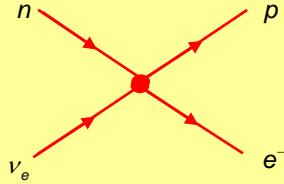


4. V-A Theory for charged current weak interactions

Fermi's original ansatz for $n \rightarrow p e^- \bar{\nu}_e$



4-fermion interaction with fermion vector currents:

$$M = \frac{G_F}{\sqrt{2}} \cdot J_{N,\mu} \cdot J_e^{\mu+} = \frac{G_F}{\sqrt{2}} \cdot (\bar{u}_p \gamma_\mu u_n) \cdot (\bar{u}_e \gamma_\mu v_\nu)$$

↳ Fermi coupling constant

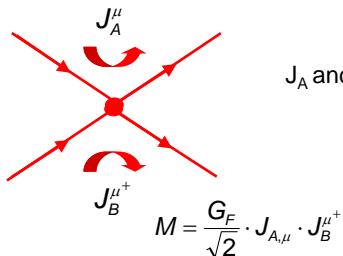
Problem: ansatz cannot explain parity violation

Most general ansatz:
Lorentz invariant form of
fermion currents

$$M = \frac{G_F}{\sqrt{2}} \cdot \sum_i (\bar{u}_p \Gamma_i u_n) \cdot (\bar{u}_e \Gamma_i v_\nu)$$

↳ Most possibilities ruled out,
left over with V-A ansatz

4.1 V-A ansatz for fundamental fermions



J_A and J_B are lepton and quark currents

$$J_\ell^\mu = \bar{u}_\ell \gamma^\mu (1 - \gamma^5) u_\nu$$

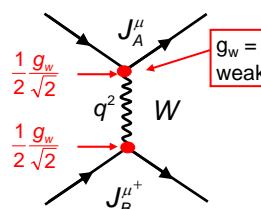
$$J_q^\mu = \bar{u}_d \gamma^\mu (1 - \gamma^5) u_u$$

Reminder

$$u_L = \frac{1}{2} (1 - \gamma^5) u$$

$$u_R = \frac{1}{2} (1 + \gamma^5) u$$

According today's understanding the 4-fermion coupling is the $q^2 \rightarrow 0$ limit of W propagator:



$$M = \frac{4}{2\sqrt{2}} \cdot J_{A,\mu} \cdot \underbrace{\frac{(g_{\mu\nu} - \frac{q_\mu q_\nu}{M_W^2})}{q^2 - M_W^2}}_{\text{for } q^2 \rightarrow 0:} \cdot \frac{1}{2\sqrt{2}} \cdot J_B^{\mu+}$$

$$\frac{G_F}{\sqrt{2}} = \frac{g_w^2}{8M_W^2} \quad \text{With } G_F \approx 1.16 \times 10^{-5} \text{ GeV}^{-2}$$

follows $w M_W \approx 80 \text{ GeV}$: $g_w \approx 0.65$

V-A coupling of leptons and quarks

Reminder

$$\bar{u}_\ell \gamma^\mu (1 - \gamma^5) u_\nu = \bar{u}_\ell \gamma^\mu u_\nu^L = (\bar{u}_\ell^L + \bar{u}_\ell^R) \gamma^\mu u_\nu^L = \bar{u}_\ell^L \gamma^\mu u_\nu^L$$

In V-A theory the weak interaction couples **left-handed lepton/quark currents** (**right-handed anti-lepton/quark currents**) with an universal coupling strength:

$$\frac{G_F}{\sqrt{2}} = \frac{g_w^2}{8M_W^2}$$

Corresponding to the weakly interacting particles ordered in left-handed doublets one finds the following left-handed fermion currents:

Lepton currents:

1. $\begin{pmatrix} \nu_e \\ e^- \end{pmatrix} \quad j_{e\nu}^\mu = \bar{u}_e \gamma^\mu (1 - \gamma^5) u_\nu$
2. $\begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix} \quad j_{\mu\nu}^\mu = \bar{u}_\mu \gamma^\mu (1 - \gamma^5) u_\nu$
3. $\begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix} \quad j_{\tau\nu}^\mu = \bar{u}_\tau \gamma^\mu (1 - \gamma^5) u_\nu$

