Improved manipulation and detection of an ultracold $^6\text{Li}^{133}\text{Cs}$ mixture towards the investigation of the Bose polaron

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Abstract  This thesis lays the groundwork for the investigation of the Bose polaron in an ultracold Fermi-Bose mixture of $^6$Li and $^{133}$Cs atoms. The effect of radiofrequency (rf) pulses of different lengths and temporal shapes on the energy spectrum of the Bose polaron is investigated theoretically for a system consisting of a single $^6$Li impurity in a $^{133}$Cs Bose-Einstein condensate. Experimentally, we establish the basis for rf spectroscopy of the Bose polaron with a new setup to create rf pulses that couple the two energetically lowest hyperfine states of $^6$Li. This new setup enables us to reach Rabi frequencies of up to 13.5 kHz and to create arbitrary temporal pulseshapes. The detection of $^6$Li and $^{133}$Cs with a dual-species high-resolution imaging system is improved and analyzed. To this end, the Doppler shift of the imaging transition due to the absorption of photons by the lithium atoms is compensated by adapting the laser frequency during imaging. Additionally, both of the two energetically lowest hyperfine states of $^6$Li can now be imaged in one experimental realization. The imaging resolution and limiting aberrations are determined for both species using noise correlation analysis.

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1 Introduction

The description of complex many-body systems in terms of quasi-particles has been hugely successful in many different areas of physics [1–3]. Of particular interest is the so-called polaron scenario. Here, one considers a single particle (impurity) immersed in a sea of many particles of a different type (bath). This problem has first been formulated by Lev Landau in the context of an electron moving in a crystal lattice [4]. The electron attracts nearby ionic crystal cores and thus polarizes the crystal, which is shown in the left panel of Figure 1.1. As the electron moves, it drags along a cloud of polarization, which changes its properties, like its effective mass and energy, away from those of a free particle. Salomon Pekar later coined the term "polaron" to describe the resulting quasiparticle consisting of the electron and its surrounding polarization [5, 6]. In 1950, Herbert Fröhlich found a simple model for this effect by describing the electron as "dressed" by phonons, the elementary excitations of the crystal lattice [7]. Even this simple model, however, can not be solved analytically.

A very well-controlled version of the polaron can be realized in the field of ultracold atomic gases, as depicted in the right panel of Figure 1.1. Here the polaron scenario is realized with a number-imbalanced mixture. The minority atoms act as impurities, while the majority atoms form the bath. Depending on the quantum statistics of the atoms forming the bath, the corresponding quasi-particle is either called Fermi or Bose polaron. One can show that, if a bosonic bath forms a Bose-Einstein condensate (BEC), the system behaves like the solid-state polaron described by Fröhlich in the case of weak interactions [8–10]. The role of the lattice phonons is then taken on by the Bogoliubov excitations of the BEC.

In a crystal the interactions between the impurity and the surrounding lattice sites are always small and attractive. In ultracold gases, however, the interactions between the impurity and the bosons can be tuned arbitrarily using so-called Feshbach resonances (FR) [12], making it possible to go beyond the polaron scenario realized in solid state systems and access the regime of strong coupling between
Chapter 1. Introduction

Figure 1.1: Visualization of the polaron scenario in solid state physics (left) and ultracold gases (right). In the case of a crystal, the impurity is an electron that attracts nearby lattice atoms, thus polarizing the crystal. In ultracold gases, the impurity is an atom that either attracts or repels the surrounding atoms. In both cases the effect of the interactions can be described by a quasiparticle that has different properties from the bare impurity. Reprinted from [11] with permission. Credit: APS/Carin Cain.

The energy shift of the impurity due to its interactions with the bath is commonly measured with radiofrequency (rf)-spectroscopy. The principle of this type of measurement is depicted in Figure 1.2. Initially the impurity atoms are prepared in a state, here denoted $|\downarrow\rangle$, that does not interact with the surrounding bosons. By subjecting the impurities to an rf pulse, they can be transferred from the non-interacting state $|\downarrow\rangle$ to a state $|\uparrow\rangle$ in which they interact with the bosons. An rf spectrum is obtained by varying the detuning $\delta$ of the rf pulse and measuring the number of atoms transferred to the interacting state. Due to the interactions with the bath, the resonance position, i.e. the rf frequency where the number of transferred atoms is maximal, is shifted away from its bare value $\omega_0$. The sign and magnitude of this shift depend on the impurity-boson interactions, which can be...
characterized by the s-wave scattering length $a$ at low temperatures. Measuring the energy shift for different scattering lengths, two characteristic branches show, one below the energy of the bare transition (attractive branch), one above it (repulsive branch).

Our system, consisting of a mixture of bosonic $^{133}$Cs and fermionic $^6$Li, is characterized by a large mass ratio of the two species, which is predicted to lead to interesting new effects in the Bose polaron spectrum. For example, the three-body correlations between two bosons and one impurity give rise to the Efimov effect [19, 20]. It manifests itself in a series of loosely bound impurity-boson-boson trimer states that exist even in a regime where no two-body bound states exist and that exhibit a universal scaling law. Levinsen et al. [21] have investigated the effect these Efimov states have on the properties of the Bose polaron using a variational Ansatz based on the expansion of the wave function in terms of up to two Bogoliubov excitations of the BEC. They found that three-body correlations significantly lower the energy of the attractive polaron branch and can even lead to an avoided crossing with the deepest Efimov state. Signatures of the Efimov
effect have not been observed in previous Bose polaron experiments, however, because the relatively small mass imbalance of the investigated mixtures \( ^{39}\text{K}-^{39}\text{K} \) and \( ^{40}\text{K}-^{87}\text{Rb} \) leads to shallow Efimov states. In our system the Efimov effect is enhanced due to the large mass imbalance between the two species [22]. This has made it possible to observe a series of Efimov states [23–25]. Sun et al. [26] have predicted signatures of the Efimov effect in the polaron spectra of a \( ^6\text{Li} \) impurity in a \( ^{133}\text{Cs} \) bath based on a virial expansion valid at high temperatures. These findings suggest that our system is particularly well-suited to investigate the interplay between few- and many-body physics in the polaron scenario.

Probing the spectrum of the Bose polaron in a mixture of ultracold gases requires advanced methods of manipulation and detection: To perform rf spectroscopy one first needs to know how large the expected frequency shift due to the interactions is. One then needs to apply precisely tailored and characterized rf pulses to resolve this frequency shift. Here, applying pulseshapes with a low amount of spectral leakage in the Fourier domain can prevent the polaron signal from being masked by the signal of the bare transition. Additionally, one needs to be able to detect small numbers of impurity atoms, making a high resolution and good signal-to-noise ratio necessary. This is especially challenging when imaging two atomic species with different wavelengths.

This thesis reports on the realization of these methods in the existing experiment for the preparation of ultracold \( ^6\text{Li}-^{133}\text{Cs} \) mixtures. In chapter 2 we theoretically explore the influence of an rf pulse on the spectrum of the Bose polaron within the framework of the truncated basis method including one excitation of the BEC. This enables us to calculate polaron spectra even beyond linear response, i.e. for a significant fraction of impurities transferred to the interacting state, and to compare the effect of different temporal shapes of the rf pulses on the polaron spectra. Chapter 3 contains a description and characterization of our new setup to generate rf pulses, as well as our first experimental efforts to observe signatures of the Bose polaron in a \( ^6\text{Li}-^{133}\text{Cs} \) mixture. Chapter 4 focuses on the analysis and improvement of our dual-species high-resolution imaging setup. To this end, we improve the signal-to-noise ratio for lithium imaging by compensating for the Doppler shift of the atoms due to absorption of photons. We employ noise-correlation analysis to precisely characterize the resolution and limiting aberrations of our setup. In chapter 5 we summarize the results of this thesis and give an outlook on future experiments.
2 Theoretical considerations for rf spectroscopy of a Bose polaron

In recent years the Bose polaron has been under intense investigation, leading to the development of many advanced theoretical approaches to describe its properties. For example, perturbation theory up to third order in the impurity-boson interactions has allowed to derive analytic results for the polaron's energy and effective mass [27]. At stronger interactions, variational theories have been very successful in describing the properties of the Bose polaron [21, 28, 29]. Numerically exact results for the polaron ground state and effective mass have been obtained using Quantum Monte-Carlo simulations [30, 31].

In this chapter we employ a variational ansatz within the truncated basis method (TBM) to the polaron system, limiting the Hilbert space of the impurity wave function to a set number of excitations of the bath. This approach has been developed in the group of Meera Parish and Jesper Levinsen at Monash university. In particular, the methods employed here follow closely those used in [32] to investigate the dynamics of the Fermi polaron, in [15] to calculate the response of the Fermi polaron in a $^{40}$K-$^6$Li mixture to Ramsey spectroscopy, and in [21] and [29] to investigate the properties of the Bose polaron including higher-order correlations. Here, we limit our calculations to include only one Bogoliubov excitation of the BEC, meaning that we do not take three-body-correlations into account.

Of particular interest is the investigation of the energy shift due to the interactions of the impurity with the bath, i.e. the self-energy of the polaron, which can be measured using rf spectroscopy as introduced in chapter 1. To describe the resulting polaron spectra theoretically, so far the spectral function of the polaron system has been considered. However, the spectral function is probed by rf spectroscopy only within linear response theory. Experimentally, a large population transfer beyond linear response is beneficial to the signal-to-noise ratio. It is still
an open question when linear response breaks down in the system and what happens to the spectrum of the Bose polaron once we go beyond the linear response regime.

To address these open questions we extend the TBM to explicitly include effects of the rf pulse that is used to measure polaron spectra experimentally. This will give us information on the Rabi frequency and pulse length needed to resolve the spectral features of the polaron at different interaction strengths. Additionally, we investigate the linearity of the response for different parameters of the rf pulse. As linear response theory is a prerequisite for the rf spectrum to represent the spectral function of the system, these considerations are of great importance for the experimental investigation of the Bose polaron. We show that our approach reproduces the spectral function, calculated also within the TBM and broadened appropriately, well for short rf pulses. We extend our method to include arbitrary temporal shapes of the rf pulses and compare spectra calculated for different pulse shapes. We perform all calculations explicitly for the mass imbalance of a $^6$Li - $^{133}$Cs mixture.

### 2.1 Model of the interactions

We consider the dynamics of a single impurity immersed in a homogeneous, weakly interacting BEC. The impurity is modeled as a two-level system of two hyperfine states that are coupled by an rf pulse. We assume the impurity to be non-interacting with the BEC in in state $|\downarrow\rangle$, while the interactions between the impurity in state $|\uparrow\rangle$ and the BEC can be varied using a Feshbach resonance.

This is a good approximation for a cold and dilute sample of $^6$Li atoms in the two lowest hyperfine states, immersed in the centre of a trapped $^{133}$Cs BEC, where the density can be assumed constant.

The bare Hamiltonian of the system is given by

$$H_0 = \sum_k \left( E_k \beta_k^+ \beta_k + \epsilon_k c_{\uparrow k}^+ c_{\downarrow k} + \epsilon_k c_{\downarrow k}^+ c_{\uparrow k} + (\epsilon_k^d + \nu_0) d_k^+ d_k \right), \quad (2.1)$$

where $c_{\uparrow k}$ ($c_{\downarrow k}$) creates (annihilates) an impurity in state $|\uparrow\rangle$ with momentum $k$, and $\epsilon_k = \frac{k^2}{2m_{\text{Li}}}$ denotes the impurity kinetic energy. $d_k^+$ ($d_k$) creates (annihilates) a
closed-channel Li-Cs dimer with the momentum $k$, kinetic energy $\epsilon_k^l = \frac{k^2}{2(m_{\text{Li}}+m_{\text{Cs}})}$ and binding energy $\nu_0$.

The interactions of the bosons with each other of the form

$$H_{BB} = g_{BB} \sum_{p,p',q} b_{p+q}^\dagger b_{p'}^\dagger q b_p q,$$

with the coupling strength $g_{BB}$ are implicitly included in the first term of $H_0$ via the Bogoliubov transformation [33]

$$b_k^\dagger = u_k \beta_k^\dagger - v_k \beta_{-k}$$

with $u_k = \sqrt{\left(\frac{\epsilon_k^B + \mu}{E_k} + 1\right)/2}$ and $v_k = \sqrt{\left(\frac{\epsilon_k^B + \mu}{E_k} - 1\right)/2}$. Here the operator $\beta_k^\dagger (\beta_k)$ creates (annihilates) an excitation with momentum $k$ following the dispersion relation $E_k = \sqrt{\epsilon_k^B (\epsilon_k^B + 2\mu)}$, where $\mu = 4\pi\hbar^2 n a_B / m_{\text{Cs}}$ is the chemical potential for the interspecies scattering length $a_B$ and $\epsilon_k^B = \frac{k^2}{2m_{\text{Cs}}}$ is the single-particle energy. We choose the energy of the interacting BEC without the impurity as zero.

The interaction between the impurity in state $|\uparrow\rangle$ and the bosons are included via the two-channel Hamiltonian [34]

$$H_{\text{int}} = g \sqrt{n} \sum_q d_q^\dagger c_q + g \sum_{q,k\neq0} d_q^\dagger c_{q-k} b_k + h.c.,$$

where the impurity and boson scatter with the coupling strength $g$ for relative momenta up to a momentum cutoff $\Lambda$, forming a closed-channel molecule. The scattering length $a$ and effective range $r_{\text{eff}}$ can be related to the bare model parameters $\nu_0, g$ and $\Lambda$ by computing the scattering amplitude $f(k)$ and comparing it to its general expansion for low energies,

$$f(k) = \frac{1}{a^{-1} - \frac{1}{2} r_{\text{eff}} k^2 + i k},$$

This gives

$$a = \mu g^2 \frac{1}{2\pi} \frac{1}{d^2 m_{\text{red}} \Lambda} - \nu_0 \quad \text{and} \quad r_{\text{eff}} = -\frac{8\pi}{m_{\text{red}} g^2},$$

where $m_{\text{red}} = \frac{m_{\text{Li}} m_{\text{Cs}}}{m_{\text{Li}} + m_{\text{Cs}}}$ denotes the reduced mass of an impurity-boson pair. The calculation is performed for example in [35] or the appendix of [36]. In practice, we set the scattering length $a$ and the range parameter $R^* = -r_{\text{eff}}/2$, which fixes
the dimer binding energy $\nu_0$. Then we choose the momentum cutoff $\Lambda$ to be as high as necessary to achieve convergent results.

