

IX. Flavor oscillation and CP violation

1. Quark mixing and the CKM matrix
2. Flavor oscillations: Mixing of neutral mesons
3. CP violation
4. Neutrino oscillations

1. Quark mixing and CKM matrix

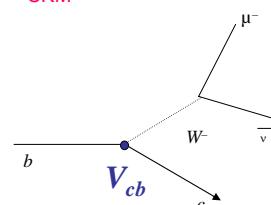
1.1 Quark mixing:

Mass eigen-states are not equal to the weak eigen-states:
Quark-mixing described by unitary **CKM matrix**,

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \underbrace{\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}}_{V_{\text{CKM}}} \cdot \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

For weak charged current one obtains:

$$J_\mu^+ \propto (\bar{u}, \bar{c}, \bar{t})(1 - \gamma_5) \gamma_\mu V_{\text{CKM}} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$



Weak states u' d' s' are orthogonal: $V_{\text{CKM}} V_{\text{CKM}}^+ = V_{\text{CKM}}^+ V_{\text{CKM}} = 1$
 V_{CKM} is unitary.

1.2 Cabibbo-Kobayashi-Maskawa matrix

Number of independent parameters:

18 parameter = 9 complex elements
 -5 relative quark phases (unobservable)
 -9 unitarity conditions

 = 4 independent parameters: 3 angles + 1 phase

PDG parametrization

3 Euler angles

1 Phase

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

where $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$

Wolfenstein Parametrization

λ, A, ρ, η

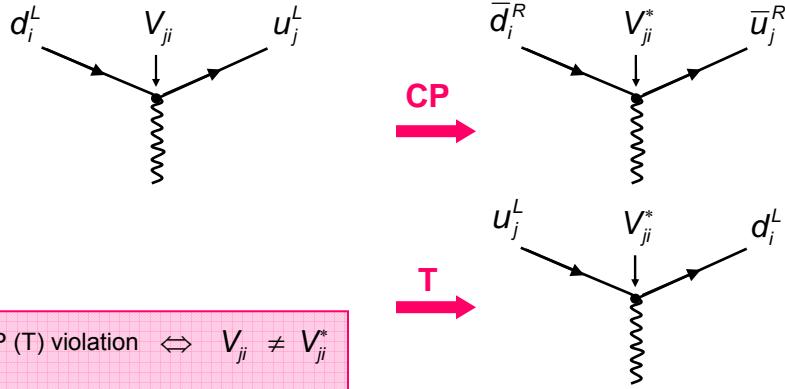
$$V_{CKM} = \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & V_{ub} = |V_{ub}| e^{-i\gamma} \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4)$$

→ hierarchy expressed by orders of $\lambda = \sin\theta_c \approx 0.22$

Magnitude of the elements:

$$\begin{pmatrix} 0.9739 \text{ to } 0.9751 & 0.221 \text{ to } 0.227 & 0.0029 \text{ to } 0.0045 \\ 0.221 \text{ to } 0.227 & 0.9730 \text{ to } 0.9744 & 0.039 \text{ to } 0.044 \\ 0.0048 \text{ to } 0.014 & 0.037 \text{ to } 0.043 & 0.9990 \text{ to } 0.9992 \end{pmatrix} \begin{array}{l} \text{PDG 2004} \\ 90\% \text{ C.L.} \end{array}$$

Komplex CKM elements and CP violation



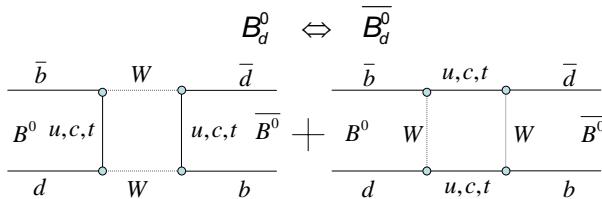
Remark: For 2 quark generations the mixing is described by the real 2x2 Cabibbo matrix \rightarrow no CP violation !!. To explain CPV in the SM Kobayashi and Maskawa have predicted a third quark generation.

2. Flavor oscillation: Mixing of neutral mesons

Neutral mesons: $|P^0\rangle: K^0 = |us\rangle \quad D^0 = |\bar{u}c\rangle \quad B_d^0 = |\bar{d}\bar{b}\rangle \quad B_s^0 = |\bar{s}\bar{b}\rangle$

 $|\bar{P}^0\rangle: \bar{K}^0 = |u\bar{s}\rangle \quad \bar{D}^0 = |\bar{u}c\rangle \quad \bar{B}_d^0 = |\bar{d}\bar{b}\rangle \quad \bar{B}_s^0 = |\bar{s}\bar{b}\rangle$

Standard Model predicts oscillations of neutral Mesons:



Transition can be described by matrix element: $\langle \bar{B}_d^0 | H_W | B_d^0 \rangle$

2.1 Phenomenological description of mixing

Schrödinger equation for unstable meson:

$$i \frac{d}{dt} |\psi\rangle = H |\psi\rangle = \left(m - \frac{i}{2} \Gamma \right) |\psi\rangle \Rightarrow \begin{cases} |\psi(t)\rangle = |\psi_0\rangle \cdot e^{-imt} \cdot e^{-\frac{1}{2}\Gamma t} \\ |\psi(t)\rangle^2 = |\psi_0\rangle^2 \cdot e^{-\Gamma t} \end{cases}$$

