Observation of Efimov Resonances in a Mixture with Extreme Mass Imbalance

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We observe two consecutive heteronuclear Efimov resonances in an ultracold Li-Cs mixture by measuring three-body loss coefficients as a function of magnetic field near a Feshbach resonance. The first resonance is detected at a scattering length of $a_-^{(0)} = -320(10)a_0$, corresponding to $\sim 7(\sim 3)$ times the Li-Cs (Cs-Cs) van der Waals range. The second resonance appears at $5.8(1.0)a_-^{(0)}$, close to the unitarity-limited regime at the sample temperature of 450 nK. Indication of a third resonance is found in the atom loss spectra. The scaling of the resonance positions is close to the predicted universal scaling value of 4.9 for zero temperature. Deviations from universality might be caused by finite-range and temperature effects, as well as magnetic field–dependent Cs-Cs interactions.

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The control of interactions in ultracold atomic systems via magnetically tunable Feshbach resonances opens up new pathways for the investigation of few- and many-body physics [1]. One intriguing example is the access to the universal regime, which is characterized by a magnitude of the scattering length a exceeding all other length scales of the system. In the limit of at least two resonant pairwise interactions, an infinite series of three-body bound states, the so-called Efimov states, exists [2–4]. Counterintuitively, these trimers persist even for $a < 0$, where the two-body potential does not support a bound state. The ratio between two subsequent trimer energies follows a discrete scale invariance with a universal scaling factor of $\exp(-2\pi/s_0)$. Here, s_0 depends only on the quantum statistics of the constituent atoms, their mass ratio, and the number of resonant interactions [3,5]. This scale invariance is also reflected in those values of a where the energy of the bound states coincides with the threshold of three free atoms for $a < 0$, resulting in enhanced three-body loss. When the position of the first resonance is given by $a_-^{(0)}$, the Nth excited state is found at the scattering length $a_-^{(N)} = a_-^{(0)} \exp(\pi N/s_0)$. It has been shown that, for homonuclear systems, $a_-^{(0)}$ depends only on the characteristic range r_0 of the interatomic van der Waals potential [6–11]. The universal scaling factor acquires a value of 22.7 for equal mass constituents and features a drastic reduction in heteronuclear mass-imbalanced systems of two heavy particles and one light particle [3,5], resulting, e.g., in a factor of 4.9 for a ^{6}Li-^{133}Cs mixture.

In ultracold atom experiments, Efimov resonances become evident in the three-body loss coefficient L_3 in the rate equation for atom loss $\dot{n} = -L_3 n^3$. Here, n denotes the number density of atoms, and $L_3 \propto C(a)a^4$. The Efimov physics are contained in the dimensionless, log-periodic function $C(a)$. Thus far, Efimov resonances have been studied in several equal mass systems [6,7,12–18], where the scaling between different resonances is predicted to follow $C(a) = C(22.7a)$. This large scaling factor demands a level of temperature and magnetic-field control that makes the observation of an excited Efimov states highly involved. There had been indication of such an excited state in a three-component Fermi gas of ^{6}Li atoms [19] exhibiting the same scaling as equal mass bosons [20]. A finite temperature model [21] suggests that the experimental conditions of current experiments [21,22] are close to temperature regimes where the observation of a second excited resonance in bosonic ^{7}Li becomes feasible. In fact, recently, two consecutive Efimov resonances have been observed in a system of three Cs atoms [23].

In heteronuclear systems, only K-Rb mixtures have so far been investigated [24,25], where a scaling factor of ~ 131 obstructs the observation of an excited Efimov state. In ^{6}Li-^{133}Cs the predicted scaling factor of 4.9 [3,5] and the ability to tune the scattering length over a large range due to broad Feshbach resonances [26,27] benefit the observation of a series of Efimov resonances.

In this Letter we present the observation of two consecutive Efimov resonances near the broad Feshbach resonance at 843 G in the energetically lowest ^{6}Li-^{133}Cs channel, via measurements of the three-body loss coefficient. The assigned scattering lengths $a_-^{(0)} = -320(10)a_0$ and $a_-^{(1)} = -1871(388)a_0$ yield a ratio of $a_-^{(1)}/a_-^{(0)} = 5.8(1.0)$, which is consistent with the ratio for two consecutive bound states in a zero temperature model [3,5]. However, our analysis shows that for our experimental conditions the first excited Efimov state is already close to the unitarity limit, which leads to broadening of the resonant feature in the three-body loss spectrum and, additionally, might cause shifts of its position. We observe another loss feature close to the anticipated position of the doubly excited Efimov state, but it is at the limit of our sensitivity of three-body loss coefficient measurements due to the unitarity-limited regime.