The states $|\uparrow\rangle$ and $|\downarrow\rangle$ are coupled by the rf pulse, resulting in the Hamiltonian (within the rotating wave approximation)

$$H_{rf} = \sum_k \delta c_{j,k}^\dagger c_{j,k} + \frac{\Omega_0}{2i} \sum_k \left( e^{i\phi} c_{j,k}^\dagger c_{j+1,k} + e^{-i\phi} c_{j,k}^\dagger c_{j-1,k} \right)$$

(2.7)

where $\delta = \omega - \omega_0$ is the frequency detuning, $\Omega_0$ the resonant Rabi frequency and $\phi$ the phase of the pulse. Since we only investigate single rf pulses here, we can take $\phi \equiv 0$.

The total Hamiltonian of the polaron system under the influence of an rf-pulse is given by $H = H_0 + H_{\text{int}} + H_{rf}$. We will now proceed to estimate the time evolution of the impurity under this Hamiltonian using the TBM.

### 2.2 Truncated basis method

At very low temperatures it is a good approximation to expand the states spanning the Hilbert space of the polaron wave function in terms of the excitations of the BEC. Here, we only include one Bogoliubov excitation, leading to the following ansatz for the form of the truncated basis states:

$$|\Phi\rangle = \left( \alpha_0 c_{j,0}^\dagger + \sum_{k \neq 0} \alpha_k c_{j-k}^\dagger c_{j+k}^\dagger + \gamma_0 c_{j,0}^\dagger + \sum_{k \neq 0} \gamma_k c_{j-k}^\dagger c_{j+k}^\dagger + \delta_0 d_{j,0}^\dagger \right) |\text{BEC}\rangle ,$$

(2.8)

where $|\text{BEC}\rangle$ denotes the ground state of the weakly interacting BEC, given by the condition $\beta_k |\text{BEC}\rangle = 0$ for $k \neq 0$, and the the weights of the different contributions $\xi_i = \{ \alpha_0, \alpha_k, \gamma_0, \gamma_k, \delta_0 \}$ are the variational parameters.

We find the eigenenergies and eigenstates of the Hamiltonian $H$ in the part of the total Hilbert space that is spanned by states of the form given in Equation 2.8, i.e. the truncated Hilbert space. This is done in a variational approach by determining the coefficients $\xi_i$ via the stationary condition

$$\frac{\partial}{\partial \xi_i} \langle \Psi | H - E | \Psi \rangle = 0.$$  

(2.9)
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We obtain the five coupled linear integral equations

\[ \alpha_0(E - \delta) = \frac{\Omega_0}{2i} e^{i\phi} \gamma_0 \]  
\[ \alpha_k(E - E_k - \epsilon_k - \delta) = \frac{\Omega_0}{2i} e^{i\phi} \gamma_k \]  
\[ \gamma_0 E = g\sqrt{n_0} - \frac{\Omega_0}{2i} e^{-i\phi} \alpha_0 \]  
\[ \gamma_k(E - E_k - \epsilon_k) = g u_k \delta_0 - \frac{\Omega_0}{2i} e^{-i\phi} \alpha_k \]  
\[ \delta_0(E - \nu_0) = g\sqrt{n_0} \gamma_0 + g \int_k u_k \gamma_k \frac{dk}{(2\pi)^3}, \]  

that we can solve numerically for the coefficients \( \xi_i \). To do this we first notice that, because we only consider isotropic s-wave interactions, \( u_k, \alpha_k \) and \( \gamma_k \) do not depend on the orientation of \( k \), so that we can reduce the equations to one dimension. In particular, the integral in Equation 2.10e becomes

\[ \int u_k \gamma_k \frac{dk}{(2\pi)^3} = \frac{1}{2\pi^2} \int u_k \gamma_k k^2 dk. \]  

Now we can solve the equations in a discretized form, meaning we can choose a grid of \( N \) points \( k_i \) up to some cutoff-momentum \( k_N = \Lambda \). This discretization is performed using the method of Gauss-Legendre discretization.

2.3 Gauss-Legendre discretization

In this section we will briefly introduce the method of Gauss-Legendre discretization following Press et al. [37]. In general the most straightforward way of discretizing an integral over a function \( f(x) \) is to divide the interval of interest, say \([-1,1]\), into equal steps, thus approximating the area under the curve \( f(x) \) by a sum of trapezoids:

\[ \int_{-1}^{1} f(x) \, dx \to \sum_{x_i} f(x_i) \Delta x, \]  

where \( \Delta x = \frac{2}{N-1} \), with \( N \) the chosen number of steps on the interval \([-1,1]\), and \( x_1 = -1, x_2 = -1 + \Delta x, ..., x_N = 1 \).
More accurate results can be obtained by giving oneself more degrees of freedom in the discretization. This can be done by no longer demanding the abscissas \( x_i \) at which the function is evaluated to be equidistant:

\[
\int_{-1}^{1} f(x) \, dx \rightarrow \sum_{x_i} f(x_i) \Delta x_i,
\]

where \( \Delta x_i \) is the weight of the evaluation at abscissa \( x_i \), which no longer needs to be the same for different \( i \).

But how do we find the abscissas and weights that give the most accurate result? A popular approach to this question is the method of Gaussian quadratures. Here you demand the discretization to be exact for a set of polynomials up to order \( N \), which is the number of discretization points you are using, and calculate the abscissas and weights from this condition. It can be shown that this method is exact for polynomials up to order \( 2N - 1 \). In the special case of the Gauss-Legendre discretization, the Legendre polynomials

\[
P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2 - 1)^n
\]

are used for determining the abscissas and weights. This method is very accurate for functions that are well approximated by polynomials.

To apply this method to arbitrary momentum intervals \([0, k_N]\), we use the transformation

\[
k = \frac{2}{1 + x} - 1 \quad \frac{2}{k_N + 1} - 1 < x < 1.
\]

The abscissas \( k_i \) and weights \( \Delta k_i \) are then calculated from the momentum cutoff \( k_N \equiv \Lambda \) and the number of points \( N \), and we obtain

\[
\int u_k \gamma_k \frac{dk}{(2\pi)^3} \rightarrow \frac{1}{2\pi^2} \sum_{i=1}^{N} u_k \gamma_k k_i^2 \Delta k_i.
\]

In practice, the number of points used for calculations was chosen by starting from a small number of points \( N \approx 20 \) for some typical set of parameters and then increasing the number of points until convergence was obtained.

By numerically solving the five equations Equation 2.10 in discretized form, we diagonalize the projection of the total Hamiltonian on the truncated Hilbert space. We obtain \( 2N + 3 \) eigenenergies \( E_j \) and eigenvectors \( |\Phi_j\rangle \) of the form Equation 2.8.
These are our starting point to calculate the time evolution of the impurity under the influence of both interactions with the BEC and the rf pulse.

### 2.4 Time evolution

We start out with a zero momentum impurity in the non-interacting state:

$$|\Psi_0\rangle \equiv |\Psi(t=0)\rangle = c_{j0}^\dagger |\text{BEC}\rangle$$  \hspace{1cm} (2.17)

This assumption that the impurity initially occupies the zero momentum state is justified for low temperatures. We calculate the wave function after it evolved with the full Hamiltonian $H = H_0 + H_{int} + H_{rf}$ for a given time $t$, by using the approximate eigenvectors and eigenenergies obtained by solving Equation 2.10:

$$|\Psi(t)\rangle = e^{-iHt} |\Psi_0\rangle \approx \sum_j e^{-iE_j t} |\Phi_j\rangle \langle \Phi_j | \Psi_0\rangle$$  \hspace{1cm} (2.18)

This way, we obtain the probability of the impurity to be in the interacting state $|\uparrow\rangle$ after a time $t$:

$$\langle \Psi(t)|n_\uparrow|\Psi(t)\rangle \approx \sum_{ij} \langle \Psi_0|\Phi_i\rangle e^{iE_i t} \langle \Phi_i|n_\uparrow|\Phi_j\rangle e^{-iE_j t} \langle \Phi_j | \Psi_0\rangle$$

$$= \sum_{ij} e^{-i(E_j-E_i)t} \gamma^*_{j0} \gamma^*_{i0} \left( \gamma_{j0} \gamma_{i0} + \sum_{k \neq 0} \gamma^*_{k} \gamma^*_{-k} \right),$$  \hspace{1cm} (2.19)

where we have used $n_\uparrow = c_{j0}^\dagger c_{j0} + \sum_k c_{j+k}^\dagger c_{j-k}$.

To calculate the polaron spectrum at a given interaction strength, we vary the detuning $\delta$ of the rf pulse. For each value of $\delta$ we diagonalize the Hamiltonian and calculate the probability $n_\uparrow$ of the impurity to be in state $|\uparrow\rangle$ according to Equation 2.19.

We can generalize these results for rf pulses of arbitrary temporal shape. In this case, we sample the now time-dependent Rabi frequency $\Omega(t)$ at $n$ equidistant points. This gives a set of $n$ Hamiltonians that only differ in the Rabi frequency. We then diagonalize every Hamiltonian within the TBM exactly as above. We
propagate the wavefunction for a time $\Delta t = \tau/n$ with each of the $n$ Hamiltonians, where $\tau$ is the pulse length:

$$|\Psi(t)\rangle = e^{-iH_n \Delta t} \ldots e^{-iH_1 \Delta t} |\Psi_0\rangle.$$  

(2.20)

This way, we obtain for the time-evolution of the probability to be in $|\uparrow\rangle$:

$$\langle \Psi(t)|n_\uparrow|\Psi(t)\rangle = \langle \Psi_0|e^{iH_1 \Delta t} \ldots e^{iH_n \Delta t} n_\uparrow e^{-iH_1 \Delta t} \ldots e^{-iH_n \Delta t} |\Psi_0\rangle$$

$$= \sum_{j_1 \ldots j_n} \sum_{i_1 \ldots i_n} \langle \Psi_0|\Phi^{(1)}_{j_1}\rangle e^{iE^{j_1}_{\uparrow} \Delta t} \langle \Phi^{(1)}_{j_1}|\Phi^{(2)}_{j_2}\rangle e^{iE^{j_2}_{\downarrow} \Delta t} \ldots \langle \Phi^{(n)}_{i_n}|n_\uparrow|\Phi^{(n)}_{i_n}\rangle$$

$$\times e^{-iE^{(n)}_{\uparrow} \Delta t} \langle \Phi^{(n)}_{i_n}|\Phi^{(n-1)}_{i_{n-1}} \ldots \langle \Phi^{(1)}_{i_1}|\Psi_0\rangle.$$  

(2.21)

Starting from Equation 2.21, we can sample $n_\uparrow$ for different pulse lengths $\tau$, or obtain spectra by varying $\delta$, as before. In the following section we compare the spectra obtained in this way both to spectra obtained with a constant Rabi frequency, and to the spectral function $A(\delta)$. The spectral function is obtained from the Fourier transform of the overlap function

$$S(t) = \langle \Psi_0|e^{iH_0 t} e^{-iH_{\text{int}} t}|\Psi_0\rangle$$  

(2.22)

within linear response theory:

$$A(\delta) = \Re \int_0^\infty \frac{dt}{\pi} e^{i\delta t} S(t).$$  

(2.23)

A proof for this well-known relation can be found for example in Appendix B of [38]. We also investigate the breakdown of linearity in the repulsive polaron branch by looking at the system’s response at long and intermediate times for different positive frequency detunings.

The rf pulses we consider here have a temporal envelope given by

$$\Omega(t) = \Omega_{\text{max}} \left(0.42 - 0.5 \cos \left(\frac{2\pi t}{\tau}\right) + 0.08 \cos \left(\frac{4\pi t}{\tau}\right) \right) \quad 0 \leq t \leq \tau,$$

(2.24)

$\Omega_{\text{max}}$ is the maximal Rabi frequency and the mean Rabi frequency experienced by the impurities during one pulse is $\Omega_{\text{mean}} = 0.42 \Omega_{\text{max}}$. Pulses of this form are called Blackman pulses. Blackman pulses are commonly used in many different applications because of their large suppression of sidelobes in the Fourier domain.
We describe our experimental setup for generating different temporal shapes of rf pulses and investigate the effects of using Blackman shaped pulses in chapter 3. Discretizing the time dependence of the Rabi frequency in the simple way described above, convergent results can be obtained by sampling as little as \( n = 9 \) time steps. We use \( n = 11 \) for the results below.

### 2.5 Results

We apply the theoretical methods outlined above to obtain explicit results for the \(^6\text{Li} - ^{133}\text{Cs}\) mixture. In our calculations, Planck's constant \( \hbar \), the BEC density \( n \), and the impurity mass \( m_{\text{Li}} \) are set to one. This means that energies are calculated in units of \( [E] = \frac{\hbar^2 n^{2/3}}{m_{\text{Li}}} \), angular frequencies correspondingly in units of \( [\delta, \Omega] = \frac{\hbar n^{2/3}}{m_{\text{Li}}} \), and times in units of \( [\tau] = \frac{2\pi m_{\text{Li}}}{\hbar n^{2/3}} \). Length scales are given in units of the interparticle spacing \( n^{-1/3} \).

In our calculations, we use a value of \( \Omega_{\text{max}} = 0.1 \) for the Rabi frequency, which corresponds to a typical experimental value of \( \Omega_{\text{max}} = 2\pi \times 1.6 \text{kHz} \) for a BEC density of \( n = 3.0 \times 10^{13} \ \text{cm}^{-3} \), and to a mean Rabi frequency of \( \Omega_{\text{mean}} = 0.042 \). We set the boson-boson scattering length to \( a_B = 0.01 \). A small value is chosen to ensure Bogoliubov theory is applicable to the bath. The exact value of \( a_B \) is not expected to have a large influence on the results. For the effective range parameter we set \( R^* = 0.115 \). This value was estimated for the Li-Cs FR at 843 G by Shimpei Endo from the resonance of the first excited Li-Cs Efimov trimer for a BEC density of \( n = 7.0 \times 10^{14} \ \text{cm}^{-3} \).

