HEIDELBERG UNIVERSITY

FP 13

Advanced Students Laboratory:

Characterisation of Silicon Photomultipliers & Measurement of the Fermi Constant

Version 1.1

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Contents

Chapter 1

Physics background

The basis of particle physics is the so-called Standard Model. It was made in the 1970s as a summary of all previous discoveries in particle physics. Up to date it is neither disproved in any important point, nor it has been yet experimentally confirmed in all fields. It is built on three pillars: the particles, the interactions and the Higgs mechanism. Particles and interactions are closely related, since the interactions are accomplished by and between particles. The Higgs mechanism gives the particles their mass and comes with an additional particle, the Higgs boson.

1.1 The Particles

In the Standard Model, elementary particles belong to three groups, which are Quarks, Leptons and the mediator particles, which are responsible for the interactions between different particles.

Flavor	**Mass** [MeV/c^2]	**el. charge**	**Flavor**	**Mass** [MeV/c^2]	**el. charge**
up	$2.2^{+0.6}_{-0.4}$	2/3 (e)	down	$4.7^{+0.5}_{-0.4}$	- 1/3
charm	1270 ± 30	2/3 (e)	strange	96^{+8}_{-4}	- 1/3
top	173210 ± 850	2/3 (e)	bottom	4180^{+40}_{-30}	- 1/3

Table 1.1.1: Quarks in the Standard Model [1].

Quarks and leptons have spin 1/2 and therefore are fermions. They are the elementary constituents of matter. Quarks carry colour charge, and they can experience - in contrast to leptons - the strong force. They also have a fractional electrical charge (see Table 1.1.1). Compositions of quarks are called hadrons. The combination of a quark and an anti-quark is called meson. In contrast, a particle is called baryon if it consists of three quarks. The masses of the quarks are very different and also very difficult to determine. They range from about 1.5 $\mathsf{MeV/c^2}$ (up quark) up to about 173 $\mathsf{GeV/c^2}$ (top quark).

Flavour	Mass	El. Charge	Flavour	Mass	El. Charge
e	0.511	-1e	ν_e	$< 2 \cdot 10^{-6}$	0
μ	105.66	-1e	ν_μ	$< 2 \cdot 10^{-6}$	0
τ	1776.86	-1e	ν_τ	$< 2 \cdot 10^{-6}$	0

Table 1.1.2: Leptons in the Standard Model [1]. Masses are stated in units of $\mathsf{MeV/c^2}$

Like quarks, also leptons come in three families: electron, muon and tauon, each with the corresponding neutrino (see Table 1.1.2). The electron is stable, while muons and tauons usually decay in a lower mass lepton, thus an electron or a muon. In contrast to neutrinos, electron, muon and tauon have a negative electrical charge, while the corresponding anti-particle has a positive electrical charge. They are subject to gravitational, electromagnetic and weak interaction. Their mass ranges from 511 $\mathsf{keV/c^2}$ to 1777 $\mathsf{MeV/c^2}$. Neutrinos are not only electrically neutral, but their mass is also close to zero.

1.2 The Gauge Bosons / Interactions

There are four fundamental interactions: the gravitational, the electromagnetic, the strong and the weak interaction. Each of them has one or more gauge bosons associated, since in quantum field theory the interactions are described as the exchange of particles. The Standard Model describes all of them except the gravitation.

Interaction	Mediator	acts on
Electromagnetic	Photon	Quarks, leptons (not neutrinos)
Strong	8 Gluons	Quarks
Weak	$W^\pm$, Z^0	Quarks, leptons

Table 1.2.1: Gauge bosons in the Standard Model.

Gravity is the weakest of the four interactions. Even if, because of its long range and because there is only positive charge (mass), it seems macroscopically very strong, it is small in relation to the mass of elementary particles. It is 10^{38}-times weaker than the strong interaction and therefore negligible in particle physics. As the topic of this experiment is the μ-decay, which takes place via the weak interaction, the discussions in the following are limited to this interaction.

When radioactivity and hence the β-decay of the neutron ($n \rightarrow p\, e\, \nu_e$) was discovered by Becquerel in 1896, it soon became clear that, due to the long decay time, the underlying process could not be explained by the strong force. Moreover, such an electromagnetic decay was extremely unlikely. This was a hint of a new interaction, namely the weak interaction. In order to preserve the conservation of energy in this process, Pauli postulated the neutrino in 1930. In general, the weak force couples both to quarks and to leptons. In this processes particles can change the electrical charge - then the exchange particle is a W^+ or a W^- - or leave the same electrical charge - through the exchange of a Z^0. The exchange particles couple to the weak charge g, which is carried by both quarks and leptons. A special feature of the weak interaction is that it is parity violating. This is for example visible in the muon decay.

The strong interaction is responsible for the cohesion between the quarks. The more they are separated from each other, the stronger is the attractive force between them. The exchange particles of the strong interaction are 8 gluons, which carry colour charge, the charge of the strong force. Thus the gluons can interact also with each other. The introduction of colour charge was necessary after the discovery of the Ω^- particle. Since it is made out of three s-quarks, they are subject to the Pauli principle and in order to be allowed there must be an additional quantum number.

1.3 The Muon

The muon is an elementary particle. It is associated with the muon neutrino and belongs to the second generation of leptons. The production of a muon is always accompanied by the production of a muon anti-neutrino ($\overline{\nu}_\mu$). In the case of an anti-muon, a muon neutrino (ν_μ) is necessary. Analogously, in the decay of a muon, a muon neutrino has to occur. The muon is roughly 207 times heavier than the electron ($m_e = 511\text{keV}/c^2$). So the muon is not the lightest particle and it can decay in an electron. Within today's knowledge the muon is a point-like particle without internal structure.

1.4 The Muon from Cosmic Rays

The experiment is carried out with cosmic muons, whose rate at the sea level is about $70\frac{particles}{m^2 \cdot s \cdot sterad}$ [2]. Primary cosmic rays consist of energetic protons. In the upper atmosphere layers they interact with molecules. This results mainly in the production of charged pions and kaons via the reactions

$$p + p \to p + n + \pi^+ \,, \tag{1.1}$$

$$p + n \to p + p + \pi^- \,, \tag{1.2}$$

$$p + p \to p + \Lambda(\to p + \pi^-/n + \pi^0) + K^+ \,. \tag{1.3}$$

The resulting pions and kaons are not stable, so they decay, among others, in

$$\pi^+ \to \mu^+ + \nu_\mu \,, \tag{1.4}$$

$$\pi^- \to \mu^- + \overline{\nu}_\mu \,, \tag{1.5}$$

$$K^+ \to \mu^+ + \nu_\mu \,. \tag{1.6}$$

These decays happen via the weak interaction and are mediated by the exchange of the $W^\pm$ bosons. An example is the Feynman diagram for the pion decay, which is shown in Figure 1.1. Because of the positive charge of the primary radiation and of the Earth's surface, there is a surplus of positive particles in the produced muons. The ratio as stated in [1] is

$$\frac{\#\mu^+}{\#\mu^-} \approx 1.275 \pm 0.05 \ \ . \tag{1.7}$$

The muons are generated at about 10 km altitude. Only because they are highly relativistic and there is time dilation they can reach the Earth's surface before decaying. The maximum of the energy distribution of the incoming muons at ground level is about 1 GeV (see Figure 1.2). Their average kinetic energy is about 2 GeV.

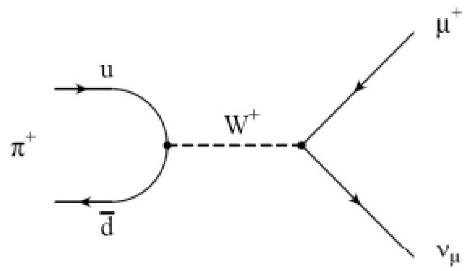


Figure 1.1: Feynman diagram of the pion decay as an example for the generation of muons.

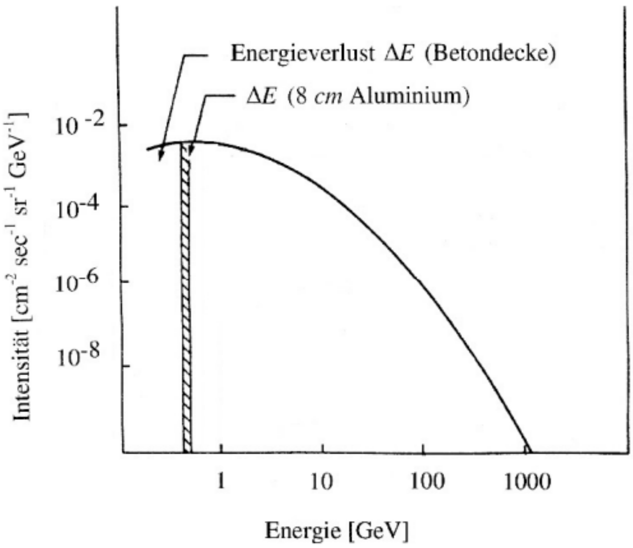


Figure 1.2: Energy spectrum of muons produced in cosmic radiation, measured at the Earth's surface.

Since muons are not subject to strong interactions, they lose their kinetic energy only via electromagnetic interactions, i.e. primarily by ionisation of atoms that are present in the air. The energy loss by bremsstrahlung is small compared to electrons because of the larger muon mass. In total, the energy loss amounts, at high kinetic energies, to $\sim 2\,\mathsf{MeV/g \cdot cm^2}$ [1]. This corresponds to a loss of energy of about $1\,\mathsf{MeV}$ in $4\,\mathsf{m}$ of air near the Earth's surface.

1.5 The Muon Decay

Muons are decelerated by interactions with matter until they decay or come to rest. Positive muons decay either as free particles, or by capturing an electron and the formation of a "Muon Atom". Negative muons have access to a further reaction channel. They can be captured by atomic nuclei, form muonic atoms and then decay via weak interactions. Same as the muon production, the muon decay happens via the weak interaction and is mediated by the exchange of the $W^{\pm}$ bosons.

