High-energy collision processes involving intense laser fields

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Outline

• **Introduction:**
  Ways to reach the MeV energy level with laser fields

• $e^+e^-$ pair creation in laser-particle beam collisions

• Exotic atoms in very intense laser fields:
  $\mu^+\mu^-$ pair creation and nuclear effects

• Laser-assisted relativistic electron-ion recombination

• **Summary**
Introduction
Efficient coupling of a laser field with a quantized system is possible when its level spacing $\Delta \varepsilon$ compares with one of these scales:

- $\Delta \varepsilon \sim \hbar \omega$  
  Resonant (multiphoton) transition
- $\Delta \varepsilon \sim eE\Delta r$  
  Quasistatic tunneling process
- $\Delta \varepsilon \sim U_p$  
  Fast electron-induced reaction
### Typical energy scales in laser physics

<table>
<thead>
<tr>
<th>Photon energy</th>
<th>Electric work</th>
<th>Ponderomotive energy</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hbar \omega$</td>
<td>$eE \Delta r$</td>
<td>$U_p = \frac{e^2 E^2}{4m \omega^2}$</td>
</tr>
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Efficient coupling of a laser field with a quantized system is possible when its level spacing $\Delta \varepsilon$ compares with one of these scales:

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- $\Delta \varepsilon \sim eE \Delta r$: Quasistatic tunneling process
- $\Delta \varepsilon \sim U_p$: Fast electron-induced reaction

Pioneering experiments on **nuclear reactions** (Ditmire/Ledingham/Umstadter) and **pair creation** (Cowan/Kühl/Chen) where $\Delta \varepsilon \sim 1$ MeV have relied on high $U_p$ of secondary electrons.
Direct laser-induced $e^+e^-$ pair creation

Pair creation requires
\[ N\hbar\omega \sim mc^2 \sim 1 \text{ MeV} \]
multiphoton regime
\[ \xi \ll 1, \text{ rate } \sim \xi^{2N} \sim I^N \]

or
\[ E \sim E_{cr} = \frac{mc^2}{e\lambda_C} \approx 10^{16} \text{ V/cm} \]
tunneling regime
\[ \xi \gg 1, \text{ rate } \sim \exp\left(-\frac{E_{cr}}{E}\right) \]

The available frequencies ($\hbar\omega_{\text{XUV}} \sim 100\text{eV}$) and field strengths ($E_{\text{PW}} \sim 10^{12} \text{ V/cm}$) are by 4 orders of magnitude too small...
$e^+e^-$ pair production in laser-particle beam collisions
Relativistic particle beam colliding with laser pulse

Exploit relativistic Doppler shift

lab frame: $\hbar \omega \approx 100 \text{ eV} \ , \ E \approx 10^{12} \text{ V/cm}$
rest frame: $\hbar \omega'$ and $E'$ enhanced by $2\gamma$
Relativistic particle beam colliding with laser pulse

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SLAC experiment:
46 GeV electron + optical laser pulse
(D. Burke et al., PRL 1997)

Pairs were produced in two-step process through an intermediate high-energy Compton photon:

$$\Omega_c + n\omega \rightarrow e^+e^-$$

(nonlinear Breit-Wheeler process)
Relativistic particle beam colliding with laser pulse

\( \hbar \omega \approx 100 \text{ eV} \), \( E \approx 10^{12} \text{ V/cm} \)

rest frame: \( \hbar \omega' \) and \( E' \) enhanced by \( 2\gamma \)

For heavy projectiles such as nuclei Compton channel strongly suppressed and pairs would be produced directly by nuclear Coulomb field:

\[
Z + n\omega \rightarrow Z + e^+e^- \\
\text{(nonlinear Bethe-Heitler process)}
\]

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\]
Laser-dressed QED approach to trident pair creation

In the presence of the laser field, the intermediate photon may reach the mass shell:

\[ k'^2 = (q - q' + nk)^2 = 0 \]

In this case, the second-order diagram decays into two first-order diagrams, which describe the consecutive two-step mechanism via Compton emission.

Furry-Feynman diagrams:

\[
S_{fi} = -i \alpha \int d^4x \int d^4y \bar{\Psi}_{q'}(x) \gamma_\mu \Psi_q(x) D^{\mu\nu}(x-y) \bar{\Psi}_{q_-}(y) \gamma_\nu \Psi_{q_+}(y)
\]
Transition from perturbative to nonperturbative regime

46.6 GeV electron beam collides with 527 nm laser beam

![Graph showing the transition from perturbative to nonperturbative regime.](Image)

- Perturbative domain
- SLAC experiment
- Nonperturbative domain
All-optical setup for laser-induced pair creation


Combine laser-accelerated electron beam with second counter-propagating laser pulse: All-optical realization of SLAC experiment to probe the tunneling regime

Hu, Müller & Keitel, PRL 105, 080401 (2010)
Nonlinear Bethe-Heitler pair creation by ultrarelativistic proton impact on an XUV laser beam