For neutral mesons $(K^0, \bar{K}^0), (D^0, \bar{D}^0), (B^0, \bar{B}^0), (B_s^0, \bar{B}_s^0)$ consider 2 compon.

$$i \frac{d}{dt} \begin{pmatrix} P^0 \\ \bar{P}^0 \end{pmatrix} = H \begin{pmatrix} P^0 \\ \bar{P}^0 \end{pmatrix} = \begin{pmatrix} H_{11} & H_{21} \\ H_{12} & H_{22} \end{pmatrix} \begin{pmatrix} P^0 \\ \bar{P}^0 \end{pmatrix} = \left(M - \frac{i}{2} \Gamma \right) \begin{pmatrix} P^0 \\ \bar{P}^0 \end{pmatrix} = \underbrace{\begin{pmatrix} m_{11} - \frac{i}{2} \Gamma_{11} & m_{12} - \frac{i}{2} \Gamma_{12} \\ m_{12} - \frac{i}{2} \Gamma_{21} & m_{22} - \frac{i}{2} \Gamma_{22} \end{pmatrix}}_{\langle B^0 | H_w | \bar{B}^0 \rangle} \begin{pmatrix} P^0 \\ \bar{P}^0 \end{pmatrix}$$

$CP \Rightarrow H_{12} = H_{21}$	$H_{11} = H_{22} = H$ $CPT \Rightarrow m_{11} = m_{22} = m$ $\Gamma_{11} = \Gamma_{22} = \Gamma$	M and Γ hermitian: $m_{21} = m_{12}^*$ $\Gamma_{21} = \Gamma_{12}^*$
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Mass eigenstates (by diagonalizing matrix)

Heavy and light mass eigenstate:

$$|P_L\rangle = p|P^0\rangle + q|\bar{P}^0\rangle \quad \text{with } m_L, \Gamma_L$$

$$|P_H\rangle = p|P^0\rangle - q|\bar{P}^0\rangle \quad \text{with } m_H, \Gamma_H$$

$$|p|^2 + |q|^2 = 1 \quad \text{complex coefficients}$$



Parameters of the mass states

$$m_{H,L} = m \pm \text{Re} \sqrt{H_{12}H_{21}}$$

$$\Gamma_{H,L} = \Gamma \mp 2\text{Im} \sqrt{H_{12}H_{21}}$$

$$\Delta m = m_H - m_L = 2\text{Re} \sqrt{H_{12}H_{21}}$$

$$\Delta \Gamma = \Gamma_H - \Gamma_L = -4\text{Im} \sqrt{H_{12}H_{21}}$$

$$x \equiv \frac{\Delta m}{\Gamma} \quad \text{und} \quad y \equiv \frac{\Delta \Gamma}{2\Gamma}$$

Neutral B mesons

$$\Delta m_d = 0.502 \pm 0.007 \text{ ps}^{-1} \text{ (PDG)}$$

$$\Delta m_s > 14.4 \text{ ps}^{-1} \text{ (PDG)}$$

$$\Delta \Gamma / \Gamma < 0.07 \text{ (90% CL)}$$

$$\Delta \Gamma_s / \Gamma_s = 0.65^{+0.25}_{-0.33} \pm 0.01 \text{ (ICHEP04)}$$

Time evolution

Generic particle (before P)

$$|B_{H,L}, t\rangle = b_{H,L}(t) |B_{H,L}\rangle \quad \text{mit} \quad b_{H,L}(t) = e^{-\Gamma_{H,L} t} e^{-im_{H,L} t}$$

$$\begin{aligned} |\psi_B(t)\rangle &= \frac{|B_L, t\rangle + |B_{H,t}\rangle}{2p} = \frac{1}{2p} \left(b_L(t) \cdot (p|B^0\rangle + q|\bar{B}^0\rangle) + b_H(t) \cdot (p|B^0\rangle - q|\bar{B}^0\rangle) \right) \\ &= f_+(t) \cdot |B^0\rangle - \frac{q}{p} f_-(t) \cdot |\bar{B}^0\rangle \quad f_\pm(t) = \frac{1}{2} \cdot [e^{-im_H t} e^{-\Gamma_H t/2} \pm e^{-im_L t} e^{-\Gamma_L t/2}] \\ |\psi_{\bar{B}}(t)\rangle &= f_+(t) \cdot |\bar{B}^0\rangle + \frac{p}{q} f_-(t) \cdot |B^0\rangle \end{aligned}$$

$$\begin{array}{ll} B^0 & P(B^0 \rightarrow B^0, t) = |f_+(t)|^2 \\ & P(B^0 \rightarrow \bar{B}^0, t) = \left| \frac{q}{p} \right|^2 |f_-(t)|^2 \end{array}$$

$$\begin{array}{ll} \bar{B}^0 & P(\bar{B}^0 \rightarrow \bar{B}^0, t) = |f_+(t)|^2 \\ & P(\bar{B}^0 \rightarrow B^0, t) = \left| \frac{p}{q} \right|^2 |f_-(t)|^2 \end{array}$$