Muons decay with an average lifetime of 2.19 μs[1] via the reactions

$$\mu^- \rightarrow e^- \nu_\mu \overline{\nu}_e \,, \tag{1.8}$$

$$\mu^+ \rightarrow e^+ \nu_e \overline{\nu}_\mu \tag{1.9}$$

and according to the exponential decay law

$$N(t) = N(t_0) \cdot e^{-\frac{t-t_0}{\tau}} \,. \tag{1.10}$$

The Feynman diagram of the decay of the negative muon is shown in Figure 1.3. When the muon decays, its rest mass is transferred to the rest mass of the electron and the kinetic energy of the electron and the neutrinos. In the limit of $m_e \ll m_\mu$, the maximum kinetic energy of the electron is $E_{max} = \frac{1}{2} m_\mu c^2$.

From the mass of the muon and its lifetime, it is possible to determine the Fermi weak interaction coupling constant G_F. It is (in natural units) given by

$$G_F^2 = \frac{192 \cdot \pi^3 \cdot \hbar}{\tau_0 \cdot (m_\mu \cdot c^2)^5} \,. \tag{1.11}$$

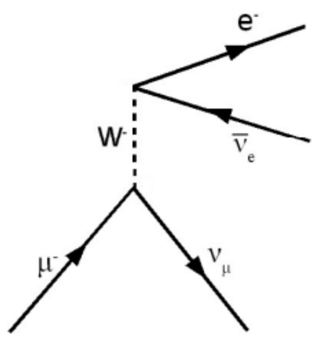


Figure 1.3: Feynman diagram of the muon decay.

Negative muons can decay also via another reaction channel, the μ-capture. If they are at rest and get into the electromagnetic field of an atom, they are

[1] The precise lifetime as given by the Particle Data Group [1] is 2.1969811± 0.0000022 μs

captured by it and they fall down in the K-shell in less than 10^{-12} s. Thus the capture of the muon

$$\mu^- + p \rightarrow n + \nu_\mu \tag{1.12}$$

competes with the muon decay. The capturing process reduces the effective lifetime of negative muons to

$$\frac{1}{\tau} = \frac{1}{\tau_0} + \frac{1}{\tau_c}\,, \tag{1.13}$$

where τ_c is the lifetime in case of pure muon capture [3]. As in this experiment we detect radiation from the cosmic rays, which is a mixture of positive and negative muons, we expect the following time dependence for the number of decaying muons:

$$N(t) = N(\mu^-, t_0) \cdot e^{-\frac{t-t_0}{\tau_0}} \cdot e^{-\frac{t-t_0}{\tau_c}} + N(\mu^+, t_0) \cdot e^{-\frac{t-t_0}{\tau_0}}\,. \tag{1.14}$$

The second part of this lab course is about the measurement of the muon lifetime τ_0. Using equation 1.11, this easily measured quantity can be used to determine G_F, which is one of the few fundamental parameters of the electroweak sector of the SM.

1.6 Current researches about Muons

For modern precision experiments it is not sufficient to use only muons coming from the cosmic rays. Therefore there are several accelerator facilities in the whole world, where intense beams of muons are produced. The properties of these beams are generally different, so it is possible to choose the most adequate one out of a large variety of beams with energies and properties depending on the experimental conditions. In these so-called meson factories, muons are produced from protons. The protons are accelerated up to typical energies between 600 MeV and 1000 MeV, then they hit a beryllium or a carbon target. Via nuclear excitation, pions are produced and afterwards decay into muons. These muons are then collected in electromagnetic guiding systems, consisting of complex magnetic dipole, quadrupole, and some solenoid fields, arranged one after another, and then are directed into the experiment. In this way it is possible to reach a typical muon flux of 10^6 per second and cm^2.

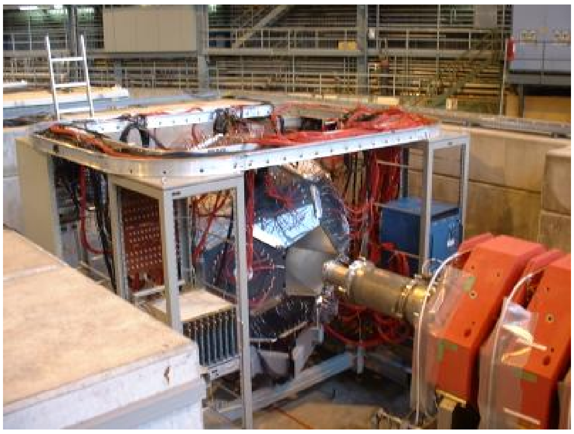


Figure 1.4: Two μLAN detector for a precision measurement of the muon lifetime at the Paul Scherrer Institute in Villingen (Switzerland)

Currently the most precise measurement of the muon lifetime with a single experiment is given by the value obtained by the μLAN experiment [4]

$$\tau = 2196.9803(22)\,\mathrm{ns}\ (1.0\mathrm{ppm})\,. \tag{1.15}$$

The μLAN experiment (detector for Muon Lifetime ANalysis) at PSI in Villingen (Switzerland) used a football-like detector. With this special form it was intended to minimise polarisation effects.

This measurement is a crucial input to the most precise calculation of the Fermi constant G_F [1]:

$$\frac{G_F}{(\hbar c)^3} = 1.1663787(6) \cdot 10^{-5}\ \mathrm{GeV}^{-2}. \tag{1.16}$$

Chapter 2

A short introduction to Particle Detection with Semiconductor Detectors

Semiconductor detectors have a wide field of applications as particle detectors, covering high resolution nuclear spectroscopy, tracking detectors, timing applications and the detection of photons. Semiconductor detectors, as their name implies, are used on crystalline semiconductor materials, most notably silicon and germanium. The basic semiconducting device that is mainly used for particle detection is a diode. In this chapter we briefly discuss properties of a diode that are important for its application as particle detector and to characterize a diode.

2.1 Semiconductors

The outer shell atomic levels of semiconductors exhibit an energy band structure as illustrated in figure 2.1. It consists of a *valence* band, a *„forbidden “energy gap* and a *conduction band*. Electrons in the conducting band can move freely, the presence of an electric field generates an electric current. On the other hands electrons in the valence band participate in covalent bonding between the lattice atoms. Conductors, semiconductors and insulators are classified by the size of the energy gap separating valence and conduction band. While in insulators the energy gap is large (i.e. a thermal excitation of an electron from the valence to the conducting gap is hardly possible) in a conductor the conducting band and the valence band are overlapping.

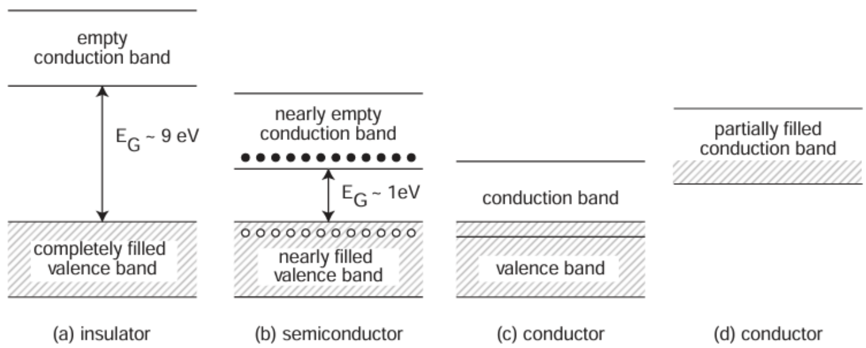


Figure 2.1: Band Structure in different solid state materials [7]

In a semiconductor at 0 K all electrons are in the valence band. At normal temperatures, however, the action of thermal energy can excite a valence electron into the conduction band, leaving a *hole* in its original position (see figure 2.2. In this state, it is easy for neighboring valence electrons to jump from its bond to fill the hole, leaving a hole in the neighboring position. Like this also the *hole* can move through the semiconductor and contributes in the presence of an electric field to the generated current. This is to be contrasted with a metal where the current is carried by electrons only.

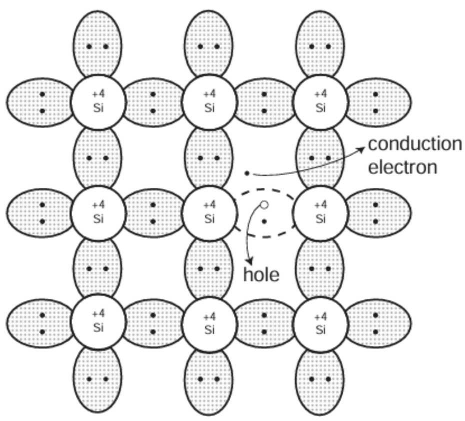


Figure 2.2: Electrons and holes moving through Silicon, thus both contributing to the current [7]

2.2 Doped Semiconductors

In a pure semiconductor crystal, the number of holes equals the number of electrons in the conduction band. This balance can be changed by introducing a small amount of impurity atoms having one more or less valence electron in their outer atomic shell. For silicon and germanium which are tetravalent, this means either pentavalent atoms or trivalent atoms. These impurities integrate themselves into the crystal lattice to create what are called *doped* or *extrinsic* semiconductors.