Attosecond laser sources based on high-harmonic generation deliver $\hbar \omega \sim 100 \text{ eV}$ at focused intensities of $10^{14} \text{ W/cm}^2$

Large Hadron Collider ($\gamma \sim 3000$-7000)
Nonlinear Bethe-Heitler pair creation by ultrarelativistic proton impact on an XUV laser beam

Attosecond laser sources based on high-harmonic generation deliver 
\( \hbar \omega \sim 100 \text{ eV} \) at focused intensities of \( 10^{14} \text{ W/cm}^2 \)

\( \hbar \omega ' \sim 1 \text{ MeV} \)
\( E' \sim 10^{-4} E_{\text{cr}} \)

Large Hadron Collider
\( (\gamma \sim 3000-7000) \)

Two-photon Bethe-Heitler pair creation becomes feasible in this setup:
About one event per second, when a bunch of \( 10^{11} \) Pb ions collides with attosecond-pulse trains of 30 fs duration at 10 kHz rep rate.

Exotic atoms in very intense laser fields
The concept of laser-driven recollisions in atomic physics

Recollision can lead to...

... scattering (ATI)
... double ionization (NSDI)
... recombination (HHG)

Corkum & Krausz, Nature Phys. 2007
Exotic atoms in very intense laser fields:

a) $\mu^+\mu^-$ pair creation from laser-driven positronium
$e^+e^-$ collisions from laser-driven positronium

Due to identical charge-to-mass ratios → dynamical response of Ps unique:

High-energy reactions by laser-driven $e^+e^-$ collisions:

- Muon production ($Mc^2 = 106$ MeV)
- Pion production ($Mc^2 = 140$ MeV)

Henrich, Hatsagortsyan & Keitel, PRL 93, 013601 (2004)

\[ eE \lambda \geq \Delta \varepsilon = 2Mc^2, \]

\[ I \geq 5 \times 10^{22} \text{ W/cm}^2 \text{ at } \lambda = 1 \mu\text{m} \]
Laser-Driven Colliders

Positronium atom in two counterpropagating laser waves:

Recolliding wave packets may be microscopically small

Process observable at high Ps density and laser rep rate

luminosity enhancement due to coherent component:

$$\mathcal{L}_m = \left[ \frac{N_e(N_e - 1)}{S_b} + \frac{N_e}{a^2} \right] f$$

Hatsagortsyan, Müller & Keitel, Europhys. Lett. 76, 29 (2006); PRA 78, 033408 (2008)
A numerical example

Ps gas density = $10^{15}$ cm$^{-3}$ [Cassidy et al., PRL 95, 195006 (2005)]

Laser pulses of $10^{23}$ W/cm$^2$, 100 fs duration, 10 µm focus, 1 kHz rep rate

Luminosity $\mathcal{L} \sim 10^{28}$ cm$^{-2}$s$^{-1}$

Event rate $R \sim 1$/min ($\sigma \sim 10^{-30}$ cm$^2$)

These numbers are comparable with those at large-scale accelerators.

Muon creation in electron-positron-photon plasma also considered in:

M. H. Thoma, Rev. Mod. Phys. 81, 959 (2009)
Exotic atoms in very intense laser fields:
  b) Nuclear effects in laser-driven muonic atoms
Nuclear effects in HHG from muonic atoms

Muonic atoms very compact due to large muon mass $M = 207m$

Muonic hydrogen
Nuclear effects in HHG from muonic atoms

Muonic atoms very compact due to large muon mass $M = 207m$

Need to consider relative motion; cutoff determined by reduced mass

$$U_p = \frac{e^2 E^2}{4\omega^2 M_{\text{red}}}$$

HHG spectra of muonic hydrogen and deuterium at 60 eV and $10^{23}$ W/cm$^2$ (ELI)
Nuclear effects in HHG from muonic atoms

Muonic atoms very compact due to large muon mass $M = 207m$

Need to consider relative motion; cutoff determined by reduced mass

$$U_p = \frac{e^2E^2}{4\omega^2 M_{\text{red}}}$$

$\varepsilon_{\text{cutoff}} = 80$ keV

HHG spectra of muonic hydrogen and deuterium at 60 eV and $10^{23}$ W/cm$^2$ (ELI)

Shahbaz, Bürvenich & Müller, PRA 82, 013418 (2010)
Laser-assisted relativistic electron-ion recombination
Radiative electron-ion recombination

What happens when the system is subject to a strong laser field?

Photon energy

$$\omega = \varepsilon_{\text{kin}} + |\varepsilon_{\text{bind}}|$$

free electron
Radiative electron-ion recombination

What happens when the system is subject to a strong laser field?