Figure 2.1 shows a comparison between the spectral response of the system after an rf pulse and the spectral function. We show results for Blackman shaped rf pulses with \( \Omega_{\text{max}} = 0.1 \) and different pulse lengths of \( \tau = 5, 10 \) and 20. We compare our results with the spectral function. It is calculated according to Equation 2.23, and to obtain smooth spectra the discrete points that are obtained are broadened with a Gaussian lineshape. According to [40], for a Blackman pulse the full-width-half-maximum of the spectral response is given by \( 2.2988/\tau \). To make a comparison to the spectral function, we obtain the corresponding standard deviation \( \sigma \) of a Gaussian via \( \sigma = \frac{2.2988}{\tau \sqrt{2\ln2}} \) and broaden the spectral functions accordingly. We show results for attractive, resonant, and repulsive impurity-boson interactions. All curves in this plot are normalized to a unit integral.
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Figure 2.1: Comparison of spectra for different pulse lengths to the corresponding spectral functions. Top: attractive interactions, Middle: resonant interactions, Bottom: repulsive interactions. Spectra are obtained from Blackman shaped rf-pulses with $\Omega_{max} = 0.1$. Spectral functions are broadened corresponding to the pulse lengths by convoluting with a Gaussian. The overall agreement is very good. All curves are normalized to a unit integral.
The agreement between the appropriately broadened spectral functions and the rf spectra obtained with Blackman pulses is excellent for all the interaction strengths that were considered. A slight underestimation of the broadening is visible in all spectra. This can be explained by the fact that the Gaussian line shape employed in the spectral function naturally has a narrower central maximum than the Fourier transform of a Blackman pulse. This could be resolved by broadening the spectral function with the Fourier transform of a Blackman pulse instead of a Gaussian.

For the longest pulse with $\tau = 20$, a maximum of about 13% of the atoms are transferred to the interacting state for attractive interactions ($a^{-1}_{\text{LiCs}} = -3$). At unitarity ($a^{-1}_{\text{LiCs}} = 0$) the maximal atom transfer is about 10%, and for repulsive interactions ($a^{-1}_{\text{LiCs}} = 3$) only 5%. Because of the relatively small atom transfer, a linear response and thus a good agreement with the spectral function is expected for the rf pulse employed here. In the following, we investigate how linear response breaks down depending on the frequency detuning.

Figure 2.2 shows the fraction of atoms transferred to the interacting state $|\uparrow\rangle$ versus the length $\tau$ of a Blackman shaped pulse with Rabi frequency $\Omega_{\text{max}} = 0.1$. We give results for a number of detunings $\delta$ across the repulsive polaron continuum. A similar investigation for the attractive branch does not make sense because it is given by a $\delta$-function within our model, which is just broadened differently for different pulse lengths. The decrease of the broadening for longer pulses can not yield information on the linearity of the system’s response.

We can see that the slope of the response is largest directly on the repulsive polaron branch at $\delta \approx 2$ and decreases for both bigger and smaller detunings. For very large detunings the response becomes nonlinear the fastest, and the fraction of transferred atoms lies below that expected for linear response. This means that for long times $\tau > 20$ the weight in the high-frequency tail of the spectrum is expected to decrease below what would be obtained within linear response.

Figure 2.3 shows a closeup of Figure 2.2 for $\tau \leq 20$, i.e. the times relevant for the spectra shown in Figure 2.1. As expected we find an approximately linear time dependence of the number of transferred atoms for all detunings.
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Figure 2.2: Time evolution of the impurity population in the interacting state for different detunings along the repulsive polaron branch. The length of Blackman pulses with $\Omega_{\text{max}} = 0.1$ was varied. The response becomes nonlinear for $\tau \gtrsim 20$. Linearity breaks down most rapidly in the high-frequency tail of the spectrum.

Figure 2.3: Time evolution of the impurity population in the interacting state for intermediate times and different detunings along the repulsive polaron branch. Closeup of Figure 2.2 for the timescales relevant in Figure 2.1. Here the response is linear for all detunings. The slope is highest directly on the repulsive branch at $\delta \approx 2$ and decreases for both higher and lower detunings.
**Figure 2.4:** Comparison of the spectrum for a long Blackman pulse (blue solid line) and the corresponding spectral function (black dashed line) at unitarity ($a_{\text{LiCs}} = 0$). The spectrum is calculated for a Blackman pulse with Rabi frequency $\Omega_{\text{max}} = 0.1$ and pulse length $\tau = 40$. This corresponds to a maximal population transfer of 35%. Small bumps in the spectrum are a sign of bad convergence. The spectrum is very similar to the spectral function even though the response of the system is no longer linear for this pulse.

**Figure 2.4** shows a spectrum at unitarity for a longer Blackman pulse of $\tau = 40$ at unitarity. These parameters correspond to a maximal transfer of 35% of the atoms to the interacting state. Even for a pulse this long, where the response is no longer expected to be linear, the spectrum is quite similar to the spectral function, with slight deviations in the repulsive polaron branch. The small bumps in the spectrum are numerical artifacts due to insufficient number of points $N$ in the discretization. Our results suggest that spectra remain close to the spectral function even for significant population transfer. However, further investigations with higher number of points and for longer times at different interaction strengths are necessary.

We can employ the generalization of the system’s time-evolution to time-dependent Rabi frequencies to compare polaron spectra obtained from different pulse shapes. **Figure 2.5** shows a comparison for spectra obtained using rectangular pulses (pulses with a constant Rabi frequency $\Omega_{\text{mean}} = \Omega_{\text{max}}$) and Blackman pulses for different impurity-boson interaction strengths. All spectra are calculated for pulses of
length $\tau = 10$ and the same mean Rabi frequency $\Omega_{\text{mean}} = 0.042$ for comparability. The spectra for both methods behave exactly as expected for all interaction strengths considered: The spectra show the same features in both cases, but for the ones calculated with rectangular pulses we find narrower central maxima of the polaron branches at the expense of spectral leakage (sidelobes). This shows that our generalization to time-dependent Rabi frequencies works well and can be used in the future to model the effect different pulse shapes have on the polaron spectrum, or to test if a certain pulse with a given shape, length and Rabi frequency is suited to resolve the spectral features of interest for some given experimental conditions.

To conclude, we have investigated the effect of rf pulses with different lengths and temporal shapes on the time evolution of the system consisting of a single $^6$Li impurity (modeled as a two-level system) in a weakly interacting $^{133}$Cs BEC. For short times one recovers the system’s (broadened) spectral function for attractive, resonant and repulsive impurity-boson interactions. We are now able to investigate the spectral response of the system beyond the linear regime, which is convenient as a large fraction of transferred atoms leads to a good signal-to-noise ratio experimentally. We analyzed the time-dependence of the response for different detunings along the repulsive polaron branch and found that linearity breaks down first for very large detunings. Extending our approach to time-dependent Rabi frequencies enabled us to compare polaron spectra for different pulse shapes. In principle one could calculate spectra for even longer pulses with an atom transfer close to unity. This would require longer computing times as convergence is not obtained in the large frequency tail of the spectrum for the number of points used here.

In the next chapter we introduce our experimental approach to rf spectroscopy of the Bose polaron. We introduce and characterize our setup to create pulses with arbitrary temporal pulse shapes. We also present our first attempt to see a polaron signal, and draw conclusions for future experiments.
Figure 2.5: Comparison between polaron spectra for Blackman (red) and rectangular (black) pulses for different interaction strengths. Top: attractive interactions, Middle: resonant interactions, Bottom: repulsive interactions. All spectra are taken for the same pulse length \( \tau = 10 \) and the same mean Rabi frequency \( \Omega_{\text{mean}} = 0.042 \). The spectra calculated for Blackman pulses show broader spectral features and no sidelobes.
3 Radiofrequency spectroscopy

Electrical and magnetic fields of different frequencies are commonly used to probe and manipulate ultracold atoms in their ultrahigh-vacuum environment. For example, we can make use of optical absorption to determine the shape of the atomic sample and the number of atoms (see chapter 4). Another very successful probing technique in the field of ultracold atoms is radiofrequency (rf) spectroscopy. Here, fields with frequencies in the kHz to GHz regime are employed to measure the energies of atomic transitions and manipulate the state the atoms are in. Because the employed frequencies are so small, the atomic recoil upon absorption or emission is negligible and only the internal state of the atom is changed.

Some examples of how rf spectroscopy has been employed in the field of ultracold atoms are measuring the mean-field energy shift due to interactions in a Fermi sea of $^6$Li atoms [41], the binding energy of dimer states [42], and the energy shift due to interactions of one atomic species with another, like in recent polaron experiments [17, 18]. It is also commonly used in ultracold atom labs for magnetic field calibration, because from a measurement of a hyperfine transition frequency the external magnetic field can be inferred. A good introductory summary on rf spectroscopy in ultracold Fermi gases can be found in [43].

In this chapter we start out by introducing a simple semiclassical model for the interaction of $^6$Li atoms with static and oscillating magnetic fields. Then we introduce and characterize our new setup to create rf pulses. This includes a new amplifier that increases the power of the field the atoms interact with. We also implemented the possibility to create arbitrary temporal pulse shapes.
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3.1 Interaction of $^6$Li with magnetic fields

$^6$Li has a nuclear magnetic moment $I = 1$ and a spin $S = 1/2$. The ground state has no angular momentum $L$, and thus $J = S$. The resulting magnetic moment interacts with a static external field via magnetic dipole interaction. This leads to the Zeeman energy shift

$$E = -\mu B = g_F \mu_B m_F B,$$

with $g_F$ the gyromagnetic ratio, $\mu_B$ the Bohr magneton and $m_F$ the projection of the magnetic moment on the quantization axis given by the external field. As long as this energy is small compared to the hyperfine interaction, $I$ and $S$ couple to the total atomic angular momentum $F$. For high magnetic fields, however, the nuclear magnetic moment and the total angular momentum decouple. This is called the Paschen-Back effect, which takes place already at relatively low fields above $\approx 50$ G for $^6$Li. In our experiments we usually work with fields ranging from a few hundred to a thousand Gauss, so $I$ and $S$ are always decoupled. Nevertheless we label the energy states of $^6$Li with the quantum numbers at zero field, $|F, m_F\rangle$.

In our experiments we prepare and manipulate the two lowest hyperfine states of $^6$Li, namely $|\downarrow\rangle = |F = 1/2; m_F = +1/2\rangle$ and $|\uparrow\rangle = |F = 1/2; m_F = -1/2\rangle$. Their energies for magnetic fields of up to 1000 G are plotted in Figure 3.1, using the analytical Breit-Rabi Formula with atomic constants taken from [44]. The splitting between the two states is $\Delta E \approx \hbar \times 75$ MHz and changes only weakly with the magnetic field in the experimentally relevant field range.

Now consider what happens if the external field is no longer static but takes on the oscillatory form $B = B_0 \cos(\omega t) e_x$. This semi-classical approach was first performed by Rabi in 1937 [45], here we follow standard textbooks like [46, 47].

If the external field is close to resonance with the relevant transition $|\uparrow\rangle \rightarrow |\downarrow\rangle$, it is a good approximation to treat the $^6$Li atom as a two-level system with the Hamiltonian $\hat{H}_0 = E_1 |\downarrow\rangle \langle \downarrow| + E_2 |\uparrow\rangle \langle \uparrow|$. Treating the magnetic dipole Hamiltonian

$$\hat{H}_1(t) = -\mu B$$

(3.2)
Figure 3.1: Level diagram of the ground state of $^6\text{Li}$ . The energies are plotted according to the Breit-Rabi formula. The two energetically lowest hyperfine states $|\downarrow\rangle = |F = 1/2, m_F = +1/2\rangle$ and $|\uparrow\rangle = |F = 1/2, m_F = 0\rangle$ are coupled by rf pulses in our experiments. A magnetic field of 1 G corresponds to $10^{-4}$ T.

as a perturbation, we can expand the atomic wavefunction in terms of the eigenstates of the unperturbed Hamiltonian:

$$\Psi(\mathbf{r}, t) = c_1(t) e^{-iE_1/\hbar} |\downarrow\rangle + c_2(t) e^{-iE_2/\hbar} |\uparrow\rangle$$  \hspace{1cm} (3.3)

and use it to solve the time-dependent Schrödinger equation

$$i\hbar \frac{\partial \Psi}{\partial t} = (\hat{H}_0 + \hat{H}_1) \Psi.$$  \hspace{1cm} (3.4)

We obtain the two coupled differential equations

$$i \dot{c}_1(t) = \frac{\Omega_0}{2} (e^{i(\omega - \omega_0)} + e^{-i(\omega + \omega_0)}) c_2(t)$$  \hspace{1cm} (3.5)

$$i \dot{c}_2(t) = \frac{\Omega_0}{2} (e^{i(\omega + \omega_0)} + e^{-i(\omega - \omega_0)}) c_1(t),$$  \hspace{1cm} (3.6)

where the Rabi frequency

$$\Omega_0 = -\frac{\langle \downarrow | \mathbf{\mu} B |\uparrow\rangle}{\hbar}$$  \hspace{1cm} (3.7)

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Figure 3.2: Rabi oscillations between the two lowest hyperfine states of $^6$Li. The error bars correspond to the standard error of the mean. From the sinusoidal fit we obtain a resonant Rabi frequency of $\Omega_0 = 2\pi \times (2.3428 \pm 0.0001)$ kHz and a contrast of $1.00 \pm 0.01$.

is defined and $\omega_0 = (E_1 - E_1)/\hbar \approx 2\pi \times 75$ MHz is the resonance frequency of the transition. Close to resonance the terms oscillating with $\omega + \omega_0$ can be neglected (rotating wave approximation). Differentiating the second line and inserting the first one we arrive at

$$\dot{c}_2(t) + i(\omega - \omega_0) c_2(t) - \left| \frac{\Omega}{2} \right|^2 c_2(t) = 0 \quad (3.8)$$

For the initial conditions $c_2(0) = 1$ and $c_1(0) = 0$ we can obtain the probability of a single atom to be in state $|\downarrow\rangle$ as

$$|c_1(t)|^2 = \frac{\Omega_0^2}{\Omega^2} \sin^2 \left( \frac{\Omega t}{2} \right), \quad (3.9)$$

where $\Omega = \sqrt{\Omega_0^2 + (\omega - \omega_0)^2}$ is the generalized Rabi frequency. Correspondingly, the probability to be in $|\uparrow\rangle$ is

$$|c_2(t)|^2 = 1 - \frac{\Omega_0^2}{\Omega^2} \sin^2 \left( \frac{\Omega t}{2} \right). \quad (3.10)$$

Thus, the population oscillates in time between the two states under the influ-
ence of an external field and we can use a $\pi$-pulse, i.e. a resonant pulse of length 
$\tau = \pi/\Omega_0$, to transfer all atoms from $|\uparrow\rangle$ to $|\downarrow\rangle$ or vice versa.

We measure Rabi oscillations in our experiment by preparing a sample of $^6\text{Li}$
atoms selectively in the state $|\uparrow\rangle$ and applying a resonant rf pulse of varying length.
We measure the number of atoms remaining in the state $|\uparrow\rangle$ via high-field absorp-
tion imaging. Typical results are shown in Figure 3.2. From a sinusoidal fit of
the data the resonant Rabi frequency $\Omega_0$ can be extracted, which is relevant for
choosing appropriate pulselengths to manipulate the atoms.

