4. Neutrino Oscillations

For massive neutrinos one could introduce in analogy to the quark mixing a mixing matrix describing the relation between mass and flavor states:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 3} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \cdot \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Pontecorvo-Maki-Nakagawa-Sakata matrix

$$\underbrace{\nu_e = U_{e1}\nu_1 + U_{e2}\nu_2 + U_{e3}\nu_3}$$

Constant for massless ν: mixing is question of convention

Massive neutrinos develop differently in time.

$$|\nu_i(t)\rangle = |\nu_i(0)\rangle e^{-iEt} = |\nu_i(0)\rangle e^{-i(p_i + \frac{m_i^2}{2p_i})}$$

(for masses $m_i << E_i$)

$\rightarrow$ there will be a mixing of the flavor states with time.

$$|\nu(t)\rangle_\alpha = \sum_i U_{\alpha i} e^{-iE_i t} |\nu_i(0)\rangle = \sum_{i,\beta} U_{\alpha i} U^*_{\beta i} e^{-iE_i t} |\nu_\beta\rangle$$

4.1 Two-Flavor mixing (for simplicity)

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \cdot \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

Time development for an initially pure $|\nu_\alpha\rangle$ beam:

$$|\nu(t)\rangle = \cos\theta e^{-iE_1 t}|\nu_1\rangle + \sin\theta e^{-iE_2 t}|\nu_2\rangle$$
$$= \left[\cos^2\theta e^{-iE_1 t} + \sin^2\theta e^{-iE_2 t}\right] \cdot |\nu_\alpha\rangle$$
$$+ \left[\cos\theta \sin\theta (e^{-iE_1 t} - e^{-iE_2 t})\right] \cdot |\nu_\beta\rangle$$

Definite momentum p; same for all mass eigenstate components

$$E_i = \sqrt{p^2 + m_i^2} = p + \frac{m_i^2}{2p}$$

$$E_2 - E_1 = \frac{m_1^2 - m_2^2}{2p} \approx \frac{\Delta m^2}{2E}$$

(assuming p_i is the same)

$t = L/\beta$ w/ $\beta \approx 1$:

$$(E_2 - E_1)t = \frac{\Delta m^2}{2E} L$$

Mixing probability:

$$P(\nu_\alpha \rightarrow \nu_\beta, t) = |\langle \nu_\beta | \nu(t) \rangle|^2 = 2(\cos\theta \sin\theta)^2 \left[1 - \cos^2 \frac{E_2 - E_1}{2} t\right]$$

$$\boxed{P(\nu_\alpha \rightarrow \nu_\beta, t) = \sin^2 2\theta \sin^2\left(\frac{\Delta m^2}{4E} L\right)} = \sin^2 2\theta \sin^2\left(\frac{1.27 \cdot \Delta m^2[eV]}{4E[GeV]} L[km]\right)$$

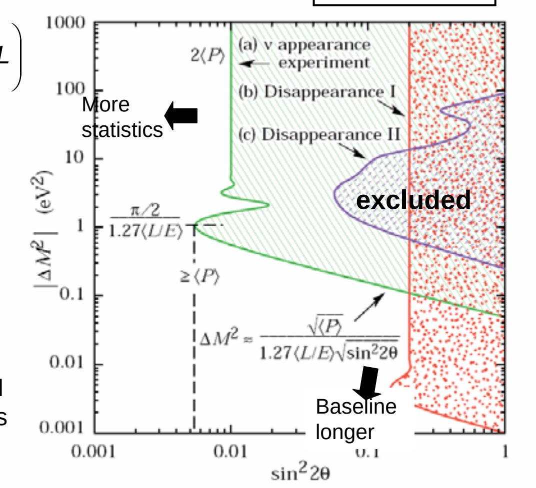
Search for Neutrino Oscillations (PDG 1996)

Exclusion plots

$$P(\nu_\alpha \to \nu_\beta, t) = \sin^2 2\theta \sin^2\left(\frac{\Delta m^2}{4E} L\right)$$

- Disappearance:
  (I) With known neutrino flux: Measurement of flux at distance L: reactor experiments,

  (II) Measure neutrino flux at position 1 and verify flux after distance L.
- Appearance:
  Use neutrino beam of type A and search at distance L for neutrinos of type B.