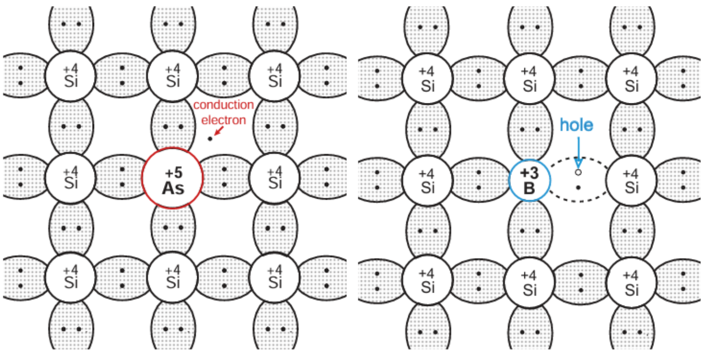


Figure 2.3: Structure of Doped silicon. Left: n-doped, Right: p-doped [7]

Figure 2.3a shows the situation for pentavalent dopant. It is also called a *donator*. Semiconductor doped with donators are called *n-type* semiconductor. A trivalent dopant on the other hand is called an acceptor. Semiconductor doped with acceptors are called *p-type* semiconductor. Typical donators are arsen, phospor and antimon. Acceptors are e.g. gallium, boron and indium. Typical doping concentrations are in the order of 10^{13} atoms/cm^3, which should be compared to density of silicon or germanium in the lattice (10^{22} atoms/cm^3).
Great use is also made of heavily doped semiconductors. Impurity concentrations can be as high as 10^{20} atoms/cm^3 leading to a high conductivity. To distinguish these semiconductors from normally doped materials, a „+ “sign after the material type is used. A heavily doped p-type semiconductor is therefore written as p^+ and a heavily doped n-type semiconductor as n^+.
When bringing p- and n-doped semiconductor materials in contact a so-called pn-boundary or pn-junction is formed. A diode is essentially made from a pn-

junction and is discussed subsequently.

2.3 The pn-Junction

The charge carrier concentration in a pn-junction is shown in figure 2.4. In the p-doped part of the crystal, holes are the dominant charge carriers (majority carriers), while in the n-doped part the majority carriers are electrons. The strong concentration gradients of the two charge carrier types at the boundary lead to a diffusion current $I_{\mathtt{diff}}$, that is, electrons of the n-doped part diffuse into the p-doped part of the crystal and holes from the p-doped part diffuse into the n-doped part. At the boundary, recombination of both carrier types occurs leading to a zone which is free of charge carriers, called the depletion zone. While there are no free charge carriers in the depletion region, the atomic cores remain ionised after e–h recombination has taken place and the region is no longer neutral but features a space charge. The p layer has a negative, the n layer a positive space-charge density $\rho(x)$. Hence the depleted region is also called space-charge region. Due to the opposite space charge in the p and in the n layer, an intrinsic electrical field is formed causing a drift current $I_{\mathtt{drift}}$ in the opposite direction to the diffusion current $I_{\mathtt{diff}}$. Figure 2.4 shows qualitatively the doping concentration, charge carrier density, space-charge density, electric field and potential across the pn-junction. Note, that the space-charge region extends further into the more weakly doped part of the semiconductor than into the more strongly doped part.

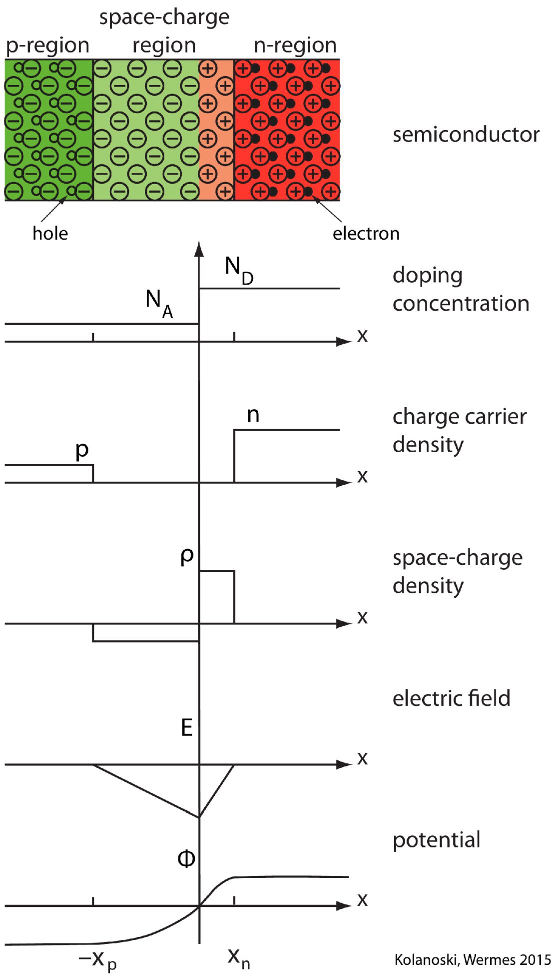


Figure 2.4: Pn-Junction and evolution of different parameters [7]

2.3.1 The pn-Junction under External Voltage

Application of an external voltage V_{ext} between the n and p sides of the junction changes the width of the depletion region depending on the size and polarity of the applied voltage. This is illustrated in Figure 2.5. If the applied voltage V_{ext} at the p side is positive relative to the n side (so-called forward bias), the electrostatic potential over the depletion zone reduces by V_{ext} and the drift current decreases in comparison to the diffusion current. Relative to the equilibrium situation more electrons diffuse from the n side to the p side and more holes from the p side to the n side, where they become minority carriers (minority carrier injection). Like this the depletion region becomes thinner. By applying reverse bias voltage with V_{ext} negative voltage at the p side or positive at the n side relative to the respective other side) the electrostatic potential is raised relative to the equilibrium state, thus counteracting the diffusion current which therefore becomes smaller. The depletion zone gets wider. For particle detection diodes (or pn junctions) operated in the reverse bias mode are used.

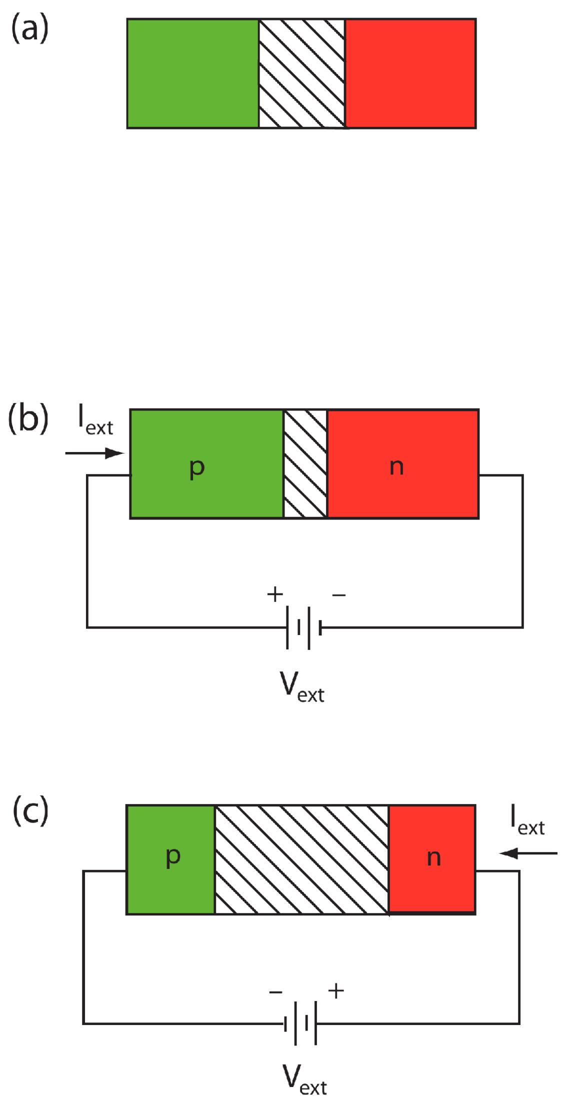


Figure 2.5: Pn-Junction with applied bias [7]

2.3.2 Particle Detection with a pn-Junction

The pn-junction of a diode can be used to detect ionizing particles. For that the diode is used in the reverse bias mode in order to obtain a depletion zone as large as possible. The depletion zone is the detection volume to detect particles. When a charged particle loses energy by ionisation or a photon is absorbed in a semiconductor detector part of the released energy is used to generate electron–hole pairs. The charge carrier pairs are separated in the electric field applied to the semiconductor and drift inside the bulk toward the electrodes on which the movement induces an induction signal. The bulk must be sufficiently free or depleted from other charge carriers. This is achieved by the reverse bias voltage. If an ionizing particle creates electron-hole pairs the electrons will start towards the n-doped region of the diode while the holes drift towards the p-doped region (see figure 2.6). In a particle detector a suitable electronics is able to detect the current produced by the charge carriers drifting towards the electrodes.

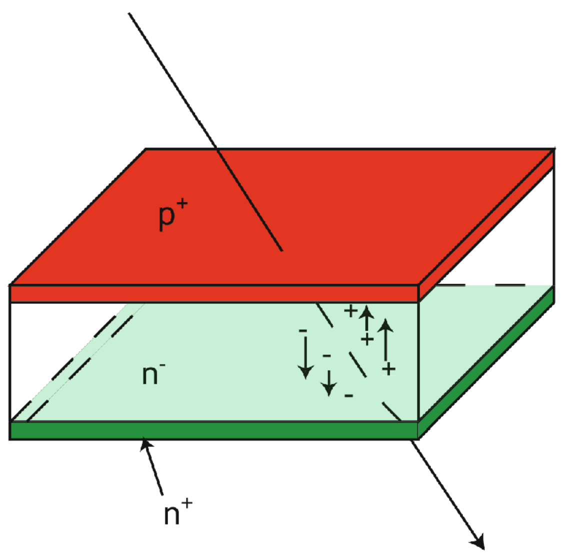


Figure 2.6: Particle passing through a diode [7]

2.4 Characteristics of a pn-Junction/Diode

2.4.1 The Shockley-Equation

To characterize a diode the current-to-voltage (I-V) curve is often used. It describes the current through a diode as a function of the bias voltage V_{ext}. Under some idealized assumptions[1] it can be written as:

$$I = I_S \left(e^{eV_{ext}/k_B T} - 1 \right).$$

This equation is known as Shockley-equation. k_B is the Boltzmann constant, T is the temperature, V_{ext} is the externally applied bias voltage and I_S is the reverse saturation current. The latter depends on the geometry of the diode, its material and the doping concentrations. The curve is shown in figure 2.7.