### 3.2 New setup for generating rf-pulses

We can see from Equation 3.9 that a high resonant Rabi frequency allows us to
drive the rf transition with short pulses. To increase the achievable Rabi fre-
quency, a new rf amplifier was implemented and characterized in the scope of
this thesis. We use the ZHL-100W-52+ from Mini Circuits. In addition to its high
output power of up to 100 W, it tolerates backreflections from the antenna. We
measure the achieved Rabi frequency for different output powers to characterize
the new amplifier. The results are shown in Figure 3.3. As the Rabi frequency
is proportional to the field amplitude (see Equation 3.7), it is proportional to the
square root of the power $P$. In the experiment, we set the power $x$ in dBm which
relates to the power $P$ in mW as

$$P(\text{mW}) = 10^{\frac{x(\text{dBm})}{10}}.$$  \hfill (3.11)

As the Rabi frequency depends on the power as $\Omega_0 \sim \sqrt{P}$, we obtain

$$\Omega_0(x) = (2.381 \pm 0.009) \times 10^{\frac{x(\text{dBm})}{20}}$$ \hfill (3.12)

for the calibration in dependence of the power $x$ in dBm. We plot the Rabi fre-
quency on a logarithmic scale to obtain a linear dependence. With the new am-
plifier we can achieve a maximal Rabi frequency of $\Omega = 2\pi \times 13.5 \text{kHz}$, enabling
us to create $\pi$-pulses as short as $37 \upmu\text{s}$.

To protect the antenna from overheating and the amplifier from too high input
powers, we build in an interlock system based on an Arduino Uno. We also imple-
ment a possibility to create arbitrary temporal shapes of the rf pulses. A schematic
drawing of the complete electronic setup used to create rf pulses is shown in Fig-
ure 3.4.
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Figure 3.3: Calibration of the Rabi frequency obtained with the new amplifier. A maximum Rabi frequency of $\Omega = 2\pi \times 13.5\text{kHz}$ can be reached. Error bars correspond to the error of the fit used to extract the Rabi frequency. The solid line is a fit to the data according to Equation 3.12.

3.2.1 Interlock system

The components used in our interlock system are shown schematically in the right part of Figure 3.4. About 1\% (20.5 dB) of the rf signal is branched off using a directional coupler (Mini Circuits ZX30-20-4) and fed to a power detector (Mini Circuits ZX47-55+). The Arduino can then calculate the power before and after the amplifier from the output of the power detector using a calibration. It controls a switch enabling it to switch off the rf power before the amplifier if either the amplifier or the antenna are in risk of being damaged. To protect the amplifier, it is not possible to send an amplitude of higher than 0 dBm to it. To prevent the antenna from overheating, pulses with a power over 15 W are only allowed to be 1000 ms long, and pulses with a power over 72 W only 200 ms. Additionally, the temperature $T$ of the antenna is modeled in each step by

$$T_{\text{new}} = T_{\text{old}} + \frac{P}{C} \Delta t - h (T_{\text{old}} - T_0),$$

(3.13)

where the heat capacity $C$ is estimated from the spatial dimensions of the antenna, the heat dissipation $h$ was estimated based on measurements performed in [48],

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and the starting temperature is set to $T_0 = 20^\circ C$. The signal is turned off if the modeled temperature reaches 40 $^\circ C$. These precautions are necessary because the antenna is positioned inside the vacuum chamber and thus can only dissipate heat back along the cables or through radiation.

### 3.2.2 Pulse shaping

Previously, rf spectroscopy on Li could only be performed using rectangular pulses in our experiment. The pulse amplitude was determined by the output level of the rf generator and the length was given by a TTL signal from the logic box. Turning the rf signal on and off to create a rectangular pulse in time is equivalent to multiplying it with a function

$$ w(t) = \begin{cases} 1 & \text{for } 0 \leq t \leq \tau \\ 0 & \text{else} \end{cases}, \quad (3.14) $$

where $\tau$ is the length of the pulse.

This is the simplest example of a so-called window or apodization function, which is generally defined as a function that has zero value outside a chosen interval. Window functions can have an arbitrary shape and can be used to smoothly bring another function down to zero at the edges of an interval. Restricting a simple sample function like in our case $\cos(\omega t)$ temporally, causes spectral content to shift away from $\omega$, which is commonly referred to as spectral leakage.
In our polaron measurements we want to resolve a weak resonance that is expected only a few hundred Hz away from the bare resonance transition. The Fourier transform of a rectangular pulse has very pronounced sidelobes compared to other pulse shapes, which makes it hard to observe small features in the vicinity of stronger resonances. However, there are other temporal pulse shapes with less spectral leakage. To exploit this in our measurements, we mix the output of the rf generator with an arbitrary function generated by the logic box. In particular we explored the option to create a so-called Blackman pulse \[39\], given by

\[
    w(t) = 0.42 - 0.5 \cos\left(\frac{2\pi t}{\tau}\right) + 0.08 \cos\left(\frac{4\pi t}{\tau}\right) \quad 0 \leq t \leq \tau. \tag{3.15}
\]

Shaping the rf signal in this way supresses the first sidelobes by -57 dB with respect to the central maximum in the spectral response, compared to only -13 dB suppression for a rectangular pulse. As a downside, the width of the central maximum increases by a factor of about three for the Blackman pulse. In general, a compromise between supression of sidelobes and broadening of the signal has to be found while considering the spectral features one wants to observe (cf. the theoretical discussion in chapter 2).

By shaping the rf pulse, we create a time dependent Rabi frequency \(\Omega(t)\). This means the atoms are subjected to a mean Rabi frequency

\[
    \Omega_{\text{mean}} = \frac{\int \Omega(t) \, dt}{\tau}, \tag{3.16}
\]

which will be smaller by a factor of 0.42 than the corresponding Rabi frequency for a rectangular pulse with the same maximal Rabi frequency and pulse length \(\tau\), because of the smaller pulse area.

In Figure 3.5 the frequency of a rf pulse of length 0.64 ms was varied to create a spectrum of the \(|\uparrow\rangle \rightarrow |\downarrow\rangle\) transition of \(^6\text{Li}\). The first spectrum was taken with a rectangular \(3\pi\) pulse and the second one with a Blackman shaped \(\pi\) pulse, because the equal length of the pulses in both cases enables us to directly compare the widths of the central maxima in the spectra. As expected the central maximum is broader in the spectrum taken with the Blackman pulse. On the other hand it can clearly be seen from the spectra that strong suppression of sidelobes has been achieved.
Figure 3.5: Rf spectra of the $|\uparrow\rangle \rightarrow |\downarrow\rangle$ transition of $^6$Li. Top: Spectrum taken with rf pulses of square temporal shape. Bottom: Spectrum taken with rf pulses of Blackman temporal shape. Sidelobes are strongly suppressed at the expense of a broader central maximum. In both measurements the peak amplitude before the amplifier corresponded to a power of $-15$ dBm and the length of the pulse was 64 ms, corresponding to a $3\pi$ pulse for the square pulse measurement and $\pi$ pulse for the Blackman pulse measurement. Error bars in both spectra correspond to the error of the mean.
Figure 3.6: Characterization of the electronics used for pulse shaping. Left: Characterization of losses in the mixer. The linear range of the mixer extends to about 0 dBm. Right: Rabi frequency calibration. The maximal achieved Rabi frequency is limited to $\Omega \approx 2\pi \times 3.6$ kHz by nonlinear losses in the mixer.

We fit the first spectrum with the spectral function given by Equation 3.10, scaled by the total number of atoms $N_0$:

$$N(\omega) = N_0 \left( 1 - \Omega_0^2 \left( \frac{\sin(\Omega_{\tau/2})}{\Omega} \right)^2 \right). \tag{3.17}$$

with $\Omega = \sqrt{\Omega_0^2 + (\omega - \omega_0)^2}$ the generalized Rabi frequency and $\omega_0$ the resonance frequency.

For the spectrum taken with the Blackman pulse, we use the Fourier transform of the pulseshape for fitting:

$$N(\omega) = N_0 \left( 1 - \frac{(0.84 - 0.36a^2(\Omega/2\pi)^2) \sin(a\Omega)}{(1 - a^2(\Omega/2\pi)^2) (1 - 4a^2(\Omega/2\pi)^2) a\Omega} \right). \tag{3.18}$$

In this case $\Omega = \sqrt{\Omega_{\text{mean}}^2 + (\omega - \omega_0)^2}$ is the temporal mean of the generalized Rabi frequency, and $a = \tau/2$ denotes half the pulselength.

Figure 3.6 shows characterization measurements of the electronics used for pulse shaping as shown in Figure 3.4. From the left panel it is obvious that the mixer...
Figure 3.7: rf pulse with Blackman shaped temporal envelope. The upper envelope of a 0.5 ms long rf-pulse was fitted with the Blackman window function given in Equation 3.15 as a cross-check to ensure the correct shape is created.

(Mini Circuits ZFM-4) introduces strong, nonlinear losses above an input power of about 0 dBm. For lower power the losses in the mixer are constant at about 5 dB. As before we use an attenuator to exploit the dynamic range of the experimental control. Taking into account the losses in the mixer, we implement a 10 dB attenuator before the mixer. In the right panel we show a Rabi frequency-calibration similar to that in Figure 3.3, only this time we use the full setup shown in Figure 3.4 to create Blackman pulses. Here, we do not see the expected exponential growth of the Rabi frequency, and our maximal achievable Rabi frequency is limited to $\Omega \approx 2\pi \times 3.6$ kHz. This is a lot less than the maximal value of 13.5 kHz we obtained previously and thus we have to conclude that the achievable Rabi frequency is severely limited by the losses in the mixer.

To ensure that the shape of the pulse we create is not distorted, we have to limit the amplitude of the pulse envelope to a value of 0.3 V. For this case we measured the signal after the mixer for the maximal input power of 10 dBm and fitted the expected Blackman shape to its envelope. The results are shown in Figure 3.7. We find from the fit that the temporal shape of the pulse is well described by a Blackman pulse.
To summarize, we have introduced and characterized a new rf setup to drive transitions between the two lowest hyperfine states of $^6$Li. A new amplifier makes it possible to reach Rabi frequencies of up to 13.5 kHz. Additionally, we have implemented the possibility to create arbitrary temporal pulsed shapes. We have shaped our pulses with a Blackman window and showed that this leads to a significant suppression of sidelobes compared to a spectrum taken with rectangular pulses. In the next section, we will focus on the application of rf spectroscopy to a mixture of $^6$Li and $^{133}$Cs as a first attempt to measure a polaronic energy shift in our system.

### 3.3 Rf spectroscopy on a $^6$Li-$^{133}$Cs mixture

After characterizing our new rf setup and the effect different pulse shapes have on the bare $|\uparrow\rangle \rightarrow |\downarrow\rangle$ transition of $^6$Li, we now discuss rf spectroscopy on a dilute lithium sample interacting with a thermal cloud of cesium atoms as a first attempt to create and probe the Bose polaron scenario in our experiment.

We create a mixture of $7 \times 10^3$ lithium and $6 \times 10^3$ cesium atoms at a temperature of 100 nK. Our trapping and cooling scheme to create this ultracold mixture is described in detail in earlier theses from our group [49–51]. We perform experiments close to the Li$|\downarrow\rangle$-Cs FR at 843 G (see blue shaded region in Figure 3.8). Here the interactions of lithium atoms in the $|\uparrow\rangle$ state with cesium are very small over the whole magnetic field range of the FR, bringing it close to the ideal scenario of an initial impurity state that is noninteracting. We thus prepare lithium initially in the state $|\uparrow\rangle$, and perform rf spectroscopy by coupling it to the interacting state $|\downarrow\rangle$ with pulses of varying detuning $\delta$, as schematically shown in Figure 1.2.

We calculate peak densities of $n_{0,\text{Li}} = 5 \times 10^{10}$ cm$^{-3}$ and $n_{0,\text{Cs}} = 3 \times 10^{11}$ cm$^{-3}$ for our samples. As the cesium density is almost an order of magnitude larger than the lithium density, we can neglect interactions between the lithium impurities. For our densities and temperature we arrive at $T/T_C \approx 4$ and $T/T_F \approx 2$, where $T_C$ is the critical temperature of cesium and $T_F$ is the Fermi temperature of lithium, meaning that both gases are far from quantum degeneracy.
Figure 3.8: $^6\text{Li} - ^{133}\text{Cs}$ and $^{133}\text{Cs} - ^{133}\text{Cs}$ scattering lengths at high magnetic fields. The blue shaded region shows the $\text{Li}|\downarrow\rangle \oplus \text{Cs}|3, +3\rangle$ FR used for our first polaron try. The green shaded region shows a $\text{Li}|\uparrow\rangle \oplus \text{Cs}|3, +3\rangle$ FR at a magnetic field where the Cs-Cs scattering length has a zero crossing, thus enabling the creation of a cesium BEC at small positive scattering lengths. Scattering lengths taken from [52–56].
A first rf spectrum is shown in Figure 3.9. We performed the measurement at a magnetic field of 844.20 G, corresponding to an impurity-boson scattering length of $a_{\text{LiCs}} \approx -1300 a_0$. We estimate the polaron self-energy by its mean field value

$$E_{mf} = \frac{2\pi \hbar^2 n_{\text{Cs}} a_{\text{LiCs}}}{m_{\text{red}}}$$ \hspace{1cm} (3.19)

and obtain a value of $E_{mf} = \hbar \times 280$ Hz for our experimental conditions. To resolve this shift away from the bare transition in our spectrum, we need to apply an rf pulse that is longer than about 12 ms. On the other hand, the length of our measurement is limited by three-body losses of cesium, which we estimate to happen on a timescale of 50 ms. We thus perform spectroscopy with a pulse length of 19.2 ms, which corresponds to a $9\pi$ pulse. We use Blackman pulses to suppress sidelobes in the spectrum that could mask the polaron signal. We observe the expected maximum of the lithium population in the interacting state at the frequency of the bare transition, but no enhancement of the population close to the expected value of the polaron signal.
Chapter 3. Radiofrequency spectroscopy

Our failure to see a polaron signal in our spectra, also at different scattering lengths, could have several reasons:

- **Spatial overlap of the clouds.** By trapping both lithium and cesium in the same optical dipole trap with a wavelength of 1070 nm, we obtain a trap depth $U_{\text{Li}}$ for lithium that is about four times larger than the trap depth for cesium $U_{\text{Cs}}$. This leads to a larger density distribution of the lithium atoms compared to the cesium atoms. Additionally, the large mass difference between the two species leads to a difference in the vertical position of the clouds that is not fully compensated in our current trapping scheme. If not all of the impurity atoms interact with the bosons, this reduces the spectral weight of the polaron branches. By mapping out a Li-Cs FR prior to our rf measurements, we were able to confirm that the two clouds had some overlap, but quantifying this requires very precise knowledge of the trapping potentials. The spatial overlap might have been so small that the number of lithium atoms transferred to the interacting state at the polaron energy was below the detection limit of our imaging system.