Quark currents:

1. $\begin{pmatrix} u \\ d' \end{pmatrix} \quad j_{du}^\mu = \bar{u}_d \gamma^\mu (1 - \gamma^5) u_u$
2. $\begin{pmatrix} c \\ s' \end{pmatrix} \quad j_{sc}^\mu = \bar{u}_s \gamma^\mu (1 - \gamma^5) u_c$
3. $\begin{pmatrix} t \\ b' \end{pmatrix} \quad j_{bt}^\mu = \bar{u}_b \gamma^\mu (1 - \gamma^5) u_t$

CKM matrix to describe the quark mixing

One finds that the weak eigenstates of the down type quarks are not equal to the their mass/flavor eigenstates:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \underbrace{\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}}_{\text{Cabibbo-Kobayashi-Maskawa mixing matrix}} \cdot \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Cabibbo-Kobayashi-Maskawa mixing matrix

The quark mixing is the origin of the flavor number violation of the weak interaction.

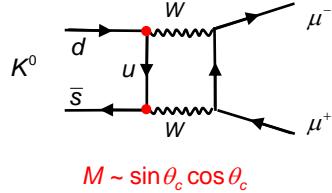
Until the early 70s, only 3 quark flavor were known. The weak transition between quarks was described by a quark doublet:

$$\begin{pmatrix} u \\ d' \end{pmatrix} = \begin{pmatrix} u \\ \cos \theta_c \cdot d + \sin \theta_c \cdot s \end{pmatrix} \quad \text{Mixing angle } \theta_c = \text{Cabibbo-Angle}$$

The mixing described automatically the suppression of $\Delta S=1$ transitions ($\sim \sin^2 \theta_c$)

Missing FCNC and GIM mechanism

FCNC in the 3 quark model: $K^0 \rightarrow \mu^+ \mu^-$



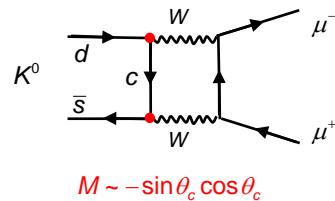
Theoretically one predicts large BR, in contradiction with experimental limits for this decay:

$$\frac{BR(K_L \rightarrow \mu^+ \mu^-)}{BR(K_L \rightarrow \text{all})} = (7.2 \pm 0.5) \cdot 10^{-9}$$

Proposal by Glashow, Iliopoulos, Maiani, 1970:

There exists a fourth quark which builds together with the s quark a second doublet:

$$\begin{pmatrix} c \\ s' \end{pmatrix} = \begin{pmatrix} c \\ -\sin \theta_c \cdot d + \cos \theta_c \cdot s \end{pmatrix}$$



Additional Feynman-Graph for $K^0 \rightarrow \mu\mu$ which compensates the first one:

Prediction of a fourth quark

4.2 Test of V-A structure in particle decays

4.2.1 Muon decay

a) Muon lifetime

$$M = \frac{G_F}{\sqrt{2}} [\bar{u}_\nu(k) \gamma_\alpha (1 - \gamma^5) u_\mu(p) [\bar{u}_e(p') \gamma^\alpha (1 - \gamma^5) v_\nu(k')]$$

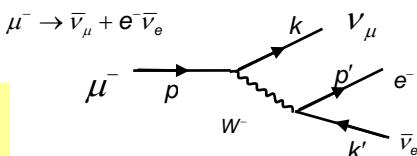
Analogous to the QED calculations of chapter III one finds after a lengthy calculation:

$$M = \frac{1}{2} \sum_{\text{Spins}} |M|^2 = 64 G_F^2 (k \cdot p')(k \cdot p)$$

Using $d\Gamma = \frac{1}{2E} |M|^2 dL$ one obtains the electron spectrum in the muon rest frame:

$$\frac{d\Gamma}{dE'} = \frac{G_F^2}{12\pi^3} m_\mu^2 E'^2 \left(3 - \frac{4E'}{m_\mu}\right)$$

with E' = electron energy



$$\frac{1}{\tau} = \Gamma = \int_0^{m_\mu/2} \frac{d\Gamma}{dE'} dE' = \frac{G_F^2 m_\mu^5}{192\pi^3}$$