Photon energy

$$\omega_\gamma = \epsilon_{\text{kin}} + |\epsilon_{\text{bind}}|$$

Quantum mechanical amplitude:

$$S_{fi} = -i \int_{-\infty}^{+\infty} dt \langle \Psi_f(t) | \hat{W} | \Psi_i(t) \rangle$$

Interaction term

$$\hat{W} = \alpha \cdot \hat{A}_\gamma$$

with

$$\hat{A}_\gamma(r, t) = \sum_{k, \rho} \sqrt{\frac{2\pi c^2}{V \omega_k}} e_{k, \rho} \left( c_{k, \rho}^+ e^{i(\omega_k t - k \cdot r)} + \text{C.C.} \right)$$
Energy spectrum of emitted photons

\[ Z = 50 \text{ (bare Sn)} \]

\[ E_i = 4.7 \text{ MeV} \]

\[ \hbar \omega_0 = 1.5 \text{ eV} \]

\[ I = 4 \times 10^{18} \text{ W/cm}^2 \]

(PHELIX @ GSI)
Energy spectrum of emitted photons

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$E_i = 4.7$ MeV

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(PHELIX @ GSI)
Energy spectrum of emitted photons

Energy of incoming electron is modulated by the field:

\[ E(\varphi) = E_i + U_p + \frac{cp_i c F_0}{E_i \omega_0} \cos \varphi \]

- \( Z = 50 \) (bare Sn)
- \( E_i = 4.7 \text{ MeV} \)
- \( \hbar \omega_0 = 1.5 \text{ eV} \)
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(PHELIX @ GSI)
Angular spectrum of emitted photons

\[ p_z(\varphi) = \frac{U_p}{c} + \frac{p_i c}{E_i} \frac{F_0}{\omega_0} \cos \varphi \]

Müller, Voitkiv & Najjari,
JPB 42, 221001 (2009)
Summary

- All-optical $e^+e^-$ pair creation via laser-accelerated electron colliding with second laser beam (two-step process)
- Multiphoton Bethe-Heitler pair creation in collisions of LHC proton beam with XUV laser pulse
- Muon pair creation feasible in the near future via strongly laser-driven positronium gas or $e^+e^-$ plasma of very high density
- Nuclear signatures in the radiative response from laser-driven muonic atoms
- Characteristic photon spectra from laser-assisted electron-ion recombination

Thank you for your attention!
Volkov states

Dirac equation:  \[
\left( ic\gamma^\mu \partial_\mu - e\gamma^\mu A_\mu - mc^2 \right) \Psi = 0
\]

Volkov solutions:

\[
\Psi_{p,s}(x) = \left( 1 - \frac{e k A}{2c(kp)} \right) u_{p,s} e^{-i(px)} e^{if(x)}
\]

\[
f(x) = \frac{e}{c(kp)} \int^{(kx)} \left[ p \cdot A(\eta) + \frac{e}{2c} A^2(\eta) \right] d\eta
\]
Theory of laser-driven muon creation

Employ Volkov states in the usual amplitude for $e^+e^- \rightarrow \mu^+\mu^-$:

$$S_{e^+e^- \rightarrow \mu^+\mu^-} = -i\alpha \int d^4x\, d^4y\, \bar{\Psi}_{p_+}(x)\gamma^\mu\Psi_{p_-}(x)$$
$$\times D_{\mu\nu}(x-y)\bar{\Psi}_{P_-}(y)\gamma^\nu\Psi_{P_+}(y)$$

Average over the momentum distribution $\Phi(p)$ in the Ps ground state:

$$S_{Ps \rightarrow \mu^+\mu^-} = \int \frac{d^3p}{(2\pi)^3} \Phi(p) S_{e^+e^- \rightarrow \mu^+\mu^-}$$
Coherent versus incoherent collisions

(a) conventional colliders: mean impact parameter $\sim$ beam size $S_b$

(b) laser-driven Ps: mean impact parameter can be microscopic ($\rho \sim$ Bohr radius $a_0$)

**Luminosity** enhancement due to coherent component:

$$\mathcal{L}_m = \left[ \frac{N_e(N_e - 1)}{S_b} + \frac{N_e}{a_0^2} \right] f$$
Comparison with HHG from highly charged ions

Similar nuclear size effects in the HHG spectra may be expected when $R_{\text{nuc}} / R_{\text{atom}}$ of similar value, e.g. about 1% for:

- muonic He$^+$
- electronic U$^{91+}$

Mass scaling leads to

$$I_p = 10 \text{ keV}$$
$$E_{\text{at}} = 1.8 \times 10^{15} \text{ V/cm}$$

Z-scaling leads to

$$I_p = 130 \text{ keV}$$
$$E_{\text{at}} = 4 \times 10^{15} \text{ V/cm}$$

→ Higher laser frequencies and intensities must be applied to U$^{91+}$.
Relativistic versus nonrelativistic spectra

![Graph showing relativistic and nonrelativistic spectra]

- \( Z = 30 \) (bare Zn)
- \( p_i = 0.3 \ mc \)
- \( \hbar \omega = 1.5 \ eV \)
- \( I = 4 \times 10^{17} \ W/cm^2 \)