Our experimental setup is similar to the one presented in Ref. [26]. In brief, we load Cs into a crossed optical dipole trap (reservoir trap) at a wavelength of 1064 nm with $1/e^2$ beam waists of 300 μm and a crossing angle of 90°. We perform degenerate Raman sideband cooling [28] in order to prepare the majority of the atoms in the energetically lowest $|F=3, m_F=3\rangle$ spin state. Here, F denotes the total atomic spin, and m_F its projection. Lithium atoms in the ground state $|F=1/2\rangle$ manifold are loaded into another crossed dipole trap (dimple trap), located 1 mm away from the reservoir trap, which operates at a wavelength of 1070 nm, has $1/e^2$ beam waists of 62 μm, and a crossing angle of ~8°. After evaporative cooling of both species in the separated traps, they are combined via a piezo-driven mirror; the reservoir trap is subsequently turned off in a slow ramp. In the final evaporation step we sympathetically cool Cs close to the $\mathrm{Li}|1/2,-1/2\rangle \oplus \mathrm{Cs}|3,3\rangle$ Feshbach resonance at 943 G [26,27], which expels all atoms in the $\mathrm{Li}|1/2,-1/2\rangle$ state from the trap. Finally, 1.6×10^4 (4×10^4) atoms remain in the $\mathrm{Cs}|3,3\rangle$ $(\mathrm{Li}|1/2,1/2\rangle)$ state. We measure trap frequencies f_x, f_y, f_z of 114 Hz × 123 Hz × 11 Hz (275 Hz × 308 Hz × 33 Hz) and temperatures around 400 nK for both species. The magnetic fields are calibrated via microwave spectroscopy of Li, with a resolution of 30 mG, limited by a residual magnetic field gradient along the long axis of the cigar-shaped trap. Day-to-day drifts on the order of 20 mG add to the systematic uncertainty of the magnetic fields.

To measure the Efimov resonances, we ramp the magnetic field from 943 G to selected values near the broad Feshbach resonance at 843 G [26], and we measure the Cs and Li atom numbers after a variable hold time by standard absorption imaging [29]. The results of our measurements are summarized in Fig. 1. The remaining number of Cs atoms for a hold time of 1 s is depicted in Fig. 1(b). To minimize the influence of systematic drifts, we randomize the order in which the points are measured. As the increasing scattering length leads to fast losses closer to the Feshbach resonance, we perform a similar scan with a reduced hold time of 400 ms for the region closer to the Feshbach resonance, as shown in Fig. 1(a). Besides rising three-body losses associated with the increasing interspecies scattering length, we observe two clear features in the Cs loss data (dashed vertical lines). Fitting Gaussian profiles with linear background yields $B_0 = 849.12(6)_{\mathrm{stat}}(3)_{\mathrm{sys}}$ G and $B_1 = 843.89(1)_{\mathrm{stat}}(3)_{\mathrm{sys}}$ G for the positions of the resonances, where the first error denotes the statistic uncertainty of the fit and the second error results from the systematic uncertainty of the absolute magnetic field value. We identify these as LiCsCs Efimov resonances. We also detect a slight increase of the atom losses at a field of $B_2 = 843.03(5)_{\mathrm{stat}}(3)_{\mathrm{sys}}$ G, where the first error contains the width of the fitted Gaussian profile. The loss feature, which indicates a possible third resonance, is shown more clearly in the inset of Fig. 1(a).