- **Inhomogeneous density distribution.** The spatial variation of the cesium density leads to a variation of the polaron energy, which in turn broadens the polaron signal in the rf spectrum. This could be overcome by creating a homogeneous cesium gas. Here the biggest challenge would be to achieve a homogeneous distribution in the vertical direction, due to the strong influence of gravity. A BEC of $^{87}$Rb atoms in a uniform potential has already been realized [57]. A simpler solution in our case is to immerse a small number of Li atoms in the central region of the Cs gas, where the variations of the density are small.

- **Cs BEC and rf parameters.** Although predictions for the polaron spectrum in Li-Cs at high temperatures have been made based on a virial expansion [26], it is not clear what happens to the spectral response when driving long rf pulses of several $\pi$. First steps to investigate this in the zero temperature limit have been discussed in chapter 2. The creation of a weakly interacting Cs BEC will not only bring our system closer to the original analogy with solid-state physics, i.e. the picture of an impurity dressed by Bogoliubov excitations, but the higher density of about $10^{13} - 10^{14}$ cm$^{-3}$ will also increase the polaron energy, thus enabling us to probe the system with shorter rf pulses. The code developed in the scope of this thesis can serve as a guide as
to which rf parameters are best to observe the polaron spectrum for a given BEC density and set of interactions. So far, a cesium BEC has only been obtained at low magnetic fields and without simultaneously trapping lithium atoms in our experiment [54, 58]. The Li\textsuperscript{||} \rightarrow Cs FR at 889 G, highlighted in green in Figure 3.8, lies close to a zero-crossing of the cesium scattering length. This means we could choose the lithium state \textket{\uparrow} as the interacting state, and probe the spectral response of the system for different interaction strengths between the impurity and the Cs BEC.
4 Improved high-resolution imaging of $^6\text{Li}$ and $^{133}\text{Cs}$

The experimental realization of the Bose polaron sets high requirements on the detection of $^6\text{Li}$ and $^{133}\text{Cs}$ atoms: Firstly, we need a good resolution to resolve a cesium BEC, which typically has a small size of only few tens of micrometers [59]. Secondly, we need to be able to detect even very small numbers of lithium atoms accurately, making a good signal-to-noise ratio necessary. In the following analysis we show that we can fulfill both of these requirements with our new dual-species imaging system.

This chapter starts out with a short introduction to absorption imaging, followed by an overview over the imaging setup. We then analyze the noise contributions for lithium imaging, and find constraints on the imaging parameters. We improve the signal-to-noise ratio by compensating for the Doppler frequency shift of the atoms due to absorption of photons from the imaging beam. Additionally, we implement imaging of both hyperfine states within the same experimental realization. In the final part of this chapter we perform noise correlation analysis for both lithium and cesium imaging. This way we obtain information about the dominant types of aberrations and the resolution of our imaging system.

4.1 Introduction to absorption imaging

In this section we give a brief overview on the standard absorption imaging technique [60] employed in our experiment to measure the density distributions of lithium and cesium. Absorption imaging is based on the illumination of an atomic cloud with coherent and (near-)resonant light. The light scattered from the atoms interferes destructively with the incident light, creating a shadow of the cloud. This shadow can be imaged onto a camera and is used to extract the density of the cloud, integrated along the direction of beam propagation.
We can describe the absorption of the laser beam by the atom cloud using the Lambert-Beer law:

\[
I_{\text{det}}(x, y) = I_0(x, y) e^{-\int \sigma(z) n(r) \, dz},
\]

(4.1)

where the \( z \)-dependence of the absorption cross-section is due to correcting for saturation:

\[
\sigma(z) = \frac{\sigma_0}{1 + \frac{I(z)}{I_{\text{sat}}} + \left(\frac{2\Delta}{\Gamma}\right)^2}.
\]

(4.2)

Here \( I_{\text{sat}} = \frac{\pi \hbar c}{3 \lambda^2} \) is the saturation intensity of the atomic transition with \( \Gamma \) the natural linewidth and \( \sigma_0 = \frac{3\Delta}{2\pi} \) the resonant absorption cross-section for the wavelength \( \lambda \).

The transmission

\[
T(x, y) = \frac{I_{\text{det}}(x, y)}{I_0(x, y)} = e^{-\int \sigma(z) n(r) \, dz}
\]

(4.3)

is obtained in praxis by taking an absorption image \( A \) together with a reference image \( R \) that does not contain the atomic cloud. A dark image \( D \) is subtracted from both images to avoid the influence of dark counts and stray light. Then, the absorption image is divided by the reference image to obtain the transmission

\[
T(x, y) = \frac{A(x, y) - D(x, y)}{R(x, y) - D(x, y)}.
\]

(4.4)

For negligible saturation and near-resonant light \( \sigma(z) \approx \sigma_0 \) and thus we can easily obtain the integrated column density:

\[
n_{2D}(x, y) \approx -\frac{1}{\sigma_0} \ln \left( T(x, y) \right).
\]

(4.5)

The absorption is also often characterized by the optical density (OD), which is given by

\[
\text{OD} = -\ln(T(x, y)) = \int \sigma(z) n(r) \, dz \approx \sigma_0 n_{2D}(x, y),
\]

(4.6)

where in the last step the same approximation as above has been made. The peak optical density and the total atom number can then be obtained by fitting an appropriate distribution function to the two-dimensional density profile.
The ability to reconstruct these quantities from the images is limited by the resolution, which characterizes the smallest length scale at which objects can still be separated in the images, and the achievable signal-to-noise ratio (SNR). We analyze and improve both of these qualities in our vertical high-resolution imaging system during the remainder of this chapter. For a better understanding, we give a short overview over the imaging setup in the following, before focusing on the SNR for imaging lithium atoms.

4.2 Imaging setup

We image cesium atoms on the $^6S_{1/2} \rightarrow ^6P_{3/2}$ transition with a wavelength of about 852 nm and lithium atoms on the $^2S_{1/2} \rightarrow ^2P_{3/2}$ transition with a wavelength of 671 nm. For both species two lasers are available, one for imaging at zero external magnetic field, and one that compensates for the frequency shift of the imaging transition at high magnetic fields of several hundred Gauss. A description of the laser systems can be found in [49], and a detailed description of high-field imaging of cesium in [61] and of lithium in [62].

In this chapter we focus on the characterization and improvement of our setup for vertical high-resolution imaging of lithium and cesium atoms, which was designed and built up by Carmen Renner and is described in detail in her thesis [59]. Here we only give a brief overview for completeness.

The shadow of the cloud is imaged on the chip of a CCD camera by two achromatic lenses in a 4f-configuration. The first lens has a focal length of 100 mm and a diameter of 30 mm, limited by spatial constraints of the experiment. The first lens sets the numerical aperture (NA) of the system to 0.15. The second lens has a focal length of 750 mm. Achromatic lenses have been chosen specifically to minimize the chromatic shift of the imaging focus between our wavelengths of 671 nm for lithium and 852 nm for cesium, with a remaining shift of about 4 mm. The setup is designed to give a magnification of 7.5. A magnification of about 7.2 has been measured before the implementation of the setup. By comparing the measured atom numbers to the atom numbers measured with our horizontal imaging system, we obtained a visual field of one camera pixel at the position of the atoms of $L_{\text{pix}} = 1.86 \mu\text{m}$, which corresponds to a magnification of $13.00 / 1.86 = 6.99$, which is close to the design value.
Chapter 4. Improved high-resolution imaging of $^6$Li and $^{133}$Cs

Figure 4.1: Drawing of the beam path for vertical dual-species imaging. The different beams for imaging both lithium and cesium at low and high magnetic fields are overlapped. The clouds are imaged onto a CCD camera by two achromatic lenses in 4$f$-configuration. The imaging light is separated from the MOT light by a mirror mounted on a translation stage. Image taken from [59].

The imaging beams are separated from the MOT beams by a two-inch mirror mounted on a translation stage, that is moved into the beam path to create a MOT and out of the beam path for imaging. This is necessary because the polarization needed for creating a cesium MOT and for imaging cesium at high magnetic fields is the same, making it impossible to separate the beams using a polarizing beam splitter. The full setup including the achromatic lenses used for imaging and the translation stage for beam separation is shown in Figure 4.1.

We use the iKon-M 934 from Andor Technology with a BEX2-DD coating because of its high quantum efficiency of about 79% for 671 nm and 77% for 852 nm. Its chip has $1024 \times 1024$ pixels with a size of $13 \mu m \times 13 \mu m$. Some characteristics of the camera, like the readout noise and the dark current, have been investigated previously in [63]. The Andor’s fast kinetics mode enables us to image both species in one experimental cycle. Therefore, we position two razorblades so that only the top 204 rows of pixels on the chip are illuminated. After illumination the collected charges are shifted down on the chip and a new image can be taken about 1 to 1.5 ms after the first one. This way four images can be taken in fast succession before having to read out the chip. In practice, we take the absorption and reference images for both lithium and cesium before reading out the chip and taking the dark images.
4.3 Improving the signal-to-noise ratio for $^6$Li

To be able to accurately detect even small numbers of $^6$Li atoms in future polaron measurements, we need to know the dominant sources of noise in our images and find optimal imaging parameters. To this end we start out with a measurement of the photon transfer curve for imaging with 671 nm and proceed by finding theoretical limits on the imaging parameters for our setup. We then introduce a way to compensate for the Doppler detuning of the atoms, and image both lithium spin states in the same experimental realization, by fast changes in the frequency of the imaging light.

4.3.1 Photon transfer curve

A well-established method to analyze the noise introduced in the imaging process is the so-called photon transfer curve [64]. The photon transfer curve can be used to measure the gain, dark counts and well depth of a CCD camera. Most importantly, it gives us information on the dominant noise contributions in our images. A photon transfer curve commonly refers to a plot of the variance of the camera counts over the mean of the counts. The mean number of counts can either be varied by changing the light intensity or the illumination time. We opt for the latter, because we have only limited laser intensity available in our imaging setup.

In the following we briefly introduce different types of noise contributions, relevant at different signal levels, going from low to high signal:

- Dark current. Even without any incident photons, electrons are excited thermally in the CCD chip and contribute to the signal. As we operate our camera at very low temperatures below $-70\,^\circ C$, this contribution is very low. It has been measured previously to be less than four electrons per pixel and second under these conditions [63], and can thus be neglected in the further discussion.

- Camera readout noise. As the charges collected on the chip are shifted for readout, additional noise is created depending on the shift speed. This has been measured to be about 12 electrons per pixel for the horizontal shift speed of 3 MHz we use, and thus also very small.
• Photon shot noise. This type of noise is not caused by the camera but is inherent in the quantized nature of the photons themselves. As we expect the photons in our laser beam of coherent light to follow Poisson statistics, we expect a noise contribution with a variance $\sigma^2$ that is proportional to the number of counts $N_e$ itself:

$$\sigma_{PSN}^2 \sim N_e$$  \hspace{1cm} (4.7)

• Fixed pattern noise. This type of noise originates from the difference in sensitivity between the individual pixels on the CCD chip, also called photoreponse nonuniformity. This type of noise leads to a variance that is proportional to the square of the signal:

$$\sigma_{FPN}^2 \sim N_e^2$$  \hspace{1cm} (4.8)

• Full well. As the pixels of the CCD can only accumulate a limited number of electrons, the variation of counts decreases rapidly above this point. For our camera, the pixel well depth is 16 bit, which corresponds to a maximum of 65536 electrons per pixel.

We obtain the photon transfer curve shown in Figure 4.2 by taking 100 images each for different exposure times between 0 and 300 $\mu$s. From each image we subtract a background image taken with the same exposure time, but without illuminating the chip to remove the influence of background light on the experimental table and dark counts. We analyze the images in a small central region of $10 \times 10$ pixels, to eliminate effects due to a possible nonuniform illumination of the chip. We normalize all images to the same mean intensity within that area. To avoid effects of fixed pattern noise, we do not evaluate the variance of the counts within one image, but instead take the variance of the counts of a single pixel over the 100 different realizations. To gain more statistics, we do this for each pixel within our region. The error bars correspond to the error of the mean over the different pixels. We perform our measurements at a gain of four, as this is the setting used in the experiment. This means that we expect a linear curve with slope 1/4 in the intermediate regime that is dominated by photon shot noise.

We fit our data according to

$$\sigma^2(N_e) = A + B N_e + C N_e^2,$$  \hspace{1cm} (4.9)
and obtain $A = 578 \pm 146$, $B = 0.32 \pm 0.26$, $C = (8.48 \pm 0.65) \times 10^{-4}$. The constant and linear term have very high uncertainty. The quality of the fit could in principle be improved by taking more data points at small imaging times. However, in our case we observed strong intensity fluctuations between the images. This means that if one takes data for imaging times close together, the mean values for different imaging times overlap, making it meaningless to sort and evaluate the images according to the imaging time. The fact that the constant offset $A$ lies above the values expected for dark current and readout noise indicates that there is some stray light impinging on the camera chip. As a background image is removed, stray light does not contribute to the mean of the counts, but does of course still contribute to the variance. The value of $B$ corresponds to the expected value for photon shot noise of $1/4$ within its uncertainty. However, the large value of $C$ makes clear that we are currently not limited by photon shot noise but by a quadratic contribution to the noise. This is due to intensity fluctuations of $\sim 10\%$ we observed in the experiment, which are caused by the AOM in
Figure 4.3: Signal-to-noise ratio based on the fit to the photon transfer curve for 671 nm in Figure 4.2. The signal-to-noise ratio increases strongly up to a few thousand counts and then saturates.

the optical setup. A similar quadratic dependence has also been found in [64] and has been attributed to fluctuations of the camera gain from pixel to pixel. While this effect would also give rise to a quadratic term, we do not expect it to play a role in our measurements since the CCD camera only has one amplifier common to all pixels.

