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Experimental methods

José R. Crespo López-Urrutia

12.10.2011

• Cross sections
• Atomic units
• Atomic and molecular beams
• Supersonic jets
• Crossed beam experiments
• Nanodroplets
• Laser cooling and trapping of atoms

José R. Crespo López-Urrutia   crespojr@mpi-hd.mpg.de
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Experimental methods

- Collision dynamics
- Cross sections
Why collision physics?

- Practical importance: many applications in technology and medicine, environmental physics
- Basic research: fundamental understanding of the observed phenomena. Difficult challenge: few (many) body systems

Only two-particle systems can be solved analytically!

Structure: H atom Dynamic problems with analytical solution:

Potential scattering

Photoabsorption in H
Collision kinematics

Conservation laws of:
- Scalars: energy, number of particles, electrical charge,...
- Vectors: momentum, angular momentum,...

\[ \mathbf{v} = 0 \]

\[ m_a \quad M_A \quad m_b \quad M_B \]

\[ \mathbf{p}_a = \mathbf{p}_b + \mathbf{p}_B \]

\[ p_{a\|} = p_{b\|} + p_{B\|} \]

\[ 0 = \mathbf{p}_{b\perp} + \mathbf{p}_{B\perp} \]

Q value:
Difference of total kinetic energy before and after collision

\[ Q = (E_B + E_b) - E_a \]

endotherm: \( Q < 0 \)
exotherm: \( Q > 0 \)
What is a (collision/reaction) cross section?

**Total cross section:**

\[ \sigma = \frac{\text{Number of reactions per scattering centre and time interval}}{\text{Current density } j \text{ of incoming particles}} \]

**Differential cross section:**

\[ \left( \frac{d\sigma}{d\Omega} \right)_\nu = \frac{\text{Flux of particles scattered in solid angle interval } d\Omega}{\text{Current density } j \text{ of incoming particles}} \]

Probability for a reaction:

\[ P = \frac{N \cdot \sigma}{A} \]
Total cross section: \[ \sigma = \int \left( \frac{d\sigma}{d\Omega} \right) \cdot d\Omega \]

Typical units of area and cross section:
- Atomic physics: \( \text{cm}^2 \); 1 a.u.\(^2 = 0.88 \cdot 10^{-16} \text{ cm}^2 \)
- Nuclear physics: 100 fm\(^2 = 10^{-24} \text{ cm}^2 = 1 \text{ barn} \)

\[ \frac{\text{Number of reactions}}{\text{time}} = \frac{\text{Projectiles}}{\text{time}} \cdot n \cdot L \cdot \sigma \]

Mean free path length: \( < \lambda > = 1/(n \cdot \sigma) \)
Differential cross section

\[
\left( \frac{d\sigma}{d\Omega} \right) = \frac{1}{2\pi \sin(\vartheta)} \frac{d\sigma}{d\vartheta} \quad \text{with} \quad d\Omega = 2\pi \sin(\vartheta) d\vartheta
\]

Projectile (incident energy $E$)

$\vartheta$

Impact parameter

Ring area: $2\pi b \cdot db$

with $d\sigma = 2\pi b \cdot db$ follows:

\[
\left( \frac{d\sigma}{d\Omega} \right) = \frac{2\pi b}{2\pi \sin(\vartheta)} \left| \frac{db}{d\vartheta} \right| = \frac{b}{\sin(\vartheta)} \left| \frac{db}{d\vartheta} \right|
\]

Dependence of scattering angle from impact parameter

$\vartheta = \vartheta(b, E)$
Franck-Hertz experiment

- Hg vapour
- Filament (e⁻ Quelle)
- Hg excitation energy: 4.9 eV

The Nobel Prize in Physics 1925

"for their discovery of the laws governing the impact of an electron upon an atom"

James Franck
- 1/2 of the prize
- Germany
- Goettingen University
- Goettingen, Germany
- b.1882
- d.1964

Gustav Ludwig Hertz
- 1/2 of the prize
- Germany
- Halle University
- Halle, Germany
- b.1887
- d.1975
Experimental methods

- Atomic units
- Atomic beams
Atomic units

Atomic units (a. u.) are a convenient "natural" measure of sizes and magnitudes. The hydrogen atom serves as a standard.