Measurement of the muon lifetime thus provides a determination of the fundamental coupling G_F

$$\tau_\mu = (2.19703 \pm 0.00004) \cdot 10^{-6} \text{ s}$$

$$G_\mu = (1.16639 \pm 0.00001) \cdot 10^{-5} \text{ GeV}^{-2}$$

Fermi constant measured in muon decays is often called G_μ

b) Test of V-A structure in the muon decay

Most general form of the matrix element for

$$M = \frac{G_F}{\sqrt{2}} \cdot \sum_{\substack{i=S,V,T \\ \lambda=L,R \\ \lambda'=\begin{cases} \pm\lambda & \text{for } S,V \\ -\lambda & \text{for } T \end{cases}}} g_{\lambda\lambda'}^i (\bar{u}_{\lambda'}(e) \Gamma^i v_{\lambda'_i}(v_e)) (\bar{u}_{\lambda_i}(v_\mu) \Gamma^i u_\lambda(v_e))$$

Chirality λ_i, λ'_i determined by Γ_i

$\lambda'_i = \begin{cases} \lambda' & i = S, T \\ -\lambda' & i = V \end{cases}$
$\lambda_i = \begin{cases} \lambda & i = V \\ -\lambda & i = S, T \end{cases}$

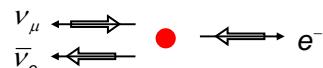
Possible current-current couplings:

$i \setminus \lambda \lambda'$	RR	RL	LR	LL	There are in general 10 complex amplitudes $g_{\lambda\lambda'}^i$ Pure V-A coupling: $g_{LL}^V = 1$ all other $g_{\lambda\lambda'}^i = 0$
S	x	x	x	x	
V	x	x	x	x	
T	x	x			

Experimental determination of $g_{\lambda\lambda'}^i$ from energy spectra and spin correlation of the decay electrons from the polarized muons

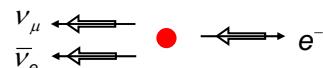
Idea:

V-A at μ vertex \Rightarrow LH v_μ



Configuration w/ max e- momentum possible

V+A at μ vertex \Rightarrow RH v_μ

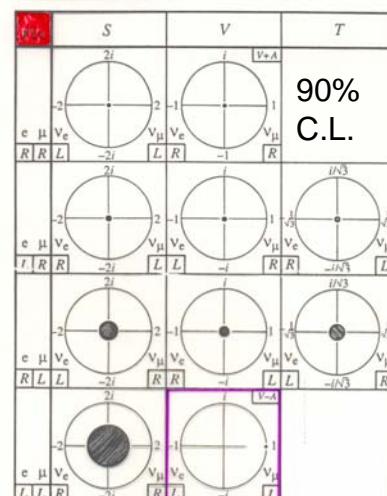


Due to angular momentum conservation not possible

Couplings in muon decay

$\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$

SIN



V-A theory is confirmed

4.2.2 Pion decay

$$\pi^+ \rightarrow \mu^+ \nu_\mu \quad \pi^+ \left\{ \begin{array}{l} \bar{d} \rightarrow W^+ \\ u \rightarrow W^+ \end{array} \right. \quad \mu^+, e^+ \nu_\mu, \nu_e$$

Naïve expectation:

Assuming the same decay dynamics the decay rate to e^+ should be much larger than to μ^+ as the phase space is much bigger.

Measurement: (PDG)

$$\frac{\Gamma(\pi^+ \rightarrow e^+ \nu_e)}{\Gamma(\pi^+ \rightarrow \mu^+ \nu_\mu)} = (1.230 \pm 0.004) \cdot 10^{-4}$$

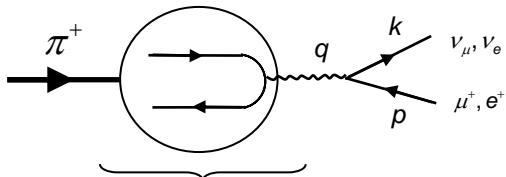
Large suppression due to a dynamic effect.