Mixing of neutral mesons

Time evolution of rates:

$$\underbrace{P(B^0 \rightarrow B^0)}_{\text{CPT}} = P(\bar{B}^0 \rightarrow \bar{B}^0) = \frac{1}{4} \left[e^{-\Gamma_L t} + e^{-\Gamma_H t} + 2e^{-(\Gamma_L + \Gamma_H)t/2} \cos \Delta m t \right]$$

$$P(B^0 \rightarrow \bar{B}^0) = \frac{1}{4} \left| \frac{q}{p} \right|^2 \left[e^{-\Gamma_L t} + e^{-\Gamma_H t} - 2e^{-(\Gamma_L + \Gamma_H)t/2} \cos \Delta m t \right]$$

$$P(\bar{B}^0 \rightarrow B^0) = \frac{1}{4} \left| \frac{p}{q} \right|^2 \left[e^{-\Gamma_L t} + e^{-\Gamma_H t} - 2e^{-(\Gamma_L + \Gamma_H)t/2} \cos \Delta m t \right]$$

Two mixing mechanisms:

- Mixing through decays
- Mixing through oscillation

$$\begin{aligned} y &= \frac{\Delta \Gamma}{2\Gamma} \approx O(1) \\ x &= \frac{\Delta m}{\Gamma} \approx O(1) \end{aligned} \quad \left. \begin{array}{l} (K^0, \bar{K}^0), (D^0, \bar{D}^0), \\ (B^0, \bar{B}^0), (B_s^0, \bar{B}_s^0) \end{array} \right\} \begin{array}{l} \text{show different} \\ \text{oscillation behavior} \end{array}$$

CP, T- violation in mixing:

$$P(B^0 \rightarrow \bar{B}^0) \neq P(\bar{B}^0 \rightarrow B^0) \Rightarrow \left| \frac{q}{p} \right| \neq 1$$

2.2 Neutral kaons

Observation of two neutral kaons K_L (long) and K_S (short) with different lifetimes:

$$\tau(K_L^0) = (51.7 \pm 0.4) \text{ ns} \gg \tau(K_S^0) = (0.089 \pm 0.001) \text{ ns}$$



CP = -1



CP = +1

K_L and K_S can be identified with the mass eigenstates:

$$|K_L\rangle = "|\mathcal{K}_{Heavy}\rangle" \equiv \frac{1}{\sqrt{2}}(|K^0\rangle - |\bar{K}^0\rangle)$$

$$|K_S\rangle = "|\mathcal{K}_{Light}\rangle" \equiv \frac{1}{\sqrt{2}}(|K^0\rangle + |\bar{K}^0\rangle)$$

$$CP|\mathcal{K}_{Heavy}\rangle = -|\mathcal{K}_{Heavy}\rangle \quad \text{Phase convention:} \\ CP|K^0\rangle = |\bar{K}^0\rangle$$

$$CP|\mathcal{K}_{Light}\rangle = +|\mathcal{K}_{Light}\rangle \quad CP|\bar{K}^0\rangle = |K^0\rangle$$

Large differences between lifetimes

$$\Delta m = (0.5303 \pm 0.0009) \cdot 10^{10} \text{ fs}^{-1}$$

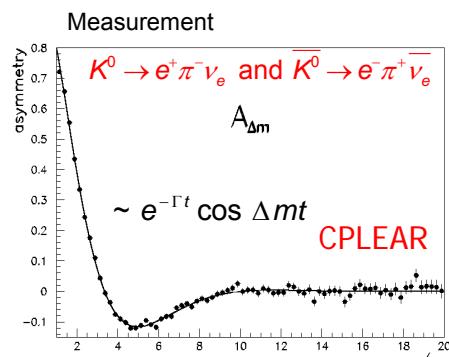
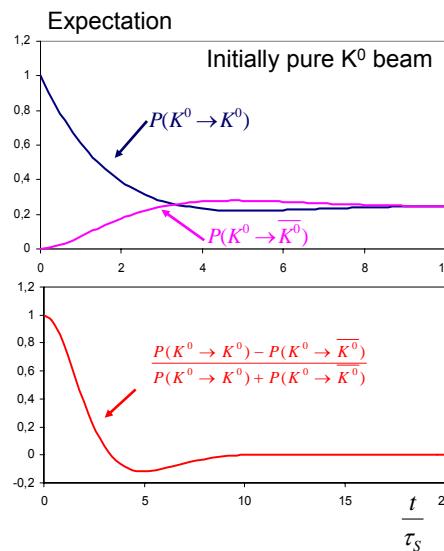
$$= (3.49 \pm 0.006) \cdot 10^{-12} \text{ MeV}$$

$$\Delta \Gamma = -11.182 \cdot 10^9 \text{ fs}^{-1}$$

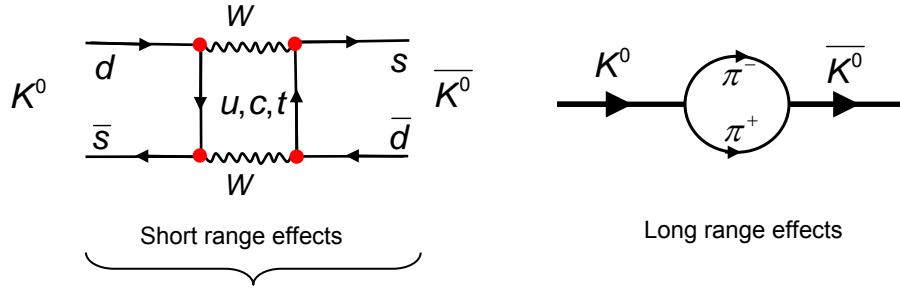
$$x = \frac{\Delta m}{\Gamma} = 0.942 \pm 0.007$$