Observation of Neutrino Oscillations

Neutrino source	Experiment	Comments
Solar neutrinos	Radio-chemical exp.: Homestake Cl exp., GALLEX, SAGE,	First observation of “neutrino disappearance” dates more than 20 years ago: “Solar neutrino problem”
	Water experiments: (Super)Kamiokande, IMB	Confirm disappearance of solar neutrinos
	Water++: SNO	Ultimate “solar neutrino experiment”: proves the oscillation of solar ν
Atmospheric neutrinos	(Super)Kamiokande	Oscillation signal
Accelerator	LSDN	Much disputed signal
	K2K	Clear disappearance signal
Reactor	KamLAND	Clear disappearance signal

The Nobel Prize in Physics 2002

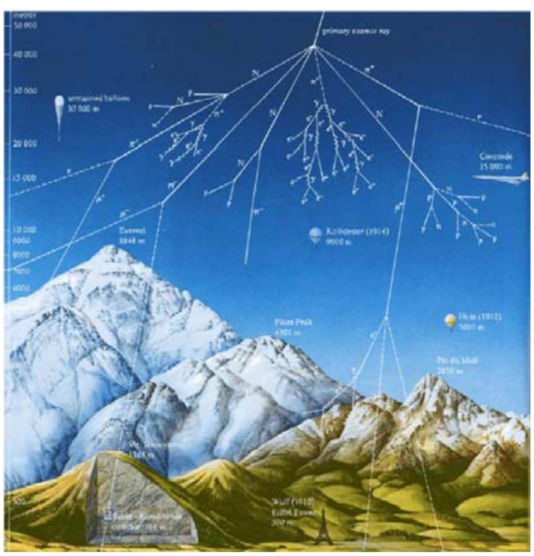
  

Raymond Davis Jr.	**Masatoshi Koshiba**	**Riccardo Giaconi**

"for pioneering contributions to astrophysics, in particular for the detection of cosmic neutrinos"

"for pioneering contributions to astrophysics, which have led to the discovery of cosmic X-ray sources"

4.2 Atmospheric neutrino problem



Cosmic radiation: Air shower

$$p + N \to \pi^{\pm}, K^{\pm}$$

$$\pi^{\pm}, K^{\pm} \to \mu^{\pm} + \nu_{\mu}(\bar{\nu}_{\mu})$$

$$\mu^{\pm} \to e^{\pm} + \nu_e(\bar{\nu}_e) + \bar{\nu}_{\mu}(\nu_{\mu})$$

⬇

$$R = \frac{\nu_{\mu} + \bar{\nu}_{\mu}}{\nu_e + \bar{\nu}_e} = 2$$

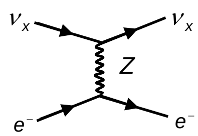
Exact calculation: R=2.1
(E_{ν}<1GeV)

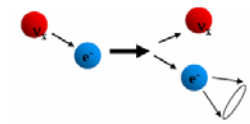
(For larger energies R>2.1)

Neutrino detection with water detectors [E_ν~O(GeV)]

Water = "active target"

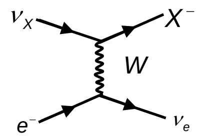
Elastic scattering





Cherenkov Light

Charged current



Kinematical limit for ν_μ: $E_\nu > m_\mu$

Detection of Cherenkov photons: Photo multiplier

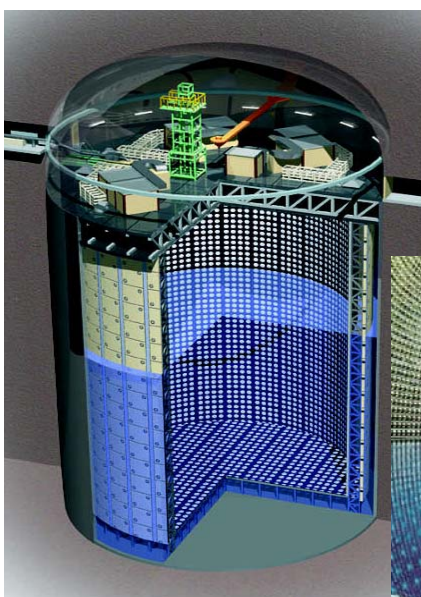
Experiments:

(Super)-Kamiokande

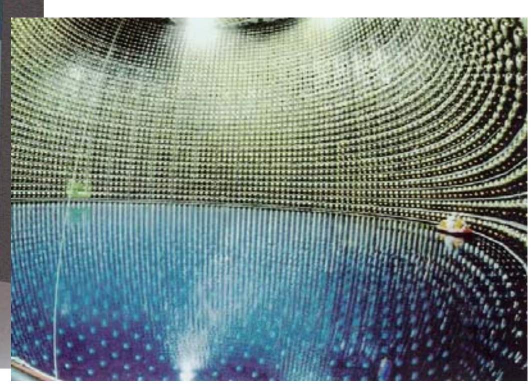
IMB

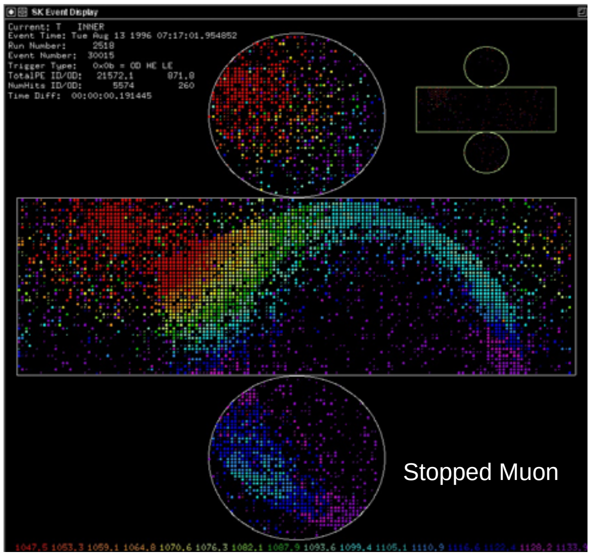
Soudan-2

Super-Kamiokande



- Largest artificial water detector (50 kt)
- Until the 2001 accident: 11000 PMTs (50 cm tubes!): 40% of surface covered with photo-cathode
- Back in operation since 2003: only 5200 PMTs left





Cherenkov cone:

$$\cos\theta = \frac{1}{\beta n}$$

$$\Leftrightarrow \theta = 42^o \; (\beta = 1)$$

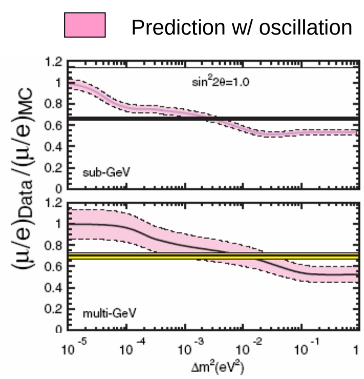
Experiment can distinguish electron and muon events, can measure energy

Ratio of muon to electron neutrinos

$$R \equiv \frac{(\mu/e)_{DATA}}{(\mu/e)_{M.C.}}$$

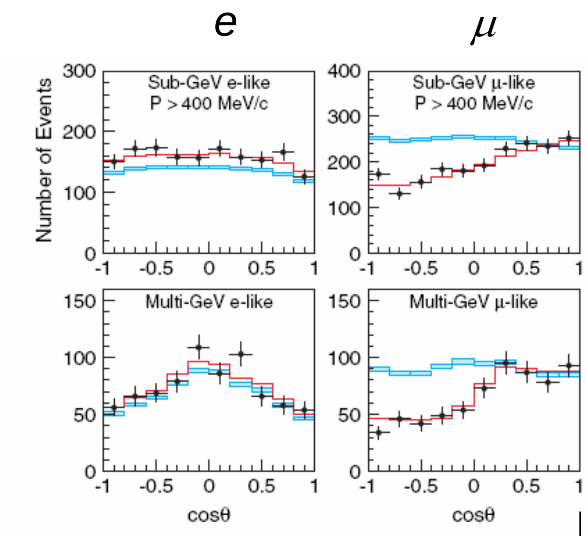
$$R_{sub-GeV} = 0.658 \pm 0.016(stat) \pm 0.032(sys)$$

$$R_{multi-GeV} = 0.702^{+0.032}_{-0.030}(stat) \pm 0.099(sys)$$



- Too few muon neutrinos observed
- Can be explained by oscillation.

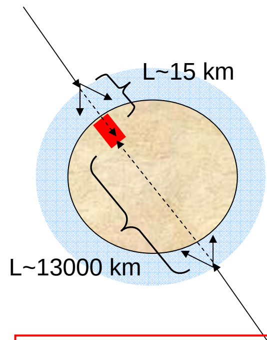
Zenith angle dependence of the neutrino flux

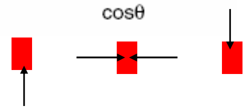


Theoretical predicton

w/o oscillation

w/ oscillation



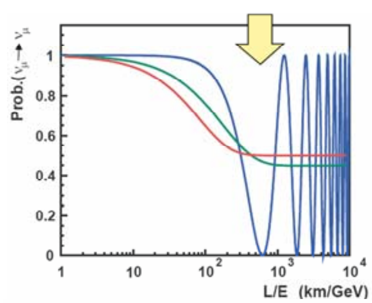


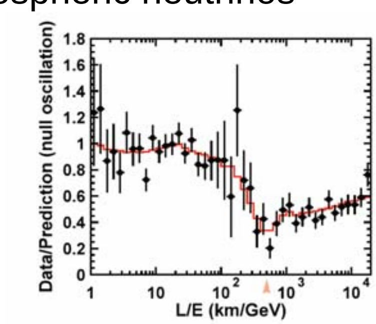
ν_μ deficit depends on angle

ν_e flux okay

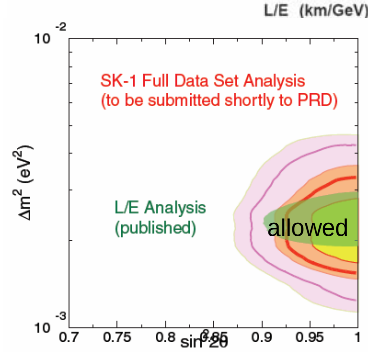
Oscillation: $\nu_\mu \leftrightarrow \nu_\tau$