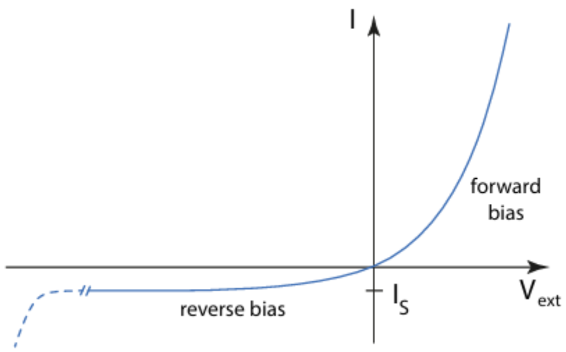


Figure 2.7: Shockley Equation i.e. current to voltage curve in a diode [7]

2.4.2 Leakage Current

For a reverse biased diode or a semiconductor detector the so-called leakage current is observed, which can have volume as well as surface contributions originating from different physical sources. A leakage current exists even in ideal diodes originating from the movement of minority carriers through the boundary by diffusion. It can be created by electrons entering the n-region from the p-region. It is described by the reverse saturation current I_S in the Shockley-equation.

[1] For details see [7]

Regarding our application the most important cause of leakage current are thermally generated electron-hole pairs in the depletion region. As their contribution is strongly temperature dependent, the T^2 dependence in Equation 2.1 is expected.

$$I_L \propto T^2 \exp(-\frac{E_G}{2k_B T}) \propto f \tag{2.1}$$

The current is proportional to the measured dark count rate of a SiPM. Therefore, this relation is useful to determine the bandgap of silicon.

2.4.3 Capacitance of a Diode

Due to the charge carrier free zone an area diode can be regarded as a plate capacitor filled with a dielectric. The capacitance to the backside is

$$\frac{C}{A} = \frac{\epsilon\epsilon_0}{d}$$

Here A is the area and d the width of the depletion zone. The relative permittivity for silicon has the value $\epsilon_{Si} = 11.9$.

2.5 Silicon Photomultipliers

This section is taken from [5], for more details see [7]

A Silicon Photomultiplier (SiPM) is a semiconducting photon detection device. SiPMs are single-photon-sensitive devices. The basic element is a single-photon avalanche diode (SPAD). SPADs are special photo-diodes possessing an additional pn-junction with high p and doping as shown in figure Figure 2.8.

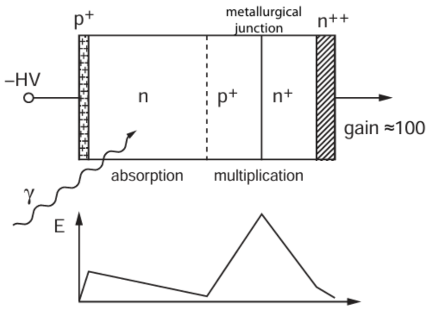
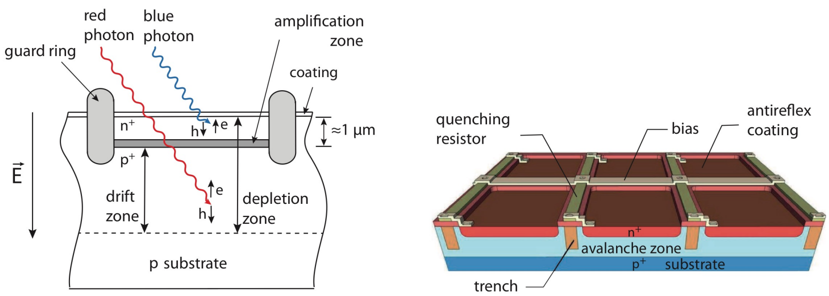


Figure 2.8: Schematic of a SPAD [7]

The electric field across the SPAD is also shown. It receives a photon signal and converts it into an electrical signal due to the photoelectric effect. A SiPM is an array of Avalanche-Photodiodes connected in parallel on a shared silicon substrate. Therefore, multiple photons at once can be registered. An Avalanche-Photodiode consists of differently doped semi-conducting parts, which are connected to a bias voltage as seen in Figure 2.9 (a).



(a) SiPM cell. (b) SiPM array.

Figure 2.9: Schematic of a SiPM [7]

The differently doped silicon build a diode. Using a reverse biasing of the diode a rather large depletion zone is created. The photon is absorbed and converted to an electron due to the photoelectric effect in the depletion zone. The depletion zone mostly extends into the p-doped area and is connected to the bias voltage. Due to the electric field of the bias voltage in the depletion zone, the created electron drifts towards a so called amplification zone, which is n-doped. There, a high electric field creates an avalanche amplification of the electron. The highly amplified electrons induces a measurable current pulse on the collection electrodes. The breakdown voltage V_{BD} of a SiPM is defined as the bias voltage V_{Bias} where the electric field is strong enough to create an avalanche. Therefore, the bias voltage defines the amplification of the SiPM. The difference of those two is defined as the over voltage $\Delta V = V_{Bias} - V_{BD}$. The height of the signal from the SiPM is proportional to the number of hit pixels in the SiPM which is the same as the number of photons it receives.

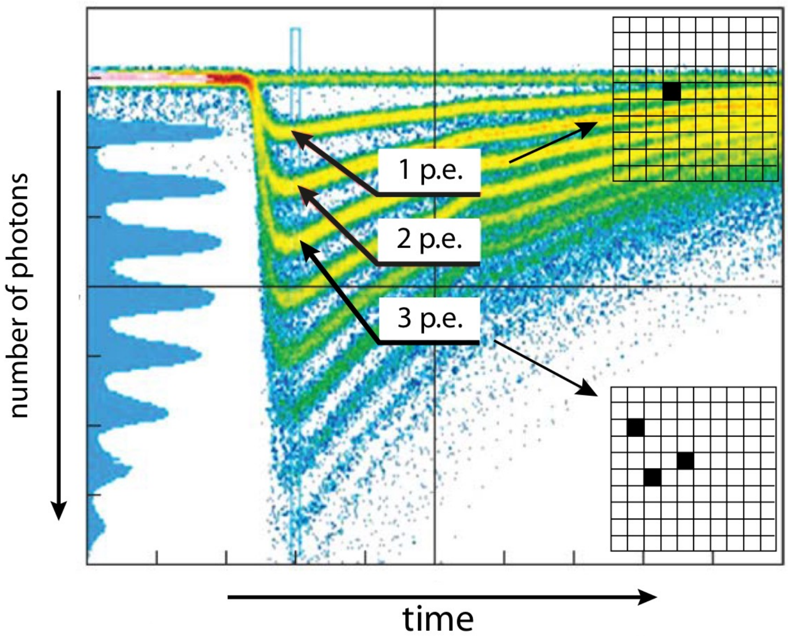


Figure 2.10: Signal Shape of the SiPM output (photo-peaks) corresponding to number of hit Pixels [7]

The number of hit pixels can then be seen as the signal heights in Figure 2.10. That are the so called photoelectronpeaks [7] . The amplitude of a signal coming from an SiPM is usually given in numbers of photoelectrons 'pe'.

The Gain of an SiPM is determined by the total charge Q divided by the elementary charge e: $G = Q/e$. By writing the charge as an integral of the current and using Ohms law, this can be written in terms of the voltage

$$G = \frac{1}{eR} \int dt U(t). \tag{2.2}$$

This relation can be used to determine the gain of a SiPM.

2.5.1 Electrical SiPM Model

This section is taken from [8].

A single SiPM cell can be modeled with an equivalent electrical circuit and two probability functions. The following parameters are defined:

- C_d: The diode capacitance
- R_S: The series resistance
- R_Q: The quenching resistance

The electrical model is presented in Figure 2.11. The rise time τ_d is given by the product of the series resistance (R_S) with the diode capacitance (C_d) and is in the order of about 100ps.

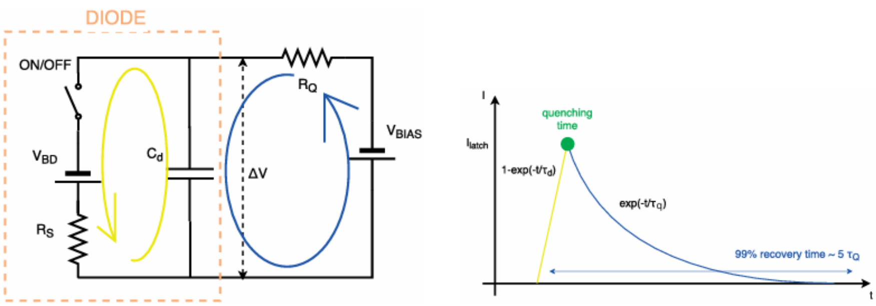


Figure 2.11: Basic SiPM electrical model (left) and it's resulting pulse shape (right) [8]

The fall time (τ_Q) is given by the product of R_Q and the diode capacitance (C_D). The typical range of τ_Q is between 10 to 200ns depending on the quenching resistor value. The figure also shows the pulse shape from this basic model and the current generation follows the scheme of Figure 2.12

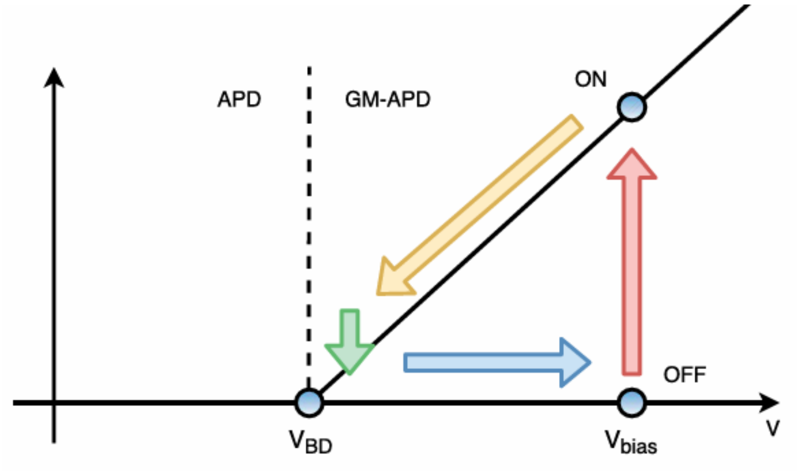


Figure 2.12: SiPM current generation steps. [8]

- P_{01}: Is the probability that an avalanche is triggered
- On condition: The switch is closed, C_d discharged to V_BD with a time constant $R_S \cdot C_d = \tau_d$ which produces a current $\Delta V/(R_Q + R_S) \approx \Delta V/R_Q$
- P_10 = Is the probability that the avalanche is stopped
- Off condition: the switch is open and the capacitor is re-charged to ΔV with a time constant of $R_Q \cdot C_d = \tau_{rec}$ (recovery time)

The recovery time is temperature dependent through the R_Q variation. For a given ΔV, the gain is simply $G = C_d \cdot \Delta V/e$.