$$y = \frac{\Delta \Gamma}{2\Gamma} = -0.9966$$

Neutral kaon system



$K^0 - \bar{K}^0$ (strangeness) oscillation in the SM



Oscillation frequency Δm :

$$\Delta m \sim \frac{G_F^2}{4\pi} m_K f_K^2 \sum_{q=u,c,t} m_q^2 |V_{qs} V_{qd}|^2 \approx \frac{G_F^2}{4\pi} m_K f_K^2 m_c^2 |V_{cs} V_{cd}|^2$$

c quark contribution dominant: although m_t^2 is very large, the factor $|V_{ts} V_{td}|^2 \sim \lambda^5$ is very small !

2.3 Neutral B Meson

2 neutral B mesons:

$B_d^0 = |d \bar{b}\rangle$ - oscillations precisely measured

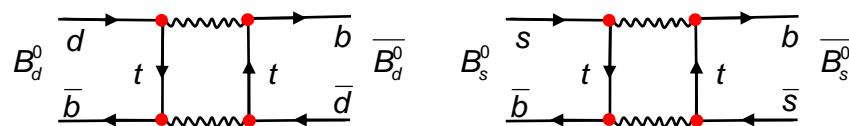
$B_s^0 = |s \bar{b}\rangle$ - oscillations not yet observed

Mixing mechanisms:

- Mixing through decay:** many possible hadronic decays $\rightarrow \Gamma$ is large

$$\Rightarrow y = \frac{\Delta\Gamma}{2\Gamma} \text{ is small} = \begin{cases} \approx 0 \text{ for } B_d^0 \\ \approx O(0.1) \text{ for } B_s^0 \end{cases} \Rightarrow \text{no mixing via decay}$$

- Mixing through oscillation** \rightarrow Significant contribution only from top loop



$$\Delta m \sim m_t^2 |V_{tb} V_{td}|^2 \sim m_t^2 \cdot O(\lambda^6)$$

$$\Delta m \sim m_t^2 |V_{tb} V_{ts}|^2 \sim m_t^2 \cdot O(\lambda^4)$$

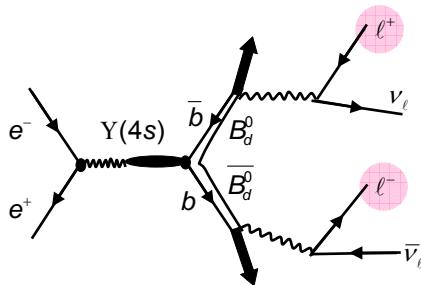
Large $\Delta m_{s,d}$: $\Delta m_s \sim 1/\lambda^2 \Delta m_d \rightarrow B_s$ osc. is about 20 times faster than B_d osc.

Discovery of B^0 mixing

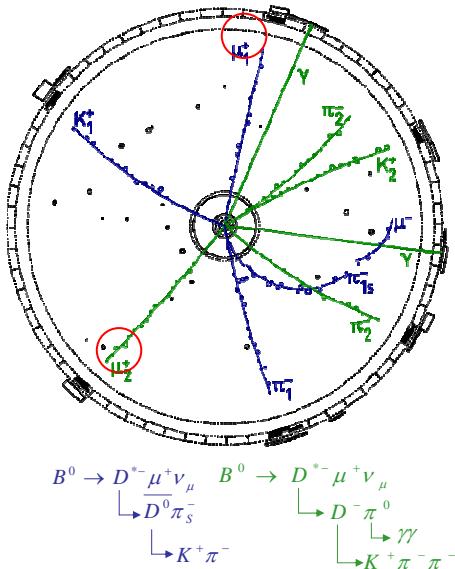
ARGUS 1987

First e^+e^- B factory at DESY:

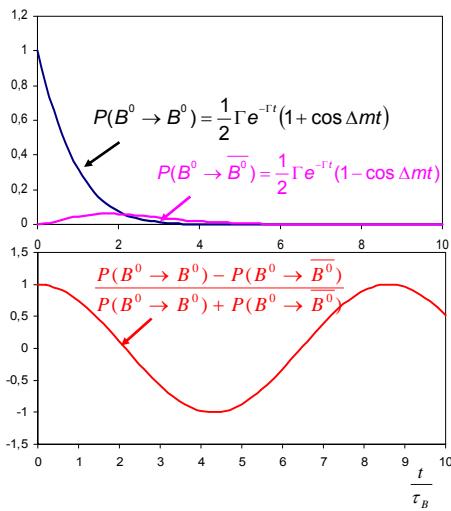
$$\text{at } \sqrt{s} = 10.58 \text{ GeV : } e^+e^- \rightarrow Y(4S) \rightarrow B^0\bar{B}^0 \quad \sigma(B\bar{B}) \approx 1 \text{ nb}$$



$$\left. \begin{array}{l} B^0\bar{B}^0 \rightarrow \ell^+\ell^- \\ B^0\bar{B}^0 \rightarrow \ell^+\ell^+ \\ B^0\bar{B}^0 \rightarrow \ell^-\ell^- \end{array} \right\} \text{mixed}$$