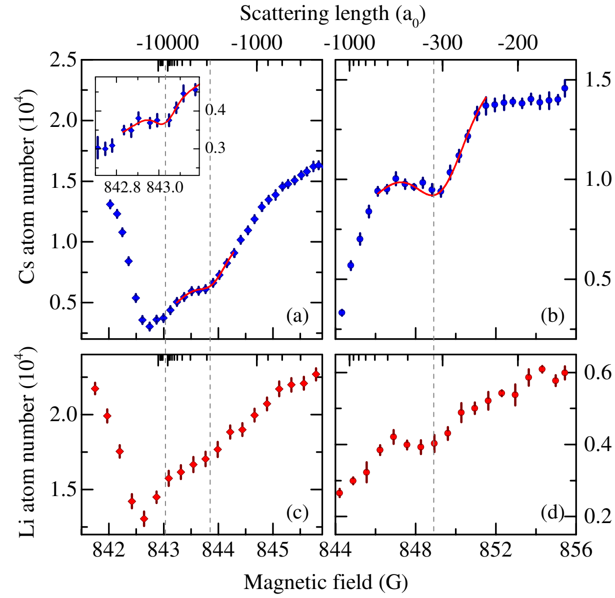

FIG. 1 (color online). Magnetic field–dependent atom loss measurements of the Li-Cs mixture at a temperature of 400 nK. The Cs atom number after a hold time of 400 ms (a) and 1 s (b) is illustrated. The inset in (a) is a zoom in to the region where a third loss feature can be seen. The Li atoms show resonances at consistent positions after a hold time of 1.2 s (c) and (d), when the initial Li atom number is reduced by a factor of 2 as compared to the measurements in (a) and (b). Each point is the mean of at least six independent measurements, and the error bars represent the standard error. The dashed vertical lines indicate the position of the Efimov resonances. The resonance positions are determined via a fit of Cs atoms with Gaussian profiles with linear background (red solid lines). The scattering length scale has been assigned via radio frequency association of universal dimers [30].

Under the experimental conditions of Figs. 1(a) and 1(b), features in the Li loss signal are too small to be discerned from fluctuations of the Li atom number, due to experimental instabilities. However, by lowering the Li atom number, which increases the fraction of lost atoms, we recover the Efimov resonances at consistent magnetic fields, as depicted in Figs. 1(c) and 1(d).

In order to verify that the observed Efimov resonances are indeed caused by a three-body process with the same ratio of Cs to Li atoms involved, the three-body loss coefficient is measured by observing the time-dependent atom losses for various magnetic fields. The dimple-trap depth is ramped up by about 10% to reduce residual evaporation; this, however, also increases the temperature to ~450 nK. At the start of the measurement we prepare a mixture of 2×10^4 (3×10^4) Cs (Li) atoms. After typical hold times on the order of a few seconds, we lose nearly all Cs atoms, while the number of Li atoms only reduces by approximately 30%. The temperature of the mixture remains unaffected within the uncertainties of our temperature determination (∼15%).

The evolution of n_{Cs} is given by the rate equation

$$\dot{n}_{\mathrm{Cs}} = -L_1^{\mathrm{Cs}} n_{\mathrm{Cs}} - L_3^{\mathrm{LiCsCs}} n_{\mathrm{Li}} n_{\mathrm{Cs}}^2 - L_3^{\mathrm{Cs}} n_{\mathrm{Cs}}^3. \quad (1)$$

Here, L_1^{Cs}, L_3^{LiCsCs}, and L_3^{Cs} are the loss coefficients for Cs background collisions, Li + Cs + Cs three-body collisions, and Cs + Cs + Cs three-body collisions, respectively. The inter- and intraspecies two-body losses are ignored in Eq. (1) because the atoms are in the energetically lowest states and thus only exhibit elastic two-body collisions. Under the conditions of the experiment the temperature dependence of the three-body loss coefficients [21] can be neglected, and n_{Li} can be assumed constant to a good approximation. Because the temperature is nearly constant, the change in density can be directly linked to the change in atom number.

Our analysis shows that the Cs-atom loss curves are well described by Li + Cs + Cs and Cs + Cs + Cs losses, while a description assuming Li + Li + Cs and Cs + Cs + Cs losses alone does not reproduce the shape of the loss curves [30]. This verifies that Li + Li + Cs three-body losses are indeed strongly suppressed due to Fermi statistics and confirms that the observed Efimov resonances originate from the Li + Cs + Cs channel, which is the only channel for this mixture that is predicted to support universal three-body bound states [3,5]. This is also reflected in the loss ratio of Li to Cs atom numbers of ∼1∶2 in the entire range of the magnetic fields probed.

In order to extract the loss rates from our measurements, we integrate Eq. (1) over the spatial coordinates and fit the solution to the time-dependent Cs atom number loss curves, with L_3^{LiCsCs} as the only fitting parameter. L_1^{Cs} and L_3^{Cs} are independently obtained from single species measurements, and therefore do not enter as fitting parameters. We note that a close-by intraspecies Efimov resonance in Cs [31] at 853 G does not influence determination of the Li-Cs Efimov resonances, since L_3^{Cs} is approximately constant in its direct vicinity and changes only in the region $a_{\mathrm{LiCs}} > -250 a_0$ [30].

The extracted three-body loss coefficient L_3^{LiCsCs} is depicted in Fig. 2. We estimate that the systematic error for the absolute value of L_3^{LiCsCs} is on the order of 80%, due to uncertainties in measured atom numbers, temperatures, and trap frequencies. Additionally, the gravitational sag reduces the spatial overlap of the atom clouds. We estimate that this effect reduces the spatial integral over the densities in Eq. (1) by ∼20%, and we include this reduction into our model. Day-to-day drifts in the beam pointing of the dimple trap might cause fluctuations of the overlap.

