2$). described by only two isospin amplitudes:

$$\begin{aligned} \mathcal{A}(\bar{B}_d \to D^+\pi^-) &= \sqrt{\frac{1}{3}}\,\mathcal{A}_{3/2} + \sqrt{\frac{2}{3}}\,\mathcal{A}_{1/2}\,, \\ \sqrt{2}\,\mathcal{A}(\bar{B}_d \to D^0\pi^0) &= \sqrt{\frac{4}{3}}\,\mathcal{A}_{3/2} - \sqrt{\frac{2}{3}}\,\mathcal{A}_{1/2}\,, \\ \mathcal{A}(B^- \to D^0\pi^-) &= \sqrt{3}\,\mathcal{A}_{3/2}\,, \end{aligned}$$

- QCDF: $\mathcal{A}_{1/2}/\mathcal{A}_{3/2} = \sqrt{2} + \text{corrections,}$ relative strong phase $\Delta\theta$ small

Isospin amplitudes from experimental data [BaBar hep-ph/0610027]

$$\left|\frac{\mathcal{A}_{1/2}}{\sqrt{2}\,\mathcal{A}_{3/2}}\right| = 0.655^{+0.015+0.042}_{-0.014-0.042}\,, \qquad \cos\Delta\theta = 0.872^{+0.008+0.031}_{-0.007-0.029}$$

Outline

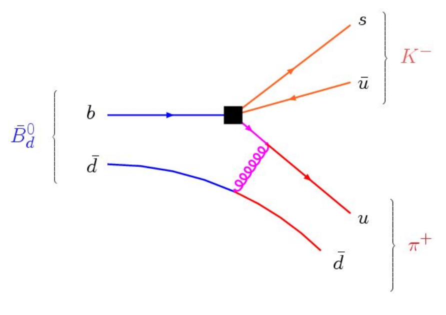
1. Introduction

2. The effective Weak Hamiltonian
   - Example: $b \to c d \bar{u}$ decay
   - Other cases

3. Hadronic matrix elements for exclusive B-meson decays
   - $B \to D^{(*)}$ form factors $\to$ HQET
   - Non-leptonic $B \to D\pi$ decays $\to$ QCDF
   - Non-leptonic $B \to \pi\pi$ and $B \to \pi K$ decays $\to$ QCDF
   - $B \to K^*$ form factors $\to$ SCET
   - QCDF corrections to $B \to K^* \mu^+ \mu^-$

$B \to \pi\pi$ and $B \to \pi K$

Naive factorization:



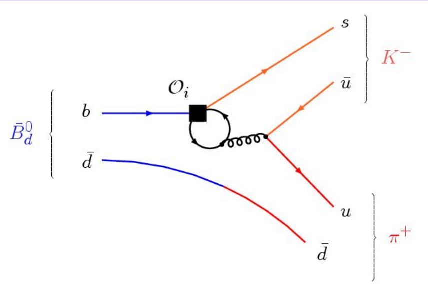
- Both final-state mesons are light and energetic.
- Colour-transparency argument applies for class-I and class-II topologies.
- $B \to \pi(K)$ form factors fairly well known (QCD sum rules)

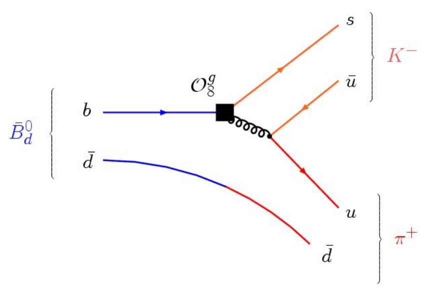
QCDF for $B \to \pi\pi$ and $B \to \pi K$ decays (BBNS 1999)

Factorization formula has to be extended:

- Vertex corrections are treated as in $B \to D\pi$
  - Include penguin (and electroweak) operators from H_{eff}.
  - Take into account new (long-distance) penguin diagrams! ($\to$ Fig.)
- Additional perturbative interactions involving spectator in B-meson ($\to$ Fig.)
  - Sensitive to the distribution of the spectator momentum ω
    $\longrightarrow$ light-cone distribution amplitude $\phi_B(\omega)$

Additional diagrams for hard corrections in QCDF (example)