Qualitative explanation within V-A theory:

$$\nu_\mu, \nu_e \leftrightarrow \bullet \leftrightarrow \mu^+ e^+ \quad J^\pi = 0$$

Angular momentum conservation forces the lepton into the “wrong” helicity state: suppressed $\sim \beta = v/c$ i.e. for vanishing lepton masses the pion could not decay into leptons.

Determination of decay rates:



Quarks in pion are bound

$$M = \frac{G_F}{\sqrt{2}} \cdot (\pi)_\mu \cdot [\bar{u}_\nu \gamma^\mu (1 - \gamma^5) v_\mu] \quad \rightarrow \quad M = \frac{G_F}{\sqrt{2}} \cdot (p_\mu + k_\mu) \cdot f_\pi \cdot [\bar{u}_\nu \gamma^\mu (1 - \gamma^5) v_\mu]$$

As the pion spin $s_\pi = 0$, q is the only relevant 4-vector:

$$q^\mu = p^\mu + k^\mu$$

$$(\pi)_\mu = q_\mu \underbrace{f_\pi(q^2)}$$

Pion form factor:

$$q^2 = m_\pi^2 : \quad f_\pi(q^2) = f_\pi(m_\pi^2) = f_\pi$$

$$\Gamma(\pi^+ \rightarrow \mu^+ \nu_\mu) = \frac{G_F^2}{8\pi} \cdot f_\pi^2 \cdot m_\pi m_\mu^2 \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)$$

$$\Gamma(\pi^+ \rightarrow e^+ \nu_e) = \frac{G_F^2}{8\pi} \cdot f_\pi^2 \cdot m_\pi m_e^2 \left(1 - \frac{m_e^2}{m_\pi^2}\right)$$

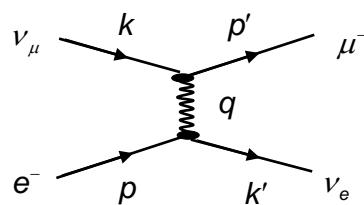
$$\frac{\Gamma(\pi^+ \rightarrow e^+ \nu_e)}{\Gamma(\pi^+ \rightarrow \mu^+ \nu_\mu)} = \left(\frac{m_e^2}{m_\mu^2} \right) \left(\frac{m_\pi^2 - m_e^2}{m_\pi^2 - m_\mu^2} \right) = 1.275 \cdot 10^{-4}$$

The prediction of the V-A theory is confirmed by the experimental observation.

4.3 Neutrino scattering

a) Neutrino-electron scattering

$\nu_\mu e^- \rightarrow \mu^- \nu_e$



$$M = \frac{G_F}{\sqrt{2}} [\bar{u}_\nu(k') \gamma^\alpha (1 - \gamma^5) u_e(p)] [\bar{u}_\mu(p') \gamma^\alpha (1 - \gamma^5) u_\nu(k)]$$