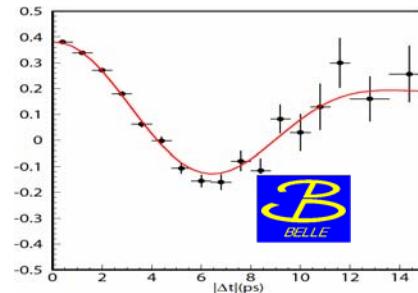


Mixing of neutral B mesons

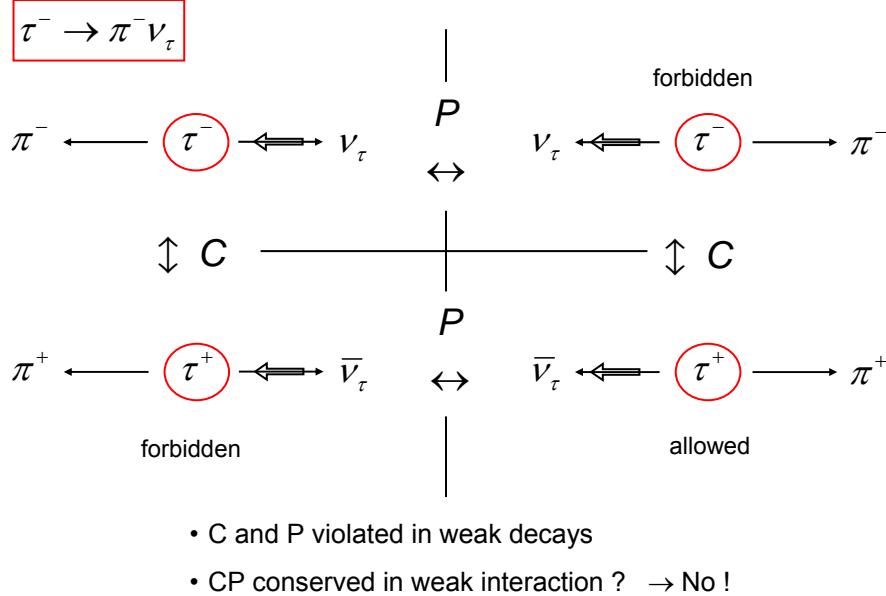


$$A = \frac{\text{unmixed} - \text{mixed}}{\text{unmixed} + \text{mixed}}$$

$$\Delta m_d = 0.511 \pm 0.005 \pm 0.006 \text{ ps}^{-1}$$



3. CP violation in the K^0 and B^0 system

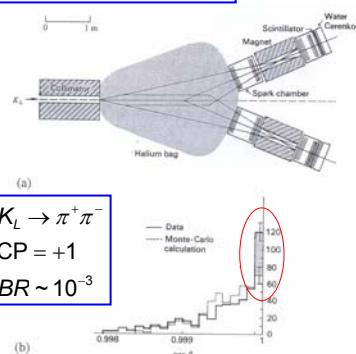


3.1 Observation of CP violation (CPV) in K_L decays

$$|K_L\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle - |\bar{K}^0\rangle) \quad \text{CP} = -1$$

should decay into 3π w/ $\text{CP}(|3\pi\rangle) = -1$
and not into 2π w/ $\text{CP}(|2\pi\rangle) = +1$

Christenson et al., 1967



- Until the year 2000 the K^0 was the only system in which CPV was observed
- In 2000, CPV was observed in the B^0 system
- So far the observed CPV is consistent with the SM prediction. But it is too small to explain the baryon asymmetry of the universe.

Fig. 7.22. (a) Arrangement of Christenson *et al.* (1964) demonstrating the CP -violating decay $K_L \rightarrow \pi^+\pi^-$. K_L decays are observed in a helium bag, the charged products being analysed by two magnet spectrometers instrumented with spark chambers and scintillators. (b) Rare two-pion decays are distinguished from the common three-pion decays by the invariant mass of the pair ($490 \text{ MeV} < M_{\pi\pi} < 510 \text{ MeV}$) and the direction, θ , of the resultant momentum vector. The $\cos\theta$ distribution is that expected from three-body decays, plus 50 events (shaded) collinear with the beam and attributed to the two-pion decay mode.