Oscillation pattern of atmospheric neutrinos





oscillation dip seen at ~500 km/GeV



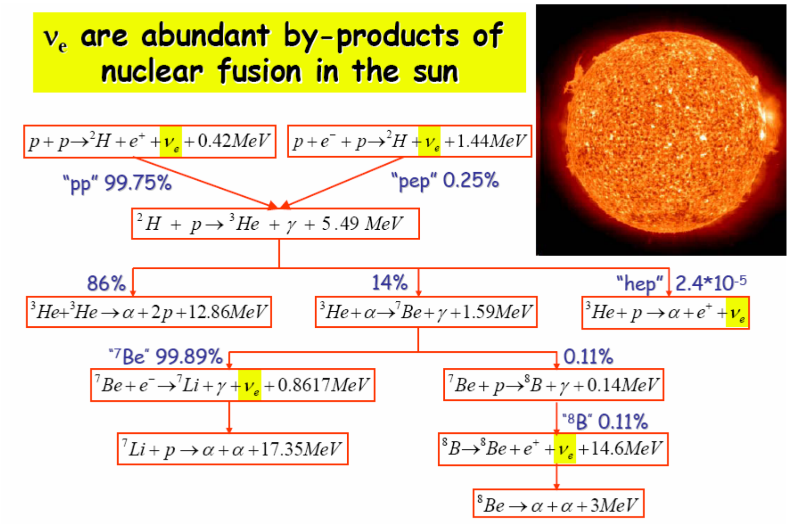
$\nu_\mu \leftrightarrow \nu_\tau$ mixing of atmos. neutrinos

$\Delta m^2 = (2.4 \pm 0.4) \times 10^{-3}$eV

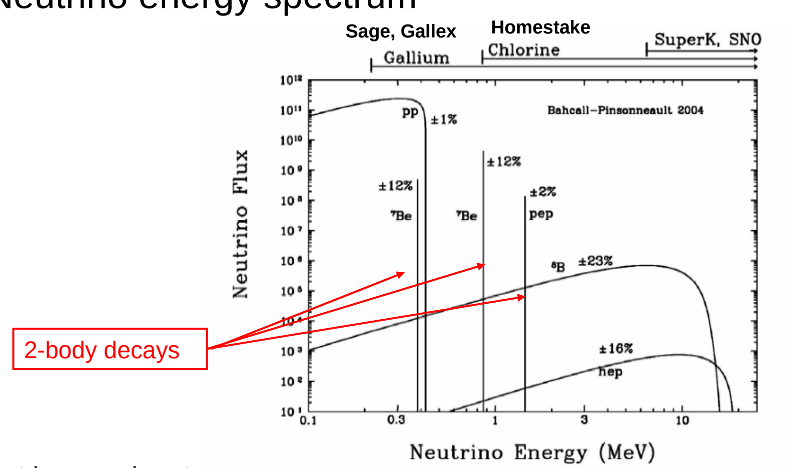
$\sin^2 2\theta > 0.92$ @ 90% C.L.

4.3 Solar neutrino problem

Neutrino production



Neutrino energy spectrum



Neutrino experiments:

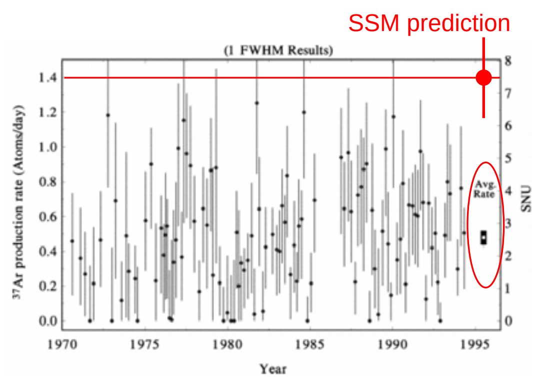
Cl_2 detectors	$\nu_e + {}^{37}Cl \to {}^{37}Ar + e$, ${}^{37}Ar \to {}^{37}Cl$ (EC)	E_ν>0.8 MeV
Ga detectors	$\nu_e + {}^{71}Ga \to {}^{71}Ge + e$	E_ν>0.2 MeV
H_2O detectors	Elastic scattering: $\nu_e + e \to \nu_e + e$	E_ν>5 MeV (detection)