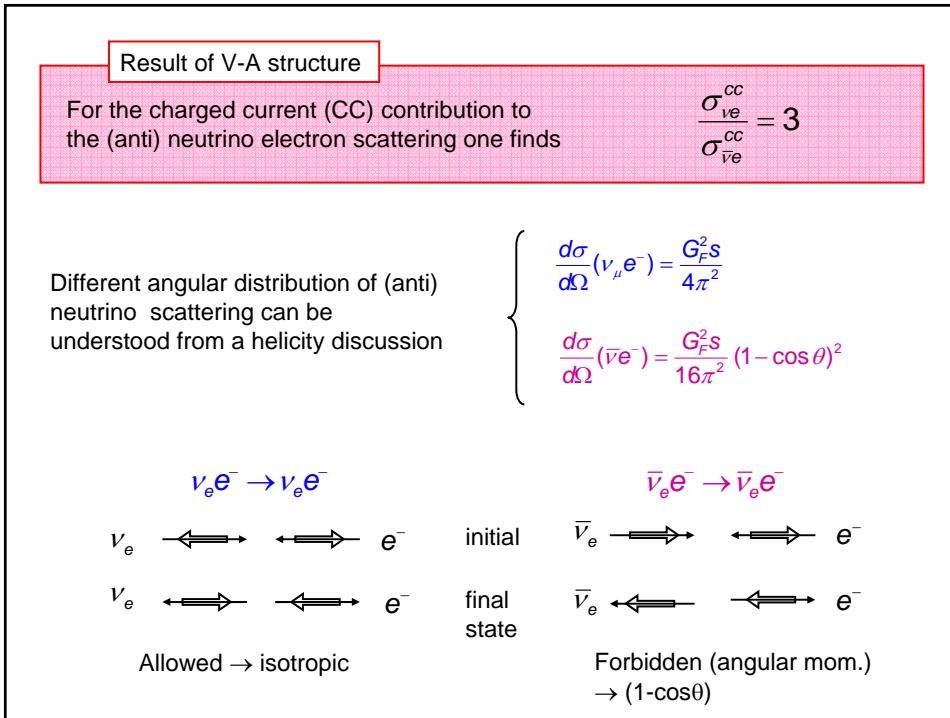
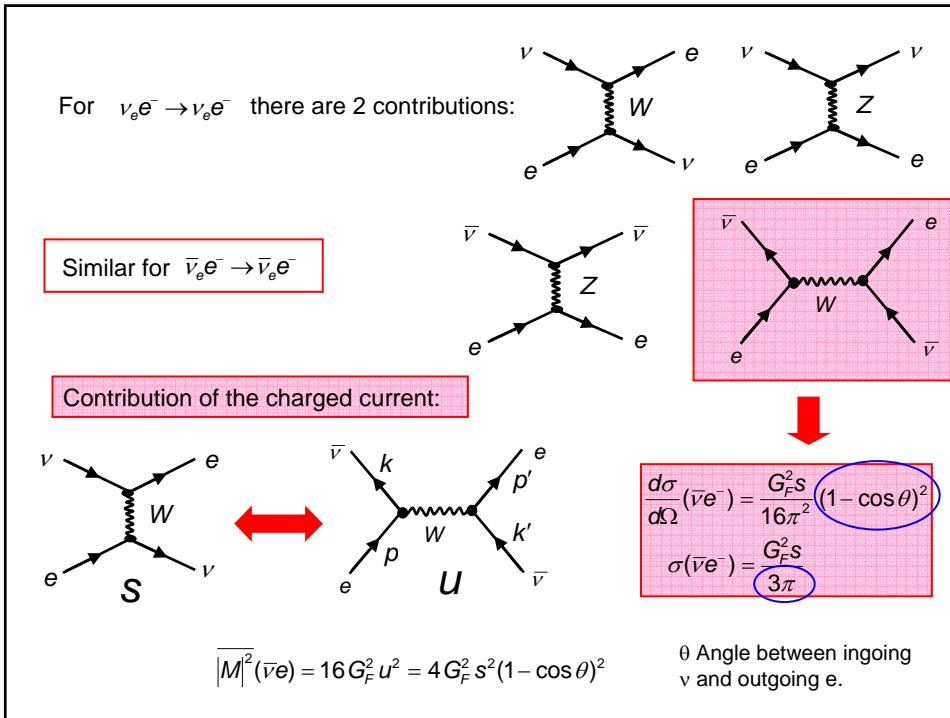
$$\overline{|M|^2} = \frac{1}{2} \sum_{Spins} |M|^2 = \dots = 64 G_F^2 (k \cdot p)(k' \cdot p') = 16 G_F^2 \cdot s^2$$