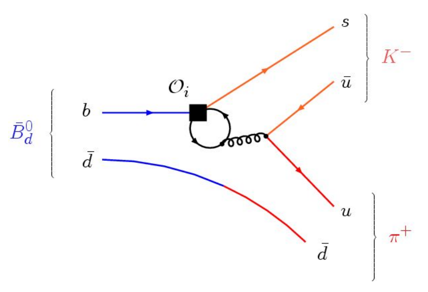


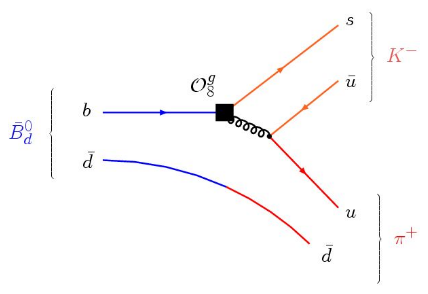


$\longrightarrow$ additional contributions to the hard coefficient functions $t_{ij}(u,\mu)$

$$r_i(\mu)\Big|_{\rm hard} \simeq \sum_j F_j^{(B\to\pi)}(m_K^2) \int_0^1 du \left(1 + \frac{\alpha_s}{4\pi}\, t_{ij}(u,\mu) + \ldots\right) f_K\, \phi_K(u,\mu)$$

Additional diagrams for hard corrections in QCDF (example)

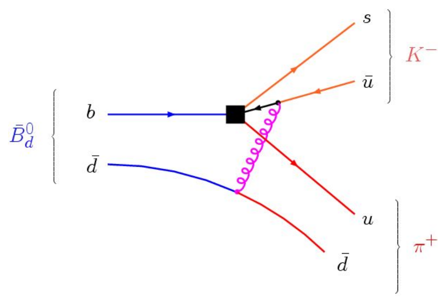




$\longrightarrow$ additional contributions to the hard coefficient functions $t_{ij}(u,\mu)$

$$r_i(\mu)\Big|_{\rm hard} \simeq \sum_j F_j^{(B\to\pi)}(m_K^2) \int_0^1 du \left(1 + \frac{\alpha_s}{4\pi}\, t_{ij}(u,\mu) + \ldots\right) f_K\, \phi_K(u,\mu)$$

Spectator corrections with hard-collinear gluons in QCDF

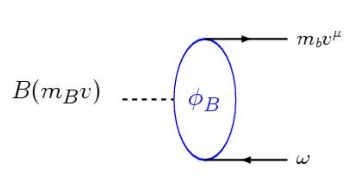


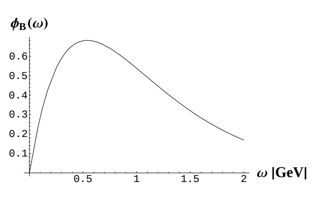
$\longrightarrow$ additive correction to naive factorization

$$\Delta r_i(\mu)\Big|_{\text{spect.}} = \int du\, dv\, d\omega \left(\frac{\alpha_s}{4\pi}\, h_i(u,v,\omega,\mu) + \ldots\right)$$
$$\times\, f_K\, \phi_K(u,\mu)\, f_\pi\, \phi_\pi(v,\mu)\, f_B\, \phi_B(\omega,\mu)$$

Distribution amplitudes for all three mesons involved!

New ingredient: LCDA for the B-meson





- Phenomenologically relevant: $\langle \omega^{-1} \rangle_B \simeq (1.9 \pm 0.2)\ \mathrm{GeV}^{-1}$

(at $\mu = \sqrt{m_b \Lambda} \simeq 1.5$ GeV)

(from QCD sum rules [Braun/Ivanov/Korchemsky])

(from HQET parameters [Lee/Neubert])

- Large logarithms $\ln m_b$ can be resummed using SCET $\surd$

Complications for QCDF in $B \to \pi\pi, \pi K$ etc.

- Annihilation topologies are numerically important.
  BBNS use conservative model estimates.
- Some power-corrections are numerically enhanced by "chiral factor"

$$\frac{\mu_\pi}{f_\pi} = \frac{m_\pi^2}{2 f_\pi\, m_q}$$

- Many decay topologies interfere with each other.
- Many hadronic parameters to vary.

$\to$ Hadronic uncertainties sometimes quite large.