From our measured photon transfer curve we can extract the signal to noise ratio, given by the mean number of counts, divided by its standard error (obtained from the variance), in dependence of the mean number of counts. The result is shown in Figure 4.3. We see that it is advantageous to image with a few thousand counts. Above that, the SNR is limited by the quadratic term in Equation 4.9. Note that the SNR could be further limited by atom number fluctuations [64], which is not investigated here. The necessary counts to achieve a good SNR can in principle be obtained either by choosing a sufficient intensity or imaging time. At our current intensity we need to image for about 100 µs to obtain about 2000 counts. However, in practice there are effects that limit the imaging time for a given intensity. The limits on the parameters for imaging lithium in our system, and how to circumvent them, will be discussed in the following.
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4.3.2 Optimal imaging parameters for $^6$Li

The imaging parameters for lithium are limited by two effects: First, when the atoms absorb a photon from the laser beam, they obtain a momentum kick, leading to the recoil velocity $v_{\text{rec}} = \frac{\hbar k}{m}$ with $k = \frac{2\pi}{\lambda}$. In our case of imaging lithium atoms with a wavelength of 671 nm, the recoil velocity is given by $v_{\text{rec}} = 9.89 \text{ cm/s}$. As the atoms scatter more and more photons, their resonance frequency changes due to the Doppler shift. Their scattering cross-section is reduced until eventually they become off-resonant. In practice this means that we underestimate the number of atoms if we scatter too many photons during the imaging process. The other limitation is given by the random walk the atoms perform due to the spontaneous emission of photons, which can lead to blurring of the images. In the following we quantify the constraints these effects impose on the imaging conditions, applying the approach in [65] to our system. We find constraints on the imaging time in dependence of the saturation parameter $s \equiv \frac{L_{\text{sat}}}{I_{\text{sat}}}$. For the $^2S_{1/2} \rightarrow ^2P_{3/2}$ transition we employ for imaging lithium, the saturation intensity is given by $I_{\text{sat}} = 2.54 \text{ mW/cm}^2$ and the natural linewidth by $\Gamma = 2\pi \times 5.87 \text{ MHz}$ [44]. Similar limitations as those derived here apply in principle also to the imaging of cesium, however they are much less restrictive because a cesium atom acquires a smaller recoil velocity of $v_{\text{rec}} = 0.35 \text{ cm/s}$ when absorbing or emitting an imaging photon due to its larger mass. Therefore we limit our considerations to the case of lithium here.

Random walk condition  The size of the visual field for one camera pixel at the position of the atoms is $L_{\text{pix}} \approx 1.86 \mu\text{m}$ in our system. In order for the random walk of the atoms to not blur the images, we demand that they should stay within an area of $L_{\text{pix}}^2$ during the duration of the imaging pulse:

$$\delta r_{\text{rec}}(t, s) < L_{\text{pix}}, \quad (4.10)$$

where $\delta r_{\text{rec}}(t, s)$ is the displacement due to the random walk. It can be described by

$$\delta r_{\text{rec}}(t, s) = \int_0^t \sqrt{\langle v_{N}^2 \rangle(t', s)} \, dt' \quad (4.11)$$

with the velocity $\langle v_{N}^2 \rangle(t, s) = \frac{2}{3} v_{\text{rec}}^2 N_{\text{sc}}(t, s)$ [47, 60]. Here $N_{\text{sc}}(t, s) = R_{\text{sc}}(s) t$ denotes the number of scattered photons for the constant resonant scattering rate.
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$R_{sc}(s) = \frac{\Gamma s}{2 \Gamma + s}$ during the time $t$. Inserting these quantities into Equation 4.10 we obtain a condition for the imaging time $t$ in dependence of the saturation parameter $s$

\[
t < \frac{3}{2^{2/3}} \left( \frac{m\lambda}{\hbar} L_{\text{pix}} \sqrt{\frac{1}{1 + s}} \right)^{2/3}. \tag{4.12}
\]

Doppler condition \hspace{1cm} The scattering rate for an ensemble of two-level atoms, including a detuning $\Delta$ of the radiation from the atomic transition, is given by

\[
R_{sc}(s, \Delta) = \frac{\Gamma}{2} \frac{s}{1 + 4 \left( \frac{\Delta}{\Gamma} \right)^2 + s}. \tag{4.13}
\]

The Doppler shift due to the velocity acquired by absorbing $N_{sc}(t, s)$ photons with the resonant frequency $\nu_0$ leads to the detuning

\[
\Delta(t, s) = 2\pi N_{sc}(t, s) \nu_0 \frac{v_{\text{rec}}}{c}. \tag{4.14}
\]

Its influence on the scattering rate is negligible if

\[
\left( \frac{\Delta(t, s)}{\Gamma} \right)^2 < 1 + s. \tag{4.15}
\]

Then, we can assume a constant scattering rate as before and arrive at the condition

\[
t < \frac{m\lambda^2}{2\pi\hbar} \frac{(1 + s)^{3/2}}{s}. \tag{4.16}
\]

Both of the conditions Equation 4.12 and Equation 4.16 are visualized in Figure 4.4. The green area shows the range of allowed imaging conditions according to these constraints. The dashed vertical line indicates our current intensity of $I \approx 0.1I_{\text{sat}}$. With this intensity we have to image for about $100\mu$s to achieve a good SNR, which places us far beyond both the Doppler and random walk limit. One solution would be to increase the intensity and image for very short times of only a few microseconds. However, the approximations made in Equation 4.5 and Equation 4.6 to extract the column density and OD from the transmission, are only valid for small intensities. For higher intensities no such approximation can be made and the extraction of the column density and OD is more involved because it depends on the value of the intensity before and after the cloud relative
Chapter 4. Improved high-resolution imaging of $^6$Li and $^{133}$Cs

Figure 4.4: Limitations on the imaging conditions for $^6$Li. The imaging time and intensity are limited by the Doppler detuning of the atoms due to absorption and their random walk due to the spontaneous emission of imaging photons. The dashed, vertical line corresponds to our current intensity of $I \approx 0.1 I_{\text{sat}}$. The green shaded area shows the range of allowed imaging conditions.

to the effective saturation intensity, which differs from $I_{\text{sat}}$ due to polarization effects and deviations of the atomic transition from a two-level system [66]. These relative intensities can be obtained by measuring the momentum transferred to the atoms during imaging [67]. Absorption imaging with high intensities $I \gg I_{\text{sat}}$ has been developed to image very dense atomic clouds [66], because in this case low absorption imaging yields a very low number of transmitted photons and thus bad signal-to-noise ratio. Apart from avoiding limitations due to the random walk and Doppler detuning, high-intensity absorption imaging also has the advantage of being robust against small intensity and frequency variations. Using a combination of high-intensity imaging and post-processing, detection of about 300 atoms with an uncertainty of less than 4 atoms has been achieved [68].

Currently the laser power available for imaging prevents us to implement high-intensity absorption imaging. However, at our current intensity we can compensate for the Doppler shift by dynamically changing the laser frequency during imaging. This will enable us to image for longer times without underestimating the atom number, thus improving the signal-to-noise ratio.
4.3.3 Compensation of Doppler shift

So far, we have not explicitly considered how the cross-section and the scattering rate reduce over time due to the Doppler effect. Because each absorbed photon leads to a detuning, which in turn increases the average time until the next photon is absorbed, the detuning and scattering rate are interdependent. This can be seen explicitly in the differential equation

\[
v'(t) = v_{\text{rec}} R_{sc}(t) = v_{\text{rec}} \frac{\Gamma}{2} \frac{s}{1 + s + 4 \left( \frac{k v(t)}{T} \right)^2},
\]

which we can solve numerically for the initial condition \( v(t = 0) = 0 \). Here we have inserted the Doppler detuning \( \text{Equation 4.14} \) with \( v(t) \equiv N_{sc}(t) v_{\text{rec}} \), and we use \( s = 0.1 \) according to the approximate intensity in our experiment. We can then insert the time-dependent velocity \( v(t) \) to obtain the time-dependent scattering rate \( R_{sc}(t) \) or cross-section \( \sigma(t) \). Of course, if we image for a certain time \( \tau \), our measured atom number and OD will correspond to the mean cross-section during that time:

\[
\sigma_{\text{eff}}(\tau) = \int_0^\tau \sigma(t) \, dt / \tau
\]

A similar approach has been applied in [69]. We show the decrease of the measured peak OD of a cloud of thermal \(^6\)Li atoms with increasing imaging time in Figure 4.5. As the OD is proportional to the cross-section close to resonance and for low intensities (cf. Equation 4.6), we simply rescale our theoretical curve for \( \sigma_{\text{eff}}(\tau) \) by a factor to mach the measured OD. We see a clear decrease in OD that is well described by our model of the Doppler detuning. The decrease is already significant for imaging times of several tens of microseconds. However, to achieve a good SNR, we need to image at least for about 100 \( \mu s \) (see subsection 4.3.1), which leads to a large underestimation of the atom number.

One way to solve this problem is to dynamically adapt the frequency of the imaging laser to stay on resonance with the atoms as they are accelerated. If we assume that the atoms stay close to resonance at all times, they experience constant acceleration \( a = R_{sc}(s, \Delta = 0) v_{\text{rec}} \). This means that we have to change the laser frequency linearly with time to compensate for the Doppler detuning. As the atoms are accelerated in the direction of the beam, the laser has to become blue-detuned
from the original transition frequency. We can now easily calculate which change in frequency is necessary to keep the scattering rate constant at the resonant value $R_{sc}(s, \Delta = 0) = \frac{\Gamma}{2(1+s)}$. We obtain the condition

$$\frac{d\nu}{dt} = \nu_0 \frac{a}{c} = \nu_0 \frac{R_{sc}(s, \Delta = 0)}{c} \frac{v_{rec}}{c}.$$  

(4.19)

and plot it in Figure 4.6 in dependence of the saturation parameter $s$. The dashed vertical line marks our experimental conditions as before. We obtain a necessary frequency change of 0.247 GHz/ms for our intensity. We achieve this frequency change experimentally by applying a current ramp to the laser diode. This is done by applying a voltage to the DC Mod input of a Toptica DL Pro 671, our lithium high-field imaging laser. We calibrate the dependence of the change in frequency on the applied voltage by observing the frequency change of the laser’s beat lock signal for a number of known voltages. A RIGOL DG4102 frequency generator is programmed remotely to create the voltage ramps. The program is fully implemented in the LabView control of the experiment. The frequency generator is operated in burst mode, meaning that a sequence can be programmed once, and is given out when the frequency generator receives a trigger from the
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Figure 4.6: Change in laser detuning necessary to compensate for the Doppler shift of the lithium atoms. As the atoms absorb photons, they are accelerated in the direction of the beam. A linear frequency shift of the imaging laser away from the original atomic transition can be used to stay resonant with the atoms. At our current intensity of $I \approx 0.1 I_{\text{sat}}$, as indicated by the dashed line, a frequency change of about 0.247 GHz/ms is necessary.

logic box. This means that, after optimizing the imaging sequence, no time is lost on programming the frequency generator.

For a chosen imaging time, the height of the frequency ramp is obtained by starting out from the calculated value of about 0.247 GHz/ms and then scanning the temporal change of the applied voltage to optimize for the maximal number of atoms detected. A schematic diagram of the frequency changes during the full sequence, together with typical values for the relevant timings and frequencies, is shown in Figure 4.7.

We use this possibility to control the laser frequency in a fast and precise way to also implement the imaging of both of the lowest hyperfine states of $^6$Li in one experimental run. All we need for this, is a fast frequency jump that corresponds to the energy difference of the two states of $\sim$76 MHz, depending on the magnetic field (see Figure 3.1), and an additional pulse of light in the imaging sequence.

Figure 4.8 shows the results of imaging for different times with compensation of
Chapter 4. Improved high-resolution imaging of $^6$Li and $^{133}$Cs

Figure 4.7: Typical changes of the laser frequency during high-field imaging of $^6$Li. The Doppler shift of the atoms during imaging is compensated by a linear change of the frequency. Both of the two lowest hyperfine states can be imaged within one experimental realization by quickly changing the laser frequency by about 76 MHz. Drawing not to scale.

the Doppler shift as described above. For each imaging time, the maximal detectable OD is plotted. The OD decreases linearly with increasing imaging time. This is because the random walk of the atoms leads to a spread of their velocity distribution, meaning that for longer times less atoms can be addressed resonantly by the laser. Even though the OD decreases much slower than without the compensation of the Doppler shift (compare Figure 4.5), we still underestimate it by about 23% when comparing the measured value at an imaging time of 100 µs to the extrapolated value at zero imaging time. The linear decrease of the OD can be easily taken into account in the evaluation procedure by experimental calibration and calculating back to the original OD.

On resonance, we expect to scatter about 1.6 photons/µs, leading to an acceleration of $a = 0.17 \, \mu m/\mu s^2$. This means that the atoms move in the direction of the beam by about $\Delta z \approx \frac{1}{2} at^2 = 850 \, \mu m$ for an imaging time of 100 µs. As the Rayleigh length of the imaging beam is only 38 µm, the atoms are accelerated out of focus during imaging. This is another reason to image with higher intensity for shorter times, as then the distance traveled by the atoms is much smaller.

Taking all the effects discussed here into account, it would be beneficial to image with an intermediate intensity of $I \approx I_{sat}$, as described in [70]. The higher intensity would enable us to image at shorter times of $\sim 10 \, \mu s$, meaning that the influence of the random walk on our images would be much less severe and the atoms would not be accelerated out of the imaging focus. For much higher intensities $I \gg I_{sat}$ the cloud would be rendered transparent to the imaging light due to the saturation, which would reduce the SNR when imaging a dilute gas like in
Figure 4.8: Reduction of measured peak optical density with increasing imaging time for compensated Doppler detuning. Despite the optimal frequency shift of the laser, the random walk of the atoms leads to a spread in their velocity distribution, meaning that not all atoms stay resonant. The grey dashed line shows to the expected dependence without compensation of the Doppler shift (cf. Figure 4.5).

our case. At an intermediate intensity $I \approx I_{\text{sat}}$, however, this is not yet the case and thus it is possible to obtain a high SNR without being limited by the random walk of the atoms or their acceleration along the beam.

To summarize, we have analyzed the signal-to-noise ratio for imaging lithium in dependence of the camera counts. We find that we are mainly limited by intensity fluctuations due to the AOM in our optical setup, and that it is beneficial to image with at least 2000 counts, which corresponds to an imaging time of 100 $\mu$s at our current intensity of $0.1 I_{\text{sat}}$. However, the Doppler shift and the random walk of the atoms limit the imaging time in dependence of the imaging intensity. At our current intensity, we find that we need to violate both the Doppler condition and the random walk condition to achieve a good signal-to-noise ratio. While the random walk of the atoms only leads to blurring of the images, we showed that the Doppler effect leads to a severe underestimation of the atom number, even for imaging times of few tens of microseconds. We compensate for the Doppler shift by dynamically changing the laser frequency during the imaging process,
making it possible to image at longer imaging times without underestimating the atom number. This way we can detect as little as few hundred atoms, which will be helpful to measure a polaron signal even if only a small fraction of lithium atoms is transferred to the interacting hyperfine state. In the following we further analyze the performance of our imaging system for both lithium and cesium.