Chapter 3

General Outline of the Experiment

The aim of the experiment is the determination of the Fermi constant from a measurement of the average lifetime of muons. For the measurement the set-up shown in Figure 3.1 is used. Muons from cosmic rays (see section 1.4) are detected by 4 layers of scintillators which are interleaved by layers of Aluminum. The thickness of the scintillators is 1 cm, the dimensions of the Aluminum absorbers is $4\ \text{cm} \times 38\text{cm} \times 30\text{cm}$. High energetic muons typically leave the set-up before they decay. Their passage through the set-up is detected by the scintillating light they produce in the scintillators. The light is detected by means of Silicon photomultipliers (SiPMs). The functionality and characteristics of SiPMs as well as the detection of particles with SiPMs in combination with

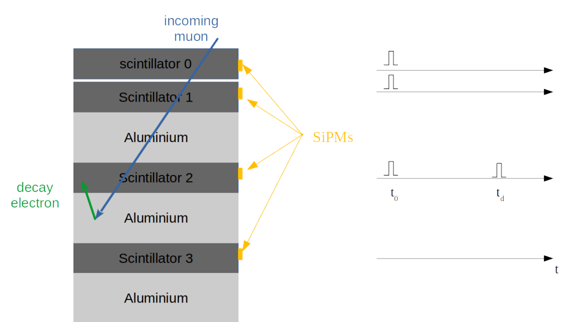


Figure 3.1: Left: Simplified sketch of the experimental set-up. Right: Timing of signals from the SiPMs for a muon decaying into an electron as indicated in the sketch on the left.

scintillators is investigated in detail in Part I of the experiment. To distinguish the passage of a muon from a dark count in a SiPM (see section 2.5) coincident signals from two or more scintillators are used. Occasionally a low energetic muon will loose its entire kinetic energy and is stopped in one of the Aluminum blocks. After some time it will decay into an electron and two neutrinos according to the Feynman diagram shown in figure 1.3. The electron released in the decay has a continuous energy spectrum from 0 to about half of the muon mass. In case it enters one of the adjacent scintillators it creates another light pulse that is detected by the SiPM with a time delay corresponding to the life time of that muon. On the right side of figure 3.1 the characteristic signal of a muon decay is shown. A muon enters the set-up and traverses the first three scintillator layers. The scintillating light produced is detected by the SiPMs and a coincident signal in scintillators 1-3 is produced at the time t_0. The muon is stopped in the Aluminum block underneath the third scintillator. It decays after some time, the decay electron enters scintillator 3 and produces at the time t_d another signal. The time difference $t_d - t_0$ corresponds to the lifetime of the muon. Note, that the decay electron could be detected also in scintillator 4.

Chapter 4

Equipment of the Experiment

4.1 Setup

The experiment uses scintillators which have circular shaped wavelength shifting fibres placed inside them.

When a particle traverses the scintillator, a light signal is produced. The wavelength shifting fibre absorbs the scintillating light and emits isotropically another photon which travels to the end of the wavelength shifting fibre. The end of the wavelength shifting fibre points to the SiPM where the light signal is collected and converted into an electrical signal. The SiPM is mounted onto a readout board. That gives different possibilities to process the signal. The SiPM Signal is amplified by an additional factor of 40 by the readout board.

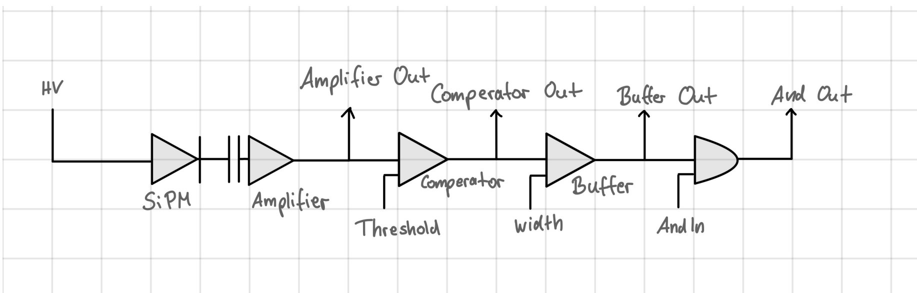


Figure 4.1: Schematic circuit of the readout of the Trigger Boards

In addition the board gives the possibility to readout different signals. This is visualised in Figure 4.1. The amplified signal can be directly accessed and measured. In addition, a tuneable threshold (th) can be applied to the signal. This gives access to the comperator signal. It gives a rectangular signal of constant height for the time duration of the amplifier overcoming the threshold. The buffer signal is a comperator signal with a constant time over threshold (ToT) and can be accessed. The amplifier signal is used to study the characteristics of the SiPM and the buffer signal is readout for the final muon measurement. Considering what is explained in chapter 3, the data of the four scintillators has to be recorded for the final measurement. This is done by a second board which records the data for a fixed time after minimal a two coincidence occurred.

4.2 Oscilloscope

The oscilloscope used in this experiment is the Rohde & Schwarz RTB2004 Digital Oscilloscope. The full instructions can be found here [6]. A brief overview of the working principle and features of the oscilloscope is given in Appendix A. It is recommended to read this and/or use it as a help during the experiment.

4.3 The Pt1000 Thermometer

To measure the temperature during the experiment a Pt1000 thermometer is used. To measure the temperature, the temperature dependence of a platinum (Pt) resistor is used. 1000 denotes the resistance at a temperature of $0°$. Online calculators are available to extract the temperature from the measured value of the resistance. The temperature error of the PT1000 sensor can be assumed to $\Delta T = 0.40°$.

Chapter 5

PART I: Characterisation of Silicon Photomultipliers

In this part the characteristic parameters of the SiPM are determined. These values can be compared to the parameters given by the supplier. You may find them online. For that search for the data sheet of the SiPM used. It is from the company Hamamatsu, the type of the SiPM (Hamamatsu calls it a MPPC) is S13360-3050CS.

5.1 Measurement of the Breakdown Voltage

The breakdown voltage V_{BD} is an important parameter to define the operation point of a SiPM. It is the voltage at which the amplification process starts resulting in a gain larger than one. The voltage at which a SiPM is operated is therefore larger than V_{BD}. A measurable quantity that is proportional to the gain is the amplitude of a signal recorded by an oscilloscope and can be used to determine the breakdown voltage. As V_{BD} and the gain at a given bias voltage depend on temperature this is done at room temperature and than repeated at lower temperature. The procedure for that measurement is the following:

1. Start by connecting the board with the SiPM mounted on it to the HV using the correct cable. The HV must not exceed 60 V. During preparation of the measurement HV and LV should be switched off.
   Important: The maximum current for the HV has to be set to 0.020A! When raising the HV you should see over the entire voltage range a 'CV'

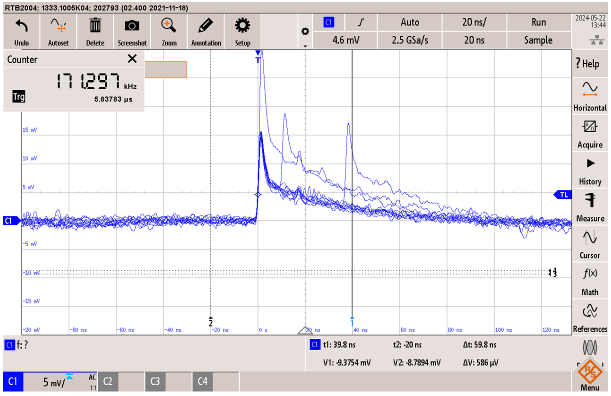


Figure 5.1: SiPM Signal on the oscilloscope with recommended settings in sample mode. The different photo-peaks are visible

in the display indicating that the power supply is working in the 'constant voltage' mode. In case this changes to 'CC', indicating the the power supply is working in the constant current mode, lower the voltage and contact the supervisor.

In addition connect the power supply of the board to a voltage of 12.0 V with a maximum current of 150 mA on the other power supply. To do so use the two banana plugs. Make sure to match the minus and plus pole correctly. Furthermore, connect the amplifier output to the oscilloscope.

2. Put the board into the fridge and make sure that the fridge is turned off and at room temperature. The fridge will be used in the next tasks. Connect the Pt1000 to a multimeter and put it in the fridge. Use scotch tape to fix it close to the SiPM. Close the fridge and put a dark cloth around it to reduce light induced noise. Connect the signal cable to the scope and terminate it with 50 Ω.