A precise determination of the field-dependent scattering length $a(B)$ is essential for a quantitative analysis. In particular, the exact value of the scattering pole of the Feshbach resonance B_{FR}, where $|a|$ diverges, plays a crucial role. We measure this value via radio frequency spectroscopy of universal dimers, which yields $B_{\mathrm{FR}} = 842.9(2)$ G and $\Delta B = 61.4(7)$ G for the resonance position and width, respectively [30]. These values are in excellent agreement with an extensive study of Li-Cs Feshbach resonances via three different models [32]. Inserting these quantities into the relation

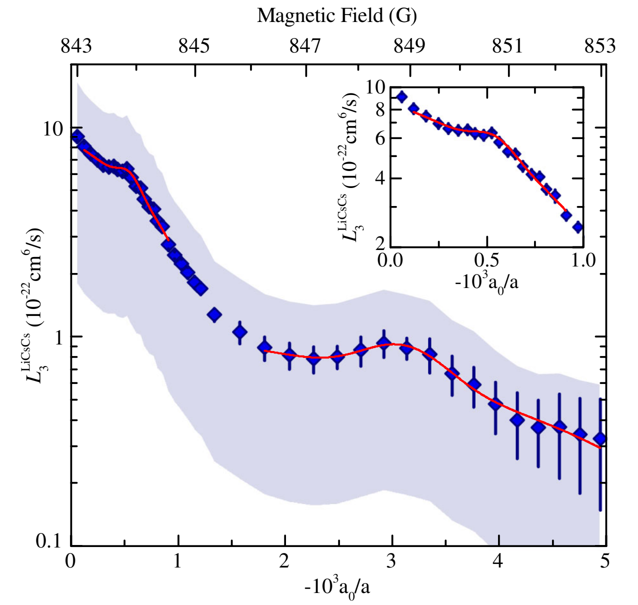

FIG. 2 (color online). Three-body loss coefficient L_3^{LiCsCs} plotted versus the inverse scattering length $1/a$. The blue diamonds show the mean of three L_3^{LiCsCs} measurements, where the error bars are given by the standard error. The red solid lines show Gaussian profiles with linear background fitted to the data to determine the position of the two Efimov resonances. The grey area illustrates the systematic error of 80% for the absolute value of L_3^{LiCsCs}. The inset shows a zoom in to the region of the first excited Efimov resonance.

$$a(B) = a_{bg}[\Delta B/(B - B_{\mathrm{FR}}) + 1] \quad (2)$$

(cf. [33]) allows us to determine the abscissa in Fig. 2, where $a_{bg} = -28.5 a_0$ [26].

We observe two distinct resonances in the L_3^{LiCsCs} measurements, which are consistent with the enhanced atom losses in Fig. 1. For large values of the scattering length, the loss rate approaches a value that is consistent with an order of magnitude estimate for the unitarity limit $L_3^{\mathrm{Lim}} \approx 10^{-21}$ cm^6/s for the temperatures in our experiments [34].

Due to the lack of a finite temperature model for heteronuclear Efimov resonances similar to the one in Ref. [21], the position of the resonances is determined via a fit of a Gaussian profile with linear background, which results in $B_0 = 848.90(6)_{\mathrm{stat}}(3)_{\mathrm{sys}}$ G and $B_1 = 843.85(1)_{\mathrm{stat}}(3)_{\mathrm{sys}}$ G. Using Eq. (2), we assign the scattering lengths $a_-^{(0)} = -320(3)_{\mathrm{stat}}(2)_{\mathrm{sys}}(10)_{\mathrm{rf}} a_0$ and

$a_-^{(1)} = -1871(19)_{\rm stat}(58)_{\rm sys}(388)_{\rm rf}a_0$ for the Efimov resonance positions, where the third error accounts for the uncertainties in the Feshbach resonance position and width as extracted from the radio frequency spectroscopy [30].

For the scaling between the first and second Efimov resonance positions, we obtain $a_-^{(1)}/a_-^{(0)} = 5.8(0.1)_{\rm stat}(0.2)_{\rm sys}(1.0)_{\rm rf}$. This value is close to the predicted scaling of 4.9 [3,5] for a zero-temperature gas in the universal limit ($|a| \gg r_0$).