3.2 CP Violation in the Standard Modell

Quarks

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$V_{ub} = |V_{ub}| e^{-i\gamma}$ see Wolfenstein parametrization

Phase angle $\neq 0$: complex CKM matrix

Different mixing for quarks and anti-quarks

Antiquarks:

$$\begin{pmatrix} \bar{d}' \\ \bar{s}' \\ \bar{b}' \end{pmatrix} = \begin{pmatrix} V_{ud}^* & V_{us}^* & V_{ub}^* \\ V_{cd}^* & V_{cs}^* & V_{cb}^* \\ V_{td}^* & V_{ts}^* & V_{tb}^* \end{pmatrix} \begin{pmatrix} \bar{d} \\ \bar{s} \\ \bar{b} \end{pmatrix}$$

Origin of CP Violation (CPV)

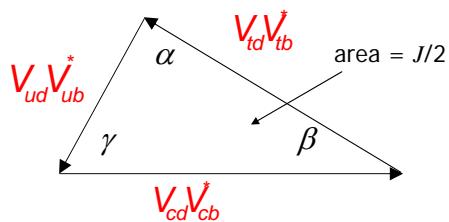
Strength of CPV: Characterized by Jarlskog invariant: $J = \text{Im} (V_{ij} V_{kl} V_{il}^* V_{kj}^*) \neq 0$

In SM: $J = \text{Im}[V_{us} V_{cb} V_{ub}^* V_{cs}^*] = A^2 \lambda^6 \eta (1 - \lambda^2/2) + O(\lambda^{10}) \sim 10^{-5}$

3.3 Unitarity Triangle

Unitary CKM matrix: $V V^\dagger = 1$

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

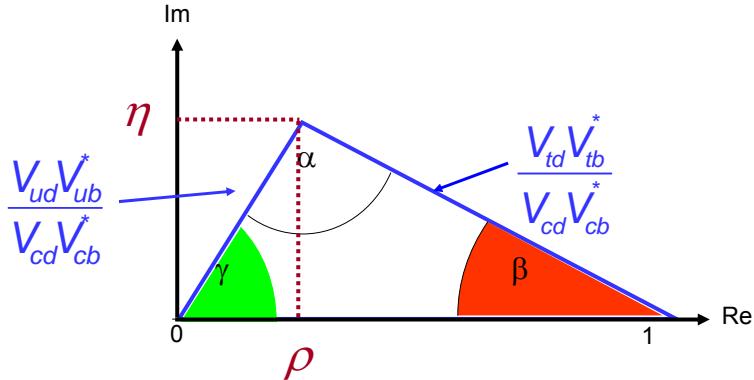


6 “triangle” relations:

$V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0$ $V_{td} V_{ud}^* + V_{ts} V_{us}^* + V_{tb} V_{ub}^* = 0$	“Real” triangles w/ similar sides: → expect large CPV effects Important for B_d and B_s decays
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Row 1x2, 2x3 Column 1x2, 2x3	Degenerated triangles (same area): → small CPV effect (Kaon decays)
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Rescaled Unitarity Triangle



$$\alpha \equiv \arg \left[-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*} \right] \quad \beta \equiv \arg \left[-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*} \right] \quad \gamma \equiv \arg \left[-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} \right]$$

3.4 “3 Ways” of CP violation in meson decays

1. CP Violation in mixing:

$$P(B^0 \rightarrow \bar{B}^0) \neq P(\bar{B}^0 \rightarrow B^0) \Rightarrow \left| \frac{q}{p} \right| \neq 1$$

Standard model prediction for B

$$\left| \frac{q}{p} \right| - 1 \approx 4\pi \frac{m_c^2}{m_t^2} \sin \beta \approx 5 \times 10^{-4}$$

Kaon system:

CP eigenstates are not identical with mass/lifetime eigenstates admixture

$$\begin{aligned} |K_1^0\rangle &= \frac{1}{\sqrt{2}}(|K^0\rangle + |\bar{K}^0\rangle) & \text{CP +1} & \quad |K_S^0\rangle = \frac{1}{\sqrt{1+|\varepsilon_m|^2}}(|K_1^0\rangle + \varepsilon_m |K_2^0\rangle) \\ |K_2^0\rangle &= \frac{1}{\sqrt{2}}(|K^0\rangle - |\bar{K}^0\rangle) & \text{CP -1} & \quad |K_L^0\rangle = \frac{1}{\sqrt{1+|\varepsilon_m|^2}}(|K_2^0\rangle + \varepsilon_m |K_1^0\rangle) \end{aligned}$$

CPLEAR 2001: $4\operatorname{Re}(\varepsilon_m) = (6.2 \pm 1.7) \cdot 10^{-3}$

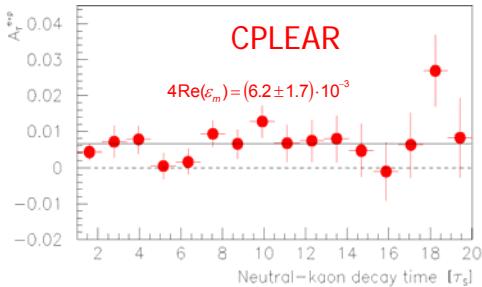
CP (T) Violation in mixing

$K^0 \bar{K}^0$ System:

$$\left. \begin{aligned} & \frac{P(\bar{K}^0 \rightarrow K^0) - P(K^0 \rightarrow \bar{K}^0)}{P(\bar{K}^0 \rightarrow K^0) + P(K^0 \rightarrow \bar{K}^0)} \cong \frac{4\text{Re}(\varepsilon_m)}{1 + |\varepsilon_m|^2} \cong 4\text{Re}(\varepsilon_m) \\ & A_{\text{exp}}(t) = \frac{\Gamma(\bar{K}_{t=0}^0 \rightarrow e^+ \pi^- \nu_e) - \Gamma(K_{t=0}^0 \rightarrow e^- \pi^+ \bar{\nu}_e)}{\Gamma(\bar{K}_{t=0}^0 \rightarrow e^+ \pi^- \nu_e) + \Gamma(K_{t=0}^0 \rightarrow e^- \pi^+ \bar{\nu}_e)}(t) \end{aligned} \right\}$$