↑
Limit $m_e \approx m_\mu \approx 0$ $s = (k + p)^2 = 2kp = 2k'p'$

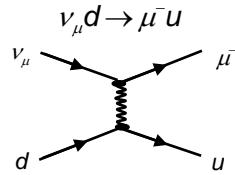
Using the phase space factor of chapter II:

$$\frac{d\sigma}{d\Omega}(\nu_\mu e^-) = \frac{1}{64\pi^2 s} \overline{|M|^2} = \frac{G_F^2 s}{4\pi^2}$$

$$\sigma(\nu_\mu e^-) = \frac{G_F^2 s}{\pi}$$

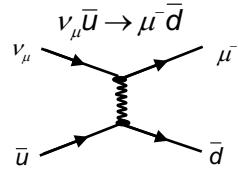


b) Neutrino-quark scattering

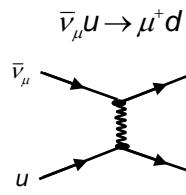


$$\frac{d\sigma}{d\Omega}(\nu_\mu d) = \frac{G_F^2 s}{4\pi^2}$$

$$\sigma(\nu_\mu d) = \frac{G_F^2 s}{\pi}$$



$$\frac{d\sigma}{d\Omega}(\nu_\mu \bar{u}) = \frac{d\sigma}{d\Omega}(\bar{\nu}_\mu u)$$



$$\frac{d\sigma}{d\Omega}(\bar{\nu}_\mu \mu) = \frac{G_F^2 s}{16\pi^2} (1 + \cos\theta)^2$$

$$\sigma(\bar{\nu}_\mu \mu) = \frac{G_F^2 s}{3\pi}$$

$$\bar{\nu}_\mu \bar{d} \rightarrow \mu^+ \bar{u}$$

$$\frac{d\sigma}{d\Omega}(\bar{\nu}_\mu \bar{d}) = \frac{d\sigma}{d\Omega}(\nu_\mu d)$$

θ Scattering angle of μ

Neutrinos only interact w/ d and anti-u quarks
Anti-neutrinos only interact w/ u and anti-d quarks

c) Neutrino-nucleon (iso-scalar) scattering

QPM: $x = \frac{Q^2}{2M\nu}$ $y = \frac{\nu}{E}$ $\nu = E - E'$

$$\frac{d\sigma(\nu_\mu N \rightarrow \mu^- X)}{dy} = \sum_i \left| \begin{array}{c} \nu_\mu \rightarrow \mu^- \\ \text{fermion loop} \\ \text{with } d \text{ and } u \end{array} \right|^2$$

$$\frac{d^2\sigma(\nu N)}{dxdy} = \sum_i f_i(x) \left(\frac{d\sigma_i(\nu q_i)}{dy} \right)_{\hat{s}=xS}$$

$$\frac{d^2\sigma(\nu N)}{dxdy} = \frac{G_F^2}{2\pi} \cdot [Q(x) + \bar{Q}(x)(1-y)^2] \quad \leftarrow 1-y = \frac{p \cdot k'}{p \cdot k} \approx \frac{1}{2}(1+\cos\theta)$$

$$\frac{d\sigma(\bar{\nu}_\mu N \rightarrow \mu^+ X)}{dy} = \sum_i \left| \begin{array}{c} \bar{\nu}_\mu \rightarrow \mu^+ \\ \text{fermion loop} \\ \text{with } u \text{ and } d \end{array} \right|^2$$

$$\frac{d^2\sigma(\bar{\nu} N)}{dxdy} = \frac{G_F^2}{2\pi} \cdot [\bar{Q}(x) + Q(x)(1-y)^2]$$

Total cross section after integration over x and y (0...1):

$$\sigma(\nu N) = \frac{G_F^2 M E_\nu}{2\pi} \cdot \left[Q_i + \frac{1}{3} \bar{Q}_i \right]$$

$$\sigma(\bar{\nu} N) = \frac{G_F^2 M E_\nu}{2\pi} \cdot \left[\bar{Q}_i + \frac{1}{3} Q_i \right]$$

$$\text{with } Q_i = \int x Q(x) dx$$

$$R = \frac{\sigma_{\bar{\nu} N}}{\sigma_{\nu N}} = \frac{1 + 3 \bar{Q}/Q}{3 + \bar{Q}/Q}$$

If nucleon consists only of valence quarks ($\bar{Q}=0$): $R=1/3$, because of V-A structure

Measurement: $R = \frac{0.34}{0.67} \Rightarrow \bar{Q}/Q \approx 0.15$

\Rightarrow There are sea quarks !