The assumption of universal behavior is not strictly justified for the first resonance, since $a_-^{(0)}$ is only a factor of ~7 (~3) larger than the Li-Cs (Cs-Cs) van der Waals length $r_0^{\rm LiCs} = 45a_0$ ($r_0^{\rm Cs} = 101a_0$), suggesting that finite range corrections might be important. The second resonance is already influenced by the unitarity limit at the temperatures achievable in the current experiment; this leads to a broadening of the resonance feature and might also cause additional shifts. In addition, the variation of the Cs intraspecies scattering length between $-1200a_0$ at B_0 and $-1500a_0$ at B_1 [31] still needs to be accounted for in the theoretical determination of the expected scaling ratio. Preliminary calculations [35] based on the model of Ref. [36] suggest that the observed ratios are consistent with a slightly larger scaling factor. However, these results are subject to ongoing investigation and are beyond the scope of this Letter.

The position of the third resonance is indicated by a modulation of the atom losses at $B_2 = 843.03(5)_{\rm stat}(3)_{\rm sys}$ G in Fig. 1, for which we assign the scattering length $a_-^{(2)} = -13.5(5.2)_{\rm stat}(3.1)_{\rm sys} \times 10^3 a_0$. The error due to systematic uncertainties in $B_{\rm FR}$ is on the order of $a_-^{(2)}$, and can even result in positive scattering lengths at B_2. In a dedicated measurement of $L_3^{\rm LiCsCs}$ performed in the vicinity of B_2, we do not resolve an additional loss feature associated with the third resonance. However, this feature is in a regime where the scattering length is on the order of the thermal wavelength, and therefore not the dominating length scale; a potential second excited Efimov resonance would be significantly reduced to the point where it could no longer be resolved. As a result, even though the losses at B_2 in Fig. 1 are likely caused by Li + Cs + Cs collisions, we cannot unambiguously validate this hypothesis at this point.

After having found two consecutive Efimov resonances in the three-body loss rate coefficient of a mixture with large mass imbalance, the next experimental step will be the creation of a Li-Cs mixture at significantly lower temperatures. The unitarity-limited regime will be pushed towards larger values of the scattering length due to its scaling with $\propto 1/T^2$, and the resonances should become narrower and exhibit smaller shifts. In the current experiment, the gravitational sag due to the large mass of Cs leads to a separation of the atom clouds, which limits the lowest achievable temperatures of the mixture. Therefore, a dedicated engineering of trap potentials with species-selective trapping forces [37] will be required. Based on a zero-range model [38] and calculations by C. Greene *et al.* [35], we estimate that temperatures on the order of 30 nK result in a unitarity-limited three-body loss coefficient of $\sim 2 \times 10^{-19}$ cm$^6/s$, which is more than 2 orders of magnitude larger than the current limit. In order to avoid the influence of the unitarity limit on the third resonance, one would require temperatures on the order of ~1 nK. Extending the model presented in Ref. [21] to the case of heteronuclear mixtures, however, might allow for a detailed analysis of the influence of unitarity at higher temperatures. An intriguing perspective is the study of finite-size effects on the Efimov trimers [39], as the size of the doubly excited Efimov trimer of ~0.2 μm is of the same order as the oscillator length of the trapping potential used in the present experiments. A second major improvement over the current investigations will be a more precise determination of the magnetic field scattering pole $B_{\rm FR}$ in order to reduce the uncertainties in the assignment of scattering lengths to the observed resonances. Here, we expect an improvement by an order of magnitude will be possible by performing additional spectroscopy on the binding energy of the universal dimer with improved magnetic field stability. A more accurate determination of the resonance positions would shed new light on the application of universal few-body theories to mixed systems with large mass imbalance, addressing, e.g., to what extent the position of the first Efimov resonances also features universal scaling, as recently found in homonuclear systems [6,7].

Note that atomic loss features in Li-Cs under comparable experimental conditions are presented in Ref. [40]. The ratios between the first and second resonance are consistent within the errors. In contrast, we refrain from discussing the third resonance in terms of universal scaling due to the unitarity regime. If we perform an analysis of the features in the loss spectra analogous to Ref. [40], where a parabola is fitted to the global loss minimum to determine $B_{\rm FR} = 842.73(1)_{\rm stat}(3)_{\rm sys}$, we obtain $a_-^{(1)}/a_-^{(0)} = 5.07(6)_{\rm stat}(13)_{\rm sys}(2)_{\rm FR}$ and $a_-^{(2)}/a_-^{(1)} = 3.79(23)_{\rm stat}(39)_{\rm sys}(6)_{\rm FR}$ for the scaling ratios (errors labeled by FR are representing the fit uncertainty of $B_{\rm FR}$).

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