## 4.4 Noise correlation analysis

Since its implementation into the main experiment, the imaging system described in section 4.2 had not been characterized. It was designed to have a resolution of 1.17 µm for cesium and 0.92 µm for lithium (standard deviation of a Gaussian fit to the point spread function). This is of the same order of magnitude as the size of a cesium BEC, which is expected to be about \(60 \times 9\) µm in our setup \[59\]. As we will need to resolve this in future polaron measurements, good alignment of the imaging system is crucial. In the scope of this thesis we carefully realigned all components of the imaging setup and characterize its performance. The resolution of an imaging system is usually measured by imaging a point-like object, like a pinhole, and characterizing the spread of its image. However, in our experiment we do not have access to the object plane of our system, because it is inside the vacuum chamber. However, using the method of noise correlation analysis developed in \[71\], we can characterize the performance of our imaging system without the need for a point-like object. This method relies on the analyzation of correlations imprinted on the images of thermal atomic clouds, which are easily available in the experiment. The key quantity extracted with this method is the point spread function, which we will introduce in the following, before giving an overview over the method and the results for our system.

### 4.4.1 The point spread function

To describe an optical system, the general aim is to find a relationship between the field distribution in the object plane, \(f(x, y)\), and detector plane, \(g(x, y)\), where the field is assumed to propagate in \(z\) direction:

\[
f(x, y) \xrightarrow{z} g(x', y')
\]  

(4.20)
In the experiment, we detect only the light intensity, but as we image with coherent light, we still need to consider the propagation of the electrical field through the optical system. This can be conveniently achieved using Fourier optics. By decomposing the field into plane waves, i.e.

$$f(x, y) = \int \int F(\nu_x, \nu_y) e^{-i2\pi(\nu_x x + \nu_y y)} d\nu_x d\nu_y, \quad (4.21)$$

with the spatial frequencies $\nu_x$ and $\nu_y$, we can reduce the light propagation to the propagation of plane waves and the superposition principle. This is a consequence of the linearity of the Helmholtz equation.

Assuming the plane waves are transformed linearly by our imaging system, we can characterize it with the so-called response function $H(x, y)$, which is the power spectrum of the field:

$$G(\nu_x, \nu_y) = H(\nu_x, \nu_y) F(\nu_x, \nu_y), \quad (4.22)$$

where $G(\nu_x, \nu_y)$ is the Fourier transform of the field in the detector plane. Equivalently, we can characterize the optical system by the Fourier transform of the response function, the point spread function psf. The Fourier convolution law tells us that the multiplication in Fourier space becomes a convolution in real space:

$$g(x', y') = \int \int f(x, y) \text{psf}(x - x', y - y') dx dy \quad (4.23)$$

We can see immediately that $g(x, y) = f(x, y)$ if the psf reduces to a delta-function, i.e. psf $= \delta(x - x', y - y')$. Because real optical systems accept only a finite range of spatial components of $F(\nu_x, \nu_y)$ due to the finite size of their components, the convolution with the psf will broaden the image of each point in the object plane. This process introduces an additional length scale to the image, below which object details cannot be resolved. By setting $f(x, y) = \delta(x - x', y - y')$ in Equation 4.23, we also see that the image of a point will be the psf.

The psf is given by the Fourier transform of the exit pupil and phase [72]:

$$\text{psf}(r) = FT \left[ p(k_r + a) e^{i\Phi(k_r + a)} \right], \quad (4.24)$$

with $\Phi$ the phase aberrations, $p$ the effective exit pupil and $a$ a shift of the exit pupil away from the optical axis. Here, $k_r = \frac{r}{\lambda d}$ denotes the coordinates in the plane.
of the exit pupil, with \( \lambda \) the wavelength of the imaging light and \( d \) the distance between the exit pupil and the detector. Note that the size of the exit pupil is not necessarily determined by the size of the last optical element of the system, but by the aperture that limits the transversal size of the propagating beam the most.

We can already see at this point that the quality and resolution of any imaging system is limited by two distinct factors:

- The finite size of the optical elements limits the acceptance of transversal modes. For any number of optical elements we can in principle think of a system for which the exit pupil is maximal and uniformly illuminated, and no aberrations are present. We call this system \textit{diffraction limited}, because its \textit{psf} is governed by the diffraction at the rims of the optical elements. In the diffraction limited system, the image of a point would contract back to a point if all transversal modes were present.

- In any real system, aberrations will be present. These distortions of the wavefront can arise from imperfect optical elements or misalignment. We call an optical system whose resolution is limited by its imaging errors \textit{aberration limited}.

In the following we analyse the performance of our imaging system starting out from the power spectrum of the intensity \( M \), which is the absolute square of the power spectrum of the field \( H \). We use the power spectrum to analyze dominant aberrations and to reconstruct the absolute value of the point spread function.

### 4.4.2 Overview over the method

We have shown that either one of the power spectrum or the point spread function can be used to describe an optical setup completely. Usually the \textit{psf} is employed because its width gives easy access to the resolution. It can in principle be measured by imaging a point-like object such as a single atom, ion, or a small BEC. However, often a point-like object is not available in the object plane of the imaging system. Autocorrelation analysis of a thermal cloud, as performed here, is a convenient way to extract the \textit{psf} in situ without the need for a small object inside the vacuum chamber and with minimal experimental effort. This method has been developed in [71]. Here we only give the main results and refer to the original work for details and derivations. Instead, we focus on the underlying
Figure 4.9: Principle of the noise correlation analysis method. For image acquisition the cloud is illuminated by a light field $E_{inc}$, creating the field distribution $E_0$ around the object plane, which is transformed by the imaging system to the field distribution $E_{det}$ at the detector. By analyzing the averaged autocorrelations of a thermal cloud of atoms, a subset of aberration parameters can be obtained from a fit to the power spectrum. These can be used to reconstruct the system’s point spread function. The method yields information on the aberrations present in the system and its resolution. Figure taken from [71].
principles of noise correlation analysis and the results obtained for our system. Reference [73] is a review on different applications of noise correlation analysis.

The reconstruction of the psf is based on absorption images of a thermal cloud of atoms, which are fast and easy to obtain in our experiment. We can extract information about our imaging system from the correlations in these images because the length scale imprinted by the imaging process separates well from the inherent length scales of the cloud:

- The density correlations $\delta n$ of the gas are imprinted on the light field used for imaging. The typical correlation length in a thermal gas is given by the thermal de Broglie wavelength
  \[ \lambda_{db} = \frac{\hbar}{\sqrt{2\pi mk_B T}} \]  
  for a gas of atoms with mass $m$ and temperature $T$. The order of magnitude is $\sim 0.1 \mu m$ for a typical temperature of $10 \mu K$.

- The resolution limit of our imaging system is given by the width of the point spread function. This is on the order of $\sim 1-10 \mu m$ for both species.

- The density distribution of the gas is non-uniform due to the initial confinement in a harmonic trap. The envelopes of our atomic clouds after expansions are on the order of $\sim 100 \mu m$.

This means that atomic correlations are much smaller than all other length scales, and any correlations we measure are due to the finite resolution of our imaging system. This is the most important prerequisite for employing autocorrelation analysis to characterize an imaging setup. The density variation of the cloud is in turn on a much larger length scale that is also not relevant for this analysis. For a different parameter regime, one can of course also extract the atomic correlations of a cold gas from autocorrelation analysis, see e.g. [74, 75].

**Figure 4.9** depicts the main steps involved in the analysis: The image of a thermal atomic cloud is recorded via absorption imaging. **Equation 4.23** shows that the imaging process can be described by a convolution with the psf, which is initially unknown. All information about our imaging system is already contained in these images and is extracted subsequently. From the discrete Fourier transform of the averaged autocorrelations in our images, we obtain the power spectrum. It gives the response of the imaging system to correlations at different spatial frequencies.
From the power spectrum we can immediately gain qualitative information about the pupil position with respect to the optical axis and dominant phase aberrations. This information is quantified by fitting the power spectrum, using Zernike polynomials as a basis, and reconstructing the point spread function from the fitting parameters. This way, we obtain the contributions of different types of aberrations and we can obtain the resolution of our imaging system from the width of the reconstructed point spread function.

In the following, we go through the different steps of the analysis, giving the most important equations as we go along, and discuss the results obtained for our imaging system. As our imaging system is designed to image both $^6$Li and $^{133}$Cs atoms, we perform the analysis for both species. As the method is the same in both cases, we show only the results from the analysis of $^{133}$Cs whenever images are shown purely to illustrate the method.

### 4.4.3 Calculation of power spectrum

In practice, we start out with 200 absorption images $A_i$ of a dilute thermal cloud of either Cs or Li atoms. For each run also a reference image $R_i$ without the cloud and a dark image $D_i$ without any imaging light is taken. We immediately subtract the dark image from both the absorption and reference image to remove the influence of stray light and obtain $A'_i = A_i - D_i$ and $R'_i = R_i - D_i$. The top panel of Figure 4.10 shows the average of absorption image over reverence image, $\langle A'_i / R'_i \rangle$. The black rectangle marks the area chosen to compute the power spectrum. Images for lithium look similar and are omitted here.

The power spectrum, which is the absolute square of the Fourier transform of the autocorrelation, is given by

$$M(k_r) = \left< |FT[I_{det}(r)] - \langle I_{det}(r) \rangle|^2 \right>,$$

(4.26)

where $\langle \ldots \rangle$ is used to denote the average over many realizations, and the average of the intensity is subtracted to obtain only the correlations due to the noise. If we didn't subtract the mean intensity, we would expect to see a Gaussian feature very close to the center of the power spectrum due to the cloud envelope.

We perform fringe removal on our absorption images, to avoid contributions of fringes in the power spectrum. This can be done by constructing an optimal ref-
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Figure 4.10: Post-processed images for Cs. Top: Average of absorption image over reference image of a thermal Cs cloud. Bottom: Typical image after fringe removal and subtraction of the mean intensity. Some spatially correlated noise is visible at the position of the cloud, while the background seems uncorrelated, except for some remaining fringes. In both images the black rectangle marks the region chosen for calculating the power spectrum.

Reference image for each image from the weighted average over all other reference images using the algorithm described in [76]. In our case, because we are not interested in possible correlations due to the cloud envelope, we use this step to also subtract the mean intensity according to Equation 4.26: Instead of constructing the optimal reference image from the reference images, we construct it from the absorption images themselves. This is a convenient way to remove fringes and subtract the mean intensity at the same time. The lower panel of Figure 4.10 shows a typical image after removing the mean intensity in this way. While the camera noise in the background of the image has a correlation length of one pixel, the noise at the position of the cloud (marked by the black rectangle) is spatially correlated over longer distances. While this images shows much less fringes than before fringe removal, as shown in the upper panel, some fringes remain visible. The fringe removal is performed for $A'_i$ and $R'_i$ independently. Afterwards we compute the power spectrum $M_A$ for the corrected absorption image and $M_R$ for the same region of interest in the corrected reference image. Then we use the difference $M = M_A - M_R$ for further analysis.

The power spectrum takes the form

$$M(k_r) \sim \left| \left\langle FT \left[ \delta n(r) \right] \right| \right|^2 \left| FT \left[ \Re \left( psf(r) e^{-i\Phi(r)} \right) \right] \right|^2$$  \hspace{1cm} (4.27)
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As the Fourier transform here reverts the Fourier transform connecting the \textit{psf} and the pupil function in \textbf{Equation 4.24}, we can gain access to pupil mismatch and phase aberrations:

$$M(k_r) \sim p(a + k_r)^2 + p(a - k_r)^2 + 2p(a + k_r)p(a - k_r) \times \cos \left( \Phi(a + k_r) + \Phi(a - k_r) - 2\Phi(a) \right),$$

with $p(k_r)$ the illumination of the exit pupil, which can have a shift $a$ away from the optical axis. $\Phi$ denotes phase aberrations that lead to amplitude modulations of the power spectrum in the intersection of the shifted pupil and its $\pi$-rotated counterpart. We fit the pupil and phase contributions to the power spectrum independently. The pupil is modeled by two intersecting circles based on the spherical symmetry of the optical elements in our system.

The resulting power spectra are shown in \textbf{Figure 4.11}. Large structures in the image correspond to small structures in the power spectrum. The very bright central few pixels are due to the length scale of the region of interest chosen in the original image. They are masked for subsequent analysis. We show in green the theoretical size of the power spectrum for a diffraction limited system, which is calculated from the respective wavelength, the magnification, and the NA. The power spectrum for lithium is larger than that for cesium because of the smaller wavelength used for imaging.

Both spectra show a high level of similarity, which suggests that no strong chromatic aberrations are present which would affect the imaging of the different species differently. From our fitting of the pupil contour it is clear that there is a small vertical shift of the pupil away from the optical axis. However this is only about $3\%$ of the vertical pupil radius and should not lead to large effect in our images. We can also immediately see that our resolution is not too far away from the diffraction limit: In horizontal direction the measured radius of the power spectrum is $15\%$ ($26\%$) smaller than theoretically expected, in vertical direction $15\%$ ($20\%$) for cesium (lithium). For an aberration free imaging system we would expect a homogeneous power spectrum up to the pupil border. Any phase aberrations lead to a modulation of the intensity in the intersection of the pupil and its $\pi$-rotated, shifted counterpart. Here, for both lithium and cesium a spherical ring structure close to the center of the spectrum is visible. This could be a sign of spherical aberrations. We also see an intensity decrease when going outwards from the center, which might be a sign of defocus. The dot-like structures with
Figure 4.11: Power spectra for cesium (top) and lithium (bottom). In both images the green circle shows the contour of the expected power spectrum for a diffraction limited imaging system. Red circles correspond to the measured exit pupil. Both spectra show similar features. See main text for discussion.

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$\pi/2$ symmetry that are visible in both spectra, for Cs as a reduction of intensity, for Li as an increased intensity, is an artefact from the power spectrum of the background. This could be due to the remaining fringes visible in Figure 4.10, which show variations on the same length scale as the noise.

In the following we reconstruct the point spread function from a fit to the power spectrum to obtain quantitative information about the leading order aberrations and the resolution of our system for both wavelengths.