3. **Important:** When you switch on the SiPM for the first time you should do this together with the supervisor. The procedure is the following: First, switch on the LV (12V). Now you should see the electronic noise on the scope. Set the trigger to the channel your SiPM is connected to. Make sure that the acquisition mode is set to sample. Set the threshold slightly above the electronic noise. Now, slowly raise the HV (also called bias voltage) to 50V. Next increase the voltage in steps of 1V until you see

SiPM signals on the oscilloscope. They should look similar to the pulses shown in like figure 5.1. To reach the seen settings a scale of 5 mV for the vertical axis and a scale of 20 ns of the horizontal axis are recommended.

4. Try to find the position of the first and second photo peak. In case you have problems to see the second photo-peak you may adjust the persistence. For that select 'acquisition' and scroll down on the screen of the oscilloscope to 'display'. Here you will find the persistence menu.

5. To perform the measurement you can start at a HV of around 52 V (check if you still see a signal, otherwise start at a higher voltage) and go up to 59 V (but not higher) in steps of 0.5 V. Measure the difference of the amplitude of the first and second photo-peak for each high voltage step. To do so you can use the two horizontal cursors of the oscilloscope. They can be turned on and off by pressing the button 'cursor'. You can change their position by using the touch screen. The difference between the two cursors can be seen on the screen indicated with ΔV.

To determine the breakdown voltage V_{bd} you can plot the data as a function of the HV for each of the performed measurements and each temperature respectively. The data should be over a wide range linear for both methods. Discuss the reasons for the deviations from the linear behavior. Fit a linear function to them and determine the breakdown voltage as the point where the count rate respectively the amplitude is zero. The measurement is repeated at low temperatures, but before the subsequent measurements described in section 5.2 and 6 should be performed.

5.2 Measurement of the Characteristics of the SiPM Circuit via the Signal Shape

As you learned in chapter 2 a SiPM can be modeled with a simple electrical circuit. Its contents can be determined by studying the signal shape of the SiPM signal. Perform the following measurements at room temperature at one HV value.

5.2.1 Measurement of the Gain and Determination of the internal Capacitance

As the gain is determined in ?? one needs to measure the integral of the voltage of the signal shape. This is going to be done by looking at the signal shape on oscilloscope and calculating the area under the signal curve.

1. Use the same setup as in the measurement before as well as the same settings on the oscilloscope. The first and the second photopeak should be fully displayed. Make sure, that you set the trigger line to about half the height of the first photo-peak in sample mode. The signal should look similar to figure 5.2. Now change the Acquisition mode to average with 1000 averages. This gives you an average signal.

2. Adjust the horizontal axis and measure the area under the curve. You can take different pictures at different horizontal and vertical scales of the signal shape and calculate the area step by step. It is helpful to start at a very small time scale. Make sure to include the whole area until the signal is back to zero.

3. Use ?? to calculate the final gain. Make sure to include the internal gain of the board on which the SiPM is mounted onto. For the resistance R in equation 2.2 you can use a value 25 Ω (Why? Discuss with the supervisor). Finally use ?? and calculate the capacitance C_D.

5.2.2 Measurement of the Rise and Recovery Time and Determination of the Resistors

The quenching resistor R_Q and the series resistor R_s are determined in ?? and ??. To determine their value the measurement of the rise time τ_D and the recovery time τ_{rec} is needed.

1. Set the oscilloscope back to the same settings as in section ??.

2. Adjust the horizontal axis such that you are able to measure the rise time which is defined as the time until the signal reaches its peak as in figure 2.11. Use the vertical cursors of the oscilloscope.

3. Perform the same step as before for the measurement of the recovery time.

The recovery time is defined as the time from the peak of the signal until it reaches zero again.

4. Calculate the quenching resistor R_Q and the series resistor R_S using ??.

5.3 Measurement of the Band Gap of Silicon

The band gap of silicon can be measured using a SiPM. This can be done via Equation 2.1. To do so, a measurement of the rate at constant gain for different temperatures has to be performed.

1. Make sure that the fridge is turned off and at room temperature. Switch off the low and high voltage of the SiPM, but leave the cables connected (connections as in section 5.1). Place the SiPM and the Pt1000 back into the fridge and cover the fridge with black cloth.

2. Set the oscilloscope settings back to the ones in section 5.1. The amplitude of the first photo-peak should be 15 mV. For that, you might have to adjust the height of the photo-peak by adjusting the high voltage. Set the trigger such, that the first and second photo-peak are clearly visible, but electronic noise is discriminated. Typically the trigger level should be set to about half the height of the first photo-peak. Switch the oscilloscope to average mode and fine-tune the high voltage again such that the amplitude of the averaged signal is again at around 15 mV.

3. Take the first measurement. Note down the temperature and the corresponding count rate.

4. Turn the fridge back on and perform a measurement every 10 Ω. Adjust the high voltage constantly meaning adjusting the high voltage such photo-peak always has the same height.

5. Plot your data i.e. f/T^2 and fit it with the corresponding fit function according to equation Equation 2.1. Determine the band gap of silicon using your fit parameter.

5.4 Summary: Measurements and data analysis for Part I

At the end of part I of this experiment you should have performed the following measurements:

1. Signal amplitude (difference between first and second photo-peak) as a function of bias voltage at room temperature and low temperature.
2. A record of the signal shape.
3. A measurement of the count rate at constant gain as a function of the temperature.

You should

1. plot the amplitude (difference between the first and the second photo-peak) as a function of the bias voltage and determine V_{BD} by extrapolating the measurement to the voltage where the amplitude vanishes.
2. use the signal shape to extract capacitance C_d, the series resistance R_S, the quenching resistance R_Q and the gain for the given biasing voltage. Compare the gain and C_d to the values given in the data sheet of the SiPM.
3. Now you can plot the gain as a function of the over voltage for given by the bias voltage minus V_{BD}. Show this result for both temperatures in the same plot. What do you observe?
4. Finally, extract the band gap from the measurement of the counting rate as a function of the temperature. Discuss the result.

Chapter 6

PART II: Measurement of the Fermi Constant

6.1 Initial measurements

Make sure that the set-up is covered light tight by black cloth. When this is done the set-up can be switched on and the initial configuration needs to be performed. This has to be done together with the supervisor. Now you should make yourself familiar with the set-up. The main components are (beside the Aluminum plates) are

1. the scintillators
2. the SiPM boards
3. an electronic box housing a Raspberry Pi (subsequently called RP4) and an FPGA
4. the oscilloscope, serving as an *online display*

This is a short description of the components:
1. The scintillators:
The scintillators are made of BC-408. To collect the light a wavelengh shifting fibre is buried in the scintillator material. When a particle traverses the scintillator the photons created are caught by the fibre and guided to the fibre end. Here a Silicon Photomultiplier detects the photons.
2. The SiPM boards:

The functionality of the SiPM boards is similar to the boards used in the first part of this lab course, i.e. it houses a SiPM, an amplifier and comparator. The difference is despite of a new and more compact design, that the threshold and HV can be controlled remotely using the FPGA.

3. The electronic box:

The readout of the data is steered by the RP4 sitting in the electronic box. Via the RP4 you initialize and steer the FPGA to set the threshold for the comparators, the HV applied to the SiPM and the trigger pattern (see subseqent section). You also start data taking via the RP4. Data are written to the RP4. When the FPGA is properly configured it receives the output of all comparators. Their state (high or low) is sampled at a frequency of 200 MHz. Once the FPGA detects a correct trigger pattern at the comparator outputs it sends a trigger signal to the *trigger out* of the electronic box.

4. The oscilloscope

The same oscilloscope as in the first part of the lab course is used. It serves mainly as an online display to verify the correct functioning of the set-up. For that connect the comparator outputs (ch0 ... ch3) to the input channels of the oscilloscope. Obey the correct order. The *trigger out* from the electronic box has to be connected to the *external trigger in* of the oscilloscope.

Make yourself familiar with the set-up:

To make yourself familiar with the set-up should log into the PC and open there a terminal. Enter the command *ssh-sipm-trg.sh.* This opens another terminal running on the RP4. Here you need two commands:

```
./trigg --lut_hits N BP
./trigg --discr_mask BP --set_thresh V
```

The parameter *BP* is very important to understand to run this experiment. It describes a bit pattern, i.e. the pattern at the input of the FPGA (i.e. the output of the comparators). If this is high, i.e. the voltage level is 2.5V it corresponds to a '1', if it is low, i.e. the voltage level is 0V it corresponds to a '0'. Now have again a look at figure 3.1. It shows a muon traversing at time t_0 scintillators Scint 0, Scint 1 and Scint 2. Then it is stopped in the next aluminum layer where it decays. The decay electron is detected at time t_d. This means the FPGA sees coincident signals at time t0 in channels 0, 1 and 2. This corresponds to the bit pattern 1110 or written in hexadecimal code 0x7 (the pre-fix 0x means that the following number has to be interpreted as a hex number).

1. What is recorded by the FPGA?
2. Give the bit pattern for a muon traversing the entire set-up, i.e. leaving a coincident signal in all 4 scintillators. How is it written in hexadecimal format.

Using the command

```
./trigg --lut_hits N BP
```

you define a bit pattern *BP* and a number of hits *N* that should be contained in the bit pattern to generate the trigger signal.
With the second command

```
./trigg --discr_mask BP --set_thresh V
```

you can set the threshold V in mV applied to all channels contained in the bit pattern *BP*. Typical values are around 1 mV.
To understand the principle of operation configure the following configurations and describe what you observe on the oscilloscope:

1. Configure the FPGA such that it triggers only on a specific single channel? Vary the threshold of this channel between 0.1 mV and 4 mV.
2. Configure the FPGA such, that it triggers in case of a coincident signal in the second and third scintillator layer. Set the threshold to 1 V
3. Configure the FPGA such, that it triggers in case of a coincident signal in the first, second and fourth scintillaor layer. Set the threshold to 1 V.

6.2 Measurement of the cosmic muon flux

The next task is to measure the flux of cosmic muons. For this the coincidence technique is used, i.e. we trigger the FPGA only in case that two signals are present at the FPGA input at the same time.