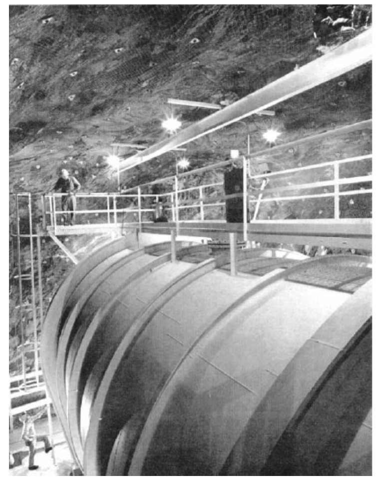
Radio-chemical experiments:

Homestake, SAGE, GALLEX

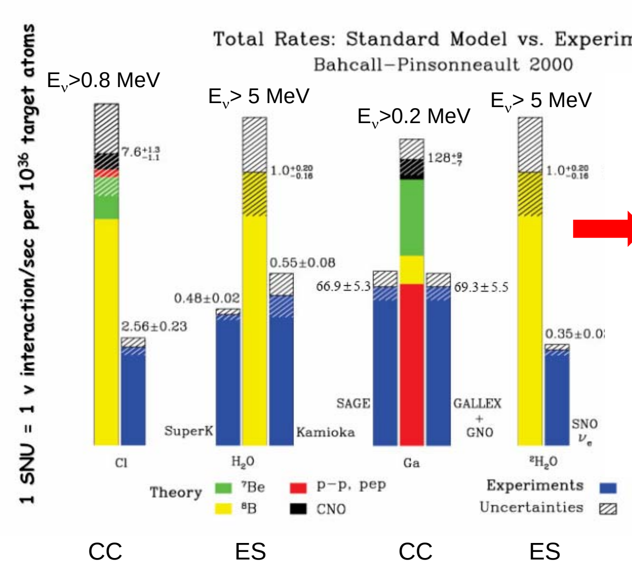
- Homestake mine, 1400 m underground
- 615 t of C_2Cl_4 (perchloroethilene) = 2.2×10^{30} atoms of ^{37}Cl
- Use ^{36}Ar and ^{38}Ar to carry-out the few atoms of ^{37}Ar (~ 1 atom/day)
- Count radioactive ^{37}Ar decays

Homestake Cl_2 experiment





Solar Neutrino Problem: Experimental summary

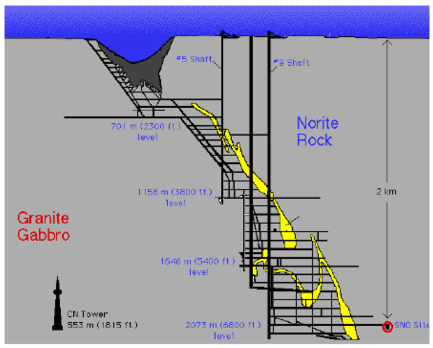


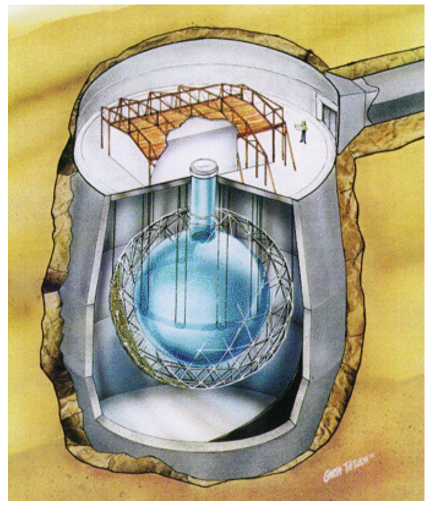
Neutrino disappearance

Can one measure the oscillated neutrinos ??

Sudbury Neutrino Observatory

- 6 m radius transparent acrylic vessel
- 1000 t of heavy water (D_2O)
- 9456 inward looking photo multipliers
- Add 2 t of NaCl to detect neutrons