**Basic constants:**

Angular momentum: $\hbar$

Electron mass: $m_0$

Elementary charge: $e$

Dielectric constant: $4\pi\varepsilon_0$
**Atomic units**

**Length:** Bohr radius

\[ a_0 = \frac{\hbar^2}{m_0e^2} \left(4\pi\varepsilon_0\right) = 0.529 \cdot 10^{-10} \text{ m} \]

**Energy:** potential energy of two elementary charges with distance \( a_0 \)

\[ E_1 = \frac{m_0e^4}{\hbar^2} \frac{1}{\left(4\pi\varepsilon_0\right)^2} = \frac{e_0^2}{a_0} \frac{1}{4\pi\varepsilon_0} = 4.3 \cdot 10^{-18} \text{ J} = 27.2 \text{ eV} \]

**Velocity:** classical velocity on 1st Bohr orbit

\[ v_0 = \frac{e^2}{(4\pi\varepsilon_0)\hbar} = \alpha c \quad \text{Fine structure constant: } \alpha = \frac{e^2}{(4\pi\varepsilon_0)\hbar c} \quad \Rightarrow \quad c = 137 \text{ a.u.} \]

**Momentum:** \( p_0 = m_0v_0 = \frac{m_0e_0^2}{(4\pi\varepsilon_0)\hbar} = 2 \cdot 10^{-24} \text{ kg m/s} \)

**Time:**

\[ \frac{a_0}{v_0} = \frac{\hbar^3}{m_0e_0^4} \left(4\pi\varepsilon_0\right)^2 = 2.4 \cdot 10^{-17} \text{ s} = 24 \text{ as} \]
Atomic units

Examples:

Momentum of an electron with 100 eV kinetic energy

\[ E_e = \frac{p_e^2}{2m} \]

\[ E_e = \frac{100}{27.2} \text{ a.u.} \iff p_e = \sqrt{2m_E E_e} = \sqrt{2 \cdot 100 / 27.2} = 2.7 \text{ a.u.} \]

Energy of a He atom with the same momentum \( p_i = 2.7 \) a.u.

\[ E_i = \frac{p_i^2}{2M_{\text{He}}} \quad M_{\text{He}} = 4 \cdot 1836 \]

\[ E_i = \frac{2.7^2}{2 \cdot 4 \cdot 1836} = 0.00049 \text{ a.u.} = 13 \text{ meV} \]

or, more directly:

\[ E_i = E_e \left( \frac{m_e}{M_{\text{He}}} \right) = 13 \text{ meV} \]
Atomic and molecular beams

An ideal target requires:
• **negligible interaction** between the particles of the target gas with the environment
• **high (but not too high) density**
• well defined **velocity** (cold gas)
• low divergence, spatial confinement

Important quantities:
• capillary or nozzle diameter \( d \) and length \( L \)
• mean path length \( \Lambda \) at pressure \( P \)

Relation of particle density and pressure (at room temperature)

\[
n \ [\text{cm}^{-3}] = 2.7 \cdot 10^{16} \cdot P \ [\text{mbar}]\]
Emission characteristics

Aperture (diameter $d$)  Nozzle (diameter $d$, length $L$)

$I(\vartheta) = \cos\vartheta$

low density  high density

$\Lambda > d$  $\Lambda > d, L$  $\Lambda > d, \Lambda < L$
Effusive atomic and molecular beams

Effusive flow: low density
⇒ collisions between the particles can be neglected

\[ \Lambda_{\text{Quelle}} \gg d \]

\( \Lambda \): mean free path length
\( d \): diameter of the source aperture

The collision rate \( R \) depends on the particle velocity \( v \), target density \( n \) and collision cross section \( \sigma \)

\[ R = v_{\text{rel}} \sigma n \]

\[ \Lambda = \frac{v}{R} = \frac{v}{v_{\text{rel}} \sigma n} \]

Example:
\( v_{\text{rel}} \sigma n \) at \( T = 800 \text{ K and } P = 1 \text{ mbar} \)

\[ \Rightarrow \Lambda = 8 \text{ mm} \]

The cross section \( \sigma \) is of the order of one atomic unit:

\[ \sigma \approx \frac{\pi a_0^2}{10^{-16} \text{ cm}^2} \]
Most probable velocity in a Maxwell-Boltzmann distribution $v_w$

Mean velocity in the gas reservoir

$$v_w = \sqrt{\frac{2kT}{m}}$$

$$\bar{v} = \left(\frac{2}{\sqrt{\pi}}\right) v_w, \approx 1.13v_w.$$

Emission characteristics of a thin aperture and of a nozzle with $L = d$

Typical flux per solid angle:
$5 \times 10^{16}$ atoms /$(s \cdot sr)$

Fig. II. 1. The angular distribution of the molecules effusing through a short circular canal. The full curve is for the length and diameter being equal and the dashed curve is for negligible length (CLA 29).
Lithium atomic (effusive) beam source with oven

a) Li oven with heated nozzle
b) Heating wires
c) Skimmer
d) Heat shielding
Stern-Gerlach’s experiment

What Wikipedia says:

The real result:
without magnetic field
with magnetic field
Rabi became interested in the molecular beam method while visiting Stern in Hamburg (Stern-Gerlach experiment) because of the possibility to influence a single atom or molecule. Rabi won the Nobel Prize in 1944 for the resonance method to record the magnetic properties of nuclei.