1. Why are the muons determined via coincidence? What does that mean? What is the most useful effect of the coincidence method?

2. Determine the coincidence time window. For this observe the output of the scintillator output on the oscilloscope.

The measurement of the cosmic muon flux is fairly simple, but a few corrections have to be applied to the measurement. The measurement principle is to count the number of coincident signals in scintillator 1 and 2. After application of some corrections the count rate divided by the active area of the scintillator gives the muon flux The corrections that need to be done take into account

1. in-efficiencies of the scintillators

2. geometrical correction to correct for the limited acceptance of the set-up.

3. accidental coincidences

The procedure for the measurement is described below. First, we set the trigger condition such that two coincident hits are present in scintillor layer 0 and 1. The trigger is started for a pre-defined measurement time as described below. The FPGA counts during that time the number of hits from all channels as well as the number of correct triggers registered. This procedure is repeated while varying the threshold applied. To ease the measurement starting and stopping of the measurement as well as setting of the correct threshold can be done by means of a shell-script called *rateMeasurement.sh*. For that verify that the threshold values and the trigger pattern is set correctly in the script. Save the script and run it by typing *./rateMeasurement.sh*. You will be prompted for a measurement time. This is the measurement time for a single threshold. In total typically 20-30 different threshold values are measured. A measurement time of 1-2 minutes per measurement point guarantees a sufficient precision to determine the flux of cosmic muons.

When the measurement is completed have a look at the file *rateMeasurement.txt*. It should look similar to this

```
60.2 1304165 1536267 1312374 14831553 8389
60.3 110547 161416 243352 1562261 758
```

```
60.4 11931 22090 76582 166015 564
60.5 3225 4517 11770 23363 572
```

The first column shows the measurement time for the data point in seconds, the second is the threshold voltage in mV, the following 4 columns give the number of registered hits in layer 0-3 and the final column shows the total number of coincident hits at that voltage.
Make a plot from this measurement showing the following variables as a function of the threshold voltage:

- the count rate of scintillator 0 and scintillator 1.
- the coincidence rate.
- the rate of accidental coincidences. The number of accidental coincidences is characterized by the background rate in each depicted scintillator N_1 and N_2 and the coincidence time window σ.

$$N_{acc} = N_1 \cdot N_2 \cdot \sigma \tag{6.1}$$

You may show all variables in a common plot, but for that the y-axis should be shown logarithmic. Discuss the results. Choose a working point, i.e. the threshold voltage you want to use for your measurement. Give the measured coincidence rate including an error. Think of a reasonable criterium to calculate the coincidence rate.

6.2.1 Measurement of the efficiency

To measure the efficiency you can start a measurement by entering the the commands `make stop`, `killall ./trigg`, `run.sh` in that order. Give the operating threshold you have chosen above in `mV`. Note down the start time. After about ten minutes stop the process by entering `Ctrl-C`. Enter again the commands `make stop` and `killall ./trigg`.
To analyze the data you first have to set-up your working environment on the PC. Enter the command `more FreigabeLink.txt` and open the given link in a browser. Create your own working directory by entering `mkdir` *`myDirectoryName`*. *myDirectoryName* is the name you choose for the directory. Next type `cd` *`myDirectoryName`* and copy all scripts you find in the link given above into

this directory. Now enter the commands `mkdir data` and `chmod u+x *.sh`. Now you copy the data from the Raspberry to the PC using the command `./copyFile.sh`. Run the Python3-script `python3 compressData.sh`. Look at the summary of the output file starting with `Double Coincidence`. Try to understand the meaning of the output. Calculate the efficiency ϵ of the second scintillator layer.

6.2.2 Corrections for the muon rate measurement

The following corrections have to be performed to extract the muon rate:

- Subtract the accidental coincidence rate from the total coincidence rate.
- Correct for the inefficiencies of both scintillators. For that assume that the efficiency of both scintillators is the same[1].
- A geometric correction has to be applied as not the entire solid angle is covered. For that reason the measured coincidence rate underestimates the cosmic muon rate by a factor of 0.923.

After all corrections have been applied calculate calculate the muon flux per square meter and compare it to the value of $70 \frac{particles}{m^2 \cdot s \cdot sterad}$ [2] given above. For that you have to take into account that the angular distribution of the cosmics follows a $\cos^2 \vartheta$ distribution. Integrate over the full solid angle (but take into account that muons come only from the top) and compare the result to your measurement. Discuss the results. For the discussion take into account figure 1.2.

6.3 Measurement of the muon lifetime

Now you can start the long term measurement to measure the muon lifetime. The command to start the measurement is the same as for the efficiency measurement: `./run.sh`.As the stopping of the muon in the set-up is a rare event a measurement time of about 24 hours is needed to achieve a good precision.
To analyze the data a Python3 script is used that first has to be completed. The script is called `dataAnalysis.py`. Using the concept of the bit pattern criteria have to be defined to divide the recorded data into three categories:

[1] It is not possible to measure the efficiency of the first scintillator like this. Why?

1. background event
2. up- and down-type muon decays

Up type events are events where the decay electron of the muon is detected in the scintillator above the stopping aluminum layer, in down-type events the electron is detected by the scintillator underneath the stopping aluminum layer. Up- and down-type events are analyzed separately as they may have different systematic errors. To test the program you first have to copy the data files using the command `./copyData.sh` followed by the command `python3 compressData.py`. The data file is written in an encoded format. The script decodes the data and selects in the next steps all events that have been triggered by one of the bit patterns defined and an additional delayed hit, i.e. at least one hit detected in one of the scintillators in a time window of 20 μs after the trigger. The data are written to a file called 'delayedHits.txt' with the following format:

0x3	0x2	2400	165
0x7	0x4	1020	165
0x3	0x2	2235	175
0x7	0x4	4325	235
0x3	0x4	1375	185
0xc	0x8	1100	180
0x7	0x4	670	175
0x7	0x8	780	190
0x7	0x4	1020	185
0x3	0x2	6015	170
0x7	0x4	5205	180
0x7	0x8	960	180

Every line corresponds to one triggered event. The first column gives the bit pattern that triggered the readout. The second column gives the bit pattern for the scintillator/SiPM that recorded the delayed hit. The third column is the time when the delayed hit is recorded nano-seconds and the last column gives the time when the trigger was registered, also in nano-seconds.

You may run now the analysis script using the command `python3 dataAnalysis.py - -trigger all` You should see a histogram that shows the time spectrum of the all delayed hits, no matter whether it is a background event, an up-type muon decay or a down-type muon decay. Before you analyze the data your task is now to assign the events to the different categories. For that open the script 'dataAnalysis.py'. Have a look at it and go to the section where you find the following lines:

```
----->
-------->
----------->
------------->
--------------->
----------------->
-----------------> define in the following lines criteria for background,
up- and down-type events, use
-----------------> for that use the varaiables triggerPattern[i] and
delayedHitPattern[i]
----------------->
--------------->
------------->
----------->
--------->
------->
----->
```

Define now a logical variable using the variables 'triggerPattern[i]' and 'delayed-Pattern[i]'. For 'i' you should use the correct bit pattern written in decimal. A non-sense example is given to demonstrate the idea. Modify it to classify correctly up-type, down-type and background events.
To study the background for our measurement we use muons that pass all 4 scintillator layers and search for delayed hits. As the muon reaches the last scintillator we know that the muon did not decay before and delayed hits in the scintillators above are caused by a noise hit or an after-pulse.
Once you translated this criterion into a logical condition and implemented it into the logical variable 'background' you may re-run the script `dataAnalysis.py` using the flag `--trigger background --layer 0`. Like this you see the time spectrum of all delayed hits for layer (i.e. scintillator) 0. Compare the results for the different layers.

- The time spectrum from the last layer and the first layers show significantly different. Can you explain the difference?

Now you are ready to have a look to the distribution of the muon decay candidates. For that run the command `python3 dataAnalysis.py --trigger down --binSize 200`. You will see a decay time distribution for all events classified as 'down-type' muon decays. To see the corresponding distribution of 'up-type decays' enter `python3 dataAnalysis.py --trigger up --binSize 200`. You may vary the bin size, possible values are `50, 100, 200, 250, 400, 500, 800, 1000, 1500, 2000`. Discuss the following questions:

- Which distribution do you expect? What do you observe? Explain the differences especially for small and large decay times. Which effects lead to a deviaton?
- Are there differences between the up-type and the down-type distribution?
- Use the tail of the distribution to estimate the background rates in units of `"background events per µs"`. Describe the procedure to calculate the background. Give also the error for the background rates. It will be used below.

Next, you should fit an exponential curve including some background to the data.

6.3.1 Fitting the decay time spectrum

Re-run the data analysis script again using the fit-option. The command is `python3 dataAnalysis.py --trigger up --binSize 200 --fit --firstBin 2`. When you are prompted for input simply enter return, the default values will be used. The fit function is simply a linear fit plus background, therefore the fit does not converge and does not give any reasonable result. The parameter `firstBin 2` defines, that the first bin is excluded from the fit.

To define the proper fit function search in the script for the following section:

```
########################################################################
#       In the following section define the fit function using
#       - two fit parameters:
#                      m: normalization factor
#                      t1: muon lifetime
#       - 3 constants (defined by the user input):
```

```
#               ratio: ratio of mu+/mu- flux
#                     muMinusLifetime: lifetime of muons in Aluminium
#                     backGround: rate of backGround events per µs
#
#       !!! Do not use different names for these parameters and do not
#           change indents!!!
#
##########################################################################
```

Write an expression that returns the value of the function according to equation 1.15 and fit the data. Perform the fit individual for down-type, up-type and the combined up- and down-type data. For the latter use the option `--fit up+down`. Choose reasonable values for the fit, i.e. choose appropriate values for the bin size and the fit range. As a rule of thumb for the bin size one can choose the bin size such, that the number of bins corresponds approximately to the square root of the number of entries in the histogram. But it should also be ensured, that there are no bins without any entries.