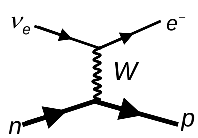


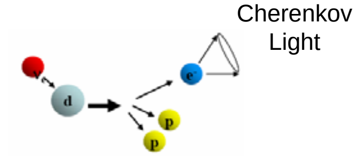
Neutrino detection with SNO

Charged current

$\sigma(\nu_\mu) = \sigma(\nu_\tau) = 0$

$\phi_\nu = \phi_{\nu_e}$

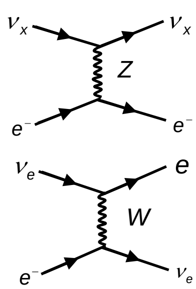


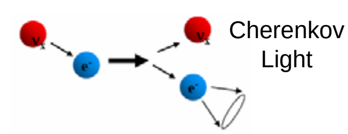


Elastic scattering

$0.154 \cdot \sigma(\nu_e) =$
$\sigma(\nu_\mu) = \sigma(\nu_\tau)$

$\phi_\nu = \phi_{\nu_e} + (\phi_{\nu_\mu} + \phi_{\nu_\tau})/6$

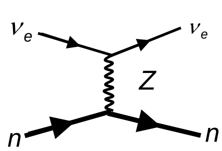


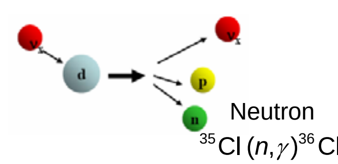


Neutral current

$\sigma(\nu_e) = \sigma(\nu_\mu) = \sigma(\nu_\tau)$

$\phi_\nu = \phi_{\nu_e} + \phi_{\nu_\mu} + \phi_{\nu_\tau}$

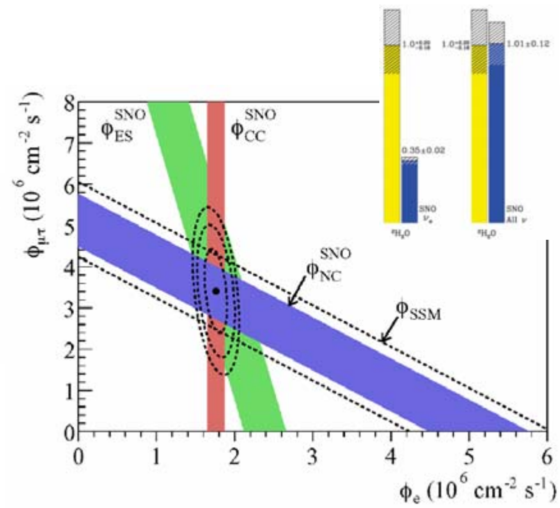




SNO Evidence for Neutrino Oscillation

$$\phi_{CC}^{SNO} = 1.76^{+0.06}_{-0.05}(\text{stat})^{+0.09}_{-0.09}(\text{syst}),$$
$$\phi_{ES}^{SNO} = 2.39^{+0.24}_{-0.23}(\text{stat})^{+0.12}_{-0.12}(\text{syst}),$$
$$\phi_{NC}^{SNO} = 5.09^{+0.44}_{-0.43}(\text{stat})^{+0.46}_{-0.43}(\text{syst}).$$

Phys. Rev.Lett. 89 (2002) 011301



Electron neutrino flux is too low.

Total flux of neutrinos is correct.

→ Interpreted as

$\nu_e \leftrightarrow \nu_\mu$ or ν_τ oscillation

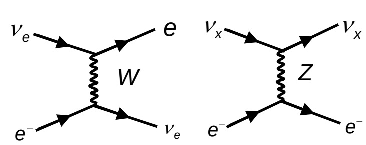
Neutrino oscillations in matter: MSW-effect

Mikhaev, Smirnov (1986), Wolfenstein (1976)

Neutrino oscillation in vacuum:

$$i\frac{d}{dt}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} = \mathbf{M}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} \qquad \mathbf{M} = \frac{m_1^2+m_2^2}{4p}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} + \frac{\Delta m^2}{4p}\begin{pmatrix}-\cos 2\theta & \sin 2\theta\\ \sin 2\theta & \cos 2\theta\end{pmatrix}$$

Neutrinos in matter:



Electrons suffer an additional potential V_e affecting the forward scattering amplitude which leads to change in the effective mass for e:

$$V_e = G_F\sqrt{2}N_e \qquad N_e\text{=electron density}$$

$$m^2 = E^2 - p^2 \rightarrow (E+V_e)^2 - p^2 \approx m^2 + 2EV_e$$

$$\Delta m_M^2 = 2\sqrt{2}G_F N_e E$$

Neutrino oscillation in matter: → additional mixing

$$\mathbf{M} \rightarrow \mathbf{M_M} = [\frac{m_1^2+m_2^2}{4p} + \frac{\sqrt{2}G_F N_e}{2}]\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} + \frac{\Delta m^2}{4p}\begin{pmatrix}-\cos 2\theta + A & \sin 2\theta\\ \sin 2\theta & \cos 2\theta - A\end{pmatrix}$$

$$A = 2\sqrt{2}G_F N_e p/\Delta m^2$$

Mixing term can be expressed using a mixing angle θ_M in matter:

$$\frac{\Delta m^2}{4p}\begin{pmatrix} -\cos 2\theta_M & \sin 2\theta_M \\ \sin 2\theta_M & \cos 2\theta_M \end{pmatrix}$$

where the mixing angle θ_M is defined as:

$$\tan 2\theta_M = \frac{\sin 2\theta}{\cos 2\theta - A} = \frac{\tan 2\theta}{1-(L_V/L_e)/\cos 2\theta}$$

$$A = L_V / L_e \qquad \begin{cases} L_V = 4\pi p / \Delta m^2 & \text{vakuum osc. length} \\ L_e = 4\pi / 2\sqrt{2} G_F N_e & \text{electron interact. length} \end{cases}$$