One of his students, Norman Ramsey, built a molecular beam to measure nuclear magnetic moments, and developed the method of successive oscillatory fields. He won the 1989 Nobel Prize for this technique and the development of atomic clocks.
Supersonic gas jets

- very low divergence
- high density
- well defined velocity (cold beam: a few K)
- rather complex apparatus (vacuum system)

Gas expands from high pressure ($P_0$ 1-100 bar) through a small nozzle into high vacuum ($P_1$)

\[ \Lambda < d \]
\[ \Lambda < L \]

What happens?

- acceleration to supersonic speed
- formation of a shock front (finite pressure $P_1$ in the chamber)
- internal energy (temperature) is transformed into kinetic energy
Supersonic gas jets

During the adiabatic expansion of the gas from initial pressure $P_0$ to $P$ through the nozzle a part of the disordered thermal motion of the particles (given by $P_0$, $T_0$) is transformed into directed translational energy.

Energy conservation gives for the enthalpy $H$ (total energy):

$$H = E_{\text{therm}} + PV = \frac{3}{2} kT + kT = \frac{5}{2} kT$$

after expansion:

$$E_{\text{kin}} = \frac{5}{2} kT$$

$$v_{\text{jet}} = \left(\frac{5kT}{M}\right)^{1/2}$$

→ Temperature decreases (adiabatic expansion)

→ Kinetic energy (directed) increases

$E_{\text{kin}} \cong 70$ meV for expansion at $T = 300$ K

$v_{\text{jet}} \cong 1700$ m/s for Helium
Supersonic expansion reduces target temperature

Thin Pinhole

Long Channel

Pressure

Temperature
Shock front

Supersonic gas jets

Figure 4.9 RF discharge source for producing dissociated atoms (Toennies et al. 1979): 1, discharge tube (Pyrex); 2, resonator; 3, moderator; 4, assembly tube; 5 and 7, gaskets; 6 and 8, support flanges.

Supersonic expansion condition: Product of pressure and nozzle diameter

\[ P_0 \cdot d \equiv 10 \ldots 1000 \text{ mbar} \cdot \text{mm} \text{ (for } T=10 \ldots 300 \text{ K)} \]

- Hydrogen discharge (30 K): \[ P_0 \equiv 10 \text{ mbar, } d \equiv 3 \text{ mm} \]
- He gas jet (300 K): \[ P_0 \equiv 30 \text{ bar, } d \equiv 30 \mu \text{m} \]
Crossed molecular beams

Experiment for reactive scattering
F + D, DF + D and F + H_2 + HF + H

Pressures are indicated: (3) liquid nitrogen cooled cold trap, (5) heater, (6) liquid nitrogen feed line, (8) tuning fork chopper, (9) synchronous motor, (10) cross correlation chopper for time-of-flight velocity analysis

Y. T. Lee, Nobel lecture 1986, Molecular beam studies of elementary chemical processes
Fig. 14. Cut-out view of the experimental apparatus for the reactive scattering of electronically excited sodium atoms with various molecules.
Liquid helium droplet sources

- Embedding molecules, clusters and weakly bound complexes in helium nanodroplets allows to study them by means of absorption and emission spectroscopy with vibrational/rotational resolution.

- In the superfluid helium droplet the guest ("dopant") molecules can rotate at very low temperatures with little interactions with the medium. The dopant immersed in the centre of the droplet does not interact with any surfaces.

Some types of dopants do not penetrate the helium droplet but orbit around it.
To study molecular spectra, large helium clusters (droplets) are formed in the nozzle. In the scattering chamber they can capture a “guest” molecule.

The clusters are irradiated with a tunable laser. When the radiation is in resonance with the guest molecule, the absorbed energy evaporates helium atoms and the mass-spectrometer signal decreases.

[Helium nanodroplets (clusters)]

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[M. Hartmann et al., Science 272, 1631 (1996)]
Helium nanodroplets apparatus
Doping helium nanodroplets with molecules
Absorption spectrum of the monomer of PTCDA molecule in helium nanodroplets (black). Blue curve: convolution with a gaussian distribution (FWHM=600 cm\(^{-1}\)). Orange curve: spectrum measured in a solution. PTCDA: perylene-3,4,9,10-tetracarboxylic-3,4,9,10-dianhydride.