Once you defined a good set of parameters of the fit extract the best estimator for the lifetime of free muons and the statistical error of the measurement.

Your next task is to investigate source of systematic errors in the measurement. For that, vary different parameters used in the fit in a reasonable range (typically $\pm 1\sigma$, if applicable. You may vary:

- The assumed values for the ratio of positive to negative muons, the background rate or the lifetime of negative muon in aluminium.
- Vary fit parameter, i.e. the bin size and the fit range in a reasonable range.
- Compare the results for up-type and down-type decays.

Give a systematic error on your lifetime measurement.

6.4 Determination of the Fermi Constant

Finally, as stated in section 1.5 the Fermi Constant can be determined from the muon lifetime. Use Equation 1.11 to determine the Fermi Constant from your measured muon lifetime. Include the statistical and the systematic error.

Determine an error and discuss the result. Compare your result with the value from Equation 1.16.

Appendix A

Oscilloscope

The oscilloscope used in this experiment is the Rohde & Schwarz RTB2004 Digital Oscilloscope. This chapter is adapted from [6]. The Front panel including the general functions of each button is depicted in the following figure.

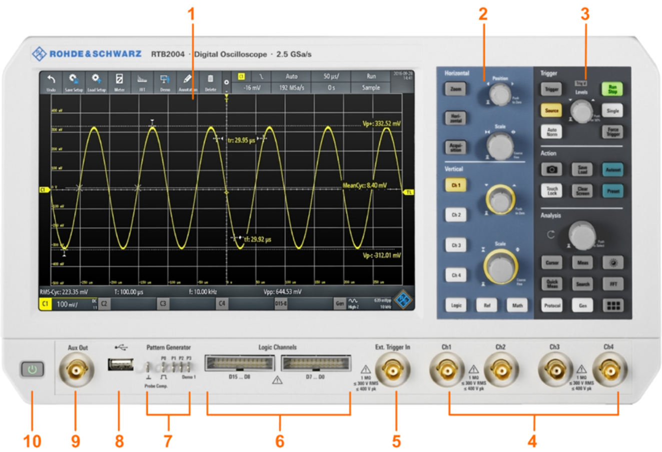


Figure A.1: Front panel of R&S RTB2004, 1 =Display, 2 =Horizontal and vertical setup controls, 3 =Trigger settings, action and analysis controls, 4 =4 Analog input channels, 5 =External trigger input, 8 =USB connector, 9 =Aux Out connector, 10 = [Standby] key

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A.1 Display Overview

The touchscreen display of the instrument shows the waveform and measurement results, and also information and everything that you need to control the instrument.

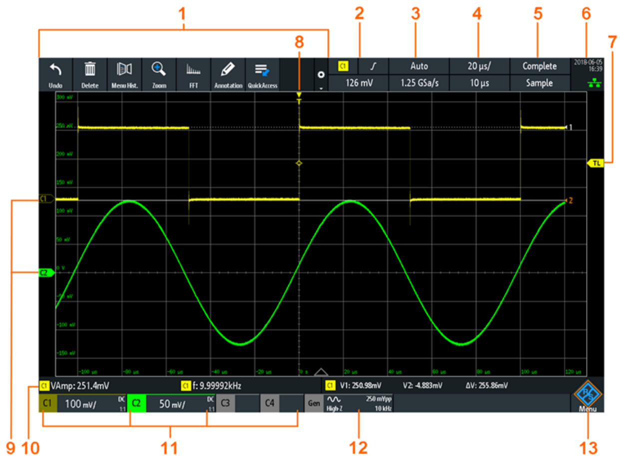

Figure A.2: Display of R&S RTB2004, 1 =Toolbar 2 =Trigger source, main trigger parameter (here: slope for edge trigger), trigger level, 3 =Trigger mode and sample rate, 4 =Horizontal scale (time scale) and horizontal position, 5 =Acquisition status and acquisition mode, 6 =Date, time, 7 =Trigger level marker, has the color of the trigger source, 8 =Trigger position marker, has the color of the trigger source, 9 =Channel markers indicate the ground levels; channel C2S is selected, i.e. it has the focus, 10 = Measurement results (here: automatic measurements on the left, cursor measurements on the right), 11 = Vertical settings of active analog channels: vertical scale, Channel 2 is selected, 13 = Menu button

By pressing the menu button on the display, you can turn on and off a side bar on the right where a few menu options are shown. You can reach more menu points by scrolling down on the display. In the following we will discuss the ones you need for this experiment.

A.2 Horizontal and Vertical Controls

Figure A.3: Horizontal and Vertical Control Buttons of the Oscilloscope

A.2.1 Horizontal Control

Scale Knob It can be turned to adjust the horizontal time scale for all signals. By pressing the knob you can switch between a fine and a coarse scale adjustment.

Position Knob The position knob changes the position of the trigger position of your signal. I.e. you can move your signal from left to right with this knob. By pressing the knob you can set it back to zero.

Zoom Button Enables or disables the zoom with the last used configurations.

Horizontal Button Opens the menu to configure all horizontal parameters.

Acquisition Button The Acquisition button opens the Acquisition menu. Its function is explained in section A.3.

A.2.2 Vertical Control

Chanel Buttons By pressing each of the four channel buttons, the respective channel is either turned on or off. If the button is illuminated in the channel colour, the channel is turned on. The effect of the key press depends on state of the channel:

- If channel is off: Turns on the channel and selects it. The rotary knobs alongside light up in the channel color.

- If the channel is on and in focus (selected): Opens the corresponding channel menu.

- If the channel is on but not in focus (not selected): Selects the channel waveform.

- If the channel is selected, and the menu is open: Pressing the key turns off the channel.

Position Knob The Position Knob changes the vertical position of the selected signal.
Scale Knob It can be turned to adjust the vertical scale for all signals. By pressing the knob you can switch between a fine and a coarse scale adjustment.

A.3 Acquisition Menu

You can access the Acquisition Menu by pressing the button Acquisition, which is located in the horizontal Controls part of the oscilloscope front. Alternatively, you can open the menu and find the Acquisition Menu there. The important part of the Acquisition Menu is the Acquire Mode. The Acquire Mode, defines what kind of signal you see on the oscilloscope. The two most relevant modes are the sample mode and the average mode. In sample mode, you just see most "real" signal reaching the oscilloscope shown in an optimal manner. Whereas in average mode you see the average of your signal. You can define the number of averages.

A.4 Cursor

You find a section "Analysis" on the front panel of the oscilloscope.

Figure A.4: Analysis Control

You can activate horizontal and vertical cursors as well as open the cursor menu by pressing the button "Cursor". Alternatively, you can open the menu and navigate until you find the cursor menu. You can adjust the position of the cursors by turning the knob located next to the "cursor" button. By pressing the knob, you can switch between the different cursors. You can also adjust the position of the cursors on the touch screen. The position of the cursors, as well as the difference between the cursors (horizontal and vertical respectively) can be seen in the right part of the Measurement results (indicated with 10 in Figure A.2). Therefore, the cursors can be used to take measurements of e.g. time distance or peak height.

A.5 Trigger Menu

Triggering means to capture the interesting part of the relevant signal. Choosing the right trigger type and configuring all trigger settings correctly allows you to detect various incidents in signals. In principle, the Trigger ensures that only a signal that overcomes a certain threshold is depicted on the oscilloscope.
You find a section "Trigger" on the front panel of the oscilloscope. You can open the Trigger menu by pressing the button "Trigger". Alternatively, you can open the Menu and select "Trigger".

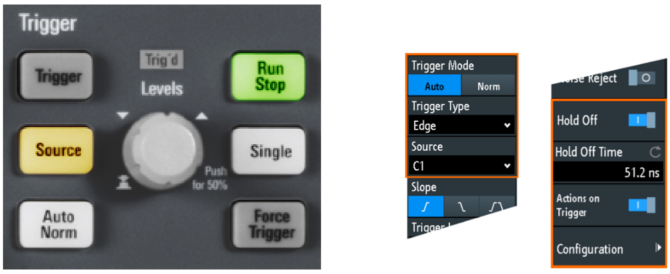

Figure A.5: Trigger Control and Trigger Menu of the Oscilloscope

Trigger Source You can select the Trigger Source, this can either be one of the four input channels of the oscilloscope or another external signal. You can change this by pressing the button "Source" or by pressing source in the Trigger menu. It will be illuminated in the channel colour.
Trigger Level You can change the Trigger Level, which is the threshold of the Trigger. The easiest way to do this, is to turn the knob next to the source button. By pressing the knob, the Trigger is set to 50 % of the signals height.
Trigger Type You can adjust the Trigger Type by changing it in the Trigger menu. The most useful mode in the case of this experiment is "Edge". In this mode the trigger, triggers on the edge of the signal. Other Trigger Types are explained in the manual [6].
Run,Stop and Single Button The "Run,Stop" button pauses and starts the signal again. The "Single" button displays a single signal.
Trigger Counter You can activate a trigger counter, which counts the number of signals above threshold by pressing the button in the lower right corner of the analysis control and then selecting trigger counter.

A.6 Action Control

The action control is located on the front panel of the oscilloscope. It is depicted in Figure A.6

Figure A.6: Action Control

A.6.1 Loading Stored Settings

The oscilloscope has the option to load previously saved settings. This is useful to restore settings for the measurements. By pressing the button "Save, Load" and then on the screen "Setup", a menu opens where different stored settings are shown. Choose the one you want to select and press load. You should now see the settings on the oscilloscopes screen.

A.6.2 Screenshot

The oscilloscope has the option to take screenshots of the current display. To do so, you have to insert a USB-Stick into the USB port in the bottom left corner of the front panel of the oscilloscope. Then, press the button with the camera symbol located in the action control section. A screenshot of the current display is then saved on the USB-Stick.

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