Matter mixing angle can go through a resonance:

$$(L_V/L_e)/\cos 2\theta = 1$$

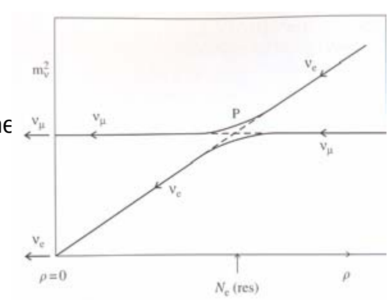
$$N_e(\text{res}) = \Delta m^2 \frac{\cos 2\theta}{2\sqrt{2} G_F p}$$

MSW effect: Very large mixing, even if vacuum mixing angle is small

As in the core of the sun, N_e is larger than the critical density the resonance condition will always be fulfilled: oscillation largely modified by matter.

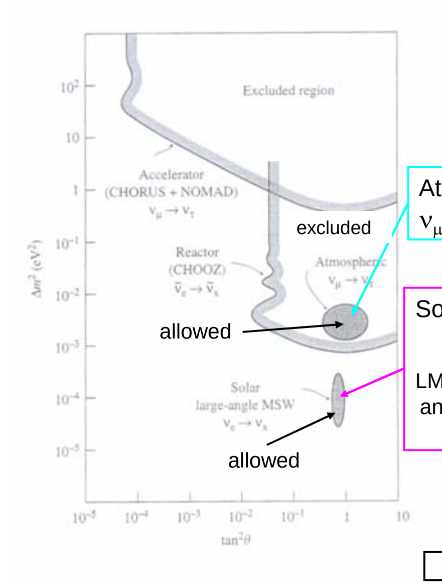
Intuitive picture of the MSW effect:

- An electron neutrino starts in the solar core, predominantly as a ν_1 mass eigenstate (vacuum).
- Extra weak potential increases the mass to the value m_2 (ν_2 eigenstate), which in vacuum would consists mainly out of ν_μ
- if solar matter density changes slowly the ν_2 mass eigenstate will pass out of sun w/o further changes: in the vacuum it will be identified with the muon neutrino.



⇨ MSW (matter) effect can explain the large energy dependent differences in the suppression factors of the neutrino flux observed in CC reactions.

Status of oscillation measurements



Atmos $\nu_\mu \rightarrow \nu_x$

$\Delta m^2 = (2.4 \pm 0.4) \times 10^{-3}$eV
$\sin^2 2\theta > 0.92$ @ 90% *C.L.*

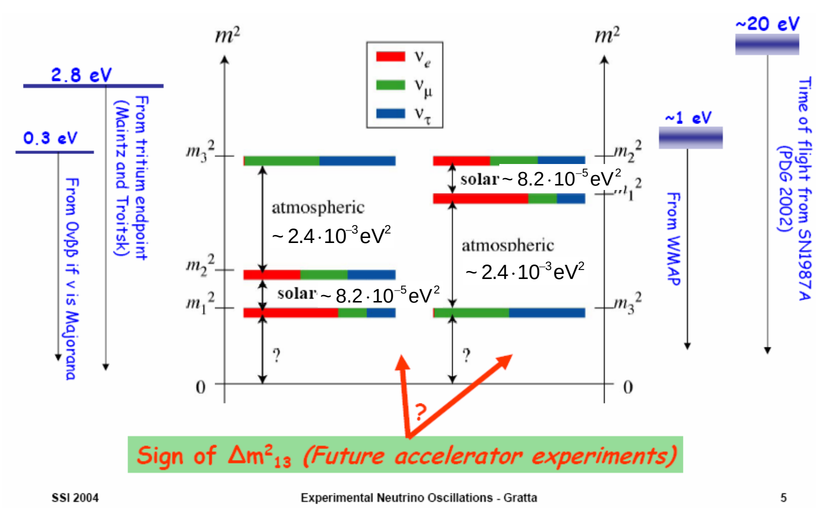
Solar+KamLAND $\nu_e \rightarrow \nu_x$ ← Long baseline "many" reactors experiment

LMA = large mixing angle: MSW effect included

$\Delta m^2 = (8.2 \pm 0.6) \times 10^{-5}$eV
$\sin^2 2\theta \approx 0.83$

⇨ Different oscillation pattern for different neutrinos – what can we learn about the masses ??

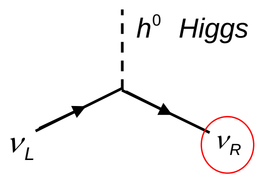
4.4 Neutrino masses



Absolute neutrino masses are not known !