Example: Predictions for $B \to \pi K$ branching fractions

Mode	Theory	S1	S2	S3	S4	Experiment
$B^- \to \pi^- \bar{K}^0$	$19.3^{+1.9+11.3+1.9+13.2}_{-1.9-\ 7.8-2.1-\ 5.6}$	18.8	20.7	24.8	20.3	20.6 ± 1.3
$B^- \to \pi^0 K^-$	$11.1^{+1.8+5.8+0.9+6.9}_{-1.7-4.0-1.0-3.0}$	14.0	11.9	14.0	11.7	12.8 ± 1.1
$\bar{B}^0 \to \pi^+ K^-$	$16.3^{+2.6+9.6+1.4+11.4}_{-2.3-6.5-1.4-\ 4.8}$	20.3	18.8	21.0	18.4	18.2 ± 0.8
$\bar{B}^0 \to \pi^0 \bar{K}^0$	$7.0^{+0.7+4.7+0.7+5.4}_{-0.7-3.2-0.7-2.3}$	6.5	8.3	9.3	8.0	11.2 ± 1.4
$B^- \to \pi^- \bar{K}^{*0}$	$3.6^{+0.4+1.5+1.2+7.7}_{-0.3-1.4-1.2-2.3}$	3.4	2.2	7.3	8.4	13.0 ± 3.0
$B^- \to \pi^0 K^{*-}$	$3.3^{+1.1+1.0+0.6+4.4}_{-1.0-0.9-0.6-1.4}$	5.5	2.6	5.4	6.5	< 31
$\bar{B}^0 \to \pi^+ K^{*-}$	$3.3^{+1.4+1.3+0.8+6.2}_{-1.2-1.2-0.8-1.6}$	5.9	2.4	6.6	8.1	15.3 ± 3.8
$\bar{B}^0 \to \pi^0 \bar{K}^{*0}$	$0.7^{+0.1+0.5+0.3+2.6}_{-0.1-0.4-0.3-0.5}$	0.6	0.4	2.1	2.5	< 3.6
$B^- \to \bar{K}^0 \rho^-$	$5.8^{+0.6+7.0+1.5+10.3}_{-0.6-3.3-1.3-\ 3.2}$	5.6	13.6	10.8	9.7	< 48
$B^- \to K^- \rho^0$	$2.6^{+0.9+3.1+0.8+4.3}_{-0.9-1.4-0.6-1.2}$	1.3	6.0	4.7	4.3	< 6.2
$\bar{B}^0 \to K^- \rho^+$	$7.4^{+1.8+7.1+1.2+10.7}_{-1.9-3.6-1.1-\ 3.5}$	4.3	13.9	12.5	10.1	8.9 ± 2.2
$\bar{B}^0 \to \bar{K}^0 \rho^0$	$4.6^{+0.5+4.0+0.7+6.1}_{-0.5-2.1-0.7-2.1}$	5.0	8.4	7.5	6.2	< 12
$B^- \to K^- \omega$	$3.5^{+1.0+3.3+1.4+4.7}_{-1.0-1.6-0.9-1.6}$	1.9	7.9	5.8	5.9	5.3 ± 0.8
$\bar{B}^0 \to \bar{K}^0 \omega$	$2.3^{+0.3+2.8+1.3+4.3}_{-0.3-1.3-0.8-1.3}$	1.9	6.6	4.5	4.9	5.1 ± 1.1
$B^- \to K^- \phi$	$4.5^{+0.5+1.8+1.9+11.8}_{-0.4-1.7-2.1-\ 3.3}$	4.4	2.5	10.1	11.6	9.2 ± 1.0
$\bar{B}^0 \to \bar{K}^0 \phi$	$4.1^{+0.4+1.7+1.8+10.6}_{-0.4-1.6-1.9-\ 3.0}$	4.0	2.3	9.1	10.5	7.7 ± 1.1

[Beneke/Neubert '03]

Outline

$B \to K^*$ form factors

Naive factorization for $B \to K^*\gamma$ and $B \to K^*\ell^+\ell^-$ decays:

- **Question :** What are the relevant operators?