### 4.4.4 Quantitative analysis and reconstruction of the psf

The phase contributions are fitted using Zernike polynomials $Z^{(j)}$, which form an orthogonal basis set assuming an elliptical pupil: $\Phi(k_r + a) = \sum A_j Z^{(j)}(k_r + a)$. Combinations of the low-order Zernike polynomials can be connected to different types of phase aberrations. This way it is possible to gain information about the types of dominant aberrations in the system from the fit to the power spectrum.

In practice we perform the fitting in two steps: In the first step we chose initial parameters manually and include polynomials of up to $j = 28$ in the fit, in the second step we use the result of the first fit as initial parameters and include polynomials up to $j = 45$. The results of the second fit are shown in Figure 4.12 for cesium. The fit for lithium looks very similar and is omitted here. The power spectrum is shown in the left panel. The central four pixels are masked because we want to exclude the effect of the final ROI from our analysis. The central panel shows the fit to the power spectrum and the right panel the residual of this fit. We see that the central plateau that falls off towards the edges of the spectrum is very well captured by the fit. The dominant rings close to the center of the spectrum, however, could not be fitted. These might be due to higher-order spherical aberrations, as there is no length scale in the absorption images that would fit the size and structure of the rings. Spherical aberrations have been observed in our imaging system even before implementation into the main experiment, and were attributed to the properties of the achromatic lenses [59]. It is possible to connect combinations of these low-order Zernike polynomials to some fundamental phase aberrations. This means that from the fitting parameters $A_j$ quantitative information about the dominant types of aberrations can be gained.
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Out of the phase aberrations modeled by our fit, we find that defocus and first order spherical aberrations are dominating. The corresponding aberration parameters are $A_d = -0.16$ ($A_d = -0.25$) and $A_{sph} = -0.13$ ($A_{sph} = -0.11$) for lithium (cesium). Some defocus is of course expected because of the chromatic shift, i.e. the difference in the focus positions of our two wavelengths, which is expected to be $\sim 4$ mm by design. However, if this was the main source of defocus in our measurements and our camera was positioned in between the two foci for lithium and cesium, we would obtain opposite signs in the corresponding aberration parameters for lithium and cesium. We repeated the noise correlation analysis at different positions of the camera, but no improvement of the defocus or the overall resolution could be achieved. This indicates that we are already quite close to the optimal camera position, and the remaining defocus stems from the relative position of the lenses in the setup. The spherical aberrations we find are most likely inherent to the lenses, as achromatic lenses had to be chosen and those don’t have good aspheric properties.

In a final step, the point spread function is obtained from the fit according to Equation 4.27. Reconstructing the psf from the aberration parameters in this way has the advantage that a direct comparison to the optimal, diffraction limited system is possible. In contrast to the power spectrum, which was given in the (inverse) image coordinates, we calculate the psf in the coordinates of the object plane, so we can compare its size to the size of the objects we want to resolve. Figure 4.13 shows a vertical and horizontal cut through the reconstructed psf in comparison
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to the psf for the diffraction limited case. Results for both imaging of $^6$Li and $^{133}$Cs are shown. The psfs for both species are very similar, which is expected because the similarity of their power spectra does not indicate a large difference between the aberrations for the two wavelengths. The difference in the width of the curves is due to the different wavelengths. The cuts in both directions look exactly the same. It is obvious that the maximum of the reconstructed psf is only about two thirds as high as theoretically possible. While the width of the central maximum is only slightly increased compared to the diffraction limited case, we can see that outside of the central maximum the psf doesn’t go down to zero between the side-lobes. This shift of spectral weight away from the central maximum of the psf is due to the aberrations discussed above and limits the resolution.

To quantify the resolution of our system we use a criterion that is inspired by how you would determine the resolution after measuring the point spread function by imaging a small object: We fit a two-dimensional Gaussian to the absolute square of the psf and give its standard deviation $\sigma_{psf}$. For comparison we also give the standard deviation $\sigma_{diff}$ for the diffraction limited case. The measured value differs from the theoretical limit by about 21% for lithium and about 13% for cesium. A common criterion to compare the aberration limited and the diffraction limited psf is the Strehl ratio, which quantifies the spectral weight in the central peak by comparison of the maxima and is given by

$$\text{Strehl ratio} = \frac{|\text{Maximum of measured psf}|^2}{|\text{Maximum of diffraction limited psf}|^2}.$$ (4.29)

For a system close to the diffraction limit we would expect a value of $\sim 0.8 - 1$. Our values of 0.49 (for cesium) and 0.43 (for lithium) show that we are not limited by diffraction at this point and there is still room for improvement. However, the obtained resolution is already quite good, especially considering the challenges that imaging with two very different wavelengths imposes on the design and implementation of a high-resolution imaging system.

<table>
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<tr>
<th>Species</th>
<th>$\lambda$ (nm)</th>
<th>$\sigma_{psf}$ ($\mu$m)</th>
<th>$\sigma_{diff}$ ($\mu$m)</th>
<th>Strehl ratio</th>
</tr>
</thead>
<tbody>
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<td>1.17</td>
<td>0.49</td>
</tr>
<tr>
<td>$^6$Li</td>
<td>671</td>
<td>1.16</td>
<td>0.92</td>
<td>0.43</td>
</tr>
</tbody>
</table>
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Figure 4.13: Horizontal and vertical cut through the point spread function for cesium (top) and lithium (bottom). The reconstructed point spread functions are shown in blue, while the red curves show the ideal, diffraction limited case.
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To summarize, we have realigned our setup for vertical imaging and characterized its performance using the method of noise correlation analysis developed in [71]. Results were obtained for both the imaging of cesium at 852 nm and lithium at 671 nm. We find that the imaging properties for the different atomic species are very similar, and a high resolution can be obtained for both. With our current resolution we will easily be able to resolve even a small BEC. The resolution is limited by defocus and spherical aberrations. To some degree both of these are inherent in the design of the setup for two wavelengths. This is because the achromatic lenses implemented are not aspheric and, despite their achromatic properties, a small remaining chromatic shift of $\sim 4$ mm between the foci for the two wavelengths is expected. We suggest to repeat the analysis performed here in regular intervals to keep track of possible changes, and realign if necessary. This is feasible because the measurements can be performed with minimal experimental effort, and key numbers like the dominant aberration parameters or the resolution are easy to compare.

In this chapter we have started out by giving a short introduction to absorption imaging in general and to our particular setup for imaging both lithium and cesium along the vertical direction. We then focused on the imaging of lithium and characterized the noise inherent in the imaging process. We found that it is beneficial to image at a high number of counts, which corresponds to long imaging times above 100 $\mu$s at our intensity. However, the imaging time is limited by the blurring of the images due to the random walk of the atoms and the underestimation of the atom number due to the detuning introduced by the Doppler shift. We were able to compensate for the latter by dynamically changing the laser frequency during imaging. This enables us to image for longer times, thus achieving a higher SNR. Further improvements could be made by imaging with a higher intensity on the order of the saturation intensity at shorter times of few microseconds. However, this would require significantly more laser power than currently available. We further analyzed the performance of our imaging system using a method based on the analysis of correlations in the images of thermal atomic clouds. We find very similar imaging conditions for lithium and cesium and excellent resolution for both. As we are now able to image very small numbers of lithium atoms and, with a resolution in the micrometer regime, are able to resolve small structures like a cesium BEC, we are well equipped to detect signatures of the Bose polaron in our system.
5 Conclusion and Outlook

In this thesis we have reported on major steps towards the investigation of the Bose polaron in a highly mass-imbalanced mixture of ultracold $^6$Li and $^{133}$Cs atoms. After giving a brief introduction to the polaron scenario, we started out in chapter 2 by explicitly including the effect of the rf pulse in the time evolution of the system consisting of a single $^6$Li impurity in a weakly interacting $^{133}$Cs BEC. Previously only the spectral function of the polaron system had been considered, which can be calculated from the Fourier transform of the overlap of the polaron state with the bare impurity within linear response theory. Including the coupling of the polaron states by the rf pulse made it possible to investigate the spectral response of the system even beyond linear response, i.e. for a significant population of the interacting state. This is relevant because in the experiment, transferring a large fraction of atoms to the interacting state yields a good signal-to-noise ratio in the detection. We compared our results for short rf pulses with the spectral function and find excellent agreement for attractive, resonant and repulsive impurity-boson interactions. We analyzed the time-dependence of the response for different detunings along the repulsive polaron branch and found that linearity breaks down first for very large detunings. Some first results for longer rf pulses and a maximal atom transfer to the interacting state of 35% indicate that the spectral response of the system might stay close to the spectral function even beyond linear response, but further analysis is necessary. Extending our approach to time-dependent Rabi frequencies enabled us to compare polaron spectra for different pulse shapes. The code developed in the scope of this thesis can serve as a guideline to choose suitable rf pulses for the spectroscopy of the Bose polaron at our experiment in the future. The results from this chapter were obtained in collaboration with Meera Parish and Jesper Levinsen at Monash University, employing methods previously developed in their group.

In chapter 3 we introduced our experimental approach to rf spectroscopy of the Bose polaron. We drive transitions between the two energetically lowest hyperfine states of $^6$Li, which serve as the non-interacting and interacting impurity state
in our experiment. The implementation of a new amplifier enables us to reach Rabi frequencies up to $\Omega = 2\pi \times 13.5 \text{kHz}$. An interlock system based on an Arduino Uno prevents the rf antenna, which is placed inside the vacuum chamber, from overheating when high powers are applied. It also protects the amplifier from damage due to too input high powers. A precise calibration of the new setup allows us to manipulate the state of the atoms in a controlled way with pulses of different powers and lengths. Additionally, we have implemented the possibility to create rf pulses with arbitrarily shaped temporal envelopes. We showed that choosing Blackman-shaped pulses is useful to suppress side lobes in the spectra. This is of importance when expecting small polaron energies, since they can easily be masked by the side lobes of the bare transition’s spectral response. We also present our first attempt to measure signatures of the Bose polaron by performing rf spectroscopy on a dilute lithium cloud interacting with a thermal gas of cesium atoms.

In order to be able to accurately detect even a small number of lithium atoms, and to resolve a cesium BEC in the future, we characterize and improve our new dual-species high-resolution imaging setup in chapter 4. To this end, we improve the signal-to-noise ratio for imaging lithium atoms. We start out by measuring the noise inherent in the imaging process in a photon transfer curve, and find that it is beneficial to obtain at least 2000 counts per pixel on average, which requires an imaging time of $100 \mu s$ at our current intensity of $I \approx 0.1 I_{\text{sat}}$. The biggest source of noise are currently fluctuations of the imaging intensity due to the AOM in our setup. When choosing appropriate imaging parameters, we need to consider two additional physical effects that can limit the quality of our images: The random walk the atoms perform due to spontaneously emitting photons in all directions leads to a blurring of the density distribution for long times. The absorption of imaging photons accelerates the atoms along the beam, leading to a Doppler shift in their transition frequency. This effect can lead to an underestimation of the atom number if the atoms become off-resonant during imaging. For both of these effects we derive constraints on the imaging time in dependence of the intensity. At our current intensity we need to violate both conditions to image at a good signal-to-noise ratio. However, we can compensate for the Doppler shift by linearly increasing the laser frequency during imaging. This way, the underestimation of the atom number is less severe and can be accounted for by calibration. We also implement the possibility to image both of the two energetically lowest lithium spin states within the same experimental realization using a fast jump of
the laser frequency of about 76 MHz. Further improvements could be obtained in the future by imaging with a higher intensity of $I \approx I_{\text{sat}}$ for shorter times of $\sim 10 \mu s$.

We apply the method of noise correlation analysis developed in [71] to analyze the performance of our imaging system after a complete realignment of its components. This method is based on the analysis of the correlations in images of thermal atomic clouds. As the natural correlation length of a thermal gas lies far below the resolution limit of typical imaging systems, any measured correlations are imprinted by the imaging system and give access to its properties. Results were obtained for both the imaging of cesium at 852 nm and lithium at 671 nm. We find that the imaging properties for the different atomic species are very similar, and a high resolution of $1.35 \mu m$ ($1.16 \mu m$), according to the standard deviation of a Gaussian fit to the point spread function, can be obtained for cesium (lithium). With our current resolution we will easily be able to resolve the density distribution of even a small BEC. The resolution is limited by defocus and spherical aberrations. To some degree both of these are inherent in the design of the setup for two wavelengths. This is because the achromatic lenses implemented are not aspheric and, despite their achromatic properties, a small remaining chromatic shift of $\sim 4 \text{ mm}$ between the foci for the two wavelengths is expected. Repeating the analysis performed here in regular intervals will allow us to keep track of possible changes in our imaging system, and realign if necessary.

As a next step in our experiment, we are aiming to create a cesium BEC and immerse lithium atoms in it. In comparison to the cesium densities in our current thermal cloud, we expect up to two orders of magnitude higher densities in a BEC. This will lead to larger energy shifts in the polaron spectrum, which in turn allows us to probe the system with shorter rf pulses. Creation of the BEC close to the zero-crossing of the Cs-Cs scattering length at a magnetic field of 880 G would enable us to use the Li-Cs FR at 889 G to vary the interactions between the impurities and the BEC. A cesium BEC has been created previously in our experiment at low magnetic fields of 21 G [54, 58], however, without simultaneously trapping lithium atoms. Simultaneous quantum degeneracy of $^6\text{Li}$ and $^{133}\text{Cs}$ has recently been reported by DeSalvo et al. [77]. They were able to confine a small number of lithium atoms inside a cesium BEC by moderate attractive interspecies interactions. This trapping scheme leads to a perfect overlap of the two species, even at very low temperatures. On the other hand, relying on the lithium-cesium interactions for trapping, limits their tuneability in experiments.
Additionally, we are aiming to realize optimal trapping conditions of our two atomic species to realize the Bose polaron scenario. Here, two points are of importance. Firstly, all of the lithium atoms should interact with the cesium atoms. Secondly, all lithium atoms should probe the same cesium density, because an inhomogeneous density distribution of the bath broadens the polaron signal. Both of these criteria could be met by trapping a few thousand lithium atoms in a small micro-trap with a waist of a few micrometers. This trap could then be positioned in the center of a much larger cesium cloud where the density variations are small. In order to not disturb the cesium bath with the micro-trap, the tune-out wavelength of cesium at 880 nm could be used.

These next steps, together with the experimental improvements and theoretical considerations presented in this thesis, will make it possible to investigate the Bose polaron and its connection with Efimov physics in our experiment.
References


References


References


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Statement of Authorship

I herewith declare that this thesis was solely composed by myself and that it constitutes my own work unless otherwise acknowledged in the text. I confirm that any quotes, arguments or concepts developed by another author and all sources of information are referenced throughout the thesis. This work has not been accepted in any previous application for a degree.

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