Neutrino masses in the Standard Model

- Neutrino Mass term: the same as for charged leptons: $\sim m\overline{\psi}_R\psi_L$



Dirac mass term

Masses of neutrinos through Yukawa coupling to Higgs: $m_\nu = \frac{\lambda_\nu \upsilon}{\sqrt{2}}$

From the vacuum expectation value of the Higgs, $\upsilon \approx 246$ GeV, follows that the Yukawa coupling must be extremely small ($<10^{-11}$) to generate the small neutrino masses. → unnatural

- Dirac mass terms imply existence of right (left) -handed (anti) neutrinos:
  → not observed: weak interaction couples only left(right)-handed (anti)neutrinos.
  = minimal extension of the Standard Model

- Lepton numbers: Because of possible mixing L_e L_μ and L_τ are not separately conserved. L is conserved !!

How can we have massive neutrinos and ensure that weak interaction couples only to ν_L and $\overline{\nu}_R$?

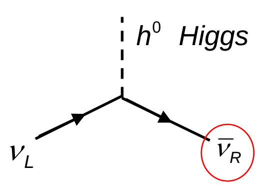
Majorana Neutrinos

- Unlike the charged leptons, neutrinos could be their own anti-particles:

Majorana Neutrinos

$$\nu = \overline{\nu} \quad \begin{cases} \overline{\nu}_L = \nu_L \\ \nu_R = \overline{\nu}_R \end{cases}$$

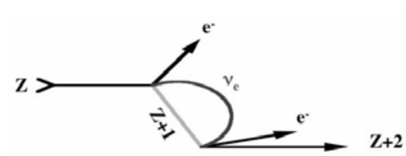
- Majorana-mass terms in addition to Dirac mass terms possible:



Majorana mass term

Mass term violates Lepton flavor conservation: $\Delta L = \pm 2$

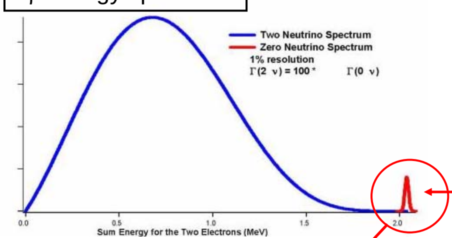
- Majorana character can be checked in neutrinoless double beta decay ($0\nu 2\beta$):



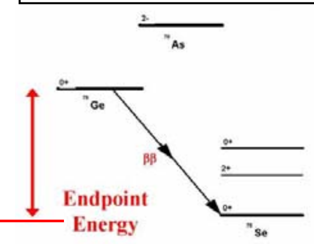
- Can we prove that neutrino is a Dirac particle

Search for $0\nu 2\beta$

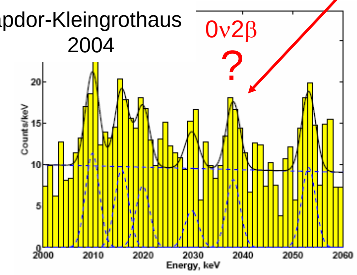
2β energy spectrum



Germanium decay (example)



Klapdor-Kleingrothaus 2004



Isotope	Half-life (years)	$m_{\beta\beta}$ (meV)	Reference
^{48}Ca	$>1.4 \times 10^{22}$	<7200-44700	[7]
^{76}Ge	$>1.9 \times 10^{25}$	<350	[8]
^{76}Ge	$>1.6 \times 10^{25}$	<330-1350	[9]
^{76}Ge	$\approx 1.2 \times 10^{25}$	≈440	[10]
^{82}Se	$>2.7 \times 10^{22}$ (68%)	<5000	[11]
^{100}Mo	$>5.5 \times 10^{22}$	<2100	[12]
^{116}Cd	$>1.7 \times 10^{23}$	<1700	[13]
^{128}Te	$>7.7 \times 10^{24}$ (geochem)	<1100-1500	[14]
^{130}Te	$>5.5 \times 10^{23}$	<370-1900	[15]
^{136}Xe	$>4.4 \times 10^{23}$	<1800-5200	[16]
^{150}Nd	$>1.2 \times 10^{21}$	<3000	[17]

Seesaw mechanism to generate light neutrinos

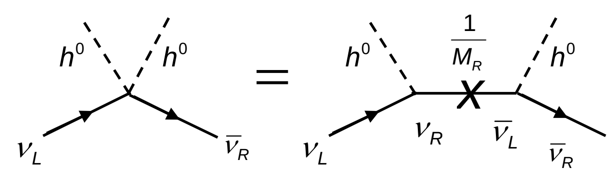
- If neutrinos are Majorana particles:

  Introduce in addtion to the Dirac mass term also a Majorana mass term for the right-handed neutrino singlet.

  The Majorana mass M_R of the right-handed neutrino can be chosen very heavy.

  Solving the mass matrix one obtains "naturally" for the "light neutrino" mass m_ν a small value:

$$m_\nu = \frac{m_D^2}{M_R}$$



Seesaw mass term for light neutrino

- Small neutrino masses can be explained … but how large is M_R?