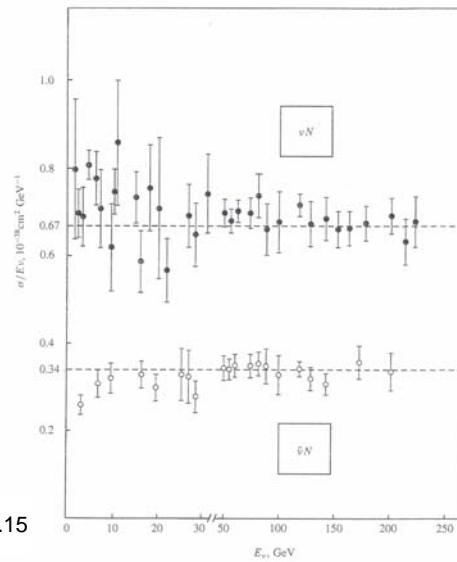


Fig. 5.13. Neutrino and antineutrino cross-sections on nucleons. The ratio σ/E_ν is plotted as a function of energy and is indeed a constant, as predicted in (5.45) and (5.46).

4.4 Problems with V-A theory

- Cross section for $\nu e^- \rightarrow e^- \nu_e$ in 4-fermion ansatz: i.e. cross section goes to infinity if $s \rightarrow \infty$: violates unitarity

$$\sigma(\nu e^-) = \frac{G_F^2 s}{\pi}$$

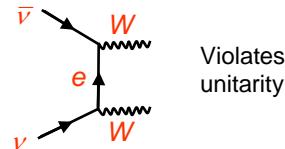
- Lee and Wu (1965) introduced a massive exchange boson. Effect of propagator:

$$\frac{G_F}{\sqrt{2}} \mapsto \frac{G_F}{\sqrt{2}} \frac{1}{1 - q^2/M_W^2} \quad \sigma(\nu e^-) \mapsto \frac{G_F^2 M_W^2}{\pi}$$

Energy behavior of cross section becomes better but still violates unitarity at very high s .

- In addition there are boson production processes of the kind:

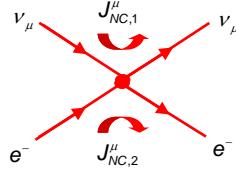
In the V-A theory they also violate unitarity !!



We need a new theory: Standard Model

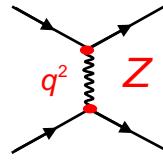
5. Structure of neutral currents

Ansatz: Four-fermion interaction



$$M = 2 \frac{4G_{NC}}{\sqrt{2}} \cdot J_{NC,1,\mu} \cdot J_{NC,2}^\mu$$

as $q^2 \rightarrow 0$ approximation of:



Experimental determination of the structure of the weak neutral currents:

$$J_{NC}^\mu = \bar{u} \gamma^\mu \frac{1}{2} (g_V - g_A \gamma^5) u$$

Neutral weak interaction couples to left- and right-handed fermion current contributions differently:

$$g_L = \frac{1}{2} (g_V + g_A) \quad g_R = \frac{1}{2} (g_V - g_A)$$

5.1 Vector and axial-vector couplings

In the Standard Model prediction for g_V and g_A :

	g_V	g_A
ν	$\frac{1}{2}$	$\frac{1}{2}$
ℓ^-	$-\frac{1}{2} + 2 \sin^2 \theta_W$	$-\frac{1}{2}$
u -quark	$+\frac{1}{2} - \frac{4}{3} \sin^2 \theta_W$	$\frac{1}{2}$
d -quark	$-\frac{1}{2} + \frac{2}{3} \sin^2 \theta_W$	$-\frac{1}{2}$

$$\text{with } \sin^2 \theta_W = 1 - \frac{M_W^2}{M_Z^2} \approx 0.223$$

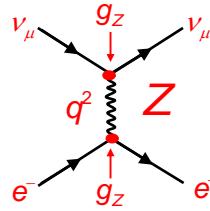
In case of the left-handed neutrinos:

$$J_\nu^\mu = \bar{u}_\nu \gamma^\mu \frac{1}{2} \cdot \underbrace{\frac{1}{2} (1 - \gamma^5)}_{\text{pure V-A structure}} u_\nu$$

pure V-A structure

5.2 Effective coupling G_{NC}

As 4-fermion interaction is the $q^2 \rightarrow 0$ approximation of a massive boson exchange:



$$M = J_{\nu,\mu} \cdot g_Z \cdot \frac{g_{\mu\nu} - q_\mu q_\nu / M_Z^2}{q^2 - M_Z^2} \cdot g_Z \cdot J_\nu$$