Frank Stienkemeier group, Freiburg University
Slowing down atoms with light

Light force due to photon scattering

- Interaction with a laser beam transfers a momentum $\hbar k$ for each absorbed photon
- Spontaneous emission is isotropic $\rightarrow$ zero net momentum transfer
- Spontaneous force is limited by saturation (maximum rate due to lifetime)

\[ \text{laser beam} \rightarrow \text{Na} \rightarrow \text{momentum transfer} \ n \cdot \hbar k \]

\[ \text{isotropic emission of} \ n \ \text{photons} \]
Optical molasses

Atoms moving toward the laser experience a force roughly proportional to their velocity: friction

Several laser beams generate a dense, viscous atom cloud
2 counterpropagating beams, tuned below resonance ($\delta<0$).

Doppler shift $\delta_v = kv$ brings 1 beam closer to resonance:

**Capture in velocity space.**

Lithium: $v_c = 4\text{m/s}; 13.3\text{mK}$

Zeeman-shift $\delta$ by B-field gradient adds position dependent detuning: Restoring force

Cooling and trapping in position space

Net force:

$$F^+\left(\delta^+(v,z)\right) + F^-\left(\delta^-(v,z)\right) \rightarrow F \approx -\eta v - \kappa z$$

**Cooling limit:** photon momentum $\hbar k$

Further cooling towards BEC: evaporation
“Single collision“ experiments: since the 1960s

Requirements:
- Crossed beams (projectile and target)
- Detectors for low-energy electrons and ions

\[
e+H \rightarrow H^++e+e
\]

H. Ehrhardt, Freiburg 1969
A modern “Franck-Hertz” experiment

- Free metal atoms are excited by electron impact
- Incident electron energy and spin are controlled
- Angle and energy dependence of the scattered or ejected electrons
- Polarization and intensity of decay photons are determined

Quantum scattering amplitudes and phases describing the interaction are determined to test theoretical models
Limitations of conventional spectrometers

Ion impact

\[ p = \sqrt{2mE} \quad v = \frac{p}{m} \]

a) 5 MeV/u \( p^+ \) \( p = 26\,000 \text{ a.u., } v = 14 \text{ a.u.} \)

b) 1 GeV/u \( U^{92+} \) \( p = 4.5 \cdot 10^8 \text{ a.u., } v = 110 \text{ a.u.} \) (relativistic)

\[ \frac{\Delta p}{p} = 10^{-5} \]

\[ \frac{\Delta p}{p} = 2 \cdot 10^{-9} \]

→ changes in projectile trajectory are not measurable!
**Double ionization**

\[ e^- + He \rightarrow He^{2+} + 3e^- \]

**Main problem:**
small solid angle

**Count rate:**

\[
\dot{N}_D = \frac{d^5 \sigma}{d\Omega_1 d\Omega_2 d\Omega_2 dE_1 dE_2} j_0 N_{T1} \Delta\Omega_1 \Delta\Omega_2 \Delta\Omega_3 \Delta E_{\text{eff}} \epsilon_1 \epsilon_2 \epsilon_3
\]

\[
\dot{N}_D = 10^{-22} \text{ cm}^2 \cdot 100 \text{ nA} \cdot 10^{11} \cdot 10^{-6} \cdot 3 \text{ eV}^2 = 0.002 \text{ 1/s}
\]

**Statistical limitations**

one hit every 10 min!

**Toroidal spectrometer**

(Université Paris XI)

**Electron gun**

Count rate: one hit every 10 min!
Imaging spectrometers

- Detection of *ions and electrons*
- Developed (ca. 1985) for target ion spectroscopy

- Recoil-Ion Momentum Spectroscopy (RIMS)
- Cold Target Recoil-ion Momentum Spectroscopy (COLTRIMS)

**Time-of-flight and position:** *full momentum information*
Large acceptance (up to $4\pi$): *multicoincidence*
Ende 13.10.2010
• Accurate measurement of the neutron magnetic moment: 
  \[ \mu_N = (-1.91304275 \pm 0.00000045) \text{ nuclear magnetons} . \]

• Test of time reversal symmetry: 
  upper limit for the neutron electric dipole moment: \((-3 \pm 5) \times 10^{-26} \text{ e\cdot cm}\)

• Hydrogen hyperfine splitting agrees with present quantum electromagnetic theory to within the accuracy of the theoretical calculation and can be used to obtain information on the proton structure.