- Consider, in particular, region of large recoil energy:

$$E_{K^*} \sim m_B/2 \gg m_{K^*} \quad \Leftrightarrow \quad q^2 = (p_B - p_K^*)^2 = m^2_{\ell\ell,\gamma^*} \ll m_b^2$$

In practice: $q^2 < 4m_c^2 \approx 6 \text{ GeV}^2$

Simplifications for $E_{K^*} \to \infty$?

$B \to K^*$ form factors

Naive factorization for $B \to K^* \gamma$ and $B \to K^* \ell^+ \ell^-$ decays:

- **Answer:** Relevant Operators:

$$\mathcal{O}_7^\gamma \propto [\bar{s}(x)\sigma^{\mu\nu}(1+\gamma_5)b(x)]\, F_{\mu\nu}(x)$$

$$\mathcal{O}_{9,10}^{V,A} \propto [\bar{s}(x)\gamma^\mu(1-\gamma_5)b(x)]\, \left[\bar{\ell}(x)\gamma_\mu(\gamma_5)\ell(x)\right]$$

$\to$ Seven independent $B \to K^*$ transition form factors for

- tensor current ($T_{1,2,3}$)
- vector current (V)
- axial-vector current ($A_{0,1,2}$)

- Consider, in particular, region of large recoil energy:

$$E_{K^*} \sim m_B/2 \gg m_{K^*} \quad \Leftrightarrow \quad q^2 = (p_B - p_K^*)^2 = m_{\ell\ell,\gamma^*}^2 \ll m_b^2$$

In practice: $q^2 < 4m_c^2 \approx 6 \text{ GeV}^2$

Simplifications for $E_{K^*} \to \infty$?

$B \to K^*$ form factors

Naive factorization for $B \to K^*\gamma$ and $B \to K^*\ell^+\ell^-$ decays:

- **Answer:** Relevant Operators:

$$\mathcal{O}_7^\gamma \propto [\bar{s}(x)\sigma^{\mu\nu}(1+\gamma_5)b(x)]\, F_{\mu\nu}(x)$$

$$\mathcal{O}_{9,10}^{V,A} \propto [\bar{s}(x)\gamma^\mu(1-\gamma_5)b(x)]\, \left[\bar{\ell}(x)\gamma_\mu(\gamma_5)\ell(x)\right]$$

$\to$ Seven independent $B \to K^*$ transition form factors for

- tensor current ($T_{1,2,3}$)
- vector current (V)
- axial-vector current ($A_{0,1,2}$)

- Consider, in particular, region of large recoil energy:

$$E_{K^*} \sim m_B/2 \gg m_{K^*} \quad \Leftrightarrow \quad q^2 = (p_B - p_K^*)^2 = m^2_{\ell\ell,\gamma^*} \ll m_b^2$$

In practice: $q^2 < 4m_c^2 \approx 6 \text{ GeV}^2$

Simplifications for $E_{K^*} \to \infty$?

Collinear Particles (in the background of soft fields)

- $E \gg m$ implies almost light-like particle momenta
- Introduce two complementary light-like vectors $n_\pm^\mu$ with

$$n_+^2 = n_-^2 = 0\,, \qquad (n_+ \cdot n_-) \equiv 2\,, \qquad \text{e.g.} \begin{cases} n_+^\mu = (1,0,0,+1) \\ n_-^\mu = (1,0,0,-1) \end{cases}$$

- Decompose momenta of light-like ("collinear") particles as

$$p^\mu = (n_+ p)\,\frac{n_-^\mu}{2} + (n_- p)\,\frac{n_+^\mu}{2} + p_\perp^\mu\,,$$

with $\quad (n_+ p) \sim \mathcal{O}(E)\,, \quad (n_- p) \sim \Lambda_{\rm QCD} \quad \Rightarrow \quad p^2 \sim p_\perp^2 \sim E\,\Lambda_{\rm QCD}$

- Introduce light-cone projectors (instead of HQET velocity projector)

$$\xi(x) \equiv \frac{\not n_- \not n_+}{4}\, q(x)\,, \qquad \eta(x) \equiv \frac{\not n_+ \not n_-}{4}\, q(x)$$