Comparison of the coupling constants in the $q^2 \rightarrow 0$ limit:

$$\frac{G_{NC}}{\sqrt{2}} = \frac{g_Z^2}{8M_Z^2} = \frac{g_W^2}{8M_W^2} \cdot \underbrace{\frac{g_W^2 M_W^2}{g_W^2 M_Z^2}}_{\rho} = \frac{g_W^2}{8M_W^2} \cdot \rho = \frac{G_F}{\sqrt{2}}$$

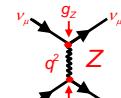
$\rho = 1$ in the SM

For the matrix element $M(v e^- \rightarrow v e^-)_{NC}$ one then finds:

$$M = \frac{G_F}{\sqrt{2}} \left[\bar{u}_v \gamma_\mu \frac{1}{2} (1 - \gamma^5) u_v(p) \right] \left[\bar{u}_e \gamma^\mu (g_V^e - g_A^e \gamma^5) u_e \right]$$

5.3 Neutrino-electron scattering (NC)

Using the matrix element above one finds:



$$\nu_\mu e^- \rightarrow \nu_\mu e^- \quad \frac{d\sigma}{dy} = \frac{G_F^2 s}{\pi} \left[\overbrace{\left(\frac{g_V^e + g_A^e}{2} \right)^2}^{g_L^2} + \overbrace{\left(\frac{g_V^e - g_A^e}{2} \right)^2}^{g_R^2} (1-y)^2 \right]$$

$$\sigma(\nu_\mu e^-) = 2mE_\nu \frac{G_F^2}{\pi} \left[\left(\frac{g_V^e + g_A^e}{2} \right)^2 + \frac{1}{3} \left(\frac{g_V^e - g_A^e}{2} \right)^2 \right]$$

$$\bar{\nu}_\mu e^- \rightarrow \bar{\nu}_\mu e^- \quad \frac{d\sigma}{dy} = \frac{G_F^2 s}{\pi} \left[\overbrace{\left(\frac{g_V^e - g_A^e}{2} \right)^2}^{g_R^2} + \overbrace{\left(\frac{g_V^e + g_A^e}{2} \right)^2}^{g_L^2} (1-y)^2 \right]$$

$$\sigma(\bar{\nu}_\mu e^-) = 2mE_\nu \frac{G_F^2}{\pi} \left[\left(\frac{g_V^e - g_A^e}{2} \right)^2 + \frac{1}{3} \left(\frac{g_V^e + g_A^e}{2} \right)^2 \right]$$

Determination of the vector/axial-vector couplings

$$\sigma(\nu_\mu e^-) = 2mE_\nu \frac{G_F^2}{3\pi} [g_V^{e2} + g_V^e g_A^e + g_A^{e2}]$$

$$\sigma(\bar{\nu}_\mu e^-) = 2mE_\nu \frac{G_F^2}{3\pi} [g_V^{e2} - g_V^e g_A^e + g_A^{e2}]$$

$$\frac{\sigma(\nu_\mu e)}{E_\nu} = [1.9 \pm 0.4 \text{ (stat.)} \pm 0.4 \text{ (syst.)}] \cdot 10^{-42} \frac{\text{cm}^2}{\text{GeV}},$$

$$\frac{\sigma(\bar{\nu}_\mu e)}{E_{\bar{\nu}}} = [1.5 \pm 0.3 \text{ (stat.)} \pm 0.4 \text{ (syst.)}] \cdot 10^{-42} \frac{\text{cm}^2}{\text{GeV}}.$$