In principle the ratio measures the T invariance:

$$\bar{K}^0 \rightarrow K^0 \xleftrightarrow{T} \bar{K}^0 \leftarrow K^0$$



$B^0 \bar{B}^0$ System:

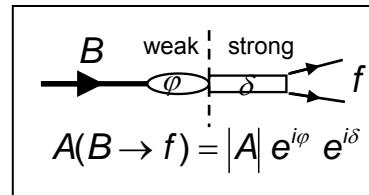
$$\begin{aligned} A_{SL} &= -0.0026 \pm 0.0067 \\ \left| \frac{q}{p} \right| &= 1.0013 \pm 0.0034 \\ \frac{4\text{Re} \varepsilon_B}{(1 + |\varepsilon_B|^2)} &= -0.0007 \pm 0.0017 \end{aligned}$$

HFAG 2004

2. Direct CPV

$$\text{Prob}(\bar{B} \rightarrow \bar{f}) \neq \text{Prob}(B \rightarrow f)$$

$$\left| \begin{array}{c} A \\ \hline B^0 \end{array} \right\rangle \rightarrow f \quad \left| \begin{array}{c} \bar{A} \\ \hline \bar{B}^0 \end{array} \right\rangle \rightarrow \bar{f} \quad \left| \frac{\bar{A}}{A} \right|^2 \neq 1$$



Standard Model:

$$A(B^0 \rightarrow X) = |A_1| e^{i\varphi_1 + i\delta_1} + |A_2| e^{i\varphi_2 + i\delta_2}$$

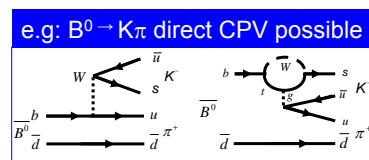
$$\bar{A}(B^0 \rightarrow X) = |A_1| e^{-i\varphi_1 + i\delta_1} + |A_2| e^{-i\varphi_2 + i\delta_2}$$

Two interfering amplitudes A_1, A_2 :

-with diff. CKM phases φ_1, φ_2

-with diff. strong phases δ_1, δ_2

$$\begin{aligned} \left| \frac{\bar{A}}{A} \right|^2 - \left| \frac{A}{\bar{A}} \right|^2 &= 4|A_1||A_2| \underbrace{\sin(\varphi_1 - \varphi_2)}_{\neq 0} \underbrace{\sin(\delta_1 - \delta_2)}_{\neq 0} \\ \left| \frac{\bar{A}}{A} \right|^2 \neq 1 &\Leftrightarrow \text{weak} \quad \text{strong} \end{aligned}$$



First observation of direct CPV in B decays

Summer 2004

$$B^0 \rightarrow K^+ \pi^-$$

$$A_{CP} = \frac{N(B^0 \rightarrow K^+ \pi^-) - N(\bar{B}^0 \rightarrow K^+ \pi^-)}{N(B^0 \rightarrow K^+ \pi^-) + N(\bar{B}^0 \rightarrow K^+ \pi^-)}$$

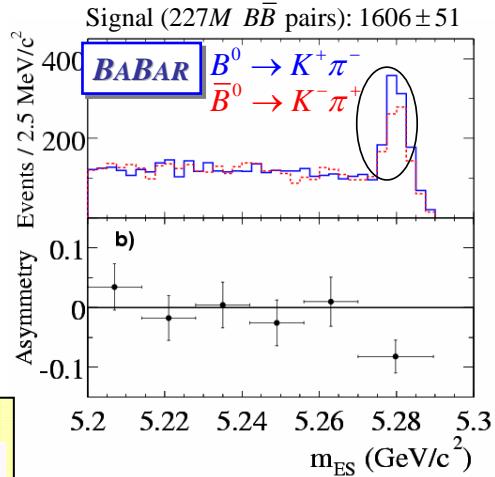
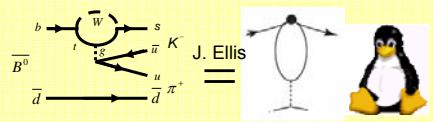
BABAR

hep-ex/0408057

$$A_{CP} = -0.133 \pm 0.030 \pm 0.009$$

4.2 σ

Effect of "penguin" contribution

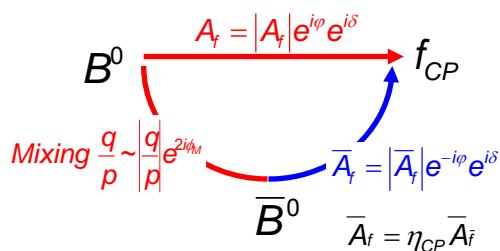


3. CPV in interference between mixing and decay

$$B^0 \rightarrow f_{CP}$$

CP eigenstate:

$$CP|f_{CP}\rangle = \eta_{CP}|f_{CP}\rangle$$



CPV if there is a phase difference λ_{CP} between the direct path and the path with mixing:

$$\lambda_{CP} \equiv \frac{q}{p} \frac{\bar{A}_f}{A_f} = \eta_{CP} \cdot \left| \frac{q}{p} \right| \cdot \left| \frac{\bar{A}_f}{A_f} \right| \cdot e^{2i(\phi_M - \phi_f)}$$

$$\text{Mixing: } \frac{q}{p} = \left| \frac{q}{p} \right| e^{2i\phi_M}$$

=1 if no CPV in decay

$$\text{If no CPV in mixing and decay: } \lambda_{CP} = -\eta_{CP} e^{2i(\phi_M - \phi_f)}$$

good approximation for $B_0 \rightarrow J/\psi K_s$

Calculation of the time-dependent CP asymmetry

$$\Gamma(B^0 \rightarrow f_{CP})(t) \propto \frac{e^{-|\Delta t|/\tau_{B^0}}}{(1+|\lambda_{CP}|^2)} \times \left[\frac{1+|\lambda_{CP}|^2}{2} - i\text{Im}(\lambda_{CP}) \sin(\Delta m_d t) + \frac{1-|\lambda_{CP}|^2}{2} \cos(\Delta m_d t) \right]$$

$$\Gamma(\bar{B}^0 \rightarrow f_{CP})(t) \propto \frac{e^{-|\Delta t|/\tau_{B^0}}}{(1+|\lambda_{CP}|^2)} \times \left[\frac{1+|\lambda_{CP}|^2}{2} + i\text{Im}(\lambda_{CP}) \sin(\Delta m_d t) - \frac{1-|\lambda_{CP}|^2}{2} \cos(\Delta m_d t) \right]$$

$$A_{CP}(t) = \frac{\Gamma(\bar{B}^0(t) \rightarrow f_{CP}) - \Gamma(B^0(t) \rightarrow f_{CP})}{\Gamma(B^0(t) \rightarrow f_{CP}) + \Gamma(\bar{B}^0(t) \rightarrow f_{CP})} = [S_f \sin(\Delta m_d t) - C_f \cos(\Delta m_d t)]$$

Time resolved

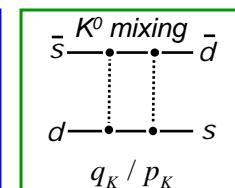
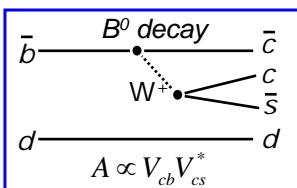
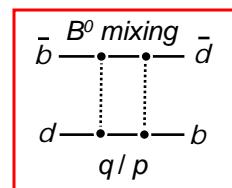
$$S_f = \frac{2\text{Im}\lambda_{CP}}{1+|\lambda_{CP}|^2} \quad C_f = \frac{1-|\lambda_{CP}|^2}{1+|\lambda_{CP}|^2}$$

Interference
= $\sin 2\beta$ for e.g. $B^0 \rightarrow J/\psi K_S$
 $\approx \sin 2\alpha_{(\text{eff})}$ for e.g. $B^0 \rightarrow \pi^+\pi^-$

indicates direct CP violation if $|q/p|=1$

3.5 SM prediction for the decay $B^0 \rightarrow J/\psi K_S$

$$\eta_{CP} = -1$$



Same for all $c\bar{c}K^0$ channels

$$\lambda_{\psi K_S} = \frac{q}{p} \frac{A_{\psi K_S}}{A_{\psi K_S}} = \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} \frac{V_{cb}^* V_{cs}}{V_{cb} V_{cs}^*} \frac{V_{cs}^* V_{cd}}{V_{cs} V_{cd}^*} = - \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} \frac{V_{cb}^* V_{cd}}{V_{cb} V_{cd}^*} = -e^{-2i\beta}$$

Beside V_{td} all other CKM elements are real

$$V_{td} \approx |V_{td}| e^{-i\beta}$$

$$\Rightarrow \begin{cases} |\lambda_{\psi K_S}| = 1 \\ \text{Im}(\lambda_{\psi K_S}) = \sin(2\beta) \end{cases}$$

no direct CPV, no CPV in mixing

Phase of decay amplitude $\varphi_f = 0$

Mixing phase $\varphi_M = \beta$

3.6 Time dependent A_{CP} for $B^0 \rightarrow J/\psi K_s$

$$A_{CP}(t) = \frac{\dot{N}(B^0 \rightarrow J/\psi K_s^0) - \dot{N}(B^0 \rightarrow J/\psi \bar{K}_s^0)}{\dot{N}(B^0 \rightarrow J/\psi K_s^0) + \dot{N}(B^0 \rightarrow J/\psi \bar{K}_s^0)} = -\eta_{CP} \sin 2(\phi_M - \phi_f) \sin(\Delta m t)$$

$B^0 \rightarrow J/\psi K_S : = \beta$

→ $A_{CP}(t) = -\eta_{CP} \sin 2\beta \sin(\Delta m t)$

Oscillation with Δm_d

If $A_{CP} \neq 0$:

- CP violation
- Measurement of the angle $\beta = \arg(V_{td})$

Time dependent CP asymmetry measurement for $B^0 \rightarrow J/\psi K_s$