Derivation of the SCET (soft collinear effective theory) Lagrangian

- Dirac Lagrangian for (massless) collinear quark in terms of $\xi(x)$ and $\eta(x)$

$$\bar{q}(x)\, i\not{D}\, q(x) \;=\; \bar{\xi}(x)\,(in_- \cdot D)\,\frac{\not{n}_+}{2}\,\xi(x) + \bar{\eta}(x)\,(in_+ \cdot D)\,\frac{\not{n}_-}{2}\,\eta(x) + \bar{\eta}(x)\, i\not{D}_\perp\, \xi(x) + \bar{\xi}(x)\, i\not{D}_\perp\, \eta(x)$$

- Classical e.o.m. for “small component” $\eta(x)$

$$(in_+ \cdot D)\,\frac{\not{n}_-}{2}\,\eta(x) = -i\not{D}_\perp\,\xi(x)$$

$$\Rightarrow \qquad \eta(x) = (in_+ \cdot D)^{-1}\, i\not{D}_\perp\,\frac{\not{n}_+}{2}\,\xi(x) = \mathcal{O}\left(\frac{p_\perp}{n_+ p}\right)\xi(x)$$

Eff. Lagrangian : $\bar{\xi}(x)\left\{(in_- \cdot D) + i\not{D}_\perp\,(in_+ \cdot D)^{-1}\, i\not{D}_\perp\right\}\frac{\not{n}_+}{2}\,\xi(x)$

- Collinear gluons: $A_c^\mu \sim p^\mu \;\Rightarrow$ nothing to expand
- Soft gluons: $A_s^\mu \sim \Lambda_{\rm QCD} \;\Rightarrow$ only $(n_- A_s) \sim (n_- A_c)$ survives for $(n_+ p) \to \infty$

Large-recoil symmetries from SCET

- Consider any $b \to s$ current with some Dirac structure Γ

$$\bar{s}(x)\,\Gamma\,b(x) \quad \simeq \quad e^{-im_b v\cdot x}\,\bar{\xi}^{(s)}(x)\,\frac{\not{n}_+\not{n}_-}{4}\,\Gamma\,\frac{1+\not{v}}{2}\,h_v^{(b)}(x)$$

(For Experts: In order to have manifest invariance under collinear and soft gauge transformations, one has to add additional Wilson line factors . . .)

- As in the case of IW-function, form factors related via trace formula

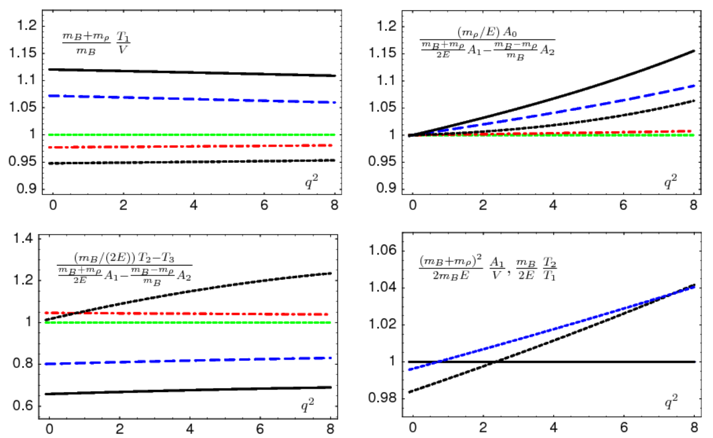
$$\langle K^*(E,\epsilon)|\bar{s}(x)\,\Gamma\,b(x)|B(v)\rangle$$
$$\simeq \quad E\cdot\mathrm{tr}\left[\left(2\not{\epsilon}_\perp^*\,\xi_\perp(E)+\not{\epsilon}_\parallel^*\,\xi_\parallel(E)\right)\,\frac{\not{n}_+\not{n}_-}{4}\,\Gamma\,\frac{1+\not{v}}{2}\,\gamma_5\right]$$

Only 2 FFs $\xi_{\perp,\parallel}(E)$ for transversely or longitudinally polarized K^*

[Charles et al. '98; Beneke/Feldmann '00]

Radiative corrections to FF relations

- Apply QCDF to determine vertex and spectator corrections.
- Use SCET machinery for RG improvement (resummation of logs)



- solid black line: NLO + LL
- short-dashed lines: light-cone sum rules

[Beneke/Kiyo/Yang '05]
[Ball et al, incl. power corrections]

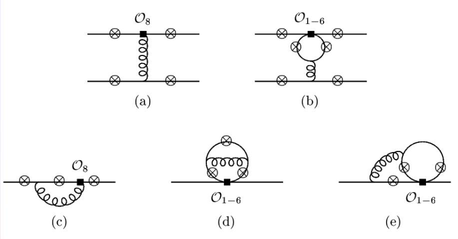
Outline

QCDF corrections to $B \to K^* \mu^+ \mu^-$

Question : Why do leptons and $B \to K^*$ not decouple?

QCDF corrections to $B \to K^* \mu^+ \mu^-$

Answer : $\ell^+\ell^-$ has the same QCD/QED quantum numbers as massive neutral meson



- Factorization formula for radiative decays (photon/lepton-pair $\sim$ emitted meson)

$$r_i^{\perp,\parallel}(\mu)\Big|_{\rm hard} = \xi_{\perp,\parallel}^{(B\to K^*)}(q^2)\left(1 + \frac{\alpha_s}{4\pi}\, t_i^{\perp,\parallel}(q^2,\mu) + \ldots\right),$$

$$\Delta r_i^{\perp,\parallel}(\mu)\Big|_{\rm spect.} = f_{K^*}^{\perp,\parallel} f_B \int du\, d\omega \left(\frac{\alpha_s}{4\pi}\, h_i^{\perp,\parallel}(u,\omega,q^2,\mu) + \ldots\right) \phi_{K^*}^{\perp,\parallel}(u,\mu)\, \phi_B(\omega,\mu)$$

Example: Hadronic uncertainties in FB asymmetry zero (SM)

- Involves only transversely polarized K^* mesons
- Ignoring radiative and power corrections, $\xi_\perp(E)$ drops out

$$\Rightarrow \quad \text{FBA zero} \quad : \quad s_0 \simeq -2 m_b m_B \frac{C_7(m_b)}{\mathrm{Re}\left[C_9^{\mathrm{eff}}(m_b, s_0)\right]} \approx 3.4^{+0.6}_{-0.5}\ \mathrm{GeV}^2$$

- Including QCF corrections, [Beneke/TF/Seidel 01/04]

$$s_0[K^{*0}] = 4.36^{+0.33}_{-0.31}\ \mathrm{GeV}^2\,, \qquad s_0[K^{*+}] = 4.15^{+0.27}_{-0.27}\ \mathrm{GeV}^2$$

(plus non-factorizable power-corrections)

Summary:
Heavy-Quark Expansion for Exclusive Decays

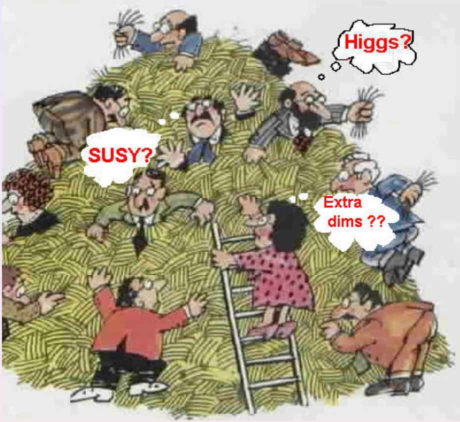
Limit $m_Q \to \infty$ and/or $E_{\rm recoil} \to \infty$:

- Simplifications due to New Symmetries:
  - Universal Isgur-Wise form factors for heavy-to-heavy decays:
    Normalization: $\xi(v \cdot v' = 1) \simeq 1 \ \to |V_{cb}|$
  - Universal form factors for heavy-to-light decays: $(q^2 \ll 4m_c^2)$
    $\to$ FBA zero

- QCD corrections to naive factorization systematically calculable, IF Color-Transparency applies (small color dipoles)
  - RG-improved perturbation theory, using SCET

- Genuinely non-factorizable corrections remain challenging
  - often (formally) suppressed by $1/m_b$ and/or CKM factors
  - numerically difficult to estimate
  - source of many controversial theoretical discussion:
    "endpoint singularities", "zero-bins", "$k_\perp$–factorization", "pQCD approach", ...

Summary



[from C. Berger's homepage]

" When looking for new physics, . . .
. . . do not forget about the complexity of the old physics ! "